Higgsology with the type II seesaw model

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Introductory motivations

$\rightarrow$ an extra motivation in retrospect:

why is the discovered 125 GeV scalar so much SM-like??

The model

The scalar sector consists of the standard Higgs weak doublet H and a colorless scalar field Δ transforming as a triplet under the $SU(2)_L$ gauge group with hypercharge $Y_\Delta = 2$:
$H \sim (1,2,1)$ and $\Delta \sim (1,3,2)$ under $SU(3)_c \times SU(2)_L \times U(1)_Y$.

$$Q = I_3 + \frac{Y}{2}$$

$$\Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix} \quad \text{and} \quad H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$$\mathcal{L} = (D_\mu H)^\dagger (D^\mu H) + Tr(D_\mu \Delta)^\dagger (D^\mu \Delta) - V(H,\Delta) + \mathcal{L}_{Yukawa} + \ldots$$

$$\mathcal{L}_{Yukawa} \supset Y_\nu L^T C \otimes i\sigma_2 \Delta L$$

$$\begin{aligned} V(H,\Delta) = & -m_H^2 H^\dagger H + M_\Delta^2 Tr(\Delta^\dagger \Delta) + [\mu(H^T i\sigma_2 \Delta^\dagger H) + \text{h.c.}] \\ & + \frac{\lambda}{4}(H^\dagger H)^2 + \lambda_1 (H^\dagger H) Tr(\Delta^\dagger \Delta) \\ & + \lambda_2 (Tr\Delta^\dagger \Delta)^2 + \lambda_3 Tr(\Delta^\dagger \Delta)^2 + \lambda_4 H^\dagger \Delta \Delta^\dagger H \end{aligned}$$

Electroweak symmetry breaking

$$\langle\Delta\rangle = \begin{pmatrix} 0 & 0 \\ v_t/\sqrt{2} & 0 \end{pmatrix} \quad \text{and} \quad \langle H\rangle = \begin{pmatrix} 0 \\ v_d/\sqrt{2} \end{pmatrix}$$

one finds after minimization of the potential the following necessary conditions:

$$M_\Delta^2 = \frac{2\mu v_d^2 - \sqrt{2}(\lambda_1+\lambda_4)v_d^2 v_t - 2\sqrt{2}(\lambda_2+\lambda_3)v_t^3}{2\sqrt{2}v_t}$$

$$m_H^2 = \frac{\lambda v_d^2}{4} - \sqrt{2}\mu v_t + \frac{(\lambda_1+\lambda_4)}{2}v_t^2$$

8 parameters $\longrightarrow$ 7 parameters with $v \equiv \sqrt{v_d^2 + 2v_t^2} = 246\text{GeV}$

$$M_Z^2 = \frac{(g^2+g'^2)}{4}(v_d^2+4v_t^2) \qquad M_W^2 = \frac{g^2}{4}(v_d^2+2v_t^2)$$

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$\rho = \frac{v_d^2+2v_t^2}{v_d^2+4v_t^2} < 1,$ but $v_t \ll v_d \to$ neutrino masses.

Higgs spectrum, couplings,...

$\rightarrow$ 10 scalar states: 7 massive physical Higgses, $h^0, H^0, A^0, H^\pm, H^{\pm\pm}$
$+$ 3 Goldstone bosons and 3 mixing angles α, β, β'

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$$h^0 = \cos\alpha\, h + \sin\alpha\, \xi^0, \quad H^0 = -\sin\alpha\, h + \cos\alpha\, \xi^0, \quad A^0 = -\sin\beta\, Z_1 + \cos\beta\, Z_2, \quad G^0 = \cos\beta\, Z_1 + \sin\beta\, Z_2$$

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$$m^2_{h^0, H^0} = \frac{1}{2}[A + C \mp \sqrt{(A - C)^2 + 4B^2}]$$

$$A = \frac{\lambda}{2} v_d^2 \quad , \quad B = v_d[-\sqrt{2}\mu + (\lambda_1 + \lambda_4) v_t] \quad , \quad C = \frac{\sqrt{2}\mu v_d^2 + 4(\lambda_2 + \lambda_3) v_t^3}{2 v_t}$$

$$m^2_{H^{\pm\pm}} = \frac{\sqrt{2}\mu v_d^2 - \lambda_4 v_d^2 v_t - 2\lambda_3 v_t^3}{2 v_t}$$

$$m^2_{H^\pm} = \frac{(v_d^2 + 2 v_t^2)[2\sqrt{2}\mu - \lambda_4 v_t]}{4 v_t}$$

$$m^2_A = \frac{\mu(v_d^2 + 4 v_t^2)}{\sqrt{2} v_t}$$

Higgs spectrum, couplings,...

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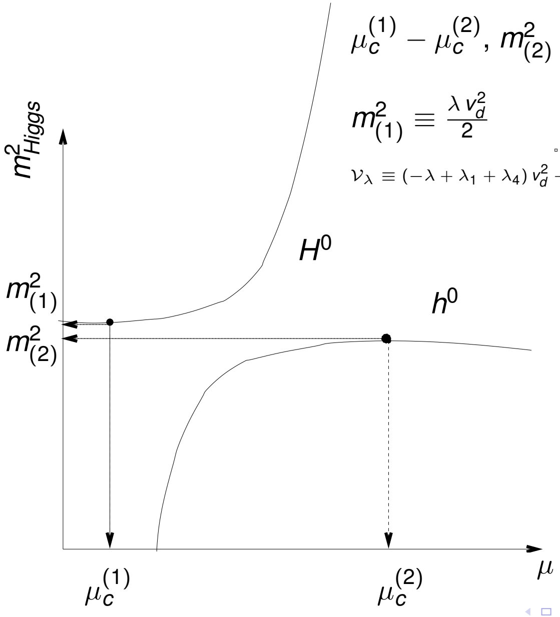
$$m_A^2 = \frac{\mu (v_d^2 + 4 v_t^2)}{\sqrt{2} v_t}$$ $\leftarrow$ would-be Majoron

Higgs spectrum, couplings,...



$$\mu_c^{(1)} - \mu_c^{(2)},\ m^2_{(2)} - m^2_{(1)} \sim \mathcal{V}_\lambda$$

$$m^2_{(1)} \equiv \frac{\lambda\, v_d^2}{2}$$

$$\mathcal{V}_\lambda \equiv (-\lambda + \lambda_1 + \lambda_4)\, v_d^2 + 4\,(\lambda_2 + \lambda_3)\, v_t^2 \quad (< 0)$$

Higgs spectrum, couplings,...

a comment:

- sometimes in the literature the EWSB equations are approximated as

$$M_\Delta^2 = \frac{2\mu v_d^2 - \sqrt{2}(\lambda_1+\lambda_4)v_d^2 v_t - 2\sqrt{2}(\lambda_2+\lambda_3)v_t^3}{2\sqrt{2}v_t} \rightarrow M_\Delta^2 \simeq \frac{\mu v_d^2}{\sqrt{2}v_t} \quad (1)$$

$$m_H^2 = \frac{\lambda v_d^2}{4} - \sqrt{2}\mu v_t + \frac{(\lambda_1+\lambda_4)}{2}v_t^2 \rightarrow m_H^2 \simeq \frac{\lambda v_d^2}{4} \quad (2)$$

  while (2) is trivially OK, (1) assumes $\mu \gg v_t$.

- $M_\Delta \sim \mu \sim M_{GUT}$ → seesaw → only SM-like h^0 at the LHC!
- $\mu \lesssim, \ll v_t$ more interesting

h^0 is the SM-like

h^0 is the SM-like

initial state (pp)	sensitivity	final state
$q\bar{q}, \gamma\gamma, Z\gamma, ZZ, W^+W^-$ [gg?]	gauge couplings, $H^\pm$, λ_1, h^0	$H^{++}H^{--}$
$q\bar{q}'$, $\gamma W^\pm$, $ZW^\pm$	gauge couplings, $H^{\pm\pm}$, $H^\mp$	$H^{\pm\pm}H^\mp$
$q\bar{q}, \gamma\gamma, Z\gamma, ZZ, W^+W^-$ [gg?]	gauge couplings, $H^\pm$, A^0, $2\lambda_1 + \lambda_4$, h^0	H^+H^-
$q\bar{q}'$, $\gamma W^\pm$, $ZW^\pm$	gauge couplings, $H^\pm$, A^0	$H^\pm H^0$
$q\bar{q}'$, $\gamma W^\pm$, $ZW^\pm$	gauge couplings, $H^\pm$, A^0	$H^\pm A^0$
$q\bar{q}'$, $\gamma W^\pm$, $ZW^\pm$	mixing suppressed [gauge couplings, $H^\pm$, A^0]	$H^\pm h^0$
$W^\pm W^\pm$	gauge couplings, $H^\pm$	$H^{\pm\pm}H^0$
$W^\pm W^\pm$	gauge couplings, $H^\pm$	$H^{\pm\pm}A^0$
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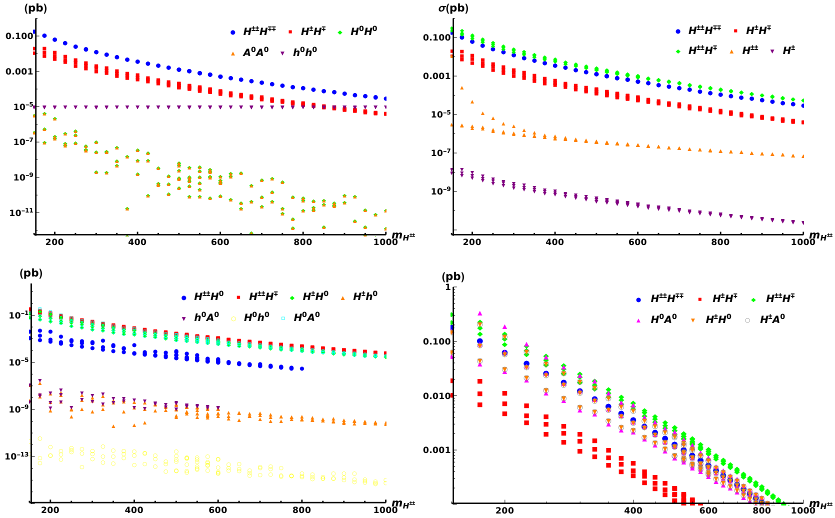
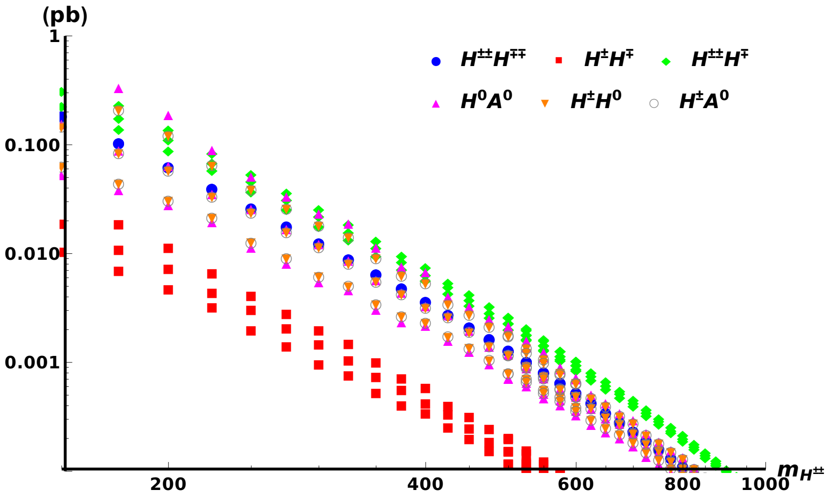
$q\bar{q}, ZZ, W^+W^-$ [gg?]	gauge couplings, $H^\pm$, A^0, $\lambda_1 + \lambda_4$, h^0	H^0H^0
$q\bar{q}, W^+W^-$	gauge couplings, $H^\pm$	H^0A^0
$q\bar{q}, W^+W^-$	gauge couplings, $H^\pm$	H^0h^0
$q\bar{q}, ZZ, W^+W^-$ [gg?]	gauge couplings, $H^\pm$, H^0, $\lambda_1 + \lambda_4$, h^0	A^0A^0
$q\bar{q}, W^+W^-$	gauge couplings, $H^\pm$	A^0h^0
SM	SM	h^0h^0

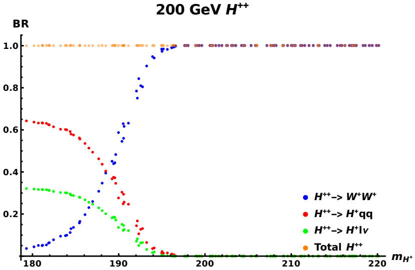
h^0 is the SM-like

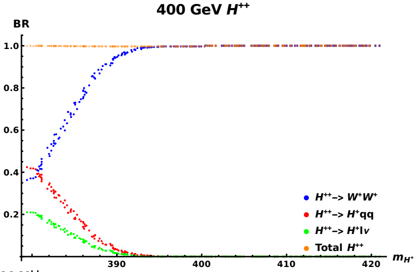


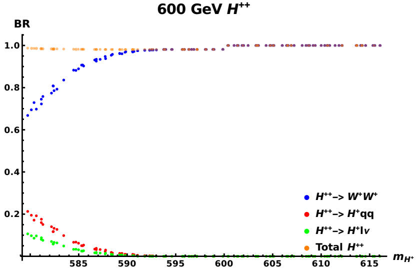
(Romain Kukla)



Dominant channels. (Romain Kukla)







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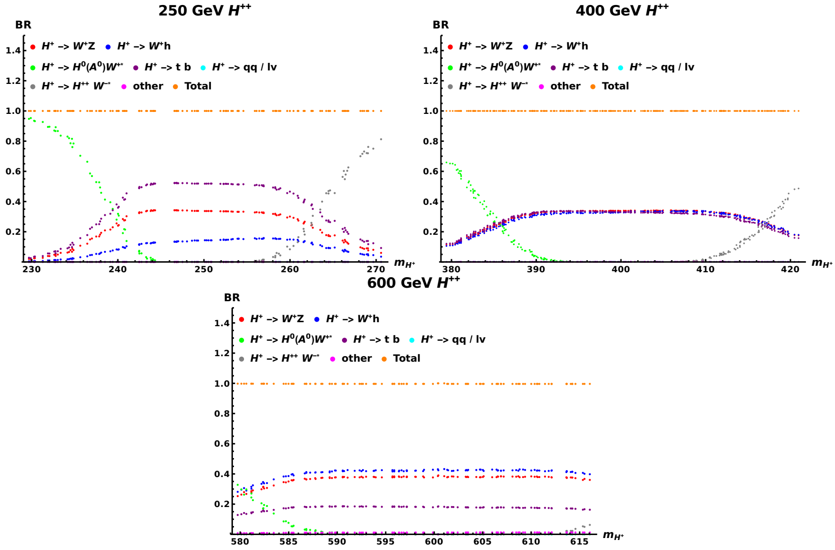

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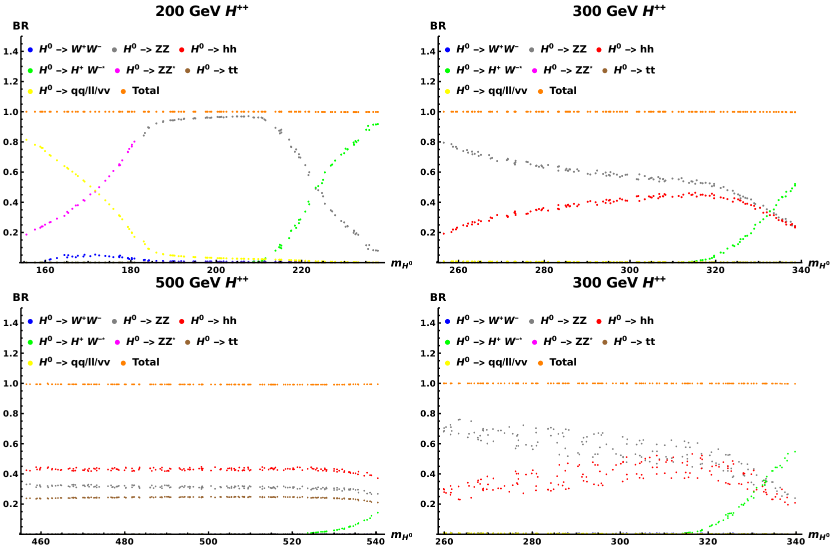


(Romain Kukla)

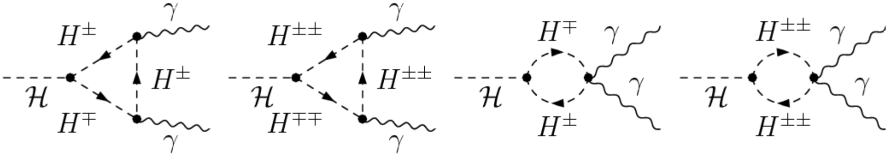
h^0 is the SM-like

- Associate production channels, $H^{\pm\pm}H^{\mp}$ and H^0A^0 are important wrt $H^{\pm\pm}H^{\mp\mp}$
- various decay channels, depending on the masses $\rightarrow$ different final states
- what can be the prospects for RUN 3? (Ana, Hanlin,...?)

$\mathcal{H}$ Into GammaGamma Study

$$\Gamma(\mathcal{H} \to \gamma\gamma) = \frac{G_F \alpha^2 M_{\mathcal{H}}^3}{128\sqrt{2}\pi^3} \Big| \sum_f N_c Q_f^2 \tilde{g}_{\mathcal{H}ff} A_{1/2}^{\mathcal{H}}(\tau_f) + \tilde{g}_{\mathcal{H}WW} A_1^{\mathcal{H}}(\tau_W)$$

$$+\tilde{g}_{\mathcal{H}H^\pm H^\mp} A_0^{\mathcal{H}}(\tau_{H^\pm}) + 4\tilde{g}_{\mathcal{H}H^{\pm\pm}H^{\mp\mp}} A_0^{\mathcal{H}}(\tau_{H^{\pm\pm}}) \Big|^2$$



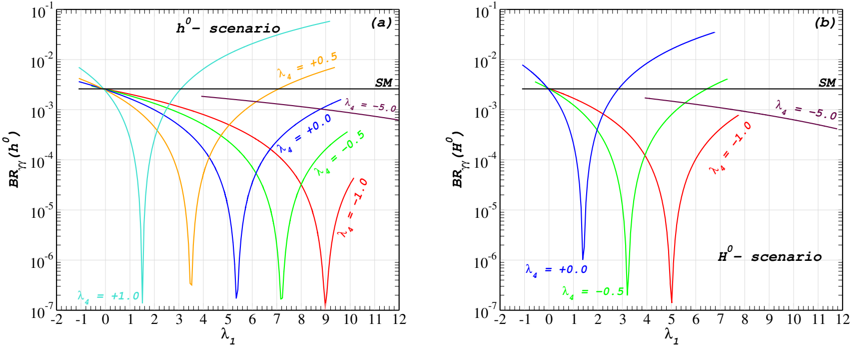
$$\tilde{g}_{\mathcal{H}H^{++}H^{--}} = -\frac{s_W}{e}\frac{m_W}{m^2_{H^{\pm\pm}}} g_{\mathcal{H}H^{++}H^{--}}, \quad \tilde{g}_{\mathcal{H}H^{+}H^{-}} = -\frac{s_W}{e}\frac{m_W}{m^2_{H^{\pm}}} g_{\mathcal{H}H^{+}H^{-}}$$

$$g_{\mathcal{H}H^{++}H^{--}} \approx -\bar{\epsilon}\lambda_1 v_d \qquad g_{\mathcal{H}H^{+}H^{-}} \approx -\bar{\epsilon}(\lambda_1 + \frac{\lambda_4}{2}) v_d$$

$\bar{\epsilon} = 1$ or $\bar{\epsilon} = \mathrm{sign}[\sqrt{2}\mu - (\lambda_1 + \lambda_4) v_t]$

relative Higgs couplings

$\mathcal{H}$	$\tilde{g}_{\mathcal{H}\bar{u}u}$	$\tilde{g}_{\mathcal{H}\bar{d}d}$	$\tilde{g}_{\mathcal{H}W^+W^-}$
h^0	$c_\alpha / c_{\beta'}$	$c_\alpha / c_{\beta'}$	$+e(c_\alpha v_d + 2s_\alpha v_t)/(2s_W m_W)$
H^0	$-s_\alpha / c_{\beta'}$	$-s_\alpha / c_{\beta'}$	$-e(s_\alpha v_d - 2c_\alpha v_t)/(2s_W m_W)$



$M_{\mathcal{H}} \simeq 125$ GeV

$M_{H^+}, M_{H^{++}} > 110$ GeV

$m_{H^0} \simeq m_{A^0} \approx 210$ GeV

$m_{h^0} \simeq m_{A^0} \approx 113$ GeV

H^0 is the SM-like

H^0 is the SM-like $\to$ light h^0, A^0

- $\frac{\mu}{v_t} \ll 1 \Rightarrow H^0$ SM-like & $m_{h^0} \simeq m_{A^0} < m_{H^0}$
- $g_{h^0 h^0 H^0} = g_{A^0 A^0 H^0} \simeq (\lambda_1 + \lambda_4) v_d$
- $|g_{Zh^0 A^0}| \approx |g_{ZZH^0}|^{SM}$

light h^0, A^0

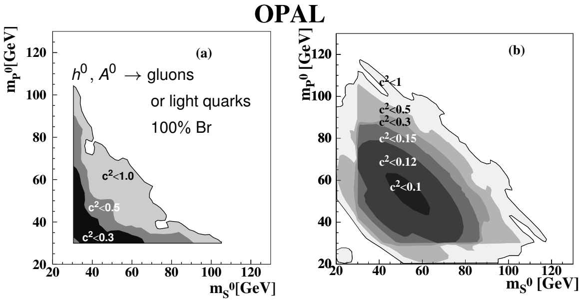
constraints to worry about:

- non-standard decay modes of the SM-like Higgs:
  - $\mathcal{H} \to h^0 h^{0(*)}, A^0 A^{0(*)}$
  - followed by $h^0, A^0 \to b\bar{b}, \nu\nu + \bar{\nu}\bar{\nu}, (gg, \tau\tau, \gamma\gamma)$, ...
- LEP2 direct search limits

  $e^+ e^- \to Z^* \to h^0 A^0$
- the Z boson width

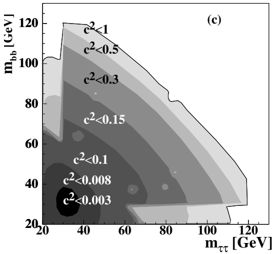
  $Z \to h^0 A^0$

light h^0, A^0



$h^0, A^0 \to b\bar{b}b\bar{b}$

100% Br



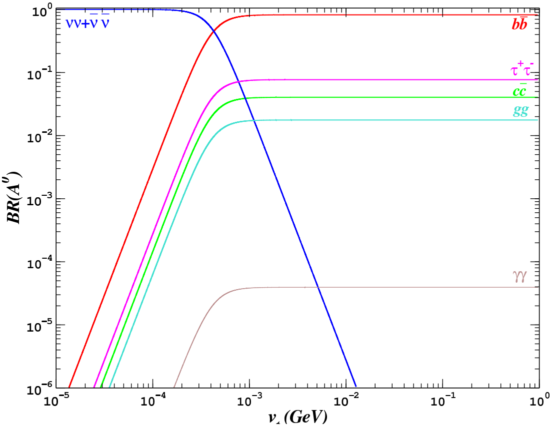
$$\sigma_{e^+e^- \to Z^* \to h^0 A^0} = (\ldots) c^2 \sigma_{HZ}^{SM}$$

$h^0, A^0 \to b\bar{b}\tau^+\tau^-$

or $b\bar{b}\tau^+\tau^-$

100% Br

light h^0, A^0



A^0: LNC decays, $4 v_t^2 / v_d^2$ suppression — h^0: $\times(\lambda_1 + \lambda_4)^2/\lambda^2$

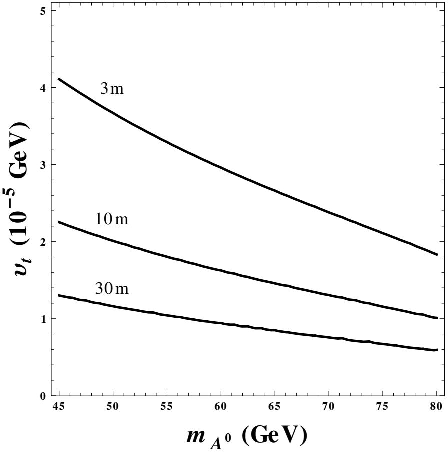
LNV decays, $2 \sum m_\nu^2 / v_t^2$ enhancement — $\times 1$

light h^0, A^0

LEP2 constraints can be evaded for sufficiently small v_t in two ways:

- branching ratios of decays in neutrinos become important or dominant
- lifetime of h^0 or A^0 becomes sufficiently long, decaying outside the detector

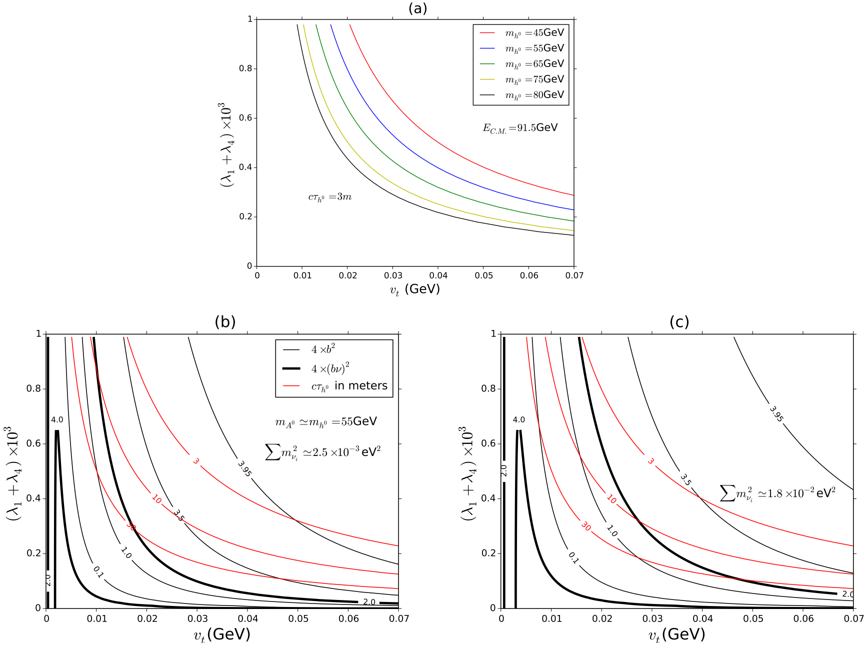
light h^0, A^0



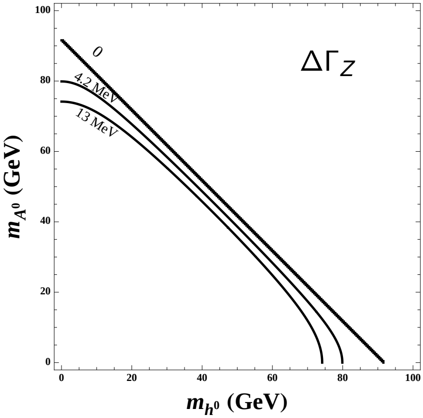
decay length $c\tau$

light h^0, A^0

$\Rightarrow$ LEP2 bounds evaded for $v_t \lesssim \mathcal{O}(10^{-3})$GeV


(a)
$m_{h^0} = 45$GeV
$m_{h^0} = 55$GeV
$m_{h^0} = 65$GeV
$m_{h^0} = 75$GeV
$m_{h^0} = 80$GeV
$E_{C.M.} = 91.5$GeV
$c\tau_{h^0} = 3m$
$(\lambda_1 + \lambda_4) \times 10^3$
v_t (GeV)
(b)
$4 \times b^2$
$4 \times (b\nu)^2$
$c\tau_{h^0}$ in meters
$m_{A^0} \simeq m_{h^0} = 55$GeV
$\sum m_{\nu_i}^2 \simeq 2.5 \times 10^{-3}$ eV^2
$(\lambda_1 + \lambda_4) \times 10^3$
v_t (GeV)
(c)
$\sum m_{\nu_i}^2 \simeq 1.8 \times 10^{-2}$ eV^2
$(\lambda_1 + \lambda_4) \times 10^3$
v_t (GeV)

light h^0, A^0



$\Gamma_Z = 2.4952 \pm 0.0023$ GeV (LEP)

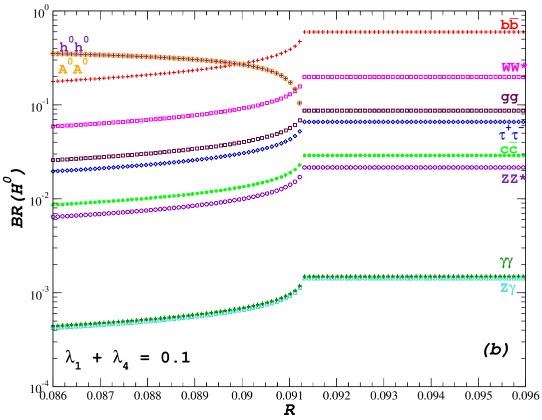
$\Gamma_Z^{SM} = 2.4961 \pm 0.0010$ (Th.)

$\Delta\Gamma_Z^{max} \simeq 4.2$MeV at the 95% C.L.

irreducible bound

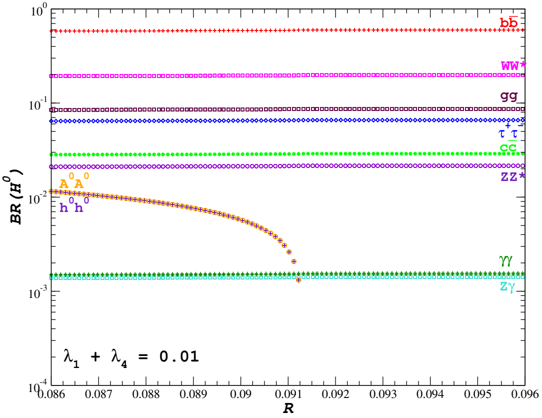
$\Rightarrow m_{h^0} \simeq m_{A^0} \gtrsim 44.3$GeV

light h^0, A^0



$$R \equiv \frac{\mu}{v_t}$$

light h^0, A^0



$$R \equiv \frac{\mu}{v_t}$$

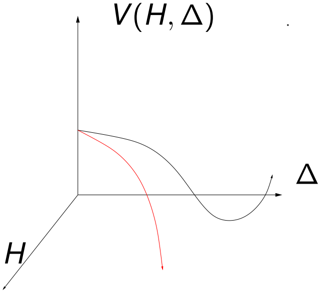
H^0 is the SM-like $\rightarrow$ light h^0, A^0

- is the scenario $44 Gev < m_{h^0}, m_{A^0} < 60 GeV$ excluded?
- tension with limits from direct search of $H^{\pm\pm}$, $H^{\pm}$, given EWPO limits, and from $H^0 \rightarrow \gamma\gamma$
- the scenario $62 GeV < m_{h^0}, m_{A^0} < 125 GeV$ not studied!

BACKUP

Dynamical constraints

Tree-level Boundedness From Below:



Dynamical constraints

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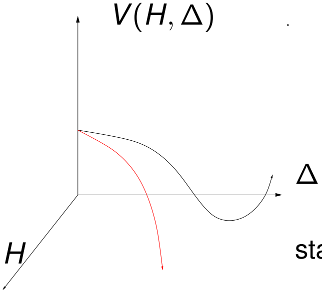

stable vacuum $\leftrightarrow V > V_{min}, \; \forall H, \Delta$

Dynamical constraints

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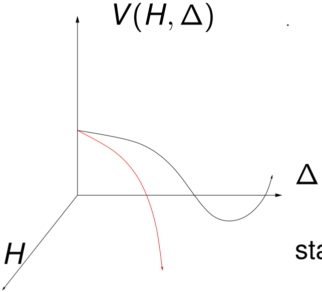
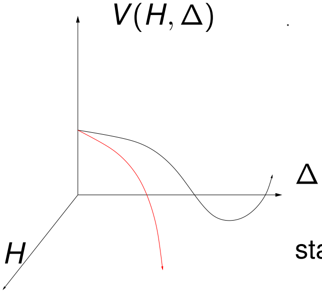


stable vacuum $\leftrightarrow$ $V > V_{min}, \ \forall H, \Delta$

keep only the quartic operators

Dynamical constraints

Tree-level Boundedness From Below:



stable vacuum $\leftrightarrow V > V_{min}, \ \forall H, \Delta$

keep only the quartic operators

$$V^{(4)}(H,\Delta) = \frac{\lambda}{4}(H^\dagger H)^2 + \lambda_1 (H^\dagger H) Tr(\Delta^\dagger \Delta) + \lambda_2 (Tr \Delta^\dagger \Delta)^2$$
$$+\lambda_3 Tr(\Delta^\dagger \Delta)^2 + \lambda_4 H^\dagger \Delta \Delta^\dagger H$$

Dynamical constraints

Tree-level Boundedness From Below: The most general solution

Dynamical constraints

Tree-level Boundedness From Below: The most general solution

$$
\begin{aligned}
r &\equiv \sqrt{H^\dagger H + Tr\Delta^\dagger \Delta} \\
H^\dagger H &\equiv r^2 \cos^2 \gamma \\
Tr(\Delta^\dagger \Delta) &\equiv r^2 \sin^2 \gamma \\
(H^\dagger \Delta \Delta^\dagger H)/(H^\dagger H Tr\Delta^\dagger \Delta) &\equiv \xi \\
Tr(\Delta^\dagger \Delta)^2/(Tr\Delta^\dagger \Delta)^2 &\equiv \zeta
\end{aligned}
$$

Dynamical constraints

<u>*Tree-level Boundedness From Below:*</u> The most general solution

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$$0 \le \xi \le 1 \quad \text{and} \quad \frac{1}{2} \le \zeta \le 1$$

Dynamical constraints

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$$V^{(4)}(r,\tan\gamma,\xi,\zeta) = \frac{r^4}{4(1+\tan^2\gamma)^2}(\lambda + 4(\lambda_1+\xi\lambda_4)\tan^2\gamma + 4(\lambda_2+\zeta\lambda_3)\tan^4\gamma)$$

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Dynamical constraints

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$$0 \leq \tan\gamma < +\infty$$

$$0 \leq \xi \leq 1 \quad \text{and} \quad \frac{1}{2} \leq \zeta \leq 1$$

Dynamical constraints

$$\lambda > 0 \;\;\&\;\; \lambda_2 + \zeta\lambda_3 > 0 \;\;\&\;\; \lambda_1 + \xi\lambda_4 + \sqrt{\lambda(\lambda_2 + \zeta\lambda_3)} > 0,$$

$$\forall\zeta \in [\frac{1}{2}, 1], \forall\xi \in [0, 1]$$

Dynamical constraints

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$$\forall\zeta \in [\frac{1}{2}, 1], \forall\xi \in [0, 1]$$

$$\lambda \geq 0 \;\; \& \;\; \lambda_2 + \lambda_3 \geq 0 \;\; \& \;\; \lambda_2 + \frac{\lambda_3}{2} \geq 0$$

$$\& \;\; \lambda_1 + \sqrt{\lambda(\lambda_2 + \lambda_3)} \geq 0 \;\; \& \;\; \lambda_1 + \sqrt{\lambda(\lambda_2 + \frac{\lambda_3}{2})} \geq 0$$

$$\& \;\; \lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2 + \lambda_3)} \geq 0 \;\; \& \;\; \lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2 + \frac{\lambda_3}{2})} \geq 0$$

Combined dynamical constraints

BFB and unitarity

Combined dynamical constraints

BFB and unitarity

$$0 \leq \lambda \leq \frac{2}{3}\kappa\pi \;\;\&\;\; \lambda_2 + \lambda_3 \geq 0 \;\;\&\;\; \lambda_2 + \frac{\lambda_3}{2} \geq 0 \;\;\&$$

$$\lambda_1 + \sqrt{\lambda(\lambda_2 + \lambda_3)} \geq 0 \;\;\&\;\; \lambda_1 + \sqrt{\lambda(\lambda_2 + \frac{\lambda_3}{2})} \geq 0 \;\;\&$$

$$\lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2 + \lambda_3)} \geq 0 \;\;\&\;\; \lambda_1 + \lambda_4 + \sqrt{\lambda(\lambda_2 + \frac{\lambda_3}{2})} \geq 0 \;\;\&$$

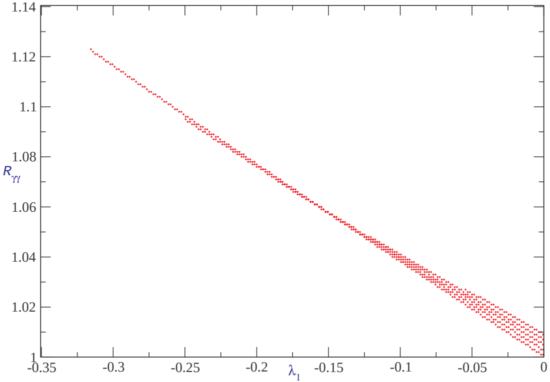
$$\lambda_2 + 2\lambda_3 \leq \frac{\kappa}{2}\pi \;\;\&\;\; 4\lambda_2 + 3\lambda_3 \leq \frac{\kappa}{2}\pi \;\;\&$$

$$\lambda_2 - 2\lambda_3 - \sqrt{(\lambda_2 - \frac{\kappa}{2}\pi)(9\lambda_2 - \frac{5}{2}\kappa\pi)} \leq \frac{\kappa}{2}\pi \;\;\&$$

$$|\lambda_4| \leq \min \sqrt{(\lambda \pm 2\kappa\pi)(\lambda_2 + 2\lambda_3 \pm \frac{\kappa}{2}\pi)} \;\;\&$$

$$|2\lambda_1 + \lambda_4| \leq \sqrt{2(\lambda - \frac{2}{3}\kappa\pi)(4\lambda_2 + 3\lambda_3 - \frac{\kappa}{2}\pi)}$$

Combined dynamical constraints



$$\lambda = 0.52, \lambda_3 = 2\lambda_2 = 0.2, v_t = 1 \text{ GeV}$$

$\Rightarrow$ exact full 5D hyper-volume: $\lambda_1 < -1, -0.5 \rightarrow$ 3permil, 3%

Ratio of branching ratios

bryy_cont_300.pdf

Akeroyd, Moretti, arXiv:1206.0535