γ and r_B... (BaBar example)

analysis with $227\times10^6 B\overline{B}$ **[PRL95, 121802 (2005)]**

$r_B(DK) = 0.12 \pm 0.08 \pm 0.03 \pm 0.04$ $r_B(D^*K) = 0.17 \pm 0.10 \pm 0.03 \pm 0.03$

$$\boldsymbol{\gamma = 70^\circ \pm 31^\circ (stat)^{+12^\circ}_{-10^\circ} (syst)^{+14^\circ}_{-11^\circ} (model)}$$

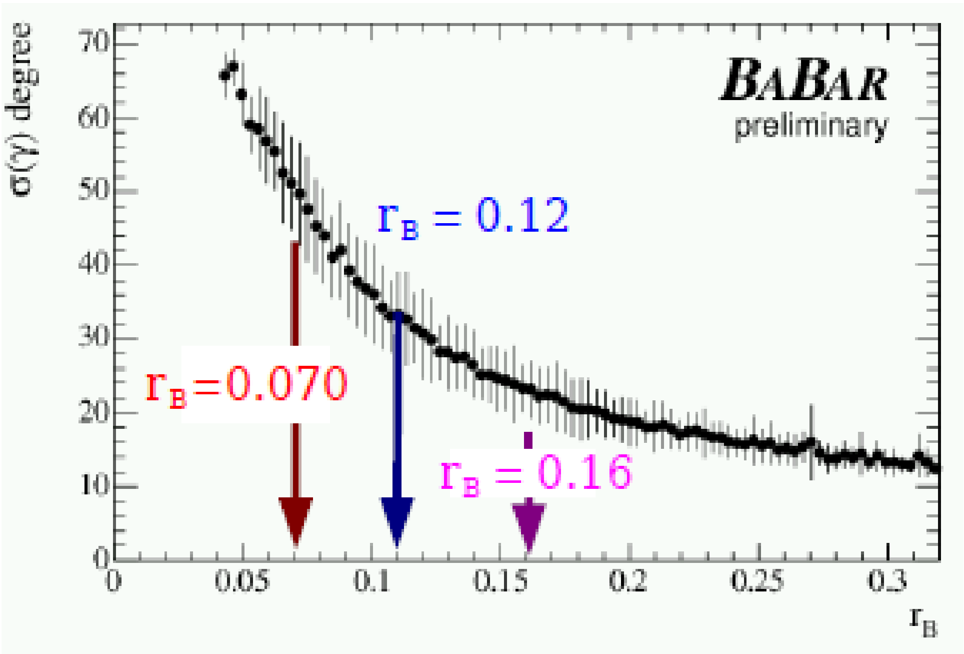
analysis with $347\times10^6 B\overline{B}$ **[hep-ex/0607104]**

$r_B(DK) < 0.140$ $0.017 < r_B(D^*K) < 0.203$

$$\boldsymbol{\gamma = 92^\circ \pm 41^\circ (stat) \pm 11^\circ (syst) \pm 12^\circ (model)}$$

uncertainty on γ scales as $1/r_B$! ($r_B = 0 \Rightarrow$ no constraint on γ)

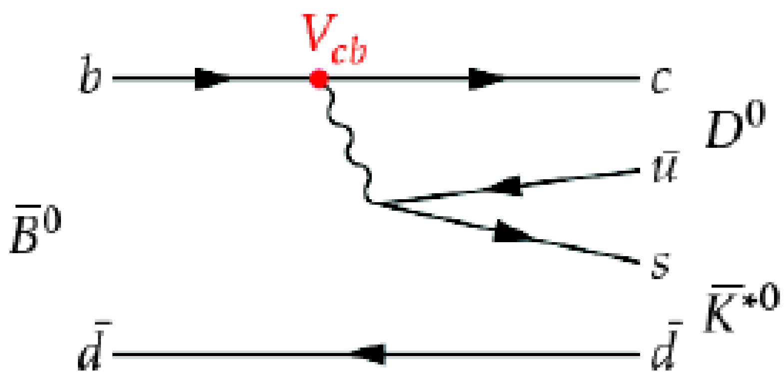
DK case

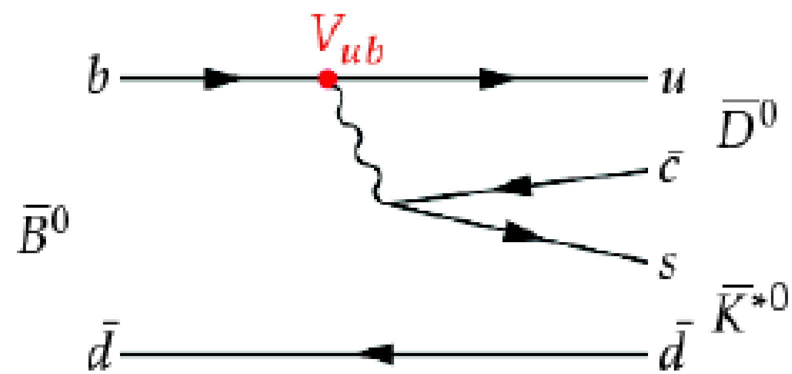


if r_B is small, the r_B(true) found is biased to higher values and the error on γ is biased toward small values

The neutral B case…

(a better r_B, but a new r_B…)





- K^{*0} refers to the $K^*(892)^0$ ⇒ charge of the kaon from $K^*(892)^0 \to K^+\pi^-$ identifies
- However lower BF…

Γ_{52}	$\bar{D}^0 K^+$	$(3.63 \pm 0.12) \times 10^{-4}$	2281
Γ_{134}	$\bar{D}^0 K^*(892)^0$	$(4.5 \pm 0.6) \times 10^{-5}$	2213

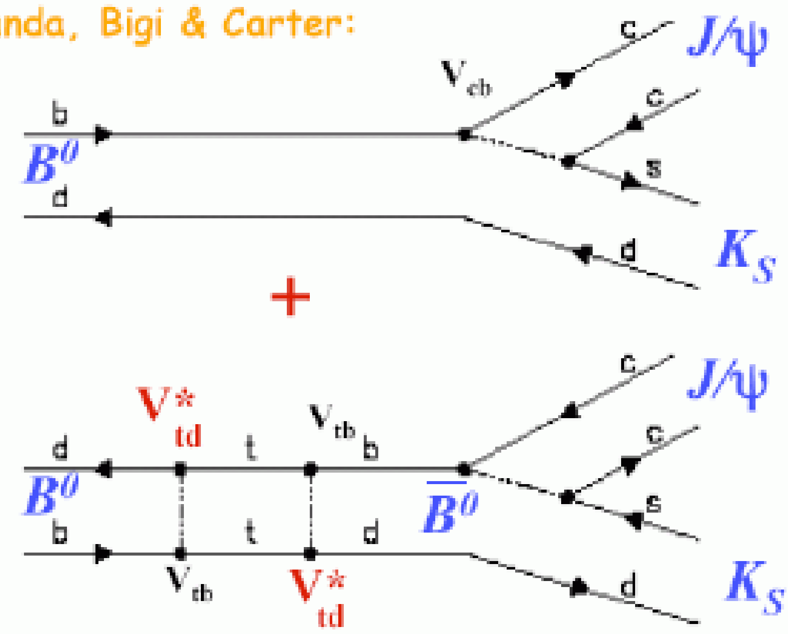
- and a new set of (r_B, δ_B)…

pollution…

Time−dependent CP asymmetries in decays to CP eigenstates

$\sin 2\phi_1$ from $B \to f_{CP} + B \leftrightarrow \overline{B} \to f_{CP}$ interf.

Sanda, Bigi & Carter:



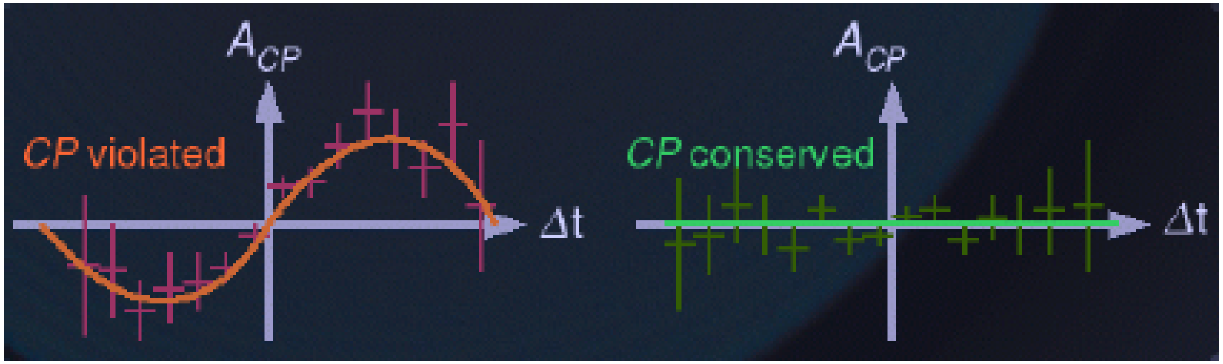
$$A_{CP}(f;t) = \frac{N(\overline{B}^0(t)\to f) - N(B^0(t)\to f)}{N(\overline{B}^0(t)\to f) + N(B^0(t)\to f)}$$

$$= \mathbf{S} \sin\Delta m_d t + \mathbf{A} \cos\Delta m_d t$$

$$= \frac{2\,\mathrm{Im}\,\lambda}{|\lambda|^2+1} \sin\Delta m_d t + \frac{|\lambda|^2-1}{|\lambda|^2+1} \cos\Delta m_d t$$

$$\lambda = \frac{q}{p}\frac{A(\overline{B}^0 \to f)}{A(B^0 \to f)} = e^{-i2\phi_1}\frac{\overline{A}_f}{A_f}$$

- $\mathbf{A} = 0$ and $\mathbf{S} = -\xi_f \sin 2\beta$ for $(c\bar{c})K_{S/L}$ $(\xi_f = \mp 1)$
- $\mathbf{A} = 0$ and $\mathbf{S} = \sin 2\alpha$ for $\pi^+\pi^-$ (if tree only)

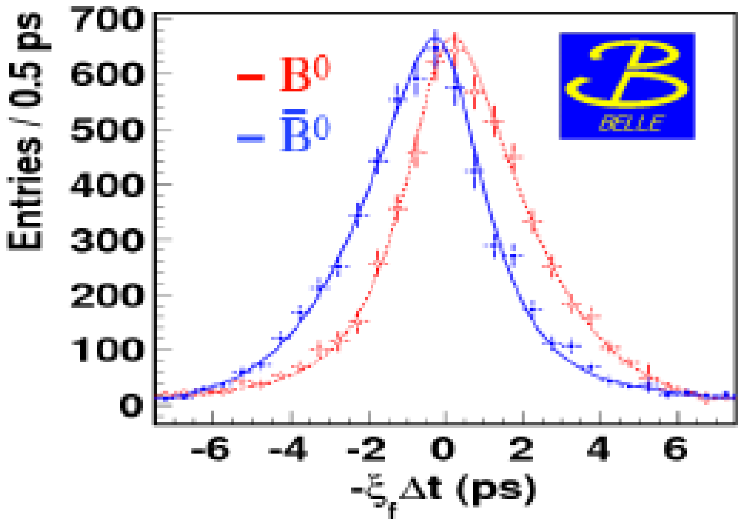


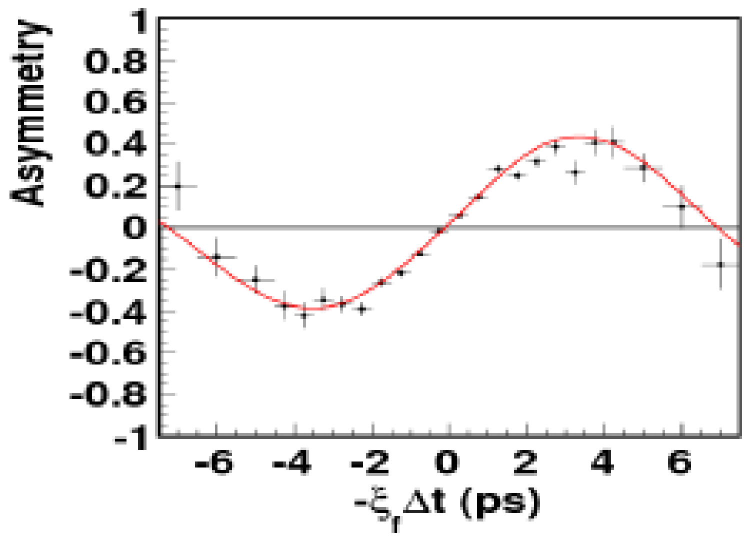
$$\mathbf{C} = -\mathbf{A}$$

$\sin 2\phi_1/\beta$ in $(c\bar{c})K^0$ ...

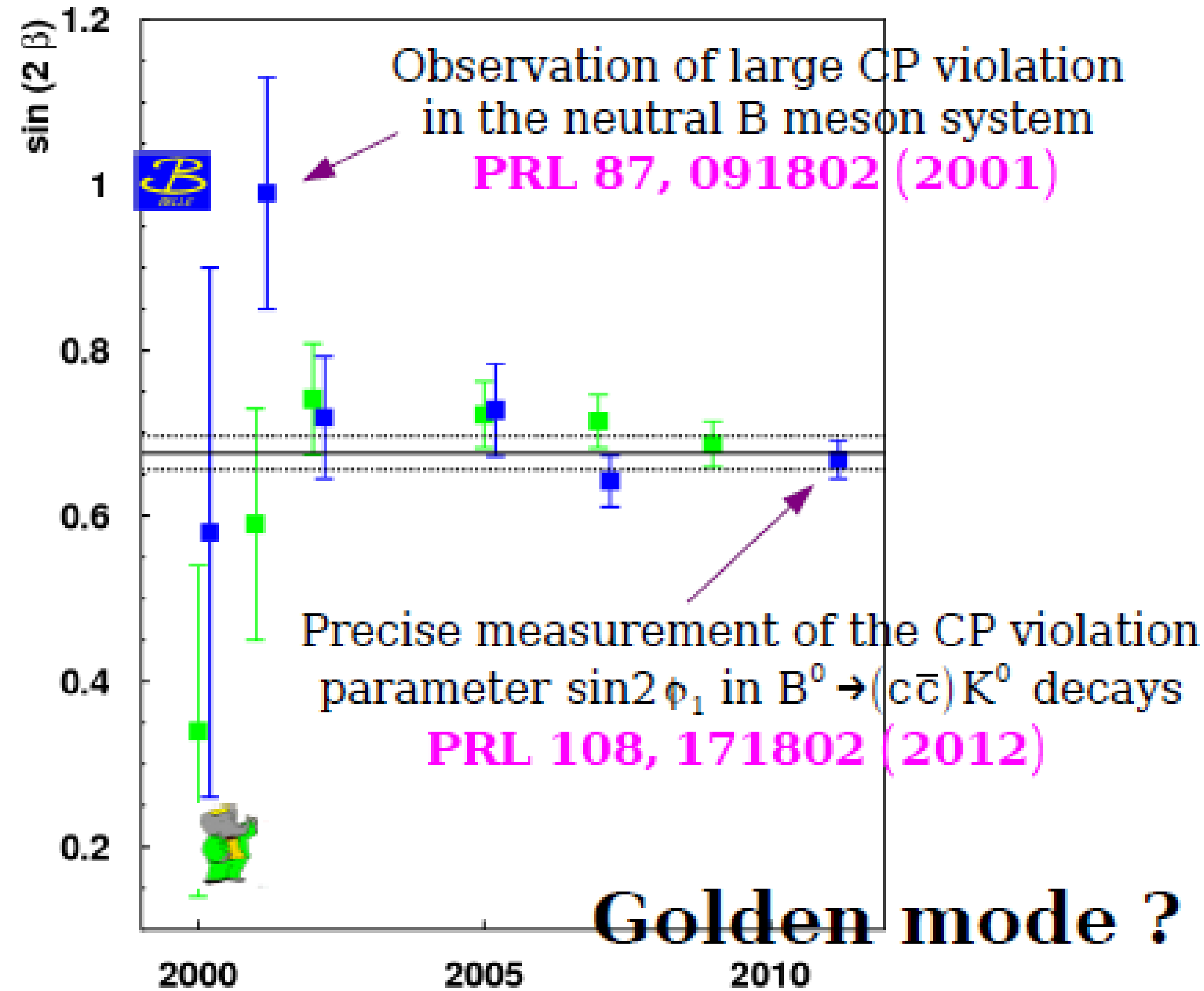
$\sin 2\beta = 0.668 \pm 0.023 \pm 0.013$

$A = 0.007 \pm 0.016 \pm 0.013$





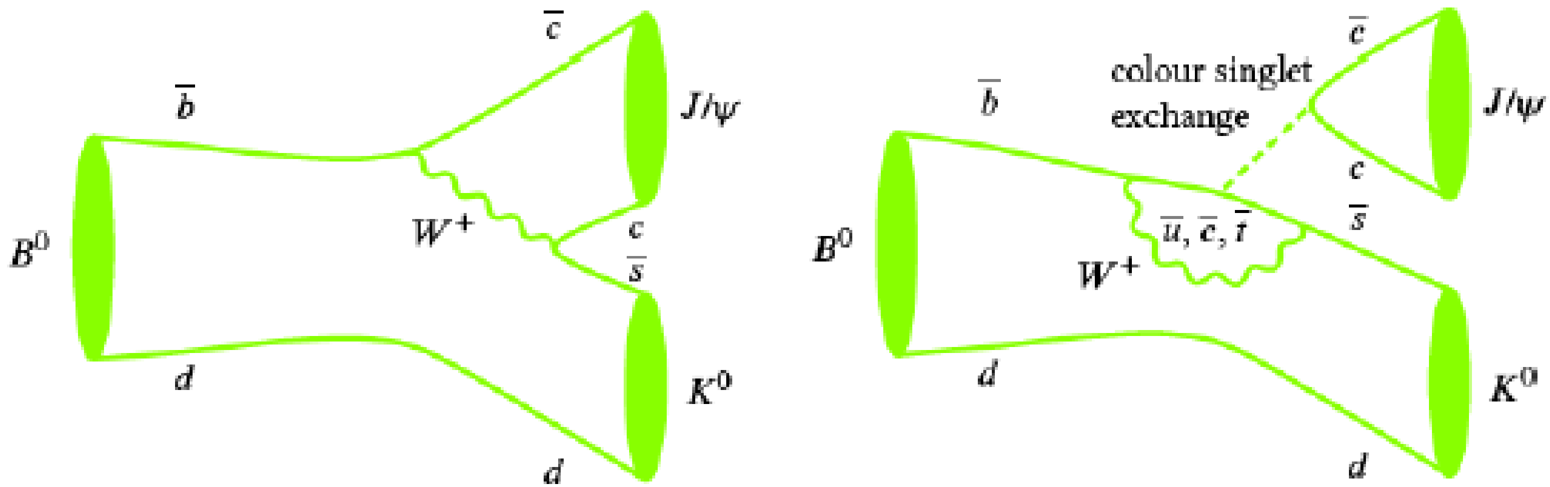
- World's most precise measurement
- anchor point of the SM
- still statistically limited !



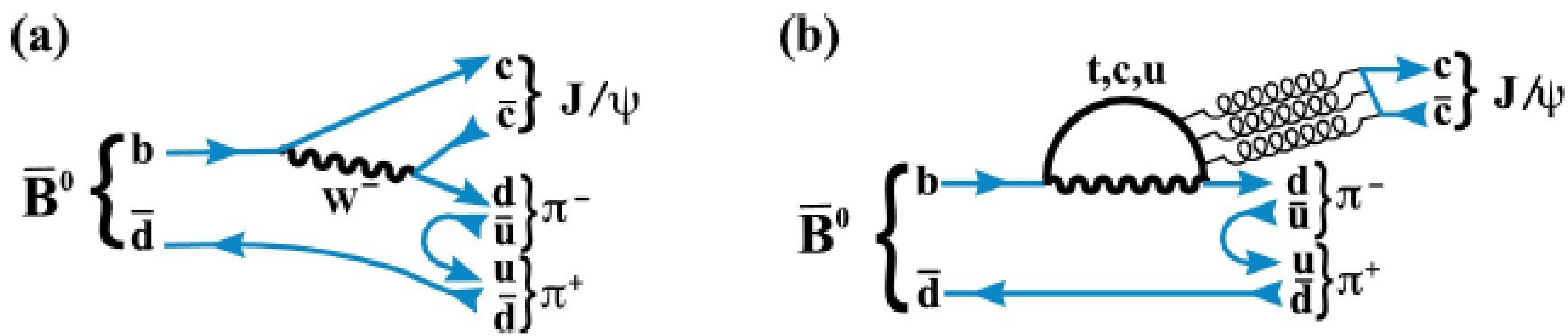
Golden mode ?

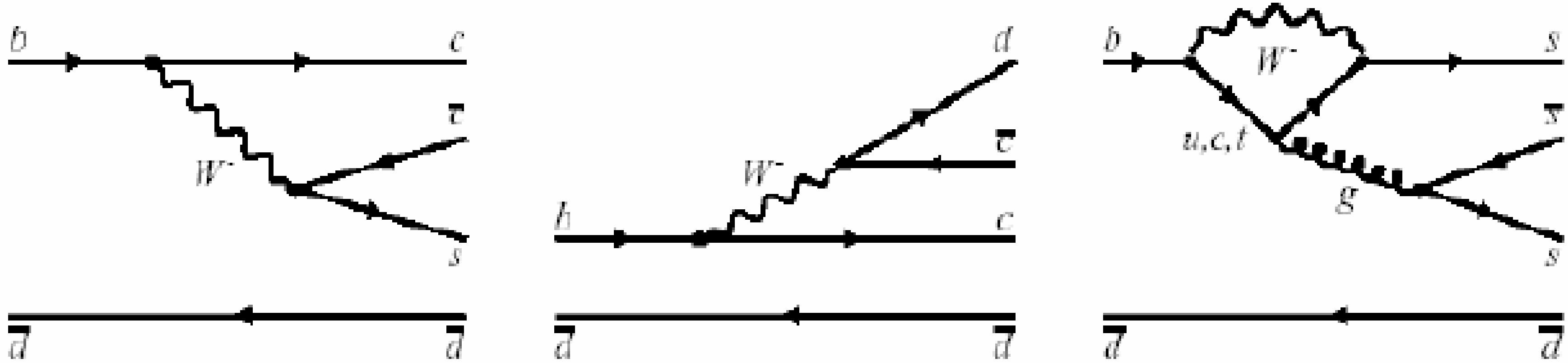
Penguin pollution in the $B^0 \rightarrow J/\psi K^0$ decays

$$\Delta S_{J/\psi K_s} \equiv S_{J/\psi K_s} - \sin 2\beta = 10^{-3}\ ?$$



$B \rightarrow J/\psi\pi^0$ or $J/\psi\pi^+\pi^-$ as an estimation of the penguin pollution





$J/\psi K_S^0, \psi(2S)K_S^0, \chi_{c1}K_S^0,$ $\eta_c K_S^0, J/\psi K_L^0,$ $J/\psi K^{*0}(K^{*0} \to K_S^0\pi^0)$	$D^{*+}D^-, D^+D^-$ $J/\psi\pi^0, D^{*+}D^{*-}$	$\phi K^0, K^+K^-K_S^0,$ $K_S^0K_S^0K_S^0, \eta' K^0, K_S^0\pi^0,$ $\omega K_S^0, f_0(980)K_S^0$

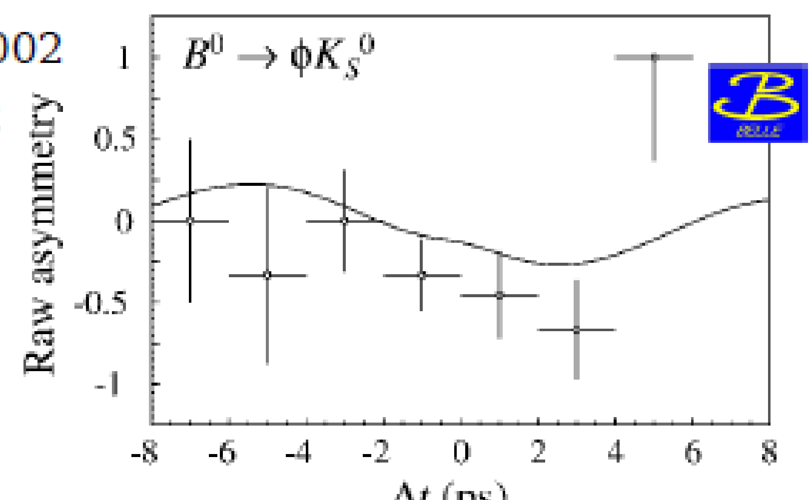
increasing tree diagram amplitude

increasing sensitivity to new physics

first reported in Moriond EW 2002

$"\sin 2\beta" = -0.73 \pm 0.64 \pm 0.22$

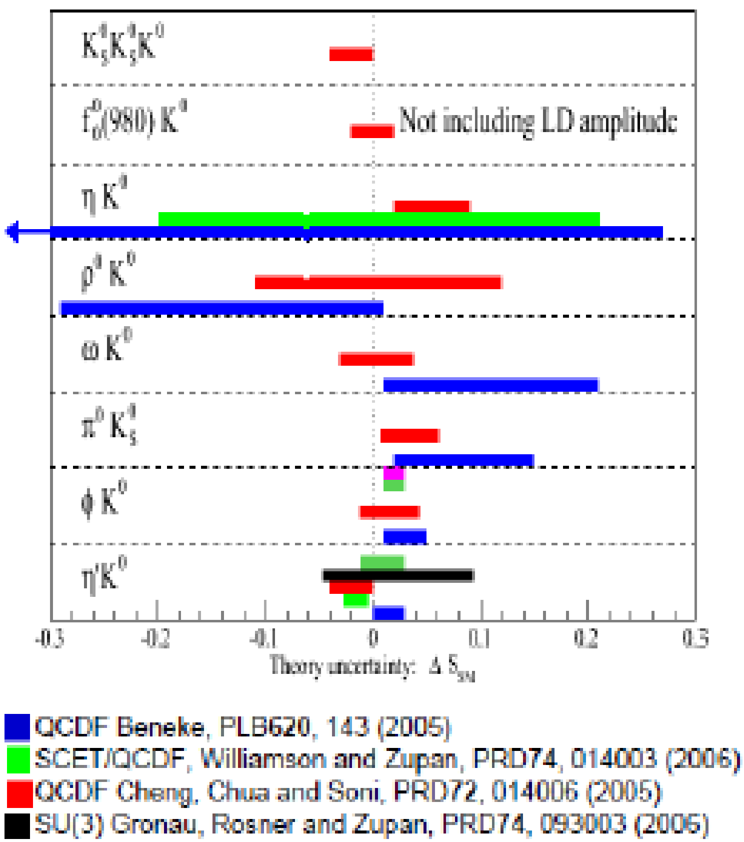
[PRD 67, 031102 (2003)]



$\sin 2\phi_1^{eff}/\beta^{eff}$ from b→s penguins

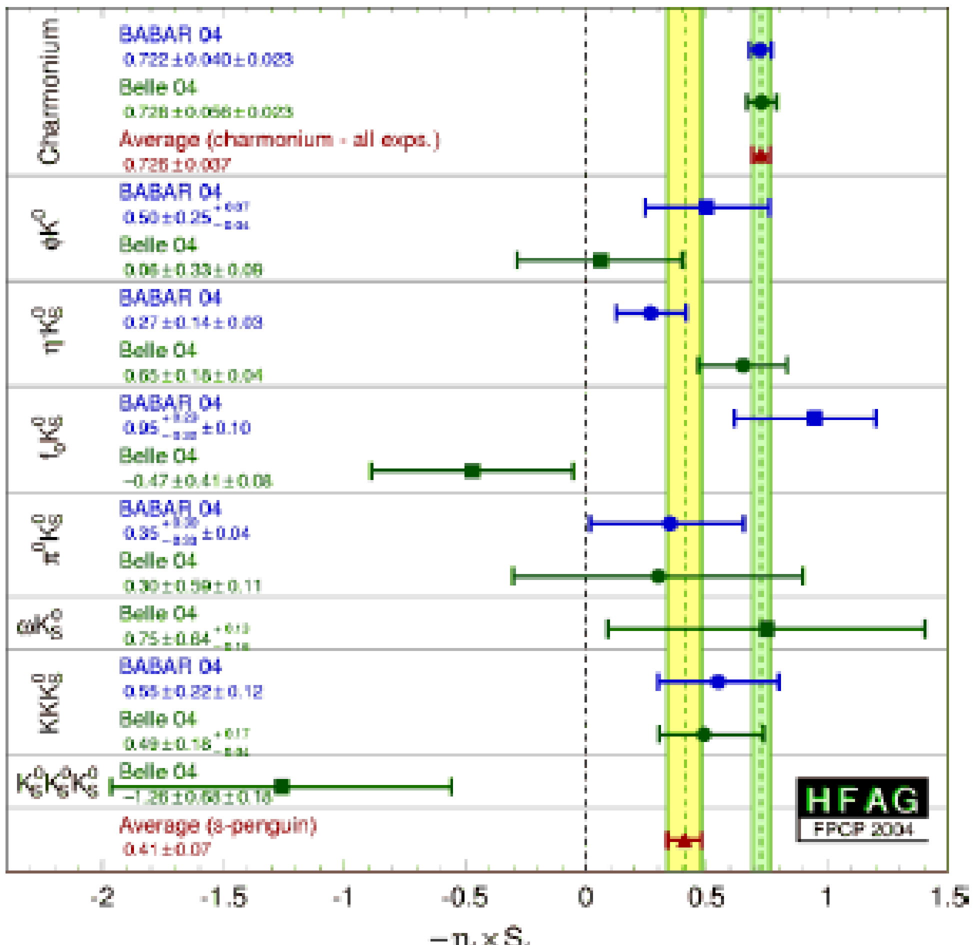
- dominant phase is the same as in $b \to c\bar{c}s$
- even in SM, possible deviations (tree pollution)
- New physics in the loop may cause deviation in the values of S and C

Theoretical prediction for ΔS



For most of the modes, theory predicts $\Delta S > 0$

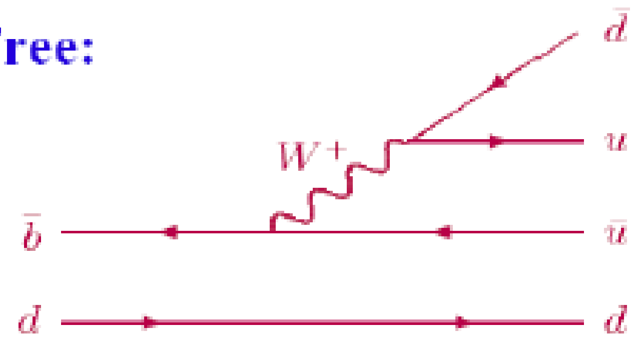
In 2004:



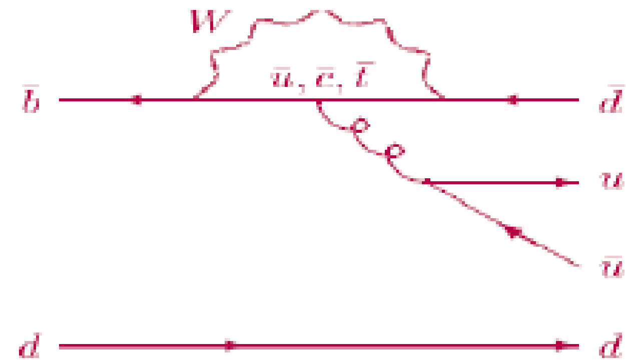
Tension between $\sin 2\beta$ from $b \to c\bar{c}s$ and $b \to q\bar{q}s$ ($\Delta S < 0$)

α determination

Tree:



Penguin:



$$A(B^0 \to \pi^+\pi^-) = T\, e^{i\gamma} + P\, e^{i\delta},\ r = |P|/|T|$$

$$A(t) = S_{\pi^+\pi^-} \sin(\Delta m t) - C_{\pi^+\pi^-} \cos(\Delta m t)$$

$$= \sqrt{1 - C^2_{\pi^+\pi^-}}\, \sin 2\alpha_{eff} \sin(\Delta m t) - C_{\pi^+\pi^-} \cos(\Delta m t)$$

from time dependent CP, we can measure α_{eff}, but we want α !

expanding in r: $$\boldsymbol{S_{\pi^+\pi^-}} = \boldsymbol{\sin 2\alpha} + 2r \cos\delta \sin(\beta + \alpha) \cos 2\alpha + O(r^2)$$

time dependent decay width:

$$\Gamma(B^0(t)) \propto \Gamma_{\pi^+\pi^-} [1 + C_{\pi^+\pi^-} \cos \Delta m t - S_{\pi^+\pi^-} \sin \Delta m t]$$

3 measurables vs. 4 unknowns: T, r, δ, γ

→ additional inputs required to determine the penguin pollution to fix r

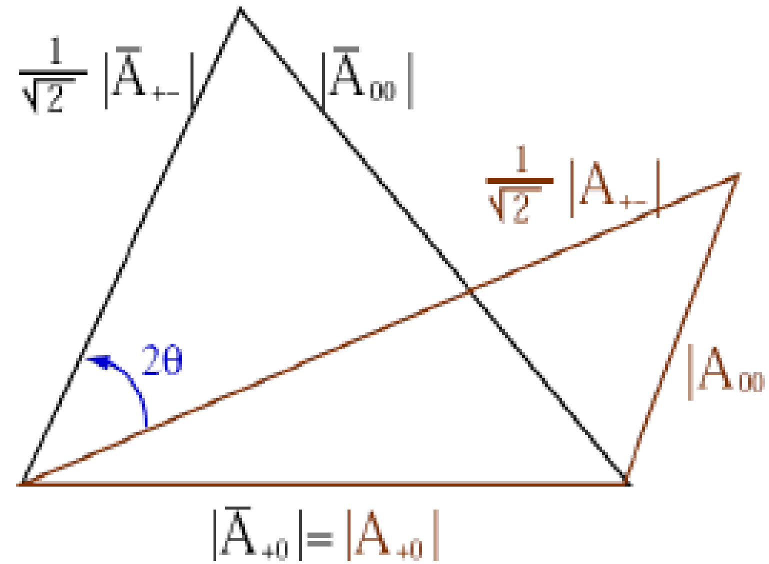
α determination with isospin analysis

[Gronau-London, PRL65, 3381 (1990)]

Isospin breaking (d and u charges different, $m_u \neq m_d$)

$$A_{+-} = A(B^0 \to \pi^+\pi^-) = e^{-i\alpha}\, T^{+-} + P$$
$$\sqrt{2}\, A_{00} = \sqrt{2}\, A(B^0 \to \pi^0\pi^0) = e^{-i\alpha}\, T^{00} + P$$
$$\sqrt{2}\, A_{+0} = \sqrt{2}\, A(B^+ \to \pi^+\pi^0) = e^{-i\alpha}\, (T^{00} + T^{+-})$$

$$A_{+-} + \sqrt{2}\, A_{00} = \sqrt{2}\, A_{+0}$$
$$\overline{A}_{+-} + \sqrt{2}\, \overline{A}_{00} = \sqrt{2}\, \overline{A}_{+0}$$



- neglecting EWP $\Rightarrow$ A_{+0} pure tree
  $|A_{+0}| = |\overline{A}_{+0}|$

$\Rightarrow$ $\Delta\alpha_{EWP} = (1.5 \pm 0.3 \pm 0.3)^\circ$

- $\pi^0 - \eta - \eta'$ and $\rho - \omega$ mixing [J.Zupan, hep-ph/0701004]
  (mass eigenstates do not coincide with isospin eigenstates)

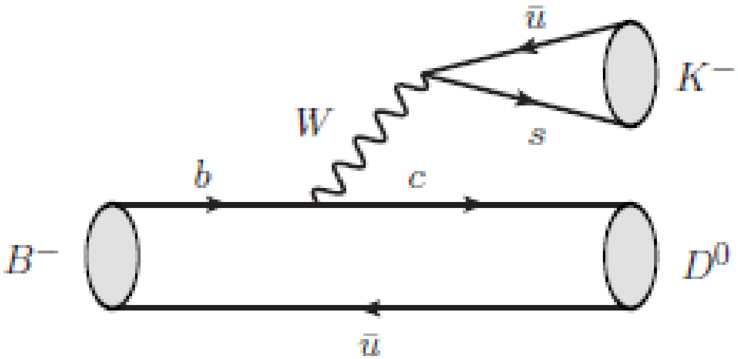
$\Rightarrow$ $|\Delta\alpha_{\pi\pi}^{\pi-\eta-\eta'}| < 1.6^\circ$

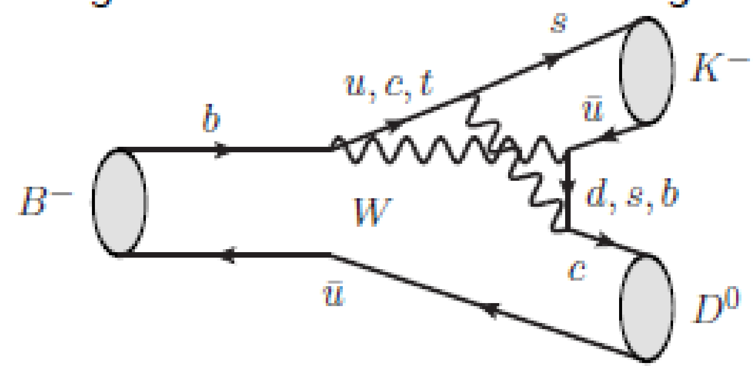
α can be resolved up to an 8-fold ambiguity ($\alpha \in [0, \pi]$)

Irreducible theory uncertainty

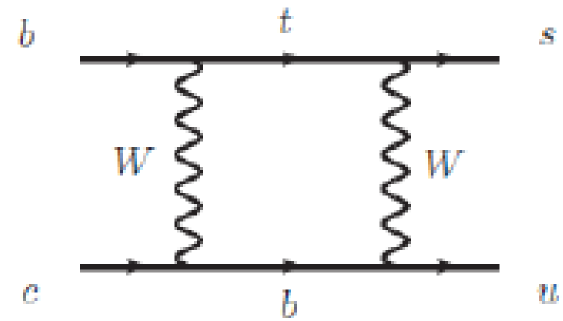
Box diagrams can change CKM structure

higer-order electroweak diagram

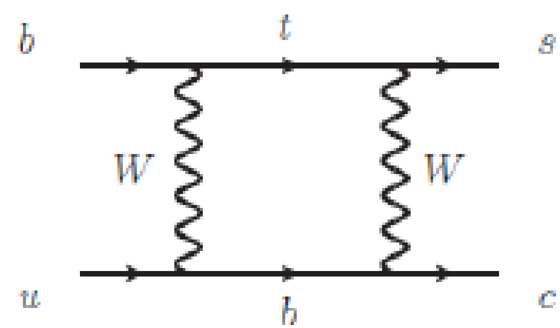




- $b \to u\bar{c}s$: :
  - tree level $\sim V_{ub}V_{cs}^*$
  - box diagram $\sim (V_{tb}V_{ts}^*)(V_{ub}V_{cb}^*)$
- same weak phase, no shift in γ



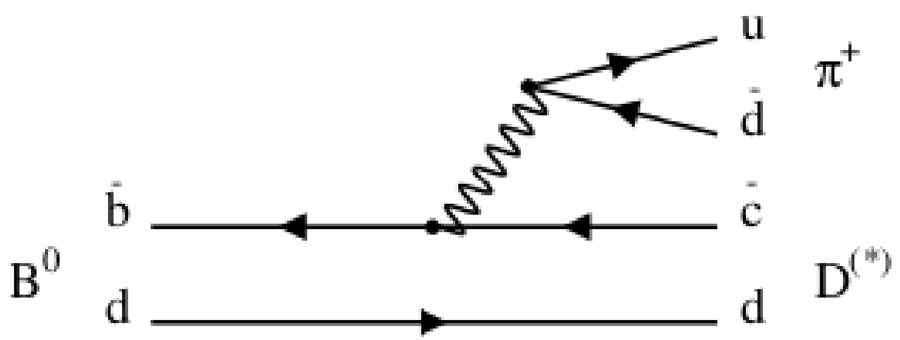
- $b \to c\bar{u}s$: :
  - tree level $\sim V_{cb}V_{us}^*$
  - box diagram $\sim (V_{tb}V_{ts}^*)(V_{cb}V_{ub}^*)$
- different weak phase, induces $\delta\gamma$

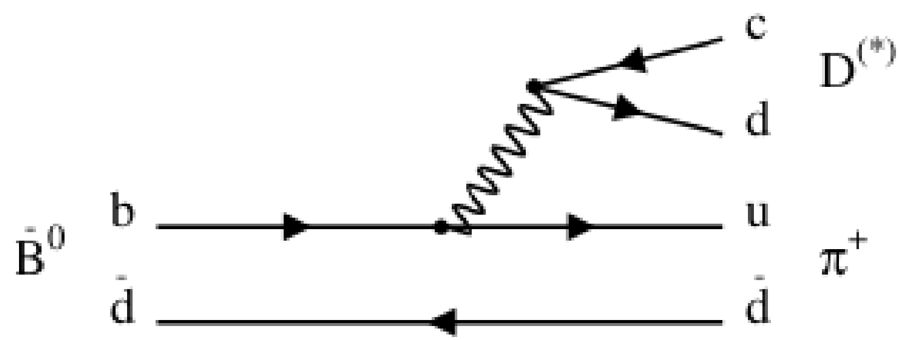


$\Rightarrow \delta\gamma/\gamma < o(10^{-7})$

pollution…

$\sin 2\beta+\gamma$ [method]



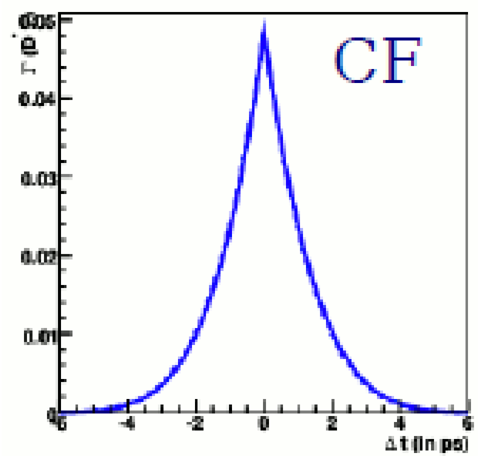


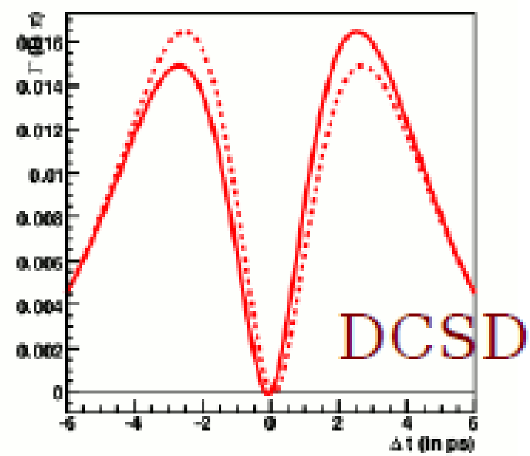
$$\Gamma(B^0 \to D^{*+}\pi^-) = \alpha[1 - C\cos(\Delta m\Delta t) - S^+ \sin(\Delta m\Delta t)],$$
$$\Gamma(B^0 \to D^{*-}\pi^+) = \alpha[1 + C\cos(\Delta m\Delta t) - S^- \sin(\Delta m\Delta t)],$$
$$\Gamma(\bar{B}^0 \to D^{*+}\pi^-) = \alpha[1 + C\cos(\Delta m\Delta t) + S^+ \sin(\Delta m\Delta t)],$$
$$\Gamma(\bar{B}^0 \to D^{*-}\pi^+) = \alpha[1 - C\cos(\Delta m\Delta t) + S^- \sin(\Delta m\Delta t)].$$

$$\alpha = e^{-|\Delta t|/\tau_{B^0}}/8\,\tau_{B^0}$$

$$S^{\pm} = -(2R/(1+R^2))\sin(2\beta+\gamma\pm\delta)$$
$$C = (1-R^2)/(1+R^2) = 1$$

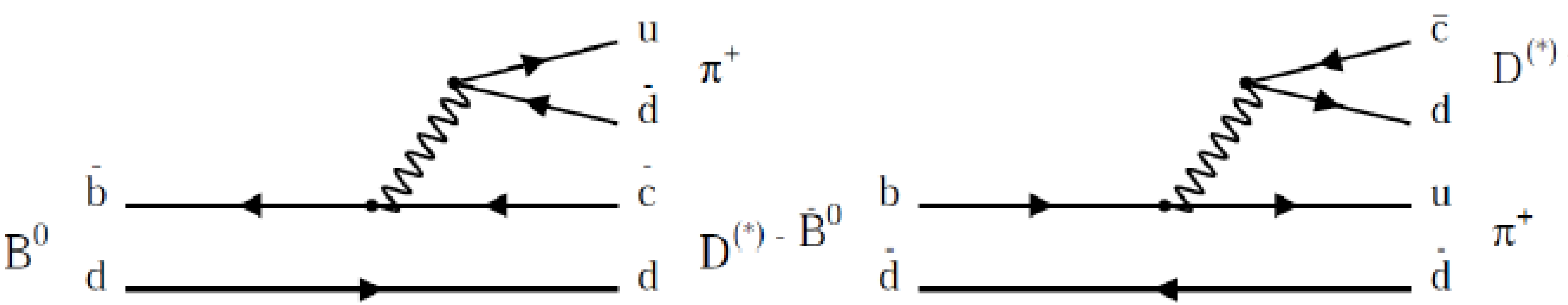
though large samples, R values expected to be small ($\simeq 0.02$)





Time - dependent measurements

- All of the measurements presented so far were time-independent
- Time-dependent measurements (mixing induced CPV) also possible:
  - $B^0 \to D^{(*)}\pi$, $B^0 \to D^{(*)}\rho$
- In order to extract γ from $B \to SS/SV$ decays, must supply $r = |A_{DCS}/A_{CF}|$ externally (expected to be ~ 1-2%), usually assuming SU(3) symmetry

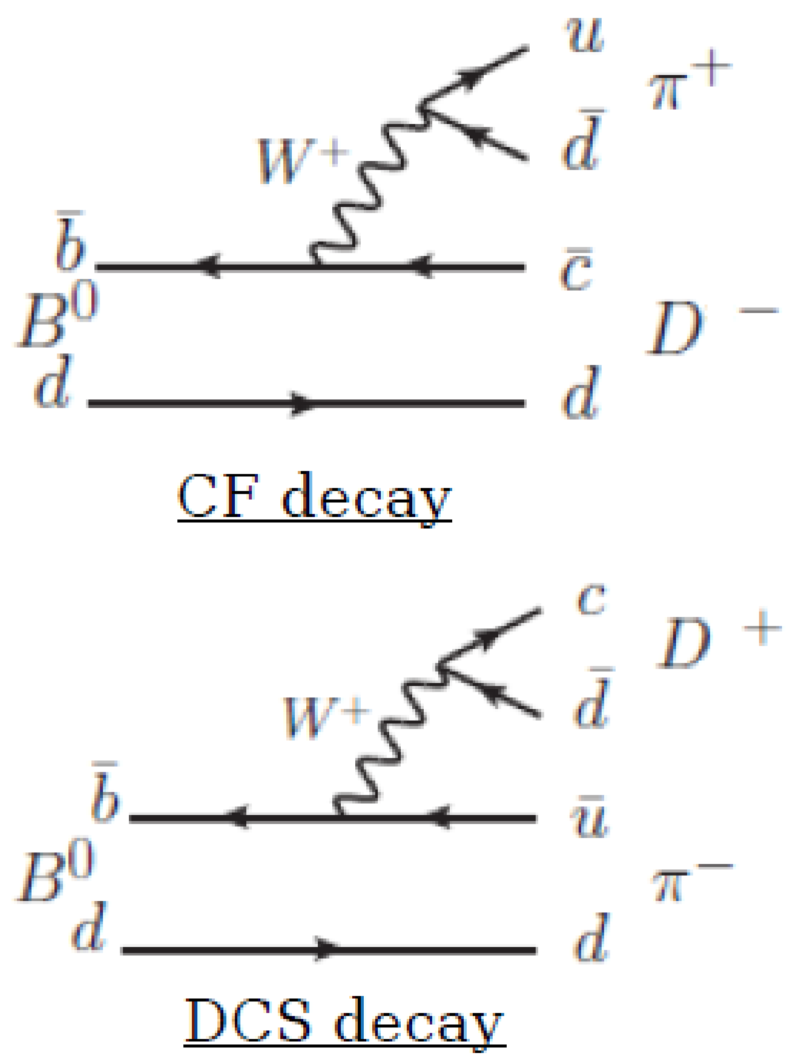


- In $B \to VV$ decays, one can extract all physics parameters from data
- Belle study: ~ 100 k evts per ab^{-1},
  3 helicity configurations: $A = \sum_\lambda A_\lambda$
  we use Cartesian coordinates $\{r_\lambda, \delta_\lambda, \phi_w\} \to \{x_\lambda, y_\lambda, \bar{x}_\lambda, \bar{y}_\lambda\}$
  $\sigma(2\beta+\gamma) \simeq 11^\circ$ for Belle II with 50 ab^{-1}

$D^{\pm}\pi^{\mp}$ time-dep analysis and $\sin 2\beta+\gamma$ extraction

$$S^{\pm} = \frac{2 r_{D\pi} \sin(\delta \pm (2\beta+\gamma))}{1+R^2}$$

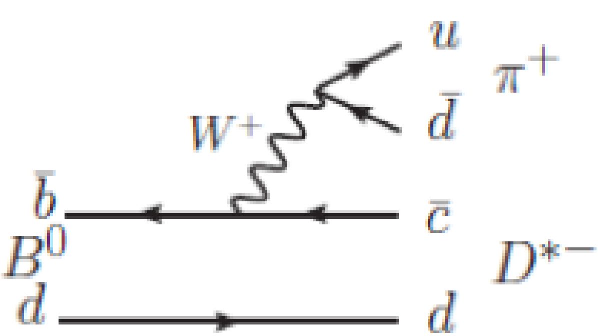
$$r_{D\pi} = \frac{|A(B^0 \to D^+\pi^-)|}{|A(B^0 \to D^-\pi^+)|}$$



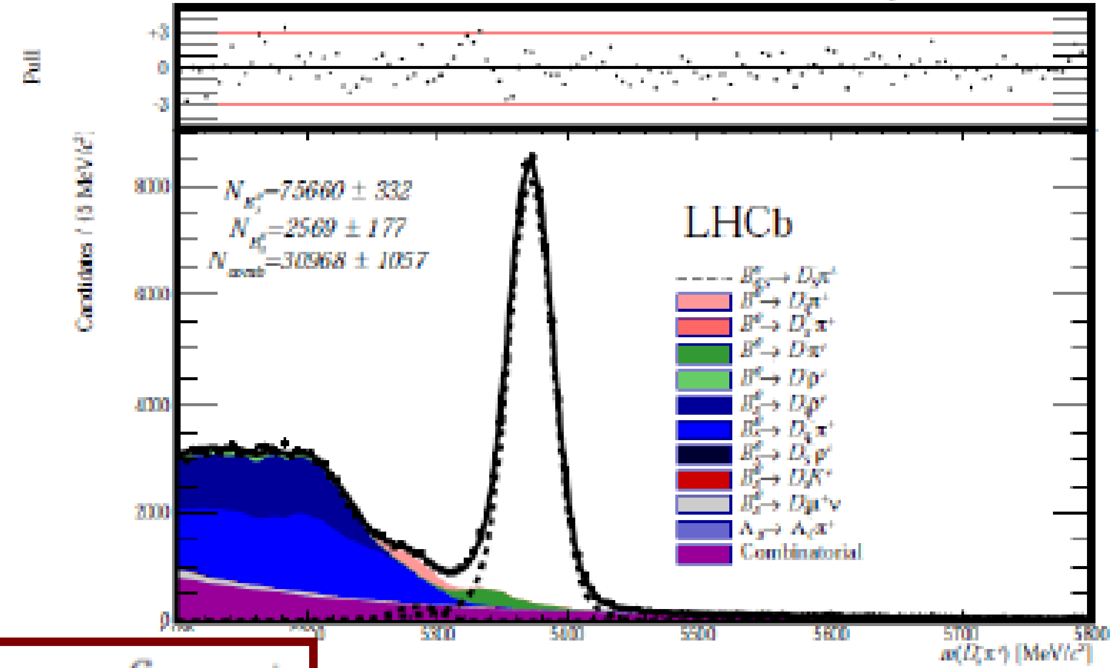
⇒ expected to be around 2%

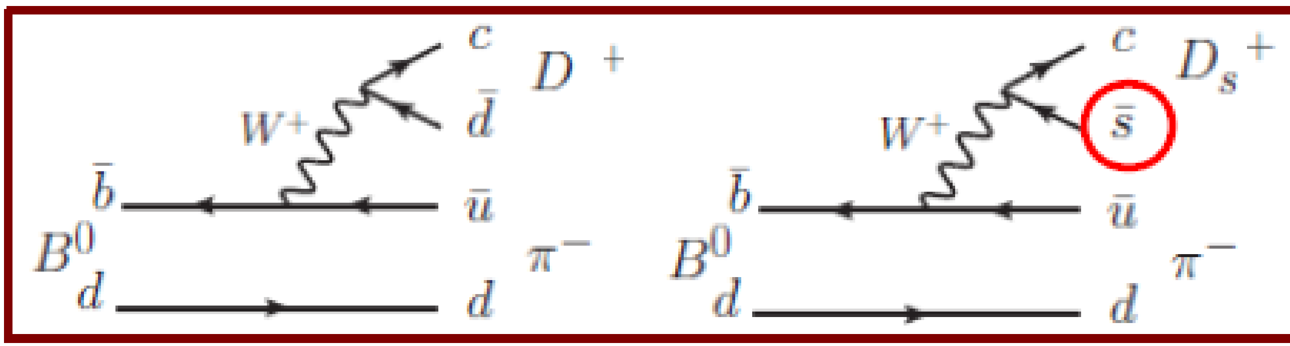
⇒ can't be measured directly from Dπ analysis, need other inputs...

$B \to D_s \pi$



LHCb-ANA-2015-037, 3 fb^{-1}



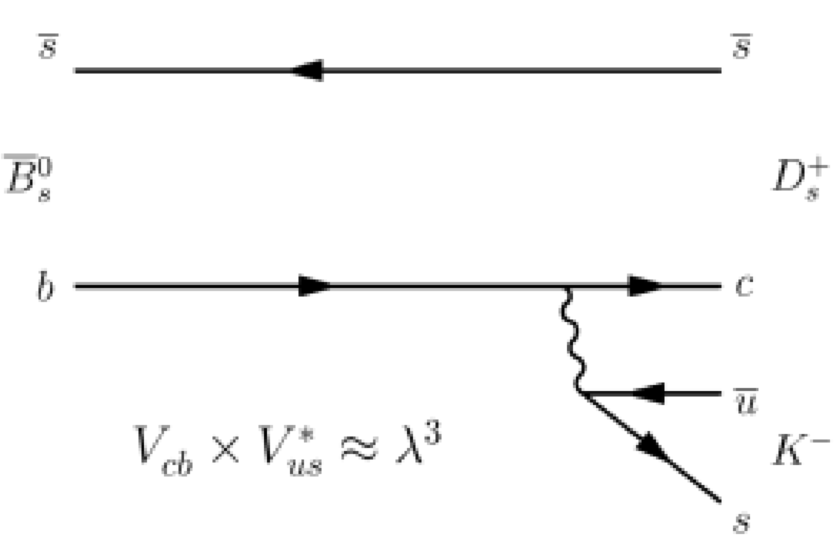


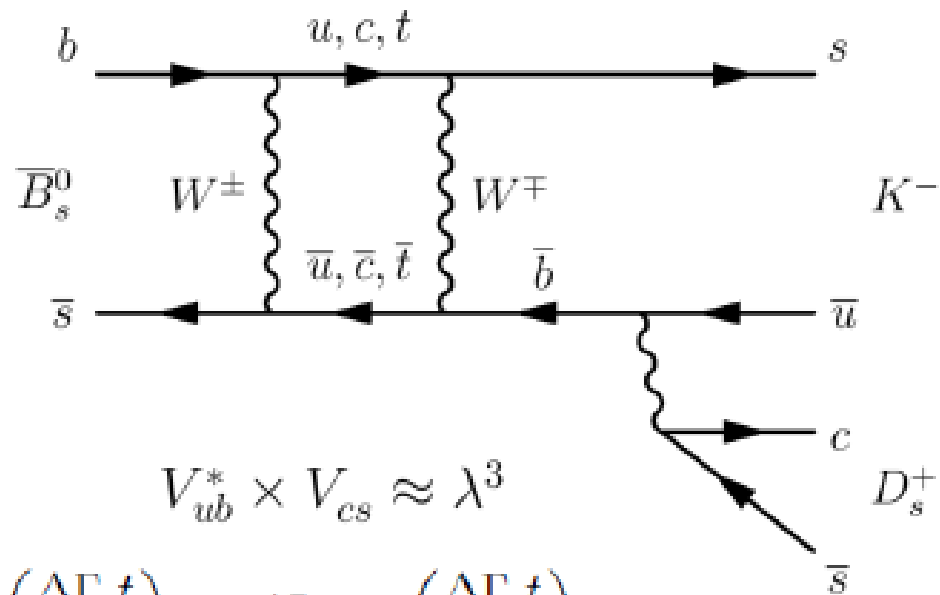
$$r_{D\pi} \simeq \tan\theta_c \frac{f_D}{f_{D_s}} \sqrt{\frac{\mathcal{B}(B^0 \to D_s^+ \pi^-)}{\mathcal{B}(B^0 \to D^- \pi^+)}}$$

$\mathcal{B}(\times 10^{-6})$	Exp.	Ref.
$25 \pm 4 \pm 2$	BaBar	arXiv:0803.4296
$19.9 \pm 2.6 \pm 1.8$	Belle	arXiv:1007.4619
$26.7 \pm 2.0 \pm 2.2$	LHCb	LHCb-ANA-2015-037

⇒ **following arXiv:1208.6463: $r_{D\pi} = (1.70 \pm 0.08 \pm 0.25\,(\text{theo.}))\%$**

Time-dependent: B_s case





$$\frac{d\Gamma_{B_s^0\to f}(t)}{dt} = \frac{1}{2}|A_f|^2(1+|\lambda_f|^2)e^{-\Gamma_s t}\left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + A_f^{\Delta\Gamma}\sinh\left(\frac{\Delta\Gamma_s t}{2}\right) + C_f\cos(\Delta m_s t) - S_f\sin(\Delta m_s t)\right],$$

$$\frac{d\Gamma_{\overline{B}_s^0\to f}(t)}{dt} = \frac{1}{2}|A_f|^2\left|\frac{p}{q}\right|^2(1+|\lambda_f|^2)e^{-\Gamma_s t}\left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + A_f^{\Delta\Gamma}\sinh\left(\frac{\Delta\Gamma_s t}{2}\right) - C_f\cos(\Delta m_s t) + S_f\sin(\Delta m_s t)\right],$$

$$C_f = \frac{1-r_{D_sK}^2}{1+r_{D_sK}^2},$$

$$A_f^{\Delta\Gamma} = \frac{-2r_{D_sK}\cos(\delta-(\gamma-2\beta_s))}{1+r_{D_sK}^2}, \quad A_{\bar{f}}^{\Delta\Gamma} = \frac{-2r_{D_sK}\cos(\delta+(\gamma-2\beta_s))}{1+r_{D_sK}^2},$$

$$S_f = \frac{2r_{D_sK}\sin(\delta-(\gamma-2\beta_s))}{1+r_{D_sK}^2}, \quad S_{\bar{f}} = \frac{-2r_{D_sK}\sin(\delta+(\gamma-2\beta_s))}{1+r_{D_sK}^2}.$$

$\Rightarrow$

- r_{D_sK}
- δ
- $\gamma - 2\beta_s$

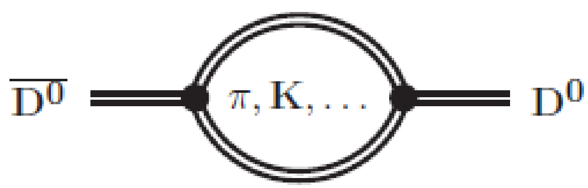
Flavour Mixing in the Charm Sector

Mass eigenstates $\neq$ flavour eigenstates

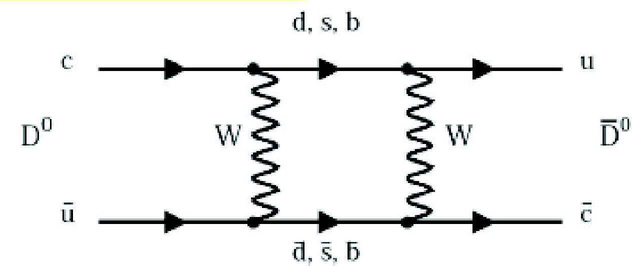
$$|D_{1,2}\rangle = p\,|D^0\rangle \pm q\,|\overline{D}^0\rangle$$

$m_{1,2}$ and $\Gamma_{1,2}$ are mass and width of $|D_{1,2}\rangle$

$p/q \neq 1 \Rightarrow$ CP violation



Long – distance contributions dominant, affected by large theoretical uncertainties



Short – distance contributions, GIM and CKM suppressed in SM

Time evolution of a $D^0 - \overline{D}^0$ system

$$i\frac{d}{dt}\begin{pmatrix}|D(t)\rangle\\ |\bar{D}(t)\rangle\end{pmatrix} = \left(M - i\frac{\Gamma}{2}\right)\begin{pmatrix}|D(t)\rangle\\ |\bar{D}(t)\rangle\end{pmatrix}$$

with M and Γ being hermitian

Solutions

$$|D^0(t)\rangle = e^{-(\Gamma/2 + im)t}\left[\cosh\left(\frac{y+ix}{2}\Gamma t\right)|D^0\rangle + \frac{q}{p}\sinh\left(\frac{y+ix}{2}\Gamma t\right)|\overline{D}^0\rangle\right]$$

$$|\overline{D}^0(t)\rangle = e^{-(\Gamma/2 + im)t}\left[\frac{p}{q}\sinh\left(\frac{y+ix}{2}\Gamma t\right)|D^0\rangle + \cosh\left(\frac{y+ix}{2}\Gamma t\right)|\overline{D}^0\rangle\right]$$

Mixing parameters

$$x = \frac{m_1 - m_2}{\Gamma_D}, \qquad y = \frac{\Gamma_1 - \Gamma_2}{2\Gamma_D}$$

$$\Gamma_D = (\Gamma_1 + \Gamma_2)/2$$

D^0 - $\overline{D}^0$ mixing

- Since D^0 mixing is small $\left(|x|, |y| << 1\right)$:

$$|D^0(t)> = e^{-(\Gamma/2 + im)t}\left[|D^0> + \frac{p}{q}\left(\frac{y+ix}{2}\Gamma t\right)|\overline{D}^0>\right]$$

- Time dependent decay rates of $D^0 \rightarrow f$:

$$\frac{dN_{D^0\rightarrow f}}{dt} \propto |<f|H|D^0(t)>|^2 = e^{-\Gamma t}|<f|H|D^0> + \frac{q}{p}\left(\frac{y+ix}{2}\Gamma t\right)|<f|H|\overline{D}^0>|^2$$

- Exponential decay modulated with x and y

  x and **y** can be obtained from measured time dependence of $\frac{dN_{D^0\rightarrow f}}{dt}$

- Shape is final state dependent

  different final states sensitive to different combinations of **x** and **y**

D^0 - $\overline{D}^0$ mixing – SM estimates

Can express (Joachim Brod)

$$\mathbf{y} = \frac{1}{2\Gamma_D} \sum_n \rho_n [<D^0|H|n><n|H|\overline{D}^0> + <\overline{D}^0|H|n><n|H|D^0>]$$

$$\mathbf{x} = \frac{1}{\Gamma_D} [<D^0|H|\overline{D}^0> + P \sum_n \frac{<D^0|H|n><n|H|\overline{D}^0> + <\overline{D}^0|H|n><n|H|D^0>}{M_D^2 - E_n^2}]$$

"Inclusive approach":

- OPE expansion in powers of "Λ/m_c"
- $x \sim y < 10^{-3}$ [Georgi 1992; Ohl et al 1993; Bigi et al 2000]
- Cannot exclude $y \sim 10^{-2}$ [Bobrowski et al 2010]
- Violation of quark-hadron duality

"Exclusive approach":

- Sum over on-shell intermediate states
- Mainly $D \to PP, PV$ leads to $x \sim y < 10^{-3}$ [Cheng et al 2010]
- $SU(3)_F$ breaking in phase space alone leads to $y \sim 10^{-2}$ [Falk et al 2002]
- Get $x \sim 10^{-2}$ from a dispersion relation [Falk et al 2004]

Results discussed in this talk ...

From HFAG page:

$D^0 \to K^+ \pi^-$ E791

$D^0 \to h^+ h^-$ E791

$D^0 \to K^+ \pi^- \pi^0$

$D^0 \to K^+ \pi^+ 2\pi^-$

$D^0 \to K_S^0 h^+ h^-$

$D^0 \to K^+ l^- \nu$ E791

$\psi(3770) \to D^0 \overline{D}^0$

★ = mixing probability $> 3\sigma$

Decays to CP-even eigenstates $D^0 \to K^+K^-, \pi^+\pi^-$

Measurement of lifetime difference between $D \to K^-\pi^+$ and $D^0 \to K^+K^-, \pi^+\pi^-$

Timing distributions are exponential (if CP is conserved)

- mixing parameter: $y_{CP} = \frac{\tau(K^-\pi^+)}{\tau(h^+h^-)} - 1$
- if CP conserved: $y_{CP} = y$

If CP is violated → difference in lifetimes of $D^0/\overline{D}^0 \to K^+K^-, \pi^+\pi^-$

- lifetime asymmetry: $A_\Gamma = \frac{\tau(\overline{D}^0 \to h^-h^+) - \tau(D^0 \to h^-h^+)}{\tau(\overline{D}^0 \to h^-h^+) + \tau(D^0 \to h^-h^+)}$
- $y_{CP} = y\cos\phi - \frac{1}{2}A_M\, x\sin\phi$
- $A_\Gamma = \frac{1}{2}A_M\, y\cos\phi - x\sin\phi$

$\phi = \arg(q/p)$

$A_M = 1 - |q/p|^2$

[S. Bergmann et al, PLB 486, 418 (2000)]

Experimental method (update with 976 fb^{-1})

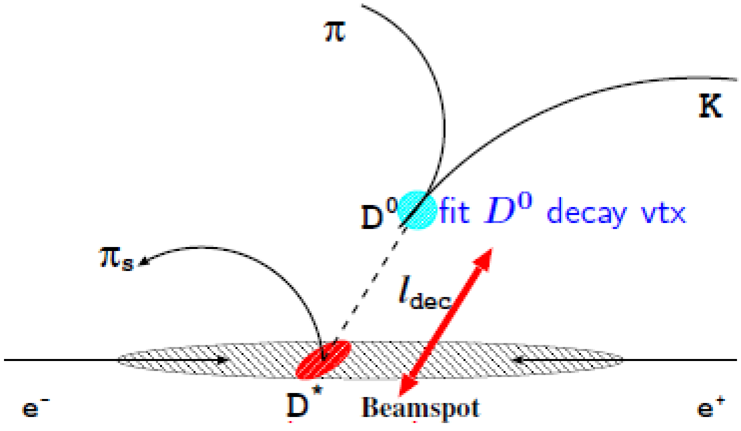
[arXiv:1212.3478; M.Staric et al, PRL 98, 211803 (2007)]

using $D^{*+} \to \pi^+ D^0$

- flavor tagging by the charge of π_{slow}
- background suppression

D^0 proper decay time measurement:

$$t = \frac{l_{dec}}{c\beta\gamma}, \qquad \beta\gamma = \frac{p_{D^0}}{M_{D^0}}$$



extrapolate production vtx

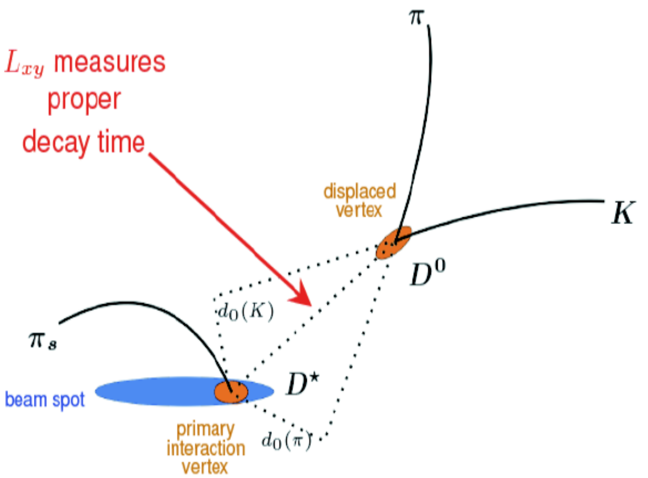
- decay time uncertainty σ_t (calculated from vtx err matrices)

To reject D^{*+} from B decays: $p^{CMS}_{D^{*+}} > 2.5\ (3.1)$ GeV/c $\Upsilon(4S)\ (\Upsilon(5S))$

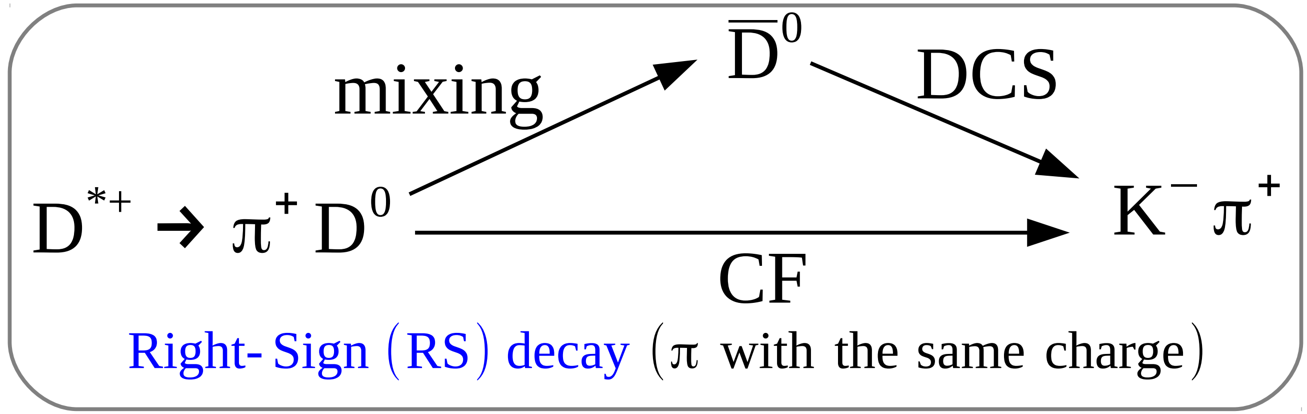
Observables:

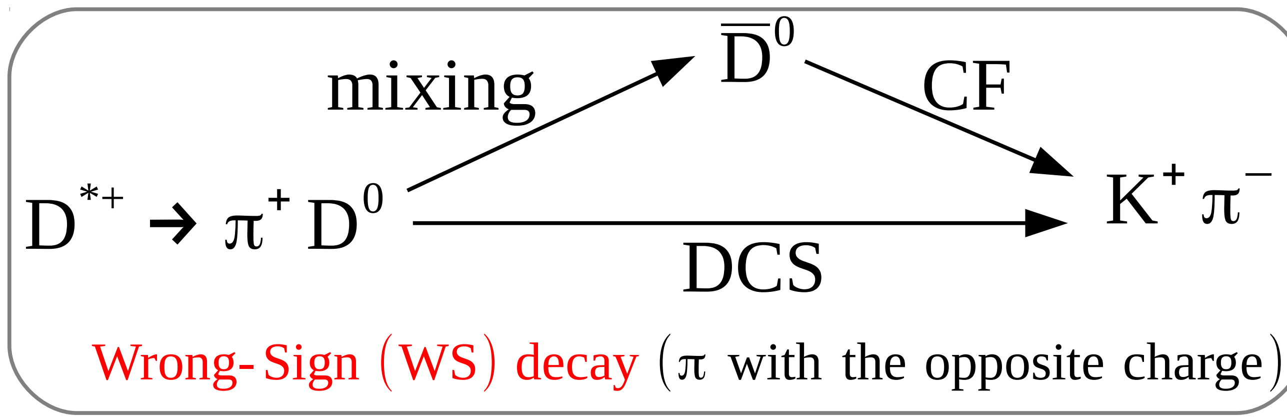
- $m = m(K\pi)$
- $q = m(K\pi\pi_s) - m(K\pi) - m_\pi$

Charm mixing in $D^0 \to K^+ \pi^-$



D^0 is tagged by $D^{*+} \to D^0 \pi^+_s$ decay





The ratio $R(t)$ of WS $D^{*+} \to D^0 \pi^+_s \to K^+ \pi^- \pi^+_s$ to RS $D^{*+} \to D^0 \pi^+_s \to K^- \pi^+ \pi^+_s$ decay rates can be approximated (assuming $|x|, |y| << 1$ and no CPV) by:

DCS to CF ratio ; mixing rate

$$R(t) = R_D + \sqrt{R_D}\, y' t + \frac{x'^2 + y'^2}{4} t^2$$

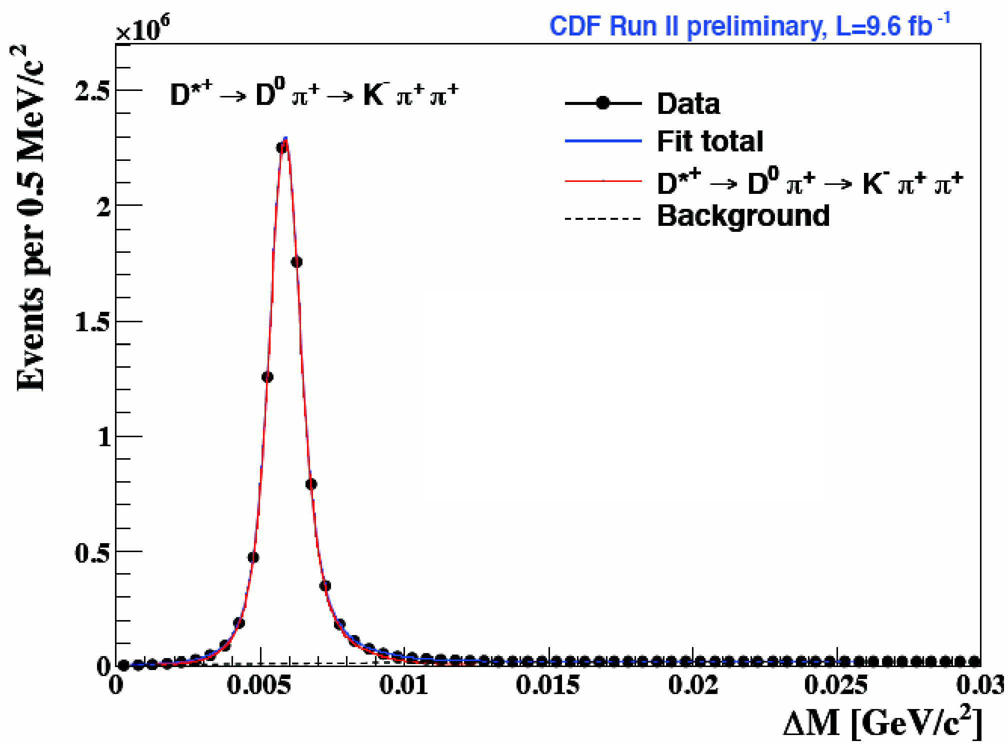
$$x' = x \cos\delta_{K\pi} + y \sin\delta_{K\pi}$$
$$y' = y \cos\delta_{K\pi} - x \sin\delta_{K\pi}$$

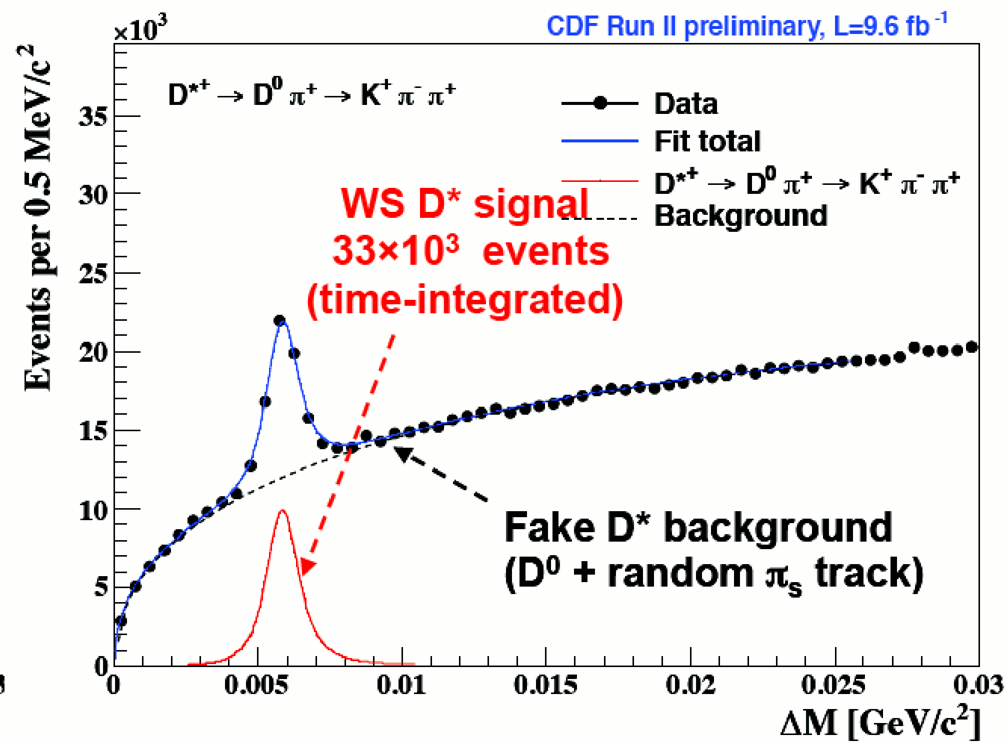
$\delta_{K\pi}$: strong phase difference btw DCS and CF amplitudes

Charm mixing in $D^0 \to K^+\pi^-$

http://www-cdf.fnal.gov/physics/new/bottom/130408.blessed-DMix_9.6fb/public_note_CDF_D_mix.pdf

Time-integrated yields ($9.6\ fb^{-1}$)





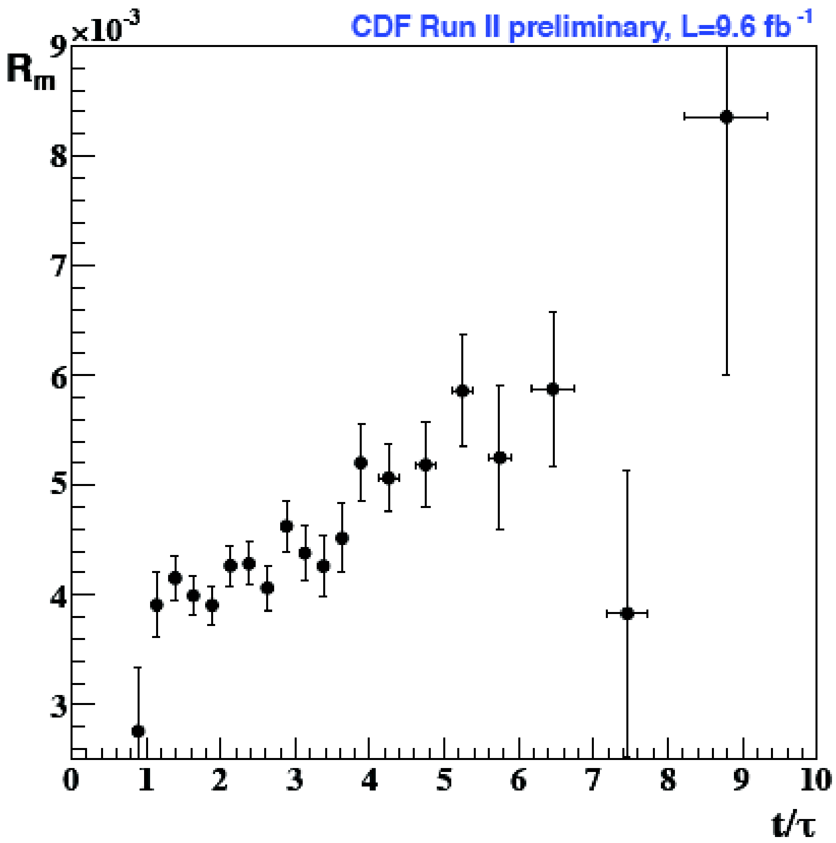
RS: $D^0 \to K^-\pi^+$
7.6 M decays

WS: $D^0 \to K^+\pi^-$
33 k decays

Charm mixing in $D^0 \to K^+ \pi^-$

http://www-cdf.fnal.gov/physics/new/bottom/130408.blessed-DMix_9.6fb/public_note_CDF_D_mix.pdf

Time-dependent fit strategy



In each decay-time bin

fit RS sample to determine signal shape's parameters

fit WS sample with signal shape fixed to RS

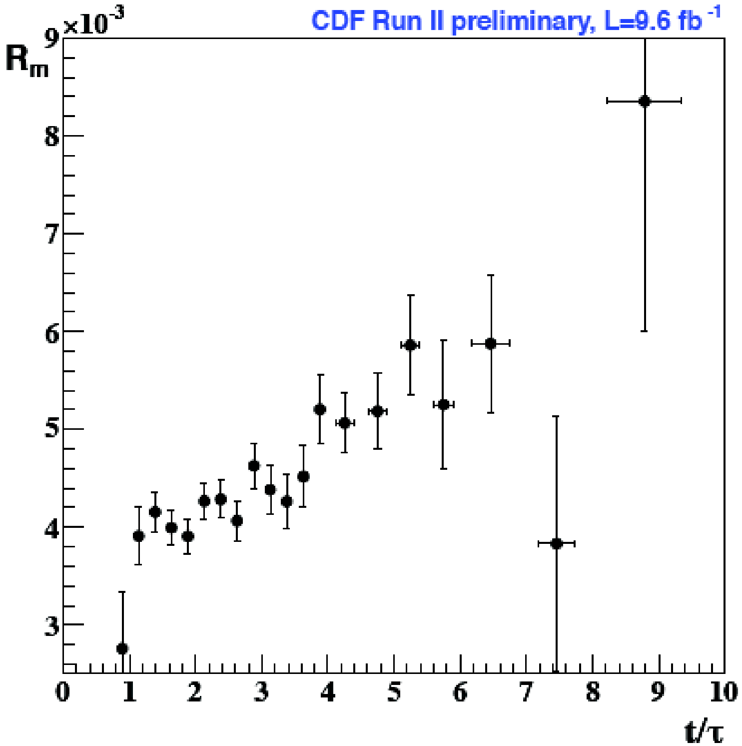
Calculate WS/RS ratio from measured yields

- charm mesons from b-hadron decays
- backgrounds from mis-identified charm decays peaking in $M(D^0 \pi_s)$

⇒ accounted for in the time-dependent fit

Charm mixing in $D^0 \to K^+\pi^-$

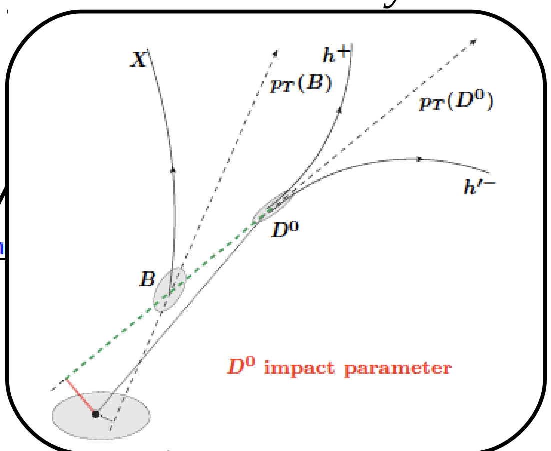
http://www-cdf.fnal.gov/physics/new/bottom/130408.blessed-DMix_9.6fb/public_note_CDF_D_mix.pdf



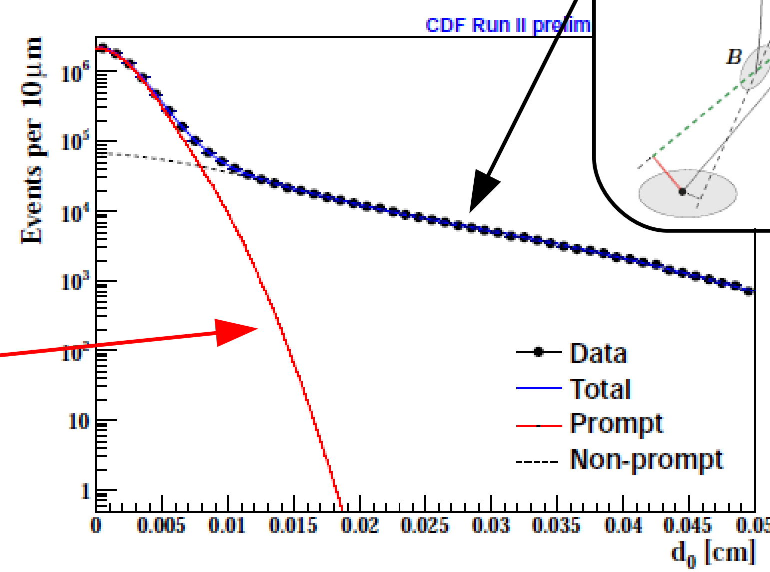
Calculate the WS/RS ratio from measured D^* yields in each decay time bin (20 bins)

$$R_m(t) = \frac{N^{WS}(t) + N_B^{WS}(t)}{N^{RS}(t) + N_B^{RS}(t)}$$

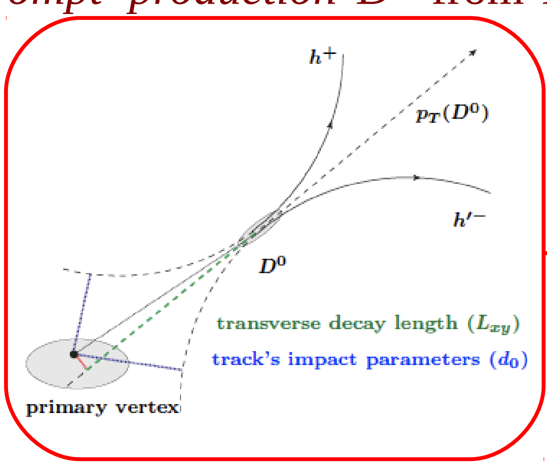
Secondary production D^0 from *B decay*



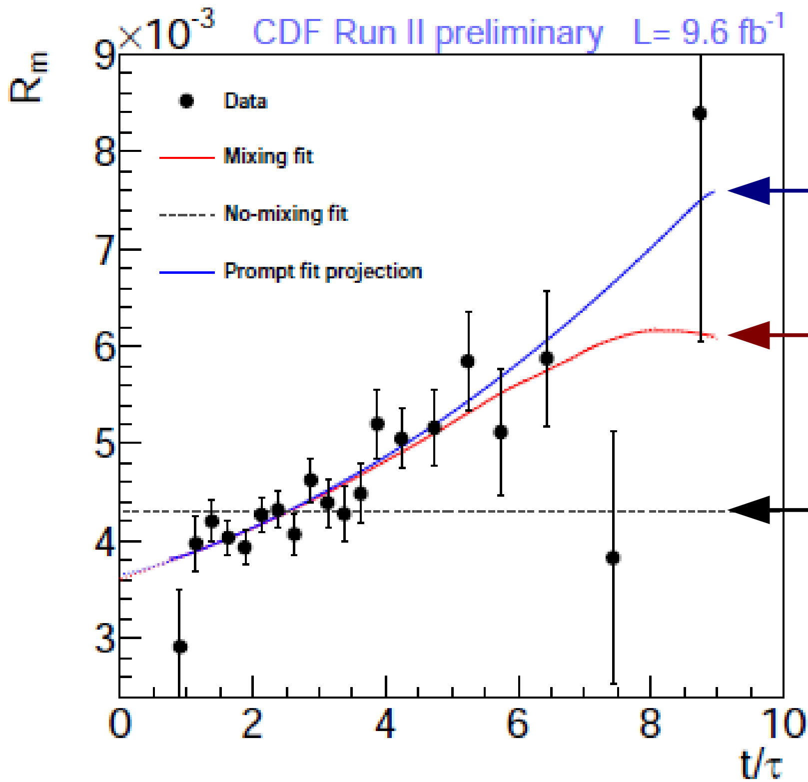
Apply $d_0(D^0) < 60\,\mu m$ to *reduce secondary D*



Prompt production D^0 from *PV*



Charm mixing in $D^0 \to K^+\pi^-$

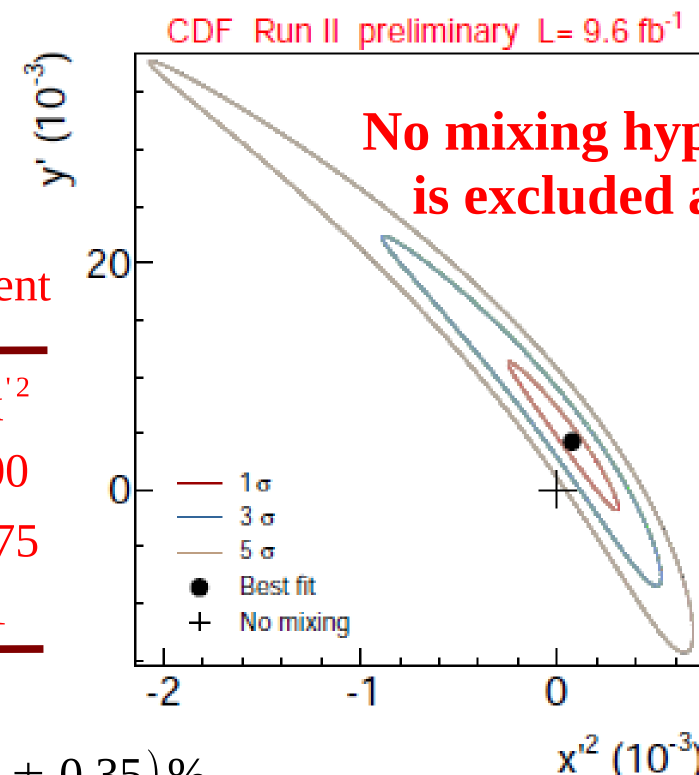


contribution from *B hadron decays is included* in *the WS/RS ratio fit*:

Projection of the prompt component of the fit, i.e. $R(t)$

Best fit, including the effect of D^* from B decays

No-mixing fit $(x'^2 = y' = 0)$



No mixing hypothesis is excluded at 6σ

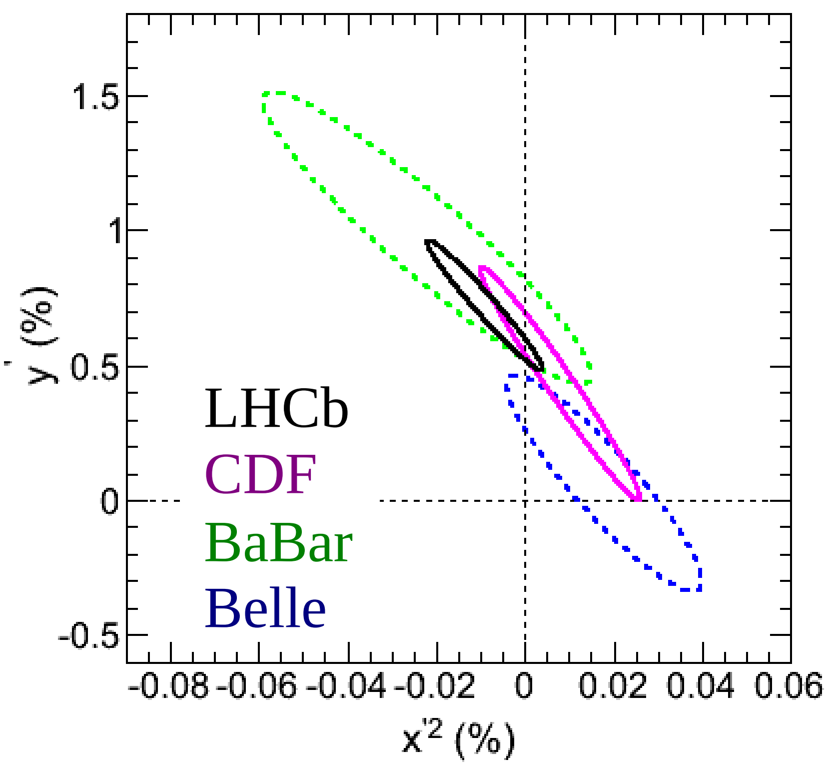
Fit type	Parameter	Fit result	Correlation coefficient		
(χ^2/ndf)		(10^{-3})	R_D	y'	x'^2
Mixing	R_D	3.51 ± 0.35	1	-0.967	0.900
$(16.9/17)$	y'	4.3 ± 4.3		1	-0.975
	x'^2	$+0.08 \pm 0.18$			1

CDF (2007) $R_D = (3.04 \pm 0.55) \times 10^{-3}$

PRL 100 (2008) 121802 $y' = (8.5 \pm 7.6)\%$, $x'^2 = (-0.12 \pm 0.35)\%$

Charm mixing in $D^0 \rightarrow K^+\pi^-$

Experiment	R_D (10^{-3})	y' (10^{-3})	x'^2 (10^{-3})	No-mixing exclusion significance
Belle PRL 96 (2006) 151801	3.64 ± 0.17	$0.6^{+4.0}_{-3.9}$	$+0.18^{+0.21}_{-0.23}$	2.0
BaBar PRL 98 (2007) 211802	3.03 ± 0.19	9.7 ± 5.4	-0.22 ± 0.37	3.9
LHCb PRL 110 (2013) 101802	3.52 ± 0.15	7.2 ± 2.4	-0.09 ± 0.13	9.1
CDF preliminary (2013)	3.51 ± 0.35	4.3 ± 4.3	$+0.08 \pm 0.18$	6.1



Charm mixing in $D^0 \to K^+\pi^-$

The ratio $R(t)$ of WS $D^{*+} \to D^0\pi^+_s \to K^+\pi^-\pi^+_s$ to RS $D^{*+} \to D^0\pi^+_s \to K^-\pi^+\pi^+_s$ decay rates can be approximated (assuming $|x|, |y| << 1$ and no CPV) by:

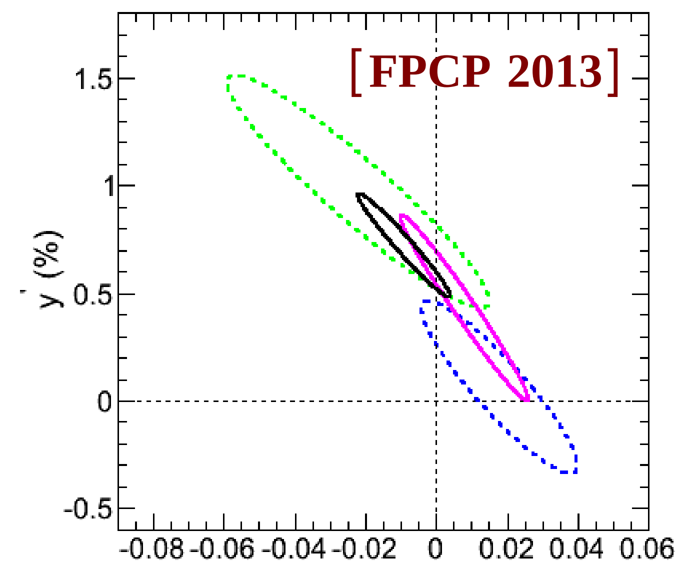
DCS to CF ratio; mixing rate

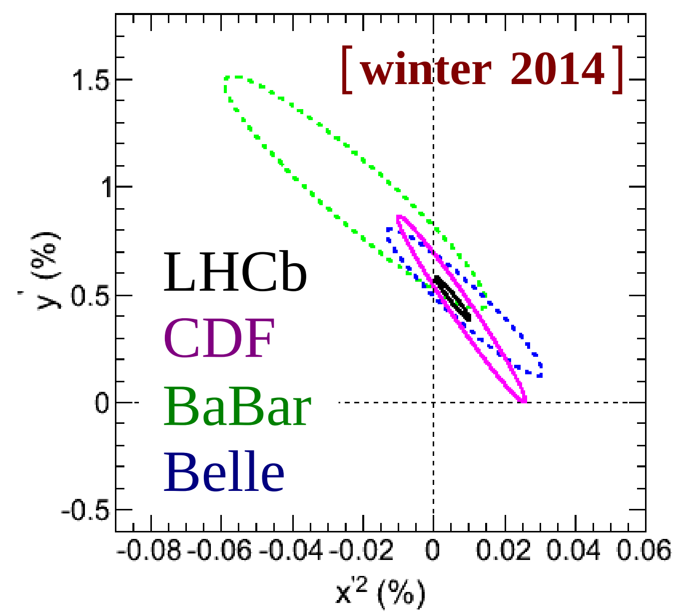
$$R(t) = R_D + \sqrt{R_D}\, y' t + \frac{x'^2 + y'^2}{4} t^2$$

$$x' = x\cos\delta_{K\pi} + y\sin\delta_{K\pi}$$
$$y' = y\cos\delta_{K\pi} - x\sin\delta_{K\pi}$$

$\delta_{K\pi}$: strong phase difference btw DCS and CF amplitudes

Exp	R_D (10^{-3})	y' (10^{-3})	x'^2 (10^{-3})	Σ
Belle PRL 112 (2014) 111801	3.53 ± 0.13	4.6 ± 3.4	$+0.09 \pm 0.22$	5.1
BaBar PRL 98 (2007) 211802	3.03 ± 0.19	9.7 ± 5.4	-0.22 ± 0.37	3.9
LHCb PRL 111 (2013) 251801	3.57 ± 0.07	4.8 ± 1.0	$+0.055 \pm 0.049$	?
CDF preliminary (2013)	3.51 ± 0.35	4.3 ± 4.3	$+0.08 \pm 0.18$	6.1





$D^0 - \overline{D}^0$ mixing

HFAG charm: A. Schwartz, B. Golob, M. Gersabeck

$\chi^2/\text{ndf} = 66.8/41$

FPCP 2013

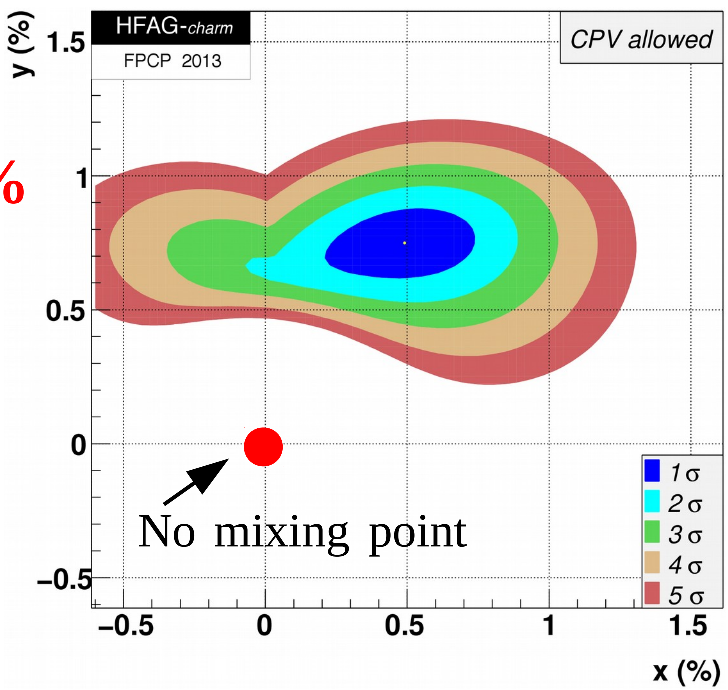
$x = \left(0.49\ ^{+0.17}_{-0.18}\right)\%$

$y = \left(0.75 \pm 0.09\right)\%$

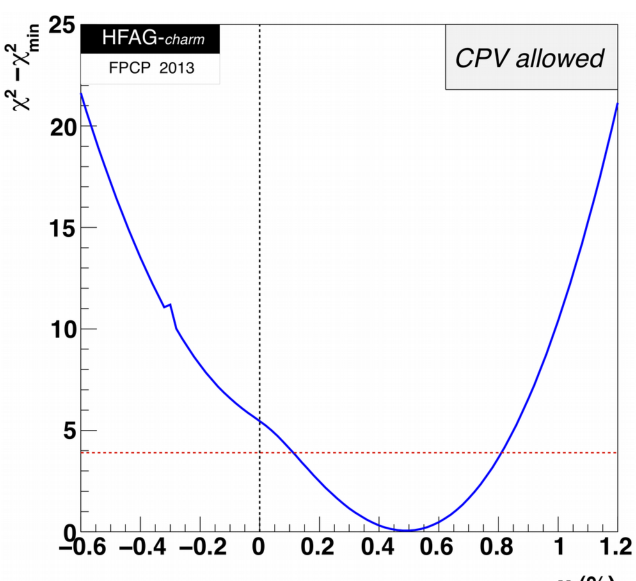
FPCP 2012:

$x = \left(0.63\ ^{+0.19}_{-0.20}\right)\%$

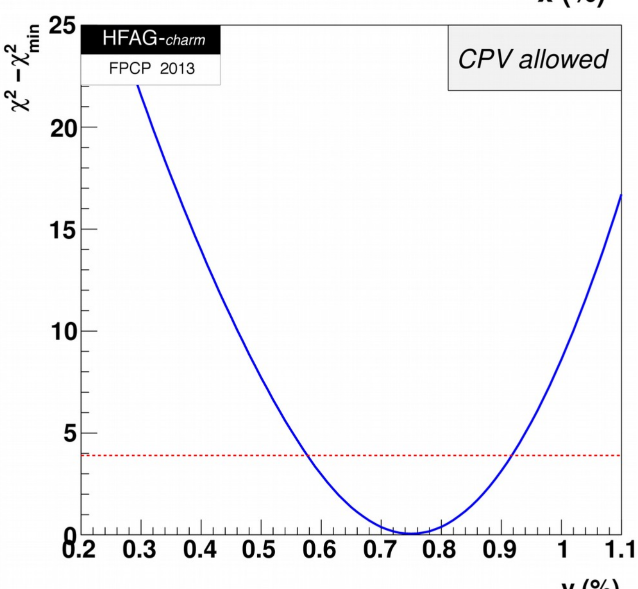
$y = \left(0.75 \pm 0.12\right)\%$



Observable	χ^2	$\sum\chi^2$
y_{CP}	2.94	2.94
A_Γ	0.03	2.97
$x_{K^0\pi^+\pi^-}$ Belle	0.85	3.82
$y_{K^0\pi^+\pi^-}$ Belle	1.68	5.50
$\lvert q/p\rvert_{K^0\pi^+\pi^-}$ Belle	0.34	5.84
$\phi_{K^0\pi^+\pi^-}$ Belle	1.04	6.88
$x_{K^0h^+h^-}$ BaBar	1.48	8.37
$y_{K^0h^+h^-}$ BaBar	0.42	8.79
$R_M(K^+\ell^-\nu)$	0.11	8.90
$x_{K^+\pi^-\pi^0}$ BaBar	6.22	15.12
$y_{K^+\pi^-\pi^0}$ BaBar	2.77	17.89
CLEOc		
$(x/y/R_D/\cos\delta/\sin\delta)$	10.83	28.72
$R_D^+/x'^{2+}/y'^+$ BaBar	7.95	36.67
$R_D^-/x'^{2-}/y'^-$ BaBar	5.82	42.49
$R_D^+/x'^{2+}/y'^+$ Belle	1.72	44.20
$R_D^-/x'^{2-}/y'^-$ Belle	0.66	44.87
$R_D/x'^2/y'$ CDF	3.41	48.28
$R_D/x'^2/y'$ LHCb	8.51	56.78
$A_{KK}/A_{\pi\pi}$ BaBar	0.72	57.50
$A_{KK}/A_{\pi\pi}$ Belle	1.55	59.05
$A_{KK} - A_{\pi\pi}$ CDF	1.66	60.70
$A_{KK} - A_{\pi\pi}$ LHCb (D^* tag)	0.00	60.71
$A_{KK} - A_{\pi\pi}$ LHCb ($B^0 \to D^0\mu X$ tag)	6.11	66.82



$x \leq 0$ excluded at 2.1σ

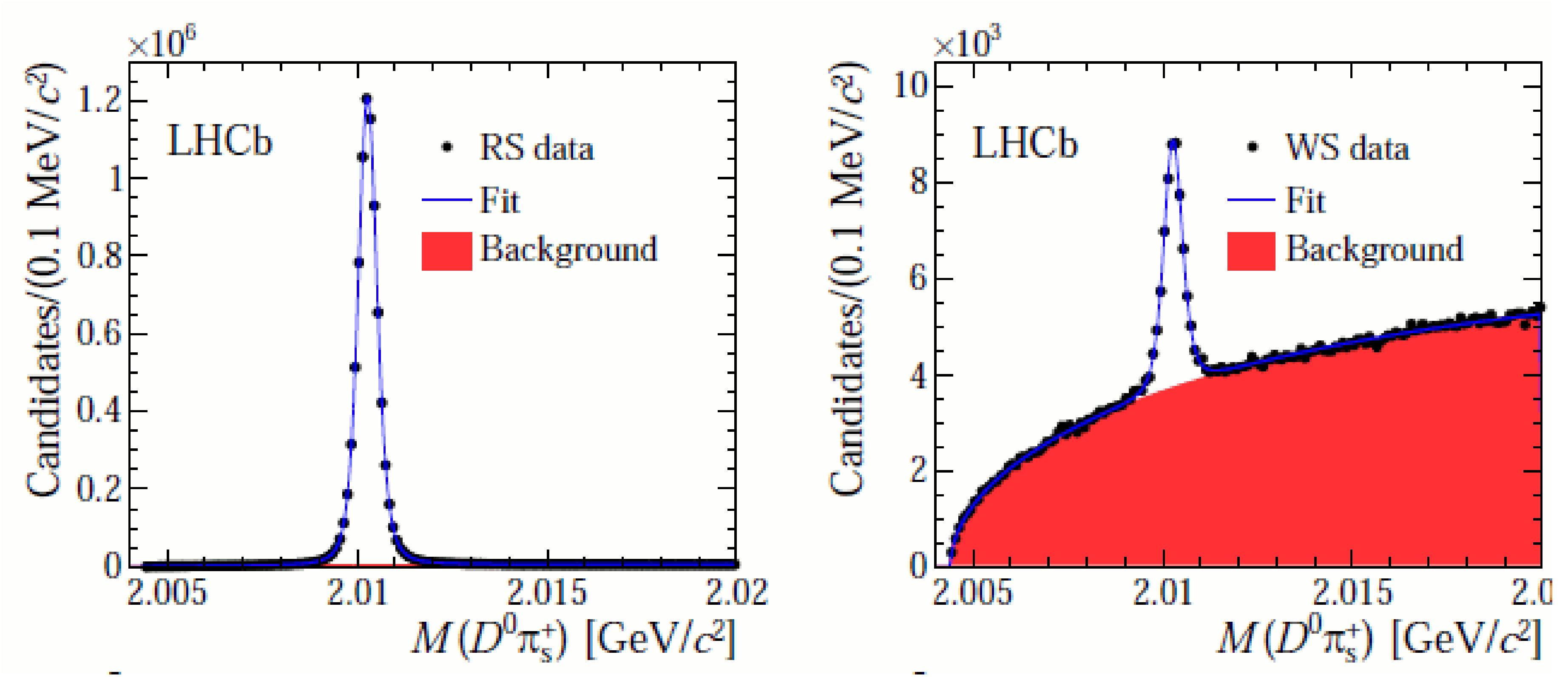


$y \leq 0$ excluded at 7.2σ

Charm mixing in $D^0 \to K^+ \pi^-$

[PRL 110, 101802 (2013), arXiv:1211.1230]

Time-integrated yields (1 fb^{-1})



RS: $D^0 \to K^- \pi^+$
8.4 M decays

WS: $D^0 \to K^+ \pi^-$
36 k decays

Charm mixing in $D^0 \to K^+\pi^-$

[PRL 110, 101802 (2013), arXiv:1211.1230]

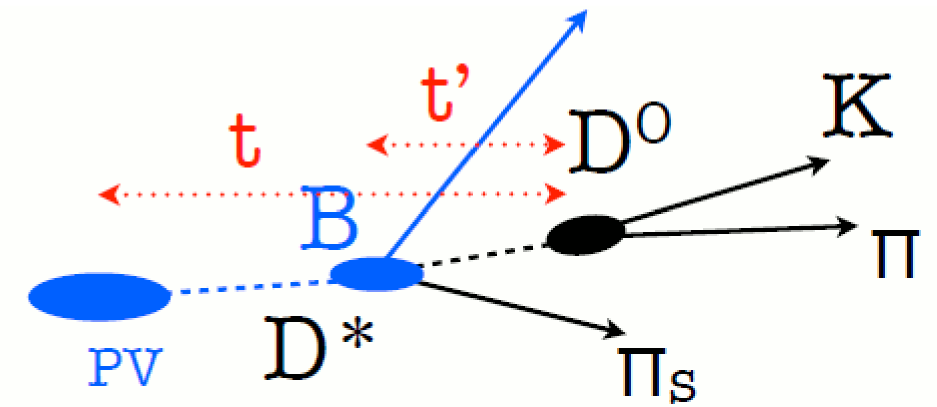
measured WS/RS ratio:

$$R^m(t) = \frac{N^{WS}(t) + N_B^{WS}(t)}{N^{RS}(t) + N_B^{RS}(t)} = R(t)\left\{1 - \underbrace{f_B^{RS}(t)\left[1 - \frac{R_B(t)}{R(t)}\right]}_{\text{bias from secondary D decays}}\right\}$$

where:

$$f_B^{RS}(t) = \frac{N_B^{RS}(t)}{N^{RS}(t) + N_B^{RS}(t)}$$

$$R_B(t) = \frac{N_B^{WS}(t)}{N_B^{RS}(t)}$$

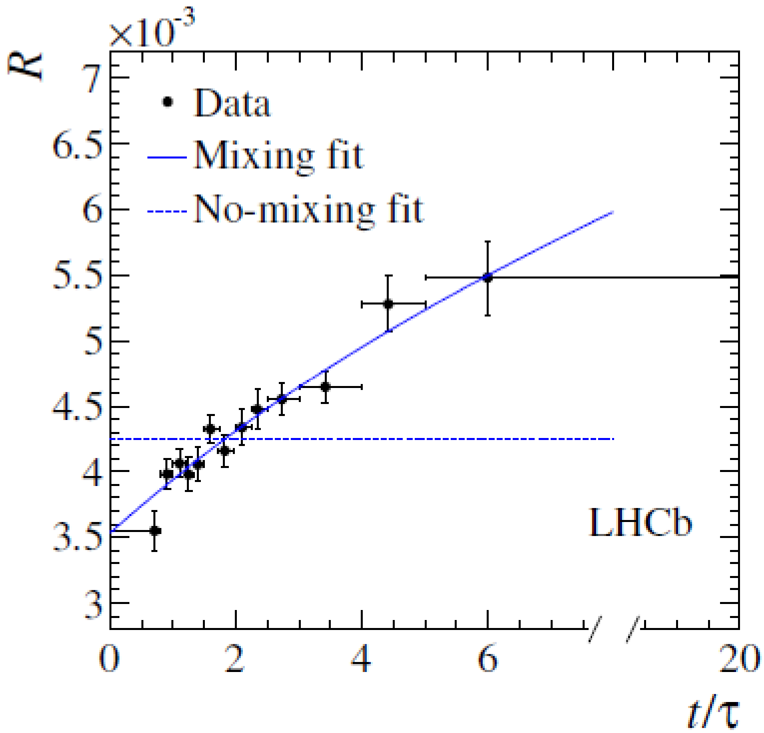


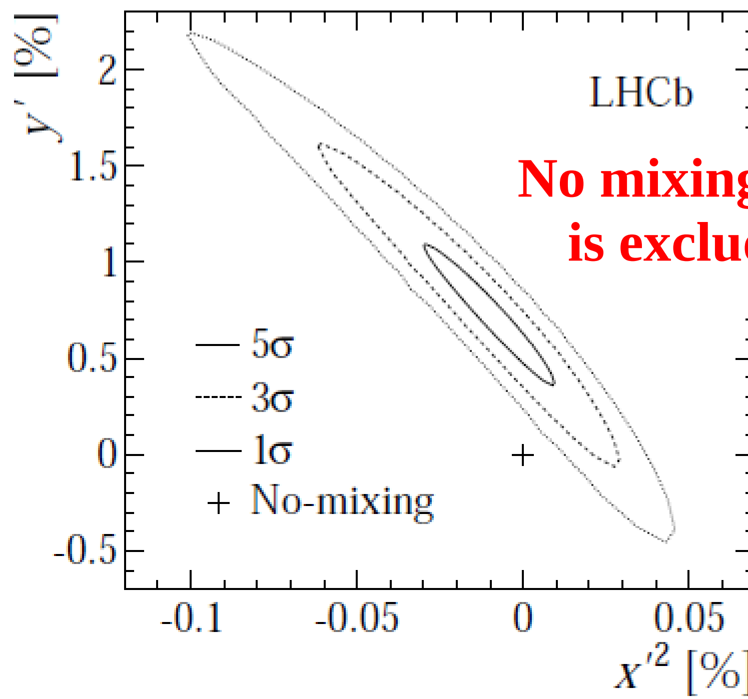
$c\tau(B) \approx 450\ \mu m$, D from B have non-zero impact parameter

cut on $\chi^2(IP)$, remaining (3%): included in the fit, shape estimated from evts reconstructed as $B \to D^*(3)\pi$, $B \to D^*\mu X$, $D^0\mu X$

Charm mixing in $D^0 \to K^+\pi^-$ (1 fb^{-1})

[PRL 110, 101802 (2013), arXiv:1211.1230]





No mixing hypothesis is excluded at 9 σ

1st observation of charm mixing from a single expt

Fit type	Parameter	Fit result	Correlation coefficient		
(χ^2/ndf)		(10^{-3})	R_D	y'	x'^2
Mixing	R_D	3.52 ± 0.15	1	-0.954	0.882
(9.5/10)	y'	7.2 ± 2.4		1	-0.973
	x'^2	-0.09 ± 0.13			1

ADS observables

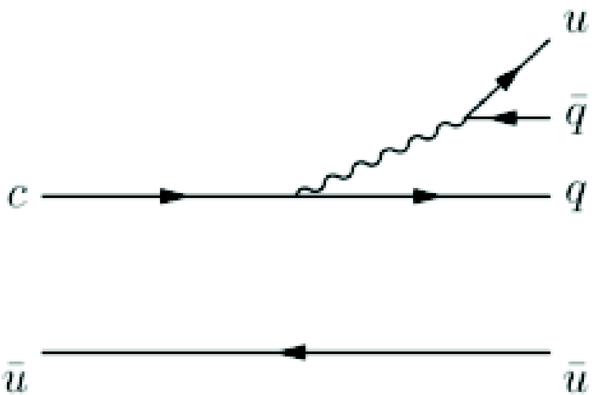
- (R_+, R_-) **instead of** (R_{ADS}, A_{ADS}) **whenever available**

Effect of D-$\bar{D}$ mixing on γ

- M. Rama, arXiv:1307.4384
- $R^{\mp} = r_B^2 + r_D^2 + 2 r_B r_D \cos(\delta_B \mp \gamma + \delta_D)$

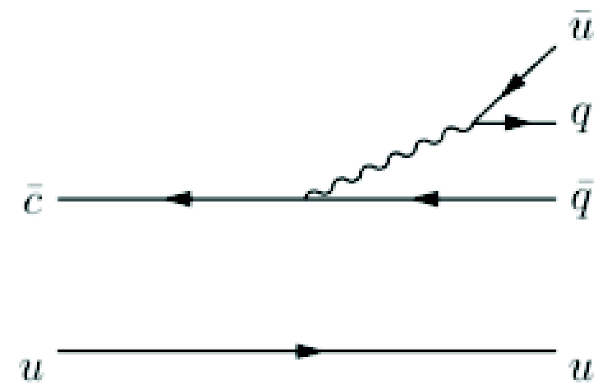
  → $R^{\mp} = r_B^2 + r_D^2 + 2 r_B r_D \cos(\delta_B \mp \gamma + \delta_D) - y r_D \cos\delta_D - y r_B \cos(\delta_B \mp \gamma) + x r_D \sin\delta_D - x r_B \sin(\delta_B \mp \gamma)$
- tried on the current LHCb average (DK): ~ 1 degree difference

CP violation in charm



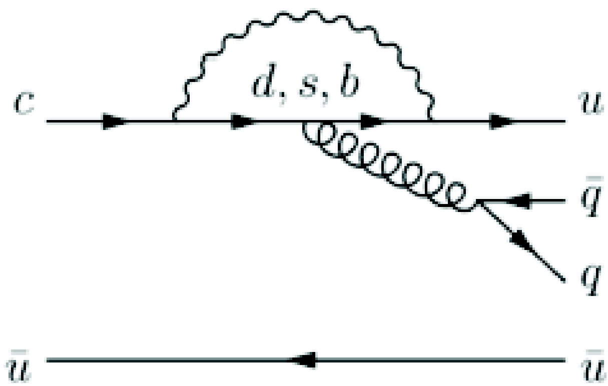
Tree level

CP



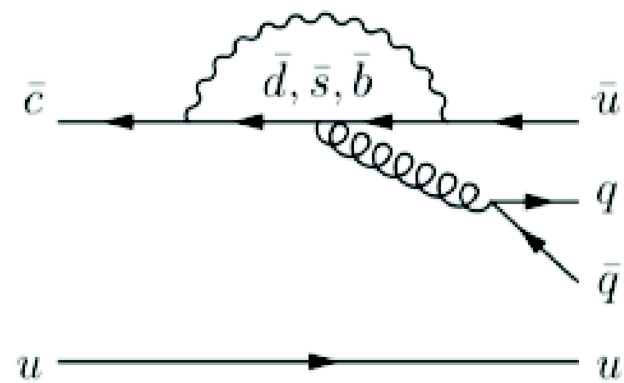
$A_1 = \rho_1 e^{i\delta_1} e^{i\theta_1}$

$\bar{A}_1 = \rho_1 e^{i\delta_1} e^{-i\theta_1}$



Loop level

CP



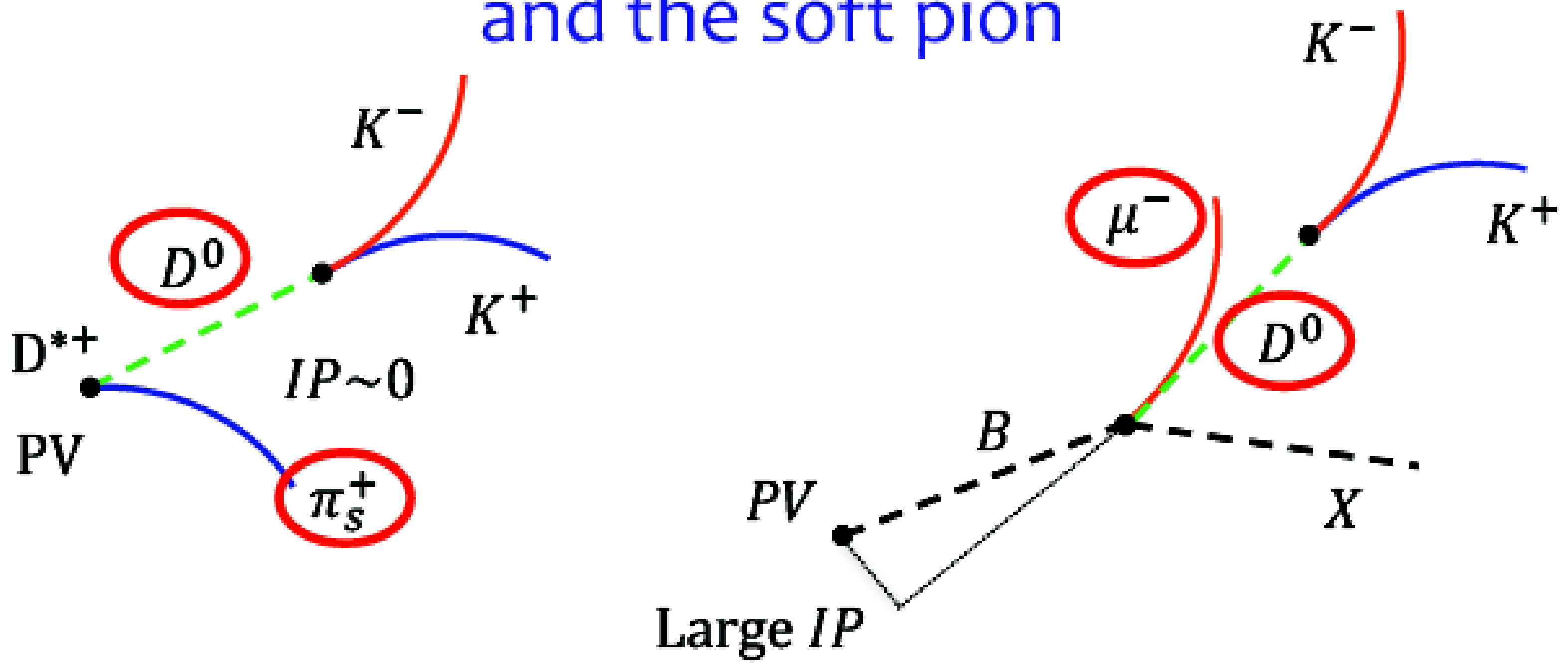
$A_2 = \rho_2 e^{i\delta_2} e^{i\theta_2}$

$\bar{A}_2 = \rho_2 e^{i\delta_2} e^{-i\theta_2}$

$$|\bar{A}_1 + \bar{A}_2|^2 - |A_1 + A_2|^2 = 4\rho_1\rho_2 \sin(\theta_1 - \theta_2) \sin(\delta_1 - \delta_2)$$

CP violation in charm

Experimentally we can tag D^0 flavour at production by means of the charge of the muon and the soft pion



CP violation in charm

ΔA_{CP} π-tagged

What we measure is the physical asymmetry plus asymmetries due both to production and detector effects

$$A_{\text{raw}}(f) = A_{CP}(f) + A_D(f) + A_D(\pi_s^+) + A_P(D^{*+})$$

CP asymmetry

Any charge-dependent asymmetry in slow pion reconstruction

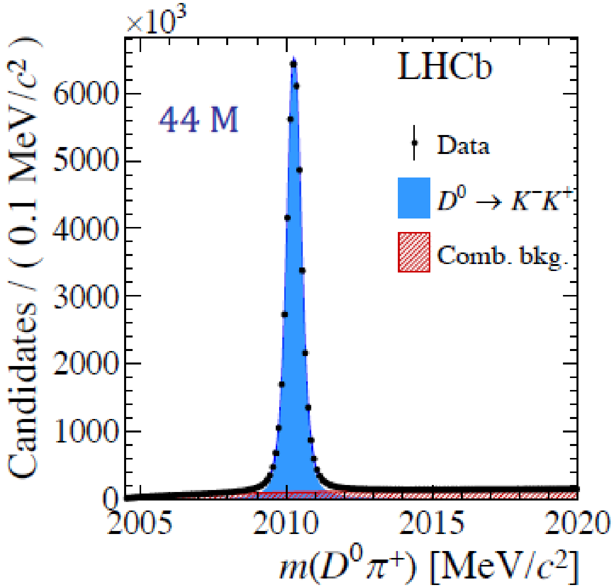
$D^{*\pm}$ production asymmetry

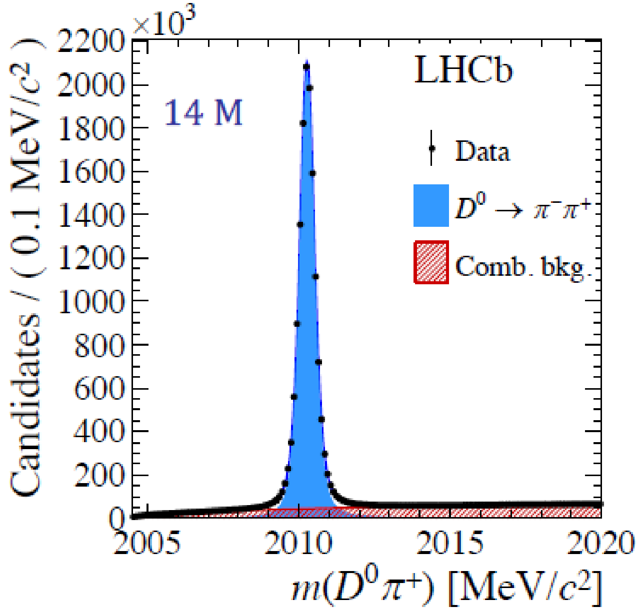
- No detection asymmetry for D^0 decays to K^-K^+ or $\pi^-\pi^+$
- … if we take the raw asymmetry difference

$$\Delta A_{CP} \equiv A_{raw}(KK) - A_{raw}(\pi\pi) = A_{CP}(KK) - A_{CP}(\pi\pi)$$

- the D^{*+} production and the slow pion detection asymmetries will cancel

CP violation in charm

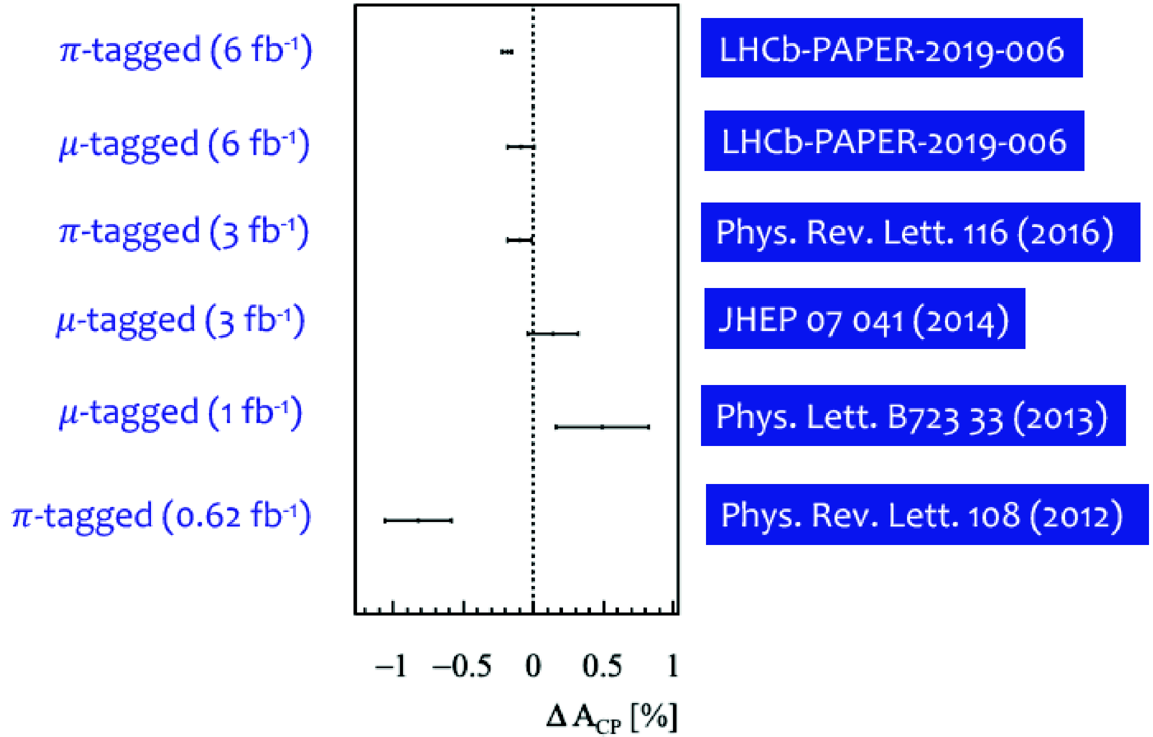




$$\Delta A_{CP} = (-15.4 \pm 2.9)\times 10^{-4}$$

5.3 standard deviations from zero

This is the first observation of *CP* violation in the decay of charm hadrons



Quantum correlated D mesons at CLEO-c

- $\Psi \to D\bar{D}$ are produced coherently in the $C = -1$ state.

$$\frac{(|D\rangle|\bar{D}\rangle - |\bar{D}\rangle|D\rangle)}{\sqrt{2}}$$

- If $\Psi(3770)$ decays into two states F and G, then decay rate (Γ) depends on their CP eigenvalue.

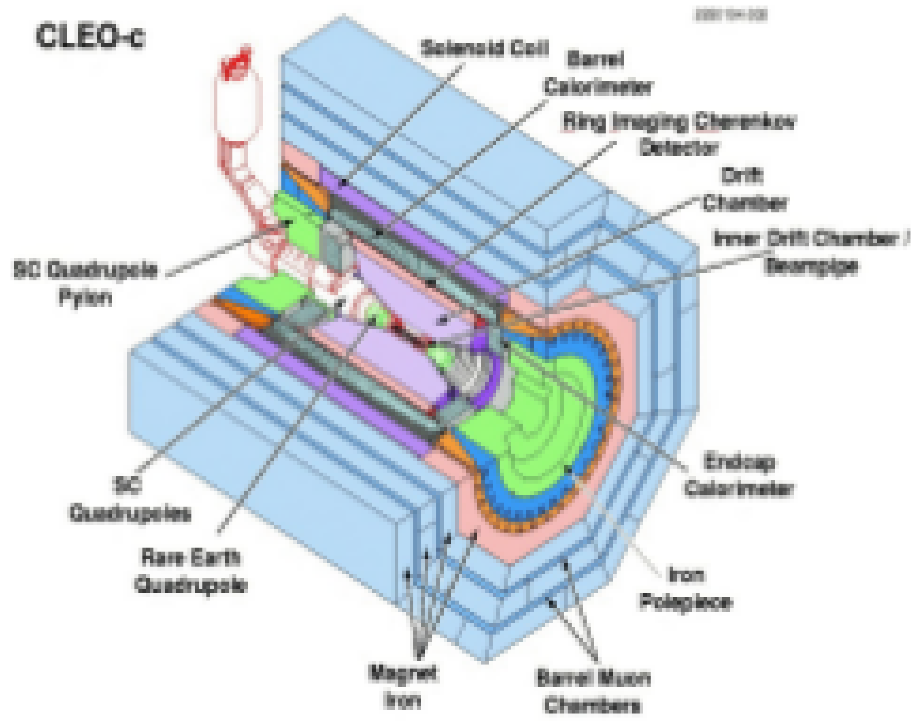


Figure : CLEO-c detector.

- $F = \mathrm{CP\ even\ (odd)}$, $G = \mathrm{CP\ odd\ (even)} \Rightarrow$ two-fold enhancement.
- $F = \mathrm{CP\ even\ (odd)}$, $G = \mathrm{CP\ even\ (odd)} \Rightarrow$ zero.
- Γ changes with F or G being quasi CP states $(\pi^+\pi^-\pi^0)$ or self conjugate states $(K_S^0\pi^+\pi^-)$.

CLEO-c data sample

A total of 818 pb^{-1} data collected at the CLEO-c - $D\bar{D}$ pairs from the $\Psi(3770)$.
One of the D mesons reconstructed to $K_S^0\pi^+\pi^-\pi^0$ (signal) and the other one to any other channel (tag).

Type	mode	yield
CP even tags	K^+K^-	200.7 ± 14.2
	$\pi^+\pi^-$	91.45 ± 9.59
	$K_S^0\pi^0\pi^0$	106.3 ± 10.9
	$K_L^0\pi^0$	357.3 ± 20.2
	$K_L^0\omega$	162.1 ± 13.7
CP odd tags	$K_S^0\pi^0$	93.97 ± 9.84
	$K_S^0\eta$	11.64 ± 3.68
	$K_S^0\eta'$	7 ± 3
Quasi CP tags	$\pi^+\pi^-\pi^0$	428.8 ± 21.7
Self conjugate tags	$K_S^0\pi^+\pi^-$	504.8 ± 23.3
	$K_L^0\pi^+\pi^-$	864.1 ± 46.1
	$K_s^0\pi^+\pi^-\pi^0$	176.4 ± 14.8
Flavour tag	$K^\pm e^\mp\nu$	1010 ± 32

CP content (F_+)

F_+ = fractional CP-even content

The double tagged yield for the signal and tag

$$M(S|T) = 2N_{D\bar{D}} \times BF(S) \times BF(T) \times \epsilon(S|T) \times [1 - \lambda_{CP}(2F_+ - 1)]\,.$$

The single tag yield

$$S(T) = 2N_{D\bar{D}} \times BF(T) \times \epsilon(T).$$

If we assume $\epsilon(S|T) = \epsilon(S)\epsilon(T)$, then we get N^+ for CP odd tag and N^- for CP even tag as follows:

$$N^{\pm} = \frac{M(S|T)}{S(T)} = BF(S) \times \epsilon(S) \times [1 - \lambda_{CP}(2F_+ - 1)]\,.$$

From these, we can calculate F_+ as

$$F_+ = \frac{N^+}{N^+ + N^-}; \qquad F_+ = 1 \Rightarrow \text{CP even}, F_+ = 0 \Rightarrow \text{CP odd}.$$

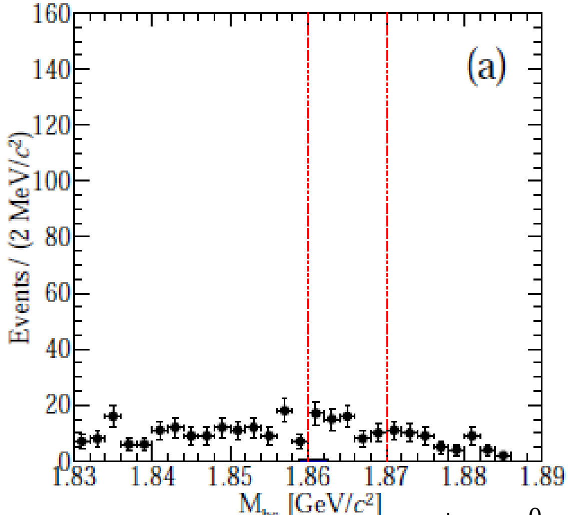
CP content (F_+) for $D \to \pi^+ \pi^- \pi^0$

[arXiv:1410.3964]

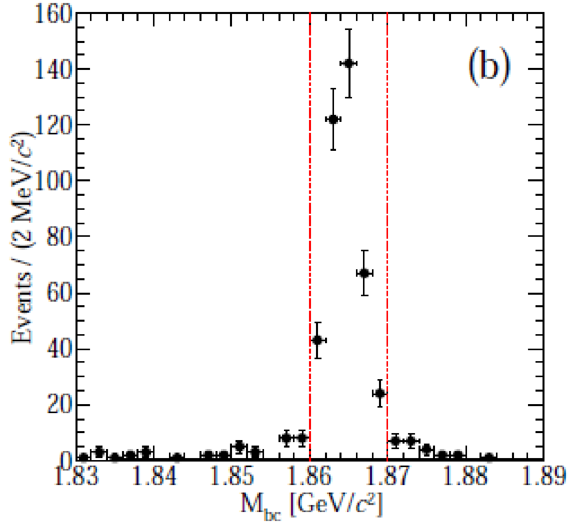
Table 1: D final states reconstructed in this analysis.

Type	Final states
Signal	$\pi^+\pi^-\pi^0$, $K^+K^-\pi^0$
CP-even	K^+K^-, $\pi^+\pi^-$, $K^0_S\pi^0\pi^0$, $K^0_L\pi^0$, $K^0_L\omega$
CP-odd	$K^0_S\pi^0$, $K^0_S\omega$, $K^0_S\eta$, $K^0_S\eta'$

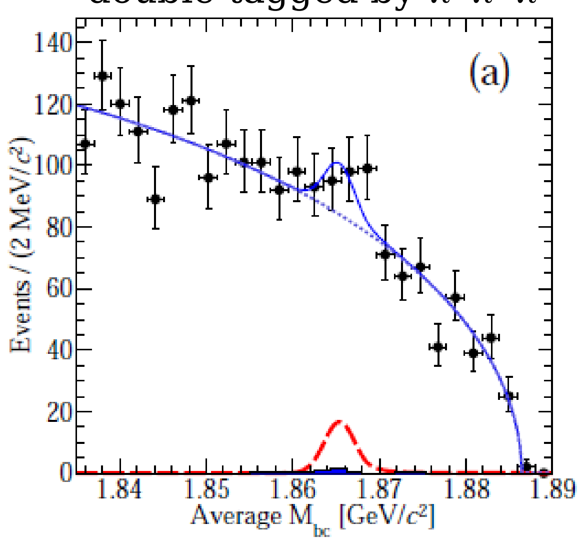
tagged by CP-even eigenstates



tagged by CP-odd eigenstates



double tagged by $\pi^+ \pi^- \pi^0$



$\Rightarrow$ **$F_+ = 0.968 \pm 0.017 \pm 0.006$**

$$A_{CP+} = \frac{2 r_B \sin\delta_B \sin\gamma}{R_{CP+}}$$

$$R_{CP+} = 1 + r_B^2 + 2 r_B \cos\delta_B \cos\gamma$$

$\Rightarrow$

$$A_{F+} = (2F_+ - 1) . \frac{2 r_B \sin\delta_B \sin\gamma}{R_{F+}}$$

$$R_{F+} = 1 + r_B^2 + (2F_+ - 1) . 2 r_B \cos\delta_B \cos\gamma$$

dilution factor of 0.936

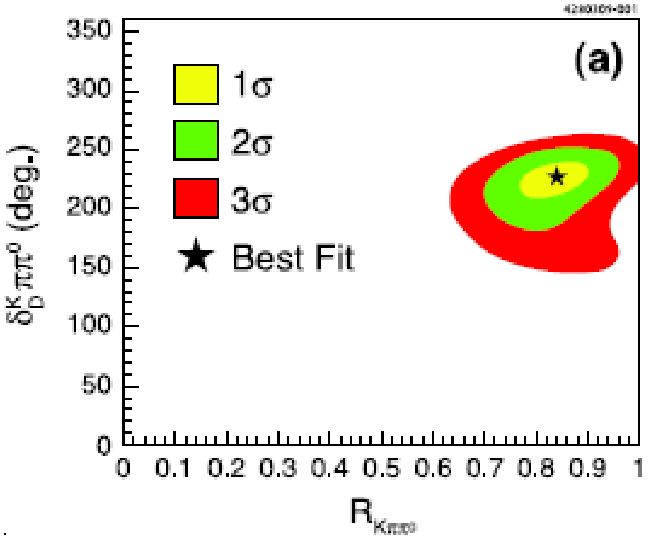
$D \to K^+ \pi^- \pi^0$

[arXiv:0903.4853]

$\neq K\pi$, amplitude ratio and vary-phase space

$D \to K^+ \pi^- \pi^0$

[arXiv:0903.4853]



$B \to D^{(*)} K^{(*)}$ Dalitz analysis

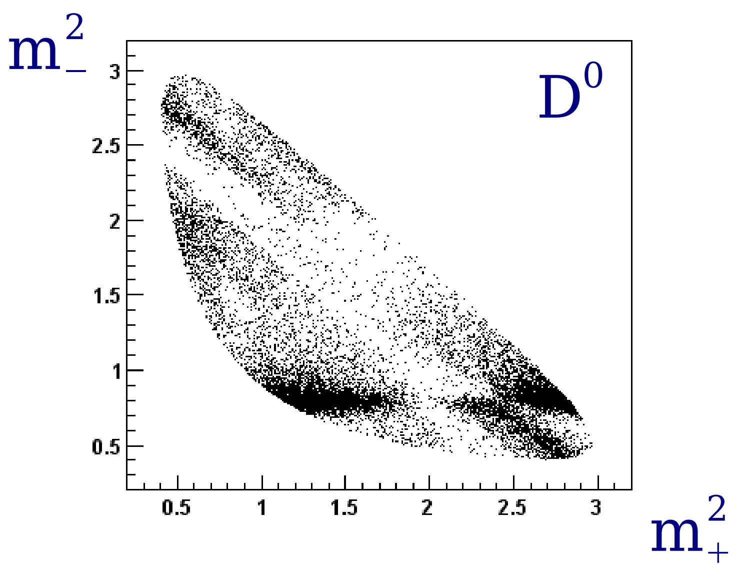
Reconstruction of three−body final states $D^0, \overline{D}^0 \to K_S \pi^+ \pi^-$

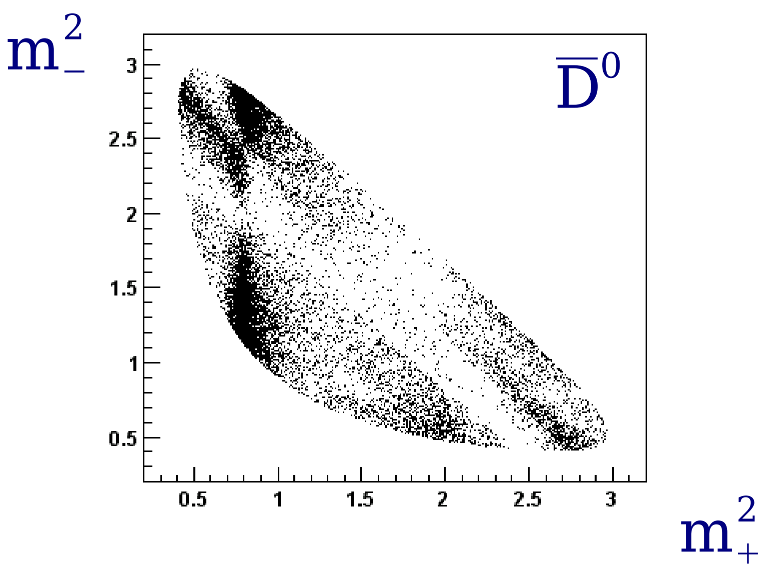
Amplitude for each Dalitz point is described as:

$$\overline{D}^0 \to K_S \pi^+ \pi^- \sim f(m_+^2, m_-^2)$$
$$D^0 \to K_S \pi^+ \pi^- \sim f(m_-^2, m_+^2)$$

$$B^+ \to (K_S \pi^+ \pi^-)_D K^+ : f(m_+^2, m_-^2) + r e^{i(\delta_B + \gamma)} f(m_-^2, m_+^2)$$

$m_- = M(K_S \pi^-)$
$m_+ = M(K_S \pi^+)$





$$B^- \to (K_S \pi^+ \pi^-)_D K^- : f(m_-^2, m_+^2) + r e^{i(\delta - \gamma)} f(m_+^2, m_-^2)$$

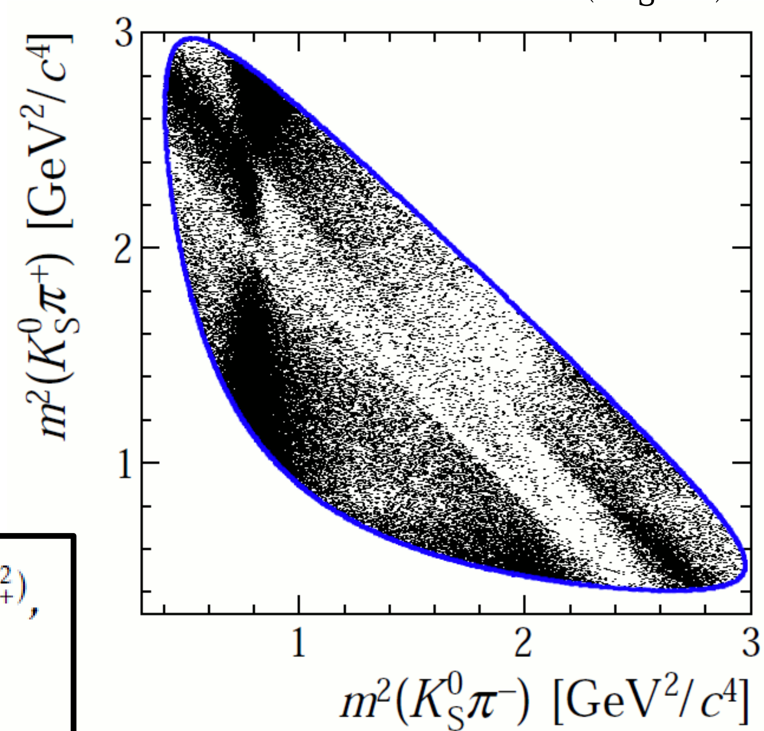
Simultaneous fit of B^+ and B^- to extract parameters r_B, ϕ_3 and δ_B

Note: 2 fold ambiguity on γ: $(\gamma, \delta_B) \to (\gamma + \pi, \delta_B + \pi)$

γ and $D \to K_S^0 \pi^+ \pi^-$

$m_+ = m(K_S^0 \pi^+)$
$m_- = m(K_S^0 \pi^-)$

best standalone meas. of γ
especially in B-factory
$K_S^0 \pi^+ \pi^-$ is $K_S^0 \rho^0$ (GLW), $K^{*+} \pi^-$...



internal structure in the Dalitz plot:
- ◦ nonresonant contributions
- ◦ resonant contributions, as $D^0 \to AR(\to BC)$ where R is short-lived intermediate resonance

$$A_D(m_-^2, m_+^2) = a_{NR}(m_-^2, m_+^2) e^{i\delta_{NR}(m_-^2, m_+^2)} + \sum_R a_R(m_-^2, m_+^2) e^{i\delta_R(m_-^2, m_+^2)},$$
$$= a(m_-^2, m_+^2) e^{i\delta(m_-^2, m_+^2)},$$

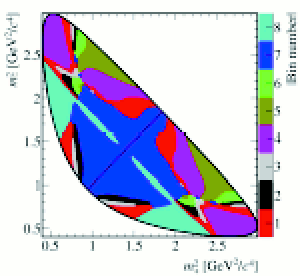
- ◦ a_{NR} and δ_{NR} are magnitude and phase of nonresonant component
- ◦ a_R and δ_R are magnitude and phase of each resonance

⇒ isobar formalism used to model $D \to K_S \pi^+ \pi^-$ decays: resonances approximated with relativistic Breit-Wigner functions, with factors to account for spins of the particles and form factors to account of the relative momenta of the decay products

Two methods for accessing the D decay information

- D dalitz plot from B decay is superposition of D^0 and $\overline{D}^0$
- it will differ between B^+ and B^-
- differences are related to r_B, δ_B and γ (or $x_\pm$, $y_\pm$)

$\Rightarrow$ two ways to deal with the varying r_D and δ_D:

Model dependent	Model independent
r_D and δ_D determined from flavour tagged decays via amplitude model	 Use CLEO data to measure average values of r_D and δ_D in bins
No interference, no direct access to phase information	Small loss in statistical precision
Systematic uncertainties due to model hard to quantify	Direct phase information, uncertainties on which are easily propagated

γ and $D \to K_S^0 \pi^+ \pi^-$

amplitudes for the CP-conjugate $D \to K_S^0 h^+ h^-$ decays:

$$A(D^0 \to K_S^0 h^+ h^-) = A_D(m_-^2, m_+^2) = a(m_-^2, m_+^2) e^{i\delta(m_-^2, m_+^2)},$$

$$A(\overline{D}^0 \to K_S^0 h^+ h^-) = \overline{A}_D(m_-^2, m_+^2) = \bar{a}(m_-^2, m_+^2) e^{i\bar{\delta}(m_-^2, m_+^2)} = A_D(m_+^2, m_-^2).$$

$\overline{A_D}(m_-^2, m_+^2) = A_D(m_+^2, m_-^2)$ occurs because final state has spin zero

amplitude of the cascade decay $B^- \to DK^-$ with $D \to K_S^0 h^+ h^-$ at the coordinate (m_-^2, m_+^2) is:

$$\begin{aligned} A(B^- \to D(\to K_S^0 h^+ h^-)K^-) \\ &\equiv A(B^- \to D^0(\to K_S^0 h^+ h^-)K^-) + A(B^- \to \overline{D}^0(\to K_S^0 h^+ h^-)K^-) \\ &= A_B A_D(m_-^2, m_+^2) + A_B r_B e^{i(\delta_B - \gamma)} \overline{A}_D(m_-^2, m_+^2) \\ &\propto a(m_-^2, m_+^2) e^{i\delta(m_-^2, m_+^2)} + r_B e^{i(\delta_B - \gamma)} \bar{a}(m_-^2, m_+^2) e^{i\bar{\delta}(m_-^2, m_+^2)}. \end{aligned}$$

partial width for the B^- decay is given by:

$$\frac{d\Gamma(B^- \to D(\to K_S^0 h^+ h^-)K^-)}{dm_-^2\, dm_+^2} = \left| A(B^- \to D(\to K_S^0 h^+ h^-)K^-) \right|^2$$

$$\propto a^2(m_-^2, m_+^2) + r_B^2 \bar{a}^2(m_-^2, m_+^2) +$$

$$2a(m_-^2, m_+^2)\bar{a}(m_-^2, m_+^2) r_B \cos[(\delta_B - \gamma) - (\delta(m_-^2, m_+^2) - \bar{\delta}(m_-^2, m_+^2))].$$

define the strong-phase difference between the D^0 and $\overline{D}^0$ amplitudes:

$$\delta_D(m_-^2, m_+^2) = \delta(m_-^2, m_+^2) - \bar{\delta}(m_-^2, m_+^2) = \delta(m_-^2, m_+^2) - \delta(m_+^2, m_-^2),$$

$$\frac{d\Gamma(B^- \to D(\to K_S^0 h^+ h^-)K^-)}{dm_-^2\, dm_+^2} \propto a^2 + r_B^2 \bar{a}^2 + 2a\bar{a}r_B \cos[(\delta_B - \gamma) - \delta_D],$$
$$\propto a^2 + r_B^2 \bar{a}^2 + 2a\bar{a}r_B[\cos(\delta_B - \gamma)\cos\delta_D + \sin(\delta_B - \gamma)\sin\delta_D].$$

Similarly, amplitude for B^+ decay:

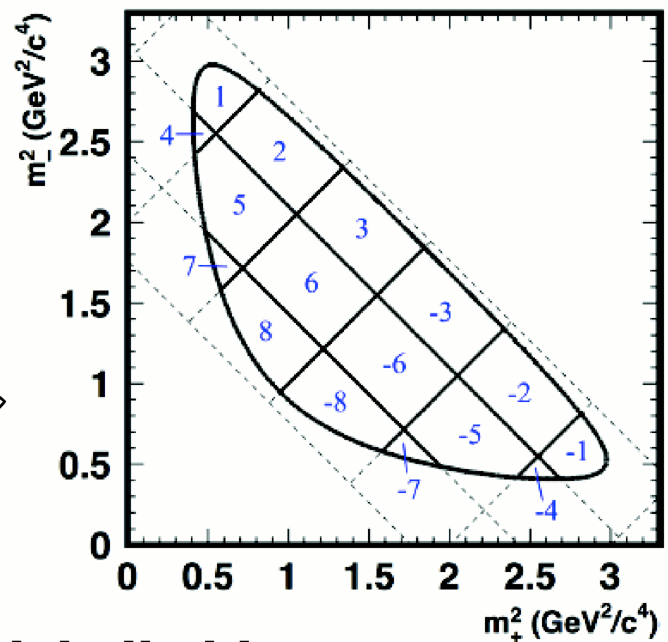
$$A(B^+ \to D(\to K_S^0 h^+ h^-)K^+)$$
$$\equiv A(B^+ \to \overline{D}^0(\to K_S^0 h^+ h^-)K^+) + A(B^+ \to D^0(\to K_S^0 h^+ h^-)K^+)$$
$$= A_B \overline{A}_D(m_-^2, m_+^2) + A_B r_B e^{i(\delta_B+\gamma)} A_D(m_-^2, m_+^2)$$
$$\propto \bar{a}(m_-^2, m_+^2) e^{i\bar{\delta}(m_-^2, m_+^2)} + r_B e^{i(\delta_B+\gamma)} a(m_-^2, m_+^2) e^{i\delta(m_-^2, m_+^2)},$$

$$\frac{d\Gamma(B^+ \to D(\to K_S^0 h^+ h^-)K^+)}{dm_-^2\, dm_+^2} \propto \bar{a}^2 + r_B^2 a^2 + 2a\bar{a}r_B \cos[(\delta_B + \gamma) + \delta_D]$$
$$\propto \bar{a}^2 + r_B^2 a^2 + 2a\bar{a}r_B[\cos(\delta_B + \gamma)\cos\delta_D - \sin(\delta_B + \gamma)\sin\delta_D].$$

in presence of CP violation and when $r_B \neq 0$, $D \rightarrow K_S^0 \pi^+ \pi^-$ DP distribution no longer same for $B^- \rightarrow DK^-$ and $B^+ \rightarrow DK^+$ decays under transformation $m_+^2 \Leftrightarrow m_-^2$, asymmetry depends of Dalitz plot coordinate strategy: measure asymmetries in integrated regions of the Dalit plot

interference term $A_D(m_-^2, m_+^2) A_D^*(m_+^2, m_-^2)$ is symmetric under exchange $(m_-^2, m_+^2) \Leftrightarrow (m_+^2, m_-^2) \Rightarrow$ define bin regions which are symmetric around the line $m_-^2 = m_+^2$

an example with 2k bins ⇒



defining integrals over the region of the Dalitz plot labelled bin i:

$$T_i = \int_i dm_-^2\, dm_+^2\, a^2(m_-^2, m_+^2),$$

$$T_{-i} = \int_{-i} dm_-^2\, dm_+^2\, a^2(m_-^2, m_+^2) = \int_{+i} dm_-^2 dm_+^2\, \bar{a}^2(m_-^2, m_+^2),$$

$$c_i = \frac{1}{\sqrt{T_{+i} T_{-i}}} \int_i dm_-^2\, dm_+^2\, a(m_-^2, m_+^2)\, \bar{a}(m_-^2, m_+^2)\, \cos\delta_D(m_-^2, m_+^2),$$

$$s_i = \frac{1}{\sqrt{T_{+i} T_{-i}}} \int_i dm_-^2\, dm_+^2\, a(m_-^2, m_+^2)\, \bar{a}(m_-^2, m_+^2)\, \sin\delta_D(m_-^2, m_+^2).$$

terms c_i and s_i contain $\cos\delta_D$ and $\sin\delta_D$

equations for the rate of events in bins i and −i of the Dalitz plots for B^- and B^+ decays:

$$\Gamma_{+i}(B^- \to D(\to K_S^0 h^+ h^-)K^-) \propto \left[T_{+i} + (x_-^2 + y_-^2)T_{-i} + 2\sqrt{T_{+i}T_{-i}}(x_- c_{+i} + y_- s_{+i})\right],$$

$$\Gamma_{-i}(B^- \to D(\to K_S^0 h^+ h^-)K^-) \propto \left[T_{-i} + (x_-^2 + y_-^2)T_{+i} + 2\sqrt{T_{+i}T_{-i}}(x_- c_{-i} + y_- s_{-i})\right],$$

$$\Gamma_{+i}(B^+ \to D(\to K_S^0 h^+ h^-)K^+) \propto \left[T_{-i} + (x_+^2 + y_+^2)T_{+i} + 2\sqrt{T_{+i}T_{-i}}(x_+ c_{+i} - y_+ s_{+i})\right],$$

$$\Gamma_{-i}(B^+ \to D(\to K_S^0 h^+ h^-)K^+) \propto \left[T_{+i} + (x_+^2 + y_+^2)T_{-i} + 2\sqrt{T_{+i}T_{-i}}(x_+ c_{-i} - y_+ s_{-i})\right].$$

$$x_\pm \equiv r_B \cos(\delta_B \pm \gamma),$$

$$y_\pm \equiv r_B \sin(\delta_B \pm \gamma).$$

parameters $T_{\pm i}$ can be determined by measuring decay rates of flavour-tagged $D^0 \to K_S^0 \pi^+ \pi^-$ decays, i.e. where D meson can be identified as D^0 or $\overline{D}^0$

measuring B→DK decay rates in each bin, 2k+3 unknowns = c_i, s_i, r_B, δ_B and γ
$k \geq 2$: greater number of equations than unknowns and γ can be determined

preferable to perform dedicated measurements of c_i and s_i, use them as inputs

c_i and s_i at charm factory

at $\psi(3770)$, $J^{PC} = 1^{--}$, decays to a $D\overline{D}$ pair (decay are quantum related)
D mesons decay to final states f_a and f_b with CP eigenvalues η_a and η_b
CP conservation requires that $\eta_a \eta_b (-1)^L = 1$, hence $\eta_a / \eta_b = -1$
$\Rightarrow$ if one D meson is reconstructed in a CP even (odd) eigenstate,
other D meson must be CP odd (even) eigenstate

measurements of c_i and s_i require that one of the D mesons decays to $K_S^0 \pi^+ \pi^-$ final state and the other decays to final state X_D

if X_D is CP even (odd) eigenstate, D meson decaying to $K_S^0 \pi^+ \pi^-$ must be CP-odd (even)
amplitude and partial width of $D_\pm$ at Dalitz plot coordinate (m_-^2, m_+^2):

$$A(D_\pm \to K_S^0 h^+ h^-) = \frac{1}{\sqrt{2}}\left(A_D \pm \overline{A}_D\right),$$

$$\frac{d\Gamma(D_\pm \to K_S^0 h^+ h^-)}{dm_-^2\, dm_+^2} = \frac{1}{2}\left(A_D^2 + \overline{A}_D^2\right) \pm A_D \overline{A}_D \cos\delta_D.$$

decay rate to bin i of the $D_\pm$ Dalitz plot:

$$\Gamma_i(D_\pm \to K_S^0 h^+ h^-) \propto \frac{1}{2}(T_i + T_{-i}) \pm \sqrt{T_i T_{-i}}\, c_i.$$

if X_D is $K_S^0 \pi^+ \pi^-$:

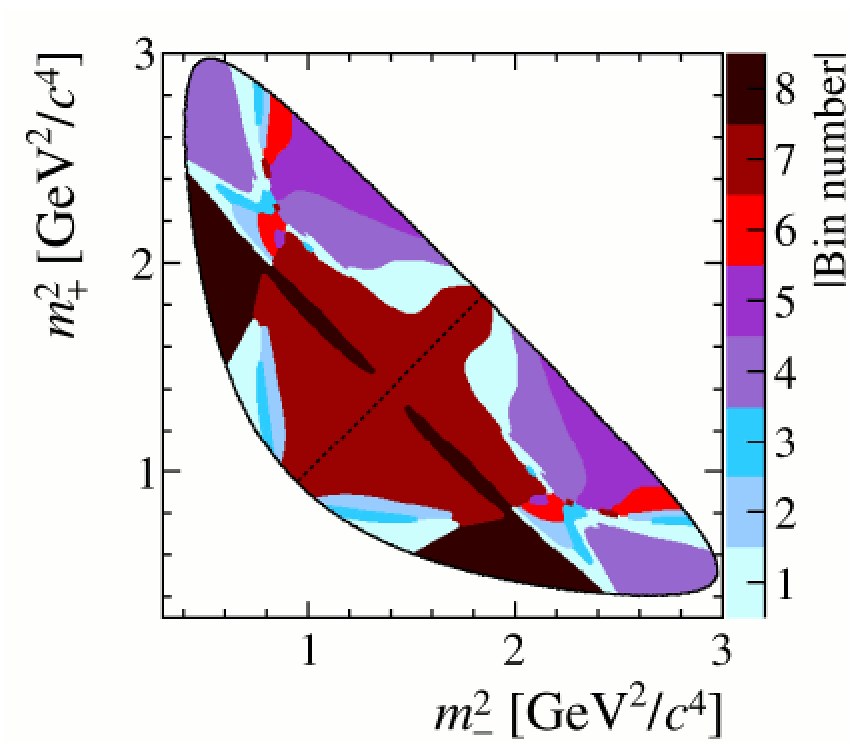
$$\Gamma_{ij} \propto T_i T_{-j} + T_{-i} T_j - 2\sqrt{T_i T_{-i} T_j T_{-j}}(c_i c_j + s_i s_j).$$

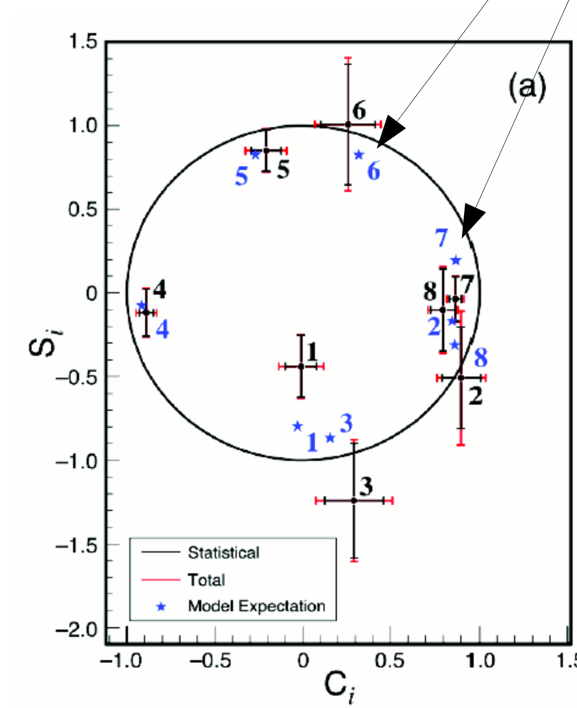
c_i and s_i at charm factory

model of the amplitude $A_D(m_-^2, m_+^2)$ determined using flavour-tagged $D \to K_S^0 \pi^+ \pi^-$ decays
$\Rightarrow$ provides prediction of $\delta_D(m_-^2, m_+^2)$ and can be used to determine binning scheme

binning: started with rectangular binning, better sensitivity to γ obtained by partionning Dalit plot into regions over which variation in $\delta_D(m_-^2, m_+^2)$ is small

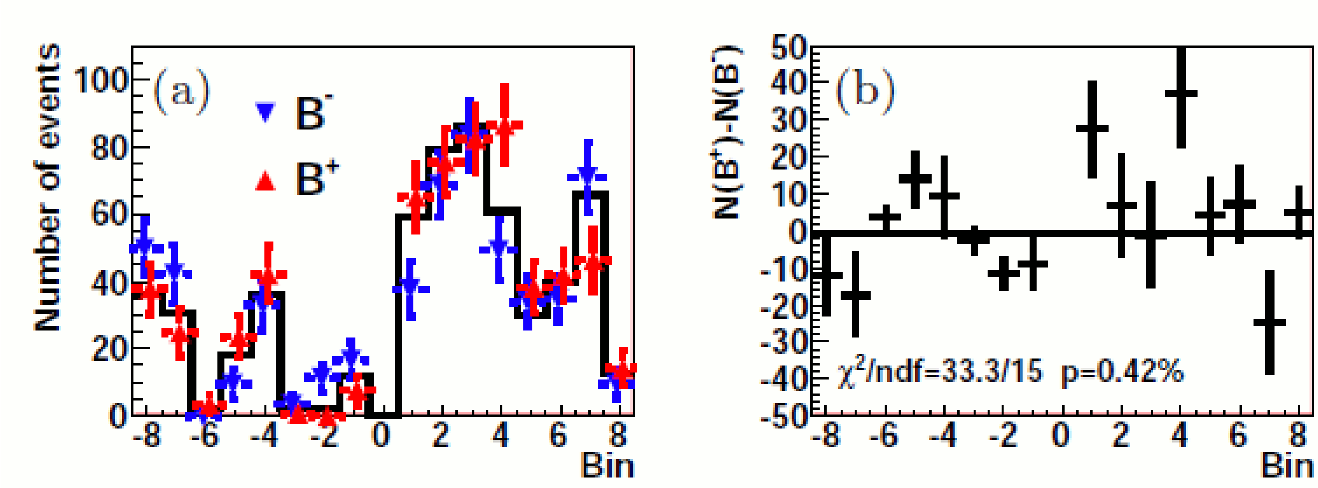
produce large non-zero values of c_i and s_i

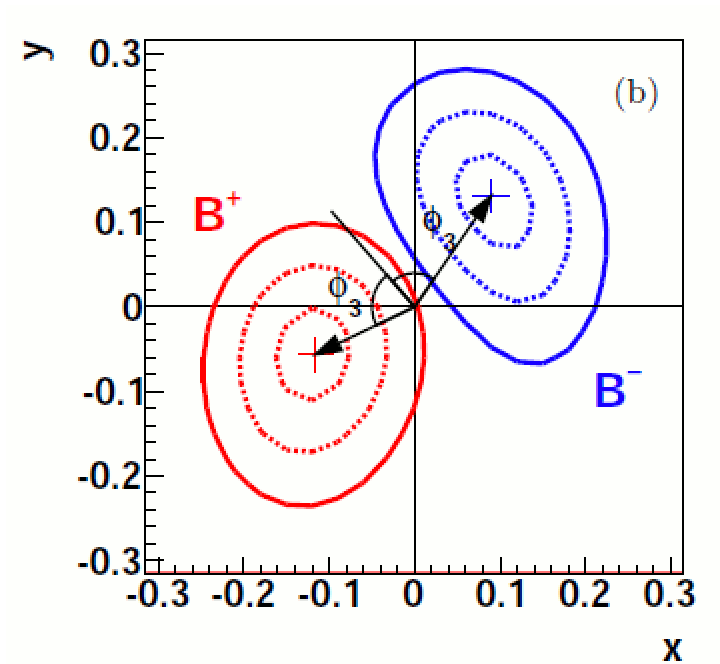


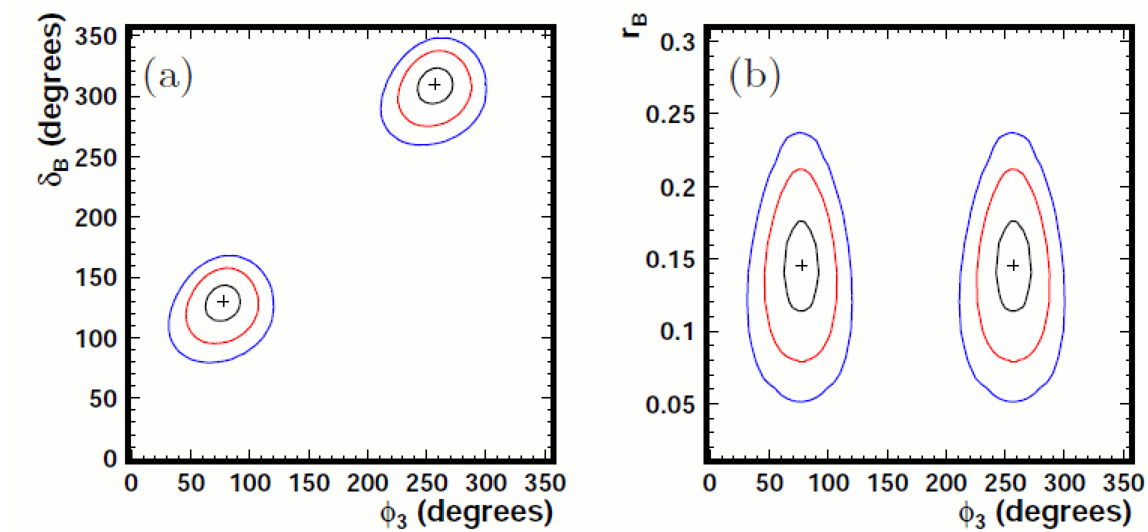


$D \to K_S \pi\pi$ and $D \to K_S^0 KK$: different binning, DK^+ and DK^{*0} different binning...

as an example, Belle arXiv:1204.6561, first model-indep γ result







$$\phi_3 = (77.3^{+15.1}_{-14.9} \pm 4.1 \pm 4.3)^\circ$$

$$r_B = 0.145 \pm 0.030 \pm 0.010 \pm 0.011$$

$$\delta_B = (129.9 \pm 15.0 \pm 3.8 \pm 4.7)^\circ,$$