Axisymmetric phase-space distribution functions for dark matter halos

Mihael Petač
Supervised by Piero Ullio

Astroparticle physics group
SISSA, Trieste

May 20, 2019

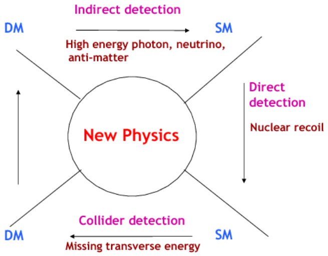
Introduction

Strong evidence for existence of particle-like DM:

- Supported by **independent observations** (e.g. galactic rotation curves, gravitational lensing, CMB)
- Alternative explanations can not account for all the phenomena

Numerous experimental searches for DM particle:

- Collider searches
- Direct detection
- Indirect detection



Current constraints

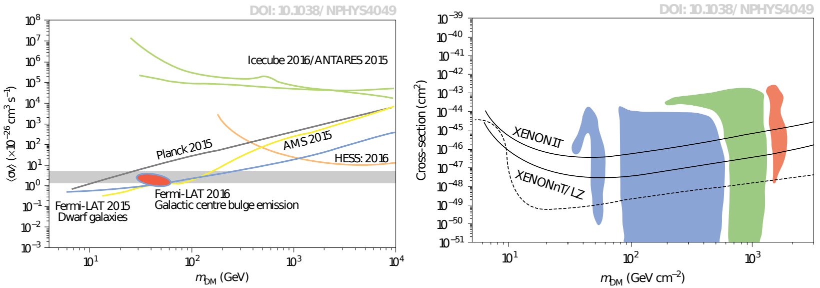


Figure: Indirect detection constraints on annihilation cross-section (left) and direct detection constraints on spin independent DM-nucleon cross-section (right).

Axisymmetric modelling

To accurately interpret **DM searches in spiral galaxies** refined models of DM distribution needed:

- Direct detection rates in Milky Way
- Indirect detection in the galactic center or nearby spirals

Generalize spherical modelling to **include additional features**:

- Presence of baryonic disc
- Halo rotation
- Halo flattening

Even if DM halo spherically symmetric, the **gravitational potential is significantly flattened due to baryonic disc**!

DM phase-space distribution

Aim: Find the solution of Boltzmann equation for the galactic DM

Assumptions:

- Stationary distribution
- Collisionless system
- Axisymmetry

$\Rightarrow$ Solution depends on **two integrals of motion**:

1. Relative energy: $\mathcal{E} = \Psi(r) - \frac{v^2}{2}$
2. Angular momentum around z-axis: $L_z = R \cdot v_\phi$

$\Rightarrow$ Solution can be split in L_z**-even and** L_z**-odd part**:

$$f(\mathcal{E}, L_z) = f_+(\mathcal{E}, |L_z|) + f_-(\mathcal{E}, L_z)$$

Axisymmetric phase-space distributions

Consider collisionless particles with **density** $\rho(R, z)$ in **relative gravitational potential** $\Psi(R, z)$:

1. For monotonic $\Psi(R, z)$ one can rewrite $\rho(R, z) \to \rho(R^2, \Psi)$
2. Phase-space distribution must reproduce the density

$$\begin{aligned} \rho(R^2, \Psi) &= \int \mathrm{d}^3 v \; f_+(\mathcal{E}, |L_z|) \\ &= \frac{4\pi}{R} \int_0^{\Psi} \mathrm{d}\mathcal{E} \int_0^{R\sqrt{2(\Psi-\mathcal{E})}} \mathrm{d}L_z \; f_+(\mathcal{E}, |L_z|) \end{aligned} \tag{1}$$

3. Take derivative with respect to Ψ

$$\frac{\partial \rho}{\partial \Psi} = 2\sqrt{2}\pi \int_0^{\Psi} \frac{\mathrm{d}\mathcal{E}}{\sqrt{\Psi - \mathcal{E}}} \; f_+ \left(\mathcal{E}, 2R^2 \left(\Psi - \mathcal{E}\right)\right) \tag{2}$$

Laplace transformation solution

Lynden-Bell D.: MNRAS123, 447 (1962)

Solve the linear integral equation with difference kernel using **Laplace Transform**:

$$\mathcal{L}\{f(t)\} \equiv \int_0^\infty \mathrm{d}t \; f(t) \; e^{-st} = \tilde{f}(s) \quad , \quad t \in \mathbb{R} \quad \text{and} \quad s \in \mathbb{C}$$

By applying $\mathcal{L}$ to Equation (2) one can show:

$$\tilde{g}(s,u) = \int_0^\infty \mathrm{d}\Psi \; \frac{s^{3/2}}{u^{1/2}} \; \rho\left(\frac{s}{u}, \Psi\right) \, e^{-s\Psi}$$

$$\text{where} \quad g(-\mathcal{E}, |L_z|) = \frac{4\pi}{|L_z|} f_+(\mathcal{E}, |L_z|)$$

Obtain g by using known **inverse Laplace transformations** (analytical solutions exist only for certain $\{\rho, \psi\}$ pairs)

Contour integral solution

Hunter C. & Qian E.: MNRAS262, 401 (1993)

Numerically-friendly implementation using **contour integrals**:

$$f_+(\mathcal{E}, L_z) = \frac{1}{4\pi^2 i\sqrt{2}} \oint_{C(\mathcal{E})} \frac{\mathrm{d}\xi}{\sqrt{\xi - \mathcal{E}}} \left.\frac{\mathrm{d}^2 \rho(R^2, \Psi)}{\mathrm{d}\Psi^2}\right|_{\substack{\Psi=\xi \\ R^2=\frac{L_z^2}{2(\xi-\mathcal{E})}}}$$

$$f_-(\mathcal{E}, L_z) = \frac{\mathrm{sign}(L_z)}{8\pi^2 i} \oint_{C(\mathcal{E})} \frac{\mathrm{d}\xi}{\xi - \mathcal{E}} \left.\frac{\mathrm{d}^2 (\rho\bar{v}_\phi)}{\mathrm{d}\Psi^2}\right|_{\substack{\Psi=\xi \\ R^2=\frac{L_z^2}{2(\xi-\mathcal{E})}}}$$

Restriction of previous integral methods to only **physically relevant domains** in the $(\mathcal{E}, L_z)$ and $\left(R^2, \Psi\right)$ planes.

Analytical continuation of ρ and Ψ to complex plane still needed!

Contour integral solution

Hunter C. & Qian E.: MNRAS262, 401 (1993)

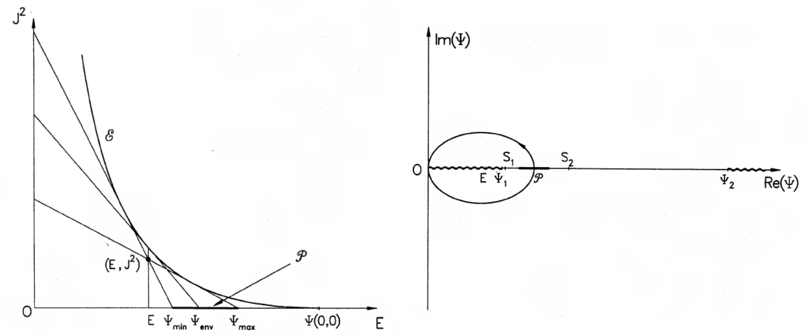


Figure: Allowed region in the $(\mathcal{E}, L_z)$ plane (left) and sketch of the contour in complex Ψ plane (right).

Sample spiral galaxy model

1. **Spherical bulge** (Hernquist potential):

$$\Psi_{\rm bulge}(R^2, z^2) = \frac{GM_b}{\sqrt{R^2 + z^2} + a_b}$$

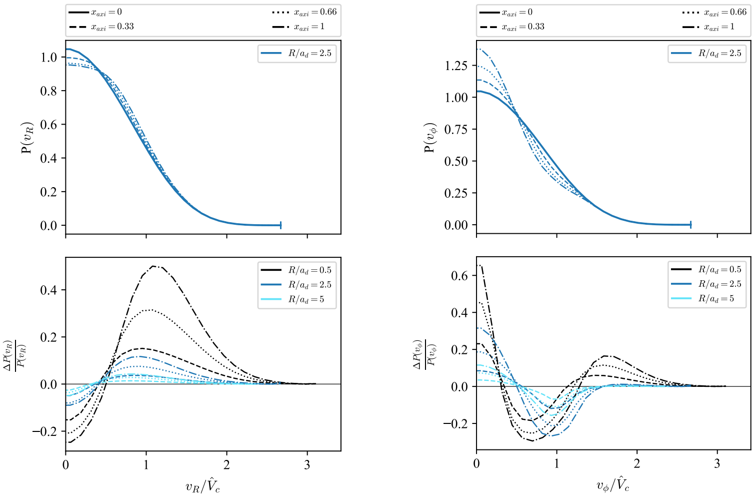
2. **Axisymmetric disc** (Myamoto-Nagai potential):

$$\Psi_{\rm disc}(R^2, z^2) = \frac{GM_d}{\sqrt{R^2 + (a_d + \sqrt{z^2 + b_d^2})^2}}$$

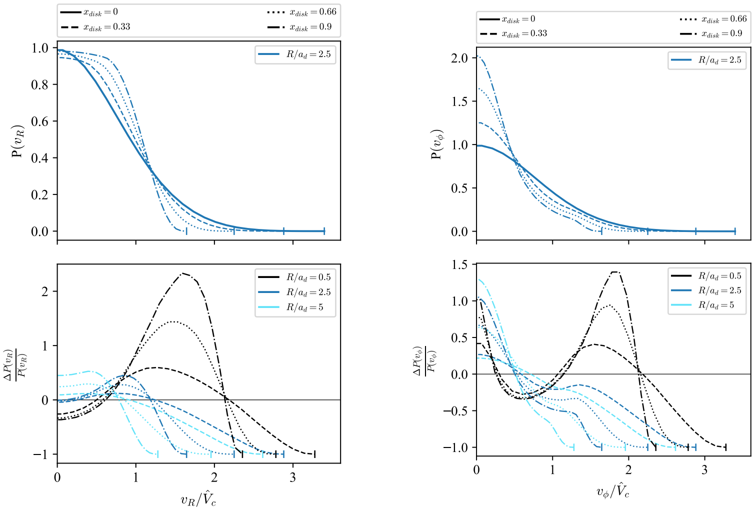
3. **Axisymmetric halo** (spherioidal NFW profile):

$$\rho_{\rm DM}(m) = \frac{\rho_s}{m/r_s \cdot (1 + m/r_s)^2} \quad \text{where} \quad m^2 = R^2 + z^2/q^2$$

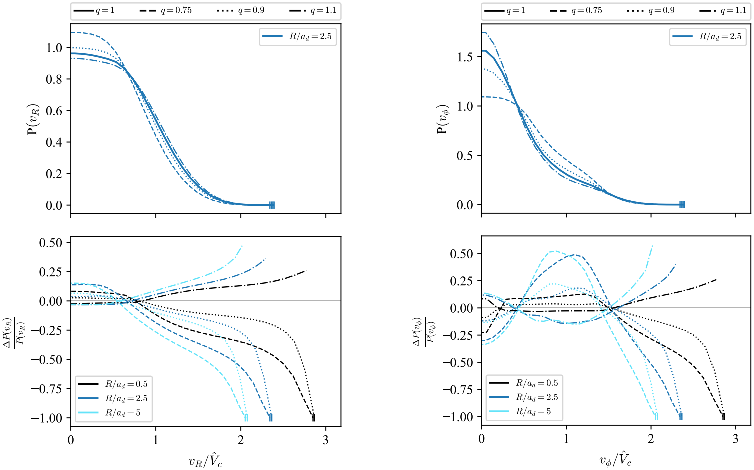
Spherical vs axisymmetric modelling



Disc normalization



Halo flattening



Velocity dispersion and anisotropy

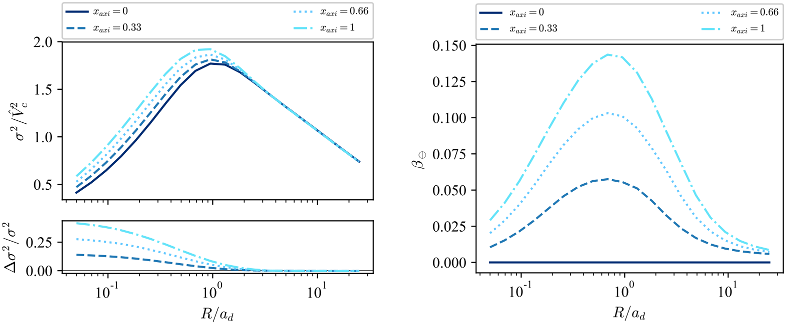


Figure: Velocity dispersion (left) and velocity anisotropy (right) for various admixture of the axisymmetric gravitational potential.

Halo rotation

Highly unconstrained

S. E. Bryan et al.: MNRAS 429, 3316 (2013)

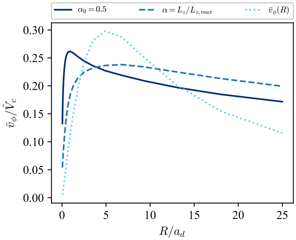
Chua et al.: MNRAS 484, 476 (2019)

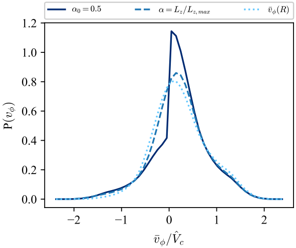
Simulations predict spin parameter:

$$\lambda(r) \equiv \frac{J(r)}{\sqrt{2}r\, M_{\mathrm{DM}}(r)\, V_c(r)} \sim 0.03 - 0.07$$

Consider several choices:

1. $f_{-}(\mathcal{E}, L_z) = \alpha_c \cdot \mathrm{sign}(L_z) \cdot f_{+}(\mathcal{E}, L_z)$
2. $f_{-}(\mathcal{E}, L_z) = \frac{L_z}{|L_{z,max}|} \cdot f_{+}(\mathcal{E}, L_z)$
3. $\bar{v}_\phi(R) = \frac{\omega R}{1+R^2/r_a^2}$





Application: Milky Way model (Bovy2015)

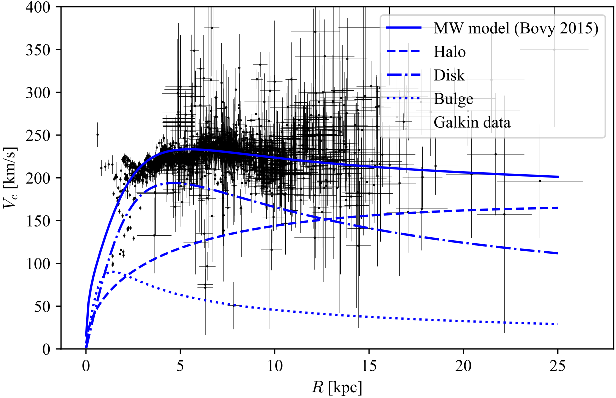


Figure: Comparison of the Milky Way model to the circular velocity data.

Direct detection

Attempts to measure **nuclear recoils** in laboratory targets due to interactions with **galactic DM**

$$\frac{\mathrm{d}R}{\mathrm{d}E_r} = \frac{1}{m_A m_\chi} \cdot \int_{|\vec{v}|>v_{\min}} \mathrm{d}^3 v \ |\vec{v}| \cdot P(\vec{v}) \cdot \frac{\mathrm{d}\sigma}{\mathrm{d}E_r} \quad , \quad v_{\min} = \sqrt{\frac{m_A E_r}{2\mu_{A\chi}^2}}$$

Spin-independent interaction: $\frac{\mathrm{d}\sigma}{\mathrm{d}E_r} = \frac{m_A \sigma_n^{\mathrm{SI}}}{2\mu_{A\chi}^2 v^2} A^2 F^2(E_r)$

For general $\frac{\mathrm{d}\sigma}{\mathrm{d}E_r}$ useful to define:

$$g(v_{\min}) \equiv \frac{1}{\rho_\odot} \int_{|\vec{v}|>v_{\min}} \mathrm{d}^3 v \ \frac{f(\vec{r}_\odot, \vec{v})}{|\vec{v}|}$$

$$h(v_{\min}) \equiv \frac{1}{\rho_\odot} \int_{|\vec{v}|>v_{\min}} \mathrm{d}^3 v \ |\vec{v}| \cdot f(\vec{r}_\odot, \vec{v})$$

Application: Direct detection

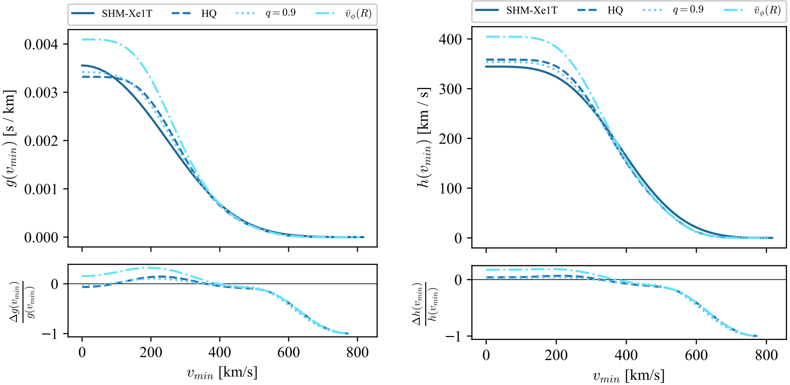


Figure: Dependence of the differential recoil rate on the DM velocity distribution for $\frac{\mathrm{d}\sigma}{\mathrm{d}E_r} \propto v^{-2}$ (left) and $\frac{\mathrm{d}\sigma}{\mathrm{d}E_r} \propto v^{0}$ (right).

Application: Cross-section limits

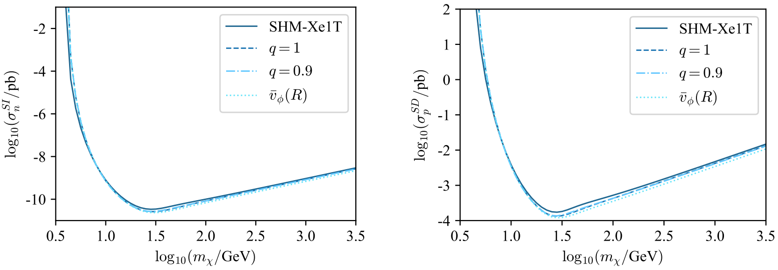


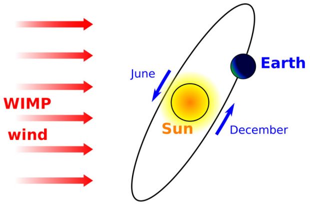
Figure: Cross-section limits for spin-independent (left) and spin-dependent (right) scattering operator.

Application: Annual modulation

Annual modulation of $\frac{\mathrm{d}R}{\mathrm{d}E_r}$ expected due to the **Earth's relative movement** with respect to the galactic rest frame.

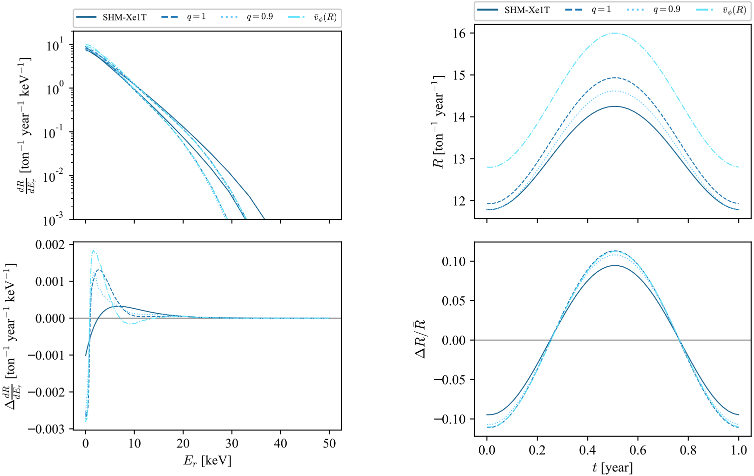
Additional boost ($|v_\oplus| \approx 30$ km/s):

$$\mathcal{E} \to \mathcal{E} - \frac{v_\oplus^2}{2}$$
$$L_z \to L_z + R_\odot(\vec{v}_\oplus \cdot \hat{e}_\phi)$$



Upon detection could provide new information regarding the Milky Way's halo and/or type of DM-nucleon coupling.

Application: Annual modulation



Outlook

Use the HQ method with **more detailed Milky Way model**:

- More flexible DM density profile (e.g. Zhao)
- Detailed baryonic modelling (thin and thick disc, gas, ...)
- Include non-relaxed components (debris flow, ...)

Derive the cross-section limits for **all 16 non-relativistic effective DM-nucleus scattering operators**.

Implications for **angular dependence of direct detection signals**.

Summary

Implementation of **novel method for phase-space reconstruction in axisymmetric systems**.

Study of **additional halo features**:

- Influence of flattened disc potential
- Halo flattening
- Halo rotation

Reinterpretation of **DM-nucleus cross-section limits**.

Effects of DM phase-space modelling on **annual modulation**.

Conclusion

Rapid **improvements in sensitivity of experiments** call for refined modelling of DM distribution within galaxies.

Detailed analysis needed for:

- **Interpretation of tensions** among different experiments (Galactic center excess, DAMA annual modulation signal)
- **Legitimate exclusion of DM models**

Precise understanding of modelling crucial upon eventual detection of DM signals.