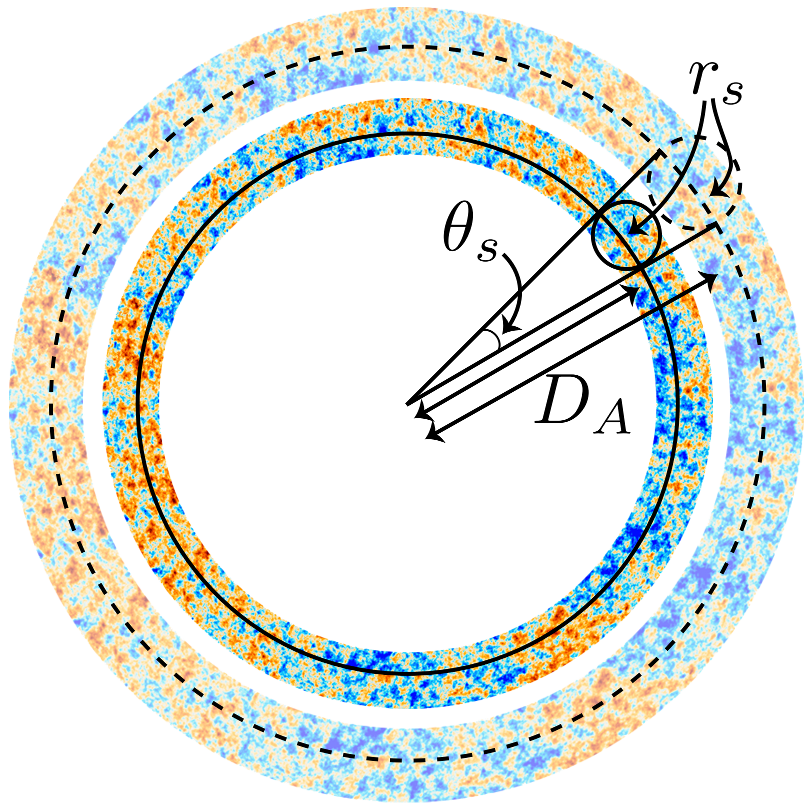
Cosmological aspects of scalar fields

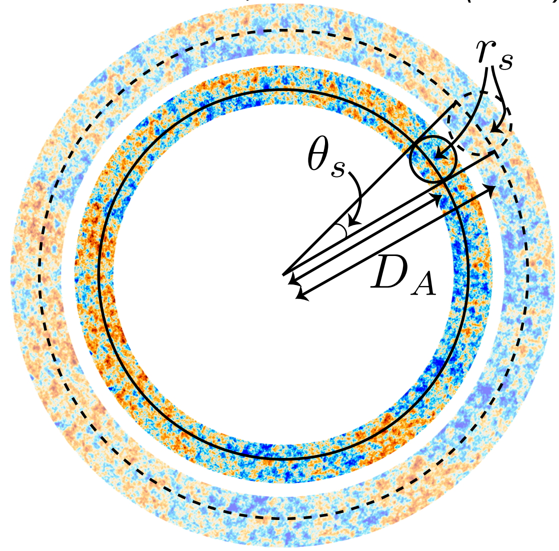


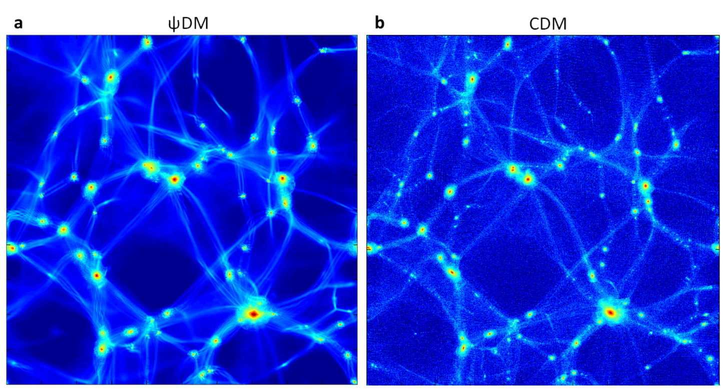
Tristan L. Smith
Swarthmore College

What can scalar fields do?

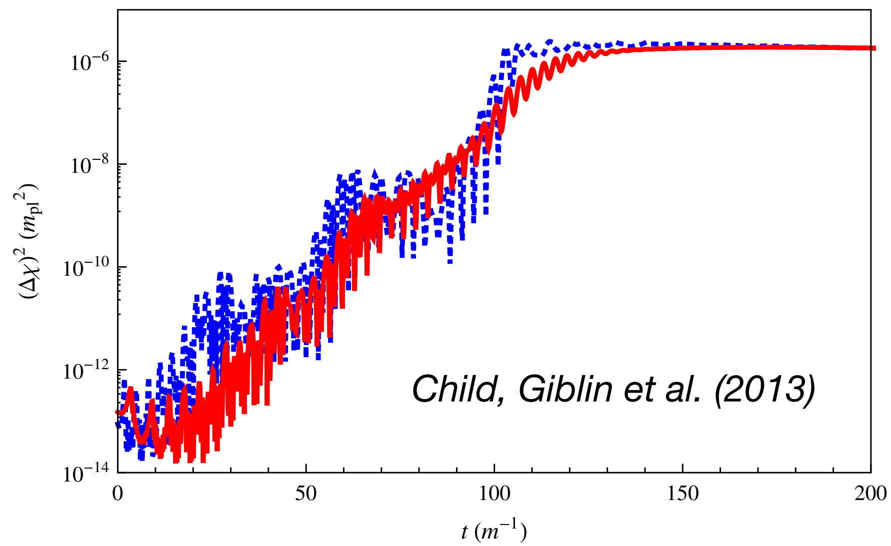
- **Inflation**
- **Quintessence**
- **Scalar field DM**
- **‘Early dark energy’**
- **Pre/re-heating**

Poulin, Smith et al. (2019)





Schive, Chiueh, and Broadhurst (2014)



Child, Giblin et al. (2013)

- **If you don’t like the QFT of scalar fields… think of them as effective models**

Scalar fields: the duct tape of the universe

Inflaton; quintessence; fuzzy DM; scalar interactions…

Higgs field… ubiquitous in string theory

Can scalar fields do *everything*?

NO! Their dynamics are actually quite constrained and beautiful

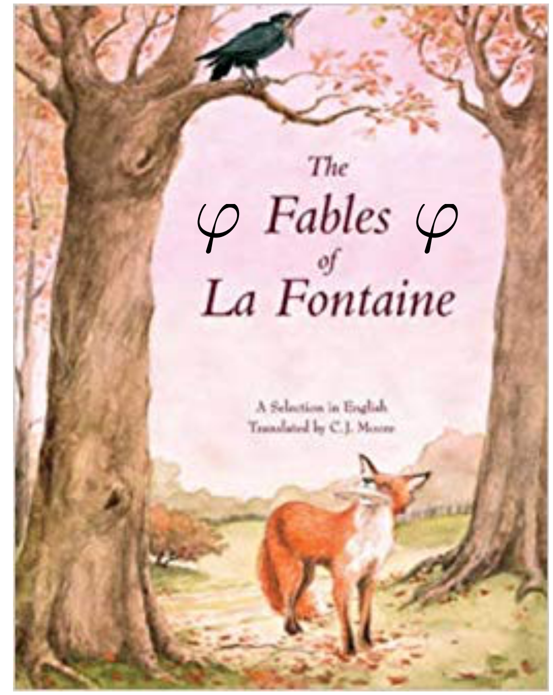
Scalar fields: the duct tape of the universe

- Slow-roll… thawing
- Attractor behavior (i.e. quintessence tracking)
- Anharmonic oscillations
- Parametric (self) resonance
- Perturbations

Focus on minimally coupled scalar fields which are initially (nearly) homogeneous- i.e. no phase-transition after inflation

Scalar fields: background evolution

$$\ddot{\varphi} + 3H\dot{\varphi} + V_{,\varphi} = 0$$



General story:

- **The field is fixed by Hubble friction**
- **Once the Hubble parameter drops enough the field starts to evolve**
- **If there is a local minimum, field *might* oscillate**
- **If not the field evolves monotonically**

Scalar fields: background evolution

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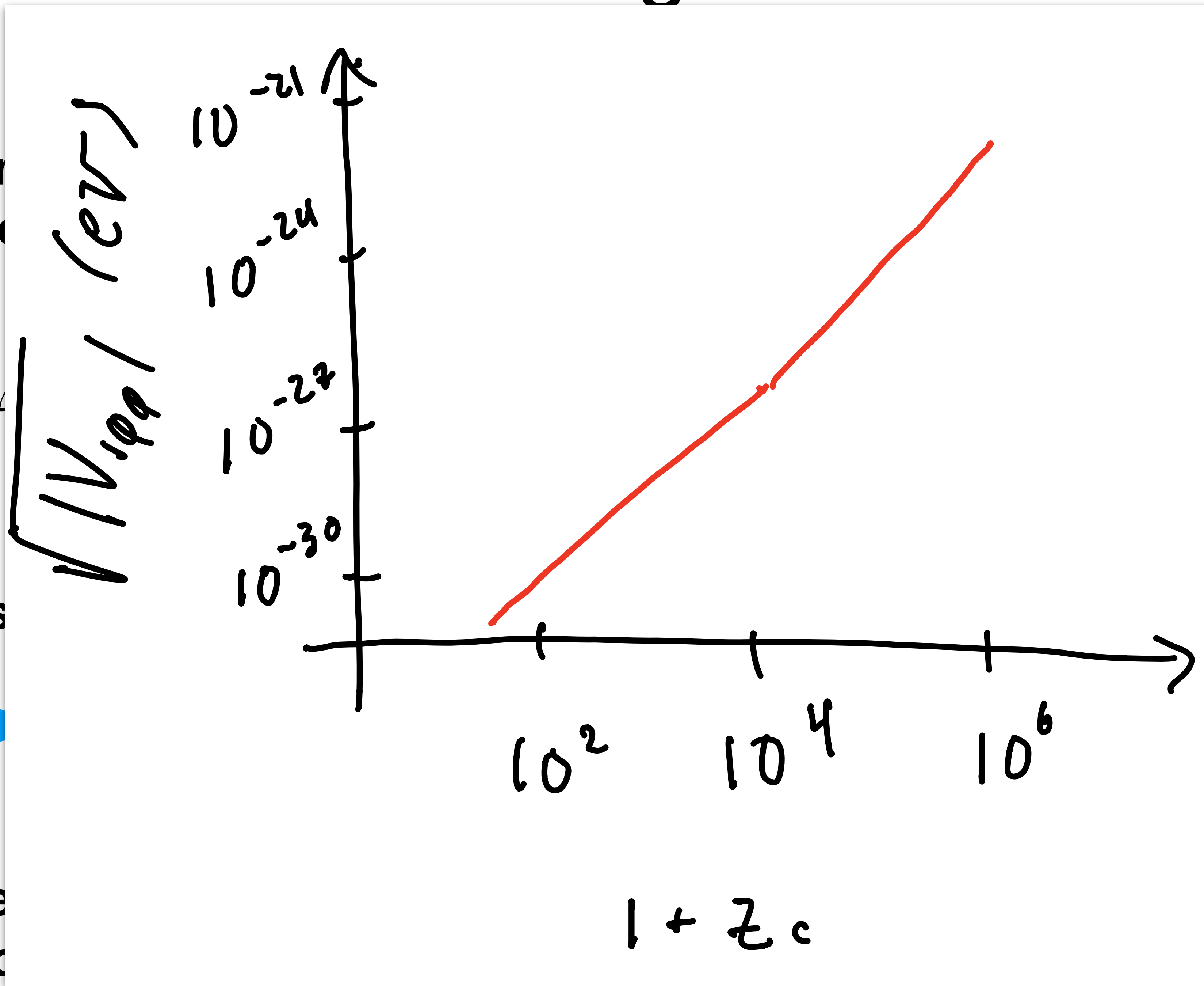
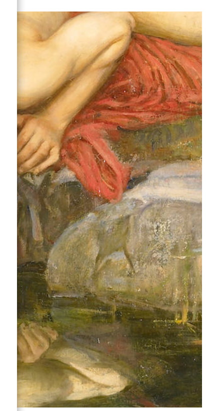


Exits



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Scalar fields: background evolution

$$\ddot{\varphi} + 3H\dot{\varphi} + V_{,\varphi} = 0$$

Also follows from energy conservation:

$$\dot{\rho}_\varphi = -3H\rho_\varphi(1 + w_\varphi)$$

$$\rho_\varphi \equiv \frac{1}{2}\dot{\varphi}^2 + V \qquad w_\varphi \equiv \frac{\frac{1}{2}\dot{\varphi}^2 - V}{\frac{1}{2}\dot{\varphi}^2 + V} \qquad H^2 = \frac{\kappa^2}{3}(\rho_B + \rho_\varphi)$$

The evolution of the field is beautifully described in a 'phase-space' *(Copeland, Liddle, and Wands 1998)*

$$X \equiv \frac{\kappa\dot{\varphi}}{\sqrt{6}H} \qquad \frac{\rho_\varphi}{\rho_{\rm tot}} = X^2 + Y^2$$

$$Y \equiv \frac{\kappa\sqrt{V}}{\sqrt{3}H} \qquad w_\varphi = \frac{X^2 - Y^2}{X^2 + Y^2}$$

Does not assume φ is subdominant!

Attractor behavior

$$X' = -X\frac{3}{2}\left[(w_B+1)Y^2 + (1-w_B)(1-X^2)\right] - Y^2\sqrt{\frac{3}{2}}\lambda(\varphi)$$

$$Y' = \frac{3}{2}Y\left[1 + X^2 - Y^2 + w_B(1-X^2-Y^2) + \frac{\sqrt{6}}{3}X\lambda(\varphi)\right]$$

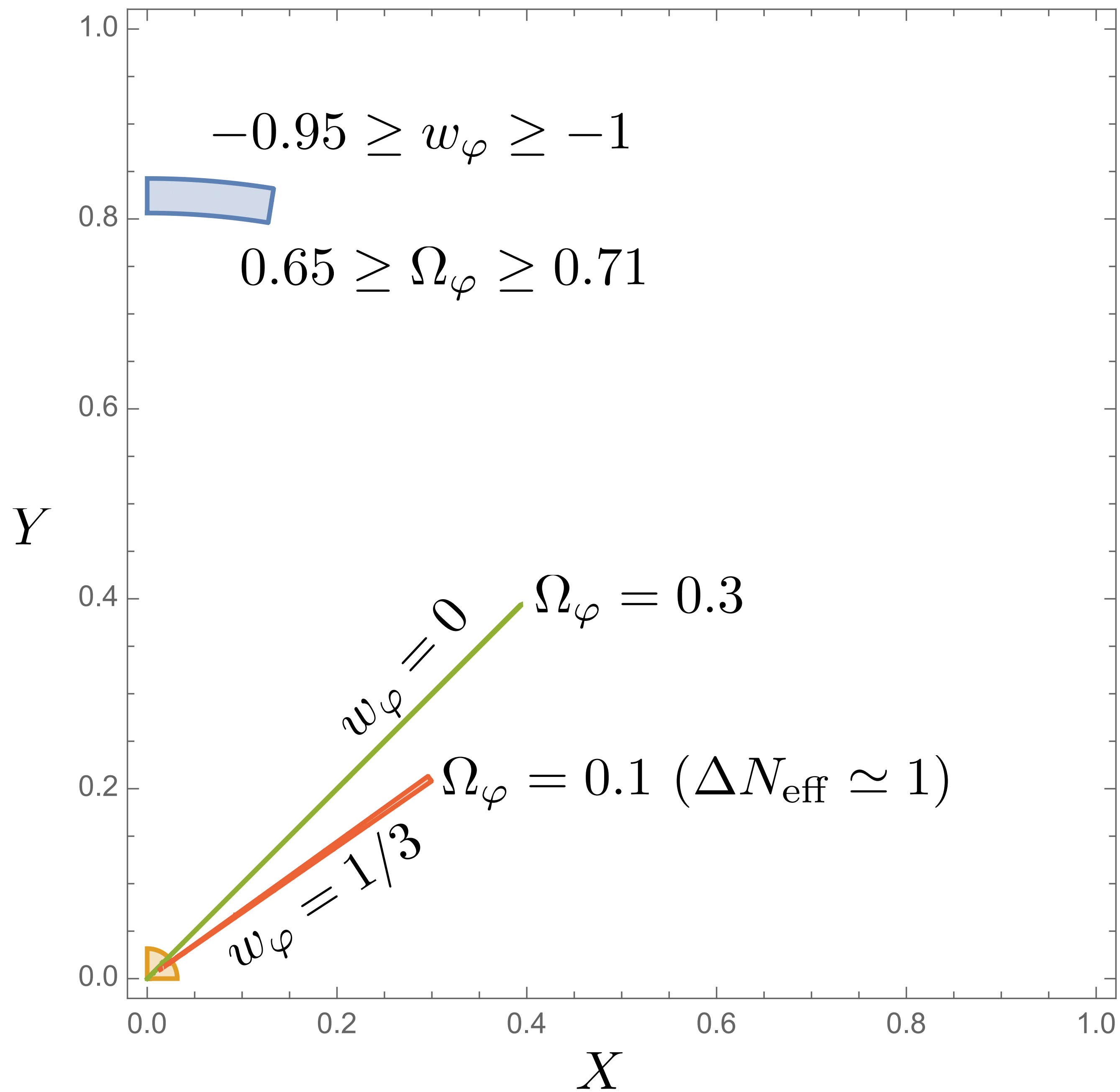
$$\lambda(\varphi) \equiv \frac{d\ln V}{d\kappa\varphi}$$

Solve for fixed points ($X' = Y' = 0$) and assess stability…

Two stable behaviors:

- $\lambda^2 > 3(w_B+1) \rightarrow \Omega_\varphi = \frac{3(1+w_B)}{\lambda^2},\ w_\varphi = w_B$ ← **Tracking!**
- $\lambda^2 < 6 \rightarrow \Omega_\varphi = 1,\ w_\varphi = \frac{\lambda^2}{3} - 1$ ← **Scalar field dominates!**

Scalar field phase-space



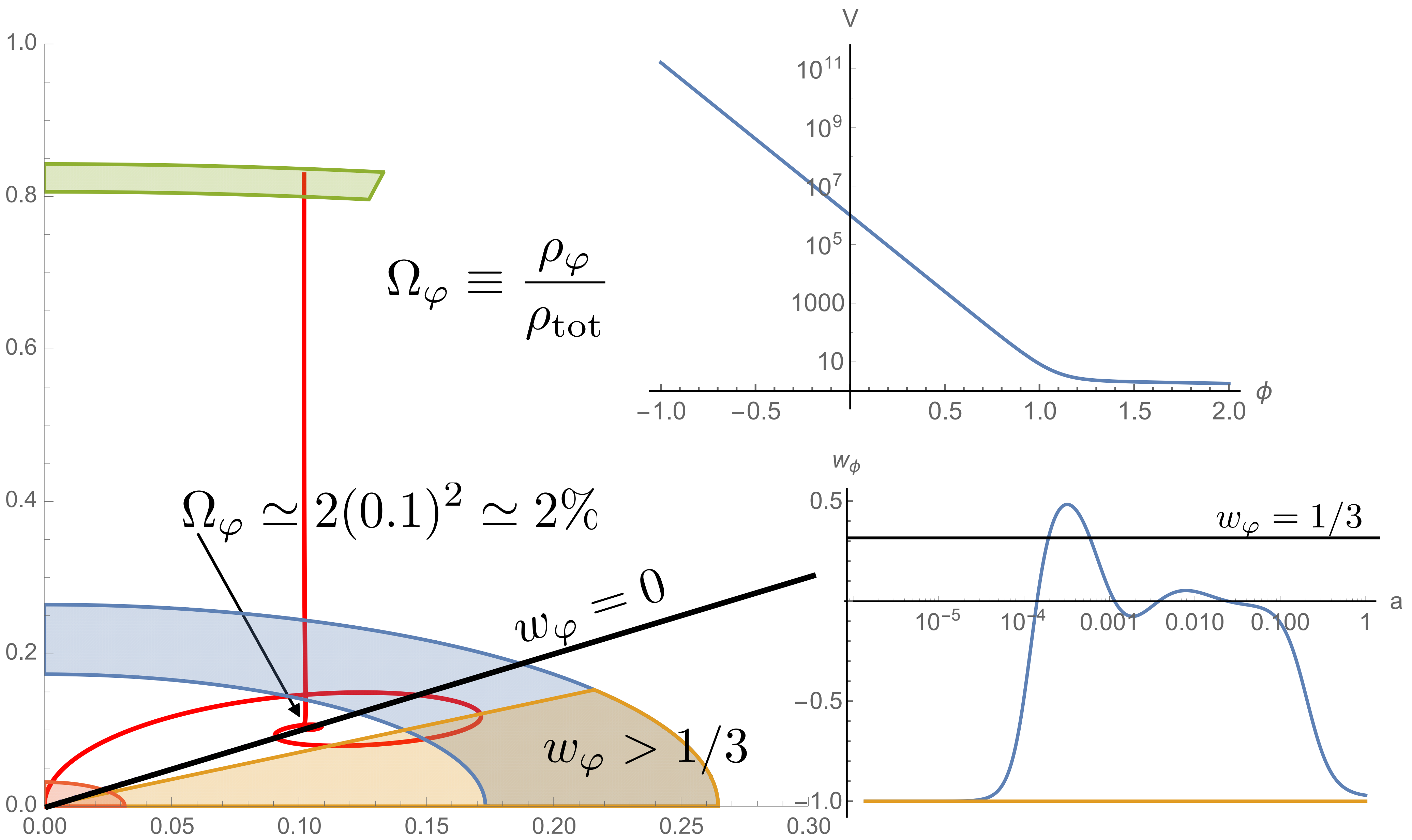
$$\Omega_\varphi \equiv \frac{\rho_\varphi}{\rho_{\mathrm{tot}}}$$

Scalar field phase-space: double exponential

$$V(\varphi) = \mu_1 e^{-\lambda_1 \varphi} + \mu_2 e^{-\lambda_2 \varphi}$$

Large (tracking) $\longrightarrow \lambda_1 = 12$

Small (domination) $\longrightarrow \lambda_2 = 0.25$



Oscillating scalar fields

$$V = V_0 \varphi^n \qquad \ddot{\varphi} + 3H\dot{\varphi} + nV_0\varphi^{n-1} = 0$$

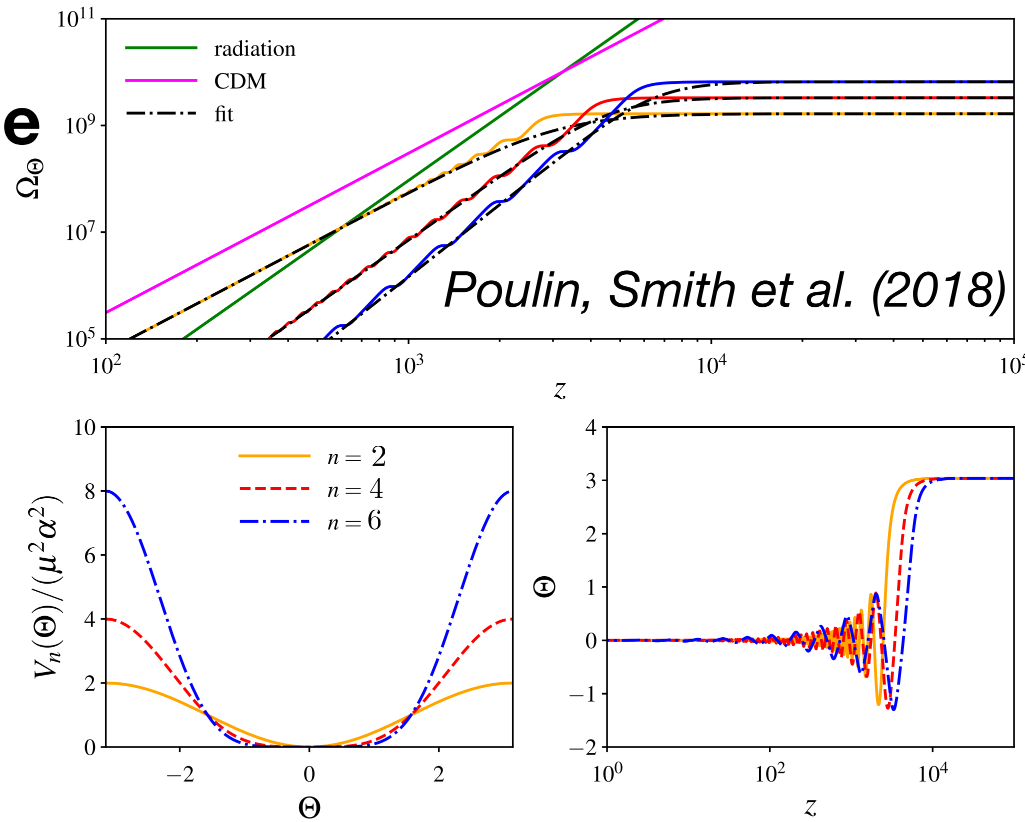
- **Damped oscillations**
- **If oscillations are much faster Hubble energy is approximately conserved**
- **'Cycle-averaging' we have the virial theorem**

$$\frac{1}{2}\langle \dot{\varphi}^2 \rangle \simeq \frac{n}{2}\langle V(\varphi) \rangle$$

$$w_\varphi \equiv \frac{\frac{1}{2}\dot{\varphi}^2 - V}{\frac{1}{2}\dot{\varphi}^2 + V}$$

- **Gives a cycle-averaged EOS**

$$\langle w_\varphi \rangle \simeq \frac{n-2}{n+2}$$



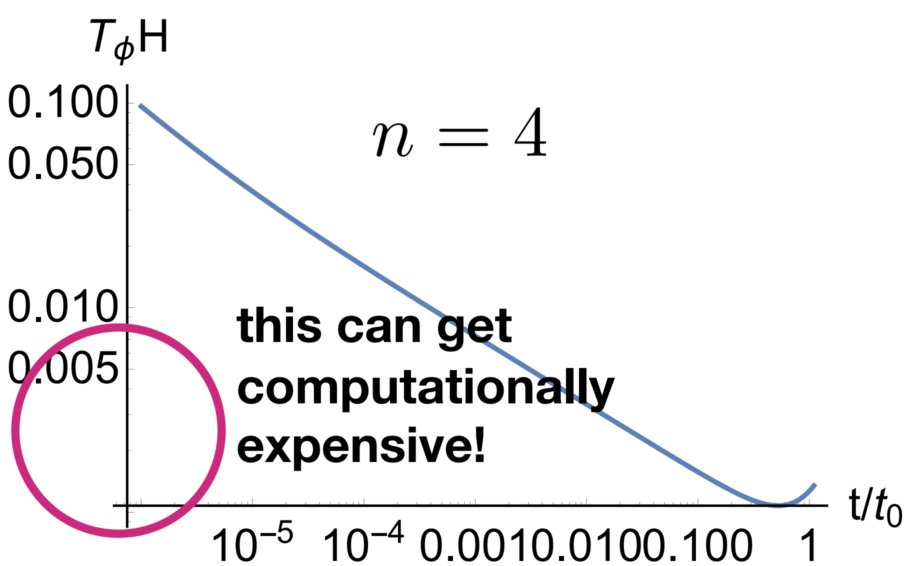
Oscillating scalar fields

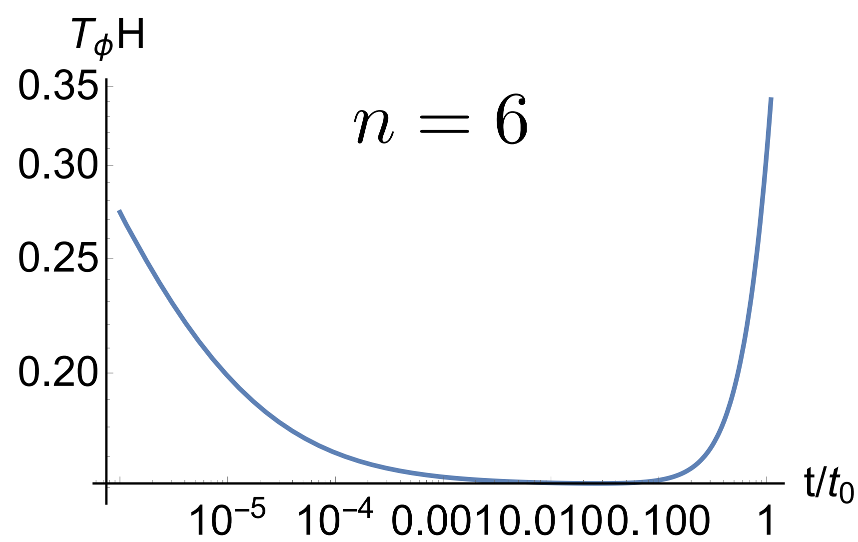
$$\ddot{\varphi} + 3H\dot{\varphi} + nV_0\varphi^{n-1} = 0 \quad \langle w_\varphi \rangle \simeq \frac{n-2}{n+2}$$

- **For n>2 the oscillations are *anharmonic***
- **Oscillation period depends on amplitude**

$$\omega_\varphi = \omega_0 a^{-3w_\varphi} \qquad n = 4 \rightarrow 3w_\varphi = 1$$

$$\omega_0 \propto V_0^{1/n} \qquad n = 6 \rightarrow 3w_\varphi = 3/2$$





~~Oscillating~~ scalar fields

$$\ddot{\varphi} + 3H\dot{\varphi} + nV_0\varphi^{n-1} = 0$$

- **We can also solve this equation for certain values of n**

$$H = \frac{2}{3(1+w_B)}\frac{1}{t} \qquad \varphi = \varphi_i \left(\frac{t}{t_i}\right)^m \rightarrow m = -\frac{2}{n-2}$$

$$w_\varphi = -1 + (1+w_B)\frac{n}{n-1}$$

Ratra & Peebles (1988)
Liddle & Scherrer (1998)

- **No oscillations (i.e. Hubble friction wins) as long as**

$$n > \frac{2(3+w_B)}{1-w_B}$$

$\rightarrow$ **radiation domination** $n > 10$

$\rightarrow$ **matter domination** $n > 6$

Perturbations!

$$\delta\ddot{\varphi} + 3H\delta\dot{\varphi} + \left[\frac{k^2}{a^2} + V_{,\varphi\varphi}\right]\delta\varphi = -2V_{,\varphi}\Psi + 4\dot{\varphi}\dot{\Psi}$$

- **Can also write as coupled first order differential equation…. i.e., conservation of perturbed stress-energy**

$$\delta'_\varphi = -(1+w_\varphi)(\theta_\varphi - 3\Phi') - 3\frac{a'}{a}\left(\frac{\delta P_\varphi}{\delta\rho_\varphi} - w_\varphi\right)\delta_\varphi$$

$$\theta'_\varphi = -\frac{a'}{a}(1-3w_\varphi)\theta_\varphi - \frac{w'_\varphi}{1+w_\varphi}\theta_\varphi + \frac{\delta P_\varphi/\delta\rho_\varphi}{1+w_\varphi}k^2\delta_\varphi + k^2\Psi$$

- **Sub-horizon perturbation determined by ‘sound speed’**

Scalar field sound-speed

$$\frac{\delta P_\varphi}{\delta \rho_\varphi} = c^2_{\mathrm{ad},\varphi} + c^2_{\mathrm{nad},\varphi}$$

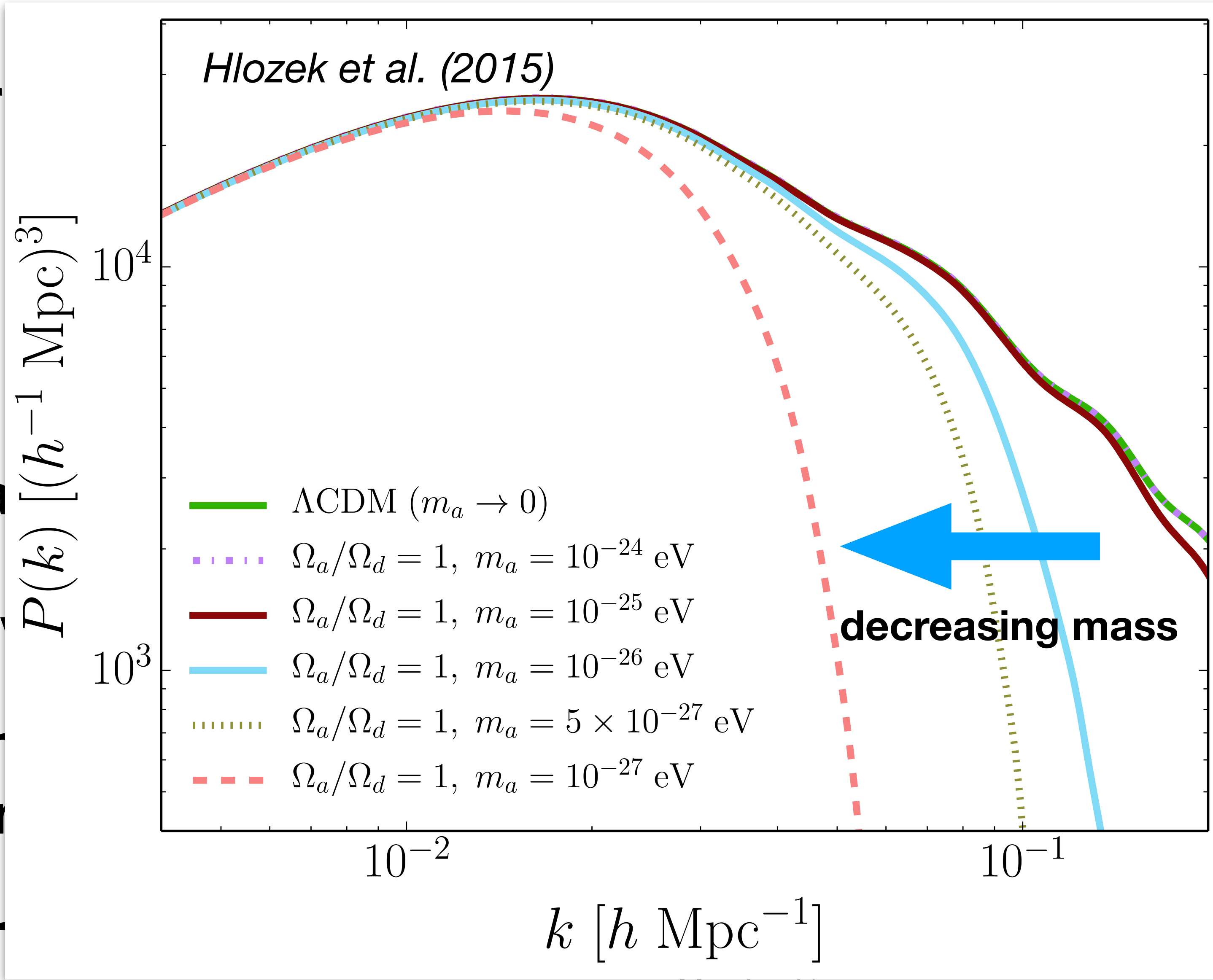
$$c^2_{\mathrm{ad},\varphi} = \frac{\dot{P}_\varphi}{\dot{\rho}_\varphi} \qquad\qquad c^2_{\mathrm{nad},\varphi} = 1$$ *Hu (1998)*

$$= 1 + \frac{2}{3H}\frac{V_{,\varphi}}{\dot{\varphi}}$$

- **Once (or if) oscillations start the fluid equations are numerically unstable (perturbed KG is stable)**

- **We can derive cycle-averaged effective sound-speed**

Cycle-averaged sound-speed



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- **Effecti ... ndent**
- **On 'sm ... ss clusteri**
- **On 'lar**

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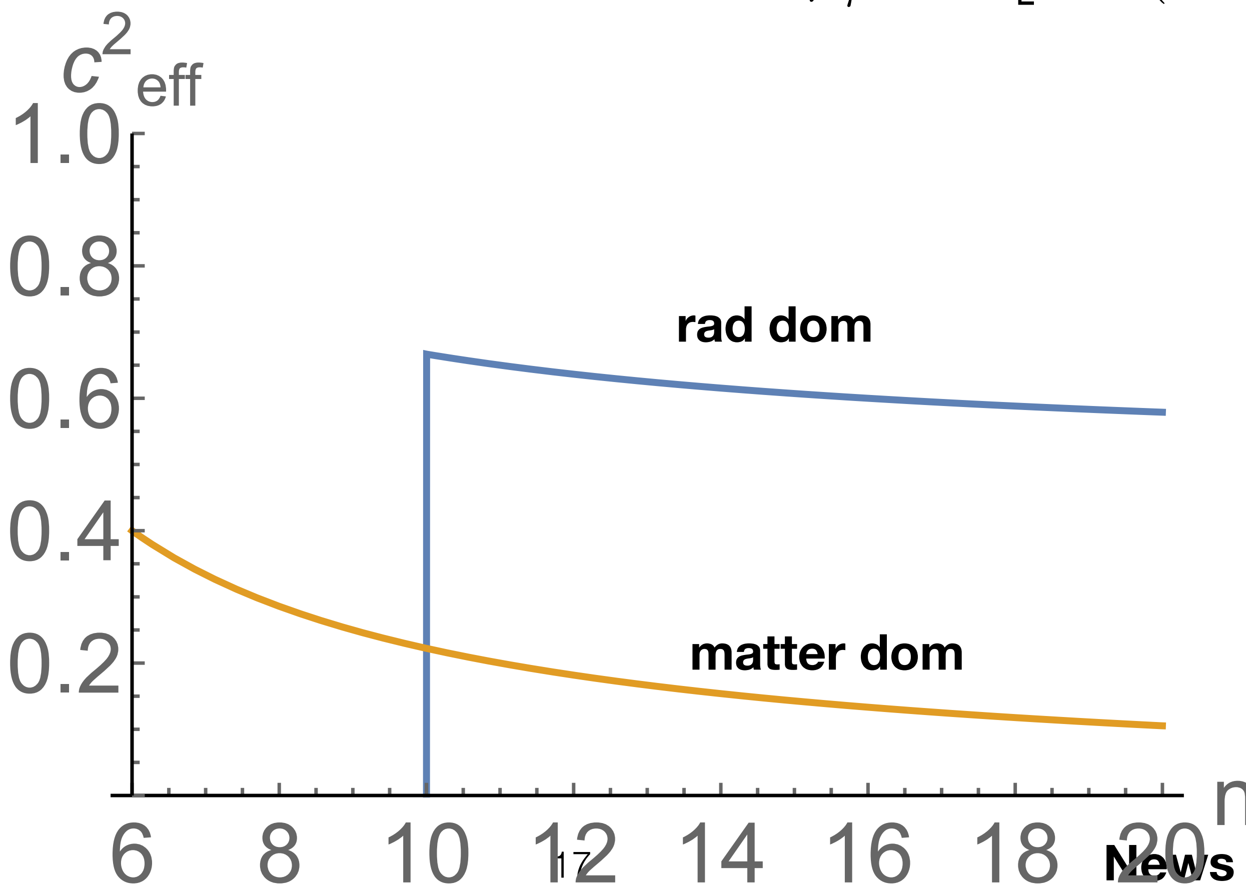
fields don't cluster

- **As mass decrease suppressed k decreases**

Scaling sound-speed

$$\frac{\delta P_\varphi}{\delta \rho_\varphi} = 2\left(1 + \frac{1}{3H}\frac{V_{,\varphi}}{\dot\varphi}\right) = 2\left(1 - \frac{|\dot V|}{3H\dot\varphi^2}\right)$$

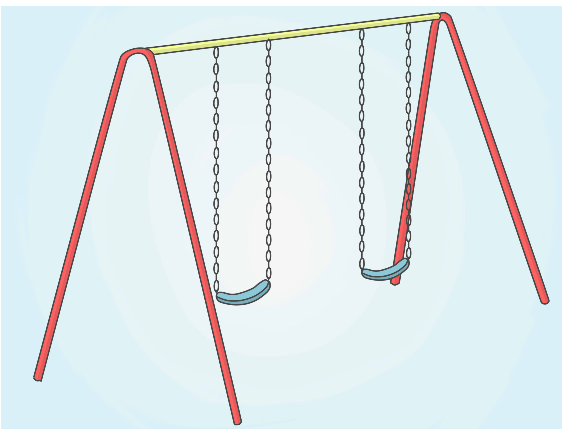
- **For power-law attractor** $V \propto \varphi^n$ $\quad \frac{\delta P_\varphi}{\delta \rho_\varphi} = 2\left[1 + \frac{2-n}{(n-1)(1+w_B)}\right]$



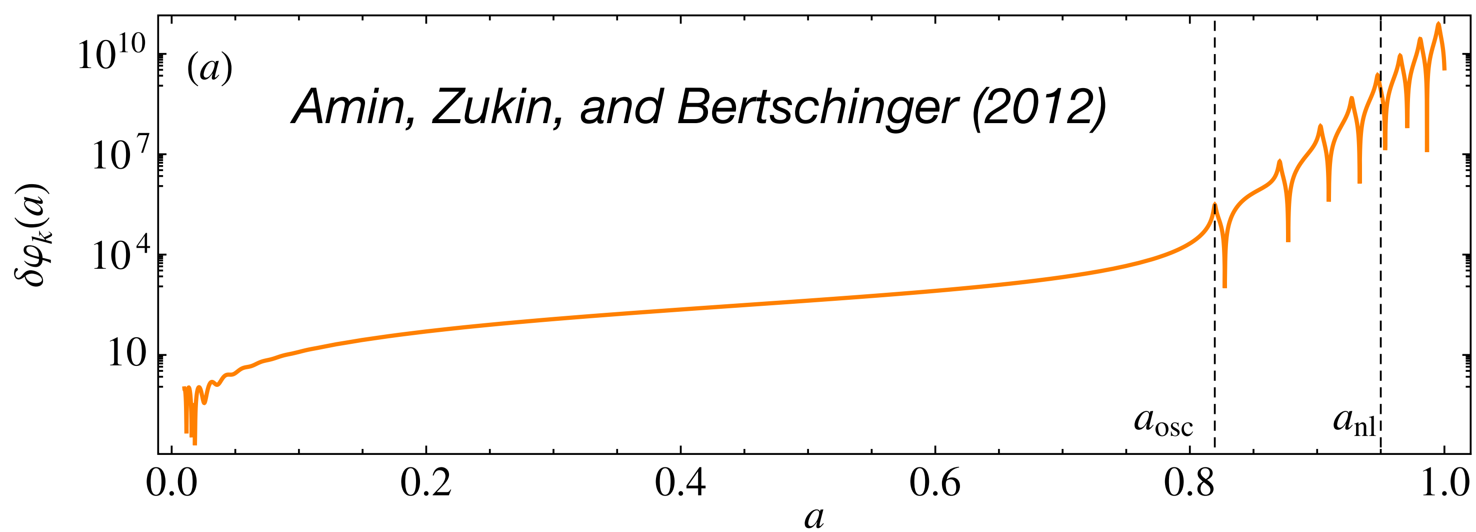
Parametric resonance!

$$\delta\ddot{\varphi} + 3H\delta\dot{\varphi} + \left[\frac{k^2}{a^2} + V_{,\varphi\varphi}\right]\delta\varphi = -2V_{,\varphi}\Psi + 4\dot{\varphi}\dot{\Psi}$$

- **When the effective frequency varies in time can get resonant growth**



$$V_{,\varphi\varphi} = V_0 n(n-1)\varphi(t)^{n-2}$$



Initial conditions

$$\delta\ddot{\varphi} + 3H\delta\dot{\varphi} + \left[\frac{k^2}{a^2} + V_{,\varphi\varphi}\right]\delta\varphi = -2V_{,\varphi}\Psi + 4\dot{\varphi}\dot{\Psi}$$

- **Solution can be divided into :** $\delta\varphi = \delta\varphi_{\rm H} + \delta\varphi_{\rm I}$

isocurvature **'adiabatic'**

- **Generically expect isocurvature (spectator field during inflation)**

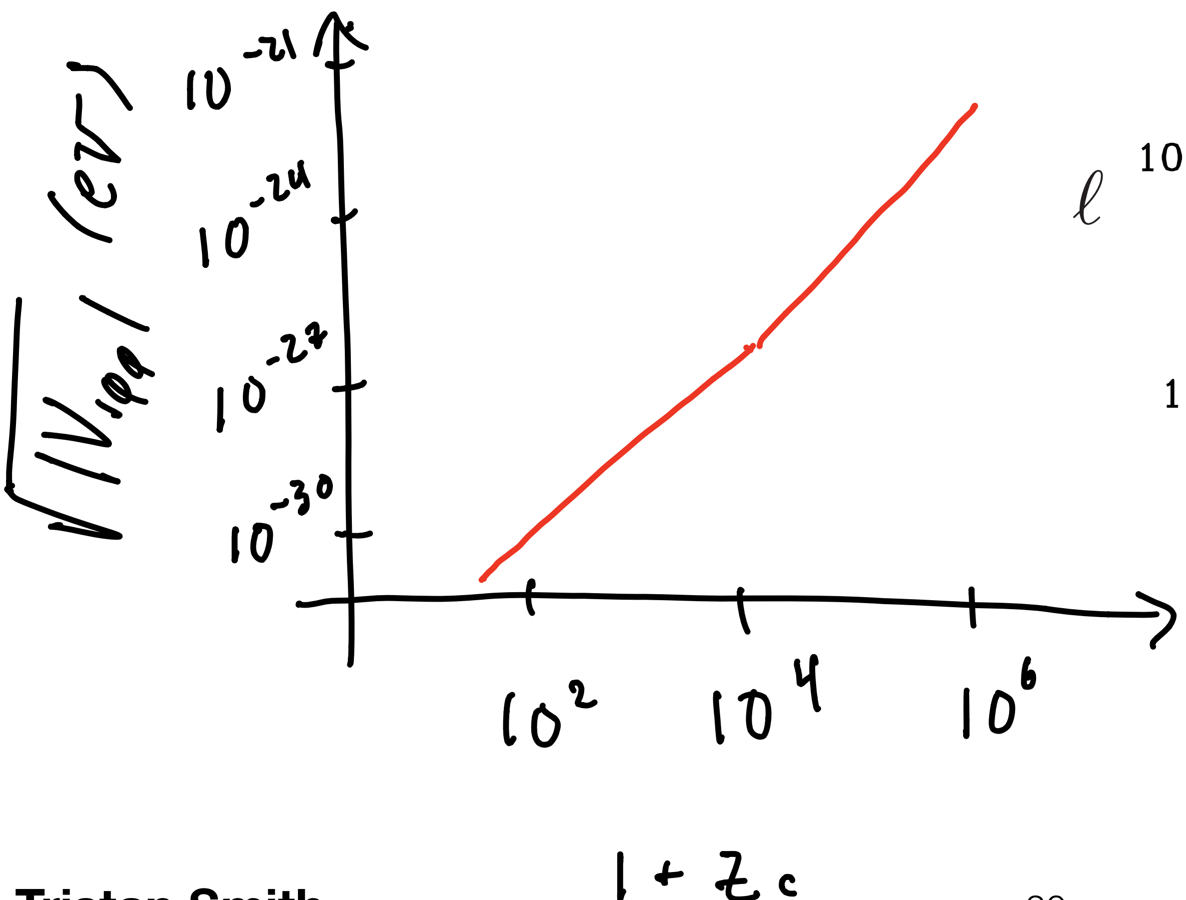
$$\langle \delta\varphi(\vec{k})\delta\varphi^*(\vec{k}')\rangle = (2\pi)^3[P_{\delta\varphi,\rm ad}(k) + P_{\delta\varphi,\rm iso}(k)]\delta_D(\vec{k} - \vec{k}')$$

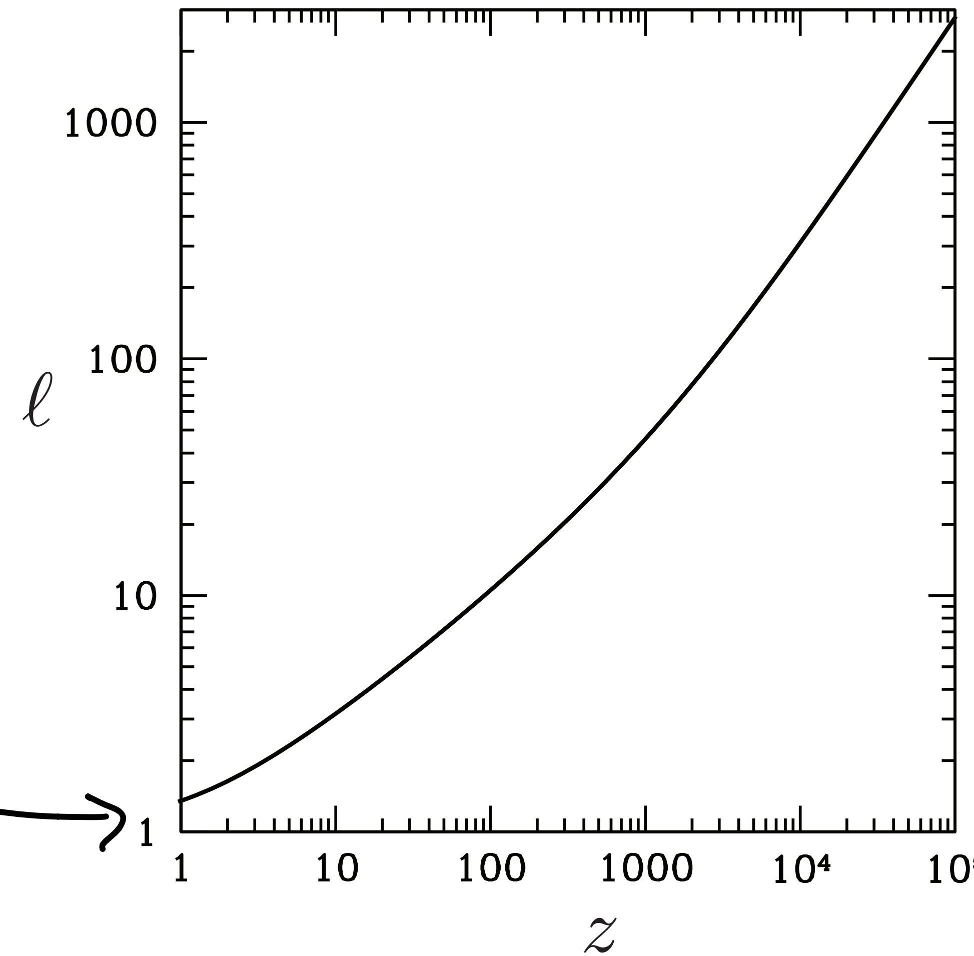
Generated by adiabatic potential sources

$$P_{\delta\varphi,\rm iso}(k) = rA_s\left(\frac{k}{k_p}\right)^{-r/8}$$

Example: ‘early dark energy’

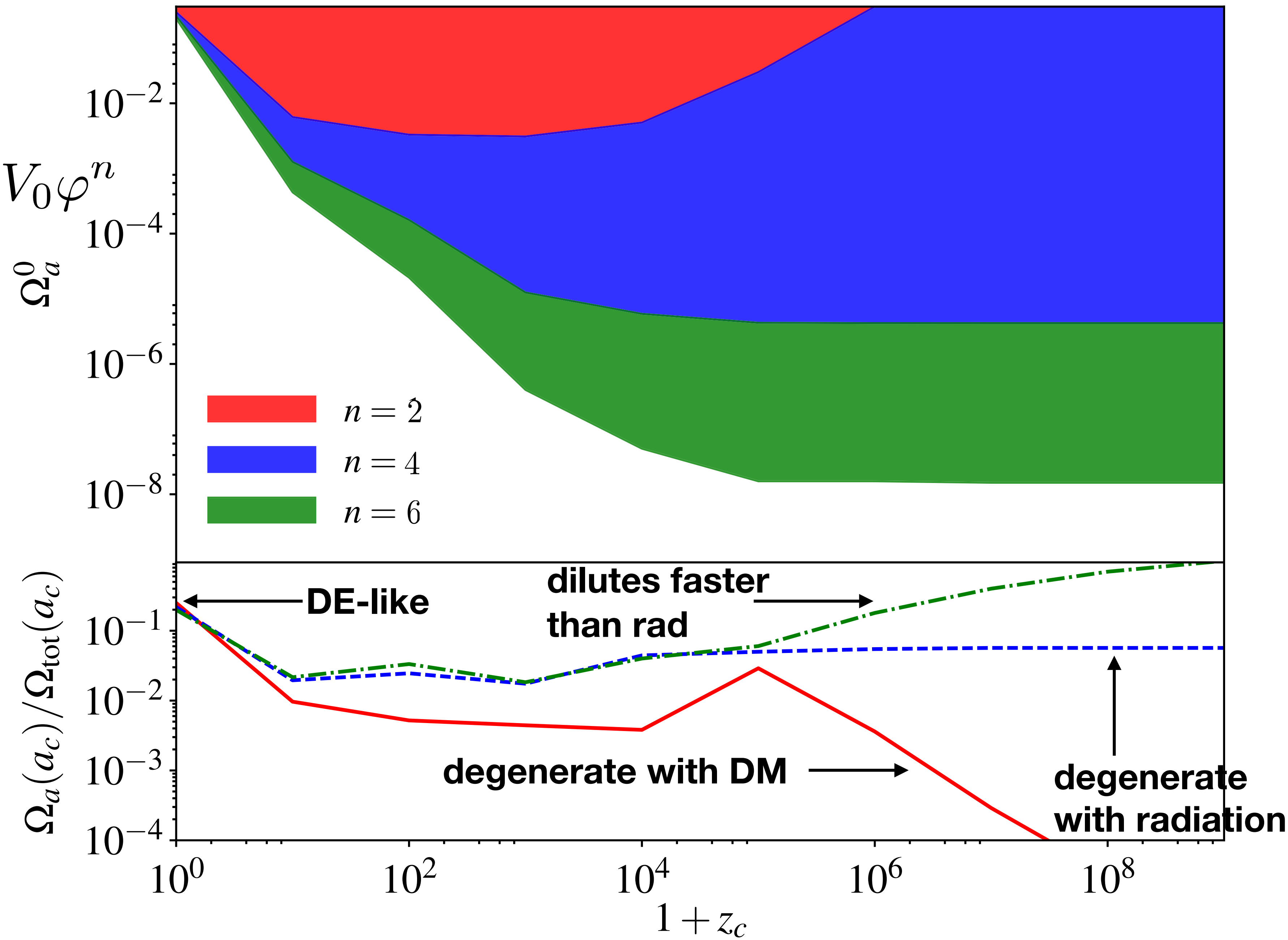
- **Look for a scalar field that becomes dynamical on scales observable in the CMB** *Poulin, Smith et al. (2018)*





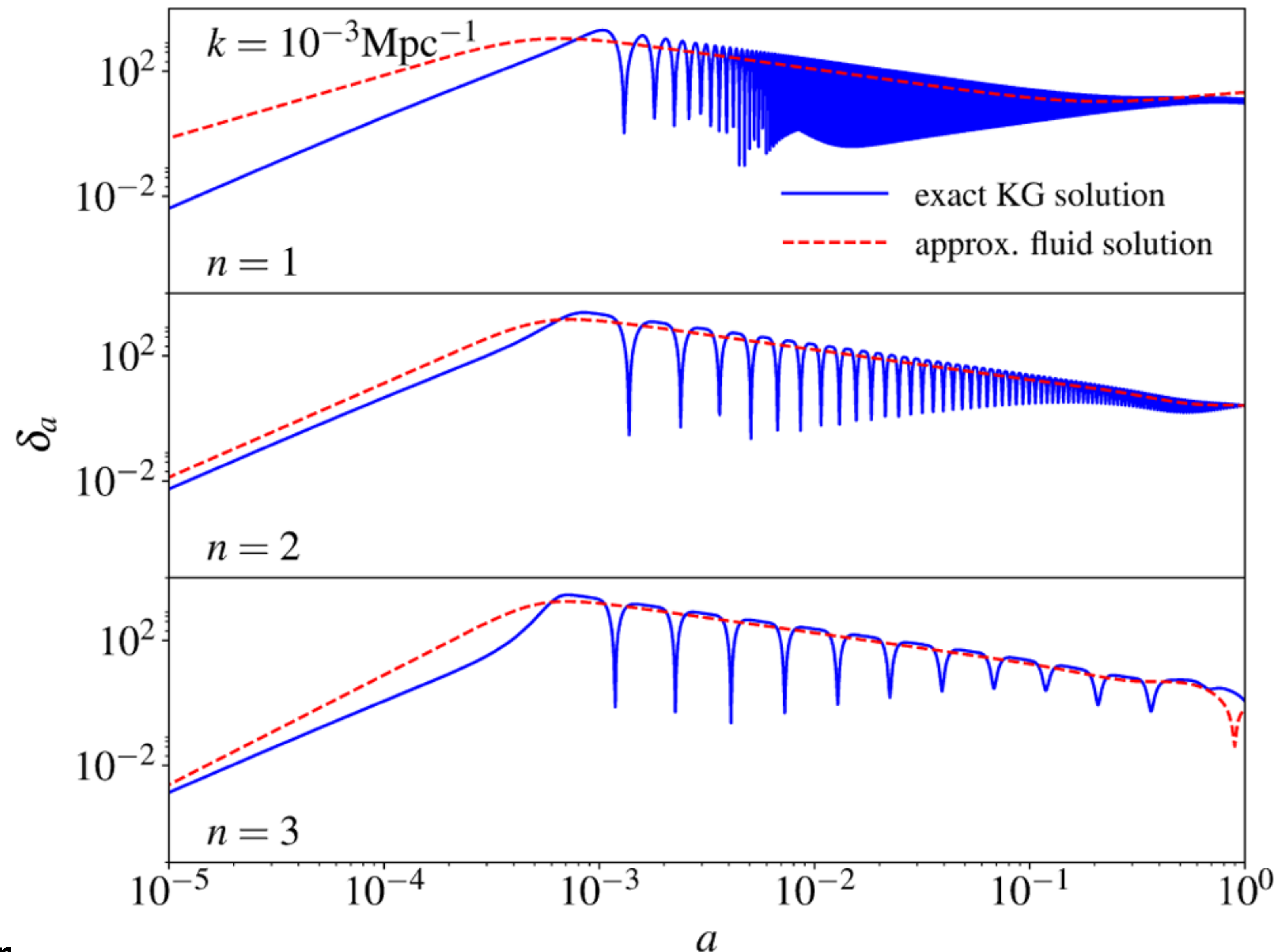
Constraints

$$V = V_0 \varphi^n$$



How good is the fluid approximation?

Scale- and time-dependent effective sound speed



Hubble tension and scalar fields

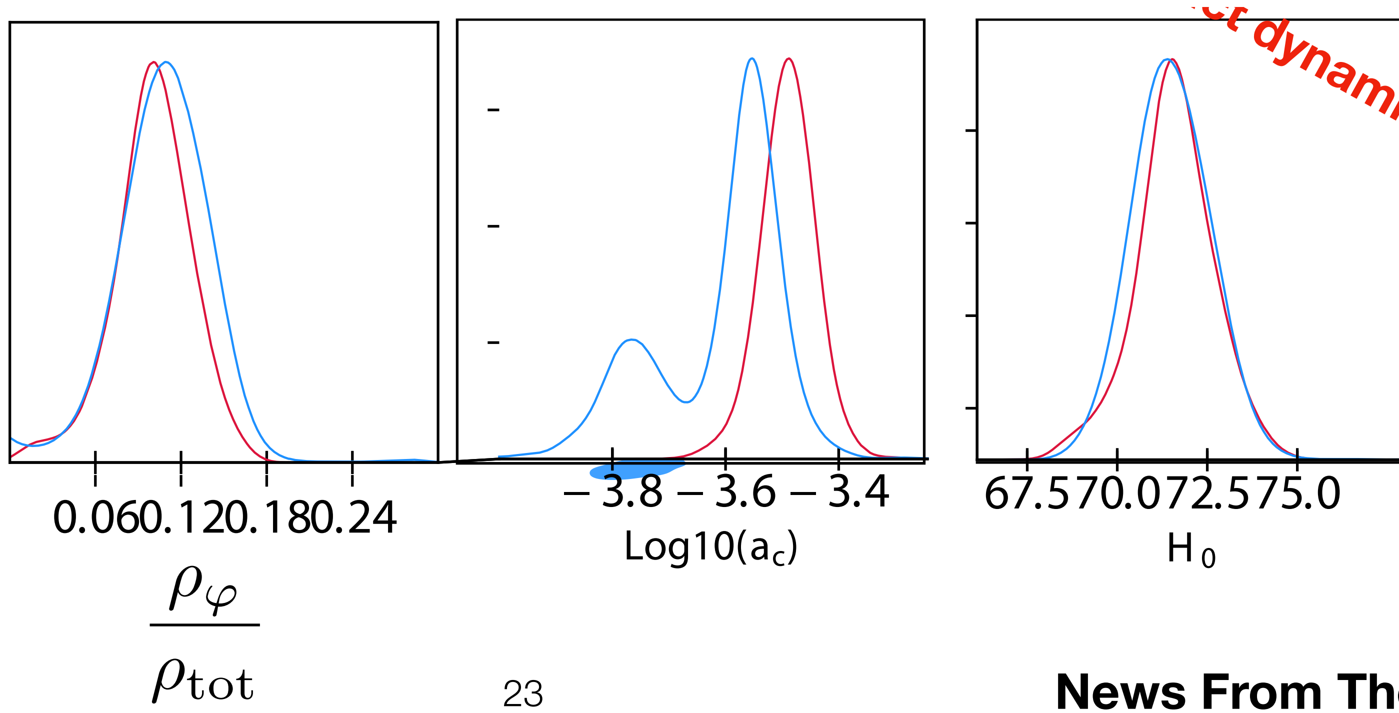
dynamics

- n = 4
- n = ∞
- n = 6
- ΛCDM

Poulin, Smith et al. (PRL in press 2019)

- n = 4
- n = 6

Smith, Poulin, and Amin (in prep)



Conclusions

- **Cosmological scalar fields a have diverse but constrained phenomenology**

- **Background dynamics described by damped (an)harmonic oscillator**

- **Perturbations are driven damped harmonic oscillators**

    - **Scale and time dependent effective sound speed**
    - **Parametric resonance**

- **As a ‘generic’ additional component scalar fields may play an important role in understanding current/future ‘tensions’**