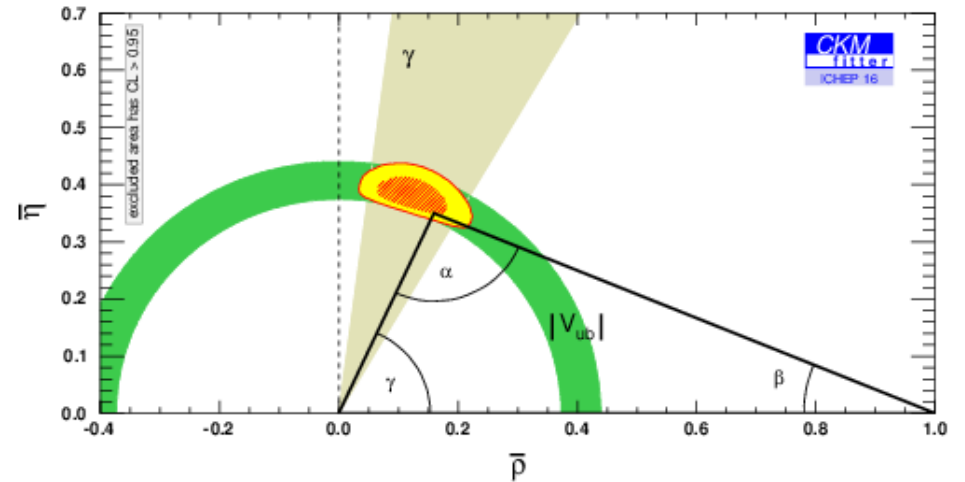
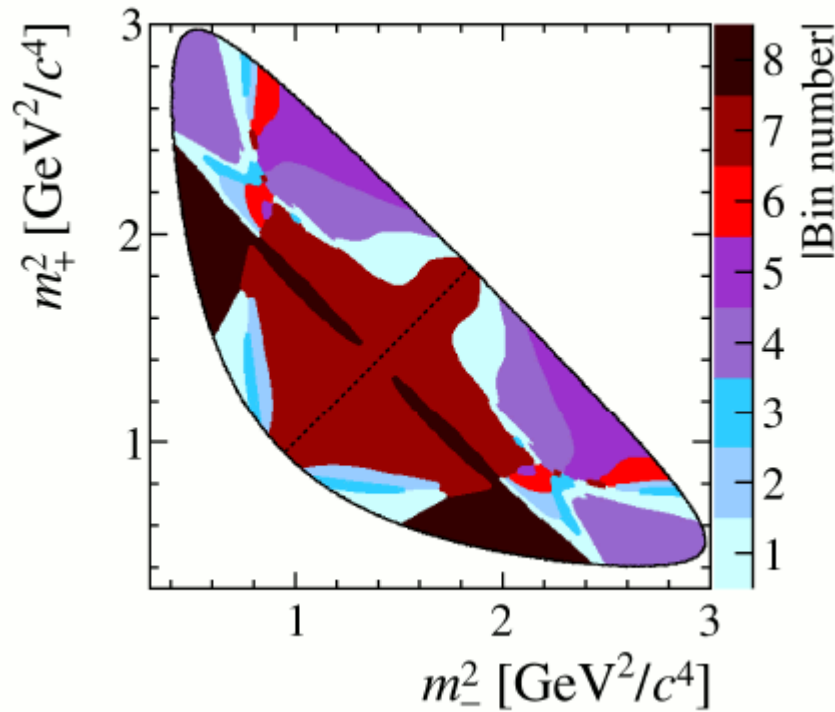
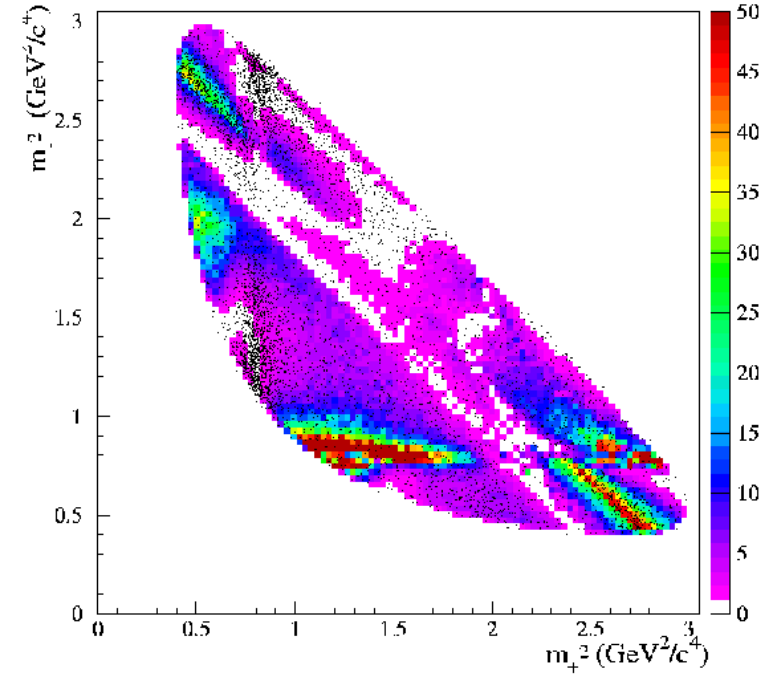
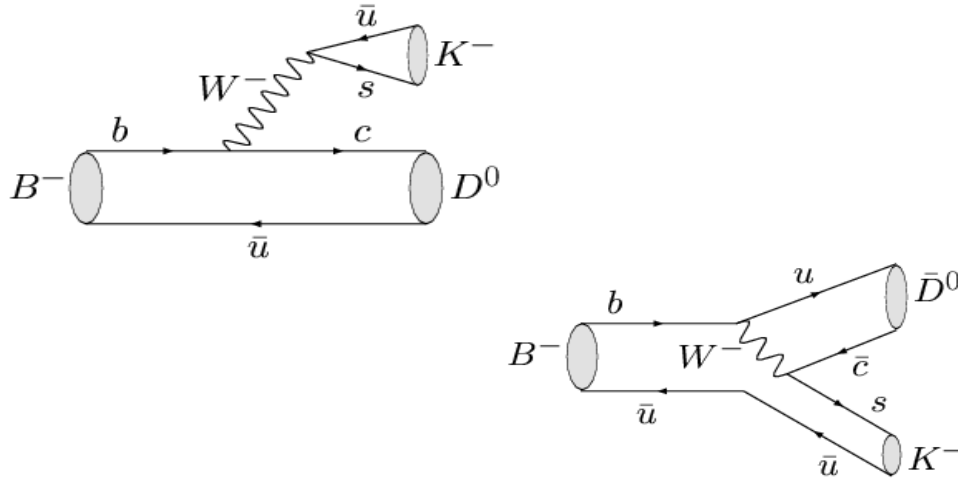
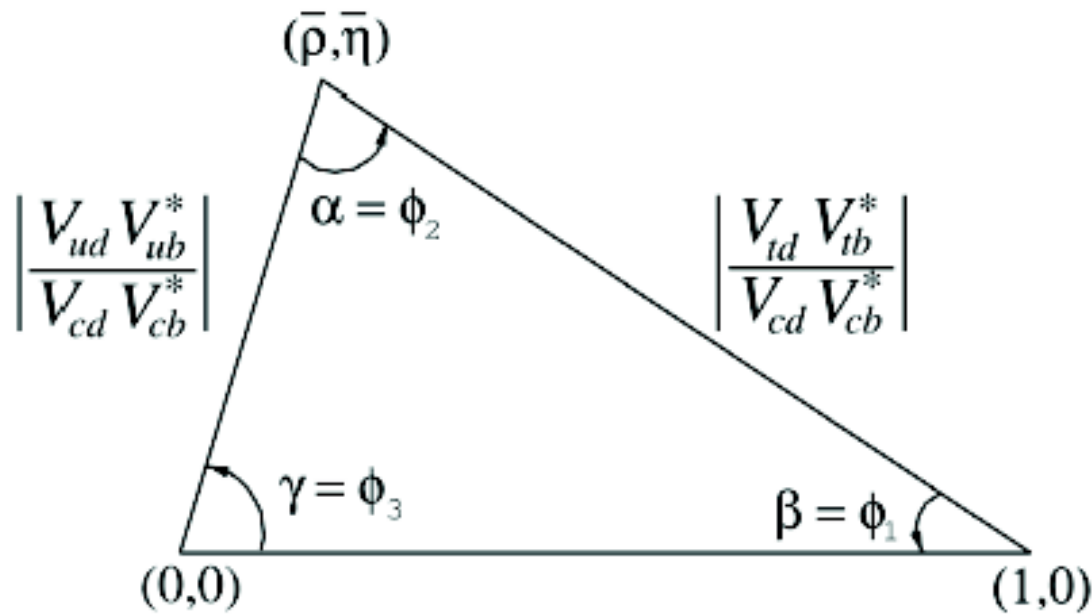


γ and $B \rightarrow DK$

K. Trabelsi
2019/05/28



Motivation

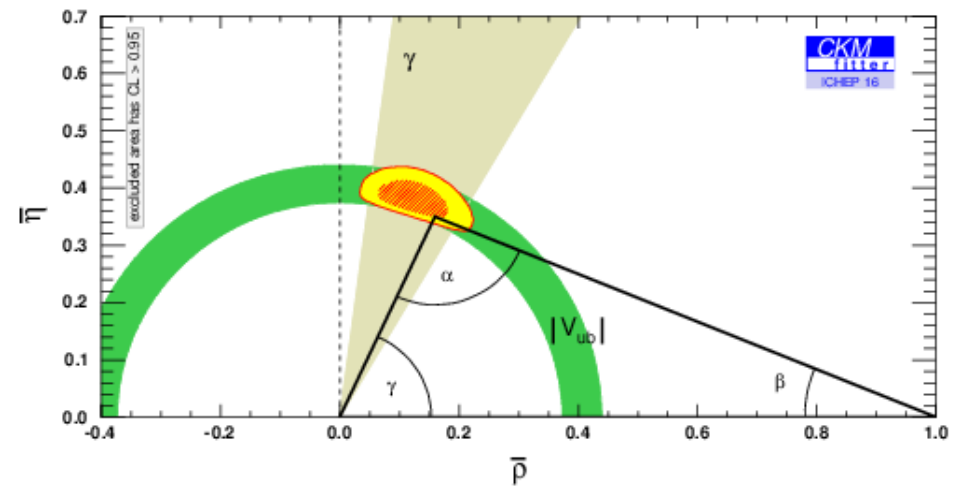
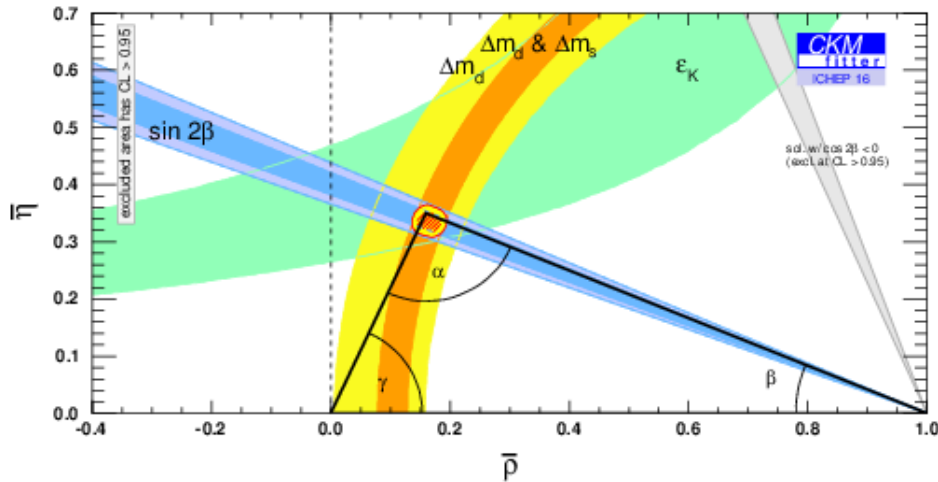


$$\alpha \equiv \arg\left(-\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*}\right), \quad \beta \equiv \arg\left(-\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*}\right), \quad \gamma \equiv \arg\left(-\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}\right)$$

In Wolfenstein parameterisation, up to order λ^4 , all the CKM elements involved are real except V_{ub}^* and V_{td} :

$$\alpha \approx \arg\left(-\frac{V_{td}}{V_{ub}^*}\right), \quad \beta \approx \arg(-V_{td}^*), \quad \gamma \approx \arg(-V_{ub}^*)$$

Motivation



Loop processes more easily altered by presence of NP
 constraints on the apex of UT currently more stringent from loop measurements

Loop vs Tree

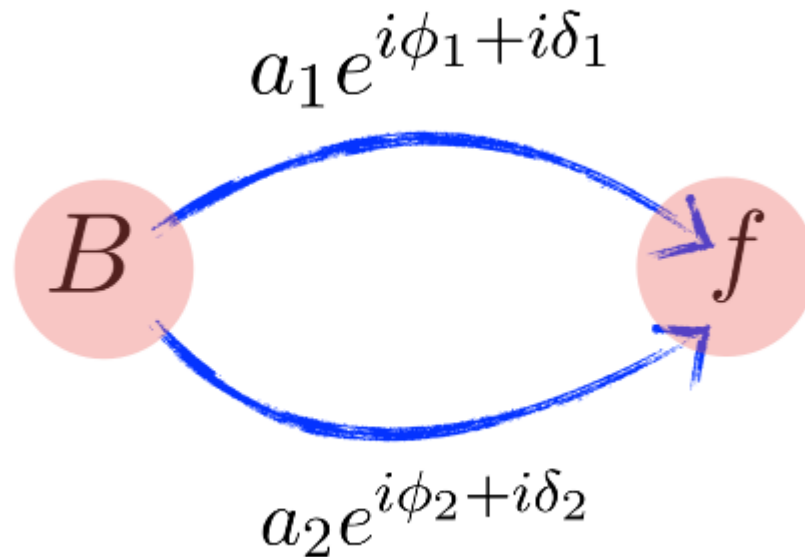
Why γ is a key goal ?

γ is least well measured parameter of UT
 Theoretically pristine
 with LHCb and Belle II the ideal degree level precision is possible

Direct CPV (CPV in decay)

$$Asym_f \equiv \frac{\Gamma(\bar{B} \rightarrow \bar{f}) - \Gamma(B \rightarrow f)}{\Gamma(\bar{B} \rightarrow \bar{f}) + \Gamma(B \rightarrow f)} = \frac{1 - |A_f/\bar{A}_f|^2}{1 + |A_f/\bar{A}_f|^2}$$

In order to have non-vanishing CP asymmetry, $Asym \neq 0$, the $B \rightarrow f$ decay amplitude needs to receive contributions from (at least) two different terms with differing weak, $\phi_{1,2}$, and strong phases, $\delta_{1,2}$



$$A_f = a_1 e^{i\phi_1 + i\delta_1} + a_2 e^{i\phi_2 + i\delta_2},$$

$$\bar{A}_f = a_1 e^{-i\phi_1 + i\delta_1} + a_2 e^{-i\phi_2 + i\delta_2}.$$

Direct CPV (CPV in decay)

$$A_f = a_1 e^{i\phi_1 + i\delta_1} + a_2 e^{i\phi_2 + i\delta_2},$$
$$\bar{A}_f = a_1 e^{-i\phi_1 + i\delta_1} + a_2 e^{-i\phi_2 + i\delta_2}.$$

The weak phases are due to CKM phase in the SM Lagrangian and change the sign under CP transformation, while the strong phases are due to on-shell rescattering of particles (pions, etc) and are thus CP even, the same as QCD interactions. The CP asymmetry is, in simplifying limit $a_2/a_1 \ll 1$,

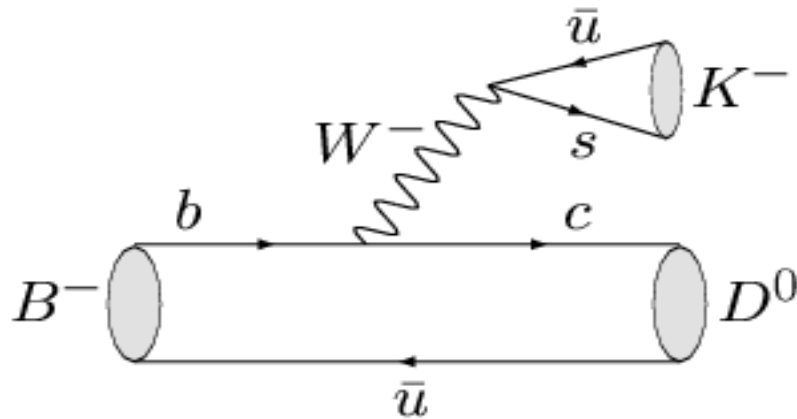
$$\mathcal{A}_f = \frac{a_2}{a_1} \sin(\phi_2 - \phi_1) \sin(\delta_2 - \delta_1) + \mathcal{O}(a_2^2/a_1^2).$$

The CP asymmetry vanishes in the limit where either

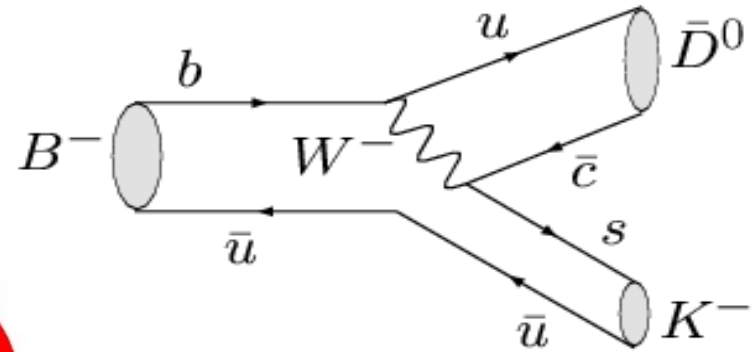
- (i) there is only one contribution to the amplitude $a_2 \rightarrow 0$
- (ii) if the weak phase difference vanishes, $\phi_2 - \phi_1 \rightarrow 0$
- (iii) if the strong phase difference vanishes, $\delta_2 - \delta_1 \rightarrow 0$

γ measurements from $B^\pm \rightarrow DK^\pm$

- Theoretically pristine $B \rightarrow DK$ approach
- Access γ via interference between $B^- \rightarrow D^0 K^-$ and $B^- \rightarrow \bar{D}^0 K^-$



color allowed
 $B^- \rightarrow D^0 K^- \sim V_{cb} V_{us}^*$
 $\sim A \lambda^3$



color suppressed
 $B^- \rightarrow \bar{D}^0 K^- \sim V_{ub} V_{cs}^*$
 $\sim A \lambda^3 (\rho + i\eta)$

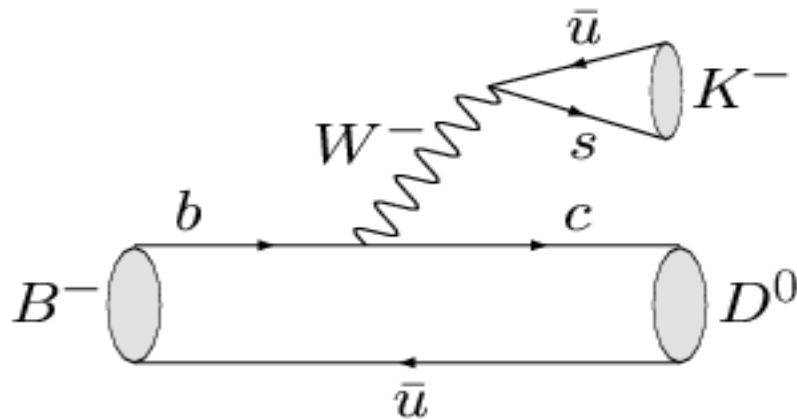
CKM elements involved are $\frac{V_{cs} V_{ub}^*}{V_{us} V_{cb}^*}$ while $\gamma \equiv \arg\left(-\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}\right)$

$$\Rightarrow \frac{V_{cd} V_{cs}}{V_{ud} V_{us}} = -1 + \frac{A \lambda^4}{2} - A^2 \lambda^5 \left(\rho + i\eta - \frac{1}{2}\right) + O(\lambda^6)$$

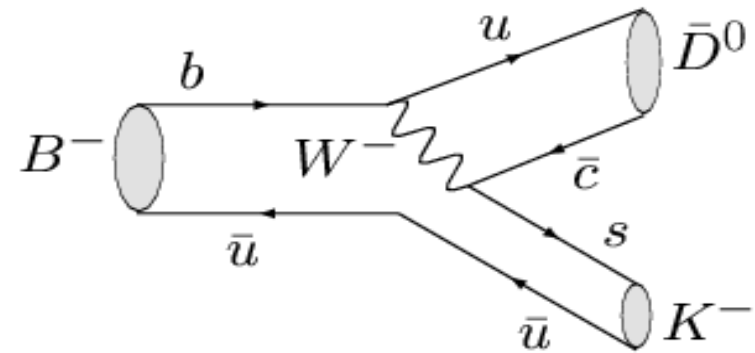
\Rightarrow leading order correction on γ is of the order $\lambda^5 \sim 10^{-4}$ (negligible)

γ measurements from $B^\pm \rightarrow DK^\pm$

- Theoretically pristine $B \rightarrow DK$ approach
- Access γ via interference between $B^- \rightarrow D^0 K^-$ and $B^- \rightarrow \bar{D}^0 K^-$



color allowed
 $B^- \rightarrow D^0 K^- \sim V_{cb} V_{us}^*$
 $\sim A \lambda^3$



color suppressed
 $B^- \rightarrow \bar{D}^0 K^- \sim V_{ub} V_{cs}^*$
 $\sim A \lambda^3 (\rho + i\eta)$

relative magnitude of suppressed amplitude is r_B

$$r_B = \frac{|A_{\text{suppressed}}|}{|A_{\text{favoured}}|} \sim \frac{|V_{ub} V_{cs}^*|}{|V_{cb} V_{us}^*|} \times [\text{color supp}] = 0.1 - 0.2$$

relative weak phase is γ , relative strong phase is δ_B

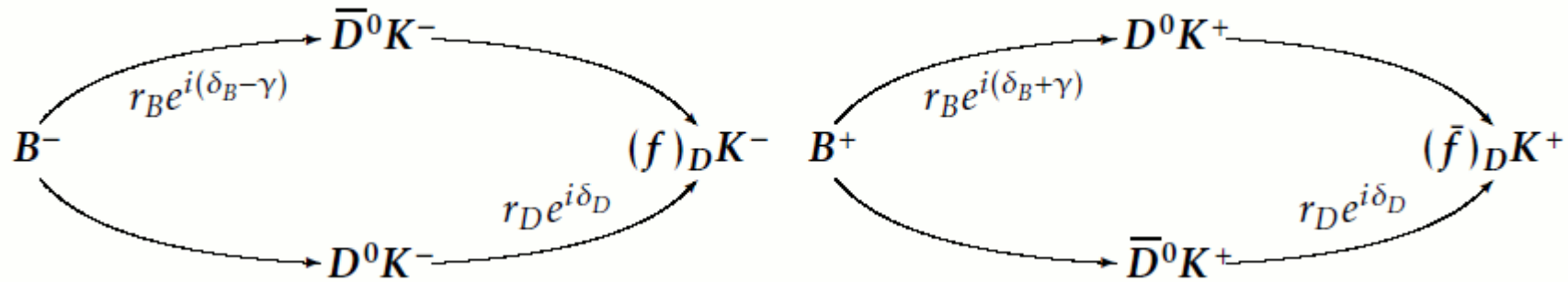
\Rightarrow for $D\pi$: same dependence to γ , but different $r_B \sim 0.01$ ($V_{us} \rightarrow V_{ud}$, $V_{cs} \rightarrow V_{cd}$)

γ measurements from $B^\pm \rightarrow DK^\pm$

- Reconstruct D in final states accessible to both D^0 and \bar{D}^0
 - $D = D_{\text{CP}}$, CP eigenstates as $K^+ K^-$, $\pi^+ \pi^-$, $K_S \pi^0$
GLW method (Gronau-London-Wyler)
 - $D = D_{\text{sup}}$, Doubly-Cabbibo suppressed decays as $K \pi$
ADS method (Atwood-Dunietz-Soni)
 - Three-body decays as $D \rightarrow K_S \pi^+ \pi^-$, $K_S K^+ K^-$
GGSZ (Dalitz) method (Giri-Grossman-Soffer-Zupan)
- Largest effects due to
 - charm mixing
 - charm CP violation

} negligible
Y. Grossman, A. Soffer, J. Zupan
[PRD 72, 031501 (2005)]
- Different B decays (DK , $D^* K$, DK^*)
 - different hadronic factors (r_B , δ_B) for each

γ , first principles...



$$A(B^- \rightarrow D^0 K^-) = A_B \quad \text{and} \quad A(B^- \rightarrow \bar{D}^0 K^-) = A_B r_B e^{i(\delta_B - \gamma)}$$

$$A(B^+ \rightarrow \bar{D}^0 K^+) = A_B \quad \text{and} \quad A(B^+ \rightarrow D^0 K^+) = A_B r_B e^{i(\delta_B + \gamma)}$$

amplitudes of the subsequent D^0 and \bar{D}^0 decays to a common final state f

$$A(\bar{D}^0 \rightarrow f) = A_D \quad \text{and} \quad A(D^0 \rightarrow f) = A_D r_D e^{i\delta_D}$$

assuming direct CPV in D decays negligibly small: $A(D^0 \rightarrow f) \equiv A(\bar{D}^0 \rightarrow \bar{f})$ and $A(\bar{D}^0 \rightarrow f) \equiv A(D^0 \rightarrow \bar{f})$

$$A(B^- \rightarrow D(\rightarrow f)K^-) \equiv A(B^- \rightarrow D^0 K^-)A(D^0 \rightarrow f) + A(B^- \rightarrow \bar{D}^0 K^-)A(\bar{D}^0 \rightarrow f)$$

$$= A_B A_D r_D e^{i\delta_D} + A_B r_B e^{i(\delta_B - \gamma)} A_D,$$

$$A(B^+ \rightarrow D(\rightarrow \bar{f})K^+) \equiv A(B^+ \rightarrow \bar{D}^0 K^+)A(\bar{D}^0 \rightarrow \bar{f}) + A(B^+ \rightarrow D^0 K^+)A(D^0 \rightarrow \bar{f})$$

$$= A_B A_D + A_B r_B e^{i(\delta_B + \gamma)} A_D r_D e^{i\delta_D}.$$

γ , first principles...

rates of $B^- \rightarrow DK^-$ and $B^+ \rightarrow DK^+$

$$|A(B^- \rightarrow D(\rightarrow f)K^-)|^2 = |A_B|^2 |A_D|^2 [r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B - \gamma - \delta_D)],$$

$$|A(B^+ \rightarrow D(\rightarrow \bar{f})K^+)|^2 = |A_B|^2 |A_D|^2 [r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \gamma - \delta_D)].$$

if D final state f is CP eigenstates (GLW method):

$r_D = 1$, and $\delta_D = 0$ (π) for CP-even (odd) eigenstate

$$A_{CP} = \frac{\Gamma(B^- \rightarrow D(\rightarrow f)K^-) - \Gamma(B^+ \rightarrow D(\rightarrow \bar{f})K^+)}{\Gamma(B^- \rightarrow D(\rightarrow f)K^-) + \Gamma(B^+ \rightarrow D(\rightarrow \bar{f})K^+)},$$

$$A_{CP+} = \frac{2r_B \sin \delta_B \sin \gamma}{1 + r_B^2 + 2r_B \cos \delta_B \cos \gamma}$$

$$A_{CP-} = \frac{-2r_B \sin \delta_B \sin \gamma}{1 + r_B^2 - 2r_B \cos \delta_B \cos \gamma}$$

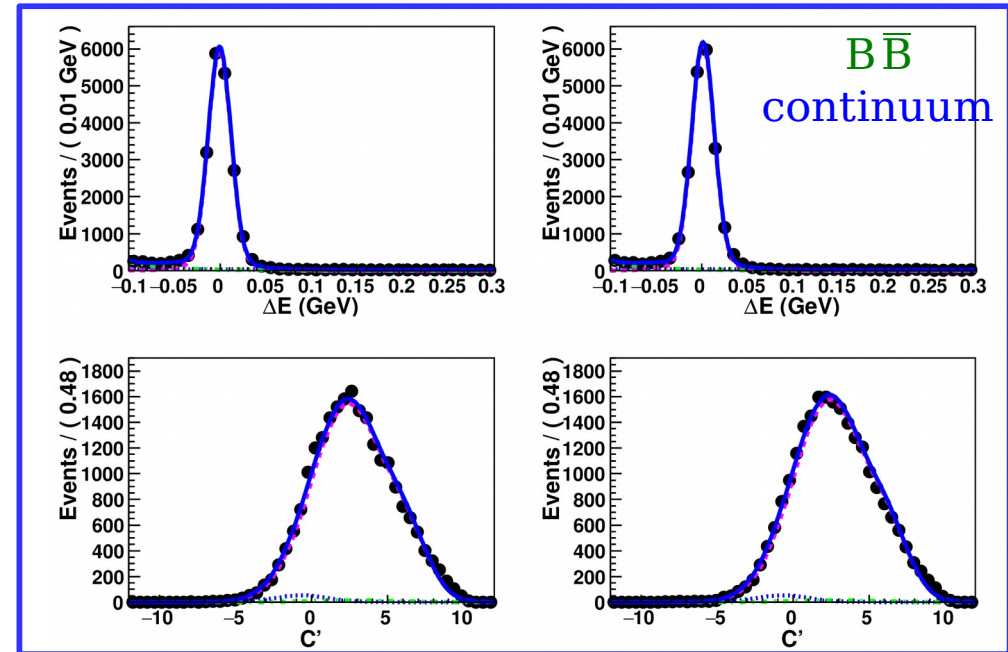
$B \rightarrow DK^\pm$ at Belle II

illustration with Belle $B \rightarrow D(K\pi)K$ analysis

$B \rightarrow D\pi$
 $B \rightarrow DK$

KID < 0.6 (pion-like)

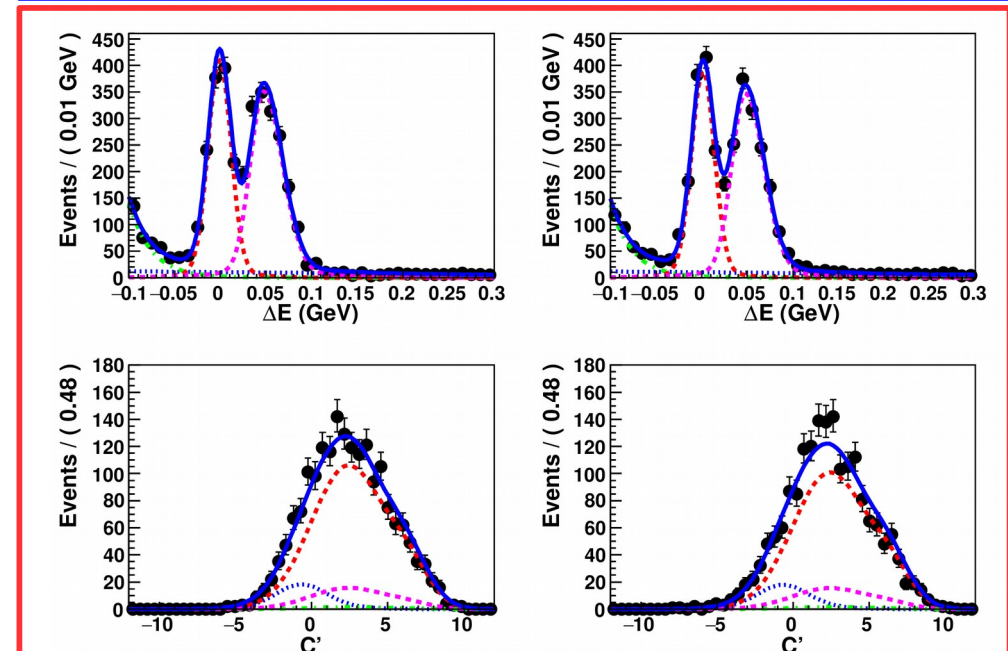
$$\begin{aligned}
 N_{\eta, KID > 0.6}^{DK} &= \frac{1}{2} (1 - \eta A^{DK}) N_{tot}^{D\pi} R_{K/\pi} \epsilon \\
 N_{\eta, KID < 0.6}^{DK} &= \frac{1}{2} (1 - \eta A^{DK}) N_{tot}^{D\pi} R_{K/\pi} (1 - \epsilon) \\
 N_{\eta, KID > 0.6}^{D\pi} &= \frac{1}{2} (1 - \eta A^{D\pi}) N_{tot}^{D\pi} \kappa \\
 N_{\eta, KID < 0.6}^{D\pi} &= \frac{1}{2} (1 - \eta A^{D\pi}) N_{tot}^{D\pi} (1 - \kappa)
 \end{aligned}$$



KID > 0.6 (kaon-like)

	kaon fake (1-ε)	kaon eff ε	pion eff (1-κ)	pion fake κ
MC	14.70 ± 0.06	85.41 ± 0.06	95.42 ± 0.03	4.47 ± 0.03
data	15.86 ± 0.40	84.32 ± 0.39	92.13 ± 0.46	7.94 ± 0.31

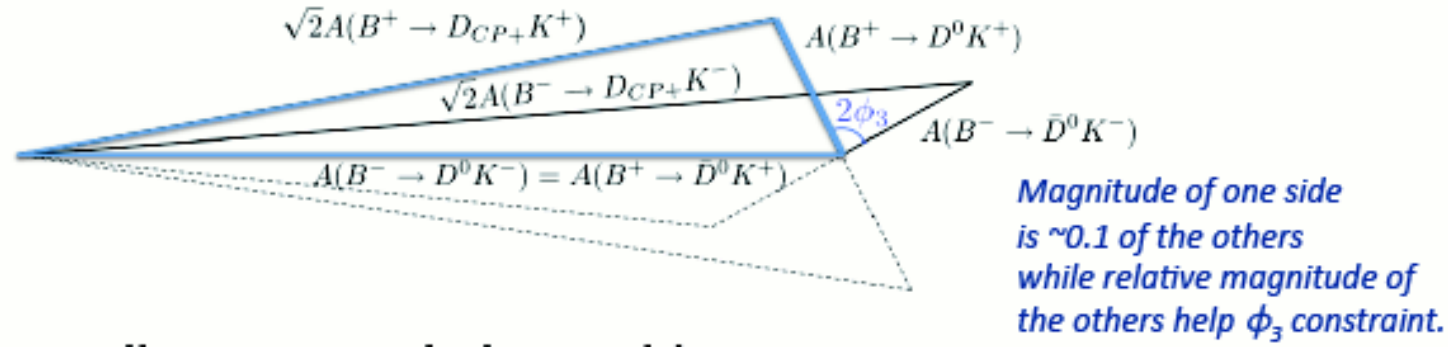
for Belle



for Belle II: performances expected to be as good (better ?) as for Belle MC...

one of the important outputs of current data taking (jury is still out)

➤ Amplitude triangle:



Usually measured observables:

$$R_{CP\pm} \equiv \frac{\mathcal{B}(B^- \rightarrow D_{CP\pm} K^-) + \mathcal{B}(B^+ \rightarrow D_{CP\pm} K^+)}{\mathcal{B}(B^- \rightarrow D^0 K^-) + \mathcal{B}(B^+ \rightarrow \bar{D}^0 K^+)}$$

$$A_{CP\pm} \equiv \frac{\mathcal{B}(B^- \rightarrow D_{CP\pm} K^-) - \mathcal{B}(B^+ \rightarrow D_{CP\pm} K^+)}{\mathcal{B}(B^- \rightarrow D_{CP\pm} K^-) + \mathcal{B}(B^+ \rightarrow D_{CP\pm} K^+)}$$

Relation between $(A_{CP+}, A_{CP-}, R_{CP+}, R_{CP-})$ and (γ, r_B, δ_B)

$$A_{CP+} = \frac{2 r_B \sin \delta_B \sin \gamma}{1 + r_B^2 + 2 r_B \cos \delta_B \cos \gamma}$$

$$A_{CP-} = \frac{-2 r_B \sin \delta_B \sin \gamma}{1 + r_B^2 - 2 r_B \cos \delta_B \cos \gamma}$$

$$R_{CP+} = 1 + r_B^2 + 2 r_B \cos \delta_B \cos \gamma$$

$$R_{CP-} = 1 + r_B^2 - 2 r_B \cos \delta_B \cos \gamma$$

⇒ look for $R_{CP\pm} \neq 1$ and $A_{CP\pm} \neq 0$

The other charged B's decays...

$$B \rightarrow D^{*0} K, D^{*0} \rightarrow D^0 \pi^0, D^0 \gamma$$

- a new set of $(r_B, \delta_B) = (r_B^*, \delta_B^*) \dots$
- η_X , CP eigenvalue of X
 - $\eta_{D^*} = \eta_D \times \eta_{\pi^0/\gamma} \times (-1)^l$, l =angular momentum between D and π^0/γ
 - $\eta_{D^*} = \eta_D$ for $D^* \rightarrow D \pi^0$, $\eta_{D^*} = -1 \times \eta_D$ for $D^* \rightarrow D \gamma$

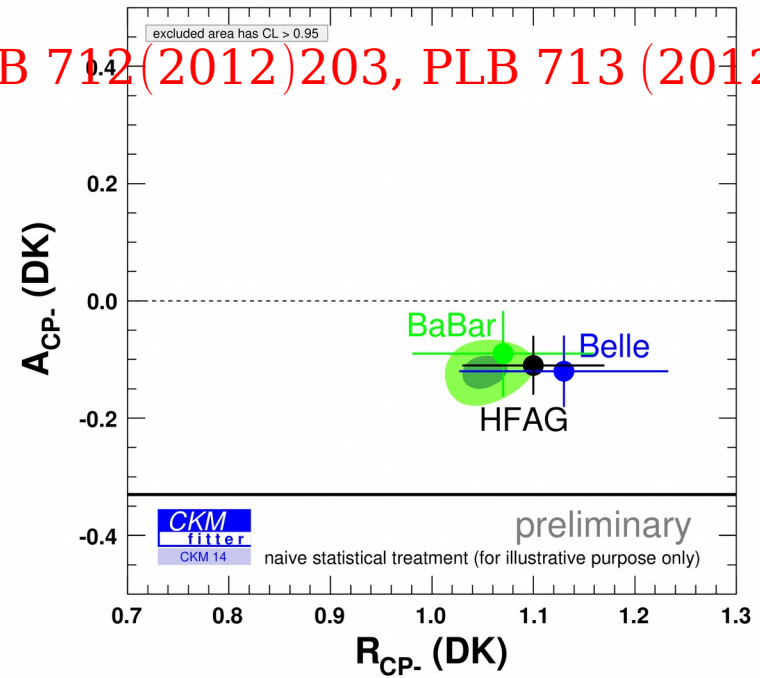
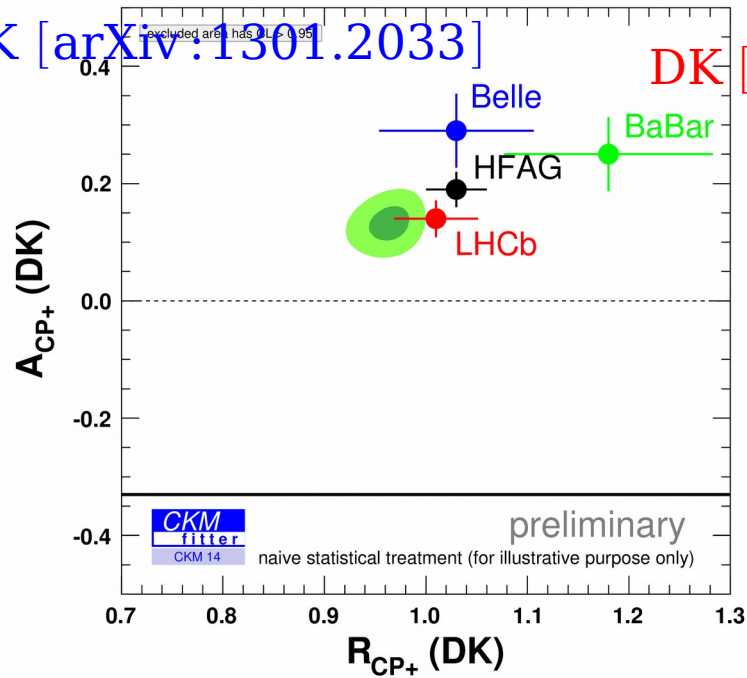
\Rightarrow shift of π between both cases

GLW observables (predictions vs measurements)

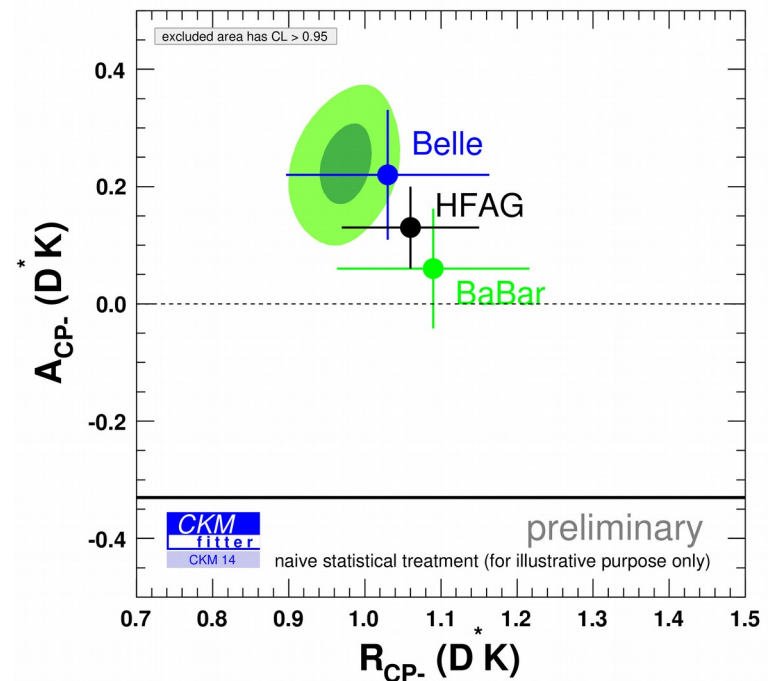
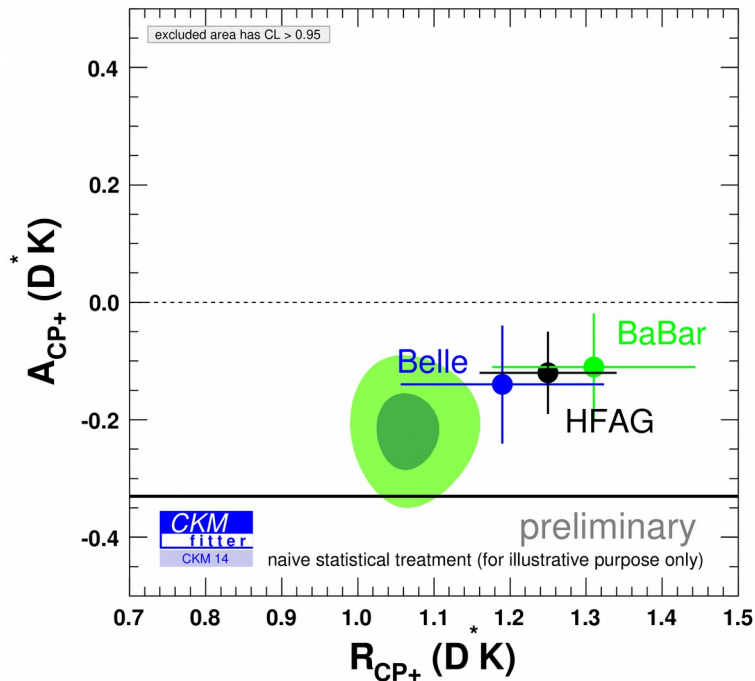
DK [PRD82(2010)072004], D^*K [PRD78(2008)092002], DK^* [PRD80(2009)092001]

DK and D^*K [arXiv:1301.2033]

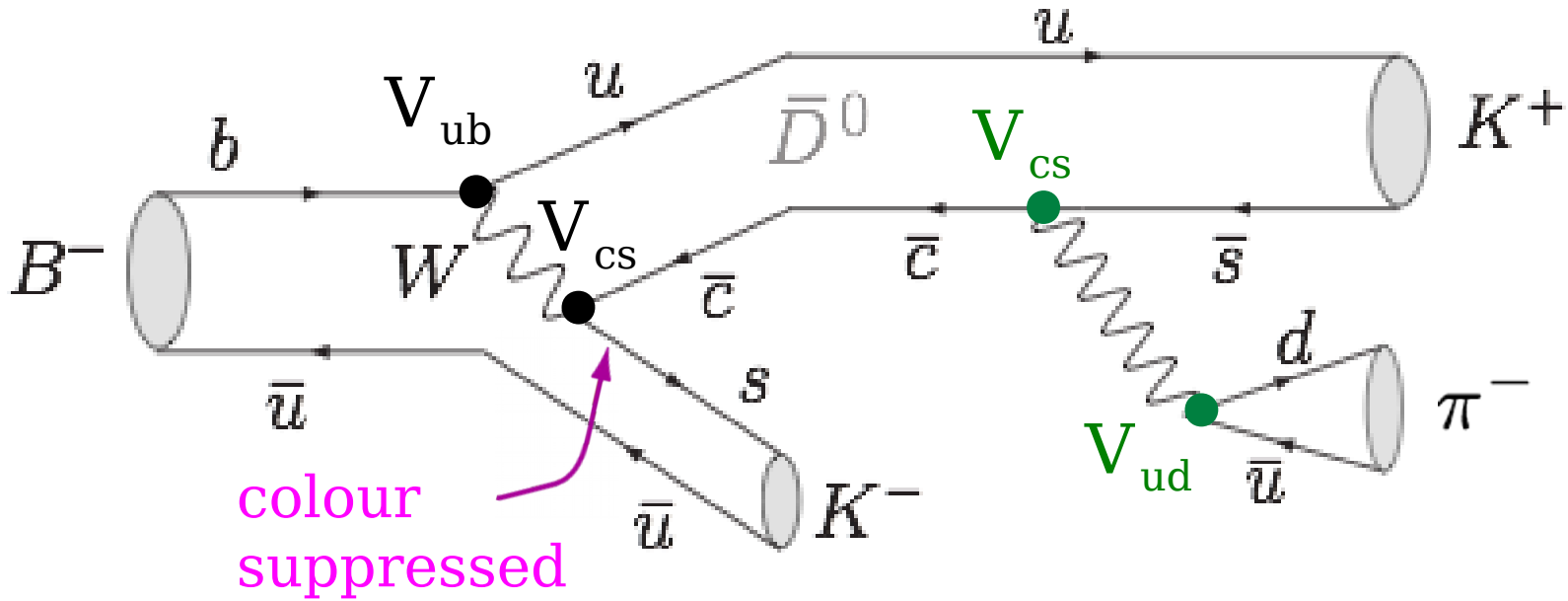
DK [PLB 712(2012)203, PLB 713(2012)351]



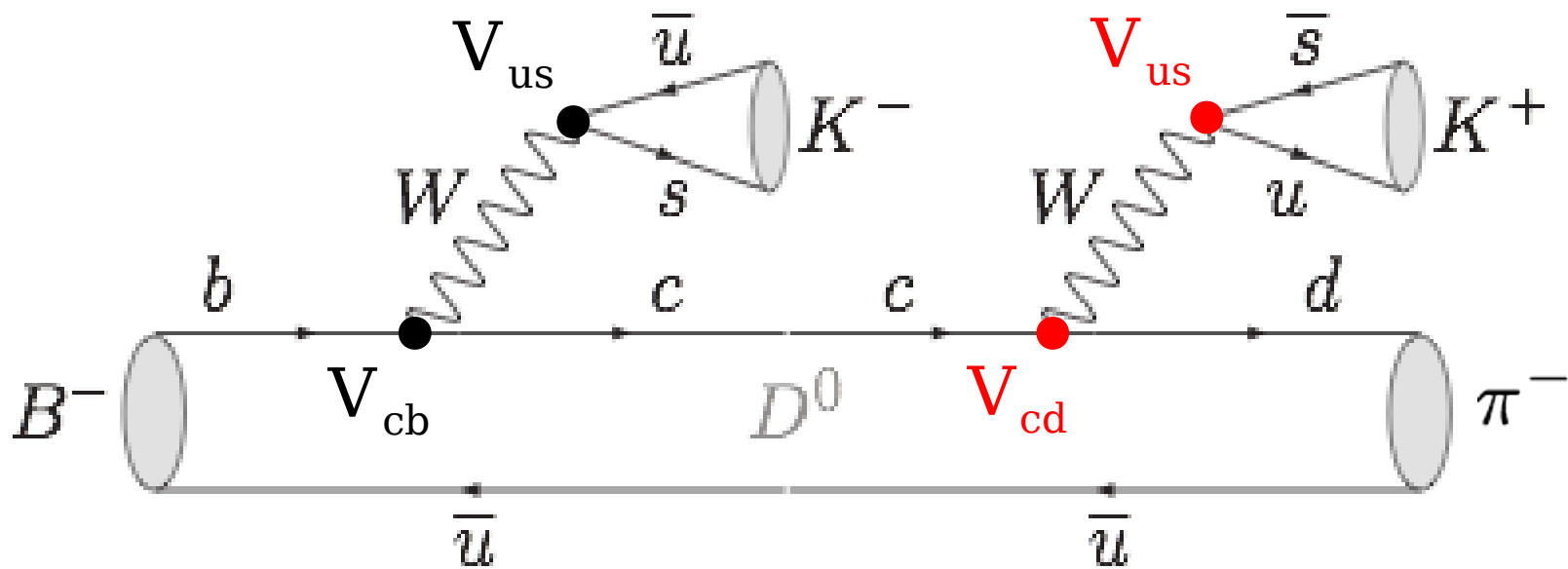
+ 22 obs.



ADS method measures ϕ_3 via the interference in rare $B^- \rightarrow [K^+ \pi^-]_D K^-$ decays



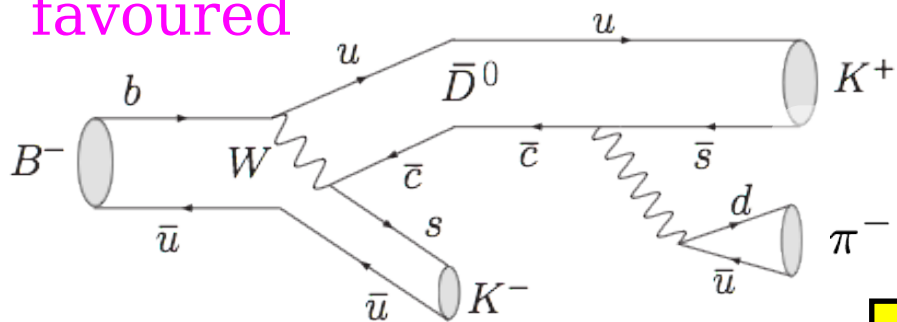
Cabibbo
favoured
D decay



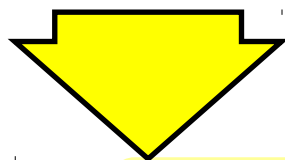
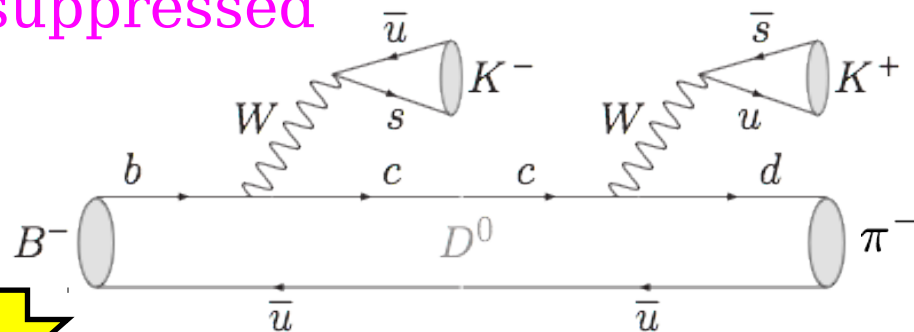
doubly
Cabibbo
suppressed
D decay

ADS rate and asymmetry (relative to the common decay):

favoured

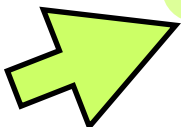
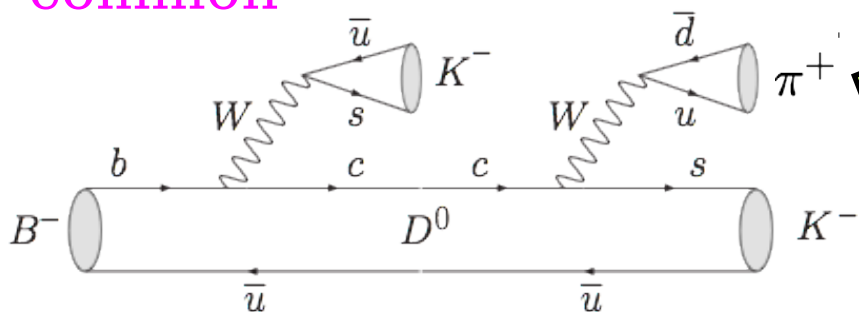


suppressed



$$\mathcal{R}_{DK} = \frac{\Gamma([K^+ \pi^-] K^-) + \Gamma([K^- \pi^+] K^+)}{\Gamma([K^- \pi^+] K^-) + \Gamma([K^+ \pi^-] K^+)}$$

common



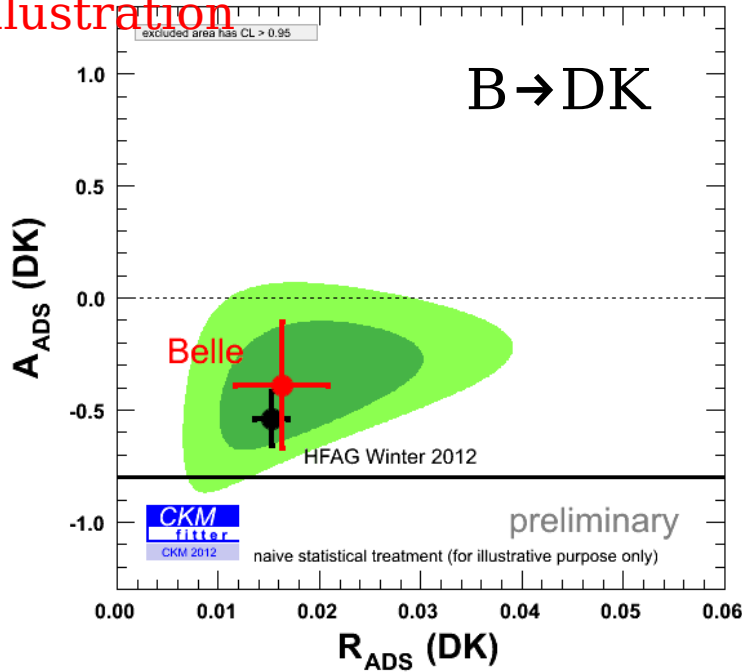
$$= r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D) \cos \phi_3$$

$$\mathcal{A}_{DK} = \frac{\Gamma([K^+ \pi^-] K^-) - \Gamma([K^- \pi^+] K^+)}{\Gamma([K^- \pi^+] K^-) + \Gamma([K^+ \pi^-] K^+)}$$

$$= 2r_B r_D \sin(\delta_B + \delta_D) \sin \phi_3 / \mathcal{R}_{DK}$$

where $r_D = \left| \frac{\mathcal{A}(D^0 \rightarrow K^+ \pi^-)}{\mathcal{A}(\bar{D}^0 \rightarrow K^+ \pi^-)} \right| = 0.0613 \pm 0.0010$

Comparison of the results obtained for $D^{(*)}K$ with expectations where "expectations" are derived from the GGSZ observables, δ_D and γ_{UT} for illustration



$$R_{ADS}(DK) = r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D) \cos \gamma$$

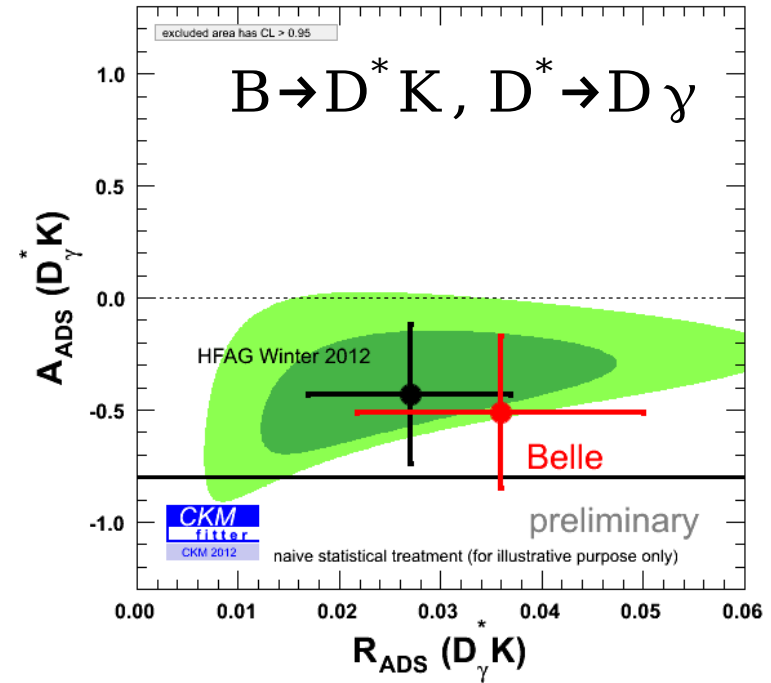
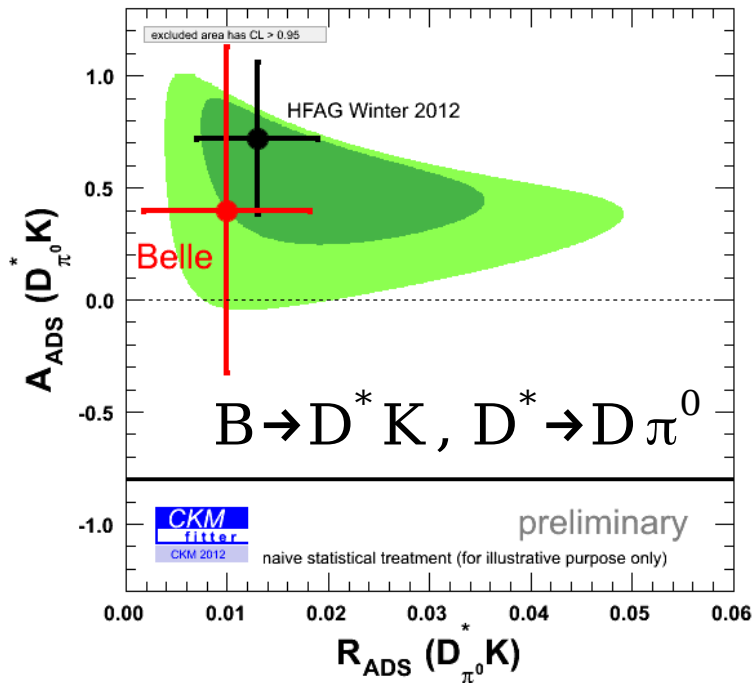
$$A_{ADS}(DK) = 2r_B r_D \sin(\delta_B + \delta_D) \sin \gamma / R_{ADS}(DK)$$

$$R_{ADS}(D_{\pi^0}^* K) = r_B^{*2} + r_D^2 + 2r_B^* r_D \cos(\delta_B^* + \delta_D) \cos \gamma$$

$$A_{ADS}(D_{\pi^0}^* K) = 2r_B^* r_D \sin(\delta_B^* + \delta_D) \sin \gamma / R_{ADS}(D_{\pi^0}^* K)$$

$$R_{ADS}(D_\gamma^* K) = r_B^{*2} + r_D^2 - 2r_B^* r_D \cos(\delta_B^* + \delta_D) \cos \gamma$$

$$A_{ADS}(D_\gamma^* K) = -2r_B^* r_D \sin(\delta_B^* + \delta_D) \sin \gamma / R_{ADS}(D_\gamma^* K)$$



Lot of interesting modes...

not used until now

D mode $2F_+ - 1$ **branching ratio**

($\times 10^{-3}$)

$K^+ K^-$ +1 3.96 ± 0.08

$\pi^+ \pi^-$ +1 1.40 ± 0.03

$\pi^0 \pi^0$ +1 0.82 ± 0.04

$K_L^0 \pi^0$ +1 10.0 ± 0.7

$K_S^0 \pi^0 \pi^0$ +1 9.1 ± 1.1

$K_S^0 \eta \pi^0$ +1 5.5 ± 1.1

$K_S^0 K_S^0 K_S^0$ +1 0.91 ± 0.13

$\pi \pi \pi^0$ 14.3 ± 0.6

$KK \pi^0$ 3.3 ± 0.1

$\pi \pi \pi \pi$ 7.4 ± 0.2

D mode $2F_+ - 1$ **branching ratio**

($\times 10^{-3}$)

$K_S^0 \pi^0$ -1 11.9 ± 0.4

$K_S^0 \eta$ -1 4.8 ± 0.3

$K_S^0 \eta'$ -1 9.4 ± 0.5

$K_S^0 K_S^0 K_L^0$ -1 1.0

$\eta \pi^0 \pi^0$ -1 unknown

$\eta' \pi^0 \pi^0$ -1 unknown

$K_S^0 K_S^0 \pi^0$ -1 < 0.6

$K_S^0 K_S^0 \eta$ -1 unknown

D mode **branching ratio** ($\times 10^{-3}$)

$K_S^0 \pi^+ \pi^-$ 28.3 ± 2.0

$K_S^0 K^+ K^-$ 4.6 ± 0.2

$K_L^0 \pi^+ \pi^-$

$K_L^0 K^+ K^-$

$K_S^0 \pi^+ \pi^- \pi^0$ 52 ± 6

$\pi^+ \pi^- \pi^0 \pi^0$ 10.0 ± 0.9

current study with Belle promising \rightarrow promising

challenging modes with K_L , two π^0 's...