

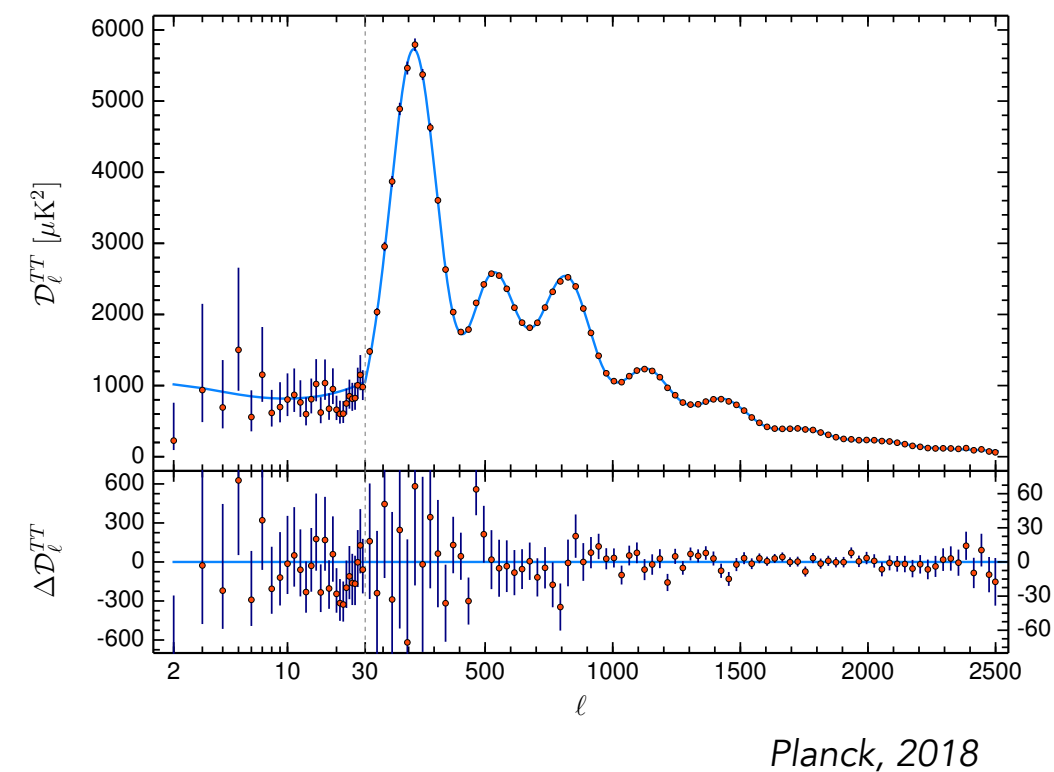
Relaxation in a Fuzzy Dark Matter Halo

Jean-Baptiste Fouvry, IAS

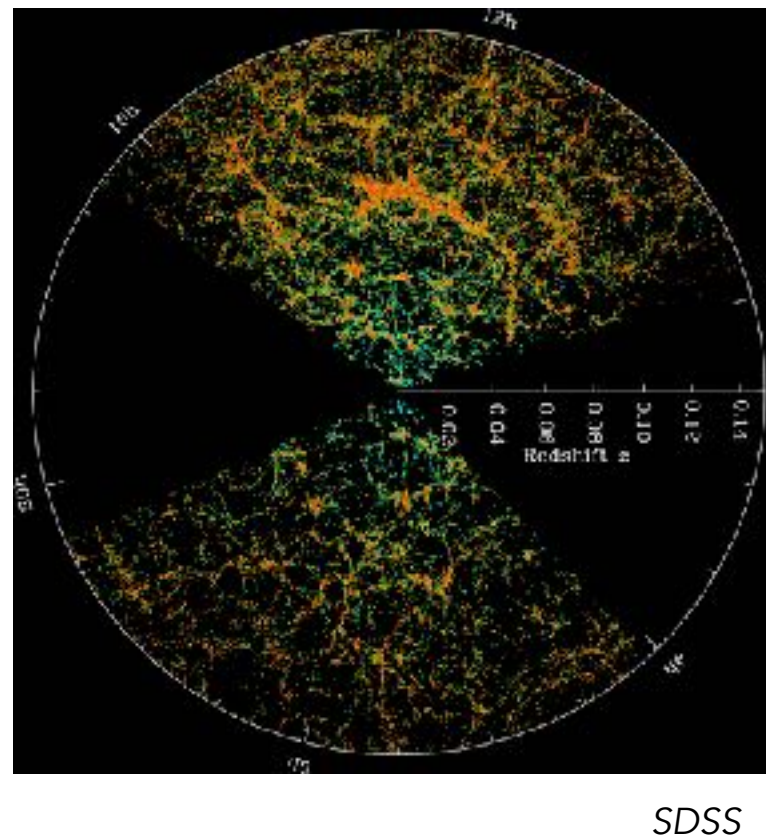
Montpellier
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In collaboration with B. Bar-Or, S. Tremaine

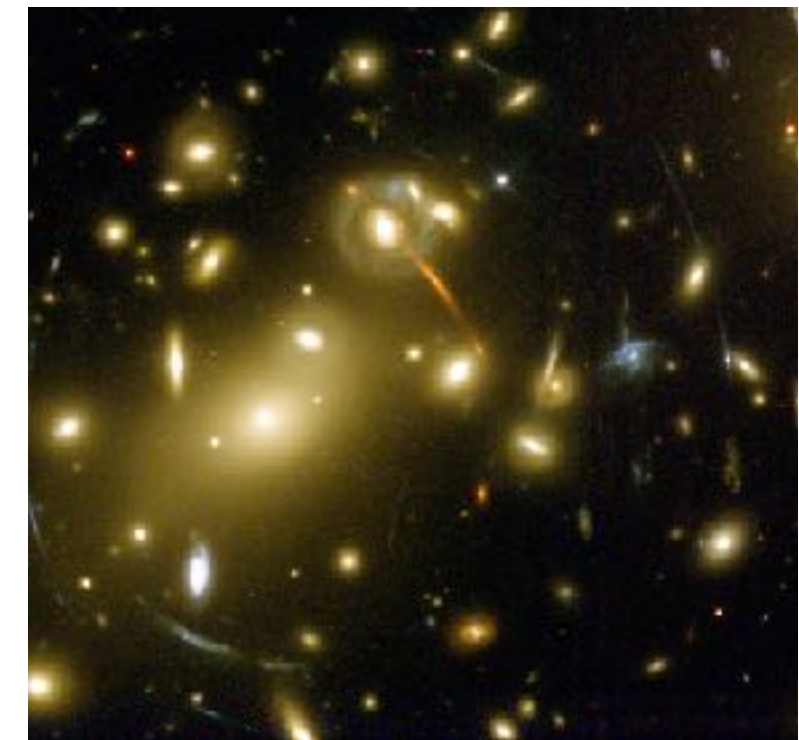
Standard Λ CDM works extremely well on cosmological scales



CMB Power spectrum

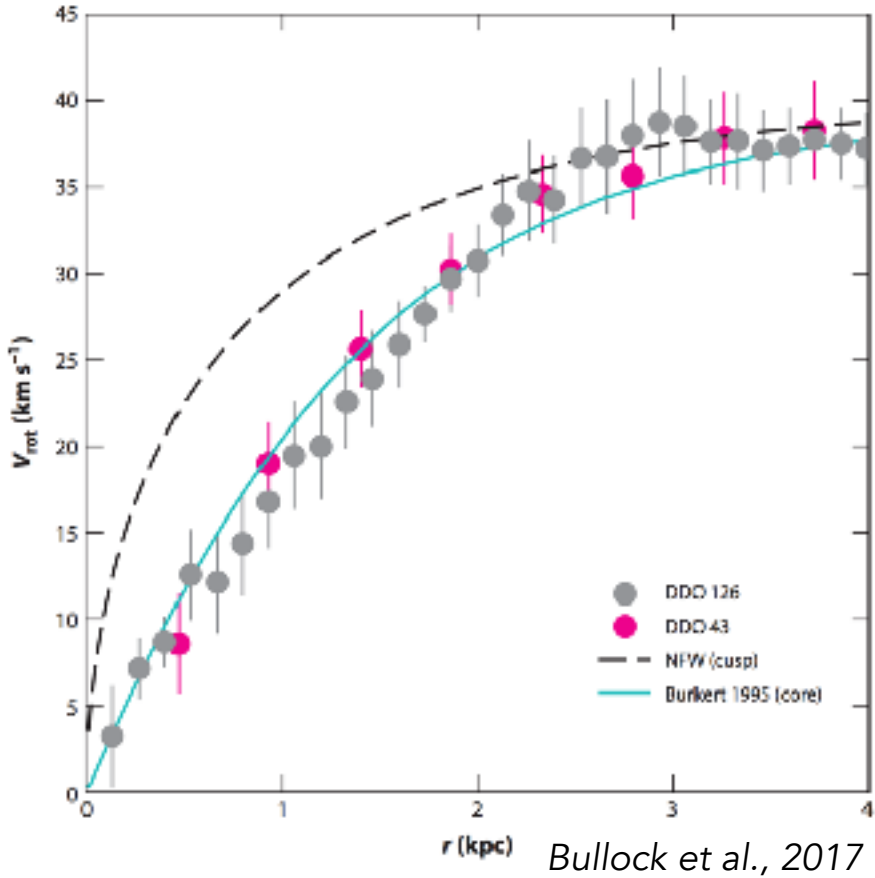
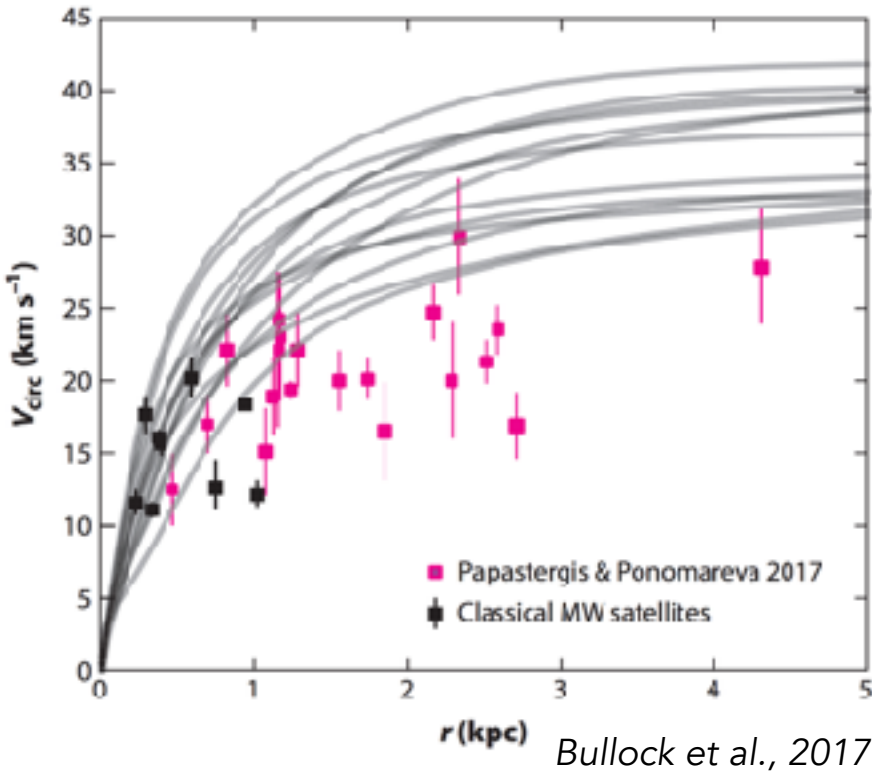
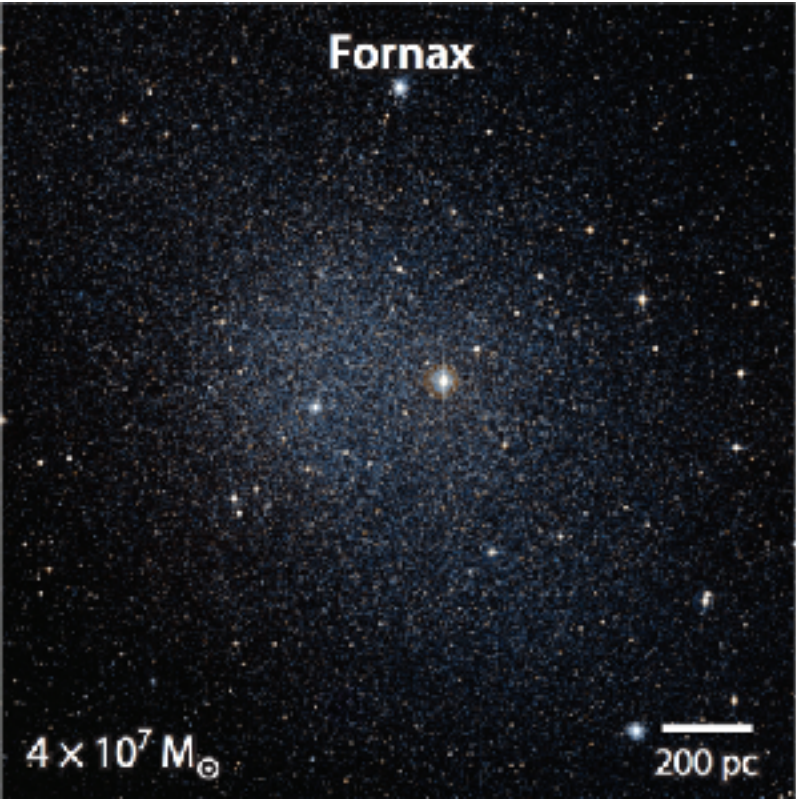


Large-scale structures



Gravitational lensing

But, Λ CDM has almost
no predictive power on small scales



Missing satellites problem

Too-big-to-fail problem

Cusp-Core problem

There are only tens of known MW satellites compared with the thousands expected DM subhalos

There are not enough massive satellites ($\sim 10^6 M_\odot$) and DM halo of ($\sim 10^{10} M_\odot$), that are too massive to have failed to form stars

Central regions of DM-dominated galaxies are less dense, less cuspy than predicted by Λ CDM

But, Λ CDM has almost no predictive power on small scales

Three possible way outs:

- CDM is the correct model for DM, but we do not understand **baryonic physics** (star formation, supernova, AGN feedback)
- There is no DM and the law of physics have to be **modified**
- CDM does not describe the behaviour of DM on **small scales** (structure formation is suppressed relative to CDM)

Missing satellites problem

There are only tens of known MW satellites compared with the thousands expected DM subhalos

Too-big-to-fail problem

There are not enough massive satellites ($\sim 10^6 M_\odot$) and DM halo of ($\sim 10^{10} M_\odot$), that are too massive to have failed to form stars

Cusp-Core problem

Central regions of DM-dominated galaxies are less dense, less cuspy than predicted by Λ CDM

Alternative: DM is made of extremely light bosons

Fuzzy Dark Matter (FDM)

Hu et al., 2000; Hui et al., 2017

+ **Light bosons**

$$m_b \sim 10^{-22} - 10^{-21} \text{ eV}$$

+ **de Broglie wavelength**

$$\lambda_{\text{dB}} = \frac{h}{m_b v} \sim 0.6 \text{ kpc} \frac{10^{-22} \text{ eV}}{m_b} \frac{200 \text{ km} \cdot \text{s}^{-1}}{v}$$

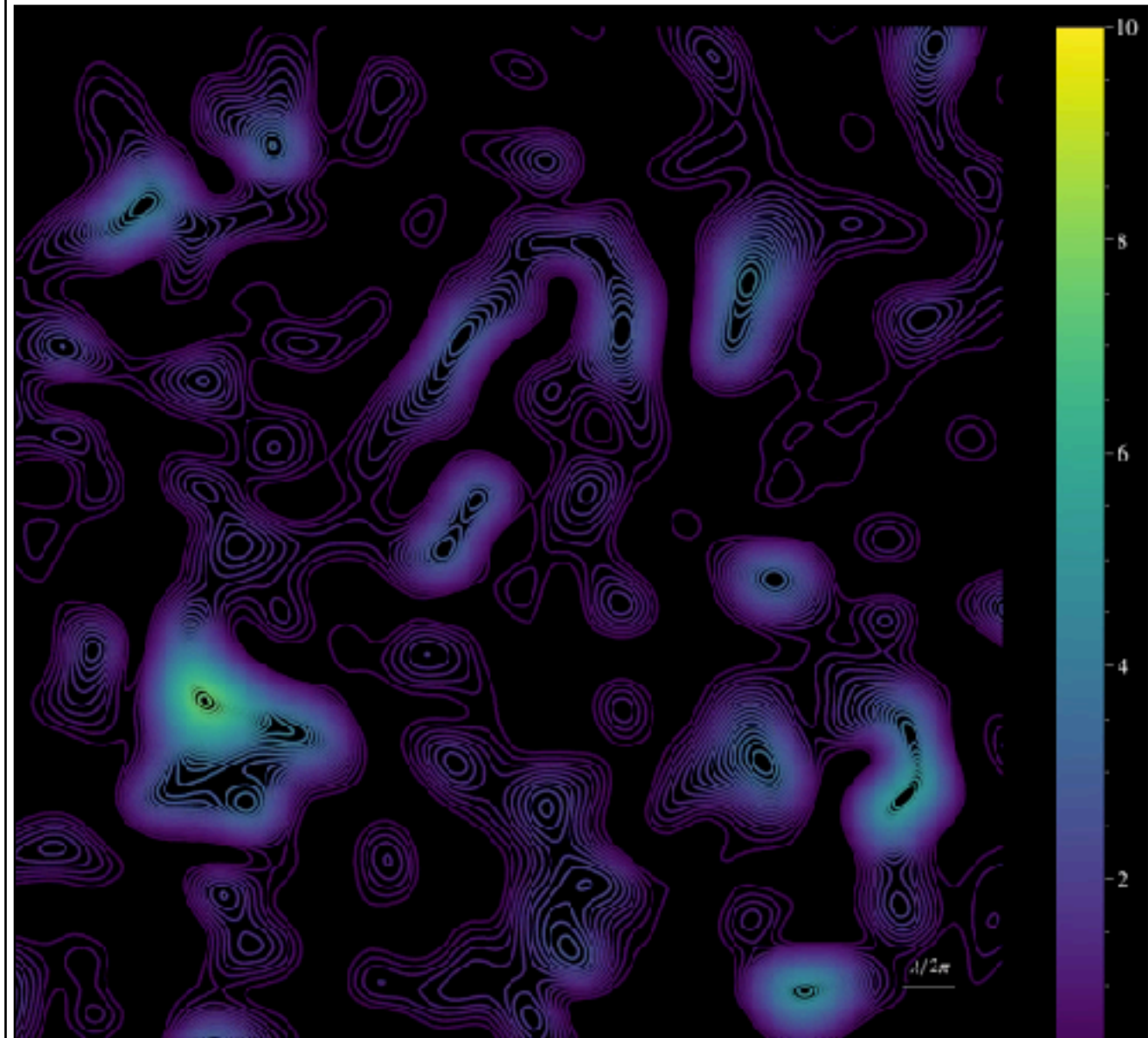
+ **Classical** field ($\rho \lambda_{\text{dB}}^3 \gg m_b$)

governed by **Schrödinger - Poisson**

$$\begin{cases} i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m_b} \nabla^2 \psi + m_b \Phi \psi \\ \nabla^2 \Phi = 4\pi G |\psi|^2 \end{cases}$$

+ **Consequences:**

- **Large scales:** identical to CDM
- **Small scales:** suppressed by Heis. princ.



Kinematic, undamped and eternal fluctuations

The soliton, i.e. Bose-Einstein condensate

+ The **soliton** occupies the halo's centre

$$r_s = 0.22 \text{ kpc} \frac{10^9 M_\odot}{M_s} \left(\frac{m_b}{10^{-22} \text{ eV}} \right)^{-2}$$

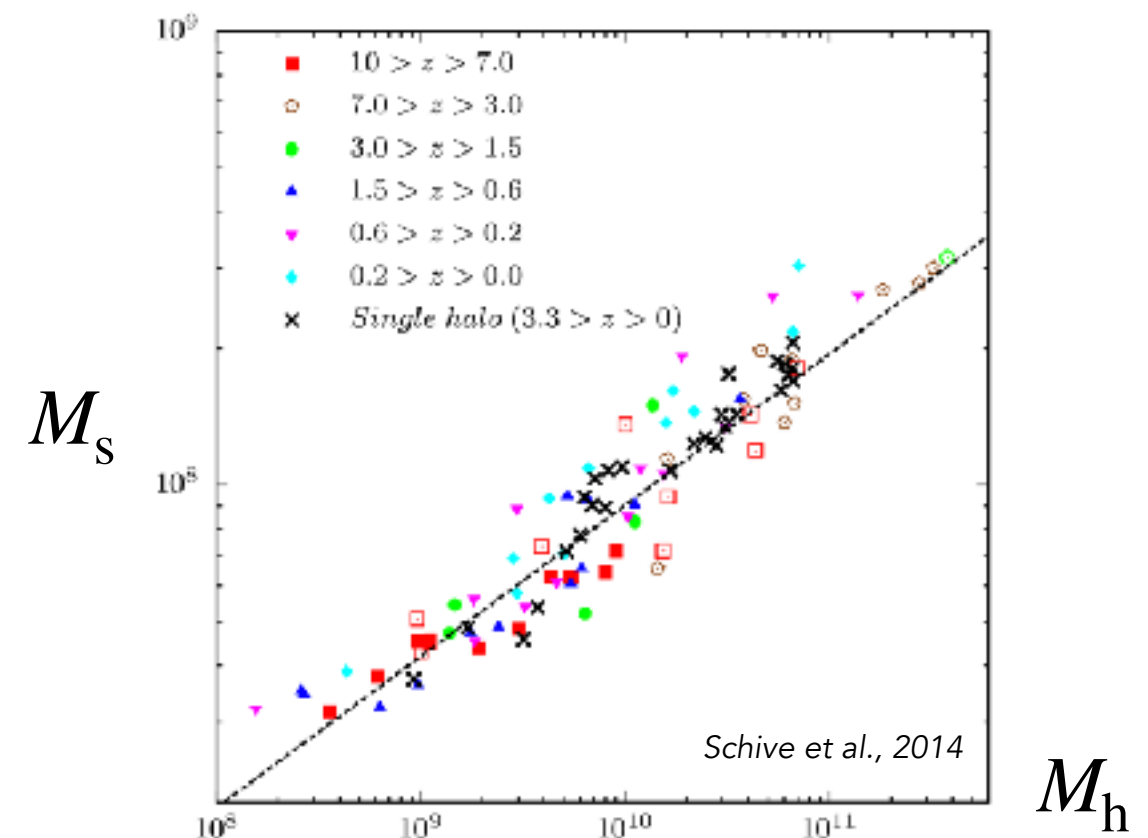
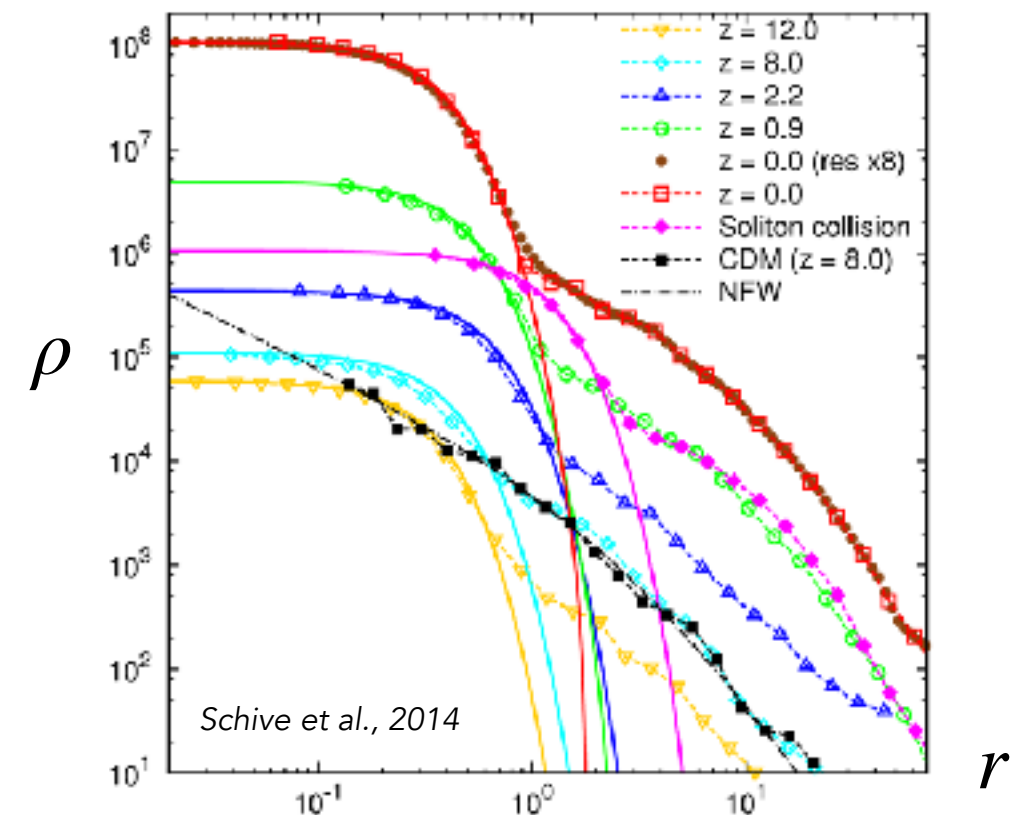
+ The soliton has a **cored profile**

$$\rho = \frac{0.019 M_\odot \text{ pc}^{-3}}{[1 + 0.091(r/r_s)^2]^8} \left(\frac{10^{-22} \text{ eV}}{m_b} \right)^2 \left(\frac{\text{kpc}}{r_s} \right)^4$$

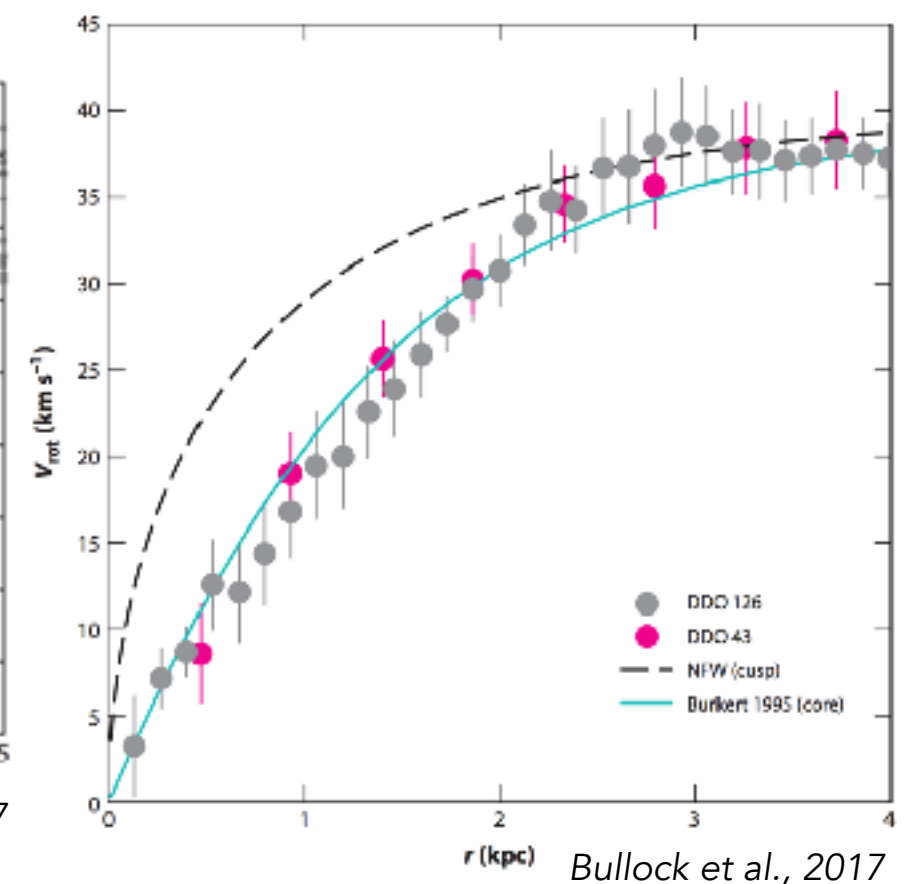
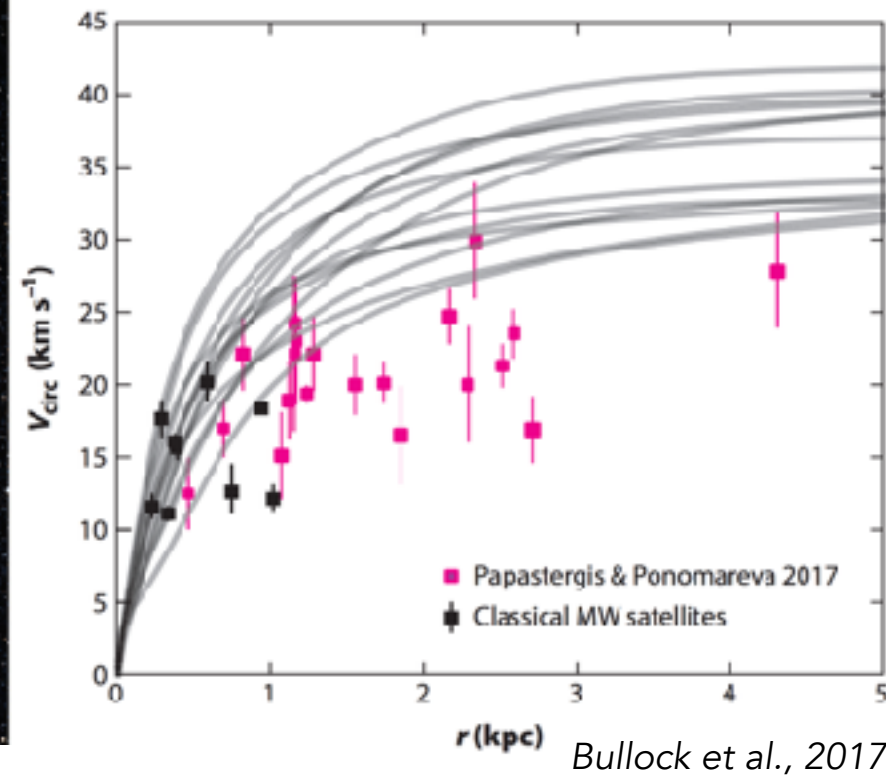
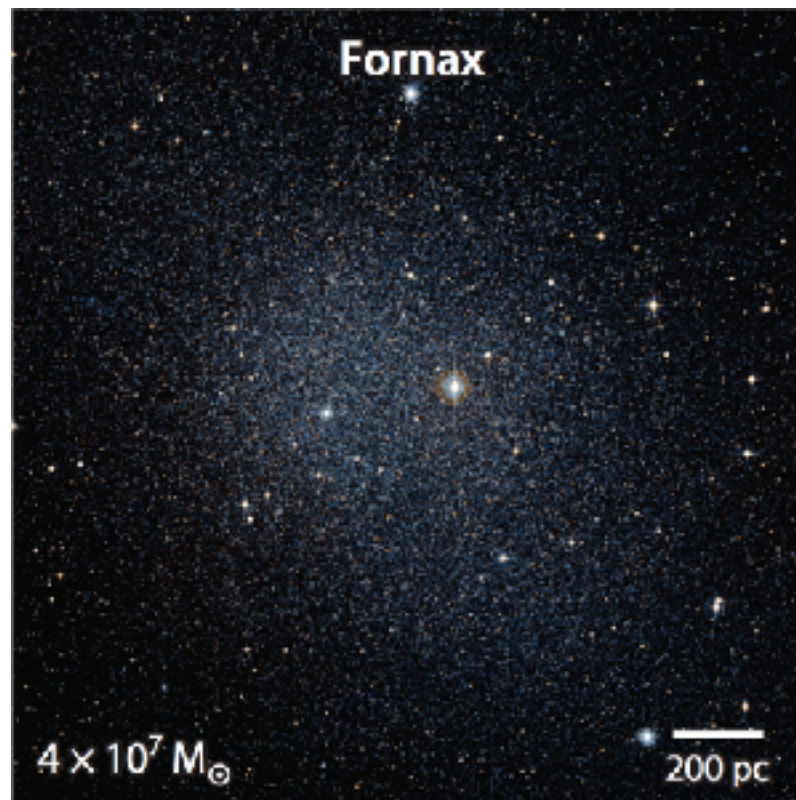
+ Numerical simulations suggest
a universal **soliton halo mass relation**

$$M_s \propto M_{\text{halo}}^{1/3} \quad \text{Schive et al., 2014}$$

+ Isolated CDM halo survives forever,
but isolated FDM halo always
eventually collapses to a soliton.



FDM's tentative answers to the small-scale problems



Missing satellites problem

Small-scale substructures
are washed out
by the **uncertainty principle**

Too-big-to-fail problem

Less massive sub-structures
in the halo.
And, the absence of cusp makes
the disruption easier

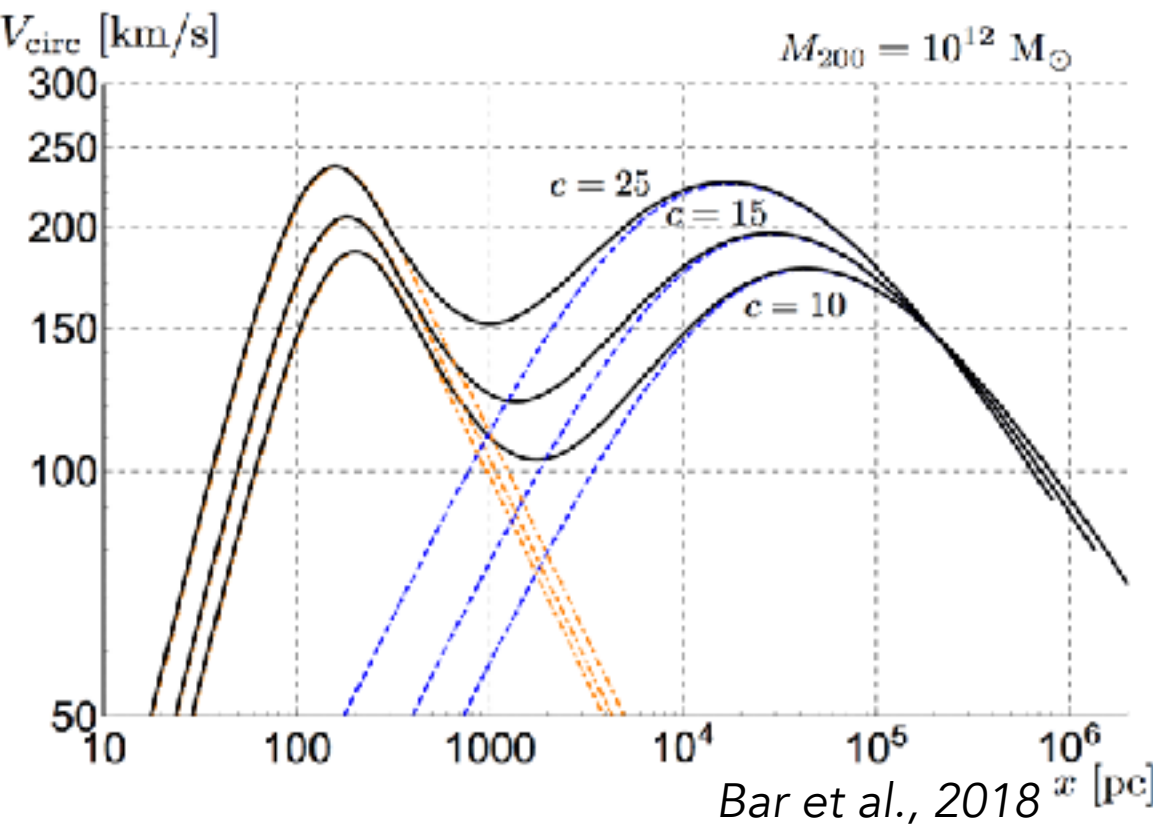
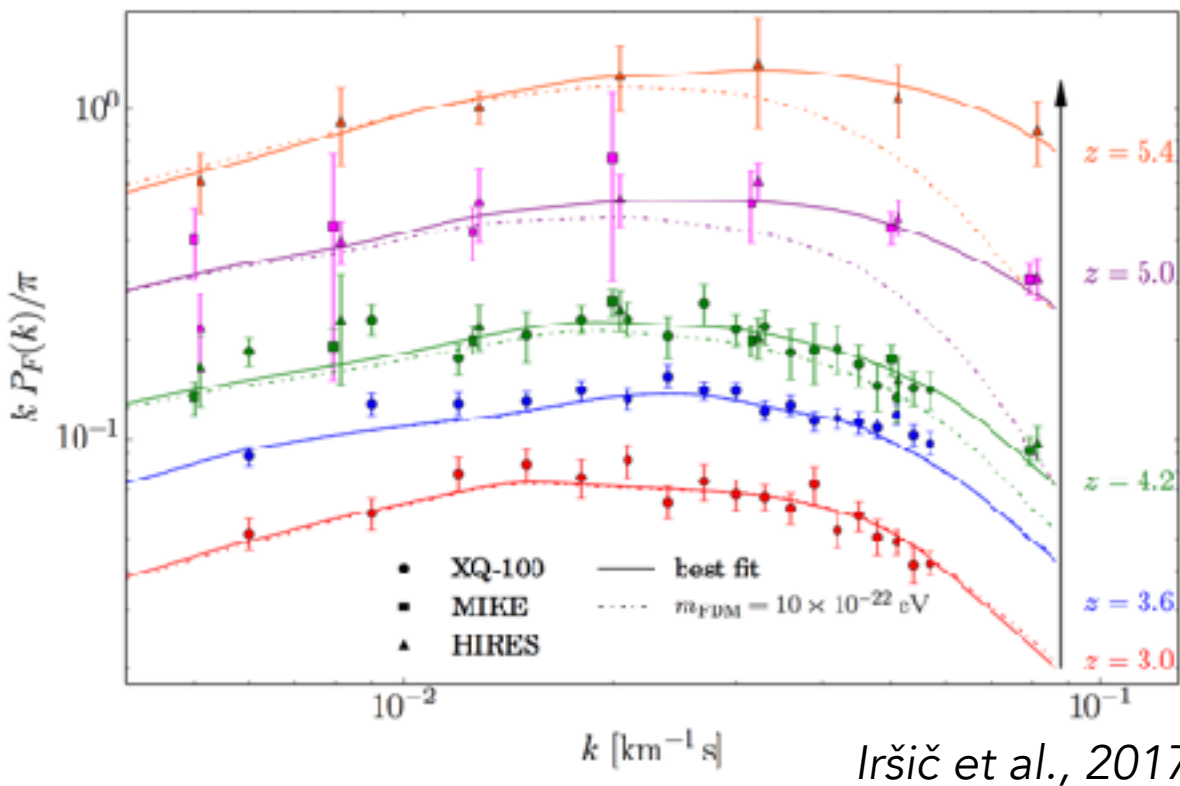
Cusp-Core problem

For low-mass halo, most of the
mass is in the soliton,
which has a **core profile**.

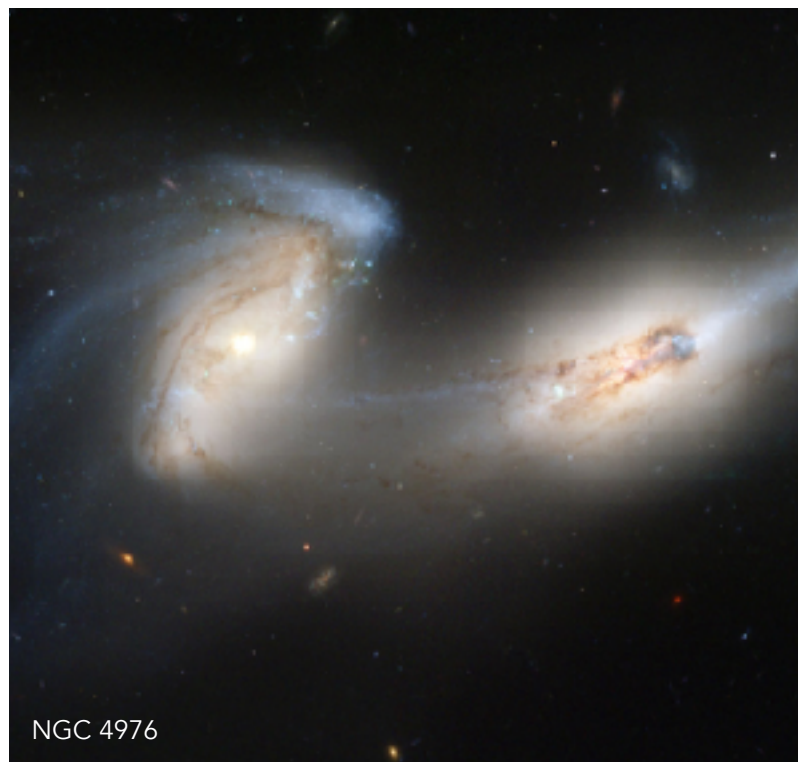
FDM already in trouble?

- + Solving **small-scale CDM problems** imposes a narrow mass range *Hui et al., 2017*
 $10^{-22} \text{ eV} \lesssim m_b \lesssim 10^{-21} \text{ eV}$

Halo densities too small *λ_{dB} too small**Behaves like CDM*
- + **Lyman- α forest observations** give a lower mass constraint *Iršič et al., 2017*
 $m_b \gtrsim 20 \times 10^{-22} \text{ eV}$
- + The soliton - halo mass relation implies a **second peak in the rotation curve**, that is not observed *Bar et al., 2018*

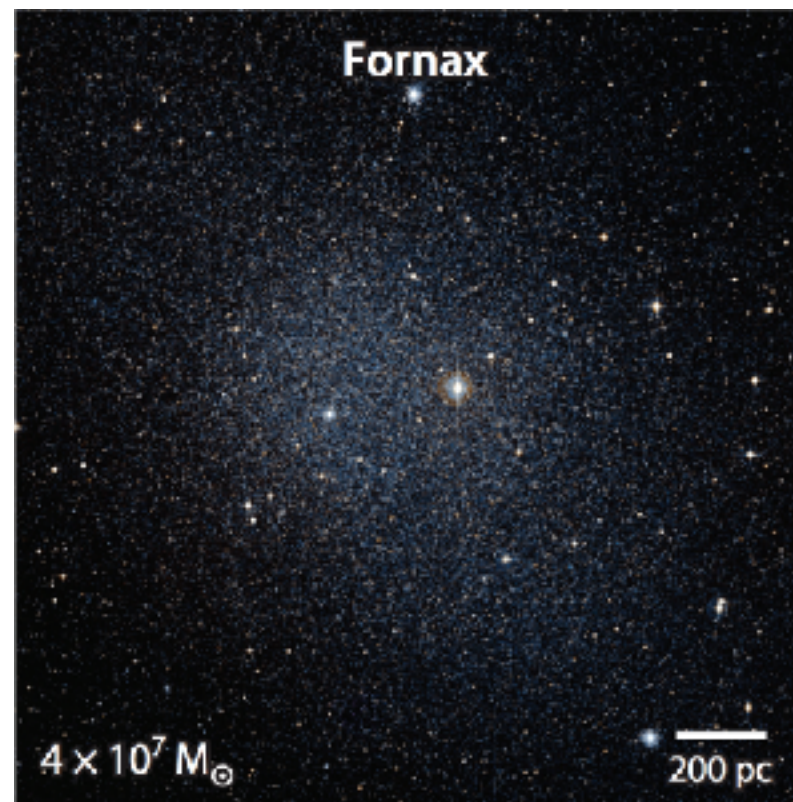


Some other puzzles with FDM



SMBH inspiral

Does FDM lead to a **stalling of the inspiral of SMBH** when they reach equipartition with quasiparticles? (Upcoming constraint from PTA)



Dynamical friction

In CDM, decay times should be < 0.1 Hubble time. Does FDM lead to the **stalling of dynamical friction**?



Bar slowdown

Bar structures should lose angular momentum to the dark halo, but in fact are rapidly rotating. Is FDM responsible for the anomalously slow rates of decay?

Observable signatures of FDM

+ FDM behaves as a collection of “**quasi-particles**” with effective mass

$$m_{\text{eff}} \propto \rho \lambda_{\text{dB}}^3$$

+ FDM is therefore “**collisional**” on small scales, **relaxation processes** can occur

$$T_{\text{relax}} \sim 1 \text{ Gyr} \left(\frac{r}{1 \text{ kpc}} \right)^4 \left(\frac{m_b}{10^{-22} \text{ eV}} \right)^3 \left(\frac{v}{200 \text{ km} \cdot \text{s}^{-1}} \right)^2$$

+ Possible signatures:

- **Inspiral of massive objects**

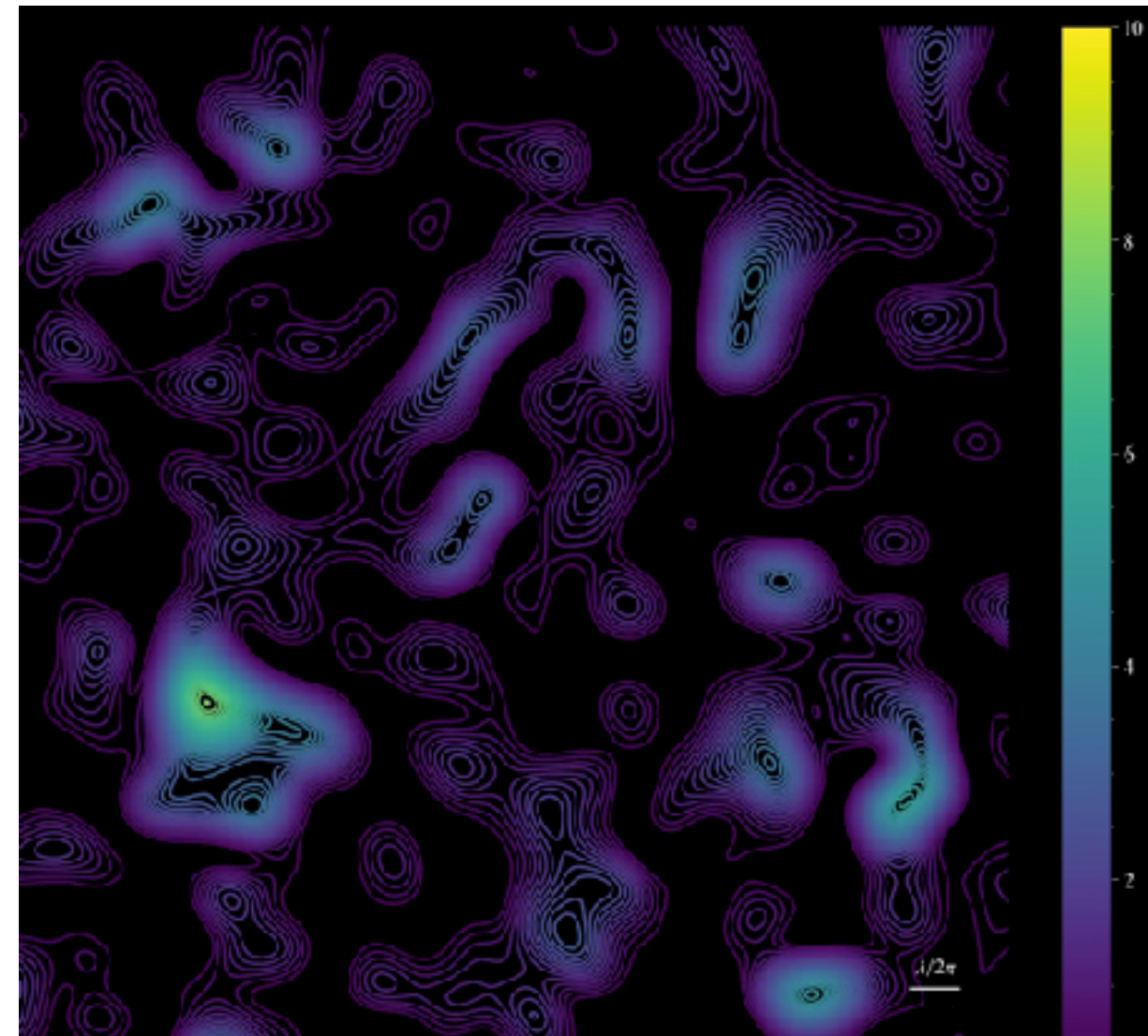
Hui et al., 2017

- Heating of galactic **stellar streams**

Amorisco & Loeb, 2018

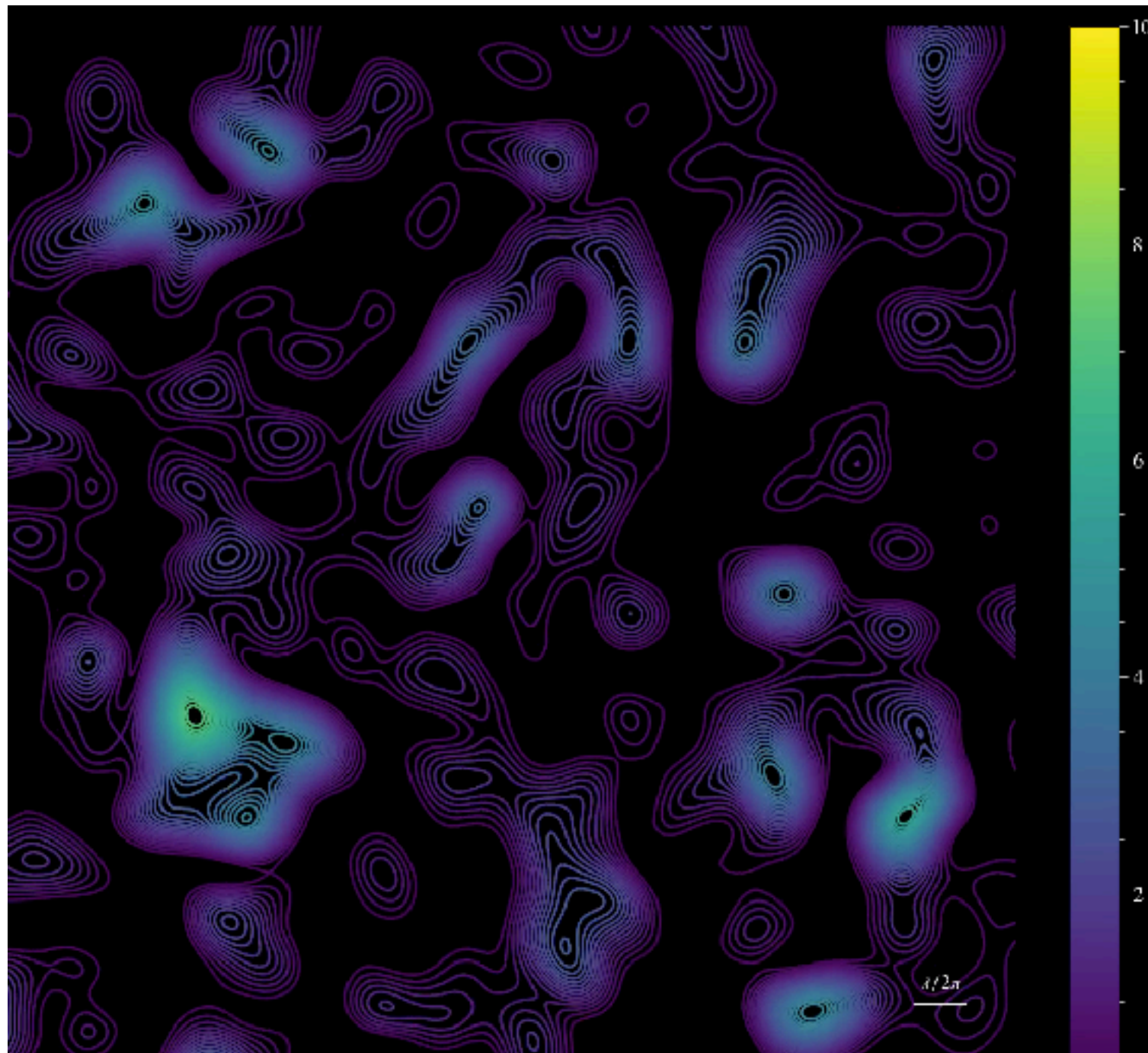
- Heating of galactic **stellar discs**

Church et al., 2019



How to describe relaxation in a FDM halo?

Relaxation processes



How does a massive object's **decay** in a FDM halo?

How does a population of (light) particles **heat up** in a FDM halo?

The classical (Chandrasekhar) case

+ Classical approximations

- Infinite and **homogenous** background
- **Local** interactions
- **Small** angle deflections

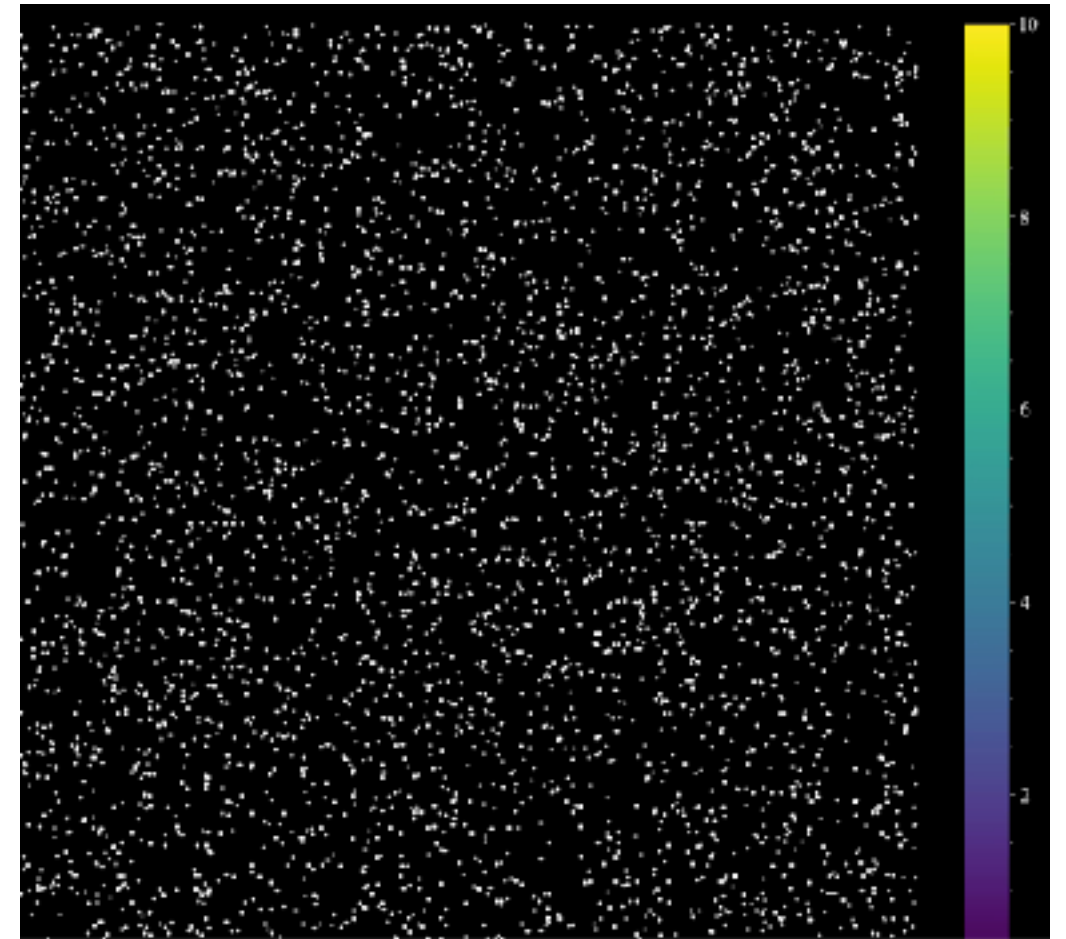
+ Computing the statistics of deflection by solving **perturbatively** the motion

$$D_1(\mathbf{v}) = \langle \Delta \mathbf{v} \rangle \propto G^2 (m_\star + m_b) \rho_b / \sigma_b \log \Lambda$$

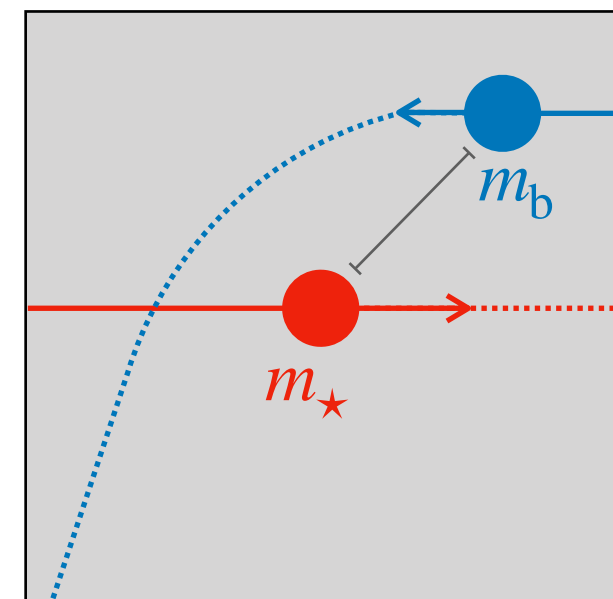
$$D_2(\mathbf{v}) = \langle (\Delta \mathbf{v})^2 \rangle \propto G^2 m_b \rho_b / \sigma_b \log \Lambda$$

+ **Coulomb logarithm**

$$\log \Lambda = \log(R_{\max}/R_{\min})$$



A classical homogeneous system



Relaxation by two-body encounters

Correlations source diffusion/heating

+ **Diffusion** is sourced by the correlation of the **potential fluctuations** *Binney&Lacey, 1988*

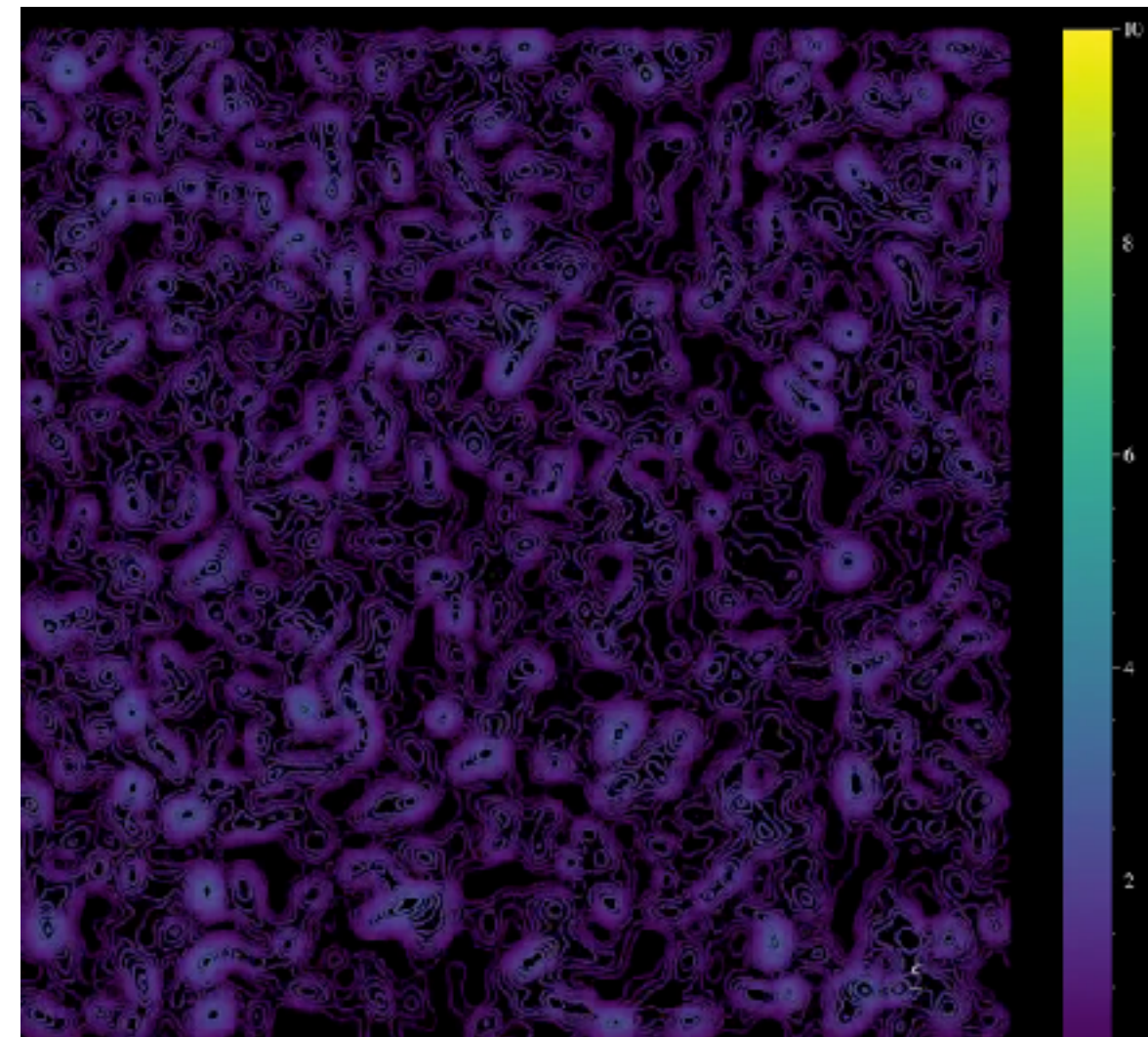
$$\langle \delta\Phi(\mathbf{r}, t) \delta\Phi(\mathbf{r}', t') \rangle = C_\Phi(\mathbf{r} - \mathbf{r}', t - t')$$

+ Diffusion coefficient for **heating**

$$D_2(\mathbf{v}) = \langle (\Delta\mathbf{v})^2 \rangle = \int d\mathbf{k} \, \mathbf{k}^2 \hat{C}_\Phi(\mathbf{k}, \mathbf{k} \cdot \mathbf{v})$$

+ Remains only to compute the correlation of **point particles on straight lines**

$$\rho(\mathbf{r}, t) = m_b \sum_{i=1}^{N_b} \delta_D(\mathbf{r} - \mathbf{r}_i - \mathbf{v}_i t) - \rho_0$$



Density fluctuations in a classical system

Linear response sources dynamical friction

+ **Friction** is sourced by the linear response to a **test perturber** *Tremaine&Weinberg, 1984*

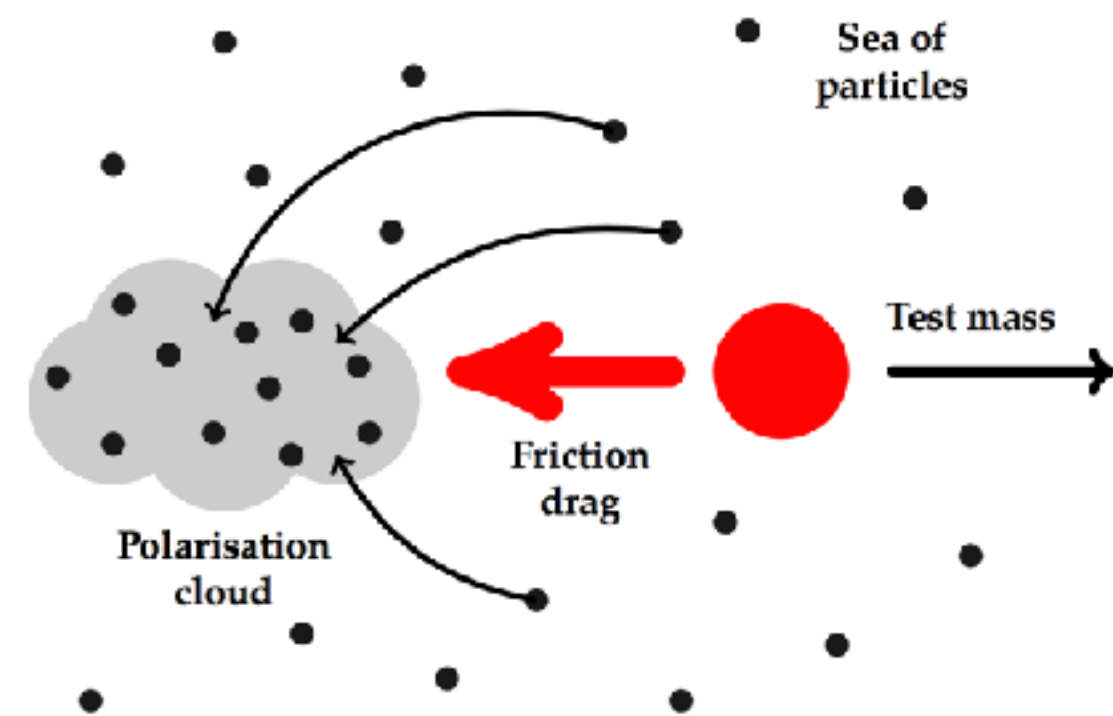
$$\frac{\partial \delta F_b}{\partial t} + [\delta F_b, \Phi_0] + [F_b, \delta \Phi_\star] = 0$$

+ Can be accelerated/damped by **polarisation**

$$\frac{\partial \delta F_b}{\partial t} + [\delta F_b, \Phi_0] + [F_b, \delta \Phi_\star + \delta \Phi_b] = 0$$

+ Remains only to compute the **backreaction** of the background's response on the test's orbit

- Effect proportional to m_\star



Fokker-Planck equation(s)

From the Master Equation

$$\frac{\partial P_{\star}(\mathbf{v}, t)}{\partial t} = - \frac{\partial}{\partial \mathbf{v}} \left[D_1(\mathbf{v}) P_{\star}(\mathbf{v}, t) \right] + \frac{1}{2} \frac{\partial}{\partial \mathbf{v}^2} \left[D_2(\mathbf{v}) P_{\star}(\mathbf{v}, t) \right]$$

First- and second-order diffusion coefficients

$$D_1(\mathbf{v}) = \langle \Delta \mathbf{v} \rangle \quad D_2(\mathbf{v}) = \langle (\Delta \mathbf{v})^2 \rangle \quad d\mathbf{v} = D_1(\mathbf{v}) dt + \eta(t) \sqrt{D_2(\mathbf{v})} d\bar{t}$$

From Kinetic Theory

$$\frac{\partial P_{\star}(\mathbf{v}, t)}{\partial t} = - \frac{\partial}{\partial \mathbf{v}} \left[D_{\text{fric}}(\mathbf{v}) P_{\star}(\mathbf{v}, t) \right] + \frac{1}{2} \frac{\partial}{\partial \mathbf{v}} \left[D_2(\mathbf{v}) \frac{\partial P_{\star}(\mathbf{v}, t)}{\partial \mathbf{v}} \right]$$

Dynamical Friction / Friction force by polarisation

$$D_{\text{fric}}(\mathbf{v}) = D_1(\mathbf{v}) - \frac{1}{2} \frac{\partial D_2(\mathbf{v})}{\partial \mathbf{v}} \quad \begin{cases} D_{\text{fric}}(\mathbf{v}) & \propto m_{\star} \dots \text{Test mass} \\ D_2(\mathbf{v}) & \propto m_{\text{b}} \dots \text{Bath mass} \end{cases}$$

Fokker-Planck equation(s)

+ In CDM, there is no diffusion, but **dynamical friction** can be important.

+ In FDM, dynamical friction is suppressed by “**softening**”

$$\Lambda_{\text{FDM}} = R/\lambda_{\text{dB}} \ll \Lambda_{\text{CDM}} = R/b_{90} \text{ with } b_{90} \simeq GM/\sigma^2$$

From Kinetic Theory

$$\frac{\partial P_{\star}(\mathbf{v}, t)}{\partial t} = - \frac{\partial}{\partial \mathbf{v}} \left[D_{\text{fric}}(\mathbf{v}) P_{\star}(\mathbf{v}, t) \right] + \frac{1}{2} \frac{\partial}{\partial \mathbf{v}} \left[D_2(\mathbf{v}) \frac{\partial P_{\star}(\mathbf{v}, t)}{\partial \mathbf{v}} \right]$$

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Stellar heating in CDM

+ Star moves in a **stochastic** fluctuating density described by the two-point correlation function

$$\langle \rho(\mathbf{r}, t) \rho(\mathbf{r}', t') \rangle = C_\rho(\mathbf{r} - \mathbf{r}', t - t')$$

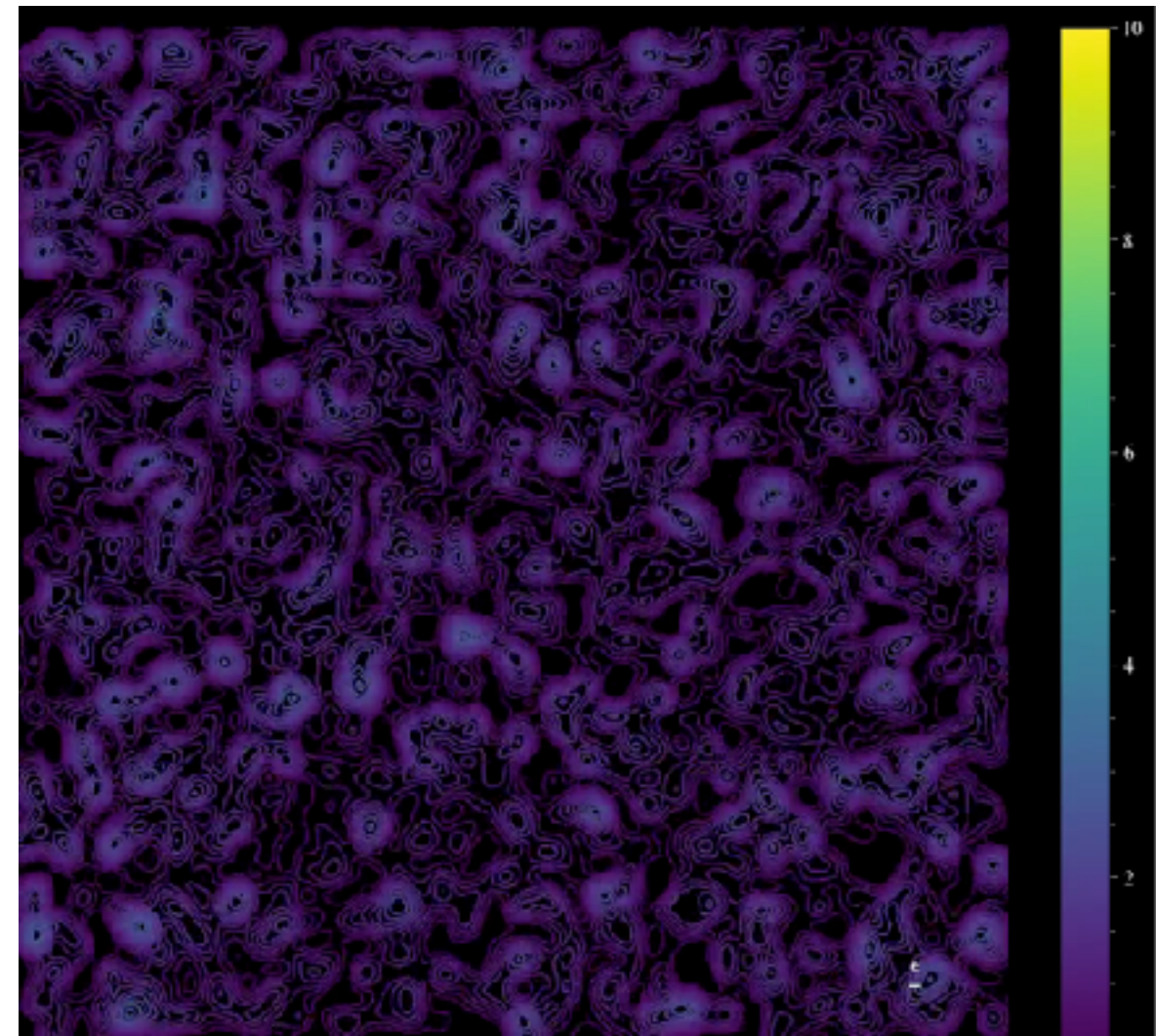
$$\hat{C}_\rho(\mathbf{k}, \omega) = \int d\mathbf{v} \delta_D(\mathbf{k} \cdot \mathbf{v} - \omega) F_b(\mathbf{v})$$

+ Diffusion coefficients only depend on the **power spectrum** of the fluctuations

$$D_2(\mathbf{v}) = G^2 \int \frac{d\mathbf{k}}{k^2} \hat{C}_\rho(\mathbf{k}, \mathbf{k} \cdot \mathbf{v})$$

+ Diffusion coefficients depend **linearly** on the background distribution function

$$D_2(\mathbf{v}) = G^2 \log \Lambda m_b \frac{\partial^2}{\partial \mathbf{v}^2} \int d\mathbf{v}' |\mathbf{v} - \mathbf{v}'| F_b(\mathbf{v}')$$



Density fluctuations in a classical system

Stellar heating in FDM

+ Star evolves in the FDM wave function, that is a sum of **plane waves**

$$\psi(\mathbf{r}, t) = \int d\mathbf{k} \varphi(\mathbf{k}) e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$$

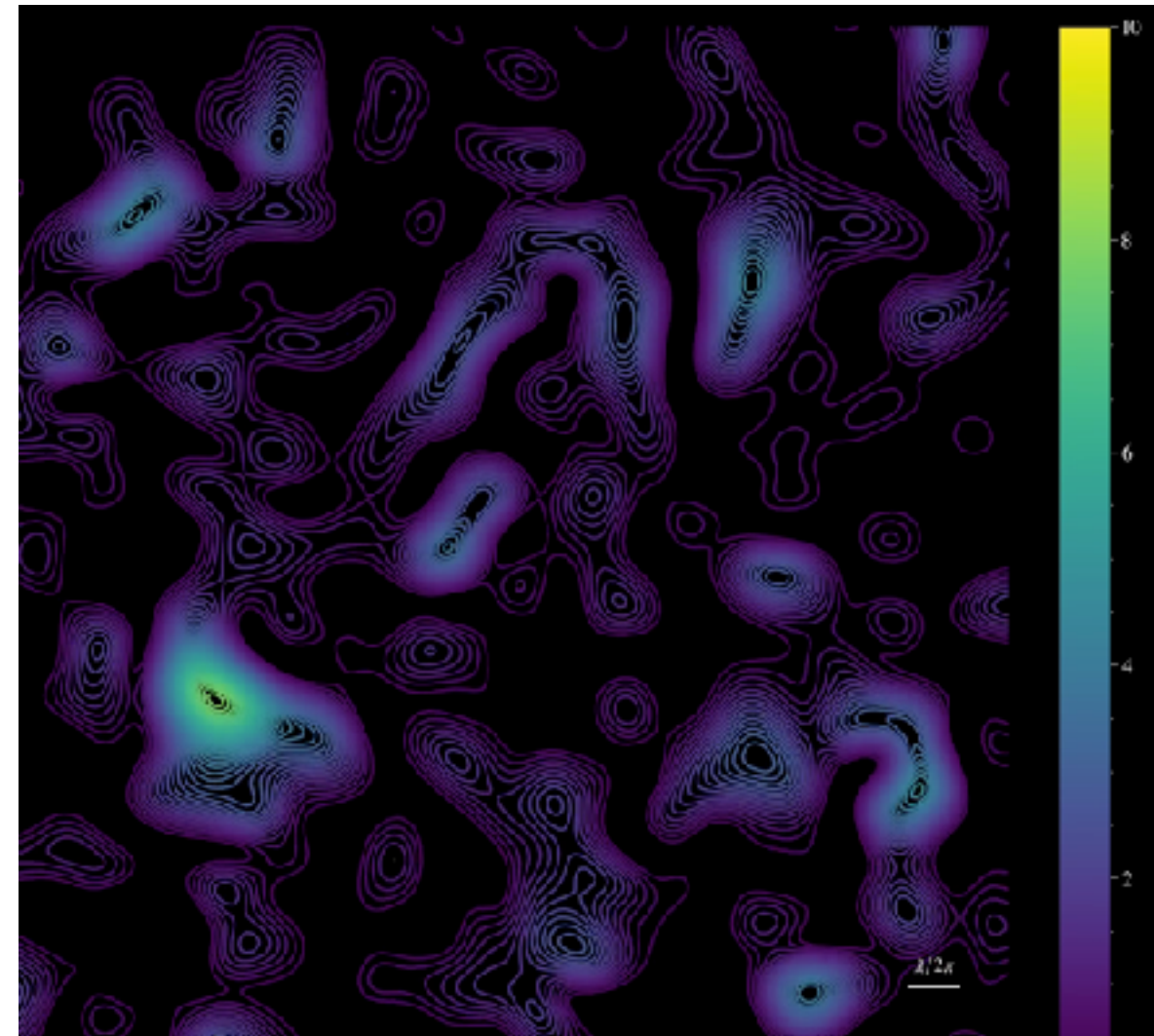
+ Fluctuations characterised by

$$\langle \varphi(\mathbf{k}) \varphi^*(\mathbf{k}') \rangle = F_{\mathbf{k}}(\mathbf{k}) \delta_{\mathbf{D}}(\mathbf{k} - \mathbf{k}')$$

with $\mathbf{v} = \hbar \mathbf{k} / m_b$ and $F_b(\mathbf{v}) d\mathbf{v} = F_{\mathbf{k}}(\mathbf{k}) d\mathbf{k}$

+ Diffusion coefficients depend **quadratically** on the FDM distribution function

$$D_2(\mathbf{v}) = G^2 \log \Lambda_{\text{FDM}} \frac{\hbar^3}{m_b^3} \frac{\partial^2}{\partial \mathbf{v}^2} \int d\mathbf{v}' |\mathbf{v} - \mathbf{v}'| F_b^2(\mathbf{v}')$$



Density fluctuations in a FDM halo

A simple reinterpretation

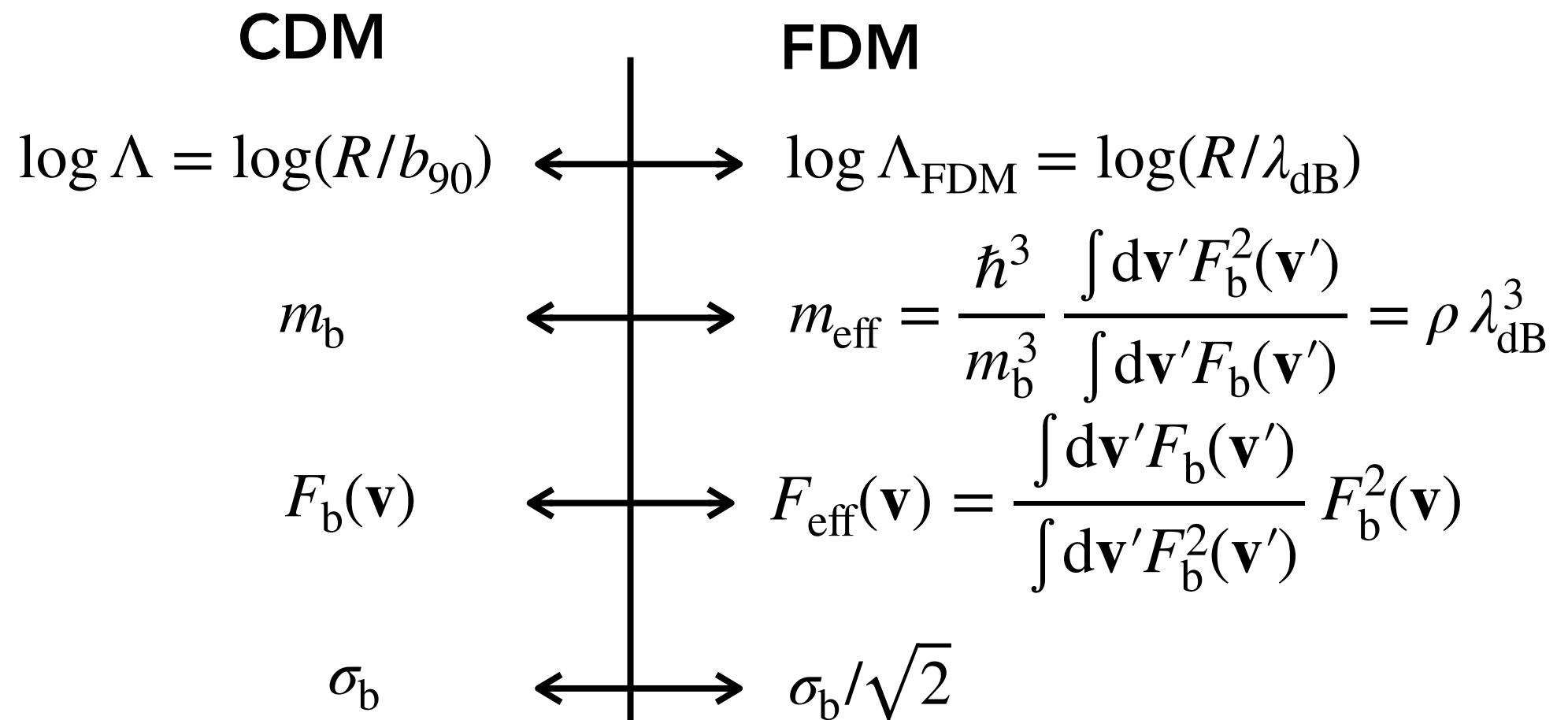
We can mimic FDM diffusion with the classical formalism

CDM

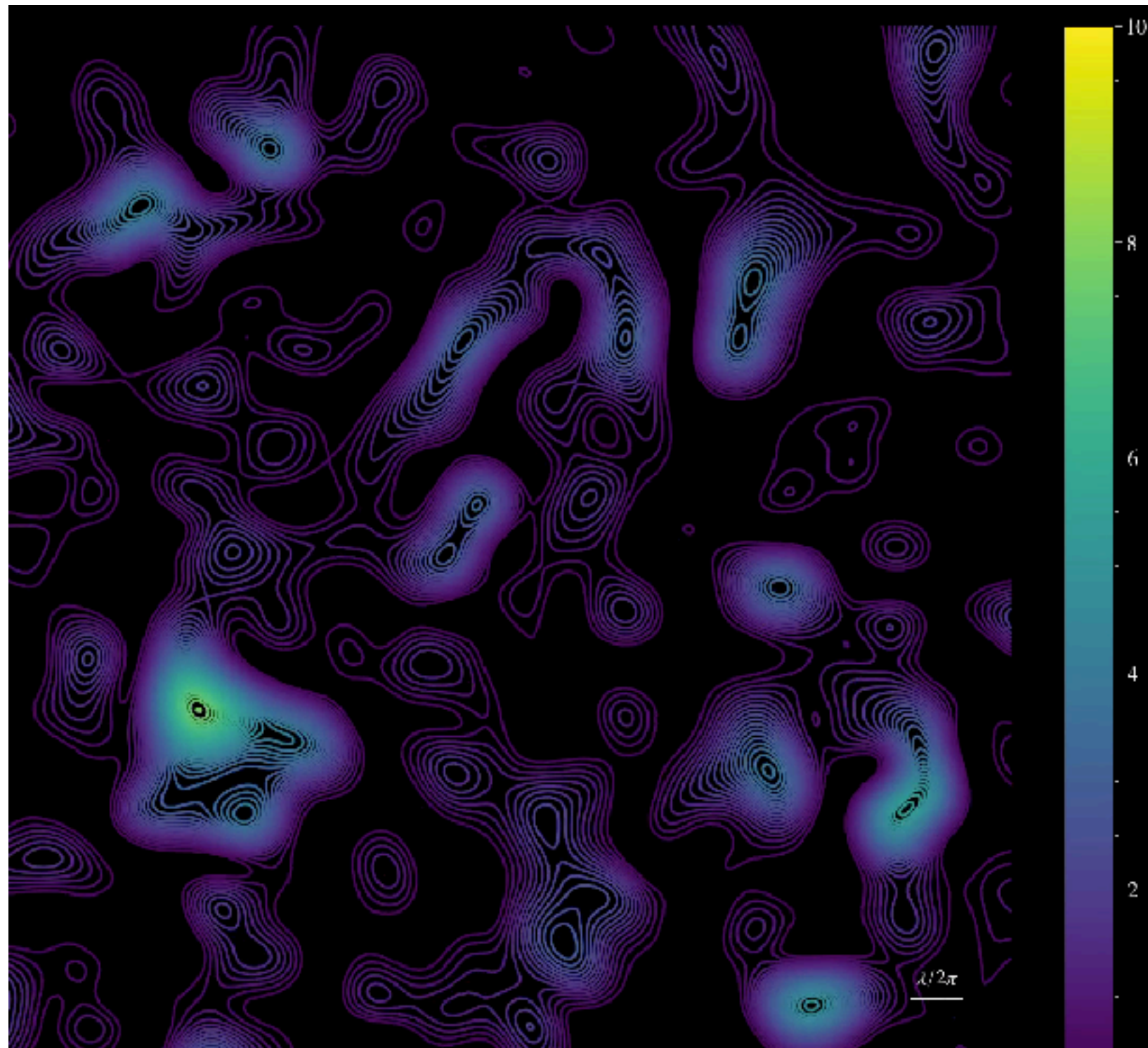
$$D_2(\mathbf{v}) = G^2 \log \Lambda m_b \frac{\partial^2}{\partial \mathbf{v}^2} \int d\mathbf{v}' |\mathbf{v} - \mathbf{v}'| F_b(\mathbf{v}')$$

FDM

$$D_2(\mathbf{v}) = G^2 \log \Lambda_{\text{FDM}} \frac{\hbar^3}{m_b^3} \frac{\partial^2}{\partial \mathbf{v}^2} \int d\mathbf{v}' |\mathbf{v} - \mathbf{v}'| F_b^2(\mathbf{v}')$$

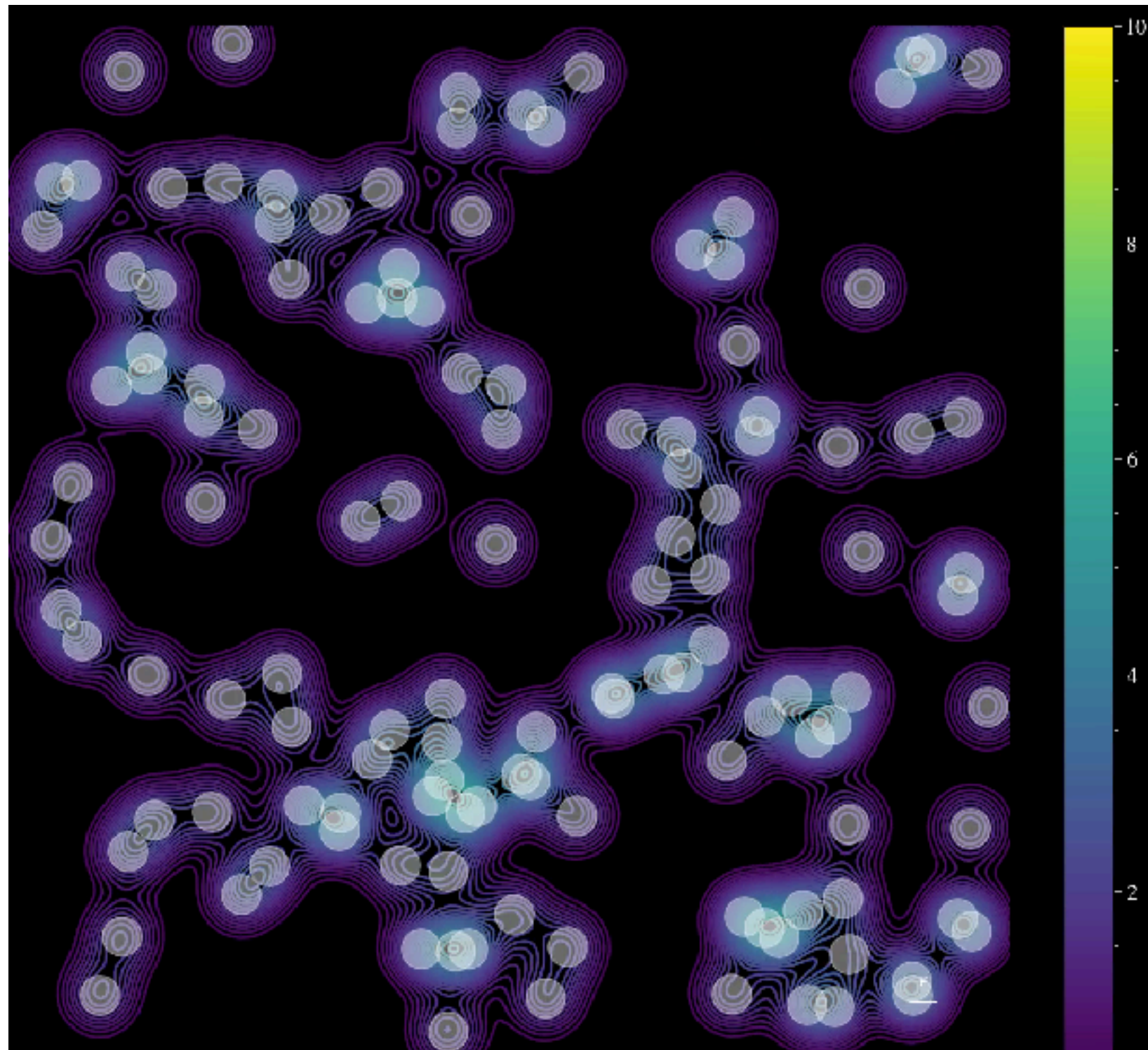


An illustration of the effective model



Density fluctuations in a **FDM** halo

An illustration of the effective model



Density fluctuations in a **classical system** with (soft) particles

Relaxation in FDM - General framework

+ Joining together **diffusion** and **friction** gives the (classical) **Landau equation**

$$\frac{\partial F_{\star}(\mathbf{v})}{\partial t} = G^2 \log \Lambda \frac{\partial}{\partial \mathbf{v}} \left[\int d\mathbf{v}' K(\mathbf{v} - \mathbf{v}') \left\{ m_b F_b(\mathbf{v}') \frac{\partial F_{\star}(\mathbf{v})}{\partial \mathbf{v}} - m_{\star} F_{\star}(\mathbf{v}) \frac{\partial F_b(\mathbf{v}')}{\partial \mathbf{v}'} \right\} \right]$$

Collision kernel

Diffusion

Friction

+ Star embedded in a **FDM halo**

$$\frac{\partial F_{\star}(\mathbf{v})}{\partial t} = G^2 \log \Lambda \frac{\partial}{\partial \mathbf{v}} \left[\int d\mathbf{v}' K(\mathbf{v} - \mathbf{v}') \left\{ \left[m_b + \frac{h^3}{m_b^3} F_b(\mathbf{v}') \right] F_b(\mathbf{v}') \frac{\partial F_{\star}(\mathbf{v})}{\partial \mathbf{v}} - m_{\star} F_{\star}(\mathbf{v}) \frac{\partial F_b(\mathbf{v}')}{\partial \mathbf{v}'} \right\} \right]$$

+ **Fuzzy limit**

$$m_b \ll \frac{h^3}{m_b^3} \sim \rho_b \lambda_{\text{dB}}^3 \sim m_{\text{eff}}$$

+ Landau equation for FDM **interacting with itself** *Uehling&Uhlenbeck, 1933; Levkov et al., 2018*

$$\frac{\partial F_b(\mathbf{v})}{\partial t} = G^2 \log \Lambda \frac{\partial}{\partial \mathbf{v}} \left[\int d\mathbf{v}' K(\mathbf{v} - \mathbf{v}') \left\{ \left[m_b + \frac{h^3}{m_b^3} F_b(\mathbf{v}') \right] F_b(\mathbf{v}') \frac{\partial F_b(\mathbf{v})}{\partial \mathbf{v}} - \left[m_b + \frac{h^3}{m_b^3} F_b(\mathbf{v}) \right] F_b(\mathbf{v}) \frac{\partial F_b(\mathbf{v}')}{\partial \mathbf{v}'} \right\} \right]$$

FDM and dynamical friction

+ The **inspiral time** can be shorter than the age of the galaxy

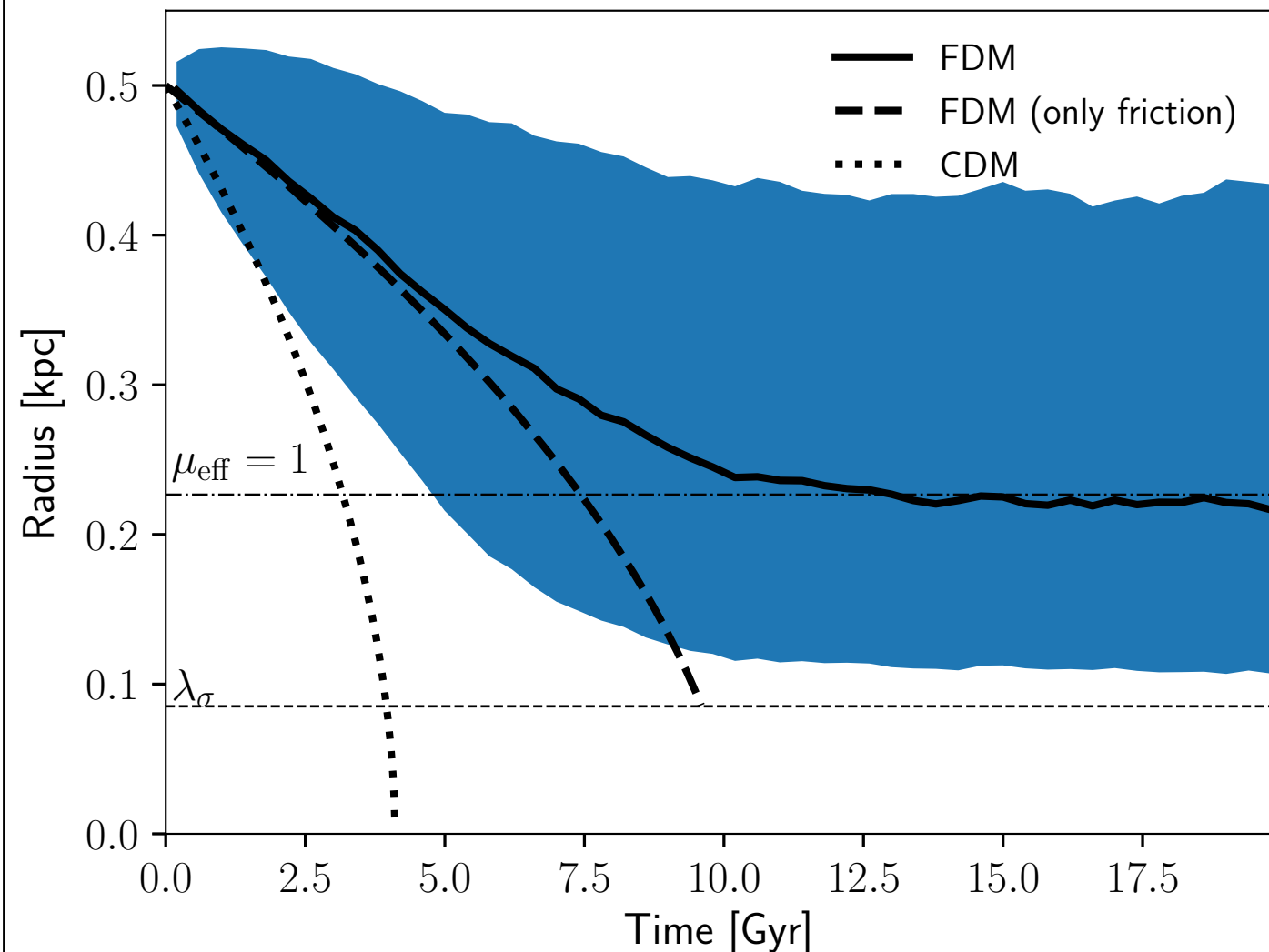
$$T_{\text{inspiral}} = \frac{33 \text{ Gyr}}{\log \Lambda} \frac{4 \times 10^5 M_{\odot}}{m_{\star}} \frac{v_c}{200 \text{ km} \cdot \text{s}^{-1}} \left(\frac{r}{0.5 \text{ kpc}} \right)^2$$

+ Friction in FDM is slower because of **softening**

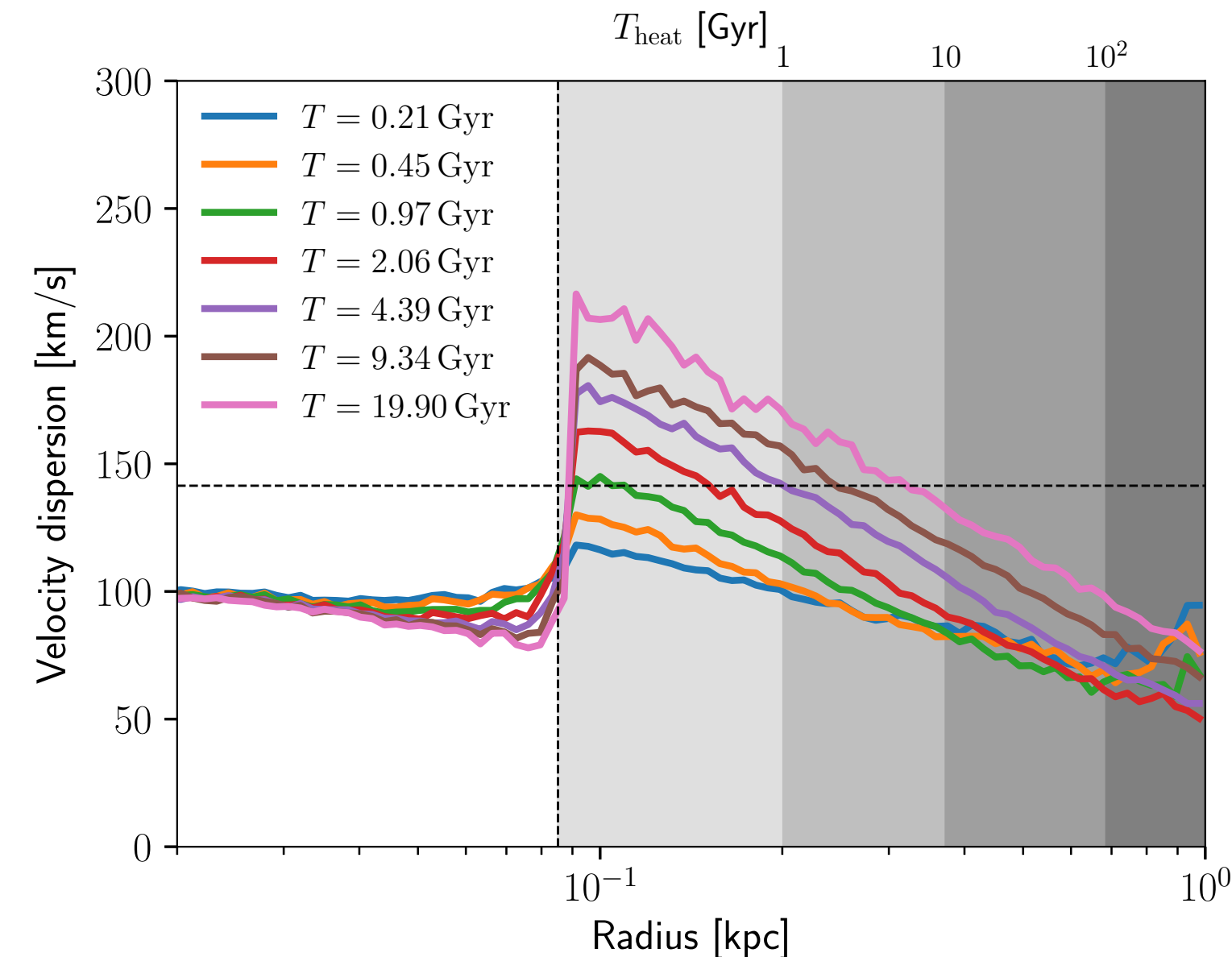
$$\log(R/\lambda_{\text{dB}}) < \log(R/b_{90})$$

+ Moreover, the inspiral will stall when the mass of the object is twice the **effective mass**

$$m_{\text{eff}} = 4 \times 10^4 M_{\odot} \left(\frac{0.5 \text{ kpc}}{r} \right)^2 \left(\frac{10^{-21} \text{ eV}}{m_b} \right)^3 \left(\frac{200 \text{ km} \cdot \text{s}^{-1}}{v_c} \right)$$



Heating the stellar population



+ Stars are **lighter** than the FDM quasi-particles

$$m_{\text{eff}} \gg m_{\star}$$

$$D_2(\propto m_{\text{eff}}) \gg D_{\text{fric}}(\propto m_{\star})$$

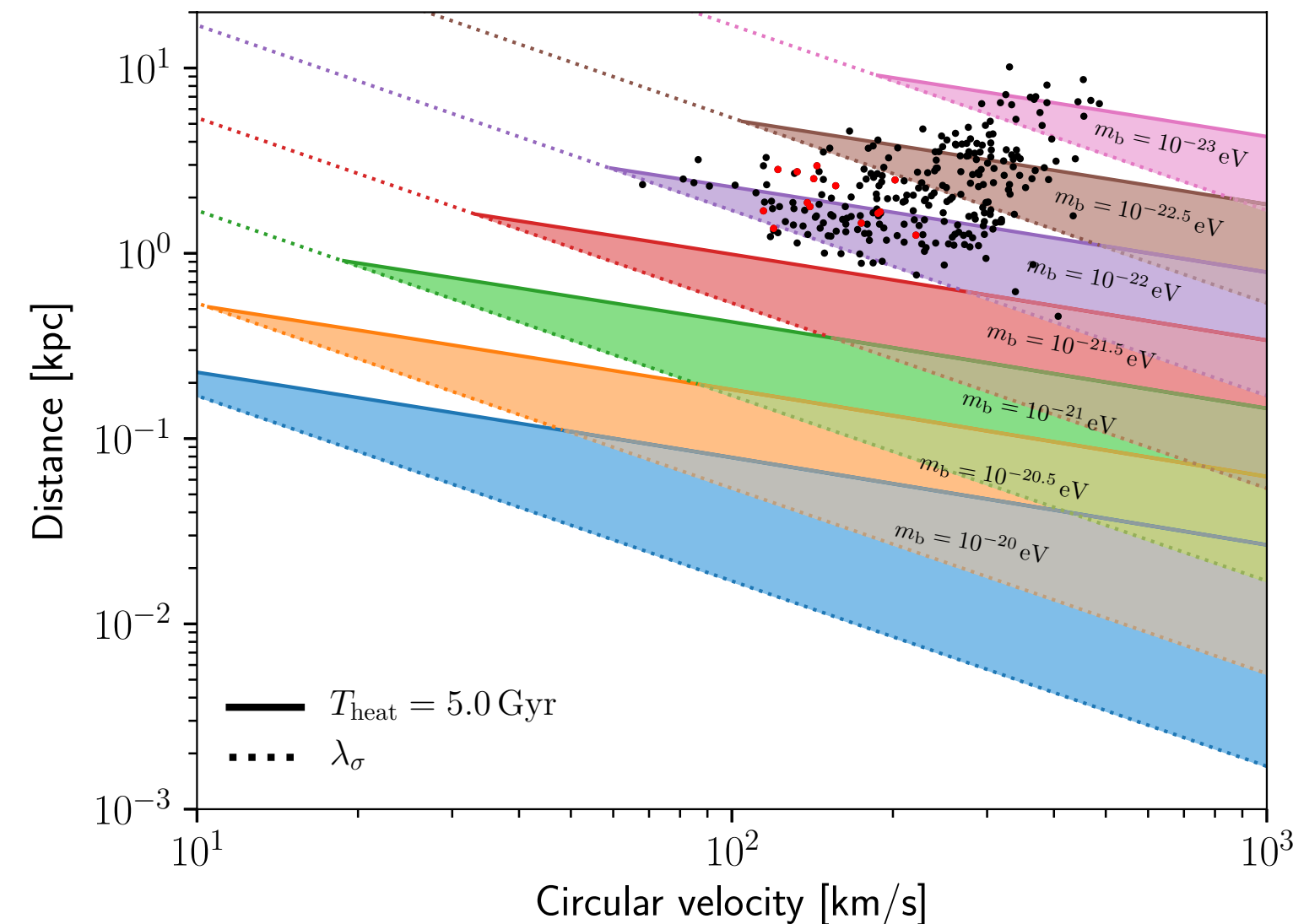
+ **Energy equipartition**

$$m_{\star} v_{\star}^2 = m_{\text{eff}} \sigma_{\text{b}}^2$$

Heating time of the stellar distribution

$$T_{\text{heat}} = \frac{3.2 \text{ Gyr}}{\log \Lambda} \left(\frac{r_{\star}}{0.2 \text{ kpc}} \right)^4 \left(\frac{m_{\text{b}}}{10^{-21} \text{ eV}} \right)^3 \left(\frac{v_{\text{c}}}{200 \text{ km} \cdot \text{s}^{-1}} \right)^2$$

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Heating time of the stellar distribution

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Conclusions

- + Extremely light bosons ("**fuzzy dark matter**") with mass $m_b \sim 10^{-22} \text{ eV}$ are distinct from CDM on galactic scales
- + On galactic scales, FDM can be **collisional**.
- + In the limit of Maxwellian velocity distribution for FDM, the gravitational interaction with FDM is equivalent to the interaction of classical particles with **effective mass** $m_{\text{eff}} \sim \rho_b \lambda_{\text{dB}}^3$
- + Far from the soliton, the effects of FDM on baryons can be described by generalising the standard (Chandrasekhar) **Fokker-Planck equation**:
 - Affects the **dynamical friction** and **inspiral** of (heavy) particles
 - Affects the **heating** of (light) stellar populations
- + Open questions:
 - On what timescales does the soliton form and grow?
 - What is the PDF of the FDM far from the soliton?
 - What is the relation between the halo velocity dispersion and the soliton mass?