Status of the galileon model from cosmological observations

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Outline

I. Presentation of the galileon model

II. Methodology

III. Constraints from cosmology

IV. On tracker solutions

V. GW170817

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Horndeski theories

- Simple principles for an extension of General Relativity :
  - Additional scalar field π coupled to the metric
  - 2[nd] order e.o.m : easy way to avoid Ostrogradski ghosts

$$\Downarrow$$

Horndeski lagrangians

$$\mathcal{L}_2^{(H)} = G_2\left(\pi, X\right)$$

$$\mathcal{L}_3^{(H)} = G_3\left(\pi, X\right)\left(\Box\pi\right)$$

$$\mathcal{L}_4^{(H)} = G_4\left(\pi, X\right)R - G_{4,X}\left(\pi, X\right)\left[2\left(\Box\pi\right)^2 - 2\pi_{;\mu\nu}\pi^{;\mu\nu}\right]$$

$$\mathcal{L}_5^{(H)} = G_5\left(\pi, X\right)G_{\mu\nu}\pi^{;\mu\nu} + \frac{1}{6}G_{5,X}\left(\pi, X\right)\left[\left(\Box\pi\right)^3 - 3\left(\Box\pi\right)\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu}^{;\nu}\pi_{;\nu}^{;\rho}\pi_{;\rho}^{;\mu}\right]$$

- Where the G_i are arbitrary functions of π and X

A particular case : the galileon

- The galileon model is a particular case of Horndeski :
  - Galilean symmetry in Minkowskii space-time (inspired by DGP, massive gravity, extra dimensions, ...) :

$$\pi \to \pi + c + b_\mu x^\mu$$

  - Simple expressions for the arbitrary functions :

$$G_2 = c_1 M^3 \pi + c_2 X, \qquad G_3 = \frac{c_3 X}{M^3}, \qquad G_4 = M_P^2 - \frac{c_4}{M^6} X^2, \qquad G_5 = \frac{3 c_5 X^2}{M^9}$$

  - The c_i are arbitrary parameters and $M^3 = M_P H_0^2$
  - Addition of direct couplings to matter : conformal and/or disformal

The galileon model

- The galileon action :

$$\mathcal{S}\left[\phi, g, \pi\right] = \mathcal{S}_{SM}\left[\phi, g\right] + \int d^4x\sqrt{-g}\left[\left(1 - 2c_0\frac{\pi}{M_P}\right)\frac{M_P^2}{2}R - \frac{1}{2}\sum_{i=1}^{5}\frac{c_i}{M^{3(i-2)}}\mathcal{L}_i - \frac{M_P}{M^3}c_G G^{\mu\nu}\pi_{;\mu}\pi_{;\nu}\right]$$

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Disformal coupling

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$$\mathcal{L}_1 = \pi$$

$$\mathcal{L}_2 = X$$

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$$\mathcal{L}_4 = X\left[2\left(\Box\pi\right)^2 - 2\left(\pi_{;\mu\nu}\pi^{;\mu\nu}\right) - \frac{1}{2}XR\right]$$

$$\mathcal{L}_5 = X\left[\left(\Box\pi\right)^3 - 3\left(\pi_{;\mu\nu}\pi^{;\mu\nu}\right)\Box\pi + 2\left(\pi_{;\mu}^{;\nu}\pi_{;\nu}^{;\rho}\pi_{;\rho}^{;\mu}\right) - 6\left(\pi_{;\mu}\pi^{;\mu\nu}G_{\nu\rho}\pi^{;\rho}\right)\right]$$

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← Non-linear lagrangians

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- Higher order lagrangians necessary to screen the galileon at small scales through Vainshtein effect

The galileon model

- A popular modified gravity model :
  - Cosmological solution with accelerated expansion
  - No effect near massive bodies due to Vainshtein screening

  $\Rightarrow$ necessary to pass tests of gravity in the solar system
  - No ghost degrees of freedom
  - Simple construction principles and limit of other well motivated cosmological models
  - Only up to six real parameters

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Galileon predictions

- Evolution in galileon gravity given by e.o.m of π and Einstein equations :

$$\frac{\delta \mathcal{S}}{\delta \pi} = 0 \qquad \text{and} \qquad G_{\mu\nu} = \kappa T_{\mu\nu}^{SM} + \kappa T_{\mu\nu}^{(\pi)}$$

- The galileon field is treated as a new fluid
- At first order $\Rightarrow$ background evolution necessary to compute cosmological distances
- At linear order $\Rightarrow$ perturbations evolution necessary to compute CMB powerspectra

Background evolution

- Cosmological background evolution :

$$\frac{dH}{d\ln a} = \frac{\omega\gamma - \lambda\beta}{\sigma\beta - \alpha\omega}$$

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functions of H and x

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$$\begin{array}{lll} c_i & \rightarrow & \bar{c}_i \equiv c_i B^i, \quad i = 2, ..., 5 \\ c_G & \rightarrow & \bar{c}_G \equiv c_G B^2 \\ x & \rightarrow & \bar{x} \equiv x/B \end{array}$$

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Perturbations evolution

- Scalar perturbations evolution in the synchronous gauge : $$0 = f_1^{eom} \cdot \bar{\gamma}'' + f_2^{eom} \cdot \bar{\gamma}' + f_3^{eom} \cdot k^2 \bar{\gamma} + f_4^{eom} \cdot k\mathcal{H}\mathcal{Z} + f_5^{eom} \cdot k\mathcal{Z}' + f_6^{eom} \cdot k^2 \eta$$

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- Barreira et al. 2013 showed that initial conditions for galileon perturbations can be taken as :

$$\gamma = \gamma' = 0 \qquad \text{at} \quad z \sim 10^{10}$$

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- A posteriori comparison to GW speed constraint from GW170817

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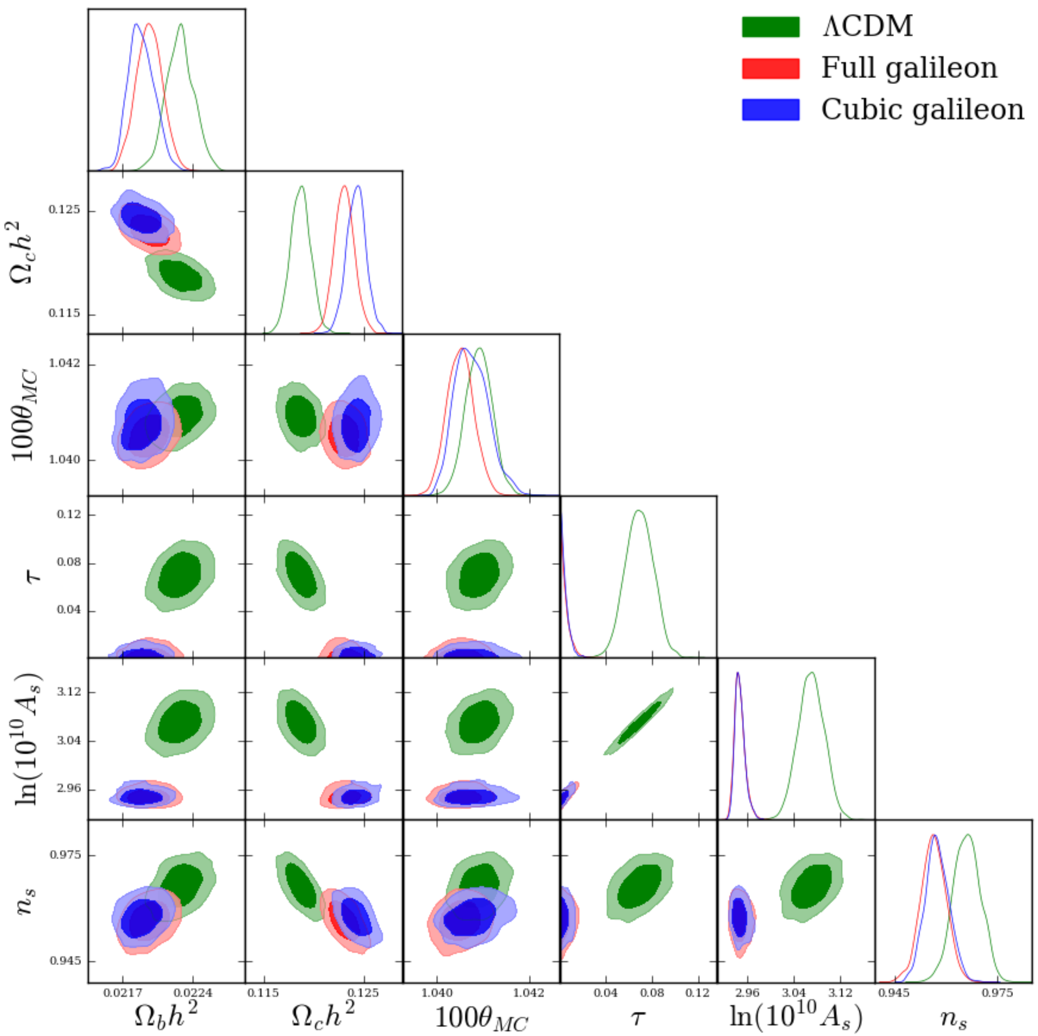

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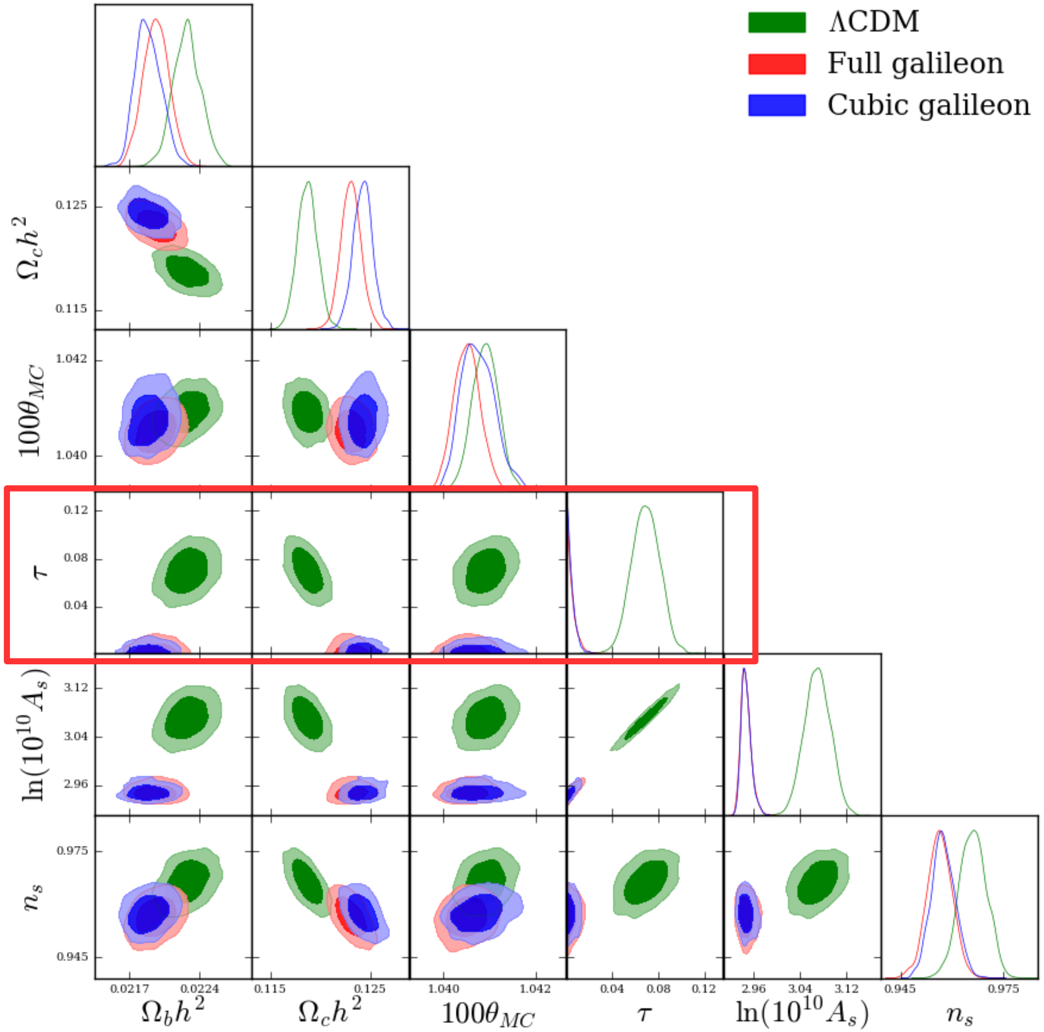

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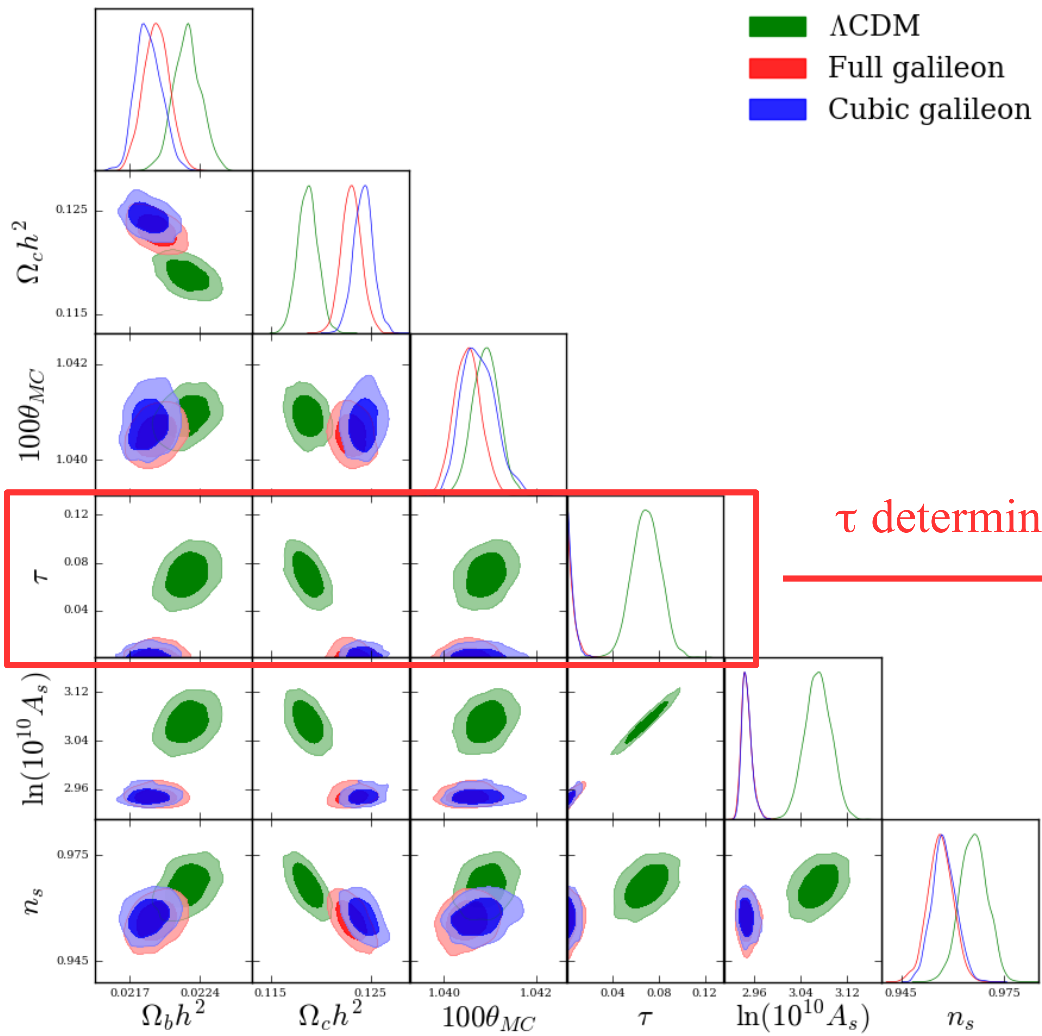

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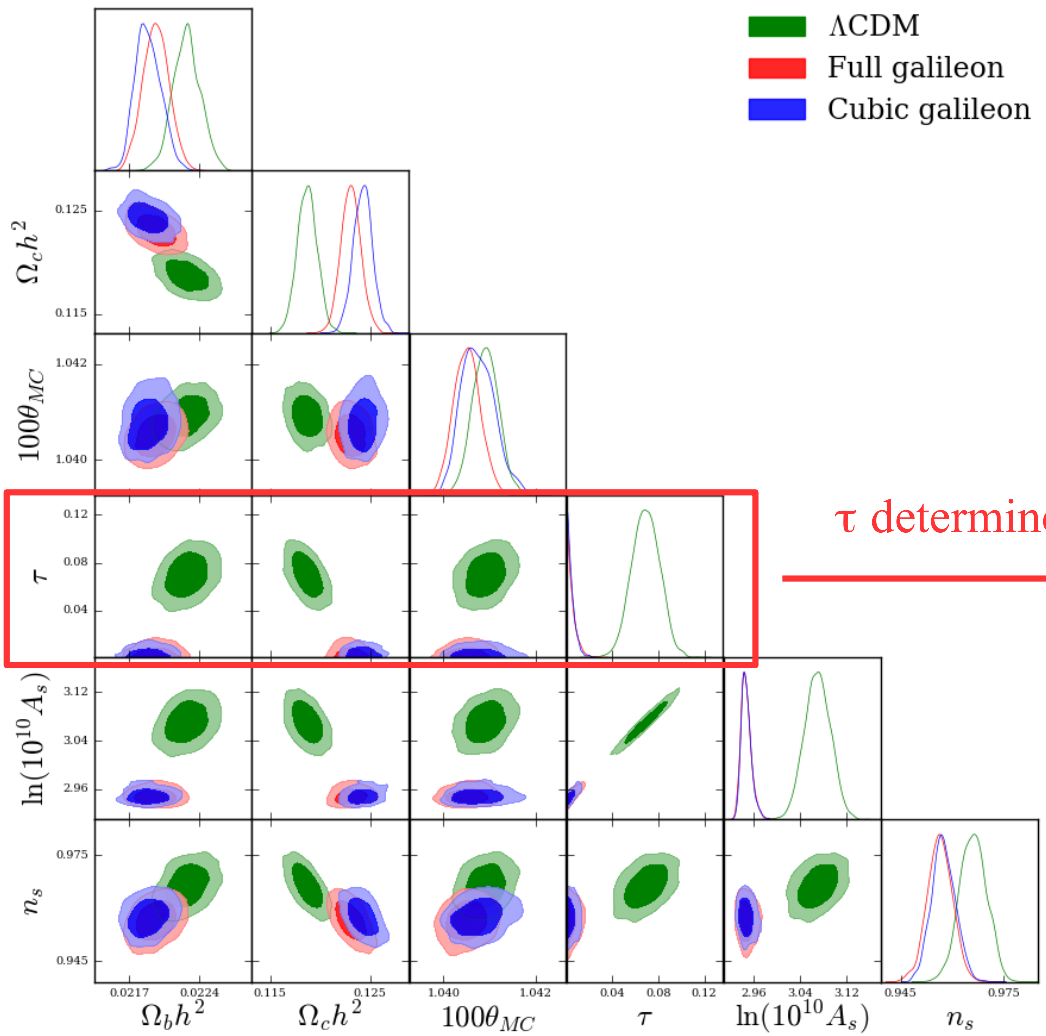

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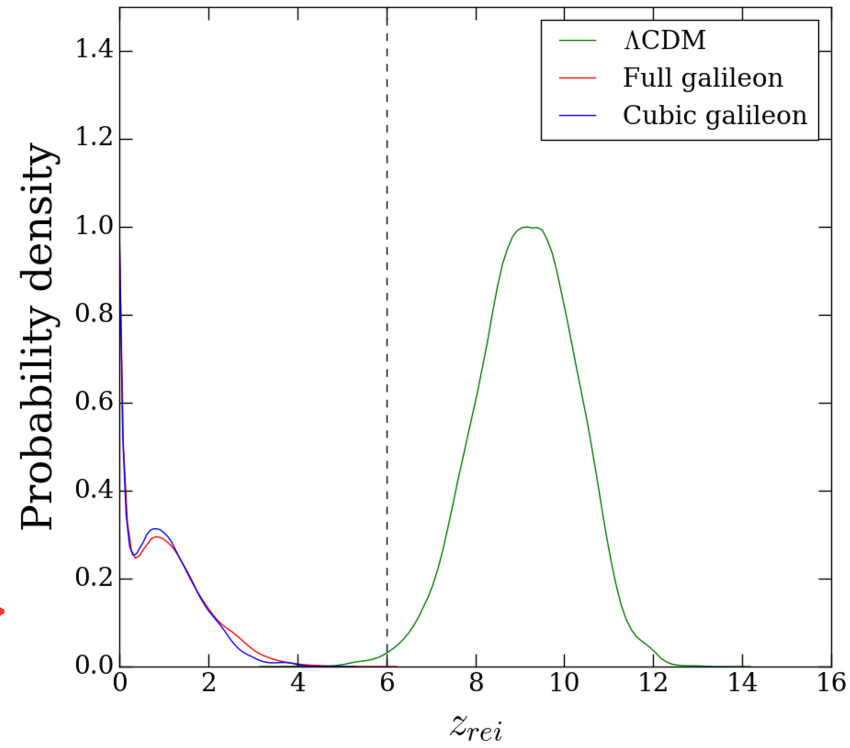

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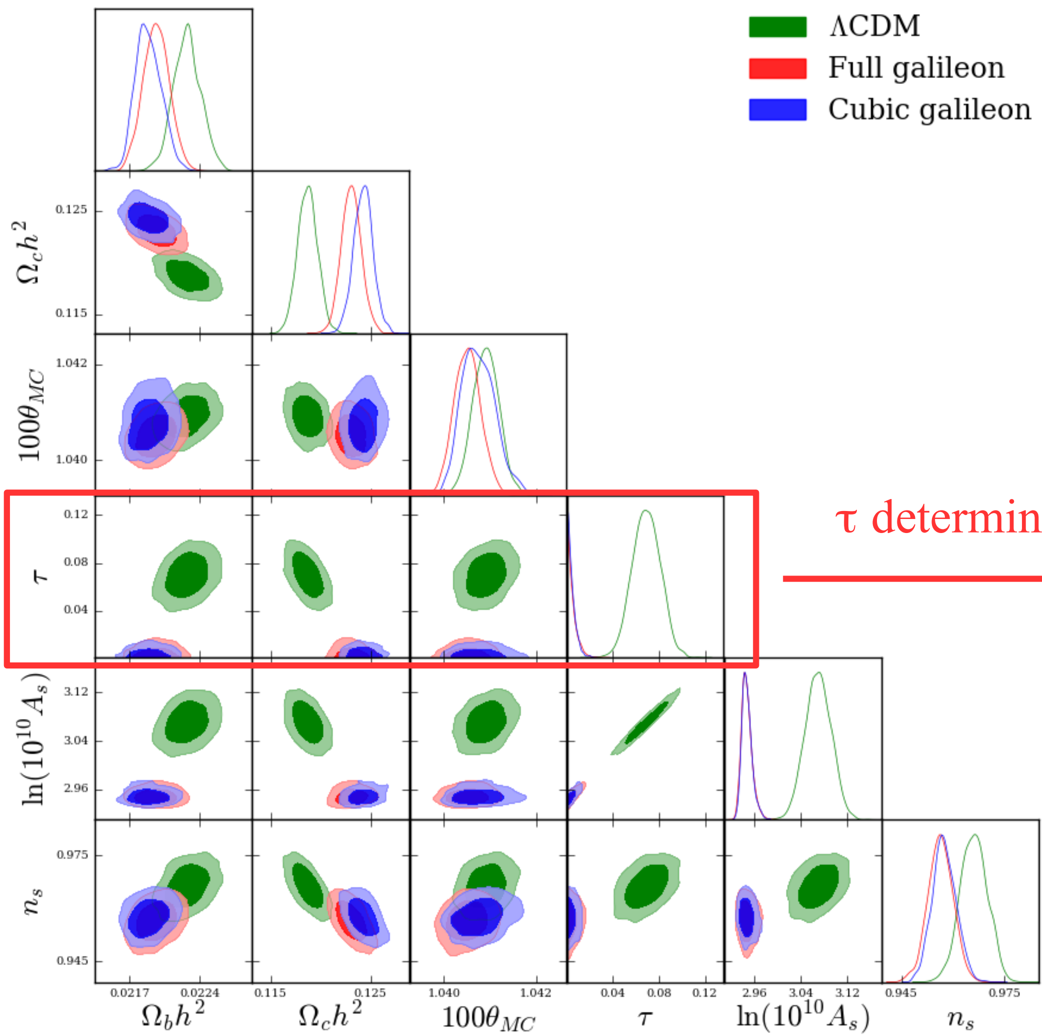
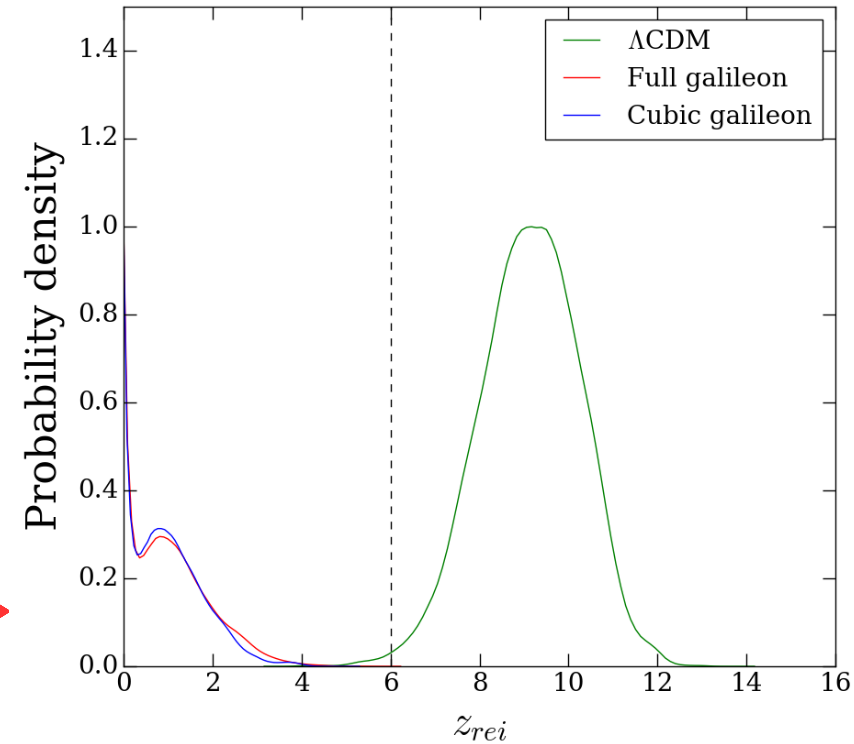


τ determines z_{rei}



- Astrophysical observations indicate that $z_{rei} > 6$ (e.g. astro-ph/0108097)

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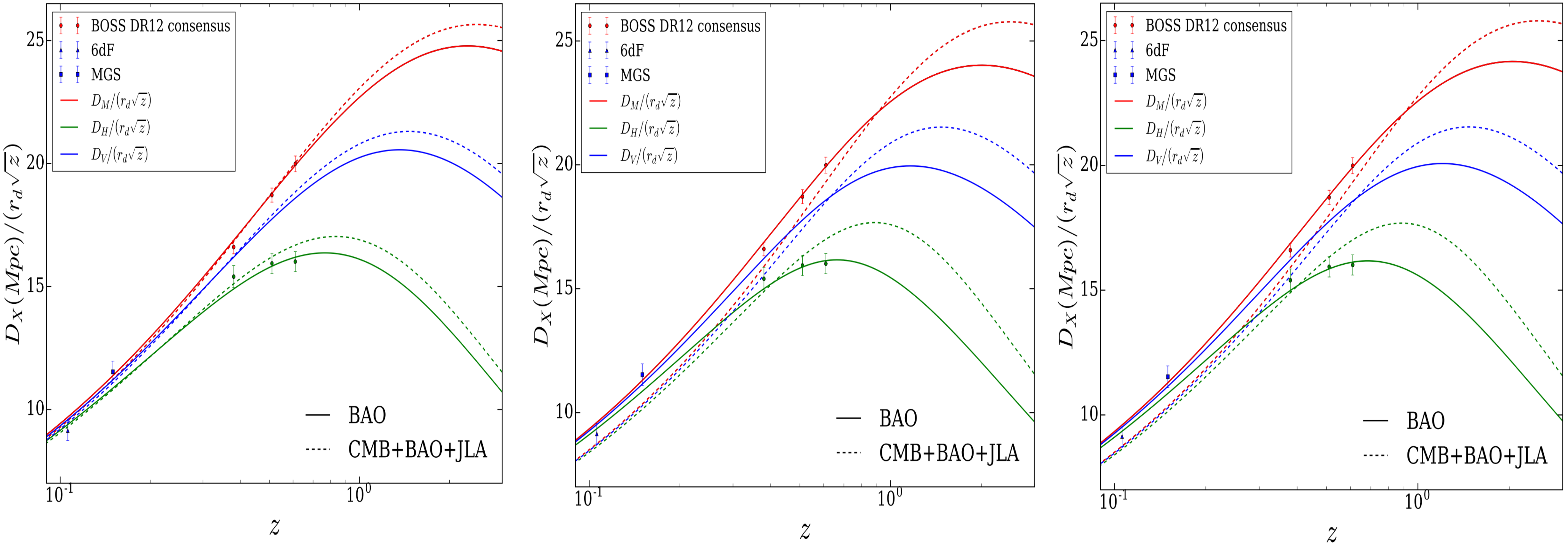
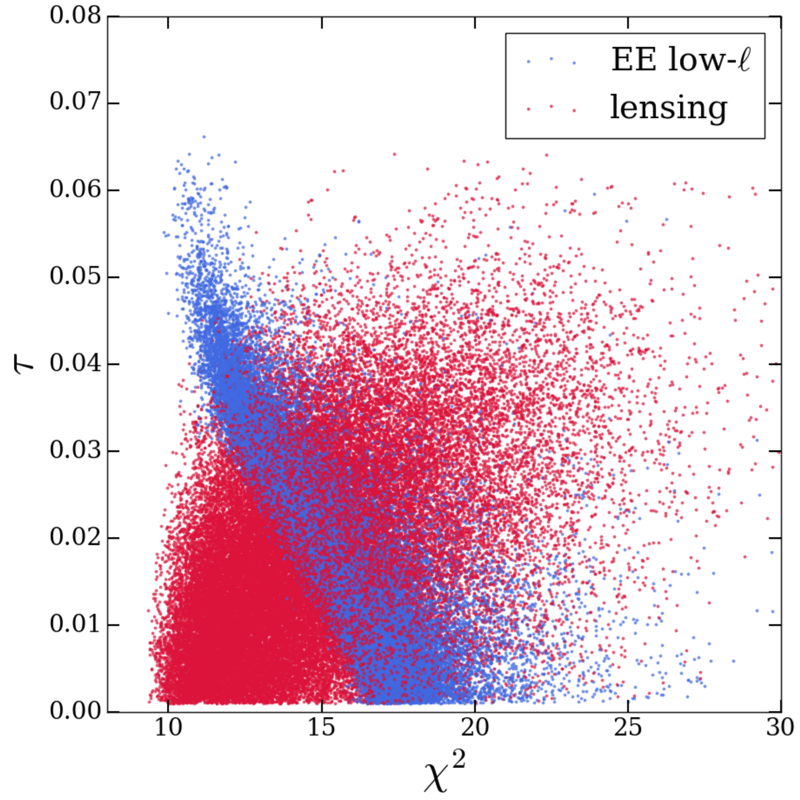


ΛCDM Full galileon Cubic galileon

Tensions in base models

- Tension on τ due to :
  - lensing
  - low-ℓ of polarization



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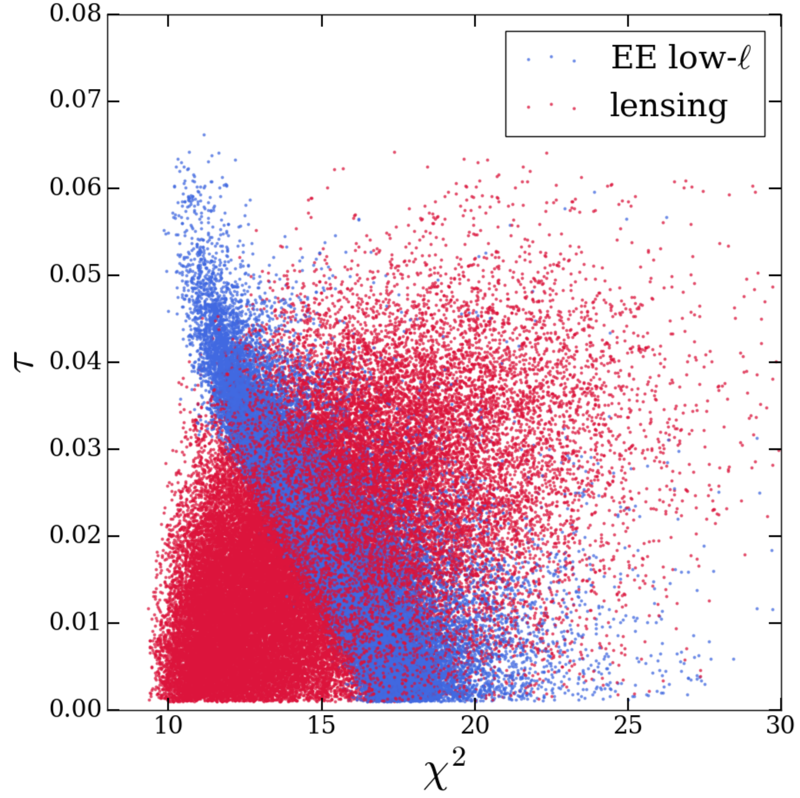

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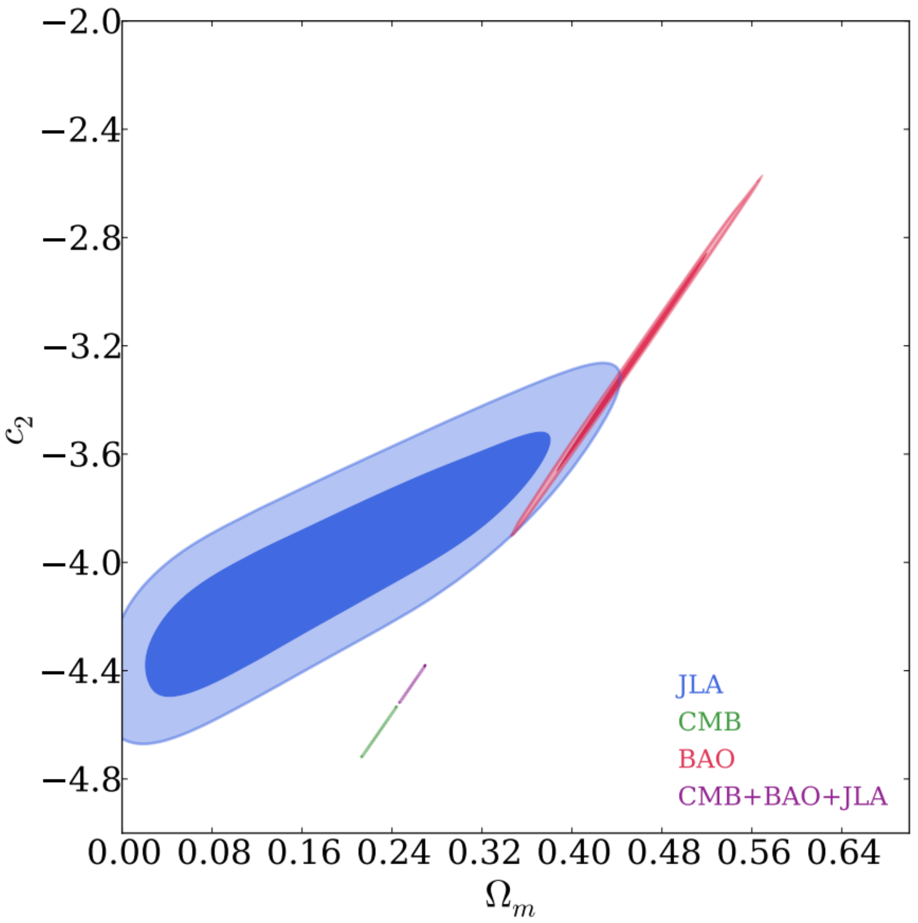

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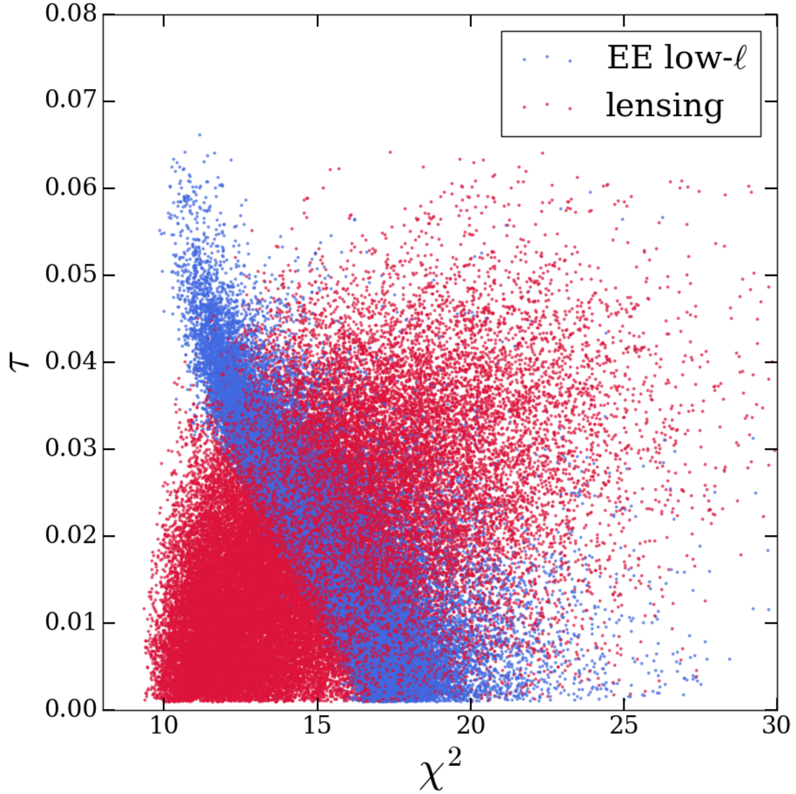

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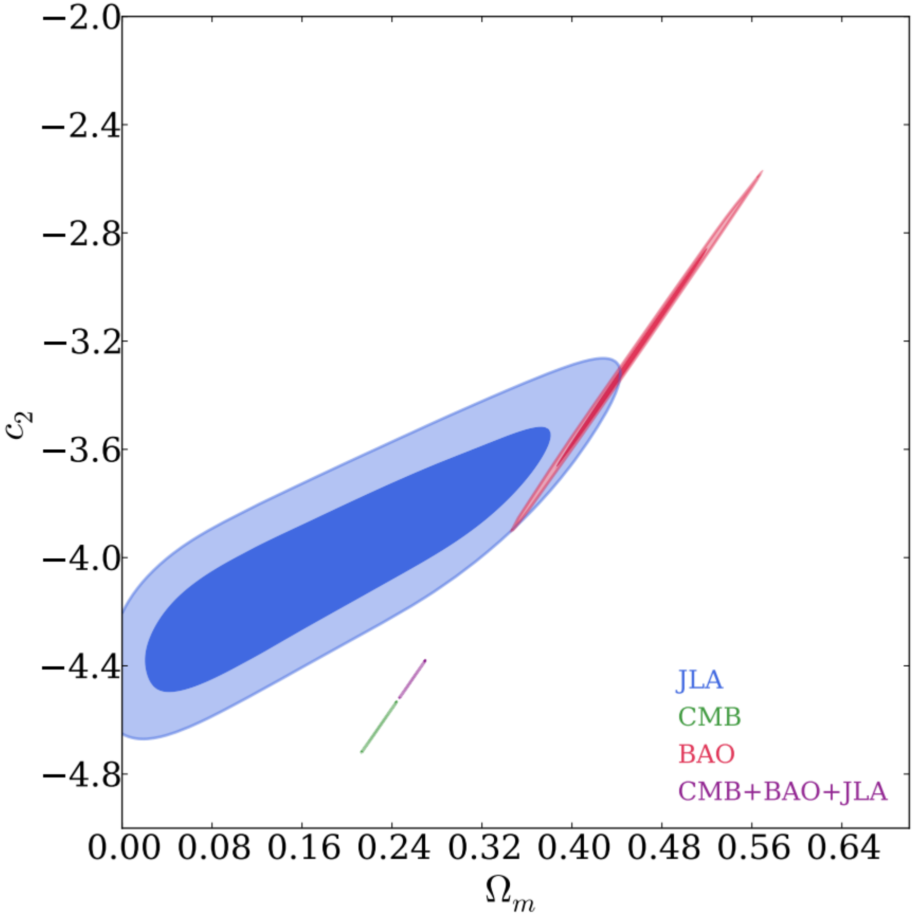
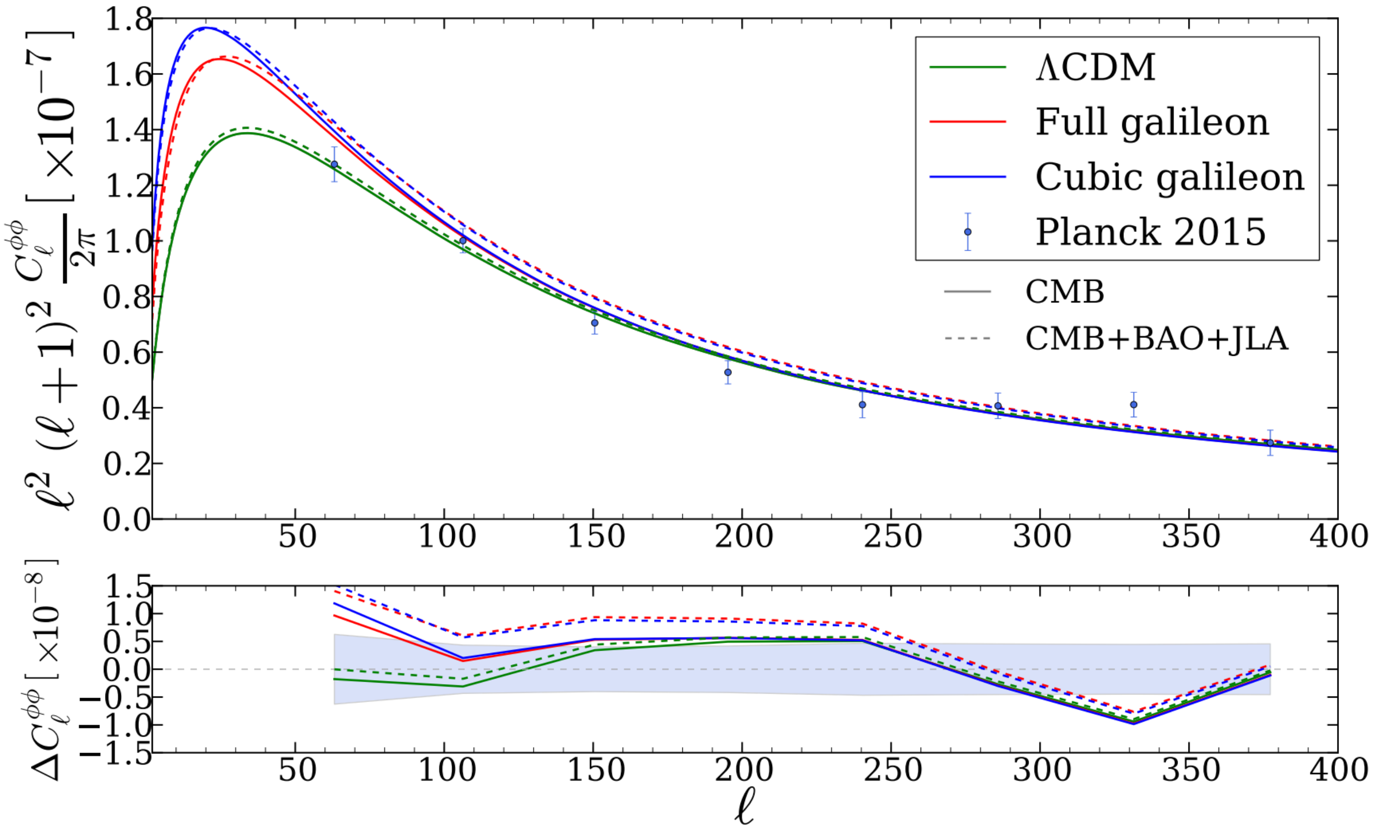


- Improve the situation with new parameters ?

Tensions in base models

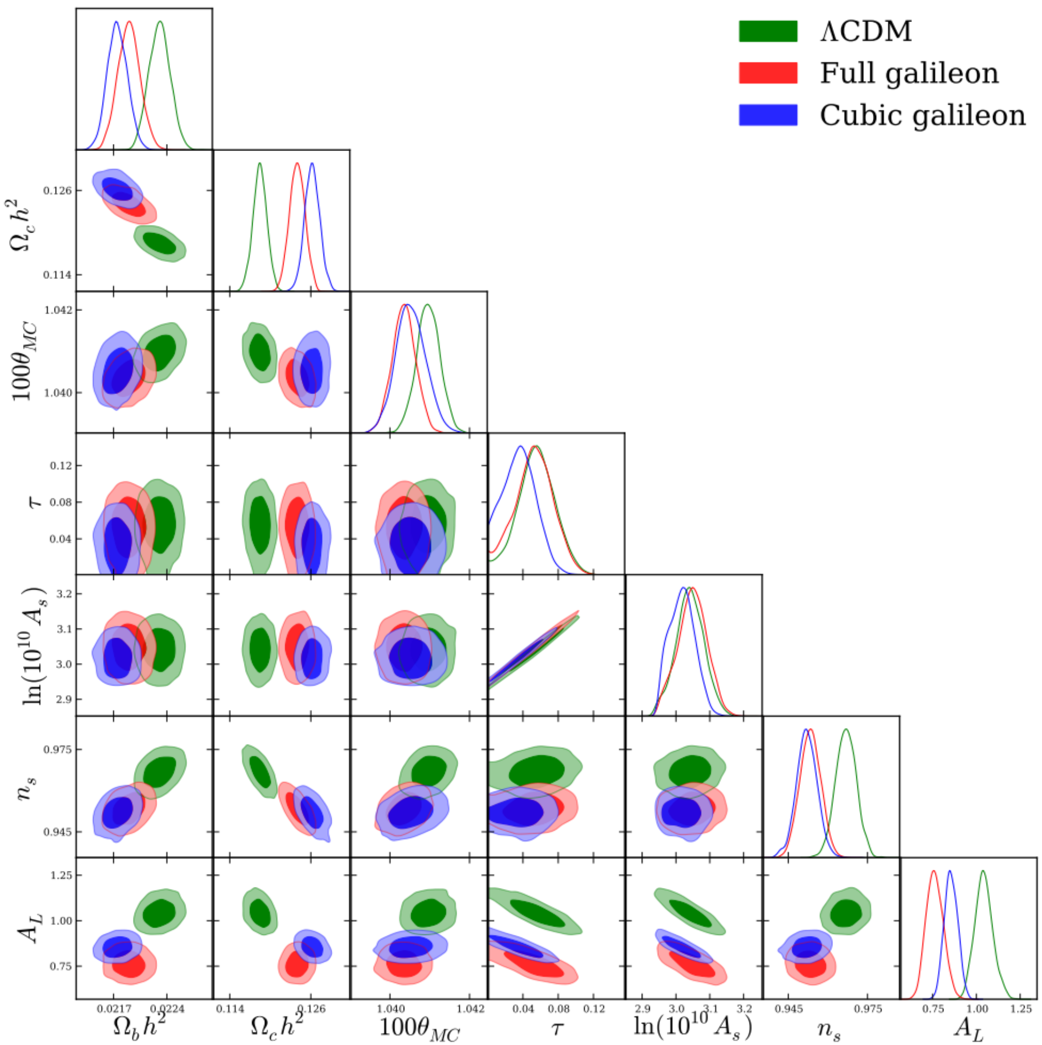
- CMB lensing power spectrum favours low A_s
- Because lensing effect stronger in galileon scenarios



- Additional parameters that have an effect on lensing normalization : A_L or Σm_ν

Extension to A_L

- Model extended to the parameter A_L :



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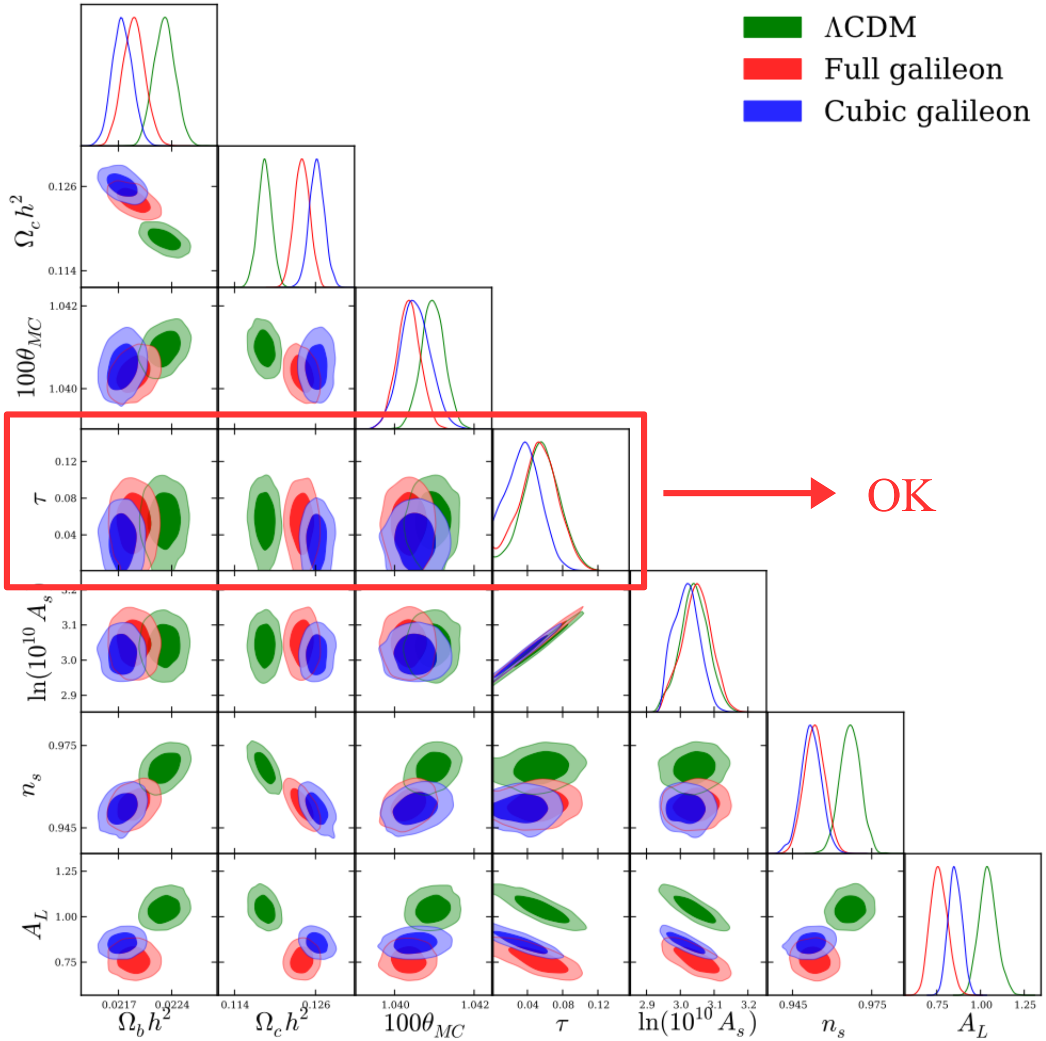

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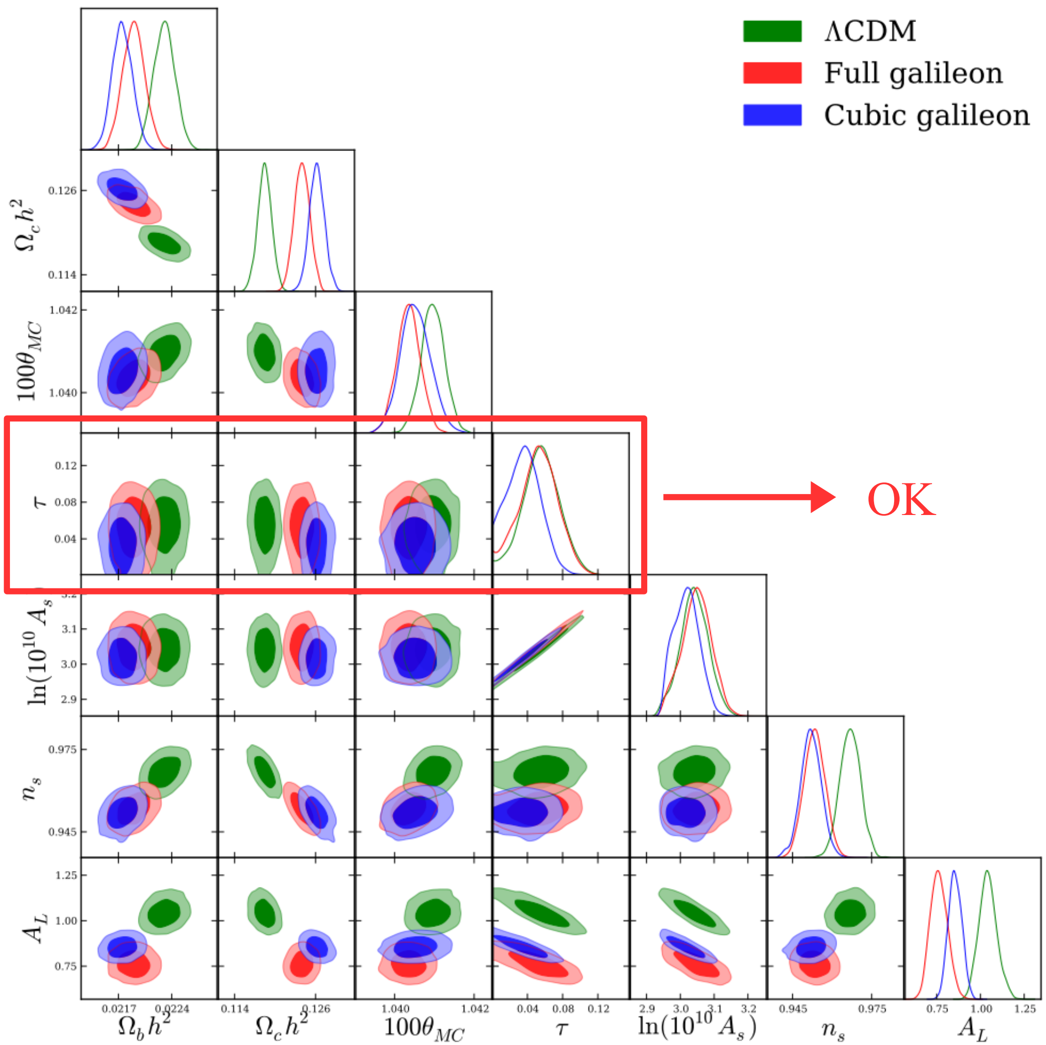
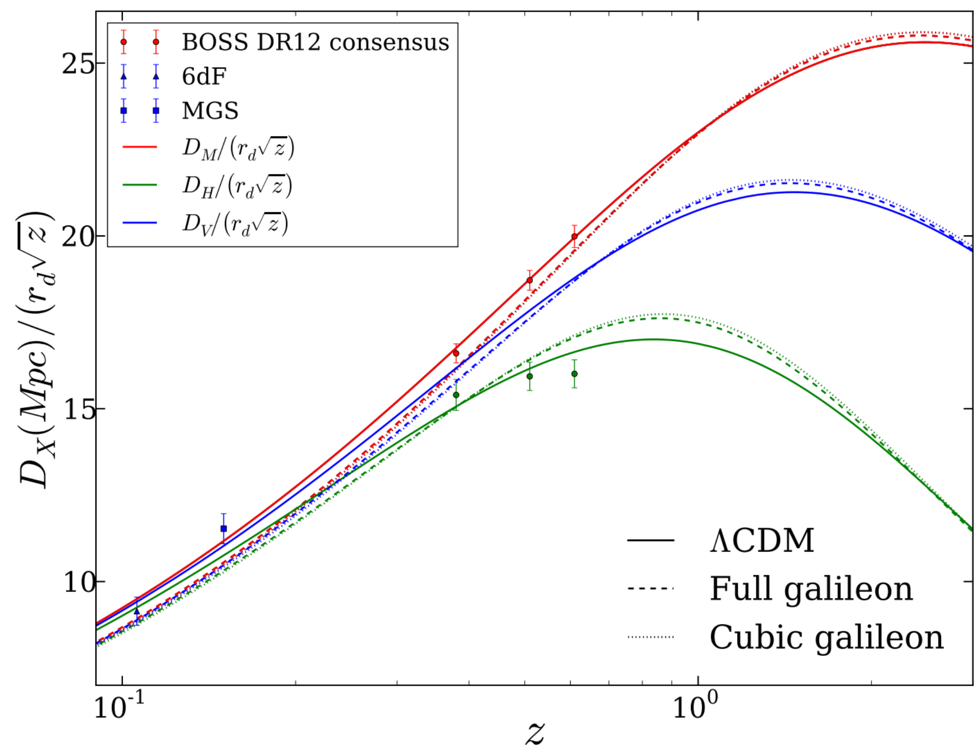


	χ^2(CMB)	χ^2(BAO)	χ^2(JLA)
ΛCDM	12945	5.2	706.6
Full galileon	12960	18.4	718.9
Cubic galileon	12984	22.5	721.6



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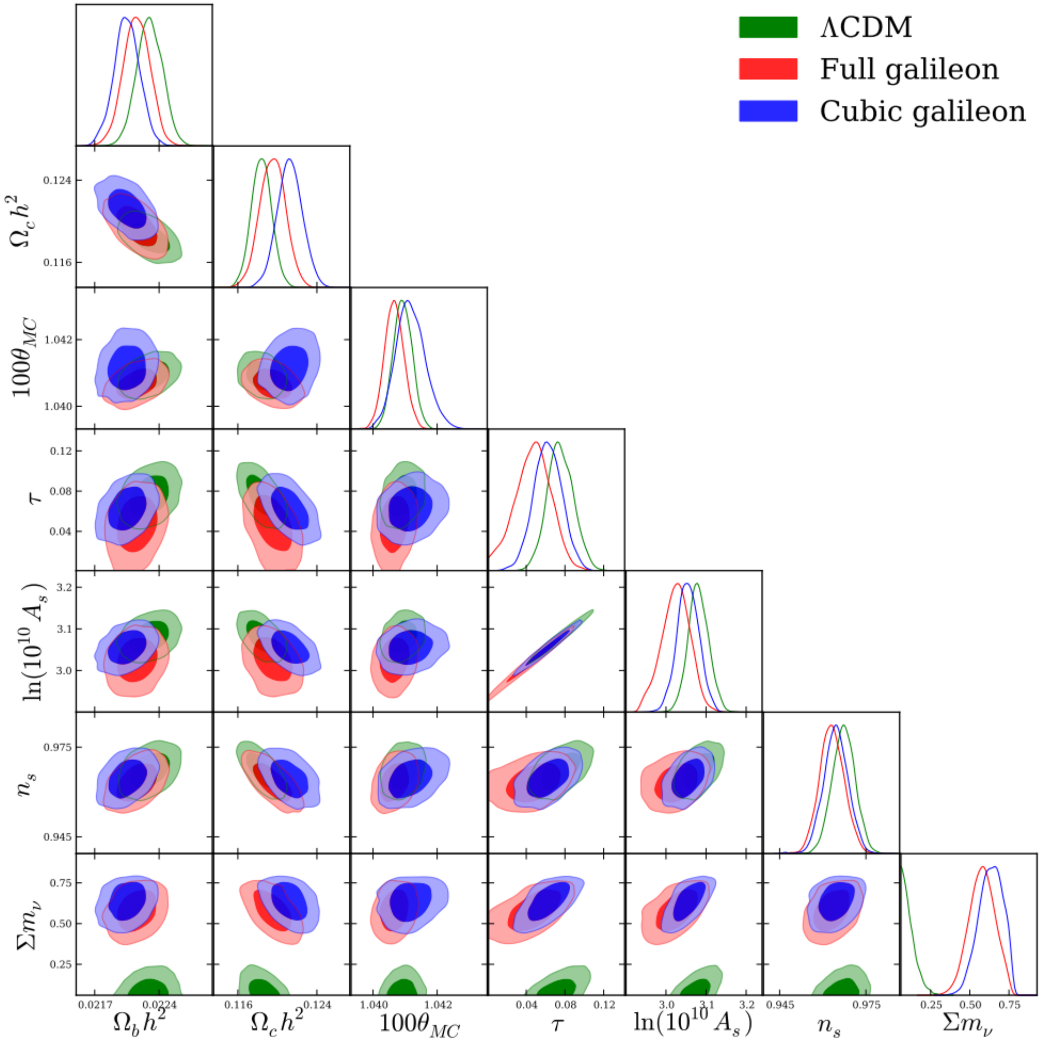

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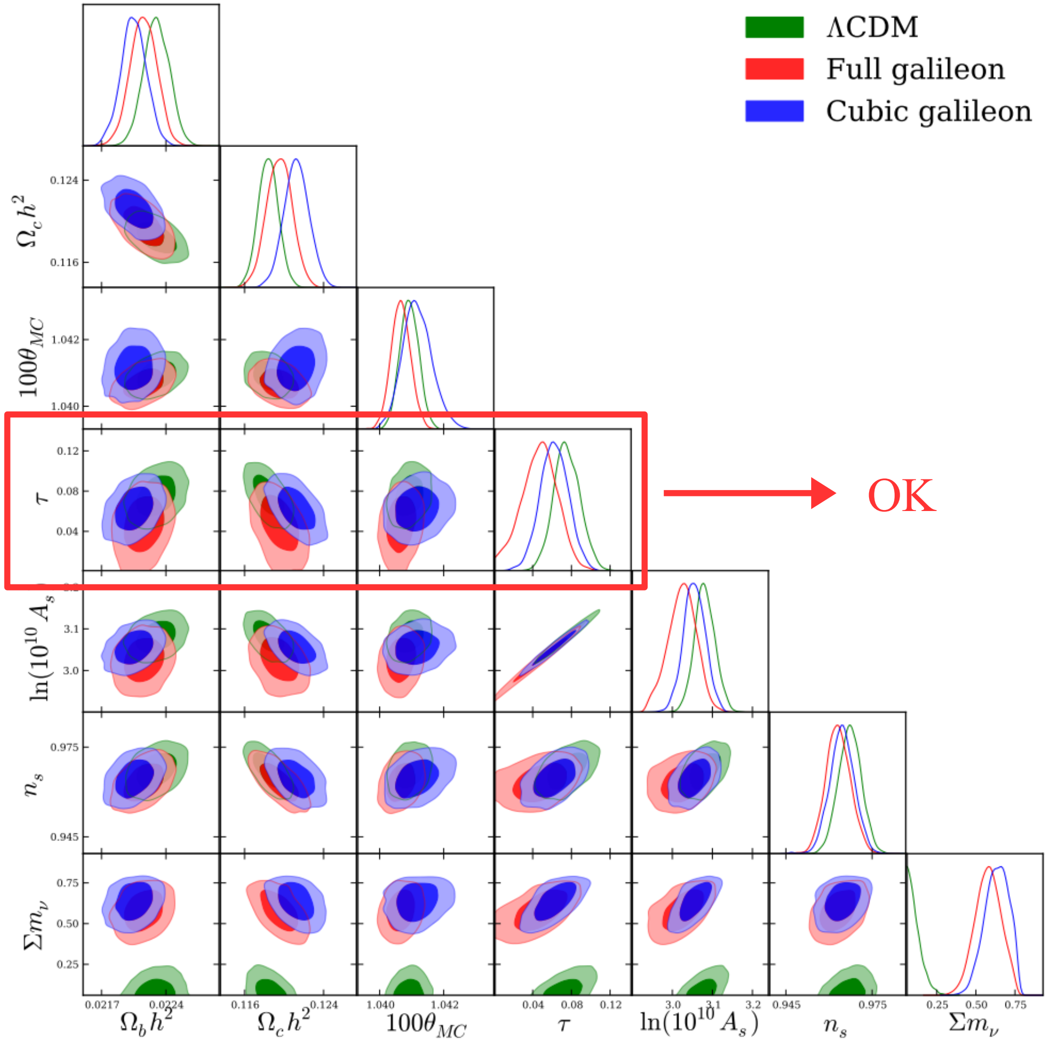

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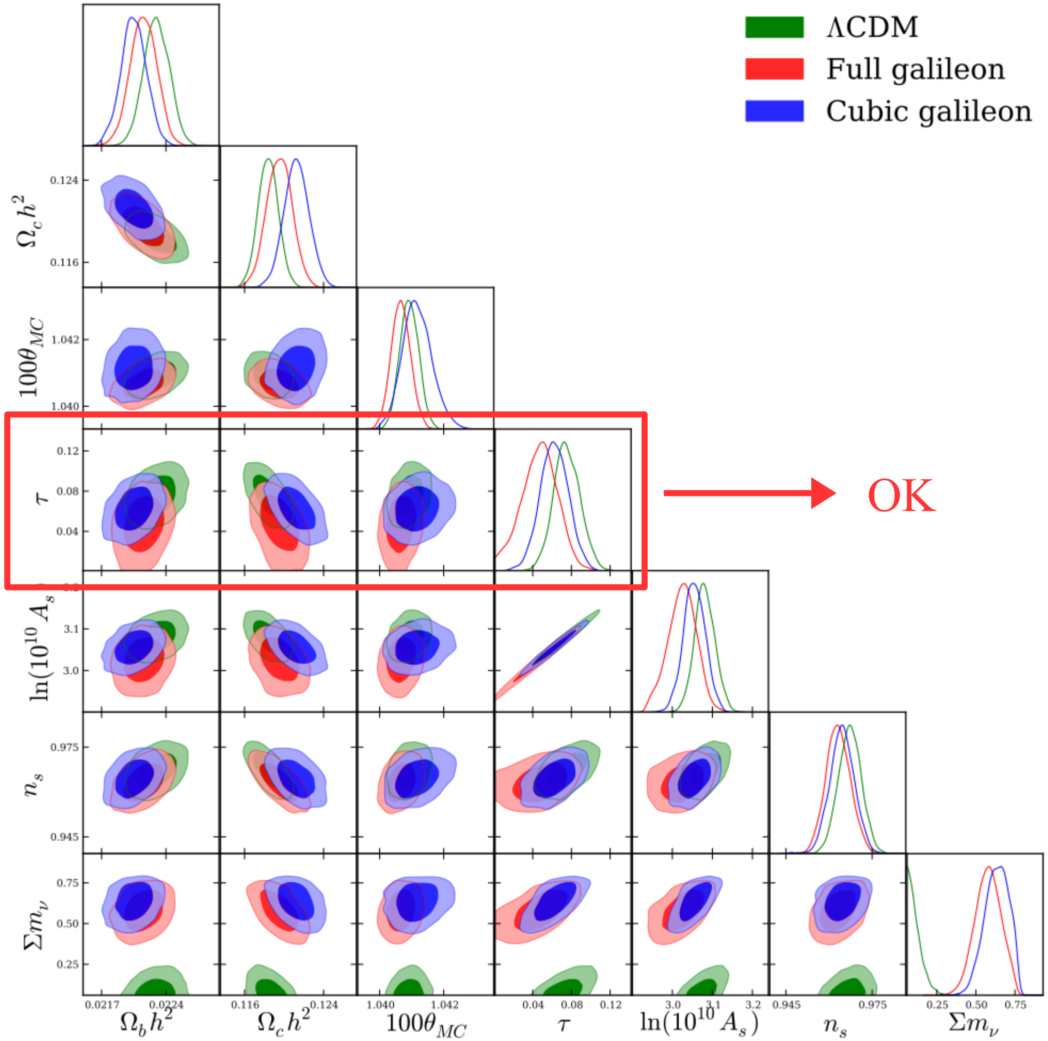
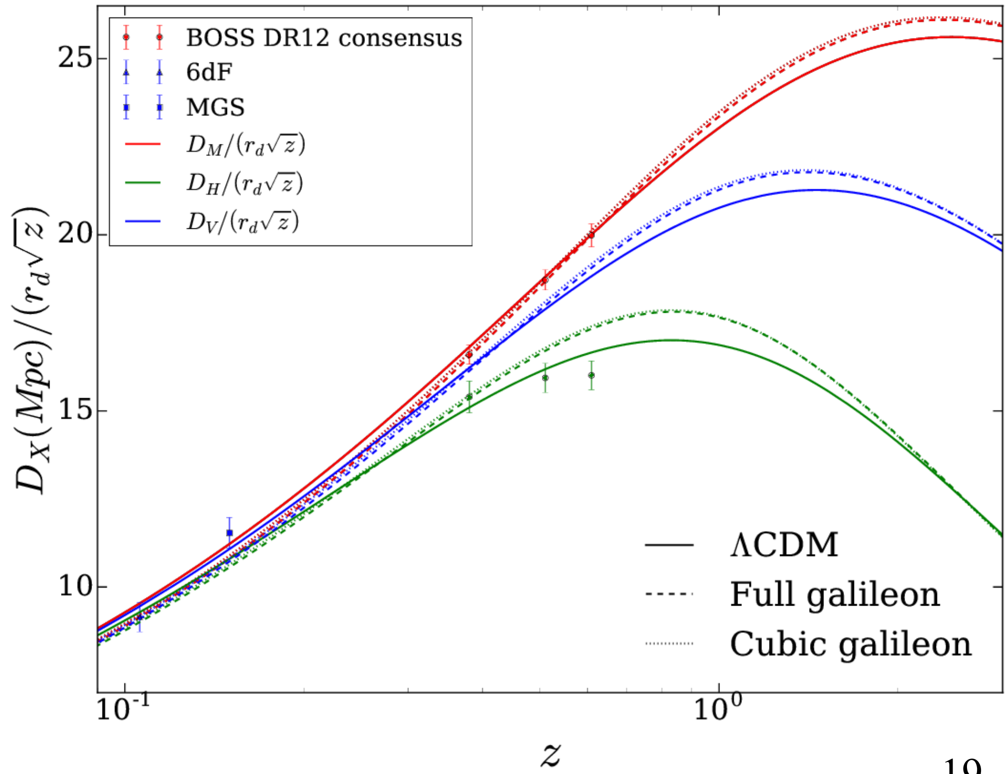


	χ^2(CMB)	χ^2(BAO)	χ^2(JLA)
ΛCDM	12946	5.5	706.7
Full galileon	12950	16.8	717.2
Cubic galileon	12963	18.3	716.5



Outline

I.Presentation of the galileon model

II. Methodology

III. Constraints from cosmology

IV. On tracker solutions

V. GW170817

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- Previously studied (see e.g. Barreira et al. 2014 or Renk et al. 2017)

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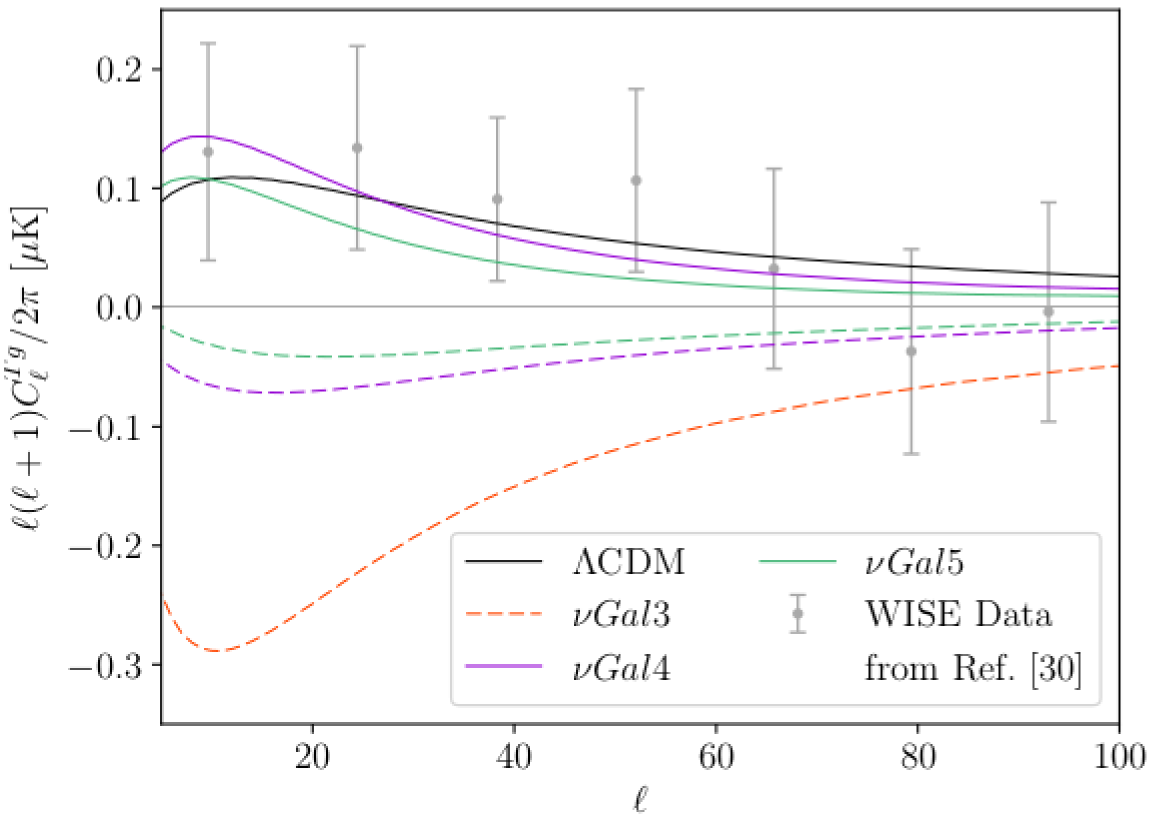

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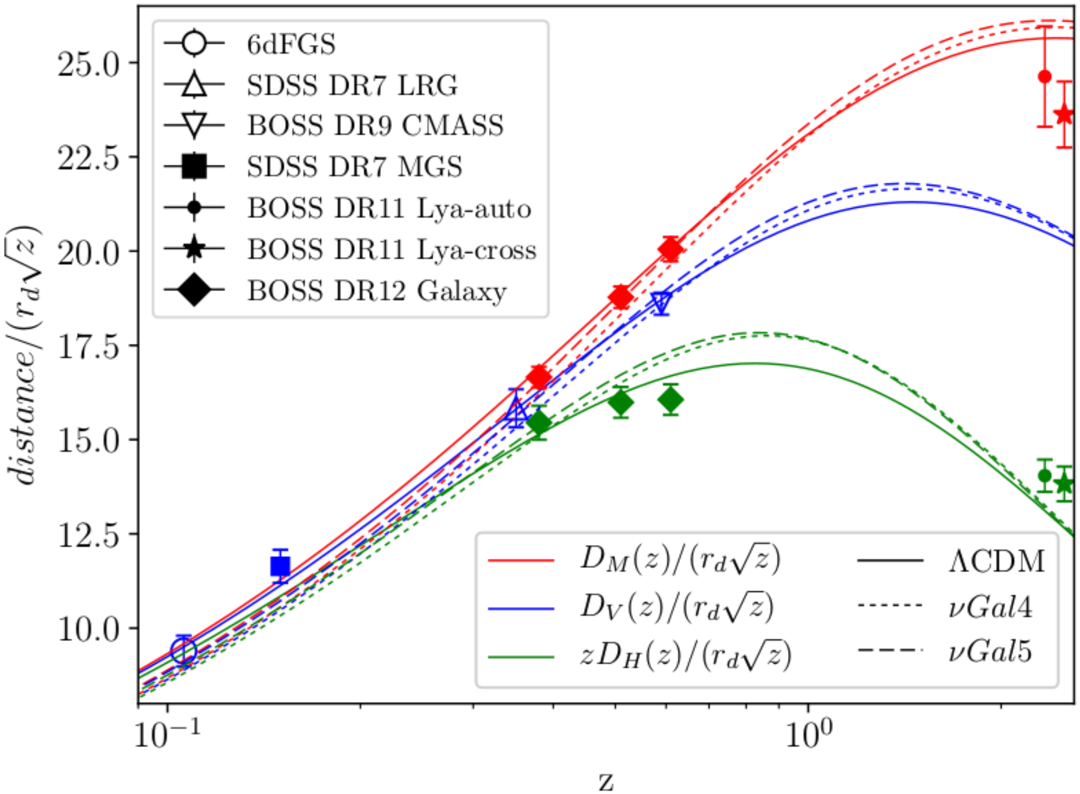

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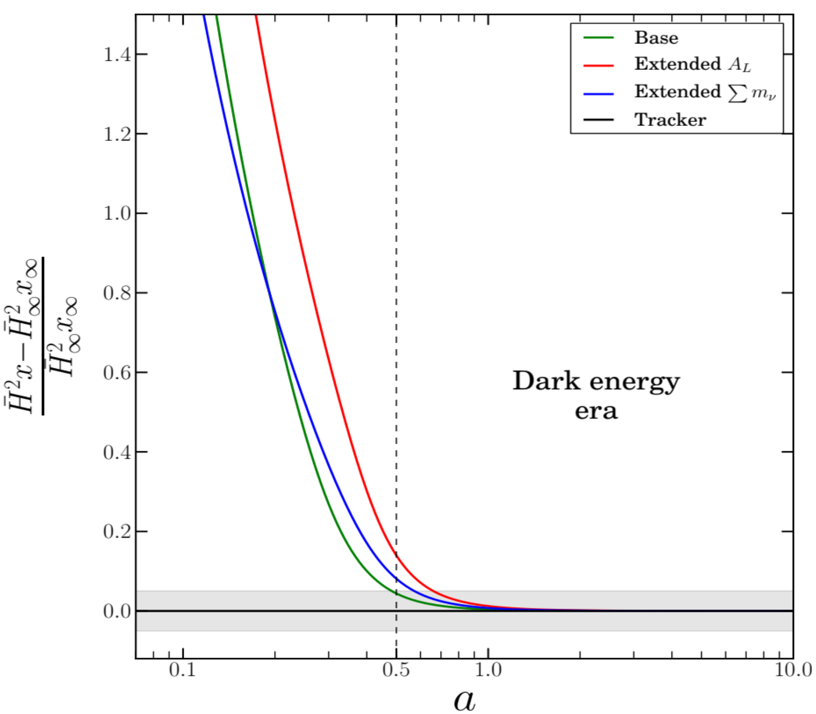
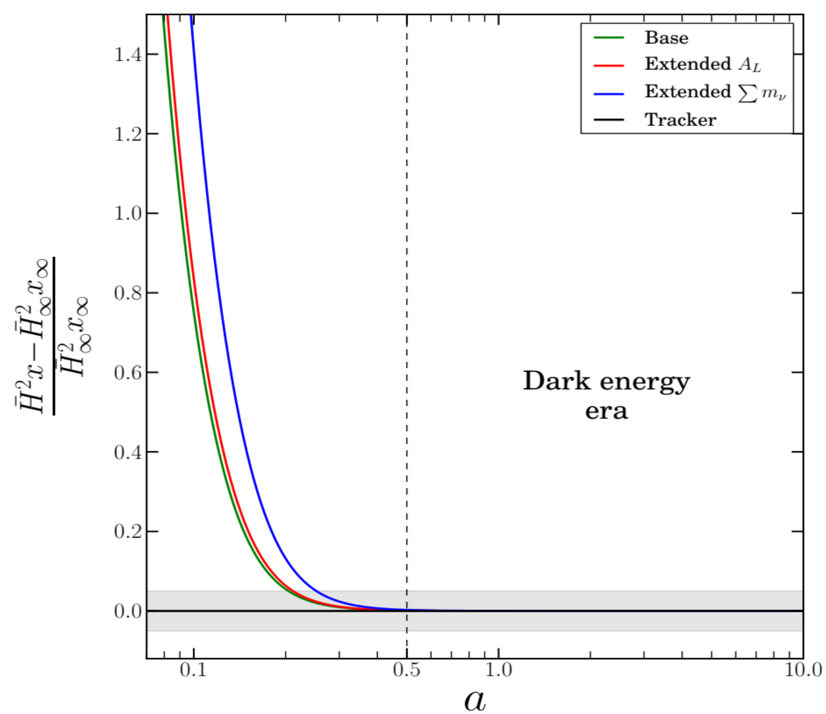


Full galileon



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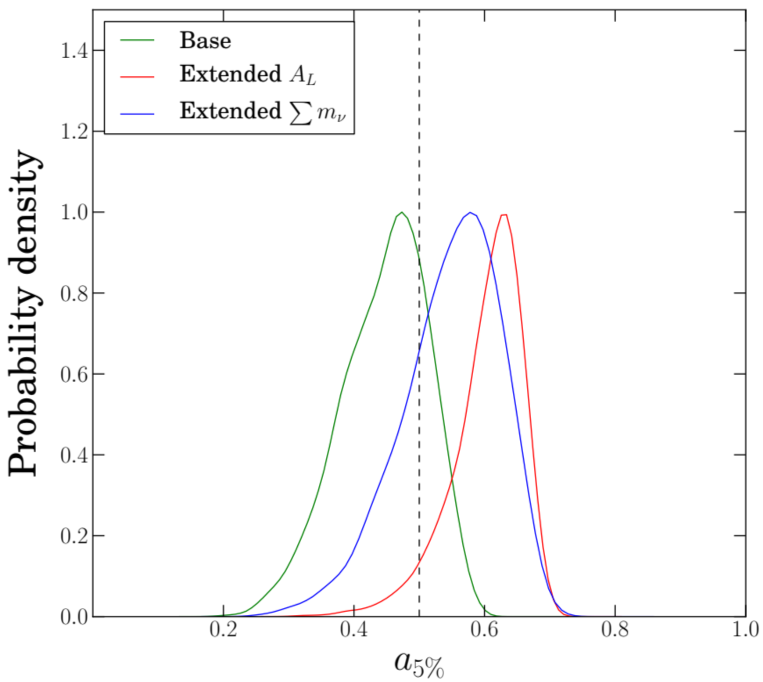

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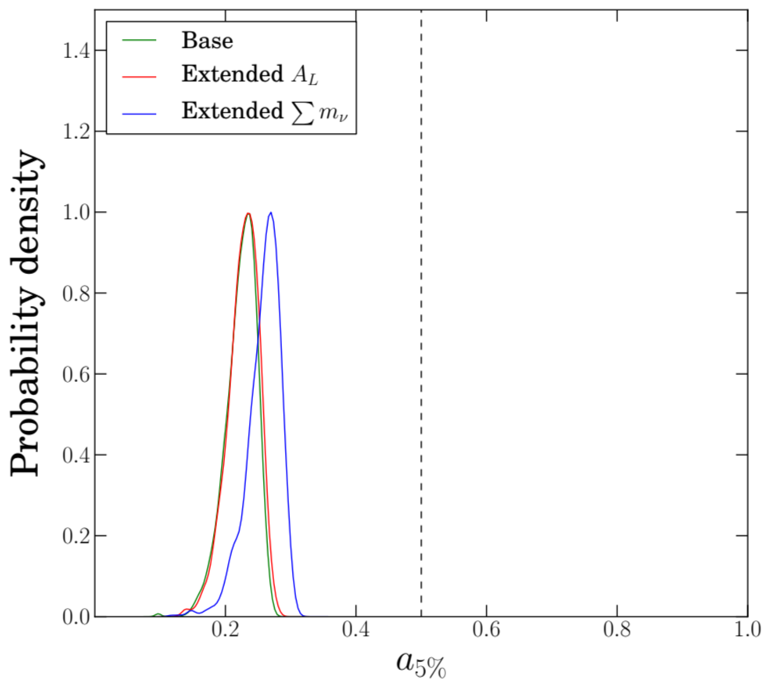

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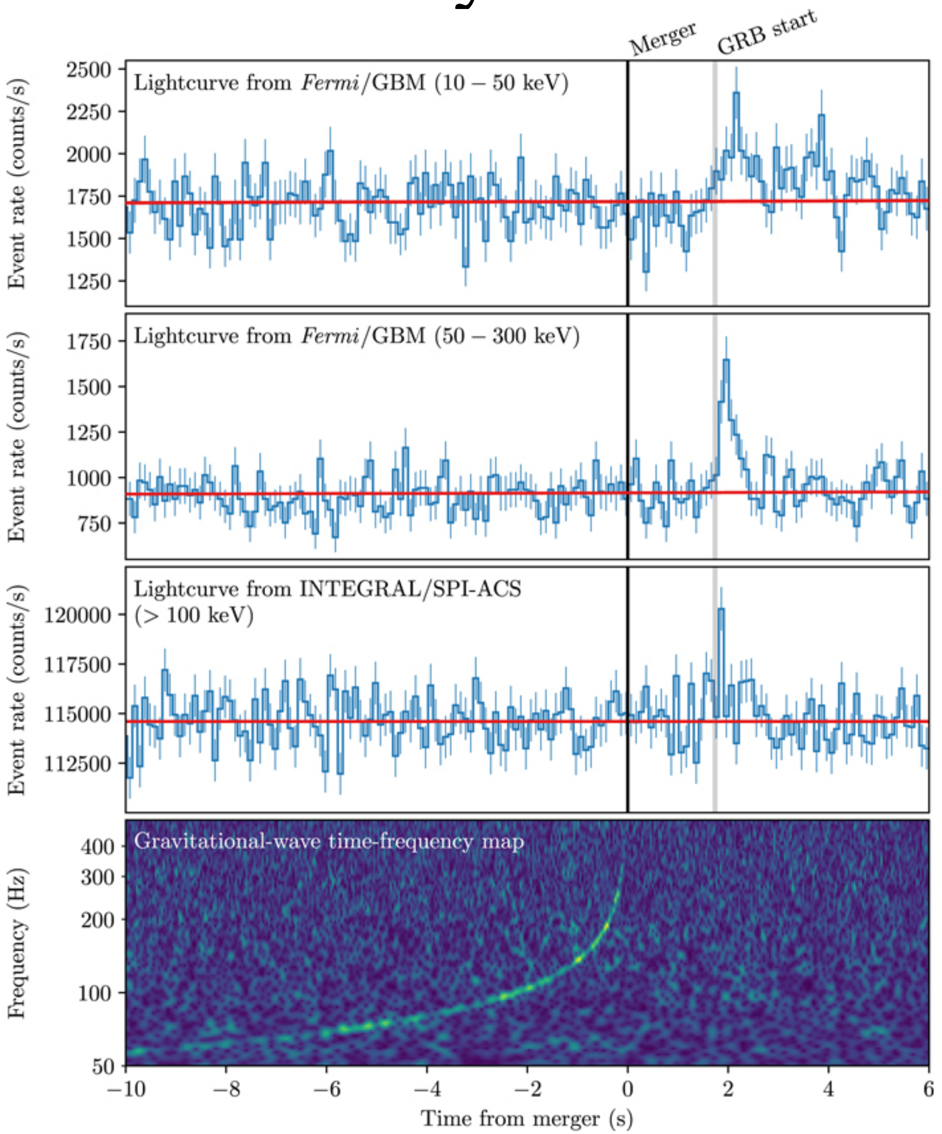
- Time delay between GW and light from GW170817



arXiv:1710.05834

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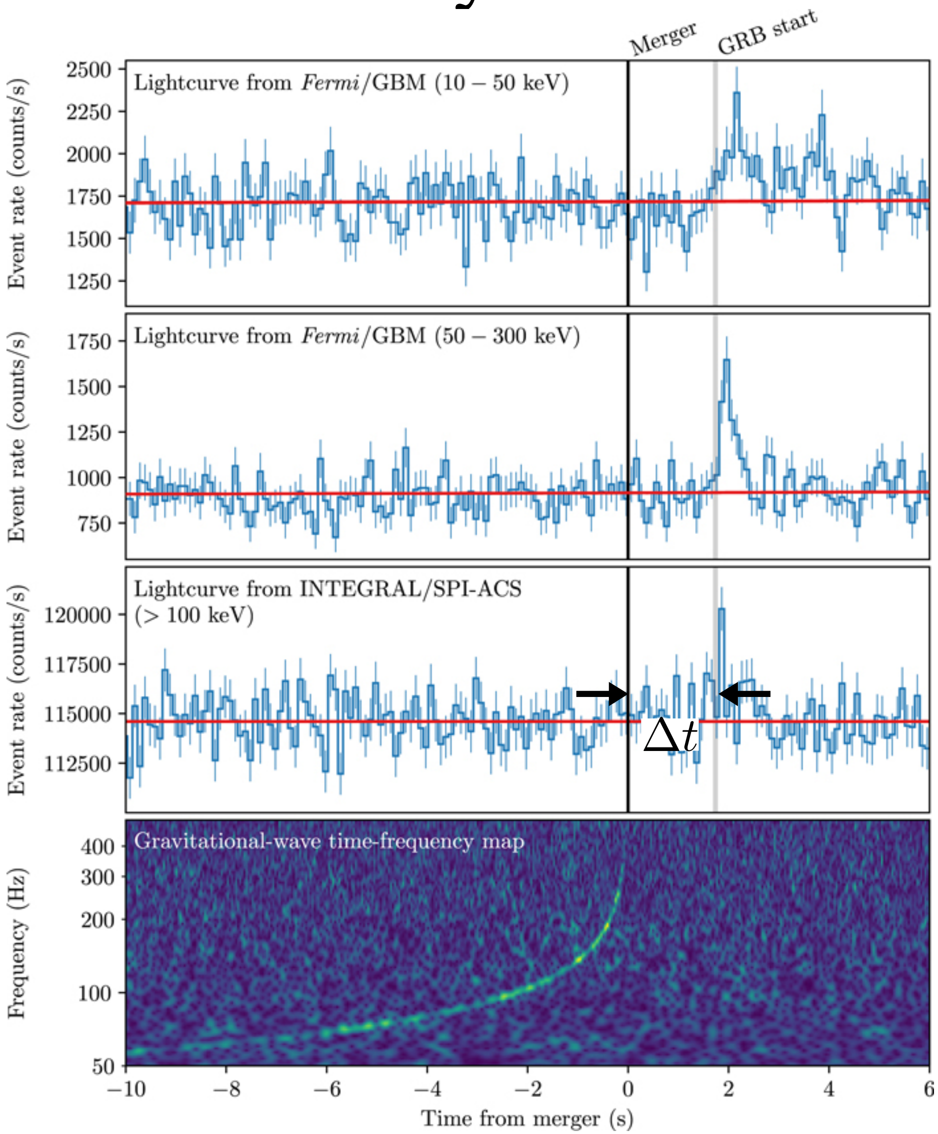

$$\Delta t = \int_{a_e}^{1} \frac{da}{aH}\left(1 - \frac{c}{c_g(a)}\right) + \delta t$$

$$= 1.74 \pm 0.05\text{s}$$

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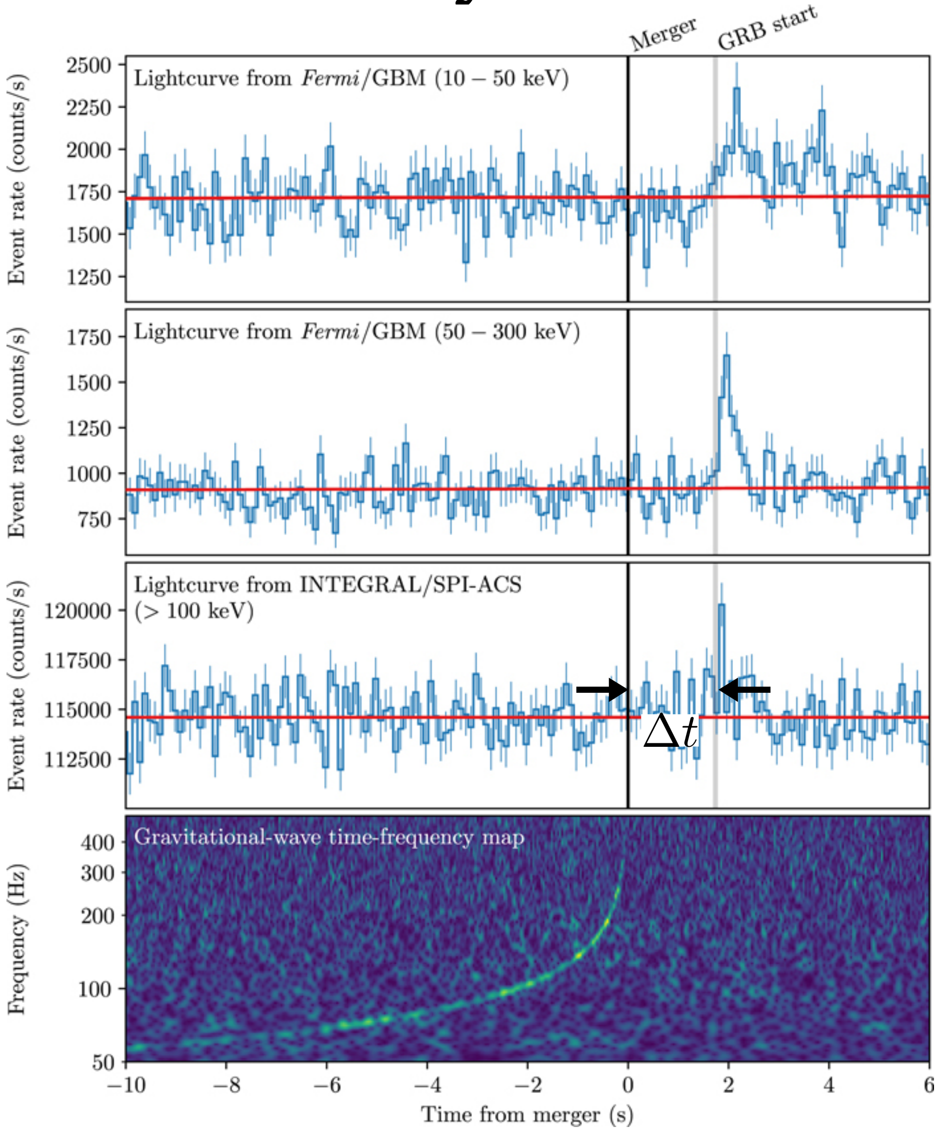

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arXiv:1710.05834

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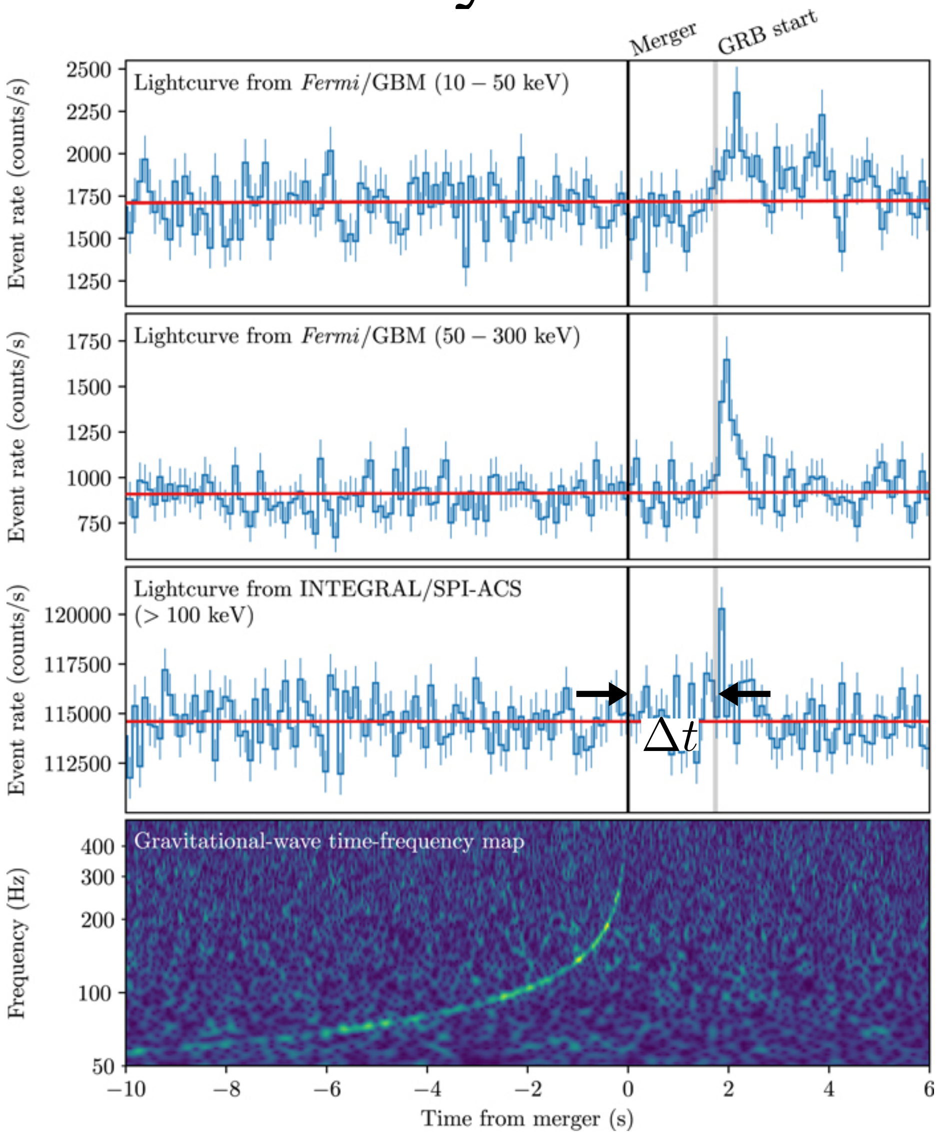

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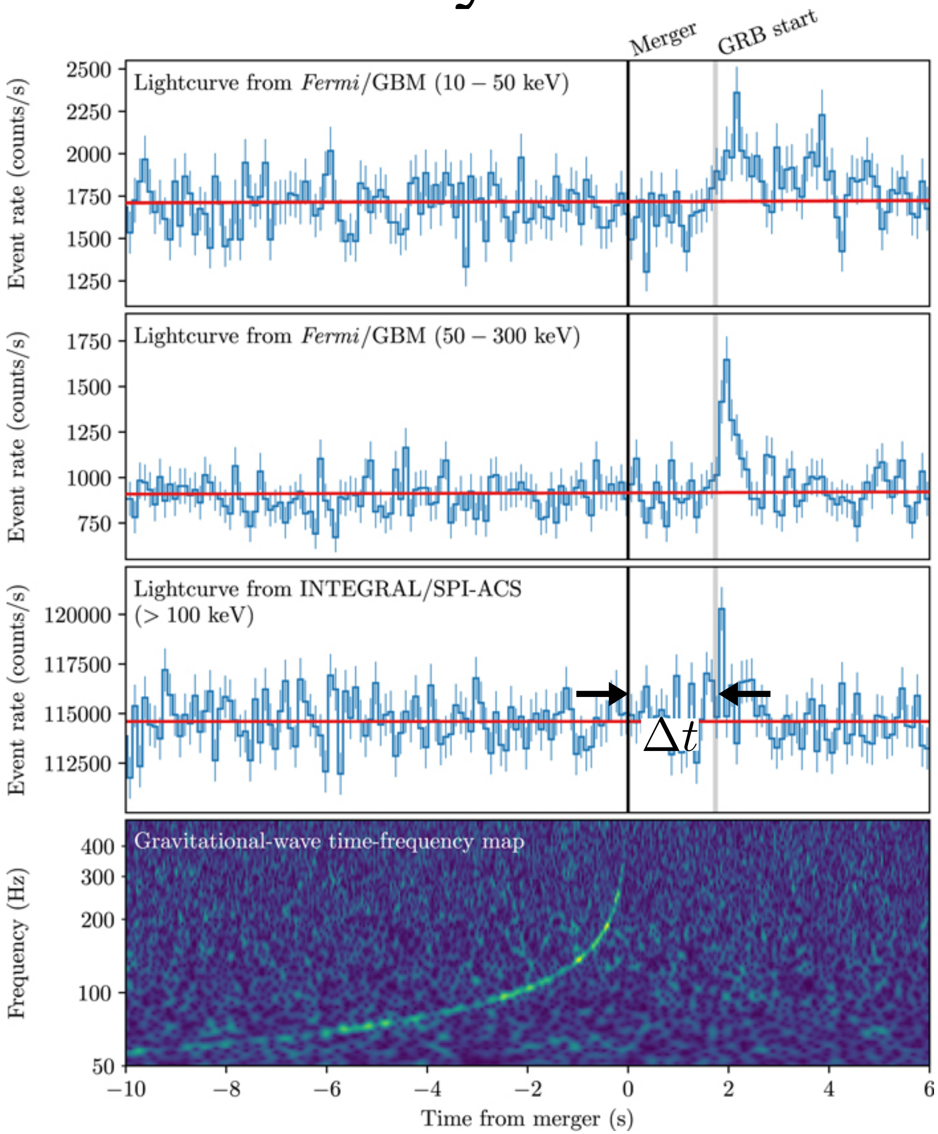

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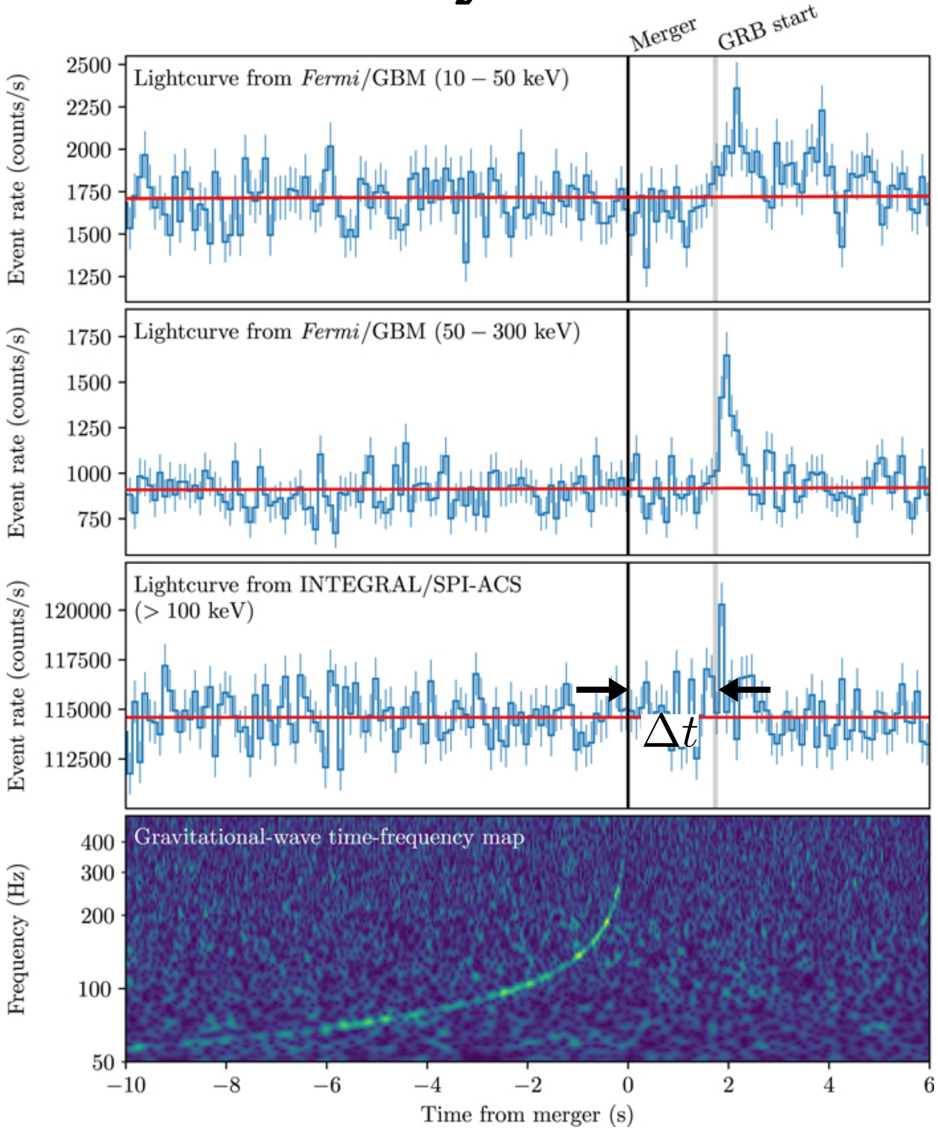

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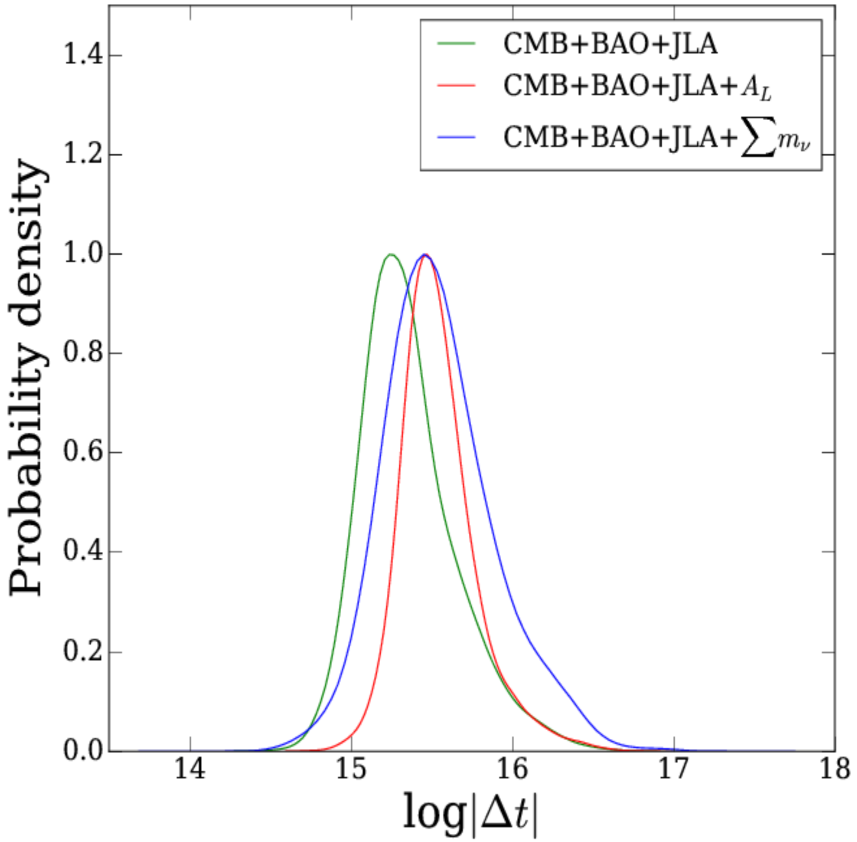
- In galileon cosmology :

$$\frac{c_g^2}{c^2} = \frac{\frac{1}{2} + \frac{1}{4}c_4\bar{H}^4x^4 + \frac{3}{2}c_5\bar{H}^5x^4 d\left(\bar{H}x\right)/d\ln a - \frac{1}{2}c_G\bar{H}^2x^2}{\frac{1}{2} - \frac{3}{4}c_4\bar{H}^4x^4 + \frac{3}{2}c_5\bar{H}^5x^5 + \frac{1}{2}c_G\bar{H}^2x^2}$$

Gravitational waves

- Modification of GW speed only due to c_4, c_5 and c_G

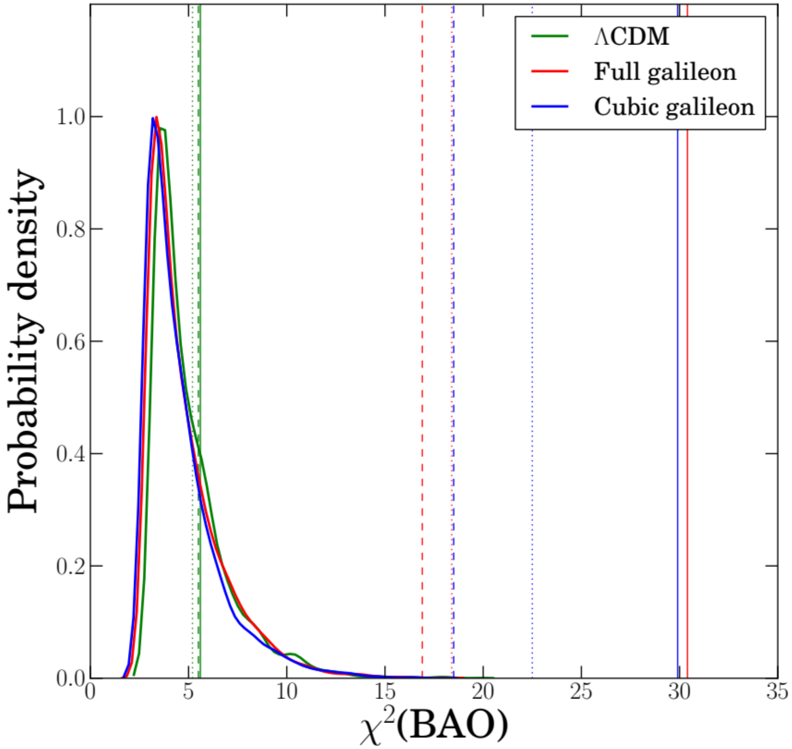
  $\Rightarrow$ affects only the full galileon model



- $\Delta t > 10^{14}$ sec ~ a few million years

Galileon status

- Status of the general galileon model (see Leloup et al. 2019) :
  - No galileon model can fit all cosmological data (especially BAO)



  - Full galileon model excluded by GW170817
  - Nevertheless, non-tracker exploration useful

Thank you !

$$\tilde{g}_{\mu\nu} = A(\pi, X)\, g_{\mu\nu} + B(\pi, X)\, \nabla_\mu \pi \nabla_\nu \pi$$

Conformal transformation

$$\pi T^\mu_\mu$$

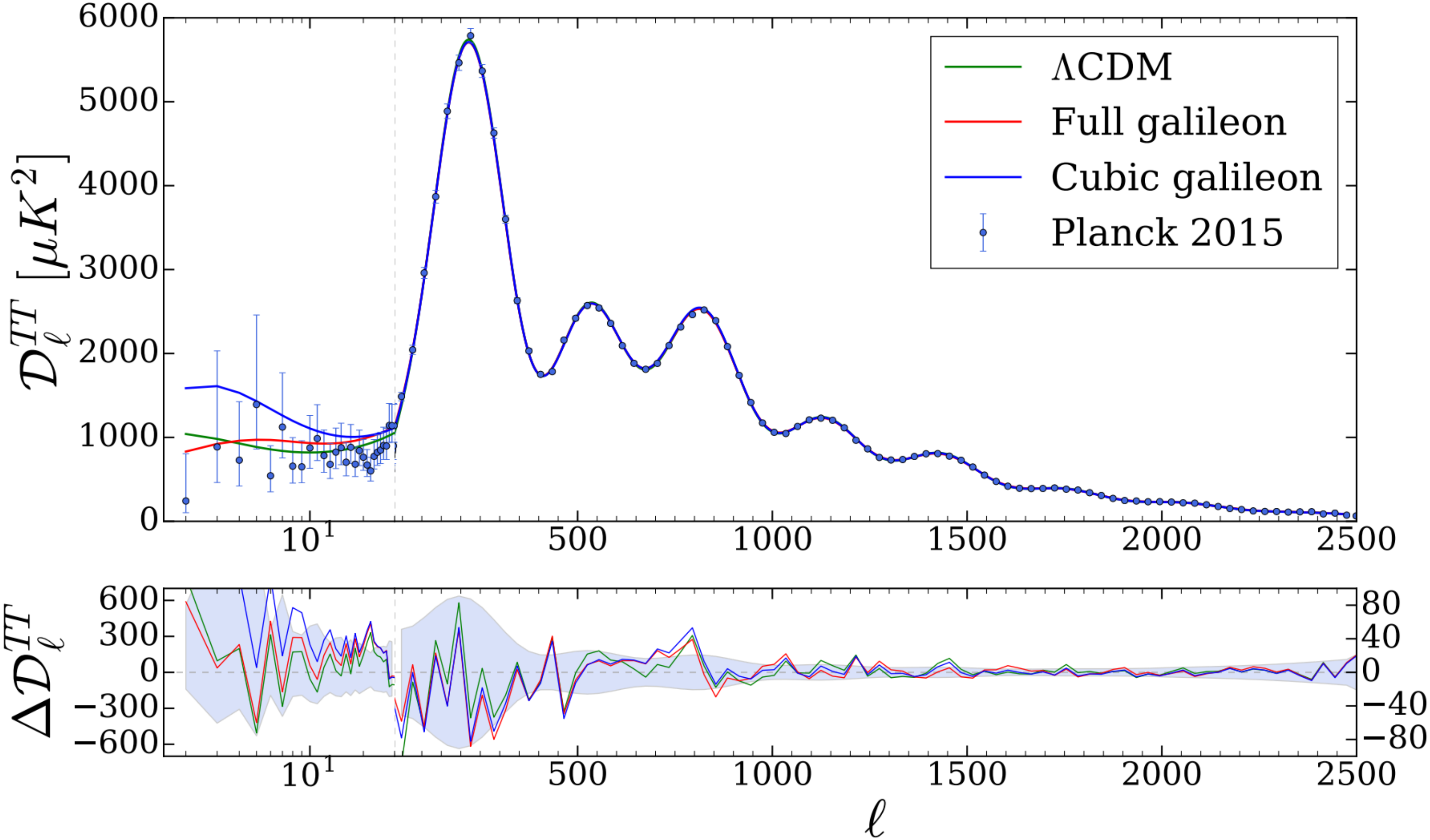
$$M_P c_0 \pi R$$

Disformal transformation

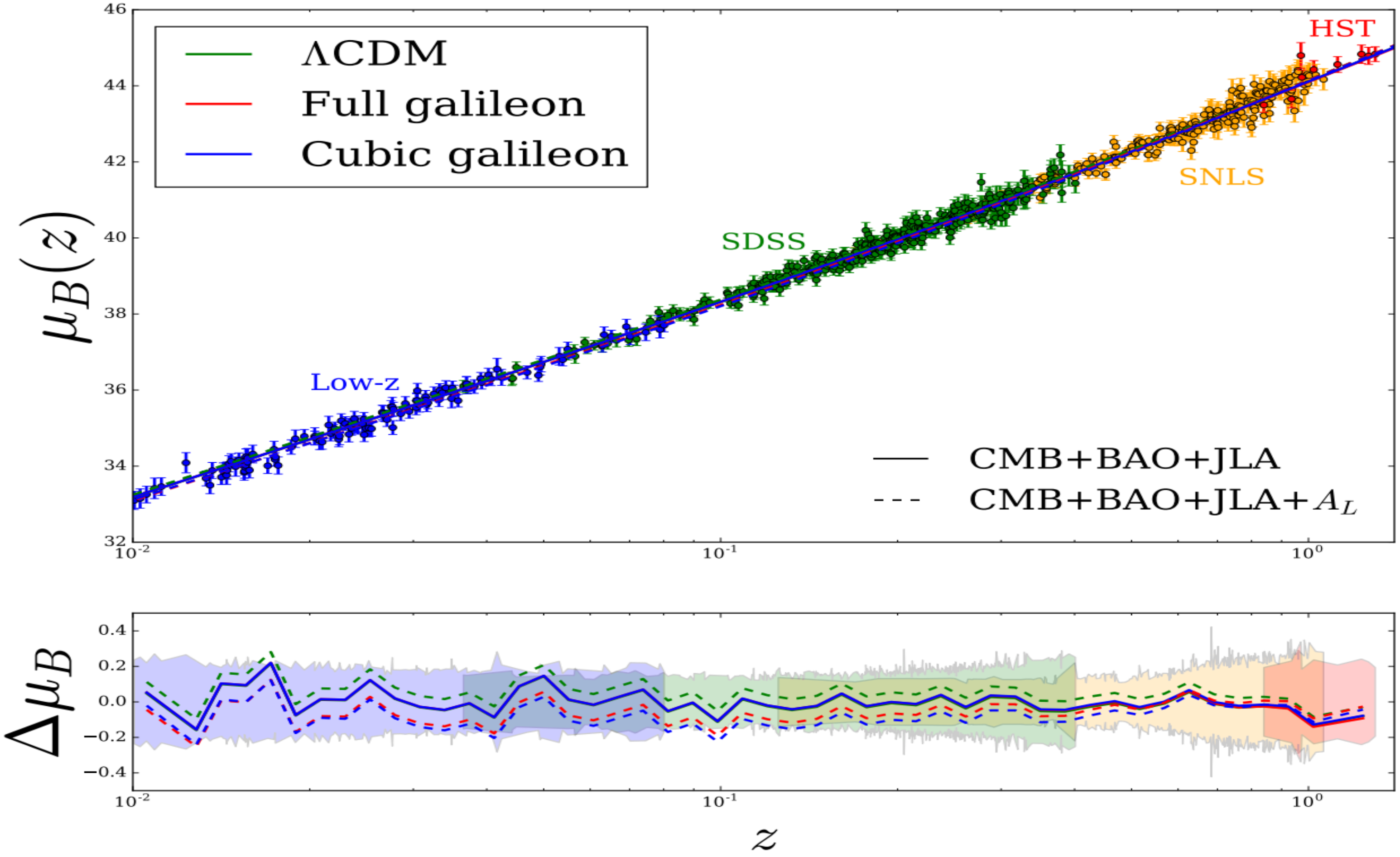
$$\nabla_\mu \pi \nabla_\nu \pi T^{\mu\nu}$$

$$\frac{M_P}{M^3} c_G G^{\mu\nu} \nabla_\mu \pi \nabla_\nu \pi$$

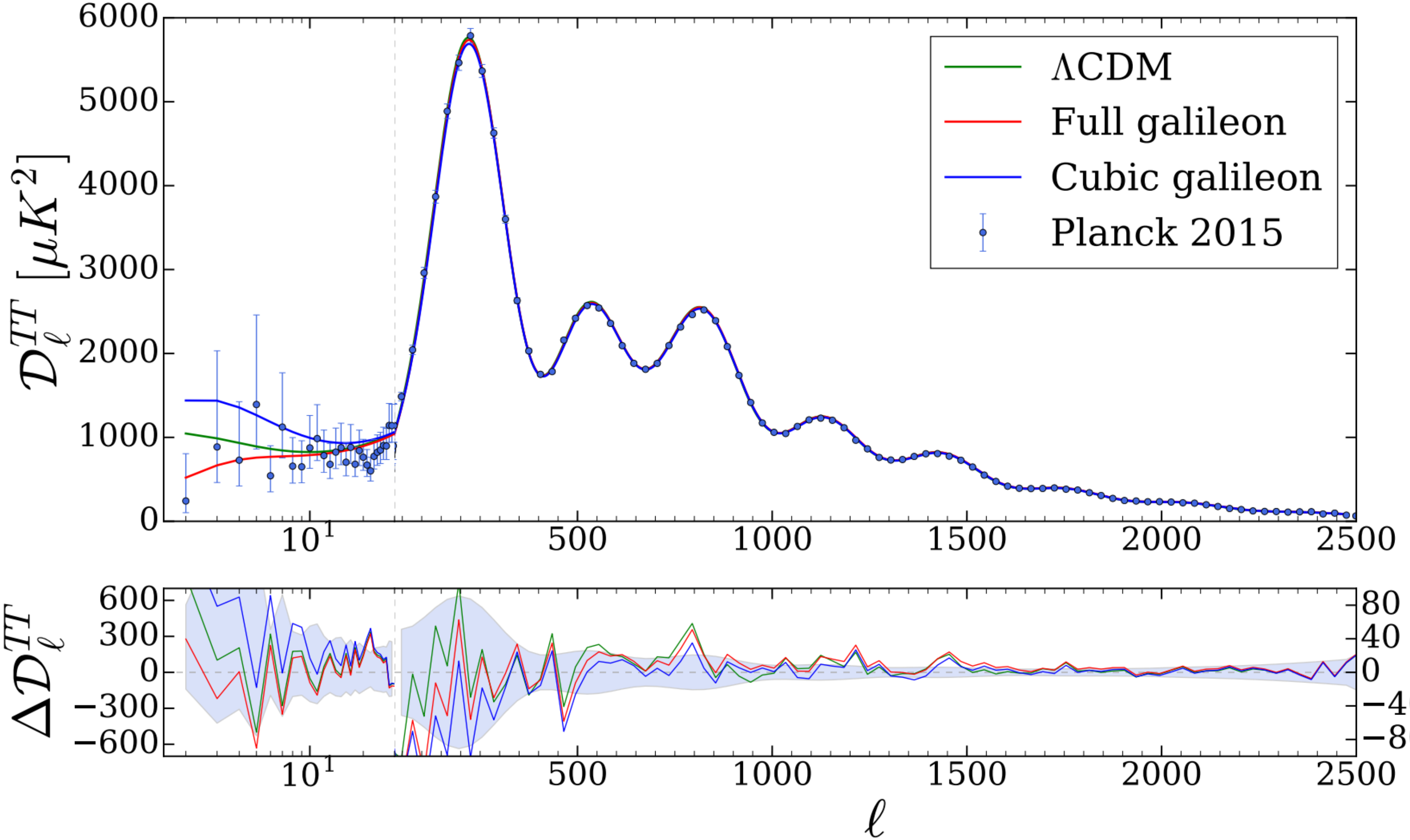
- TT powerspectrum with A_L



- SN hubble diagram with A_L



- TT powerspectrum with Σm_ν



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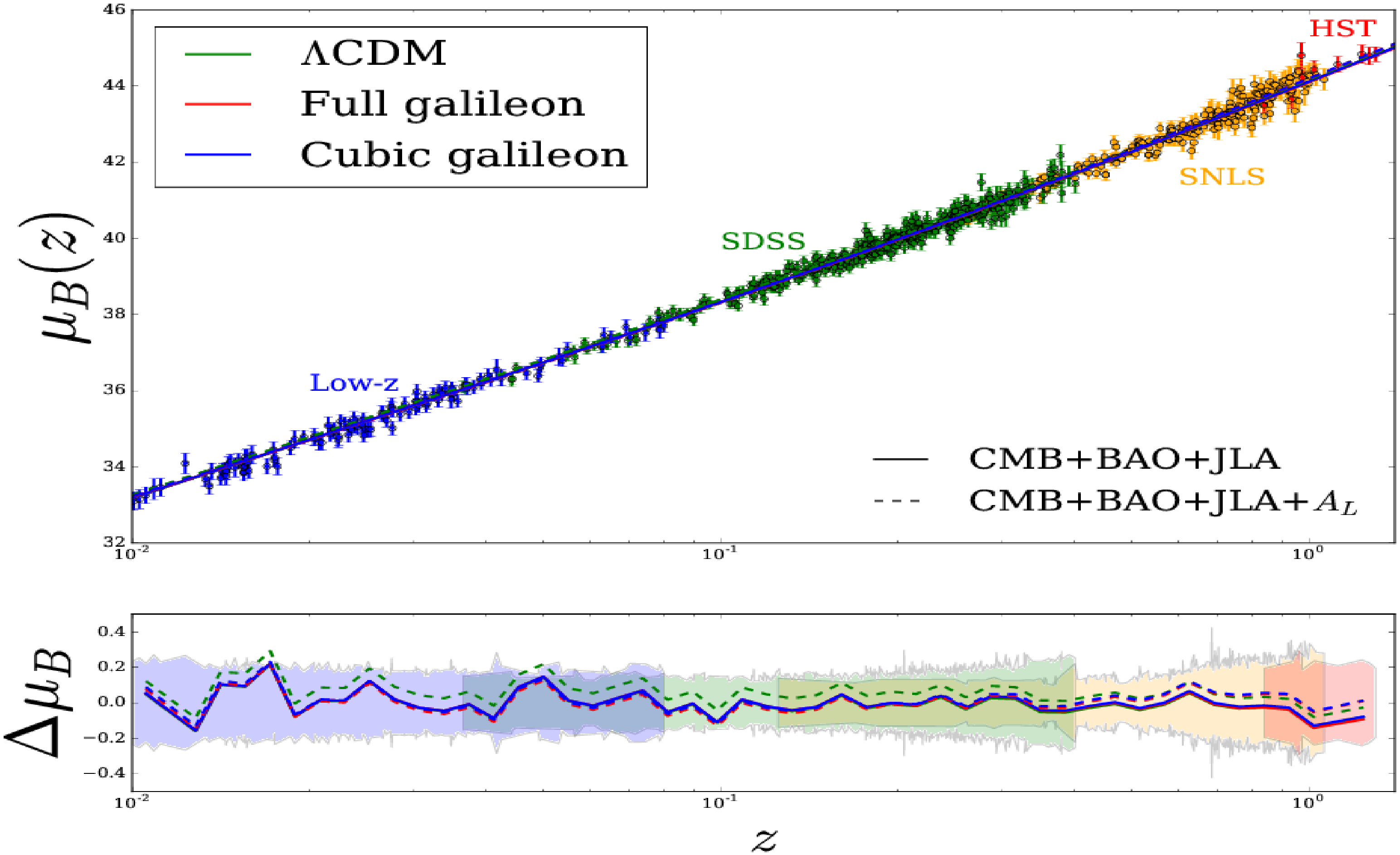
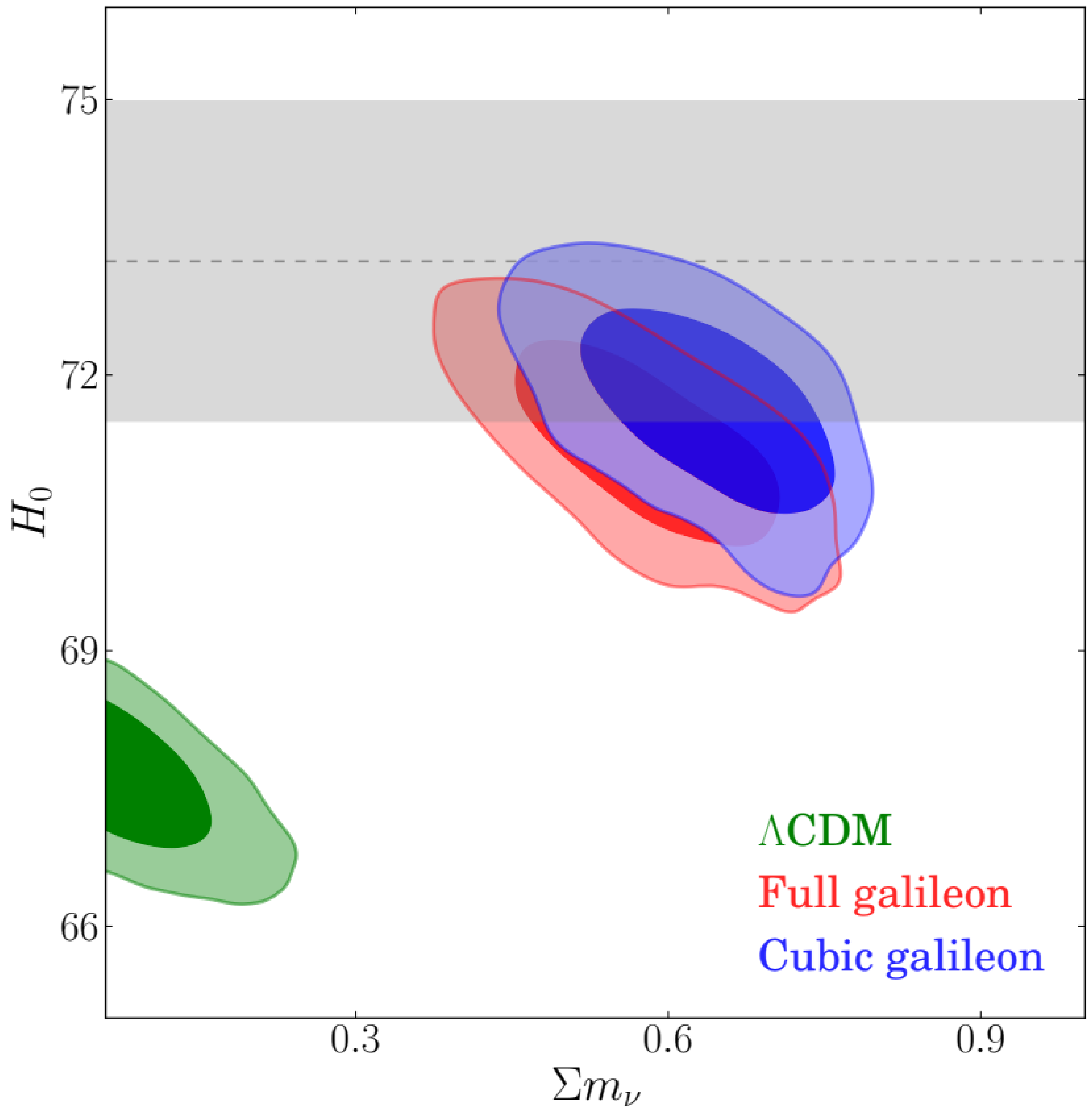
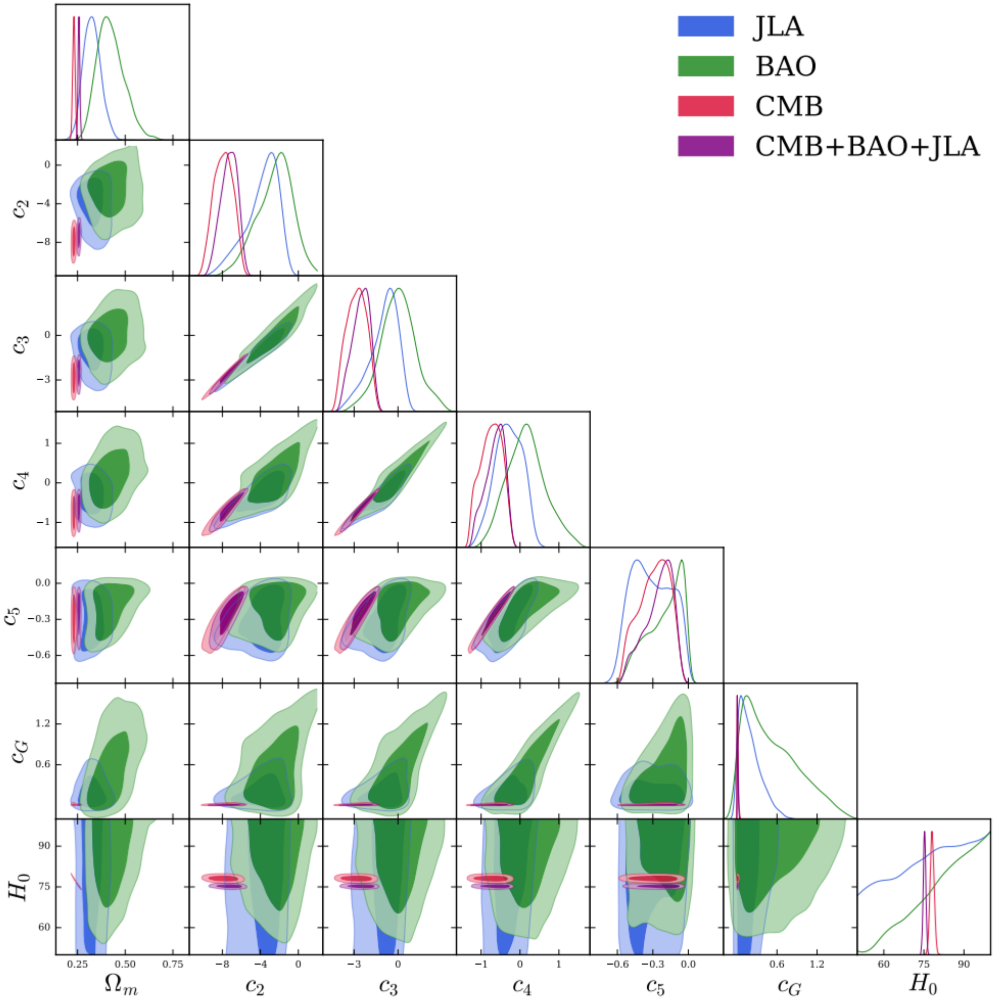
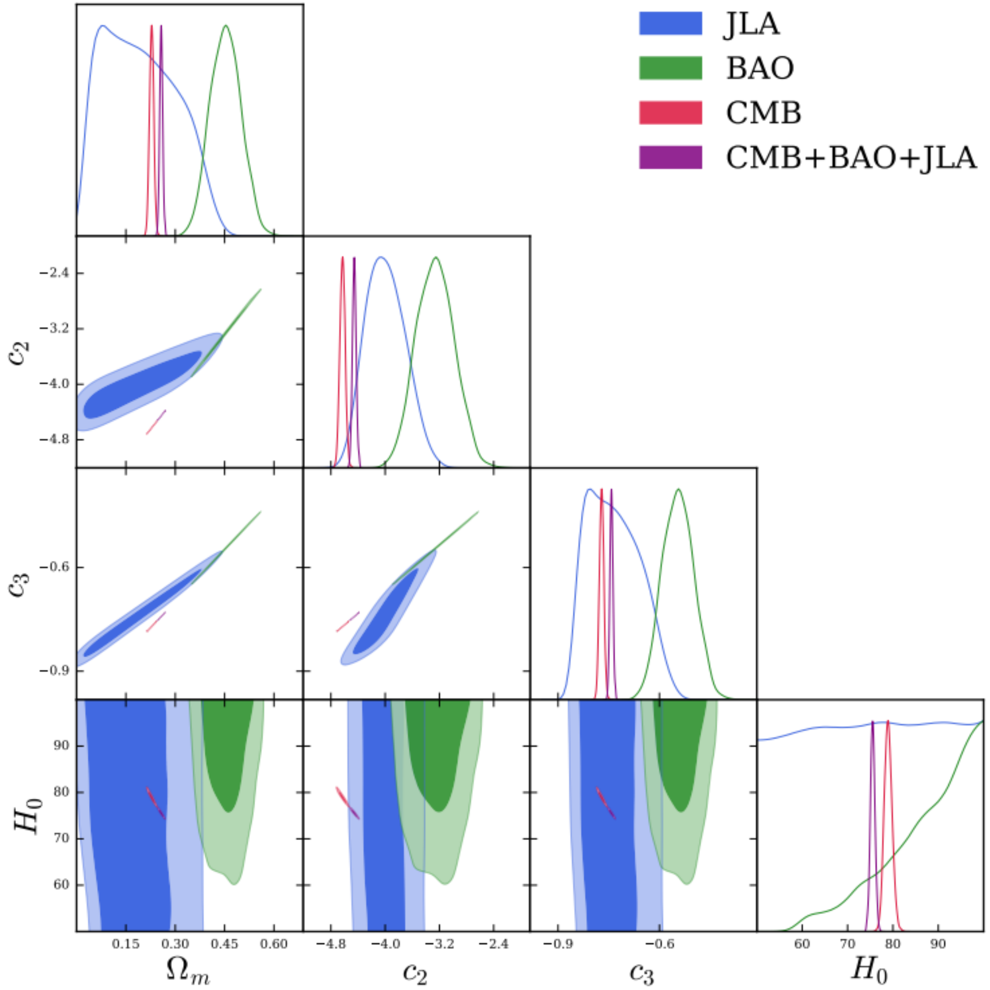




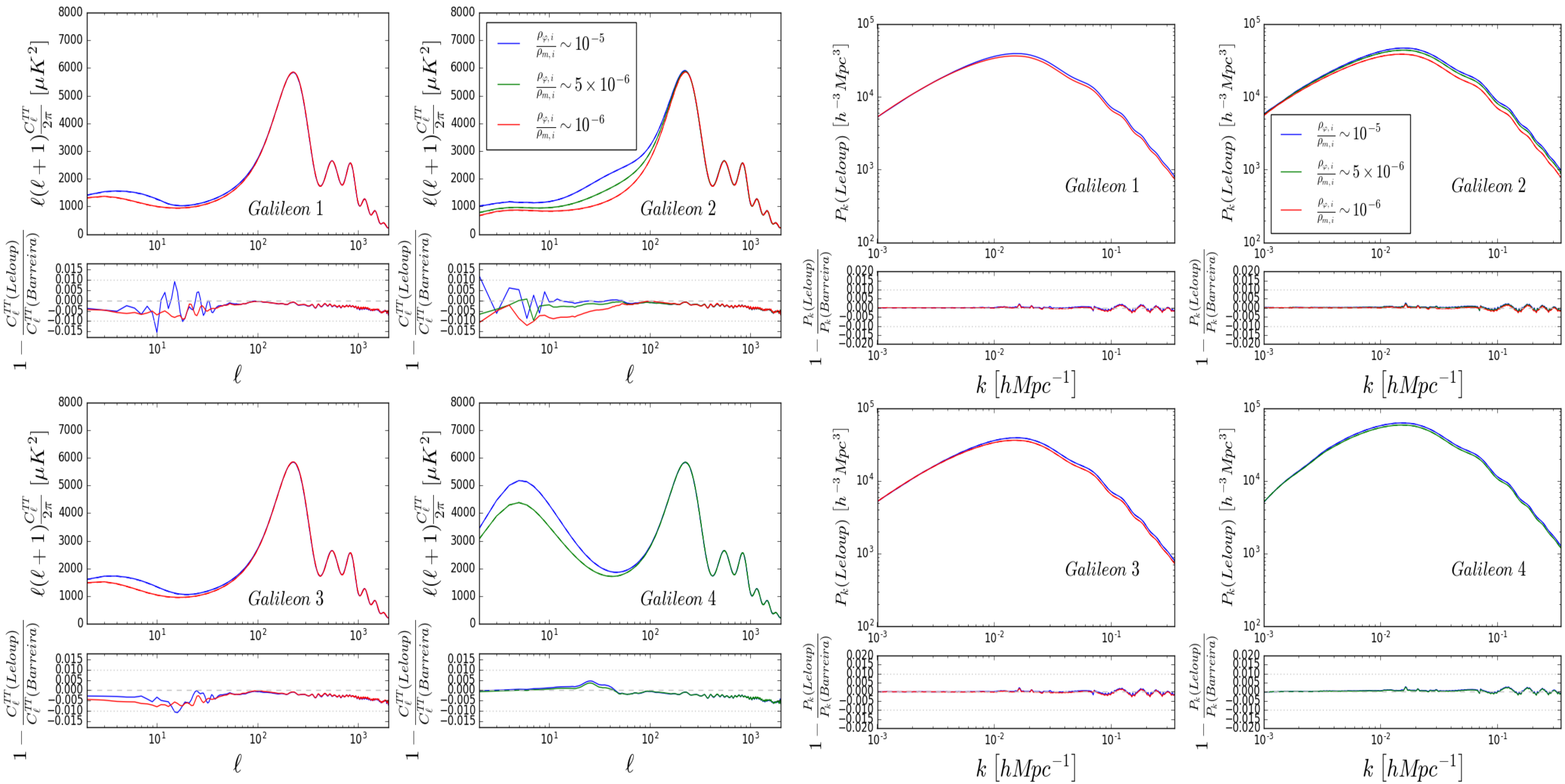
- CI on H0 from Riess et al. 2016

➢ Constraints on galileon parameters





➢ Validation of CAMB



$$
\begin{aligned}
\alpha &= \frac{c_2}{6}\bar{H}x - 3c_3\bar{H}^3x^2 + 15c_4\bar{H}^5x^3 - \frac{35}{2}c_5\bar{H}^7x^4 - 3c_G\bar{H}^3x \\
\gamma &= \frac{c_2}{3}\bar{H}^2x - c_3\bar{H}^4x^2 + 5\frac{5}{2}c_5\bar{H}^8x^4 - 2c_G\bar{H}^4x \\
\beta &= \frac{c_2}{6}\bar{H}^2 - 2c_3\bar{H}^4x + 9c_4\bar{H}^6x^2 - 10c_5\bar{H}^8x^3 - c_G\bar{H}^4 \\
\sigma &= 2\bar{H} + 2c_3\bar{H}^3x^3 - 15c_4\bar{H}^5x^4 + 21c_5\bar{H}^7x^5 + 6c_G\bar{H}^3x^2 \\
\lambda &= 3\bar{H}^2 + \frac{\Omega_\gamma^0}{a^4} + \frac{p_\nu}{M_{Pl}^2H_0^2} + \frac{c_2}{2}\bar{H}^2x^2 - 2c_3\bar{H}^4x^3 + \frac{15}{2}c_4\bar{H}^6x^4 - 9c_5\bar{H}^8x^5 - c_G\bar{H}^4x^2 \\
\omega &= 2c_3\bar{H}^4x^2 - 12c_4\bar{H}^6x^3 + 15c_5\bar{H}^8x^4 + 4c_G\bar{H}^4x
\end{aligned}
$$

1. $\chi^G = f_1^\chi \cdot \gamma + f_2^\chi \cdot \gamma' + \frac{1}{\kappa a^2}\left(f_3^\chi \cdot k\mathcal{H}\mathcal{Z} + f_4^\chi \cdot k^2\eta\right)$ with :

$$
\begin{aligned}
f_1^\chi &= \frac{k^2}{\kappa a^2}\left[-2\frac{c_3}{a^2}x^2\bar{\mathcal{H}}^2 + 12\frac{c_4}{a^4}x^3\bar{\mathcal{H}}^4 - 15\frac{c_5}{a^6}x^4\bar{\mathcal{H}}^6 - 4\frac{c_G}{a^2}x\bar{\mathcal{H}}^2\right] && \text{(A.1)} \\
f_2^\chi &= \frac{H_0}{\kappa a^2}\left[c_2x\bar{\mathcal{H}} - 18\frac{c_3}{a^2}x^2\bar{\mathcal{H}}^3 + 90\frac{c_4}{a^4}x^3\bar{\mathcal{H}}^5 - 105\frac{c_5}{a^6}x^4\bar{\mathcal{H}}^7 - 18\frac{c_G}{a^2}x\bar{\mathcal{H}}^3\right] && \text{(A.2)} \\
f_3^\chi &= -2\frac{c_3}{a^2}x^3\bar{\mathcal{H}}^2 + 15\frac{c_4}{a^4}x^4\bar{\mathcal{H}}^4 - 21\frac{c_5}{a^6}x^5\bar{\mathcal{H}}^6 - 6\frac{c_G}{a^2}x^2\bar{\mathcal{H}}^2 && \text{(A.3)} \\
f_4^\chi &= \frac{3}{2}\frac{c_4}{a^4}x^4\bar{\mathcal{H}}^4 - 3\frac{c_5}{a^6}x5\bar{\mathcal{H}}^6 - \frac{c_G}{a^2}x^2\bar{\mathcal{H}}^2 && \text{(A.4)}
\end{aligned}
$$

2. $q^G = f_1^q + \frac{1}{\kappa a^2} f_2^q \cdot k^2\left(\sigma - \mathcal{Z}\right)$ with :

$$
\begin{aligned}
f_1^q &= \frac{k}{\kappa a^2}\Big[c_2H_0x\bar{\mathcal{H}}\bar{\gamma} - \frac{c_3}{a^2}\left(-2x^2\bar{H}^2\bar{\gamma}' + 6H_0x^2\bar{\mathcal{H}}^3\bar{\gamma}\right) + \frac{c_4}{a^4}\left(-12x^3\bar{\mathcal{H}}^4\bar{\gamma}' + 18H_0x^3\bar{\mathcal{H}}^5\bar{\gamma}\right) \\
&\quad - \frac{c_5}{a^6}\left(-15x^4\bar{\mathcal{H}}^6\bar{\gamma}' + 15H_0x^4\bar{\mathcal{H}}^7\bar{\gamma}\right) - \frac{c_G}{a^2}\left(-4x\bar{\mathcal{H}}^2\bar{\gamma}' + 6H_0x\bar{\mathcal{H}}^3\bar{\gamma}\right)\Big] && \text{(A.5)} \\
f_2^q &= \frac{c_4}{a^4}x^4\bar{\mathcal{H}}^4 - 2\frac{c_5}{a^6}x^5\bar{\mathcal{H}}^6 - \frac{2}{3}\frac{c_G}{a^2}x^2\bar{\mathcal{H}}^2 && \text{(A.6)}
\end{aligned}
$$

3. $\Pi^G = f_1^\Pi + \frac{1}{\kappa a^2}\left(f_2^\Pi \cdot k\mathcal{H}\sigma - f_3^\Pi \cdot k\sigma' + f_4^\Pi \cdot k^2\phi\right)$ with :

$$f_1^\Pi = \frac{k^2}{\kappa a^2}\left[\frac{c_4}{a^4}\left(4x^3\bar{\mathcal{H}}^4\bar{\gamma} - 6x^2\bar{\mathcal{H}}^3\left(x\overset{o}{\bar{\mathcal{H}}}\right)\bar{\gamma}\right) - \frac{c_5}{a^6}\left(12x^4\bar{\mathcal{H}}^6\bar{\gamma} - 3x^4\bar{\mathcal{H}}^5\overset{o}{\bar{\mathcal{H}}}\bar{\gamma} - 12x^3\bar{\mathcal{H}}^5\left(x\overset{o}{\bar{\mathcal{H}}}\right)\bar{\gamma}\right)\right.$$
$$\left. +2\frac{c_G}{a^2}\bar{H}\left(x\overset{o}{\bar{\mathcal{H}}}\right)\bar{\gamma}\right] \tag{A.7}$$

$$f_2^\Pi = \frac{c_4}{a^4}\left(3x^4\bar{\mathcal{H}}^4 - 6x^3\bar{\mathcal{H}}^3\left(x\overset{o}{\bar{\mathcal{H}}}\right)\right) - \frac{c_5}{a^6}\left(12x^5\bar{\mathcal{H}}^6 - 3x^5\bar{\mathcal{H}}^5\overset{o}{\bar{\mathcal{H}}} - 15x^4\bar{\mathcal{H}}^5\left(x\overset{o}{\bar{\mathcal{H}}}\right)\right)$$
$$+2\frac{c_G}{a^2}x\bar{H}\left(x\overset{o}{\bar{\mathcal{H}}}\right) \tag{A.8}$$

$$f_3^\Pi = \frac{c_4}{a^4}x^4\bar{\mathcal{H}}^4 + 3\frac{c_5}{a^6}x^4\bar{H}^5\left(x\overset{o}{\bar{\mathcal{H}}}\right) \tag{A.9}$$

$$f_4^\Pi = -\frac{c_4}{a^4}x^4\bar{\mathcal{H}}^4 - \frac{c_5}{a^6}\left(-6x^5\bar{\mathcal{H}}^6 + 6x^4\bar{H}^5\left(x\overset{o}{\bar{\mathcal{H}}}\right)\right) + 2\frac{c_G}{a^2}x^2\bar{\mathcal{H}}^2 \tag{A.10}$$

4. $0 = f_1^{eom} \cdot \bar{\gamma}'' + f_2^{eom} \cdot \bar{\gamma}' + f_3^{eom} \cdot k^2\bar{\gamma} + f_4^{eom} \cdot k\mathcal{H}\mathcal{Z} + f_5^{eom} \cdot k\mathcal{Z}' + f_6^{eom} \cdot k^2\eta$ with :

$$f_1^{eom} = c_2 - 12\frac{c_3}{a^2}x\bar{\mathcal{H}}^2 + 54\frac{c_4}{a^4}x^2\bar{\mathcal{H}}^4 - 60\frac{c_5}{a^6}x^3\bar{\mathcal{H}}^6 - 6\frac{c_G}{a^2}\bar{\mathcal{H}}^2 \tag{A.11}$$

$$\begin{aligned} f_2^{eom} = H_0 \Bigg[& 2c_2\bar{\mathcal{H}} - \frac{c_3}{a^2}\left(12x\bar{\mathcal{H}}^2\overset{o}{\bar{\mathcal{H}}} + 12\bar{\mathcal{H}}^2\left(\overset{o}{x\bar{\mathcal{H}}}\right)\right) + \frac{c_4}{a^4}\left(-108x^2\bar{\mathcal{H}}^5 + 108x^2\bar{\mathcal{H}}^4\overset{o}{\bar{\mathcal{H}}} + 108x\bar{\mathcal{H}}^4\left(\overset{o}{x\bar{\mathcal{H}}}\right)\right) \\ & - \frac{c_5}{a^6}\left(-240x^3\bar{\mathcal{H}}^7 + 180x^3\bar{\mathcal{H}}^6\overset{o}{\bar{\mathcal{H}}} + 180x^2\bar{\mathcal{H}}^6\left(\overset{o}{x\bar{\mathcal{H}}}\right)\right) - 12\frac{c_G}{a^2}\bar{\mathcal{H}}^2\overset{o}{\bar{\mathcal{H}}}\Bigg] \end{aligned} \tag{A.12}$$

$$\begin{aligned} f_3^{eom} = {} & c_2 - \frac{c_3}{a^2}\left(4x\bar{\mathcal{H}}^2 + 4\bar{\mathcal{H}}\left(\overset{o}{x\bar{\mathcal{H}}}\right)\right) + \frac{c_4}{a^4}\left(-10x^2\bar{\mathcal{H}}^4 + 12x^2\bar{\mathcal{H}}^3\overset{o}{\bar{\mathcal{H}}} + 24x\bar{\mathcal{H}}^3\left(\overset{o}{x\bar{\mathcal{H}}}\right)\right) \\ & - \frac{c_5}{a^6}\left(-36x^3\bar{\mathcal{H}}^6 + 24x^3\bar{\mathcal{H}}^5\left(\overset{o}{\bar{\mathcal{H}}}\right) + 36x^2\bar{\mathcal{H}}^5\left(\overset{o}{x\bar{\mathcal{H}}}\right)\right) - \frac{c_G}{a^2}\left(2\bar{\mathcal{H}}\right) \end{aligned} \tag{A.13}$$

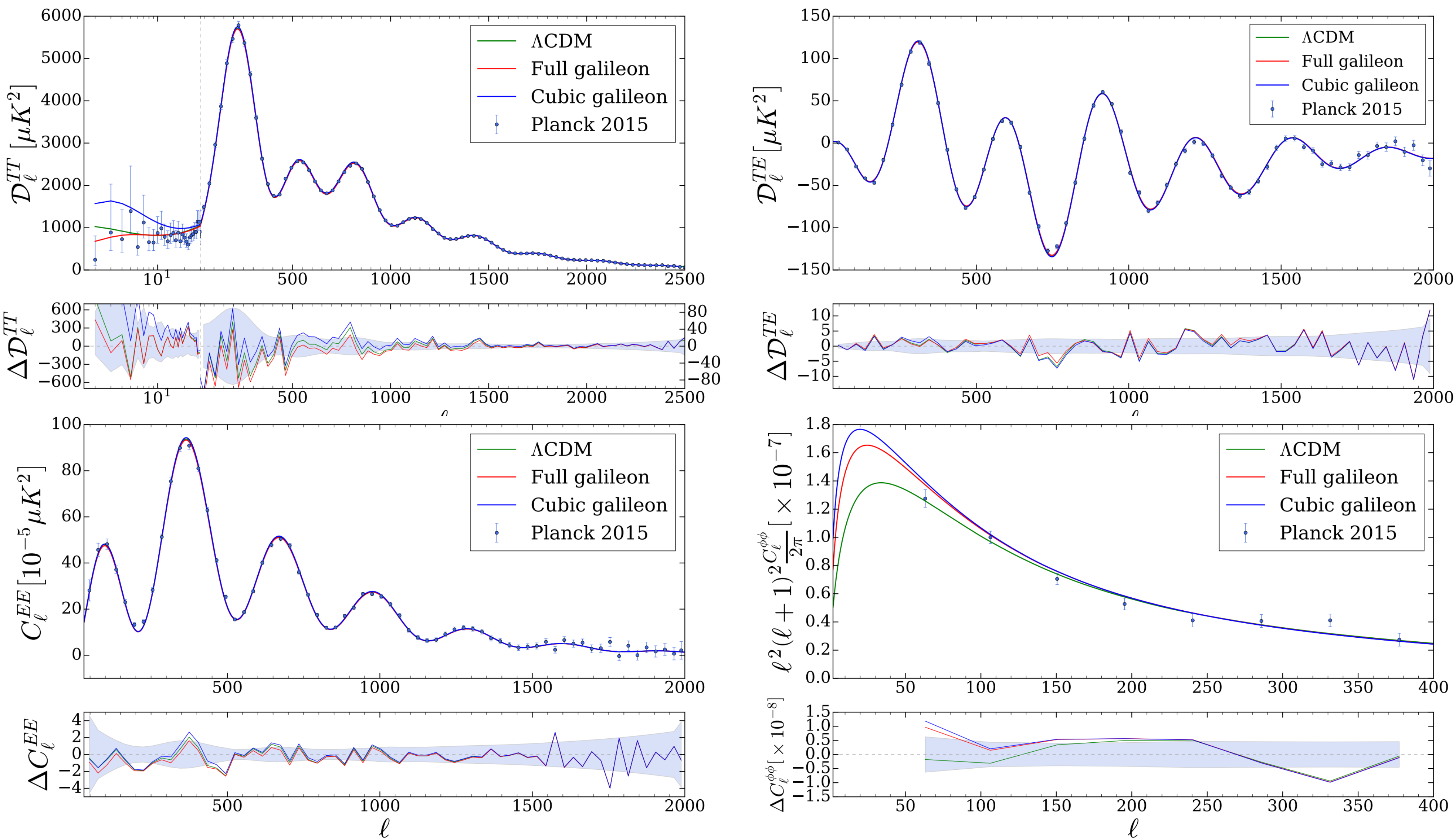
$$\begin{aligned} f_4^{eom} = {} & c_2 x - \frac{c_3}{a^2}\left(6x^2\bar{\mathcal{H}}^2 + 4x\bar{\mathcal{H}}\left(\overset{o}{x\bar{\mathcal{H}}}\right)\right) + \frac{c_4}{a^4}\left(-6x^3\bar{\mathcal{H}}^4 + 12x^3\bar{\mathcal{H}}^3\overset{o}{\bar{\mathcal{H}}} + 36x^2\bar{\mathcal{H}}^3\left(\overset{o}{x\bar{\mathcal{H}}}\right)\right) \\ & - \frac{c_5}{a^6}\left(-45x^4\bar{\mathcal{H}}^6 + 30x^4\bar{\mathcal{H}}^5\overset{o}{\bar{\mathcal{H}}} + 60x^3\bar{\mathcal{H}}^5\left(\overset{o}{x\bar{\mathcal{H}}}\right)\right) - \frac{c_G}{a^2}\left(6x\bar{\mathcal{H}}^2 + 4x\bar{\mathcal{H}}\overset{o}{\bar{\mathcal{H}}} + 4\bar{\mathcal{H}}\left(\overset{o}{x\bar{\mathcal{H}}}\right)\right) \end{aligned} \tag{A.14}$$

$$f_5^{eom} = -2\frac{c_3}{a^2}x^2\bar{\mathcal{H}}^2 + 12\frac{c_4}{a^4}x^2\bar{\mathcal{H}}^4 - 15\frac{c_5}{a^6}x^4\bar{\mathcal{H}}^6 - 4\frac{c_G}{a^2}x\bar{\mathcal{H}}^2 \tag{A.15}$$

$$\begin{aligned} f_6^{eom} = {} & \frac{c_4}{a^4}\left(-4x^3\bar{\mathcal{H}}^4 + 6x^2\bar{\mathcal{H}}^3\left(\overset{o}{x\bar{\mathcal{H}}}\right)\right) - \frac{c_5}{a^6}\left(-12x^4\bar{\mathcal{H}}^6 + 3x^4\bar{\mathcal{H}}^5\overset{o}{\bar{\mathcal{H}}} + 12x^3\bar{\mathcal{H}}^5\left(\overset{o}{x\bar{\mathcal{H}}}\right)\right) \\ & -2\frac{c_G}{a^2}\bar{\mathcal{H}}\left(\overset{o}{x\bar{\mathcal{H}}}\right) \end{aligned} \tag{A.16}$$

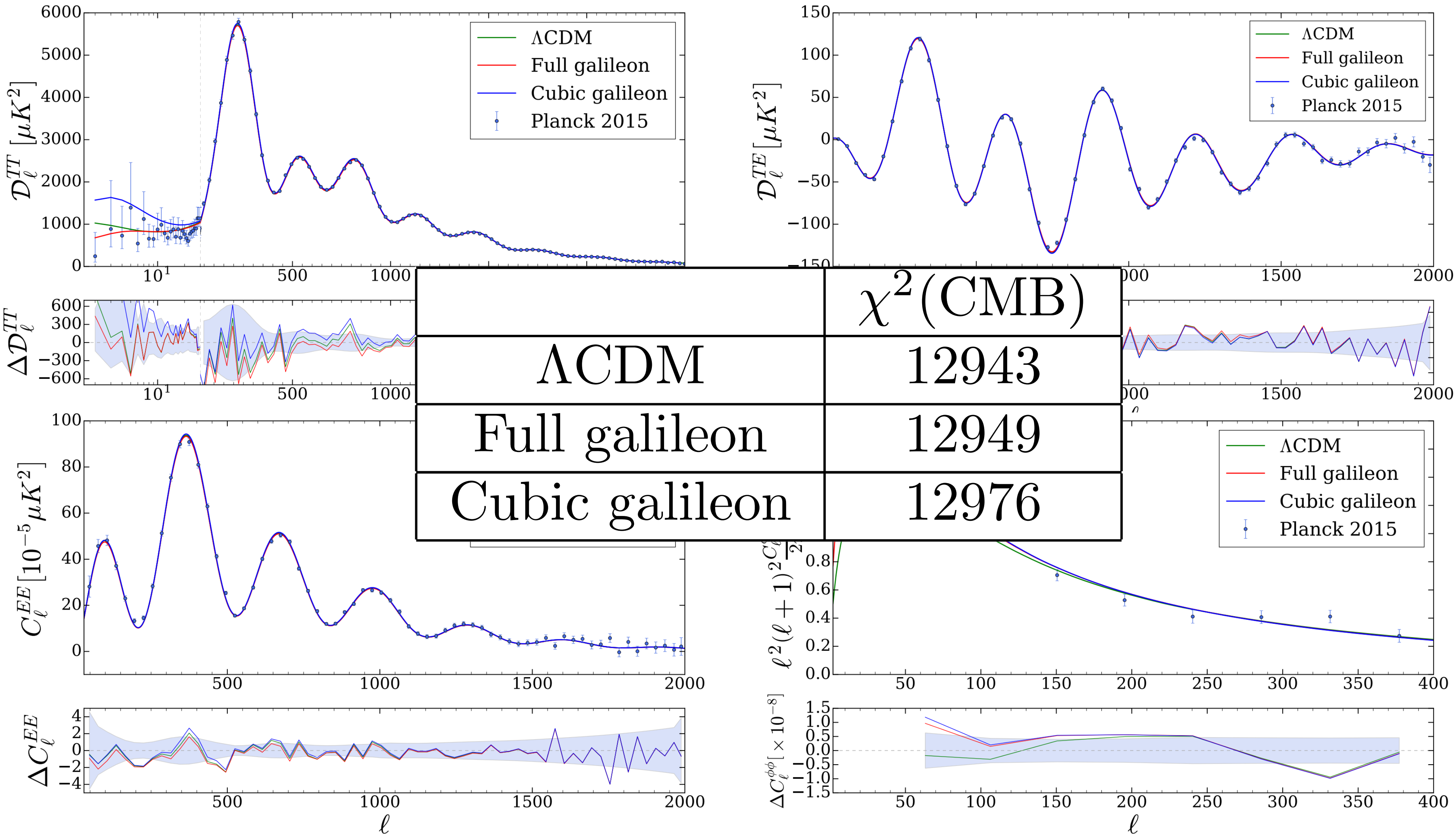
Base models

- Fit to CMB data only :



Base models

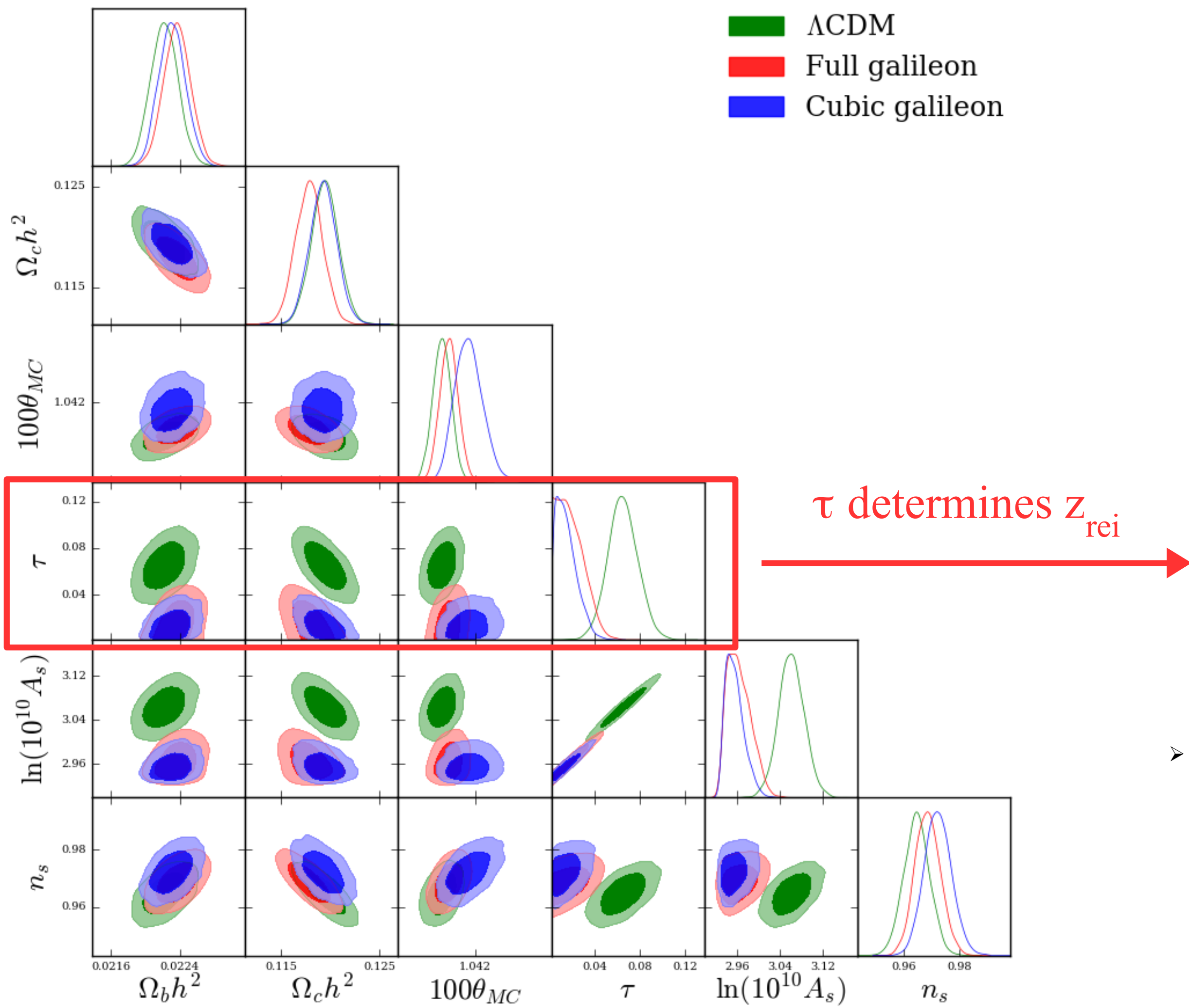
- Fit to CMB data only :

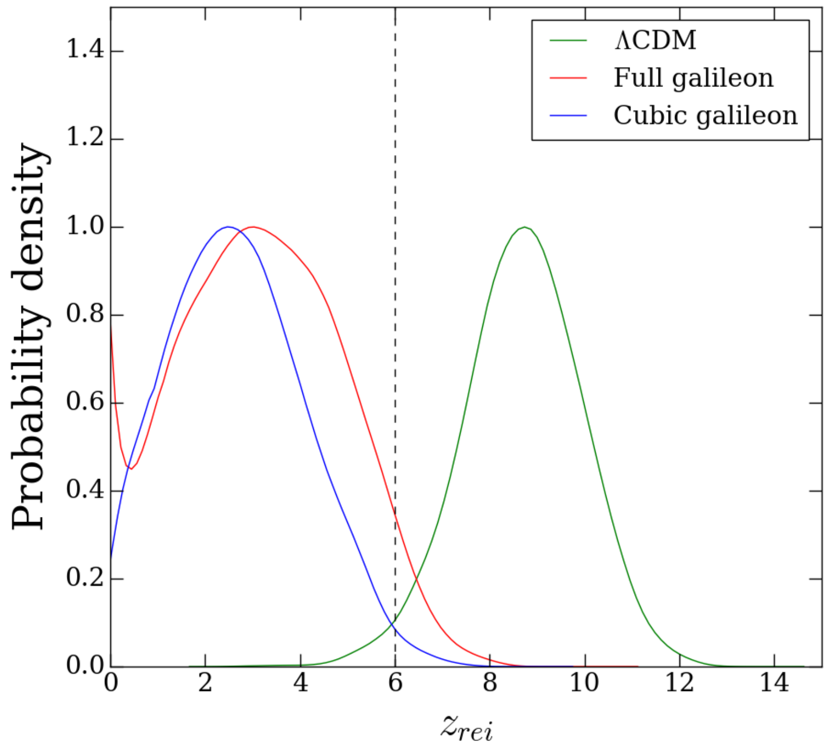


	χ^2(CMB)
ΛCDM	12943
Full galileon	12949
Cubic galileon	12976

Base models

- Fit to CMB data only :

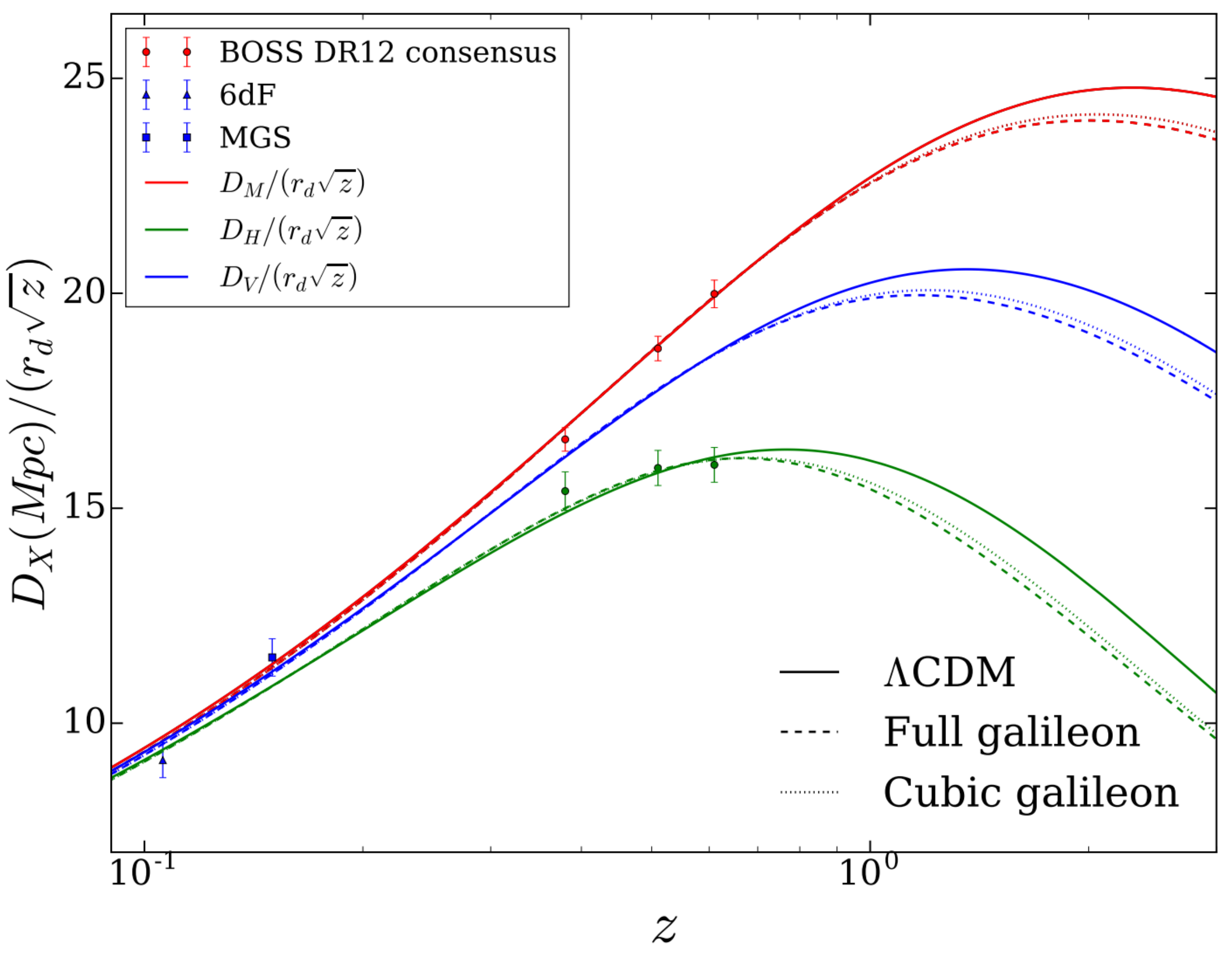




- Astrophysical observations indicate that $z_{rei} > 6$ (e.g. astro-ph/0108097) :
  - Full galileon : ~1.5σ
  - Cubic galileon : ~3.5σ

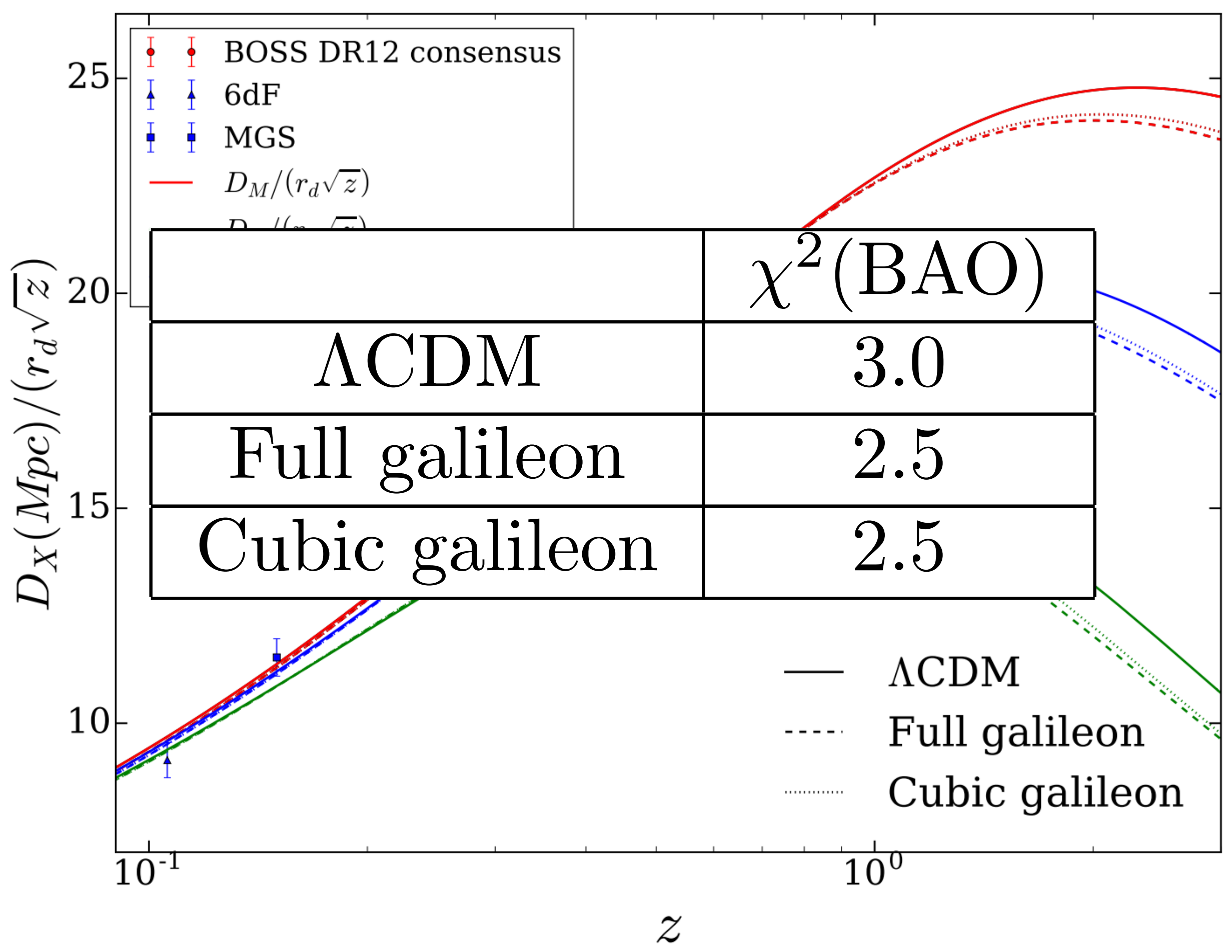
Base models

- Fit to BAO data only :



Base models

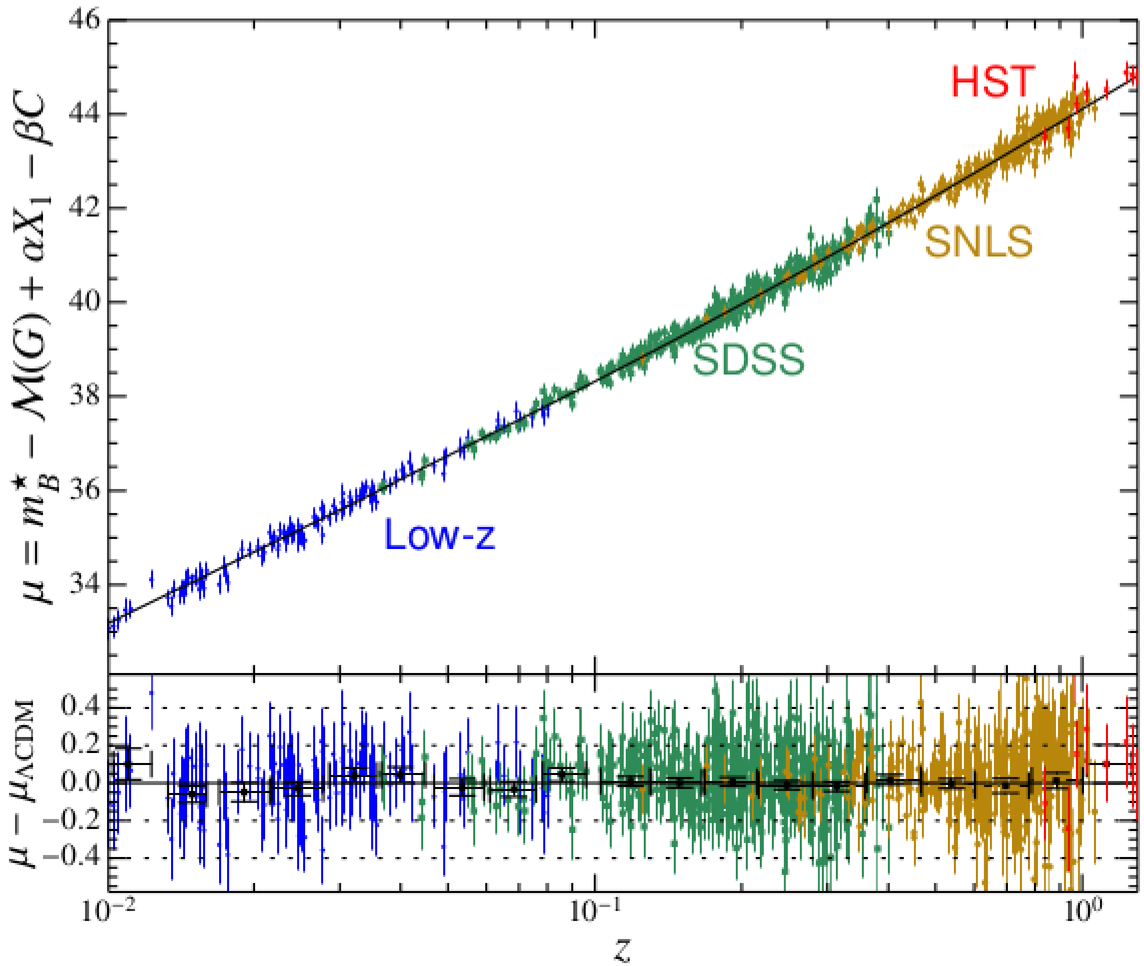
- Fit to BAO data only :



	χ^2(BAO)
ΛCDM	3.0
Full galileon	2.5
Cubic galileon	2.5

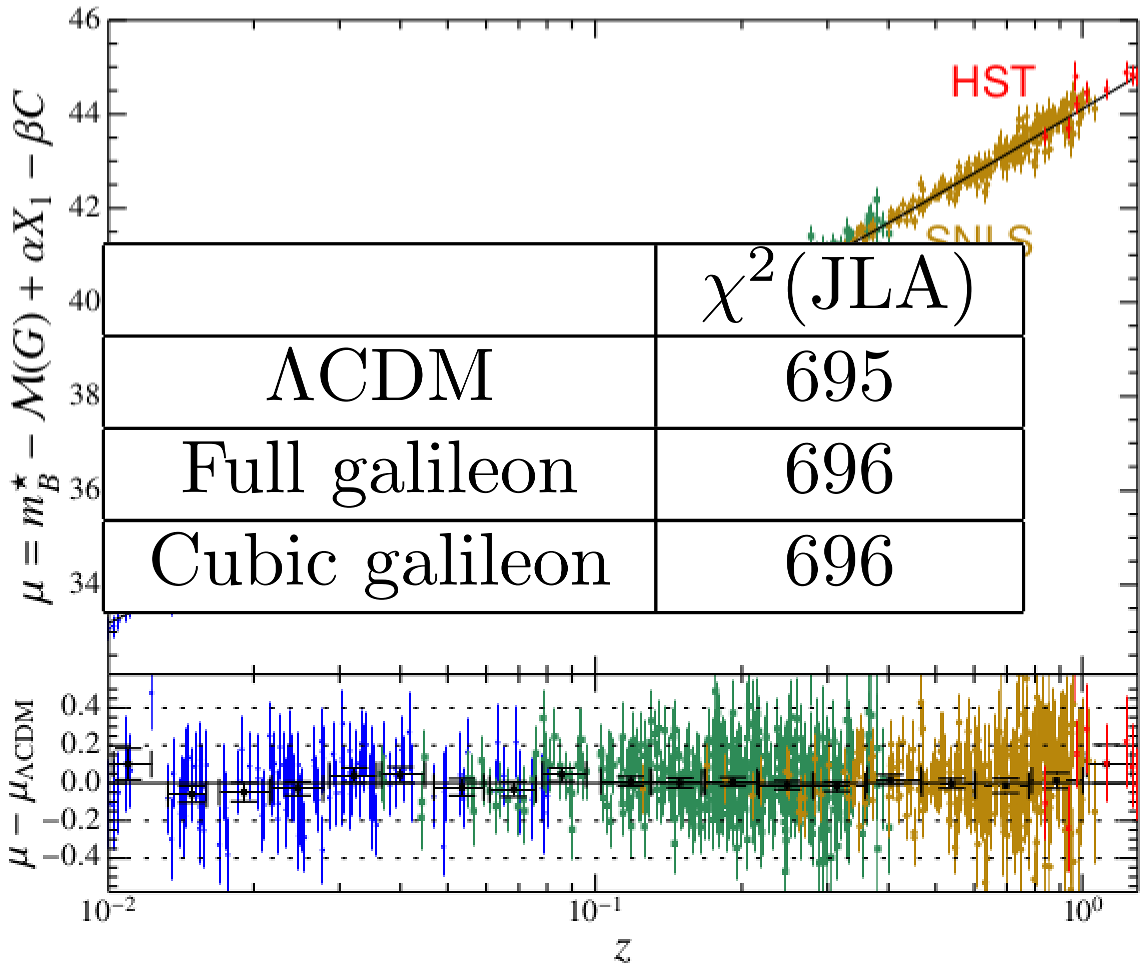
Base models

- Fit to JLA data only :



Base models

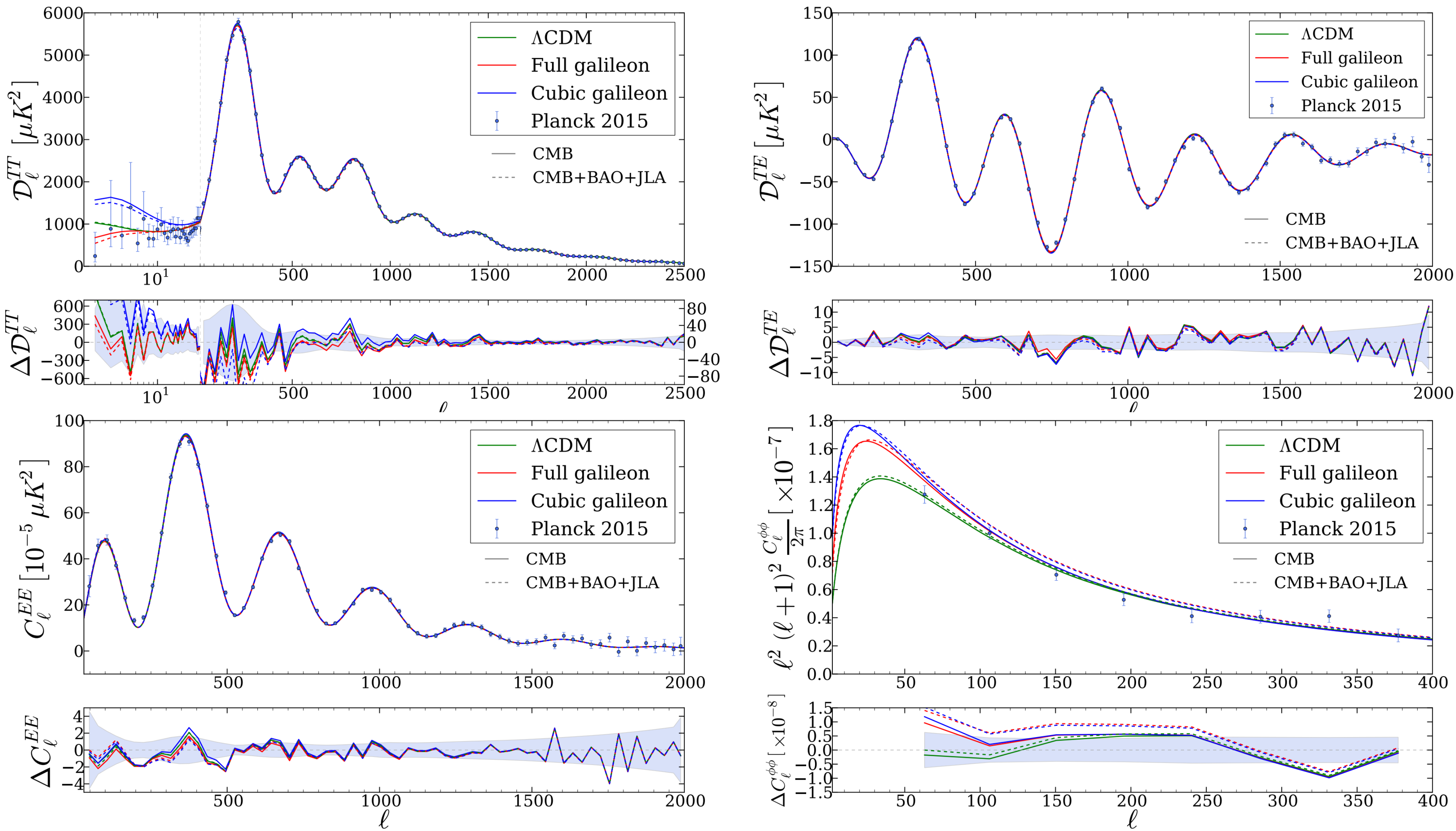
➢ Fit to JLA data only :



	χ^2(JLA)
ΛCDM	695
Full galileon	696
Cubic galileon	696

Base models

- Fit to CMB+BAO+JLA data :



Base models

- Fit to CMB+BAO+JLA data :