IS THERE A HOT ELECTROWEAK PHASE TRANSITION AT LARGE HIGGS MASSES?

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1 Why we are interested in EW transition?

High-temperature and dense matter of elementary particles appears in several areas of physics. The most familiar example is the Universe at the early stages of its expansion. The Big Bang theory states that the Universe was hot and dense in the past, with a temperature ranging from a few eV up to the Planck scale $\sim M_{Pl} \sim 10^{19}$ GeV. It is believed that the Universe during its evolution went through several phase transitions, associated with different (GUT, electroweak, QCD) mass scales. The high-temperature phase transitions, typical for grand unified theories (GUTs), may be important for cosmological inflation and primordial density fluctuations. Topological defects (such as monopoles, strings, domain walls) can naturally arise at the phase transitions and influence the properties of the Universe we observe today. The first-order electroweak (EW) phase transition is a crucial element for electroweak baryogenesis; it may also play a role in the formation of the magnetic fields observed in the Universe. The QCD phase transition and properties of the quark-gluon plasma are essential for the understanding of the physics of heavy-ion collisions. The QCD phase transition in cosmology may influence the spectrum of the density fluctuations.

The high temperature behaviour [1] of the electroweak matter and properties of the EW phase transition depend crucially on the Higgs mass. This has cosmological and, perhaps, experimental consequences. Electroweak baryogenesis (for a recent review, see [2]) requires a strongly first order electroweak phase transition which occurs only in some part of the parameter space of the electroweak theory. That, in turn, may provide some information on the masses and couplings of yet unknown particles. The absence of the sufficiently strong first order phase transition would be a strong argument in favour of GUT origin of the baryon asymmetry of the Universe.

During last several years a great progress was achieved in understanding of the electroweak phase transition. I think it is fair to say that the problem has been solved by a combination of perturbative and non-perturbative methods. In this talk I am going to review these recent developments.

2 Symmetry properties of EW theory

At first glance, one should hardly expect any *qualitative* changes in high temperature electroweak phase transition with m_H. One can argue that at $T = 0$ the SU(2)×U(1) symmetry is "broken", and intermediate W and Z bosons are massive, while at large enough temperatures $T \gg M_W$ the symmetry is "restored", and gauge bosons are massless. This looks like a symmetry argument which ensures the existence of the first or second order finite temperature phase transition in the theory, independently on the value of the Higgs mass. In fact, this argument is wrong. First, even at high temperatures there are no massless vector excitations (the previous statement was based on the tree approximation). Second, the gauge symmetry is never "broken" – all physical observables by construction are gauge-invariant. Moreover, there is no gauge-invariant local order-parameter that can distinguish between the "broken" (Higgs) and "restored" phases [3,4]. Thus, there is no gauge symmetry restoration at high temperatures (contrary to the gauge symmetry case, global symmetries may be broken or restored), but there can be (but not necessarily are) phase transitions.

This general consideration suggests the phase diagram for the SU(2)–Higgs model shown in Fig. 1. If $m_H < m_H^{crit}$ the phase transition between "symmetric" and "broken" phases is of the first kind; at $m_H < m_H^{crit}$ the phase transition is of the second kind, and at $m_H < m_H^{crit}$ the phase transition is absent. However, the previous argument (absence of symmetry breaking) says nothing about the value of m_H^{crit} – it may very well be zero or infinity. Of course, some computations are necessary in order to clarify the phase structure. A one-loop perturbative analysis, valid at small Higgs masses, $m_H < m_W$ allows to rule out $m_H^{crit} = 0$ [5], but cannot distinguish between finite and infinite value of the critical mass.

In [6] a strong non-perturbative evidence that the line of the first order phase transition indeed ends at some critical Higgs mass, $m_H^{crit} \simeq 75$ GeV was presented. In fact, the qualitative statement that the phase transition completely disappears in some part of the parameter space of the the theory has quite a general character and is true also for different extensions of the standard model, including the supersymmetric one. The details of this consideration can be found in refs. [6,7], I just explain shortly why the problem of EW phase transition is non-perturbative, then sketch the logic of its solution, and present the results.

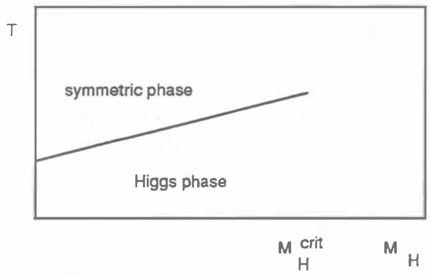

Figure 1: *Phase diagram of the electroweak theory.*

3 Infrared problem

The electroweak theory is weakly coupled: tree, or one-loop computations are usually good enough for describing all weak reactions we observe experimentally. Why do not use perturbation theory at high temperatures? There is a deep physical reason why it breaks down at high T. At zero temperature we apply perturbation theory for consideration of processes where only a small number of particles participate. Thus, the expansion parameter is roughly α_W. At high temperatures, the number of particles, taking part in collisions, may be large. Moreover, for bosonic degrees of freedom there is a well-known Bose amplification factor, associated with the bosonic distribution $n_B(E) = (\exp(E/T) - 1)^{-1}$, where $E = \sqrt{k^2 + m^2}$ is the particle energy. So, the expansion parameter becomes $\alpha_W n(E)$, which is large at $E < \alpha_W T$[8] and small in the opposite limit. Thus, all "low energy" phenomena cannot be described by perturbation theory, just because electroweak interactions are strong in the infrared region.

In fact, perturbation theory breaks down in the most interesting place, namely at the temperatures where different phase transitions are expected. This is because to describe the phase transition we should be able to compute the properties of the symmetric and broken phases simultaneously, but in the symmetric phase any perturbative infrared cutoff is absent, and expansion parameter is large.

If perturbation theory breaks down a natural inclination would be the use of direct numerical non-perturbative methods, such as lattice Monte Carlo simulations. This approach does not work, however, for theories containing chiral fermions, since we do not know how to put these on the lattice. Thus, theories such as the EW theory or grand unified models cannot be studied on the lattice with their complete particle content. This problem does not appear in pure bosonic models or in theories containing vector-like fermions, such as QCD. These models can be simulated on the lattice, but computations are often very demanding[9]. Quite ironically, the computations are *more* time consuming for *weaker* coupling constants. This can be seen as follows. At high temperatures, the average distance between particles is of the order of T^{-1}, and it is clear that the lattice spacing a must be much smaller than this distance, $a \ll T^{-1}$. At the same time, the lattice size Na, where N is the number of lattice sites in the spatial direction, must be much larger than the infrared scale, described above, i.e. $Na \gg (\alpha_W T)^{-1}$. Therefore, the lattice size is required to be rather large, $N \gg \frac{1}{\alpha_W}$, the larger the smaller the coupling constant is.

4 Effective field theory approach

The main idea of the effective theory approach to high-temperature field theory is the factorization of weakly coupled high-momentum modes, with energy $E \gg \alpha_W T$, and of strongly coupled infrared modes with energy $E < \alpha_W T$, and the construction of an effective theory for infrared modes only. The construction of the effective field theory is perturbative, while its analysis may be non-perturbative. Thus, a combination of perturbative and non-perturbative methods is to be used to solve the problem.

In refs. [7] it has been shown that in a weakly coupled electroweak theory and in many of its extensions (supersymmetric or not) the hot EW phase transition can be described by an SU(2)×U(1)+Higgs model in three Euclidean dimensions. Dimensional reduction has its own limitations. For example, for the MSM the 3d approximation is accurate to within few percent for 30 GeV $\lesssim m_H \lesssim$ 250 GeV. At the lower end of this inequality the high temperature expansion breaks down because the phase transition is very strongly first order and particle masses in the broken phase are $\sim T$ [1]. The upper end is the usual condition for the applicability of perturbation theory in the scalar sector of the MSM. In the MSSM the latter condition is satisfied automatically. Hence, the 3d description is valid for a wide range of the phenomenologically interesting part of the parameter space of the MSM and MSSM [7,10]. The 4d lattice simulations at sufficiently small Higgs masses of a pure bosonic model were carried out in refs. [9]. Whenever the comparison between 3d and 4d simulations is possible, they are in agreement, indicating the correctness of the dimensional reduction beyond perturbation theory.

The effective 3d action for soft strongly interacting bosonic modes with $k \ll g_W T$, describing the high-temperature EW theory is:

$$L = \frac{1}{4}G^a_{ij}G^a_{ij} + \frac{1}{4}F_{ij}F_{ij} + (D_i\Phi)^\dagger(D_i\Phi) + \bar{m}_3^2\Phi^\dagger\Phi + \bar{\lambda}_3(\Phi^\dagger\Phi)^2, \tag{1}$$

where G^a_{ij} and F_{ij} are the SU(2) and U(1) field strengths, respectively, Φ is a scalar doublet, and D_i is a standard covariant derivative in the fundamental representation. The four parameters of the 3d theory (scalar mass $\bar{m}_3^2$, scalar self-coupling constant $\bar{\lambda}_3$, and two gauge couplings $\bar{g}_3$ and $\bar{g}'_3$) are some functions of the initial parameters and temperature.

The effective action is three-dimensional, because at high temperatures the fourth, Euclidean time dimension is compact, $0 < \tau < 1/T$ and shrinks down when T is large. It does not contain fermions since their 3d masses are "superheavy", $m_f^3 \sim \pi T$. It does not contain zero components of the gauge fields – triplet and singlet of SU(2) – because these are "heavy" ($m_D \sim g_W T$ due to the Debye screening of electric fields in plasma), and can be integrated out. One may wonder where are the other scalars, typical for the extensions of the Standard Model. The answer is that all extra scalar degrees of freedom are naturally "heavy" (mass $\sim g_W T$) near the phase transition temperature and can be integrated out. (In the case when both scalars are light near the critical temperature, a more complicated model, containing two scalar doublets, should be studied. However, this case requires fine tuning.)

[1] This actually refers to the 4d SU(2)+Higgs theory without fermions. In the full MSM the transition never gets that strong [7], but on the other hand higher-order Yukawa corrections are becoming large at $m_H \sim 30$ GeV.

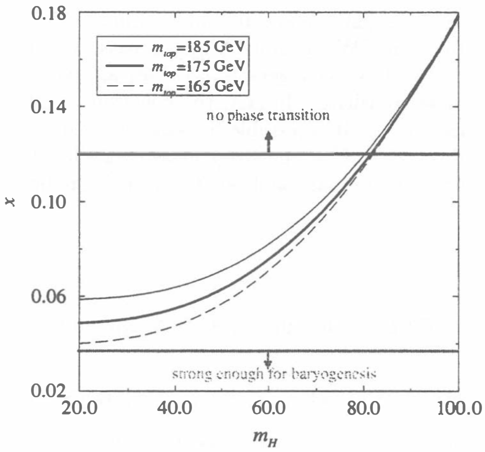

Figure 2: *The value $x = \lambda_3/g_3^2$ as a function of the physical Higgs mass m_H and the top quark mass $m_{\rm top}$ near the critical temperature defined from taking $m_3^2 = 0$. In general, x depends on the Higgs mass, the top mass, and logarithmically on the temperature.*

5 Absence of the phase transition

To study the electroweak phase transition it is sufficient to study an SU(2)×U(1) gauge–Higgs theory in 3d. This 3d theory is defined by one dimensionful parameter $g_3^2 \sim g^2T$ and three dimensionless ratios

$$x \equiv \frac{\bar\lambda_3}{\bar g_3^2}, \qquad y \equiv \frac{\bar m_3^2}{\bar g_3^4}, \qquad z = (\frac{\bar g_3'}{\bar g_3})^2. \tag{2}$$

The dimensionful coupling constant can be chosen to fix the energy scale. Therefore, the phase state of this theory is completely defined by the two numbers x and y, since z is fixed experimentally by the measurement of the Weinberg angle. For the MSM the dependence of the parameter x on the mass of the Higgs boson near the critical temperature (near $y = 0$) is shown in Fig. 2.

The parameter y change with the temperature of the system while x stays almost constant. Thus, to check if there is a first order phase transition at some particular Higgs mass one should fix the parameter x and vary y. Lattice simulations confirmed the existence of the first order phase transition at small Higgs masses, $m_H < 75$ GeV, and allowed to compute reliably the parameters of transition [7]. At larger Higgs masses the system behaves very regularly: there are no jumps in different order parameters, and correlation lengths in the system stay finite [6](second order phase transition implies infinite correlation lengths). Thus, m_H^{crit} is finite, see Fig. 2.

6 Conclusion

There exist a critical Higgs mass above which there is no high temperature electroweak phase transition. For MSM, its value is close to 80 GeV, for more complicated models the specific number depends on the model parameters. It is quite unlikely that there are any cosmological consequences coming from the EW epoch if the Higgs mass exceeds the critical value.

The requirement of the EW baryogenesis provides an even stronger constraint on the strength of the EW phase transition. In fact, the constraint [11)] does not hold for any Higgs mass in the MSM [7)] (see Fig. 2). It is possible to satisfy this constraint in a specific portion of the parameter space of the MSSM [12)]: the Higgs mass is smaller than the Z mass, the lightest stop mass is smaller than the top mass, and $\tan\beta < 3$. This prediction can be tested at LEP2.

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