

The shape of the Photon Transfer Curve of CCD sensors

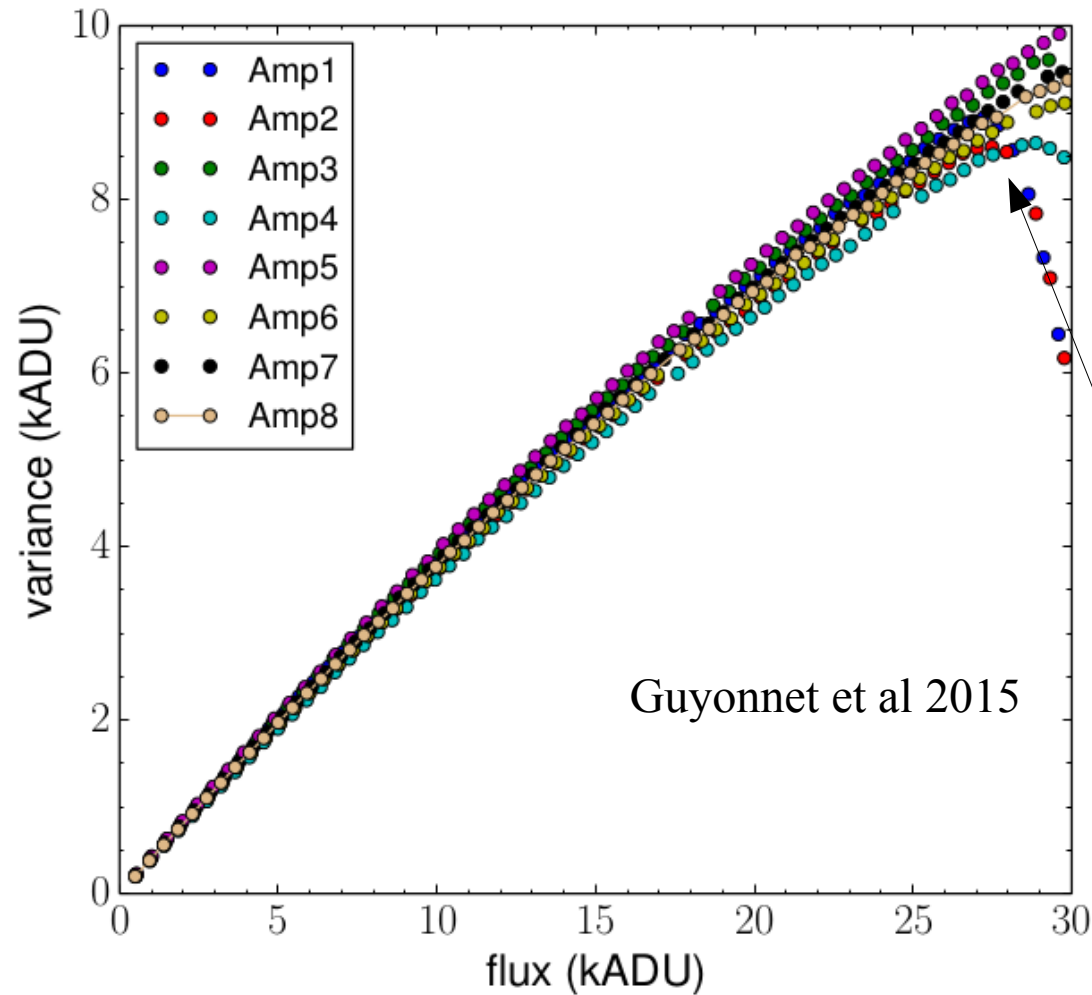
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Photon Transfer Curve

Variance of
uniform
exposures



LSST
E2V
CCD

Saturation

Naively, we expect

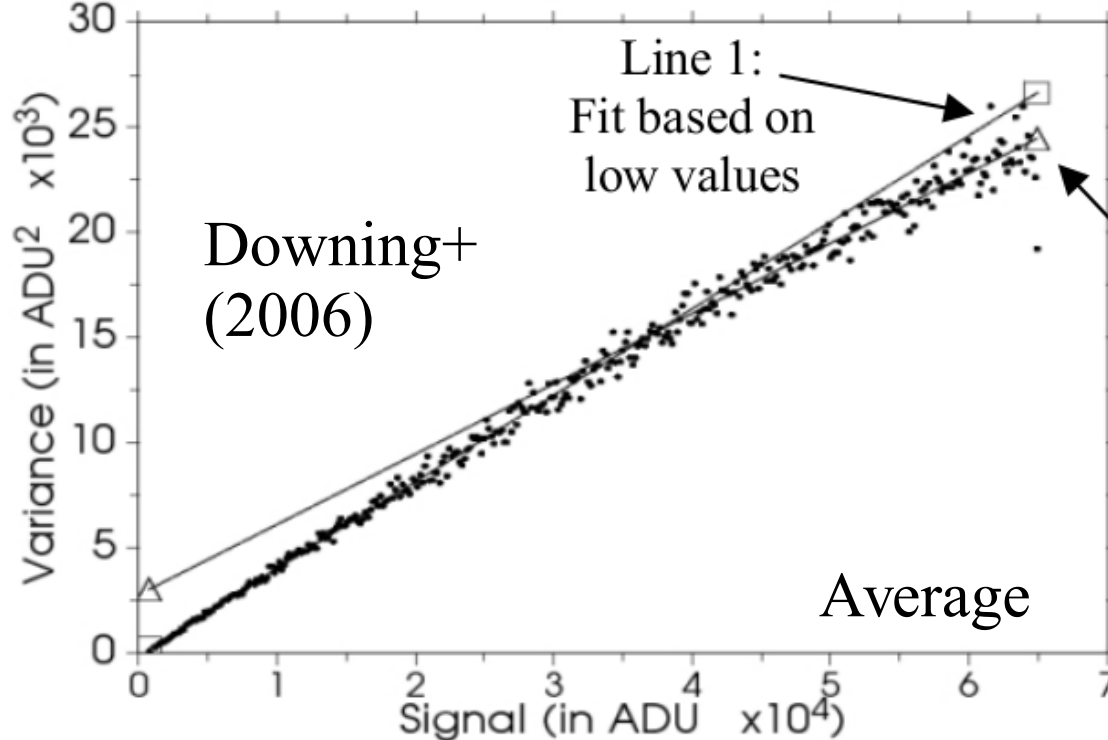
- Poisson contribution
- plus a small read noise

→ it should be a straight line

Average of the exposure

The first publication :

Variance

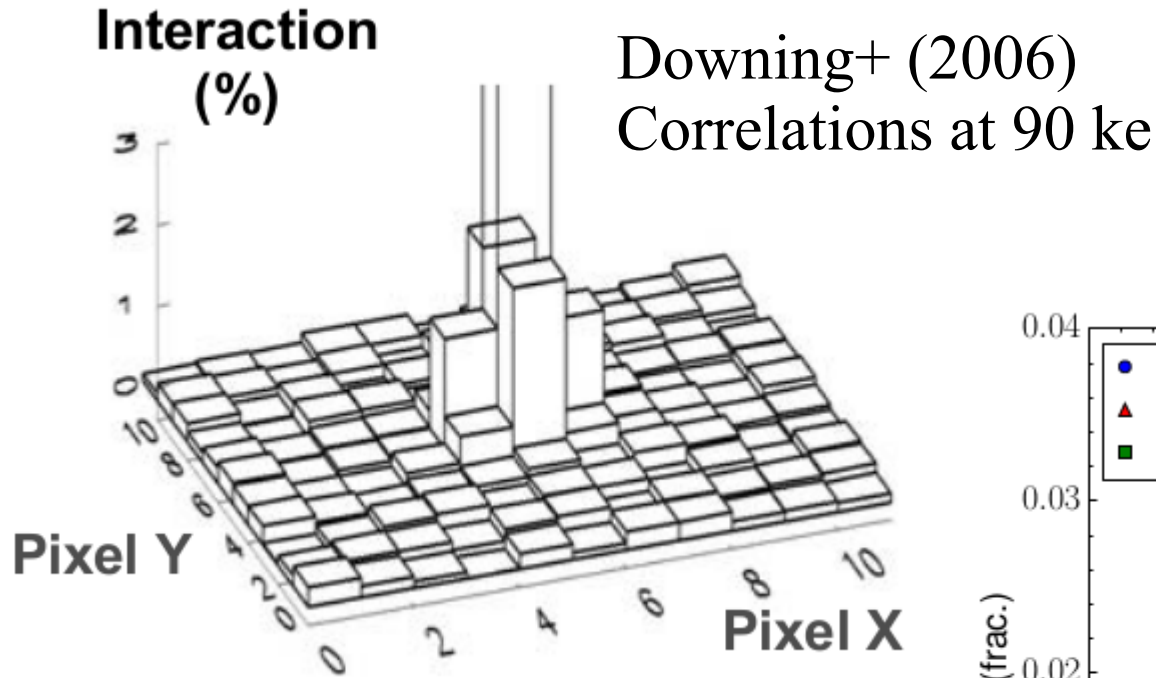


e2v CCD44-82

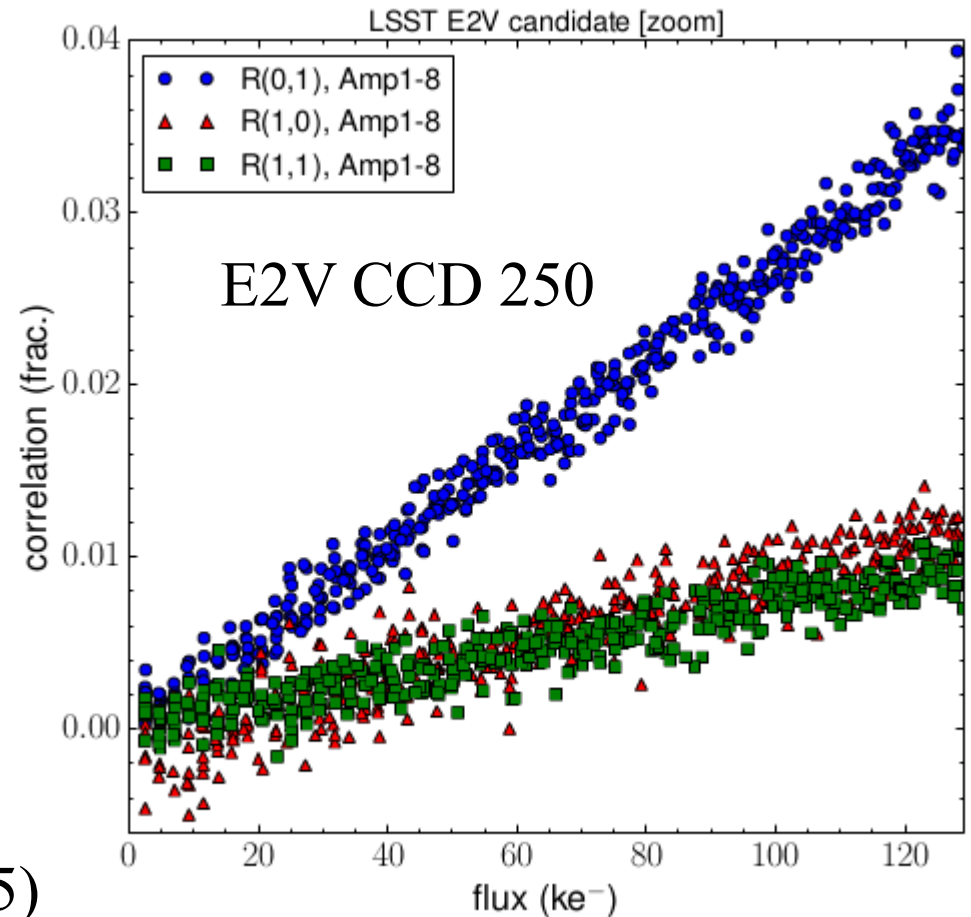
The variance of flat fields is **not** proportional to their average

- Not due to non-linearity of the video chain
- Present on all tested sensors
- Associated to covariances of neighboring pixels

Covariances/Correlations in flat fields



Correlations increase roughly linearly, and can reach a few %.



Guyonnet+(2015)

Interpretation

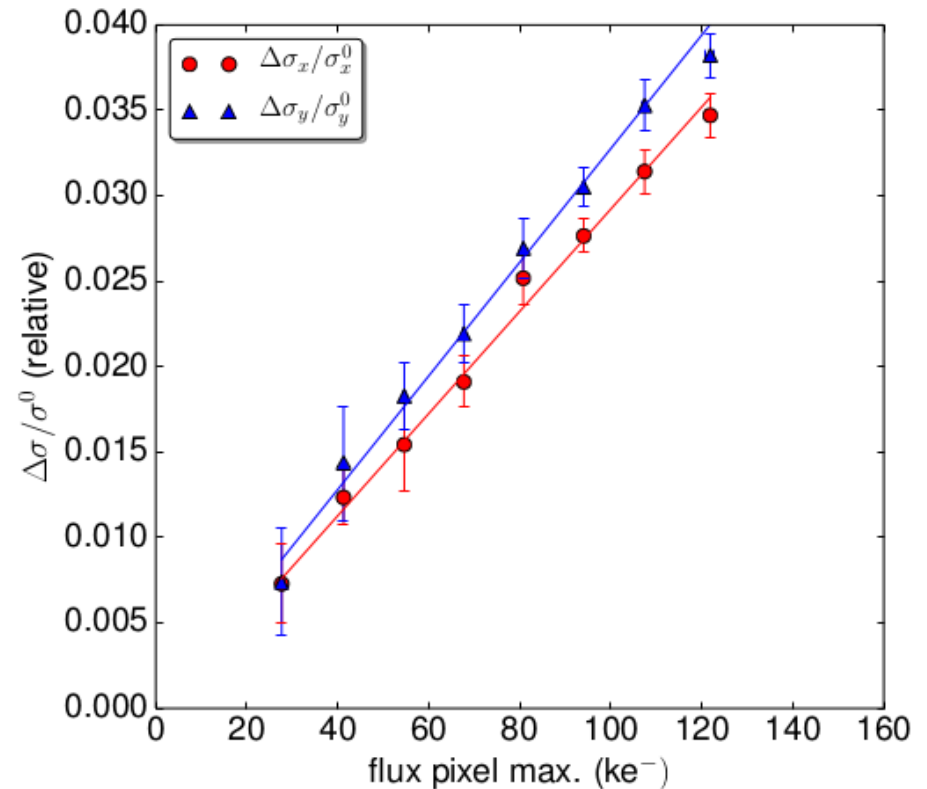
The brutal fact : in flat fields, variances do not add up

- Incoming charges have to be sensitive to what happened earlier.
- Electrostatic forces can do that
- They can also perturb “structured” (e.g. science) images

Brighter Fatter

Spot sizes increase with total (or peak) flux. In an anisotropic way.

The size of the effect varies with chip type and operating voltages



(a) LSST - E2V 250 - Spots 550 nm



(a) 200-s exposures

(b) 20-s exposures

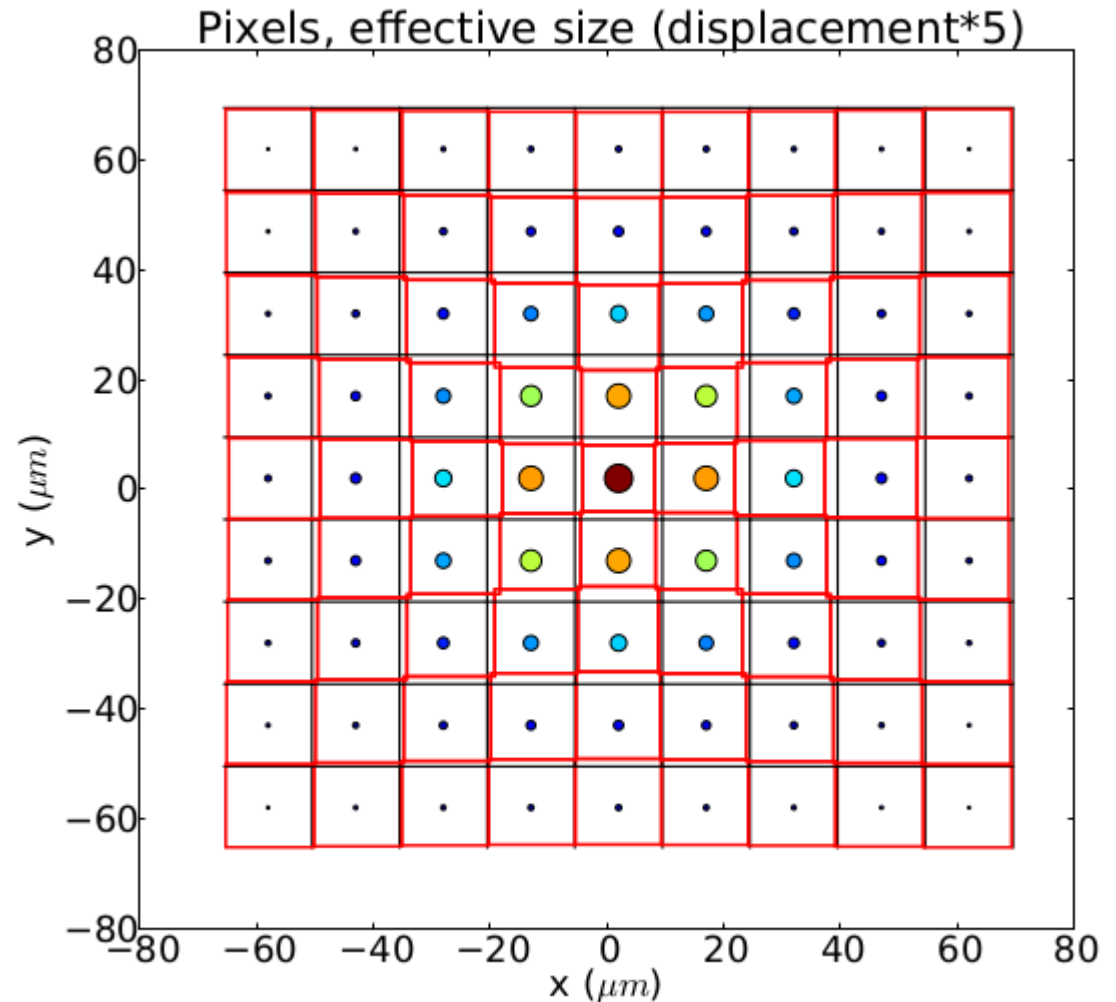
(c) subtraction (a)-(b)

Guyonnet+ (2015)

Star shapes do not evolve with flux, but pixel shapes do

Gaussian star
Rms = 1.6 pixel
Peak = 100 ke

(Guyonnet+ 2015)



Summary of facts

- The size of the effects (BF & flat field correlations) is compatible with electrostatic effects within the sensor (Laige+17)
- The chromaticity of the effects is weak if not undetectable
- Flat-field correlations are roughly linear with flux
- PTC is essentially never linear.
- With fully depleted sensors, ignoring the effect is not an option (in particular for WL)

BF Correction or handling schemes

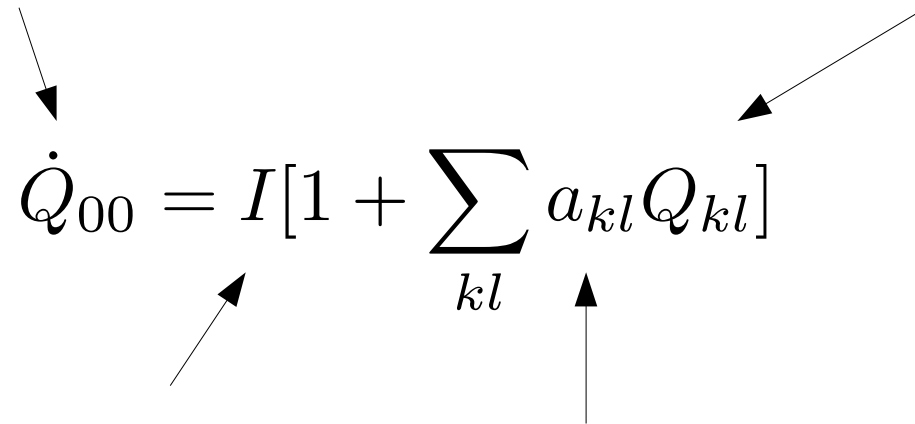
- Measure correlations/covariances
- Constrain some (crude) model of electrostatic influences
- Compute how much charge was deflected and put it back where it belongs:
 - Guyonnet et al (2015)
 - Gruen et al (2015)
 - Coulton et al (2018)

Limitations

- All approaches assume that pixel boundary shifts are proportional to source charges.
 - This is just an hypothesis, Andy Rasmussen (1608.01964) argues that it is significantly wrong.
- All approaches assume that the slope of correlations encodes the relative change of pixel area
 - This is just Taylor
- Covariances are tricky to measure, and polluted by extra contributions...
 - To be detected and removed
- The scheme assumes that images are well sampled, which is wrong for the best IQ HSC images

Dynamics (in flat fields)

- Incoming currents are affected by stored charges


$$\dot{Q}_{00} = I[1 + \sum_{kl} a_{kl} Q_{kl}]$$

Average current
per pixel

By how much a stored charge
alters a pixel area (at lag k,l).

- There is here a linearity hypothesis : pixel boundaries shift by amounts proportional to the cause (the stored charge).

From interaction to covariances

$$\dot{Q}_{00} = I \left[1 + \sum_{kl} a_{kl} Q_{kl} \right]$$

$$\sum_{kl} a_{kl} = 0$$

Charge/area conservation.
Sum runs over positive
and negative lags

Time evolution of covariances :

$$\dot{C}_{ij} = \delta_{i0} \delta_{j0} V_I + 2I \sum_{kl} a_{kl} C_{i-k, j-l}$$

↑
Poisson variance per unit time

For $a = 0$, we get Poisson: $C_{00}(t) = V_I t$

Solution of the differential equation (1)

$$\dot{C}_{ij} = \delta_{i0}\delta_{j0}V_I + 2I \sum_{kl} a_{kl}C_{i-k,j-l}$$

$$\dot{\mathbf{C}} = \delta V_I + 2I \mathbf{C} \otimes \mathbf{a}$$

Fourier space

$$\tilde{\dot{\mathbf{C}}} = V_I + 2I\tilde{\mathbf{a}}\tilde{\mathbf{C}}$$

Solution

$$\tilde{\mathbf{C}}(t) = \frac{V_I}{2I\tilde{\mathbf{a}}} [e^{2I\tilde{\mathbf{a}}t} - 1]$$

Taylor

$$\begin{aligned} \tilde{\mathbf{C}}(t) &= V_I t \left[1 + I\tilde{\mathbf{a}}t + \frac{2}{3}(I\tilde{\mathbf{a}}t)^2 + \frac{1}{3}(I\tilde{\mathbf{a}}t)^3 + \dots \right] & \mu &\equiv It \\ \tilde{\mathbf{C}}(\mu) &= V \left[1 + \tilde{\mathbf{a}}\mu + \frac{2}{3}(\tilde{\mathbf{a}}\mu)^2 + \frac{1}{3}(\tilde{\mathbf{a}}\mu)^3 + \dots \right] & V &\equiv V_I t \end{aligned}$$

Solution of the differential equation (2)

$$\dot{C}_{ij} = \delta_{i0}\delta_{j0}V_I + 2I \sum_{kl} a_{kl}C_{i-k,j-l}$$

Taylor


$$\tilde{C}(t) = V_I t \left[1 + I\tilde{a}t + \frac{2}{3}(I\tilde{a}t)^2 + \frac{1}{3}(I\tilde{a}t)^3 + \dots \right]$$

$$\tilde{C}(\mu) = V \left[1 + \tilde{a}\mu + \frac{2}{3}(\tilde{a}\mu)^2 + \frac{1}{3}(\tilde{a}\mu)^3 + \dots \right] \quad \begin{array}{l} \mu \equiv It \\ V \equiv V_I t \end{array}$$

Direct
space

$$C(\mu) = V \left[\delta_{i0}\delta_{j0} + a\mu + \frac{2}{3}TF^{-1}[(\tilde{a})^2]\mu^2 + \dots \right]$$

Noise
terms

$$C_{ij}(\mu) = \frac{\mu}{g} \left[\delta_{i0}\delta_{j0} + a_{ij}\mu + \frac{2}{3}[a \otimes a]_{ij}\mu^2 + \frac{1}{3}[a \otimes a \otimes a]_{ij}\mu^3 + \dots \right] + n_{ij}/g^2$$


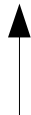
Solution

$$C_{ij}(\mu) = \frac{\mu}{g} \left[\delta_{i0}\delta_{j0} + a_{ij}\mu + \frac{2}{3}[a \otimes a]_{ij}\mu^2 + \frac{1}{3}[a \otimes a \otimes a]_{ij}\mu^3 + \dots \right] + n_{ij}/g^2$$

- Beyond second order, all curves are “mixed” (in direct space): every lag involves all “a” values.
- For the PTC a fair approximation is that a_{00} dominates (and is negative) and :

$$C_{00} = \frac{1}{2g^2 a_{00}} [\exp(2a_{00}\mu g) - 1] + n_{00}/g^2$$

Questioning the linearity assumption

$$\dot{Q}_{00} = I(1 + \sum_{kl} a_{kl}(1 + b_{kl} * I * t)Q_{kl})$$


Linearity violation
“Next to Leading Order” terms



$$C_{ij}(\mu) = \frac{\mu}{g}[\delta_{i0}\delta_{j0} + a_{ij}\mu + \frac{2}{3}[a \otimes a + ab]_{ij}\mu^2 \\ + \frac{1}{6}(2a \otimes a \otimes a + 5a \otimes ab)_{ij}\mu^3 + \dots] + n_{ij}/g^2$$

Poisson's revenge

$$\dot{C}_{ij} = \delta_{i0}\delta_{j0}V_I + 2I \sum_{kl} a_{kl} C_{i-k,j-l}$$

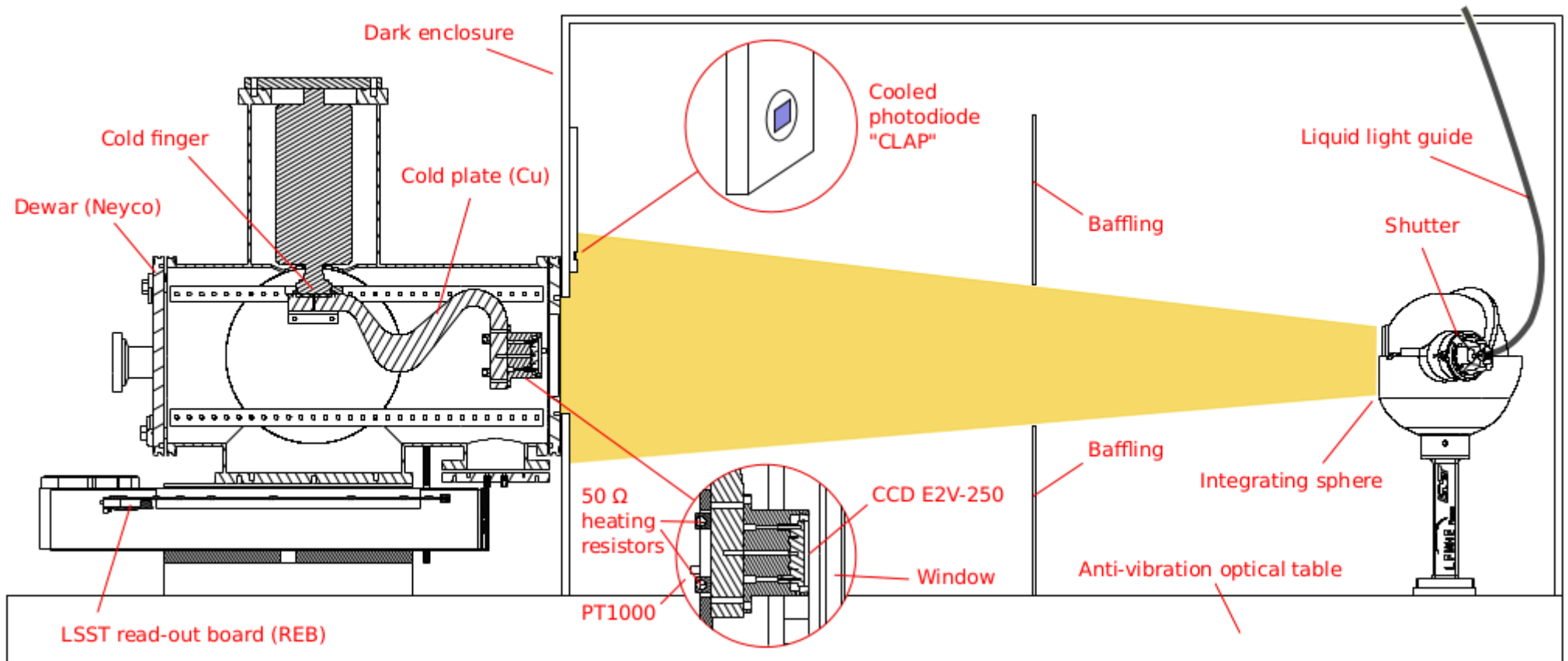
$$\text{Sum rule : } \sum_{kl} a_{kl} = 0$$

$$\begin{aligned} \sum_{ij} \dot{C}_{ij} &= V_I + 2I \sum_{ij} \sum_{kl} a_{kl} C_{i-k,j-l} \\ &= V_I + 2I \sum_{ij} a_{ij} \sum_{kl} C_{kl} \\ &= V_I \end{aligned}$$

If one sums variance and covariances, the Poisson behavior is recovered.

Data Analysis (E2V CCD 250)

- 1000 flat fields pairs at $0 < \mu < 10^5$ electrons

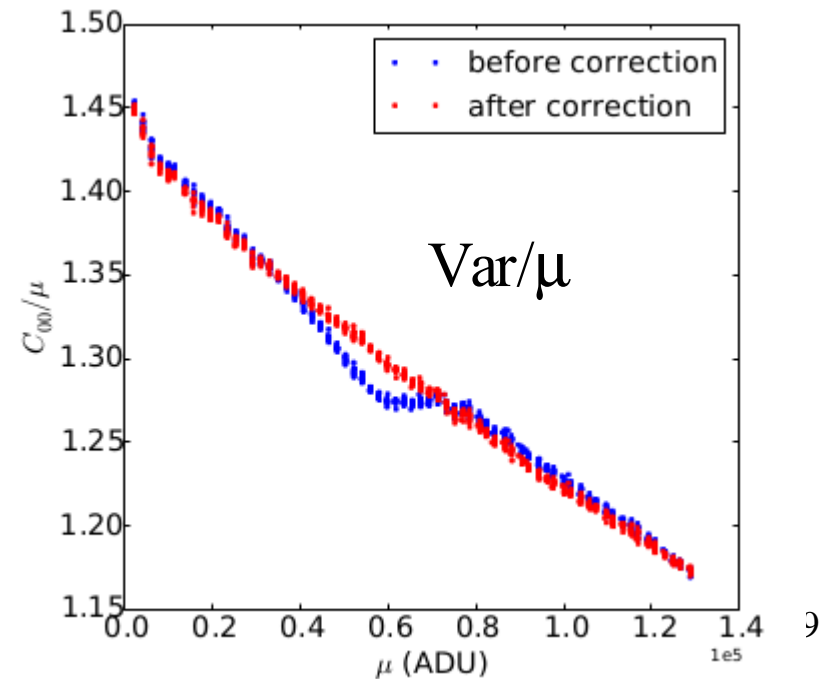
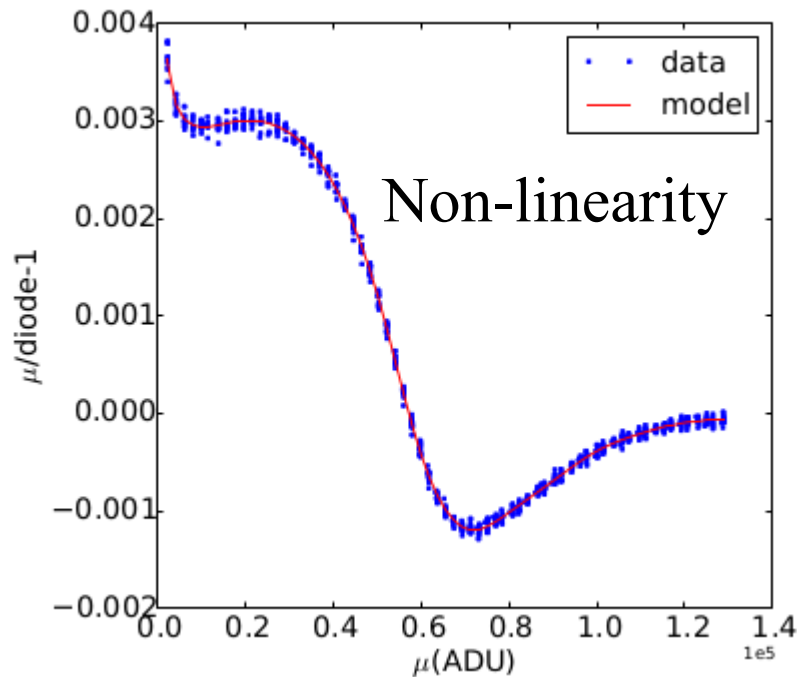
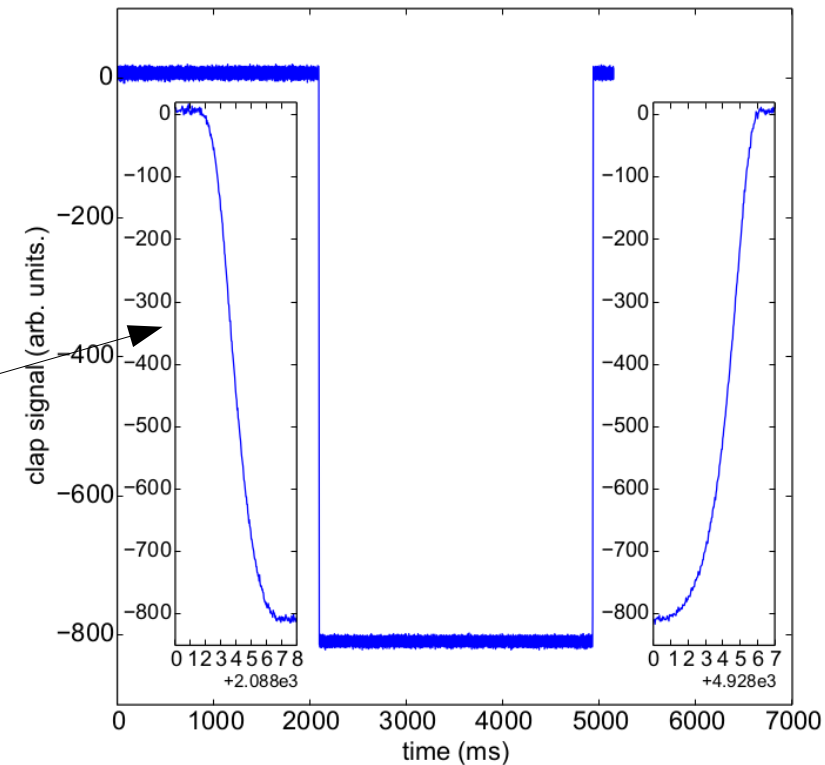


We first have to correct for: non-linearity & deferred charges

Non-linearity

The light received by the CCD is measured using an “amplified” photo-diode

We tune the integrated charge by varying the open-shutter time

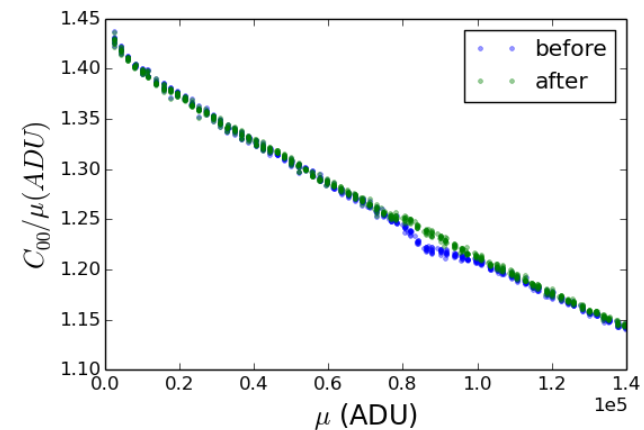
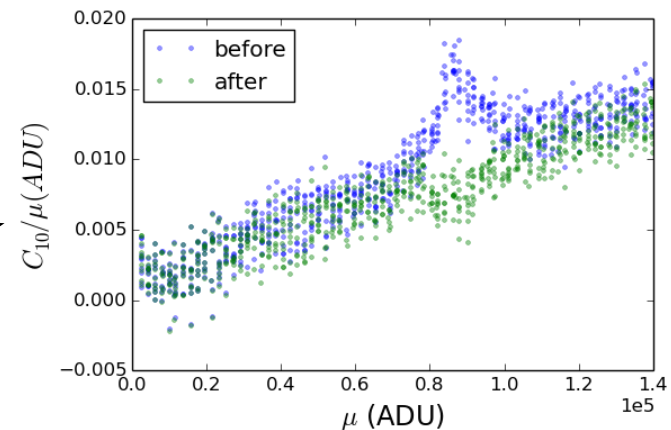
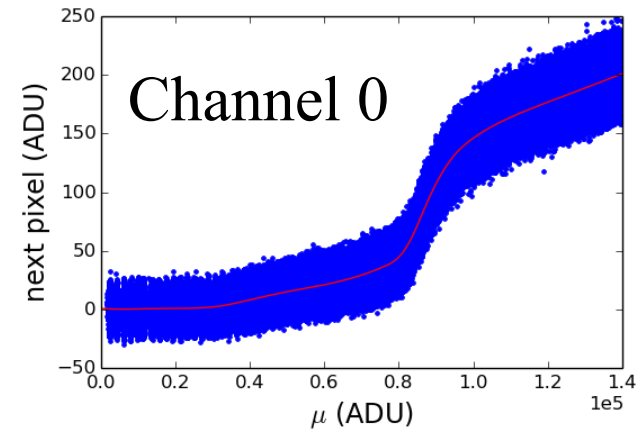


Deferred charge correction

First serial overscan pixel

C_{10}/μ : nearest serial neighbor covariance

Variance/ μ



- Different for each video channel
- Small over-correction (?)
- Reduces the correlation slope ($\sim a_{10}$) by $\sim 10\%$ (for this channel)

Fit results : Variance (PTC)

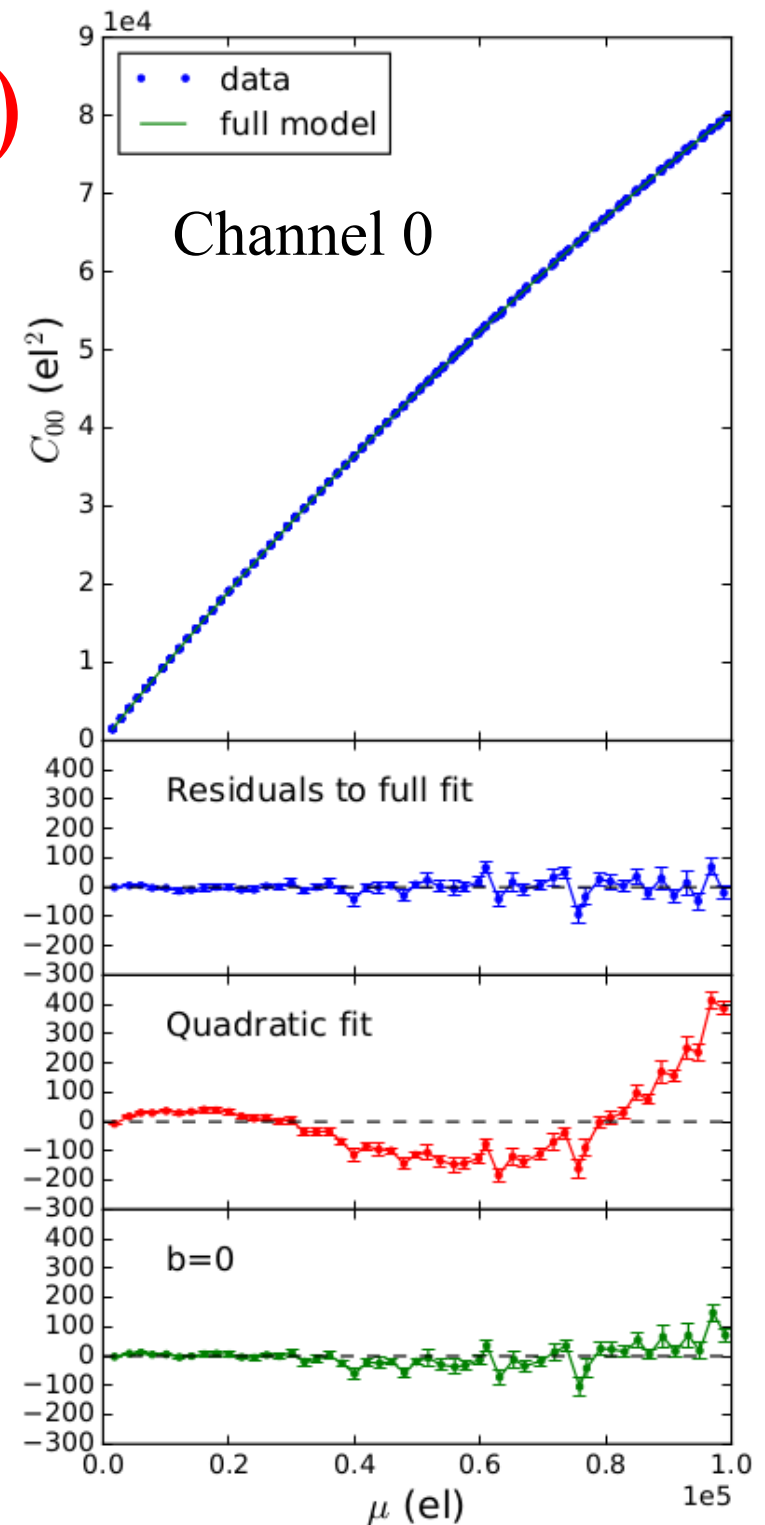
$$C_{ij}(\mu) = \frac{\mu}{g} [\delta_{i0}\delta_{j0} + a_{ij}\mu g + \frac{2}{3}[\mathbf{a} \otimes \mathbf{a} + \mathbf{a}\mathbf{b}]_{ij}(\mu g)^2 + \frac{1}{6}(2\mathbf{a} \otimes \mathbf{a} \otimes \mathbf{a} + 5\mathbf{a} \otimes \mathbf{a}\mathbf{b})_{ij}(\mu g)^3 + \dots] + \frac{n_{ij}}{g^2}$$

16 channels,
8x8 a_{ij} & 8x8 b_{ij}

Full fit (a and b)

All channels :

	χ^2_{full}/N_{dof}	χ^2_2/N_{dof}	gain	a_{00}	RO noise
value	1.23	4.04	0.713	$-2.376 \cdot 10^{-6}$	4.54
scatter	0.10	0.27	0.020	$0.032 \cdot 10^{-6}$	0.43



C01:

parallel nearest neighbor

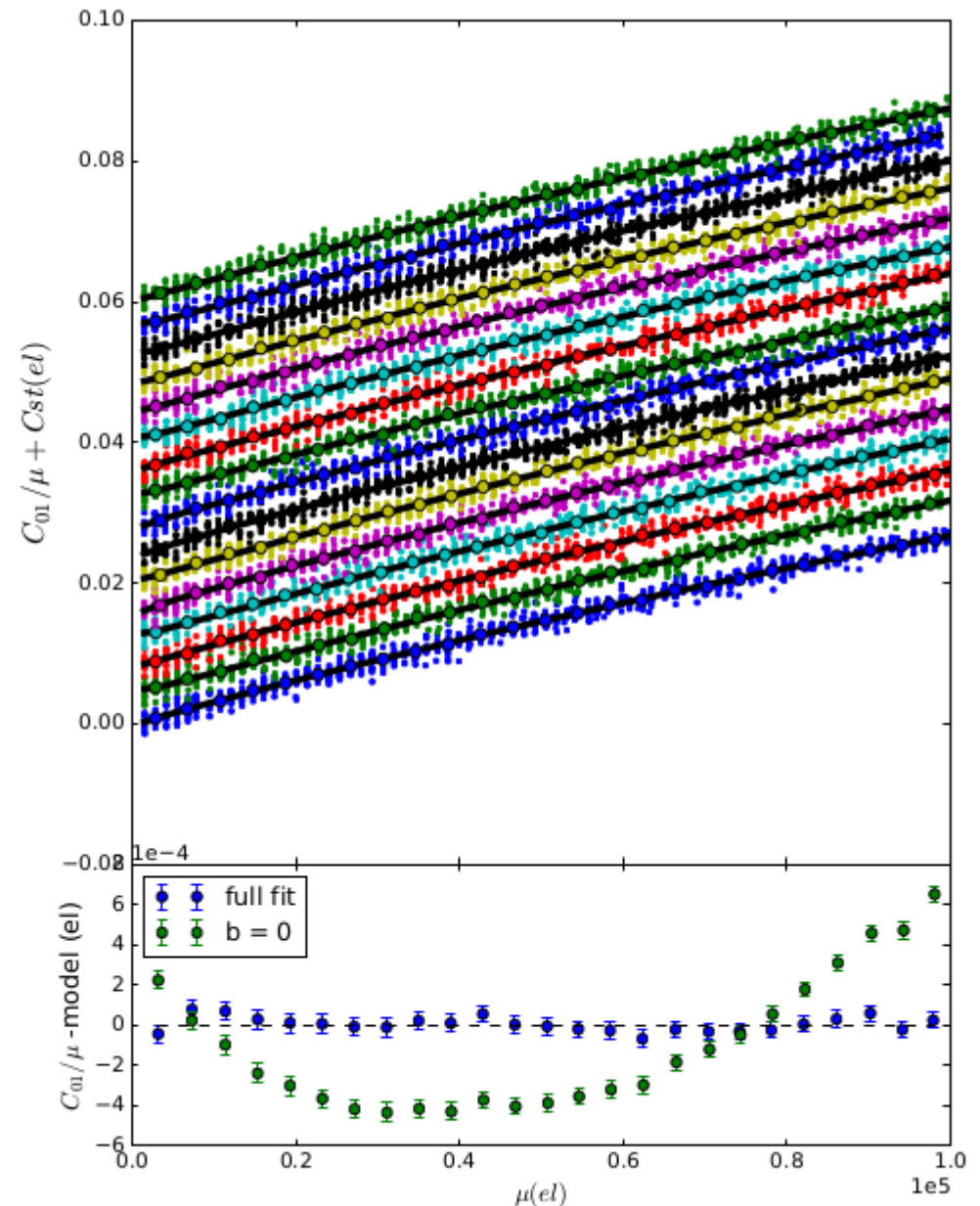
- The curvature of C_{01}/μ can be seen by eye
- $b=0$ is highly disfavored

	a_{01}	b_{01}	χ^2/N_{dof}
value	3.32e-07	1.71e-06	1.03
scatter	5.87e-09	2.87e-07	0.05

Scatter is twice
as much
as expected from
shot noise

Good
fits

C01+ Cst for the 16 channels



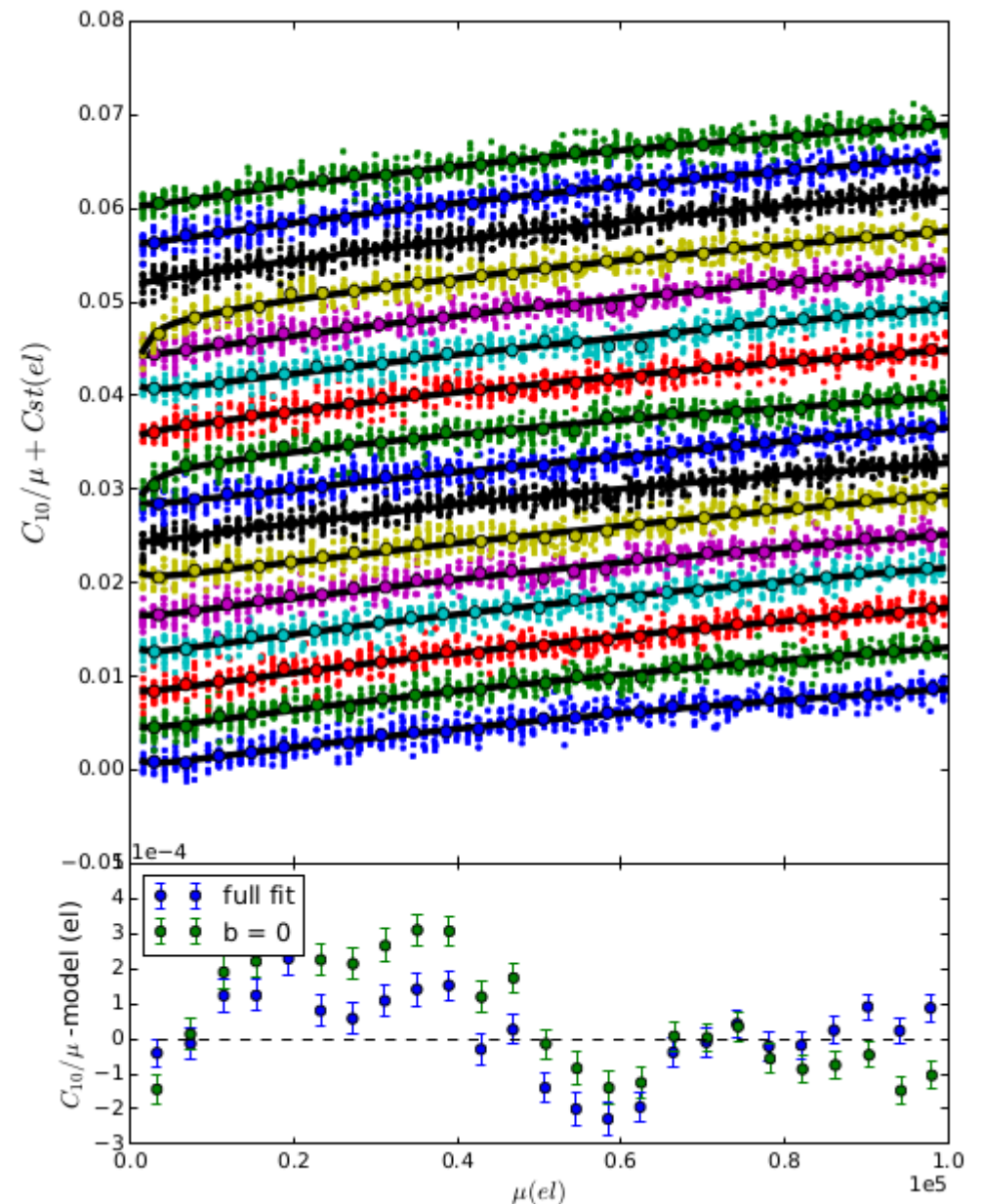
C10

serial nearest neighbor

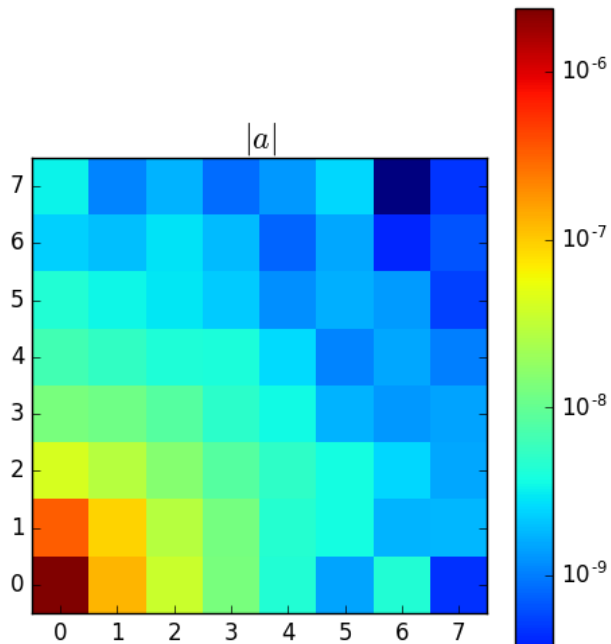
- Noisier than C01
- Much more scatter that seems real
- $b = 0$ still disfavored but much less striking

	a_{10}	b_{10}	χ^2/N_{dof}
value	$1.26 \cdot 10^{-7}$	$-1.77 \cdot 10^{-6}$	1.03
scatter	$0.08 \cdot 10^{-6}$	$0.97 \cdot 10^{-6}$	0.07

C10+ Cst for the 16 channels

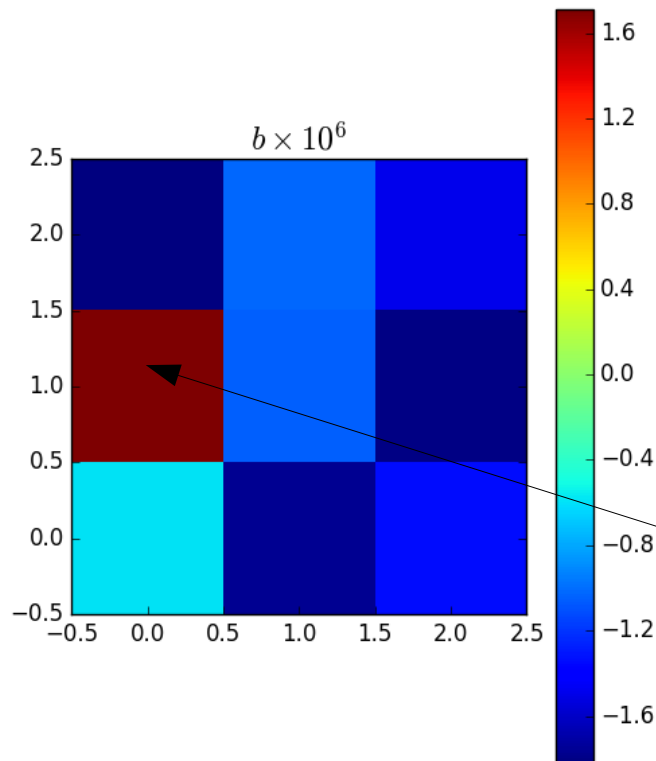


Fit results



Expressed
in el^{-1}

Average over
16 channels

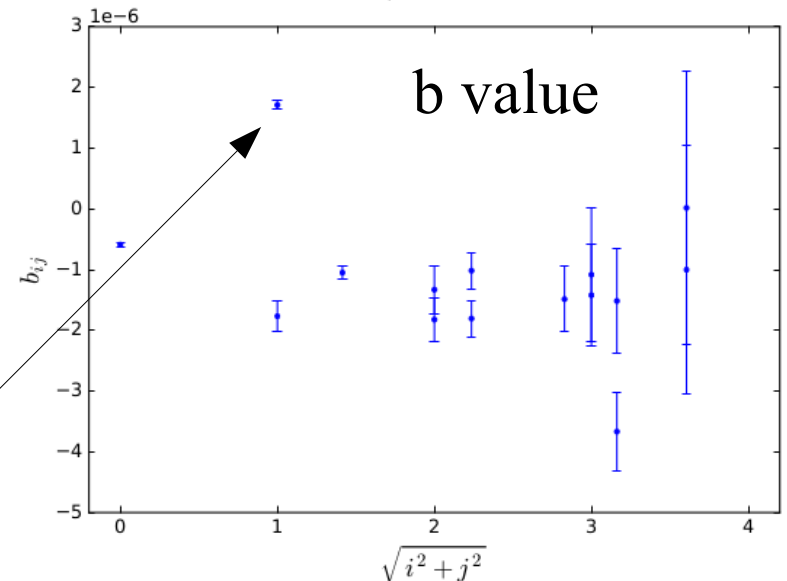
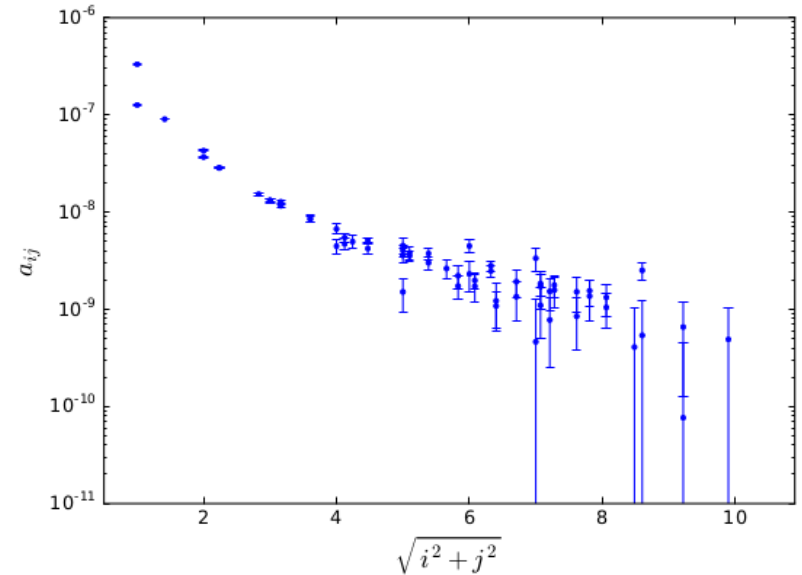


$a_{00} = -2.3 \cdot 10^{-6}$,
 $\Sigma a_{ij} = -3 \cdot 10^{-8}$,
so we miss
 $\sim 1.5\%$

$b_{10} > 0$

and all the others negative

a value vs distance



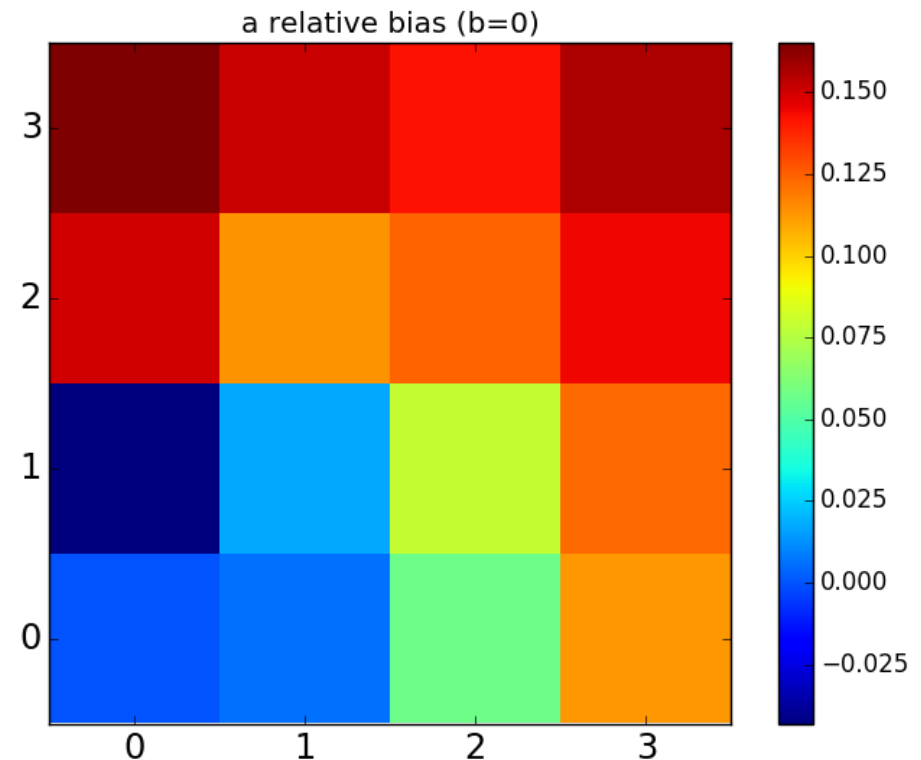
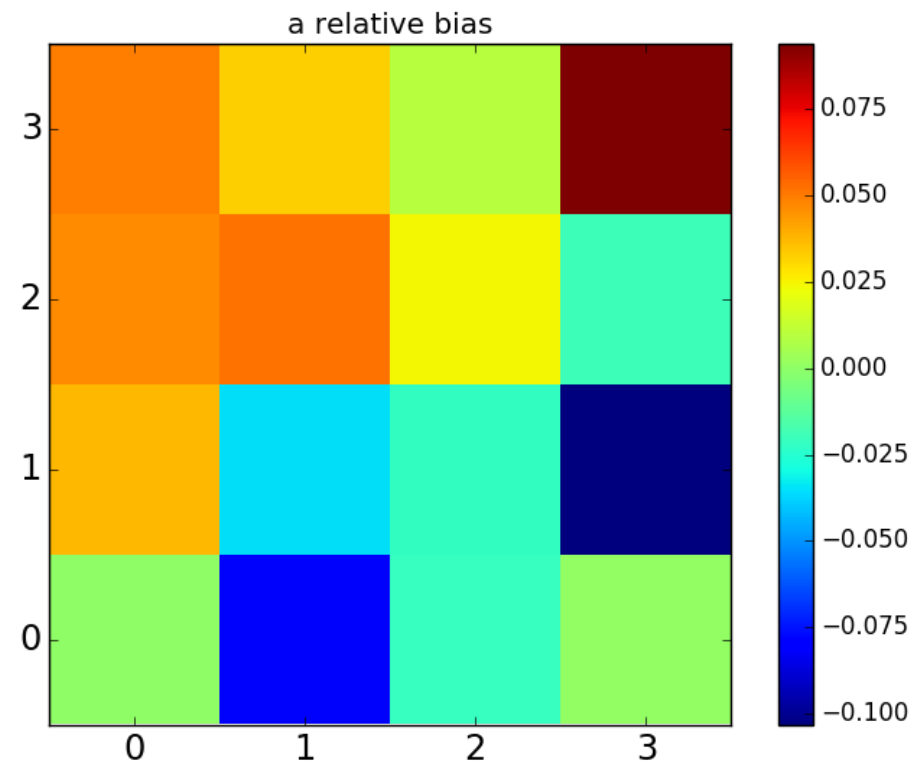
Comparison with the “standard” approach

Standard way (Antilogus + 2014):

$$a_{ij}\mu g = \frac{C_{ij}}{C_{00}}$$

At some (high) illumination

Difference between the full fit and the “standard” way : 10% peak to peak.



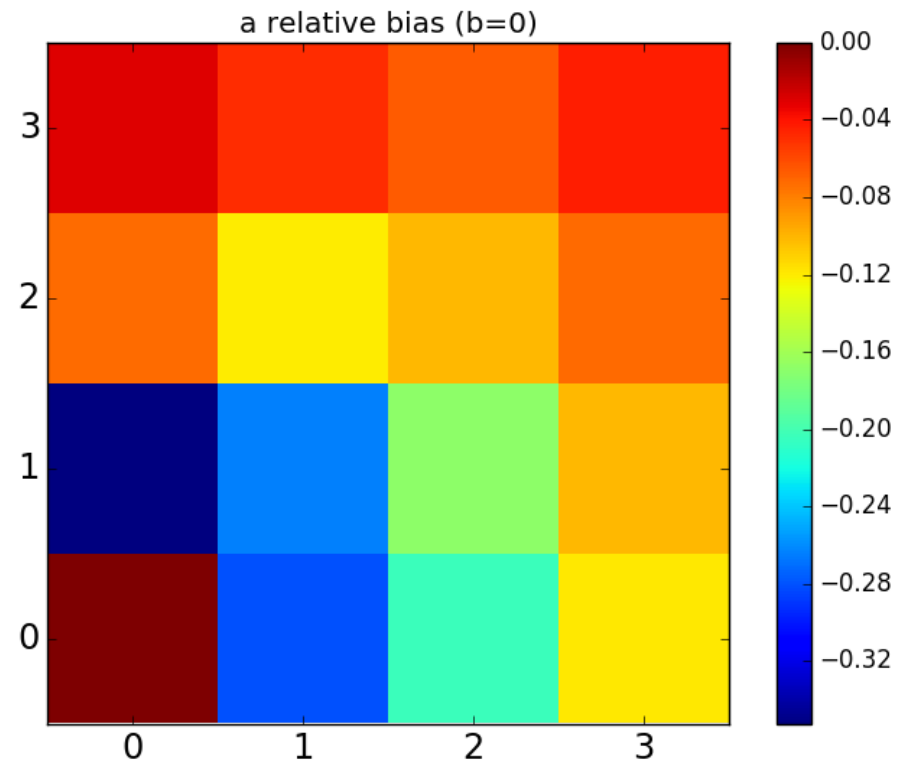
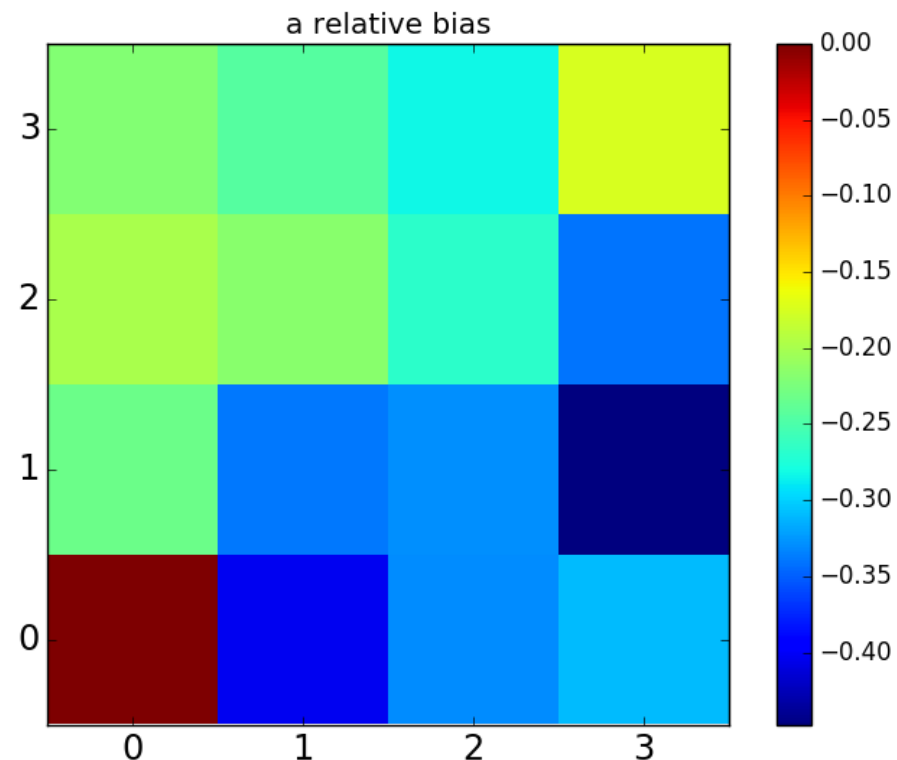
Comparison with the current “DM approach”

DM way (says Craig Lage):

$$C_{ij} = a_{ij} \mu^2$$

Possibly at several flux values.

Difference between the
full fit and the DM
way : 20% peak to peak.



Summary of the LSST (e2v) study

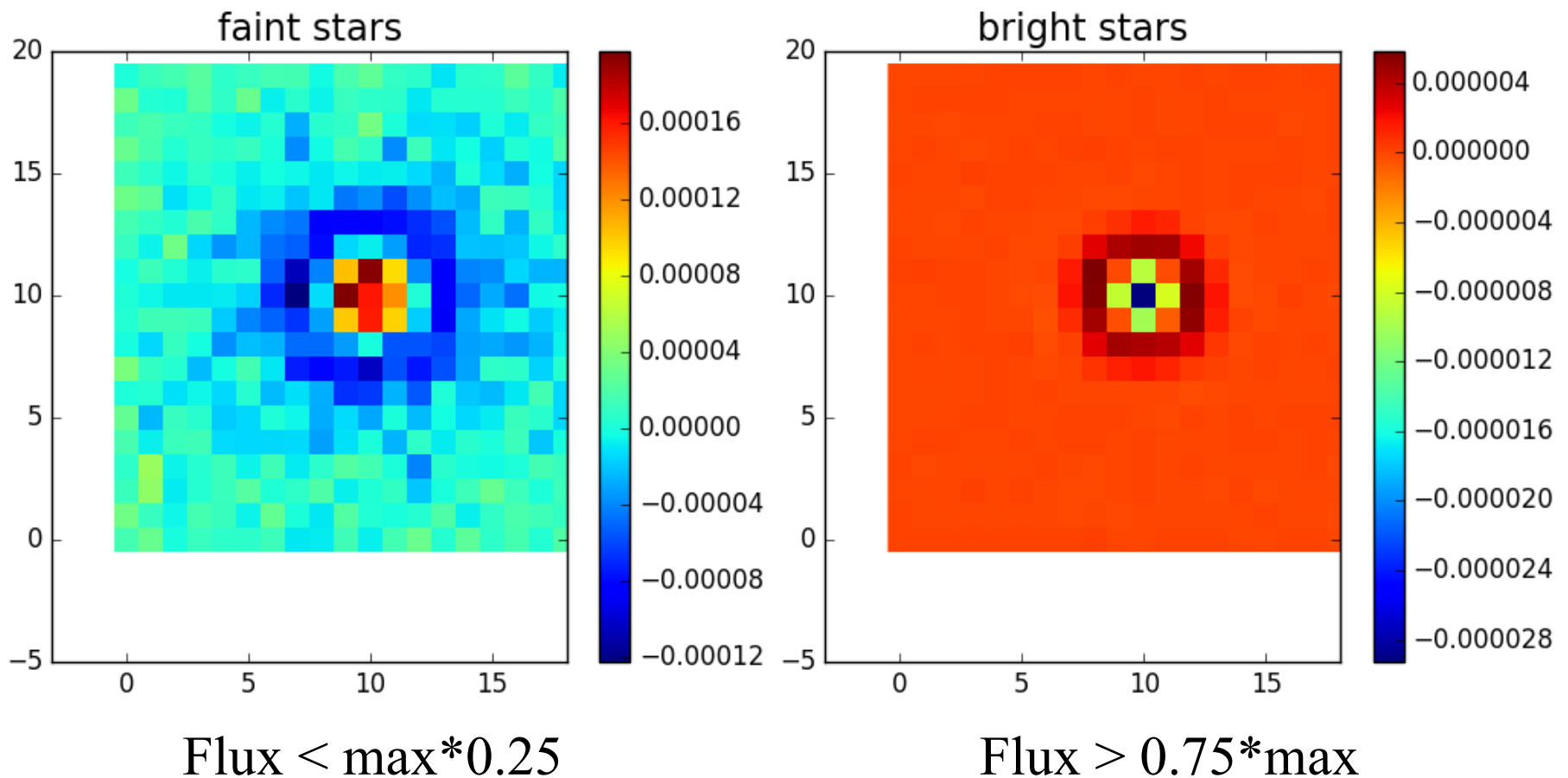
- We have developed a model for the PTC/Cov curve shapes.
- The expected shapes depend on the assumed dynamics (e.g. area alterations scale as source charges), and hence allow us to constrain the dynamics.
- With the “standard way”, systematic offsets of BF predictions by $\sim 10\%$ should not come as a surprise.
- Significantly worse with the DM way.

HSC ?

- I never had access to a sizable sample of HSC flat pairs.
- Augustin Guyonnet studied flats from Suprime Cam (~same chips) where the BF effect was found to be large.
- The current handling of BF for HSC leaves about 10% of the effect in (Mandelbaum + 17)
- No tests of the method in good IQ conditions.
- There are suspicions that the NLO effects are in fact generic.

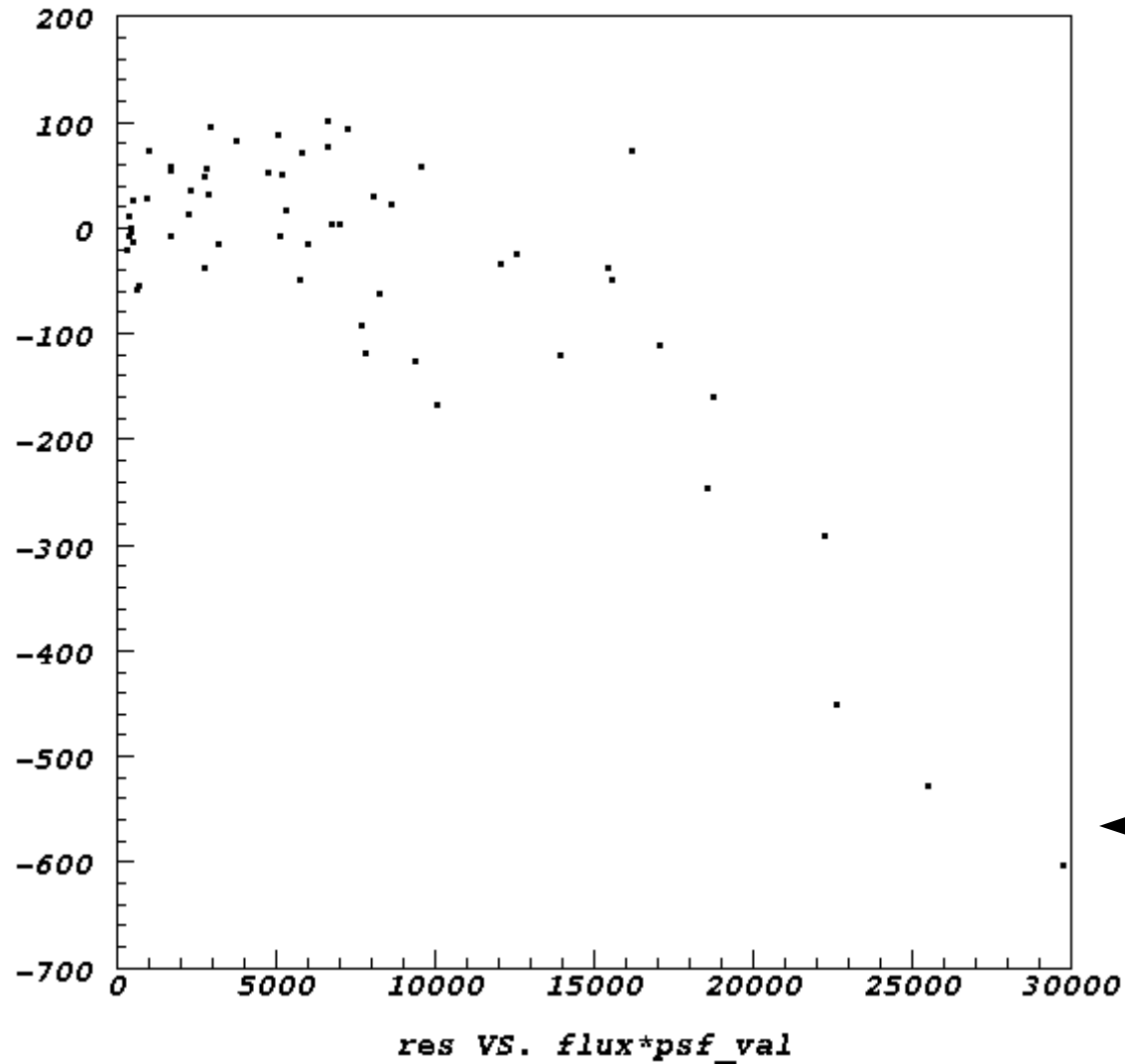
HSC processing without BF handling

- PSF residuals: $(\text{pixel}/\text{flux} - \text{PSF})$ (average seeing)



HSC PSF residuals

Residual of
central
pixel



Star peak flux

HSC tentative plans

- Integrate the BF effect into the PSF (forward modeling)
- Try to get constraints on BF (including possible NLO terms):
 - From flatfield pairs,
 - From observed star shapes.
- Refactoring the PSF modeling code to handle BF. If successful, PIFF should be the target.
- Non-linearity of the video chains ?

Covariances

