

# Modified gravity and dark energy in higher-order scalar-tensor theories

Marco Crisostomi



In collaboration with:

Christos Charmousis, Ruth Gregory, Kazuya Koyama, David  
Langlois, Matt Lewandowski, Karim Noui, Dani Steer, Niko  
Stergioulas, Gianmassimo Tasinato, Filippo Vernizzi

# DHOST

$$\begin{aligned}
 L = & K + G_3 \square \phi + G.R + A_1 \phi_{\mu\nu} \phi^{\mu\nu} + A_2 (\square \phi)^2 + A_3 (\square \phi) \phi^\mu \phi_{\mu\nu} \phi^\nu \\
 & + A_4 \phi^\mu \phi_{\mu\rho} \phi^{\rho\nu} \phi_\nu + A_5 (\phi^\mu \phi_{\mu\nu} \phi^\nu)^2
 \end{aligned}$$

[Langlois, Noui]  
[MC, Koyama, Tasinato]

Impose the degeneracy conditions

$$\begin{aligned}
 L = & K + G_3 \square \phi + G.R + A_1 [\phi_{\mu\nu} \phi^{\mu\nu} - (\square \phi)^2] + A_3 (\square \phi) \phi^\mu \phi_{\mu\nu} \phi^\nu \\
 & + f(G, A_1, A_3) \phi^\mu \phi_{\mu\rho} \phi^{\rho\nu} \phi_\nu + g(G, A_1, A_3) (\phi^\mu \phi_{\mu\nu} \phi^\nu)^2
 \end{aligned}$$

5 free functions

# DHOST

$$L = K + G_3 \square \phi + G R + A_1 [\phi_{\mu\nu} \phi^{\mu\nu} - (\square \phi)^2] + A_3 (\square \phi) \phi^\mu \phi_{\mu\nu} \phi^\nu \\ + f(G, A_1, A_3) \phi^\mu \phi_{\mu\rho} \phi^{\rho\nu} \phi_\nu + g(G, A_1, A_3) (\phi^\mu \phi_{\mu\nu} \phi^\nu)^2$$

5 free functions

## EFT of DE

[Langlois, Mancarella, Noui, Vernizzi]

Expand around FRW in the unitary gauge

$$S^{\text{quad}} = \int d^3x dt a^3 \frac{M^2}{2} \left\{ \delta K_{ij} \delta K^{ij} - \left( 1 + \frac{2}{3} \alpha_L \right) \delta K^2 + (1 + \alpha_T) \left( R \frac{\delta \sqrt{h}}{a^3} + \delta_2 R \right) \right. \\ \left. + H^2 \alpha_K \delta N^2 + 4H \alpha_B \delta K \delta N + (1 + \alpha_H) R \delta N + 4\beta_1 \delta K \delta \dot{N} + \beta_2 \delta \dot{N}^2 + \frac{\beta_3}{a^2} (\partial_i \delta N)^2 \right\}$$

Non-linear perturbations:

$\alpha_V$



# Gravitational wave constraints

$$L = K + G_3 \Box \phi + G R + \cancel{A_1 [\phi_{\mu\nu} \phi^{\mu\nu} - (\Box \phi)^2]} + A_3 (\Box \phi) \phi^\mu \phi_{\mu\nu} \phi^\nu \\ + f(G, A_1, A_3) \phi^\mu \phi_{\mu\rho} \phi^{\rho\nu} \phi_\nu + g(G, A_1, A_3) (\phi^\mu \phi_{\mu\nu} \phi^\nu)^2$$

Speed of gravity = Speed of light

GW170817

$$\ddot{h}_{ij} + (3 + \alpha_M) H \dot{h}_{ij} + (1 + \cancel{\alpha_T}) k^2 h_{ij} = 0 \quad \alpha_V = -\alpha_H$$

$$L = (G - \cancel{X A_1}) K_{ij} K^{ij} + G {}^{(3)}R + \dots$$

# Gravitational wave constraints

$$L = K + G_3 \Box \phi + G.R + \cancel{A_1 [\phi_{\mu\nu} \phi^{\mu\nu} - (\Box \phi)^2]} + \cancel{A_3 (\Box \phi) \phi^\mu \phi_{\mu\nu} \phi^\nu}$$

$$+ f(G, A_1, A_3) \phi^\mu \phi_{\mu\rho} \phi^{\rho\nu} \phi_\nu + \cancel{g(G, A_1, A_3) (\phi^\mu \phi_{\mu\nu} \phi^\nu)^2}$$

No decay of GW in DE

[Creminelli, Lewandowski, Tambalo, Vernizzi]

$$\alpha_H + 2\beta_1 = 0$$

$$A_3 = g = 0$$

# What is left?

$$L = \underbrace{K}_{\alpha_K} + \underbrace{G_3}_{\alpha_B} \square \phi + \underbrace{G}_{\alpha_M} R + \frac{3G_X^2}{2G} \phi^\mu \phi_{\mu\rho} \phi^{\rho\nu} \phi_\nu$$

$$\alpha_K \quad \alpha_B \quad \alpha_M \quad \beta_1$$

What about:

1 - Screening

2 - Self-acceleration

# Screening

$$L = K + G_3 \square \phi + G_* R + \frac{3G_X^2}{2G} \phi^\mu \phi_{\mu\rho} \phi^{\rho\nu} \phi_\nu$$

[MC, Koyama]

[MC, Lewandowski, Vernizzi]

$$\alpha_K \quad \alpha_B \quad \alpha_M \quad \beta_1$$

gravitational potentials for a spherically symmetric matter source

$$\Phi' = \frac{G_*(1 + \varepsilon_\Phi)m}{r^2}, \quad \Psi' = \frac{G_*(1 + \varepsilon_\Psi)m}{r^2}$$

**CASSINI**  $-0.2 \times 10^{-5} < \varepsilon_\Psi - \varepsilon_\Phi < 5.5 \times 10^{-5} \longrightarrow 0 \leq \beta_1 \lesssim 10^{-5}$

Region  $(\alpha_M, \alpha_B, \beta_1) \longrightarrow \varepsilon_\Phi = \varepsilon_\Psi$

Vainshtein 

**Hulse-Taylor**  $-2.5 \times 10^{-3} \leq \varepsilon_\Phi \leq 7.5 \times 10^{-3} \longrightarrow 0 \leq \beta_1 \lesssim 10^{-2}$

# Screening ?

$$L = K + G_3 \square \phi + G.R + \frac{3G_X^2}{2G} \phi^\mu \phi_{\mu\rho} \phi^{\rho\nu} \phi_\nu$$

[MC, Koyama]

[MC, Lewandowski, Vernizzi]

$$\alpha_K \quad \alpha_B \quad \alpha_M \quad \beta_1$$



[Creminelli, Tambalo, Vernizzi, Yingcharoenrat]

$$\text{Combination } (\alpha_M, \alpha_B, \beta_1) \lesssim 10^{-2}$$



$$G_3$$

$$0 \leq \beta_1 \lesssim 10^{-2}$$

# Self-acceleration

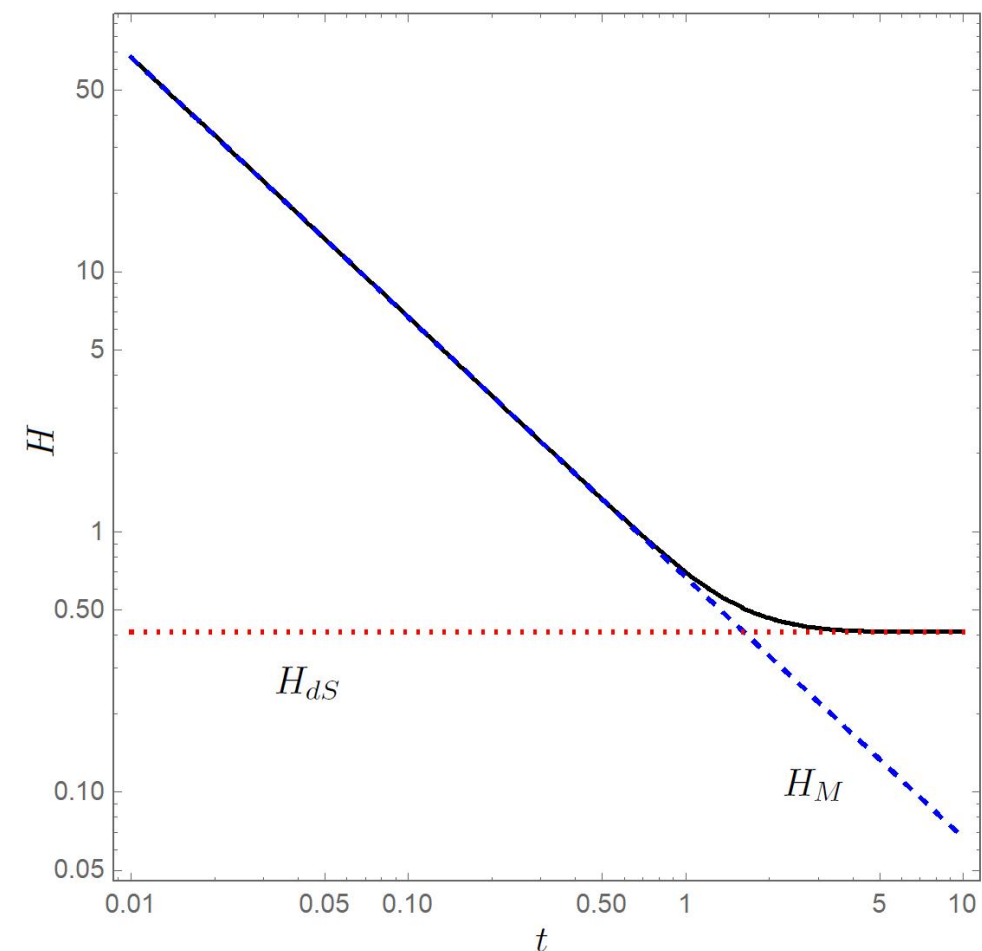
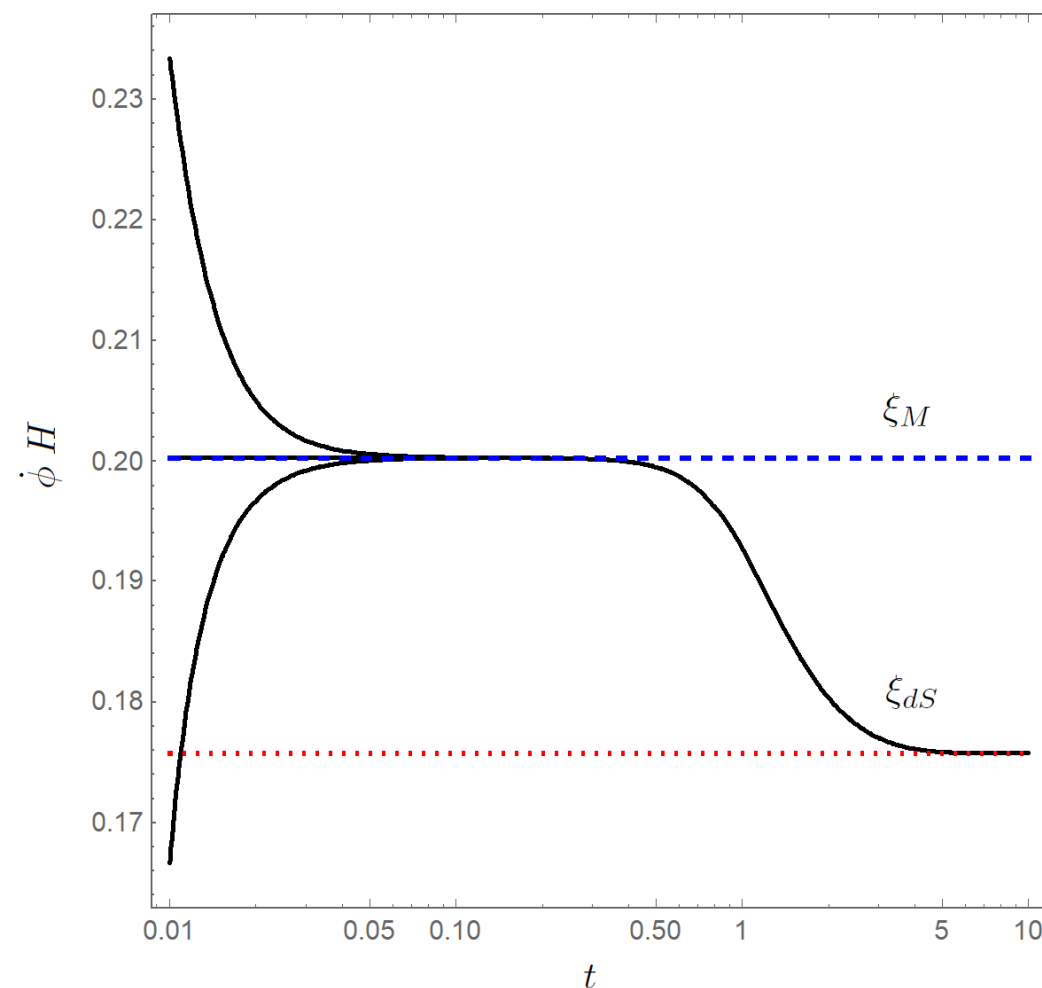
[MC, Koyama]

[MC, Koyama, Langlois, Noui, Steer]

$$L = K + G_3 \square \phi + G R + \frac{3G_X^2}{2G} \phi^\mu \phi_{\mu\rho} \phi^{\rho\nu} \phi_\nu$$

Background effect

$$K = c_2 X, \quad G_3 = \frac{c_3}{\Lambda^3} X, \quad G = \frac{M_P^2}{2} + \frac{c_4}{\Lambda^6} X^2 \quad c_2, c_3, c_4 \sim \mathcal{O}(1)$$



# Self-acceleration

[MC, Koyama]

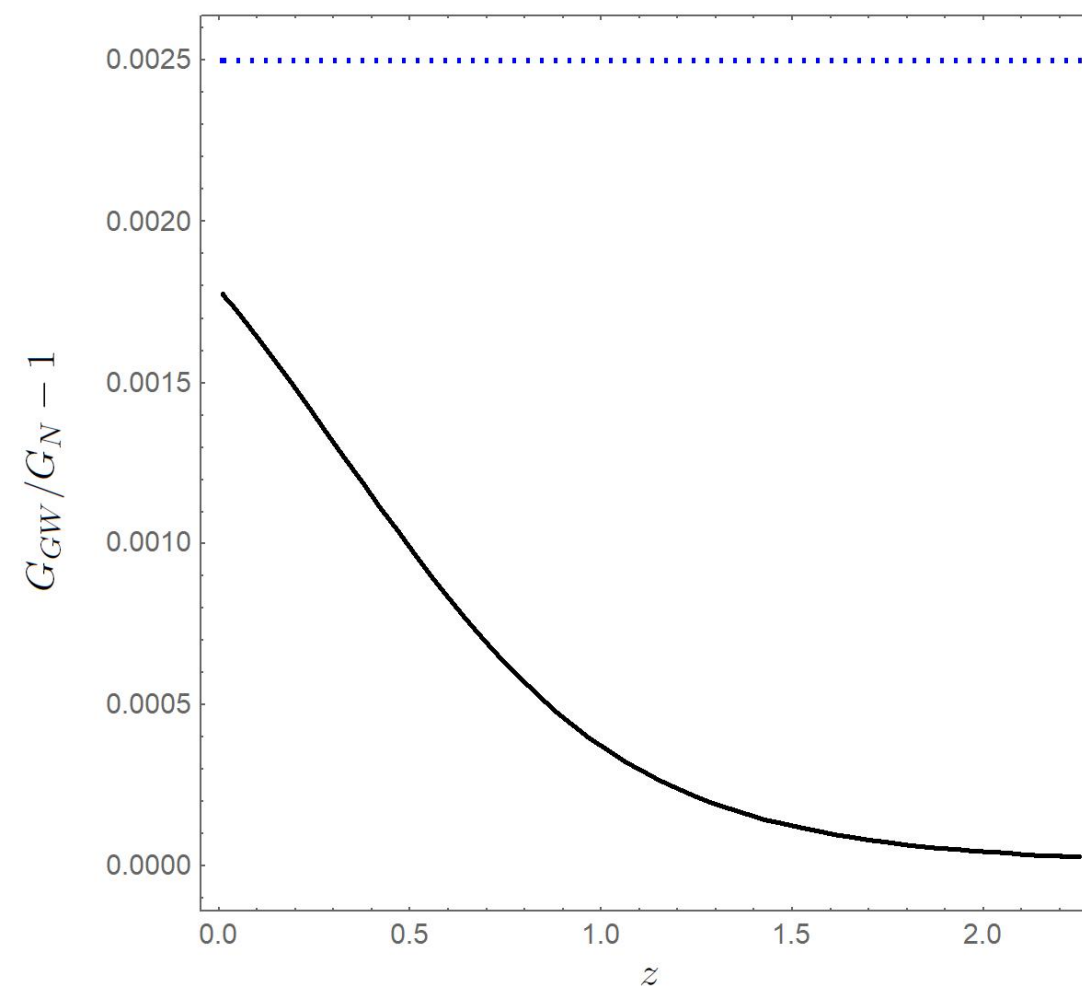
[MC, Koyama, Langlois, Noui, Steer]

$$L = K + G_3 \square \phi + G R + \frac{3G_X^2}{2G} \phi^\mu \phi_{\mu\rho} \phi^{\rho\nu} \phi_\nu$$

Background effect

$$K = c_2 X, \quad G_3 = \frac{c_3}{\Lambda^3} X, \quad G = \frac{M_P^2}{2} + \frac{c_4}{\Lambda^6} X^2$$

$$c_2, c_3, c_4 \sim \mathcal{O}(1)$$



# Signatures in LSS

## Consistency relations in LCDM

[Kehagias, Riotto]

[Peloso, Pietroni]

[Creminelli, Norena, Simonovic, Vernizzi]

$$\langle \Phi_{\vec{q}}(\eta) \delta_{\vec{k}_1}(\eta_1) \cdots \delta_{\vec{k}_n}(\eta_n) \rangle'_{q \rightarrow 0} = P_{\Phi}(q) \sum_a \mathcal{O}_a \langle \delta_{\vec{k}_1}(\eta_1) \cdots \delta_{\vec{k}_n}(\eta_n) \rangle'$$

## Consistency relations are broken in DHOST

[MC, Lewandowski, Vernizzi]

$$\delta^{(2)}(\vec{k}, t) = \int_{\vec{k}_1, \vec{k}_2}^{\vec{k}} F_2(\vec{k}_1, \vec{k}_2; t) \delta^{(1)}(\vec{k}_1, t) \delta^{(1)}(\vec{k}_2, t)$$

$$F_2(\vec{k}_1, \vec{k}_2; t) = \underbrace{A_{\alpha}(t)}_{1 + \text{DHOST}} \alpha_s(\vec{k}_1, \vec{k}_2) + \underbrace{A_{\gamma}(t)}_{\text{Always modified}} \gamma(\vec{k}_1, \vec{k}_2)$$

$$\lim_{q \rightarrow 0} \frac{B(q, k_2, k_3)}{P_{11}(q)P_{11}(k)} \approx -2A_{\alpha} \left( \frac{1}{2} \frac{\vec{q} \cdot \vec{k}}{q^2} - \frac{1}{2} \frac{\vec{q} \cdot \vec{k}}{q^2} \right) = 0$$

Because of translation invariance!

$$\lim_{q \rightarrow 0} \frac{B^{\text{mml}}(q, k_2, k_3)}{P_{11}(q)P_{11}(k)} \approx (L_{\text{lens}} A_{\alpha} - A_{\alpha}^{\text{lens}}) \frac{\vec{q} \cdot \vec{k}}{q^2}$$

Enhanced for different tracers



# Signatures in LSS

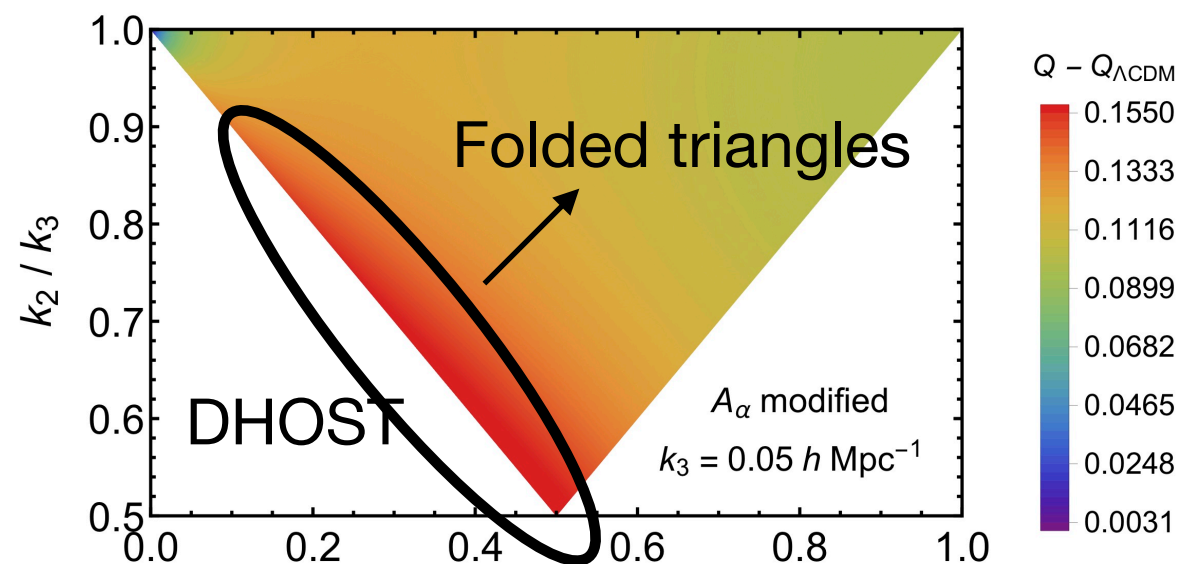
Consistency relations are broken in DHOST

[MC, Lewandowski, Vernizzi]

$$F_2(\vec{k}_1, \vec{k}_2; t) = A_\alpha(t) \alpha_s(\vec{k}_1, \vec{k}_2) + A_\gamma(t) \gamma(\vec{k}_1, \vec{k}_2)$$

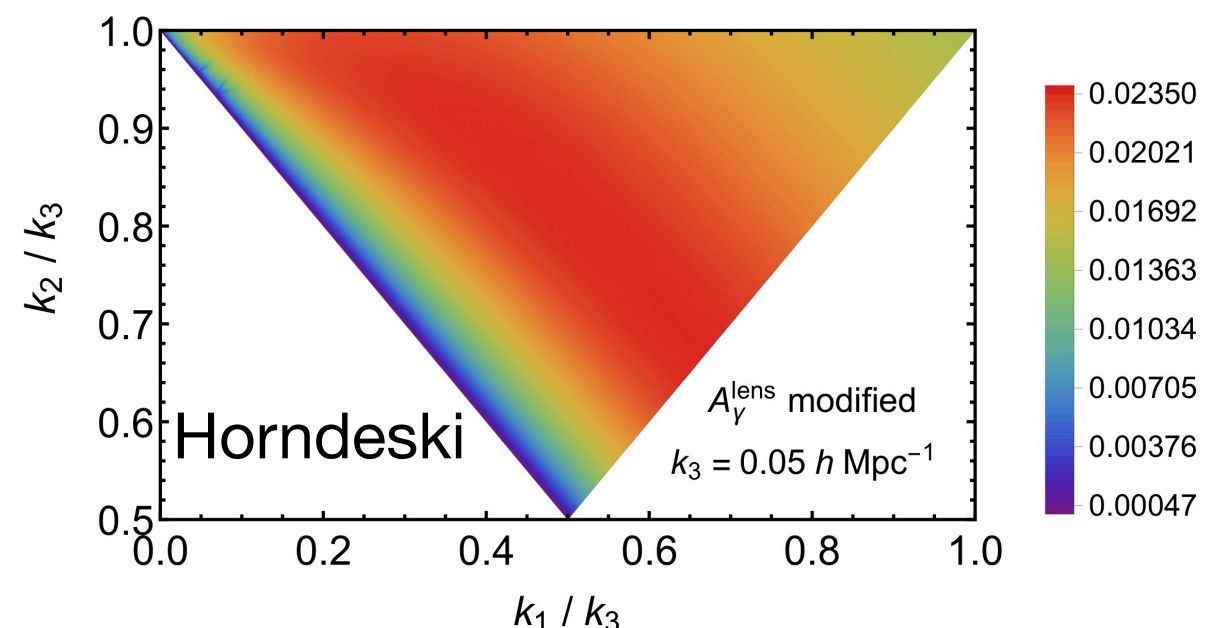
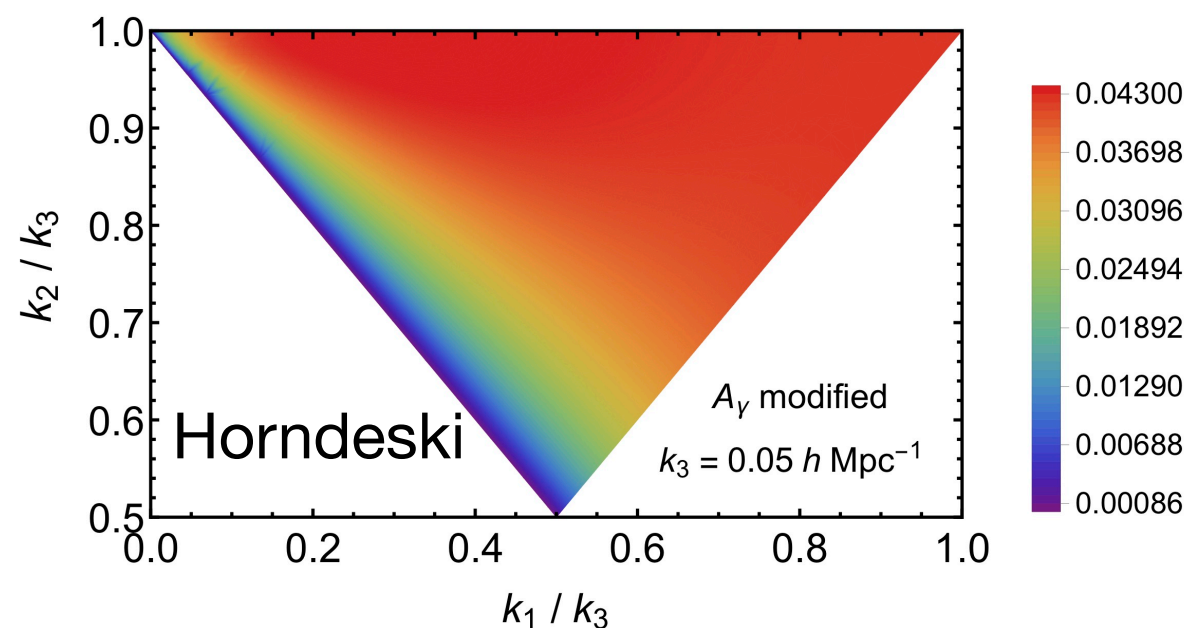
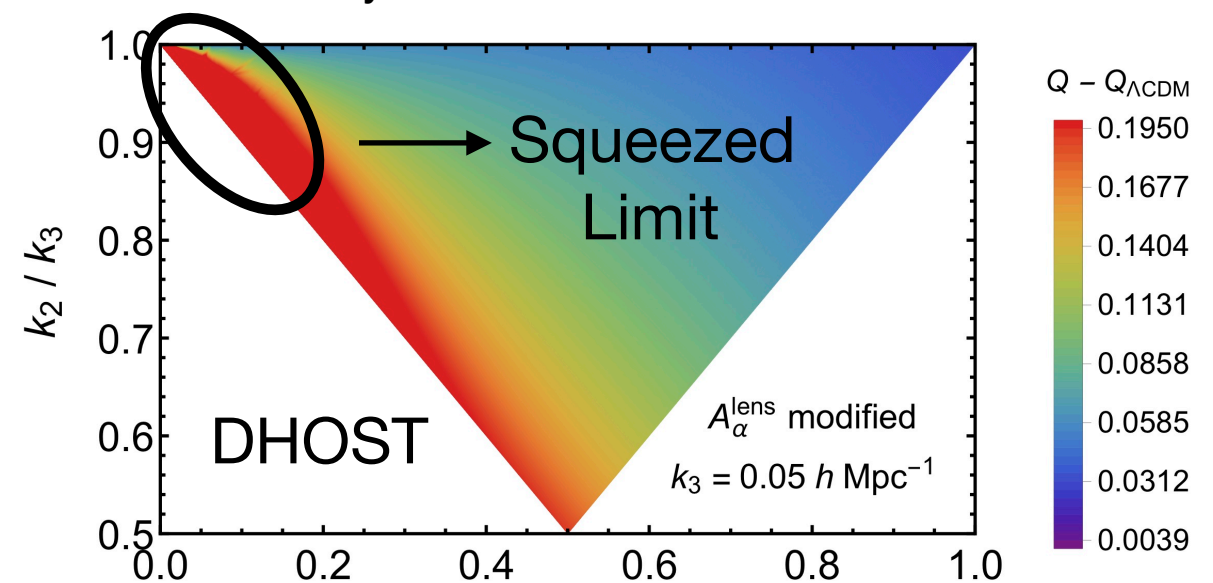
**Auto-correlation**

1 + DHOST



**Cross-correlation**

Always modified



# Stealth Black Holes

[Charmousis, MC, Gregory, Stergioulas]

$$L = K + G_3 \square \phi + G R + \cancel{A_1 [\phi_{\mu\nu} \phi^{\mu\nu} - (\square \phi)^2]} + A_3 (\square \phi) \phi^\mu \phi_{\mu\nu} \phi^\nu$$

$$+ f(G, A_1, A_3) \phi^\mu \phi_{\mu\rho} \phi^{\rho\nu} \phi_\nu + g(G, A_1, A_3) (\phi^\mu \phi_{\mu\nu} \phi^\nu)^2$$

$X \equiv \phi_\mu \phi^\mu = \text{const.} \longrightarrow$  Relation between  $\phi$  and the geodesic  $x^\mu(\lambda)$

$$\phi^\mu \leftrightarrow \frac{dx^\mu}{d\lambda}$$

$$\nabla^\mu X = 0 \longrightarrow \frac{d^2 x^\mu}{d\lambda^2} + \Gamma_{\rho\sigma}^\mu \frac{dx^\rho}{d\lambda} \frac{dx^\sigma}{d\lambda} = 0$$

**Hamilton-Jacobi  
method**

$$\frac{\partial S}{\partial x^\mu} = p_\mu = g_{\mu\nu} \frac{dx^\nu}{d\lambda}$$

$$\phi \leftrightarrow S$$

# Rotating Black Holes

[Charmousis, MC, Gregory, Stergioulas]

$$S = -E t + \cancel{I_z} \varphi + S_r(r) + S_\theta(\theta)$$

**Carter 1968**

$$S_r = \pm \int \frac{\sqrt{R}}{\Delta_r} dr, \quad S_\theta = \pm \int \frac{\sqrt{\Theta}}{\Delta_\theta} d\theta \quad \left\{ \begin{array}{l} \Delta_r = \left(1 - \frac{r^2}{\ell^2}\right) (r^2 + a^2) - 2Mr \\ \Delta_\theta = 1 + \frac{a^2}{\ell^2} \cos^2 \theta \end{array} \right.$$

$$\begin{aligned} \Theta &= a^2 m^2 \sin^2 \theta (\Delta_\theta - \eta^2), \\ R &= m^2 (r^2 + a^2) (\eta^2 (r^2 + a^2) - \Delta_r) \end{aligned}$$

$$\eta = \eta_c \quad \longrightarrow \quad R(r_0) = 0$$

# Rotating Black Holes

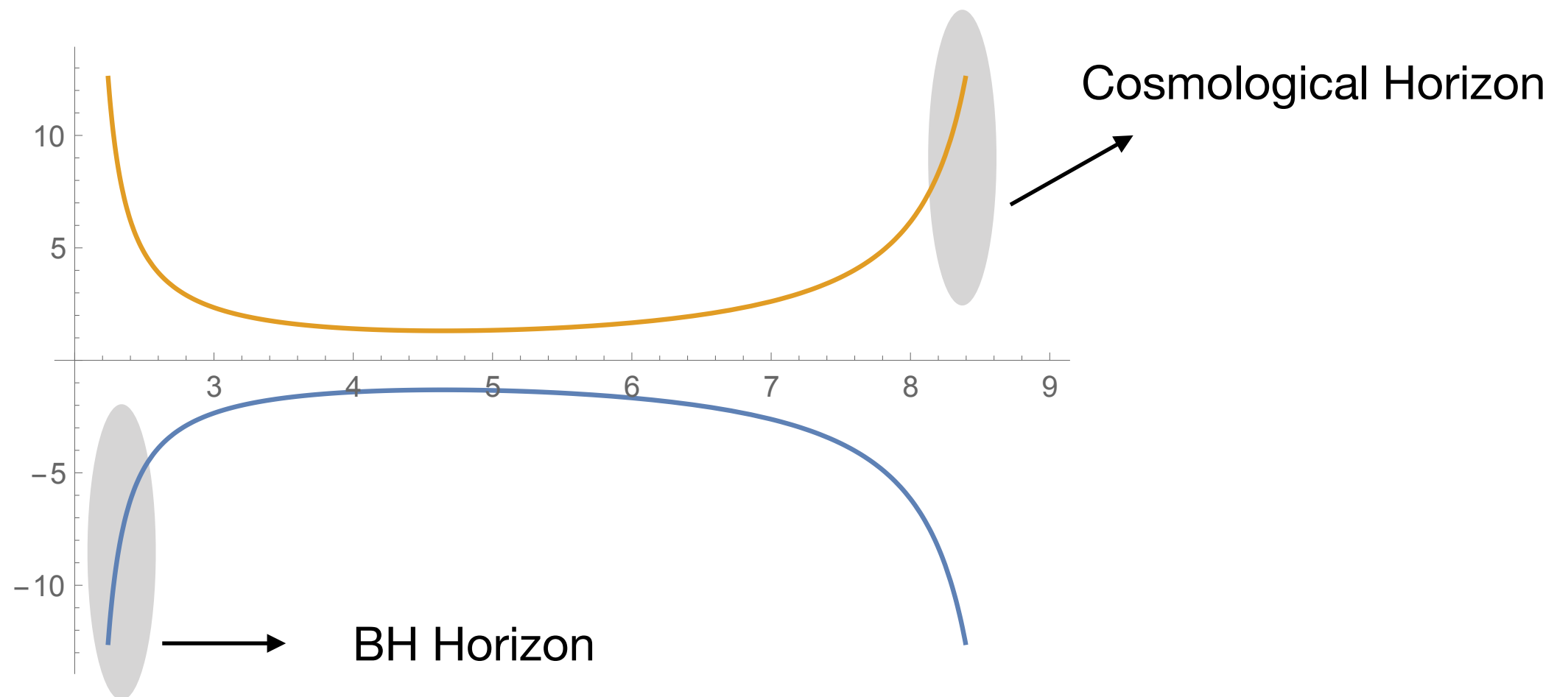
[Charmousis, MC, Gregory, Stergioulas]

$$S = -E t + \cancel{I_z} \varphi + S_r(r) + S_\theta(\theta)$$

**Carter 1968**

$$S_r = \pm \int \frac{\sqrt{R}}{\Delta_r} dr$$

$$R = m^2(r^2 + a^2) (\eta^2(r^2 + a^2) - \Delta_r)$$



# Rotating Black Holes

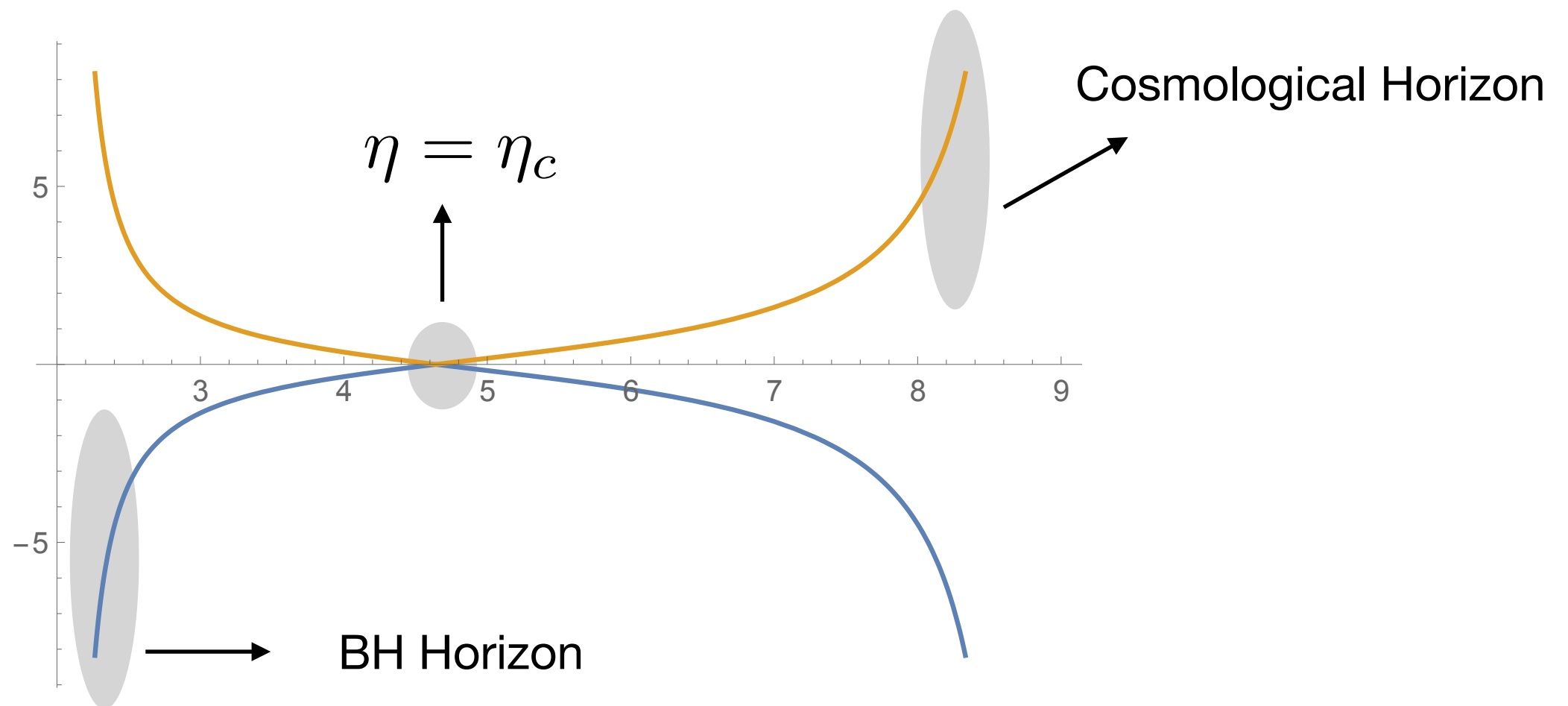
[Charmousis, MC, Gregory, Stergioulas]

$$S = -E t + \cancel{I_z} \varphi + S_r(r) + S_\theta(\theta)$$

**Carter 1968**

$$S_r = \pm \int \frac{\sqrt{R}}{\Delta_r} dr$$

$$R = m^2(r^2 + a^2) (\eta^2(r^2 + a^2) - \Delta_r)$$



# Rotating Black Holes

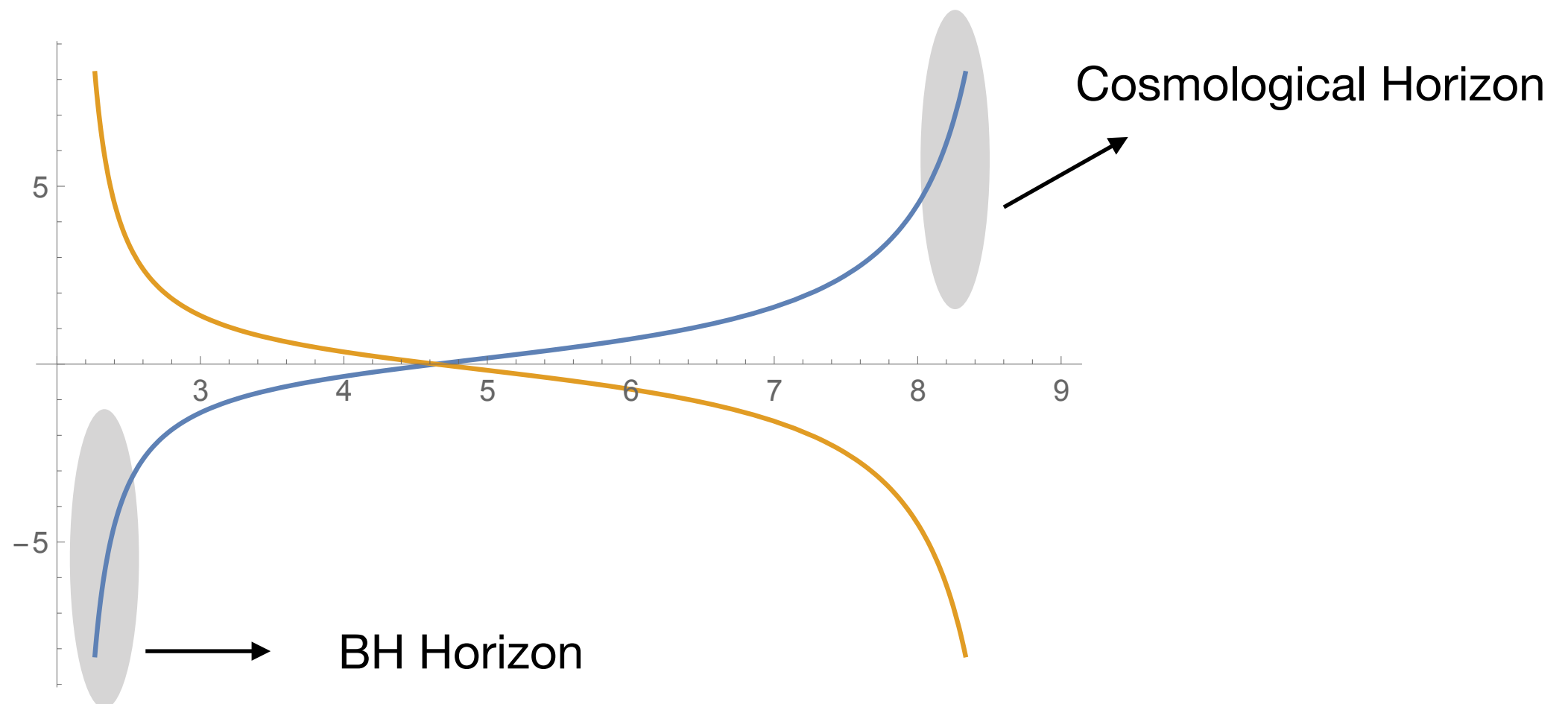
[Charmousis, MC, Gregory, Stergioulas]

$$S = -E t + \cancel{I_z} \varphi + S_r(r) + S_\theta(\theta)$$

**Carter 1968**

$$S_r = \pm \int \frac{\sqrt{R}}{\Delta_r} dr$$

$$R = m^2(r^2 + a^2) (\eta^2(r^2 + a^2) - \Delta_r)$$



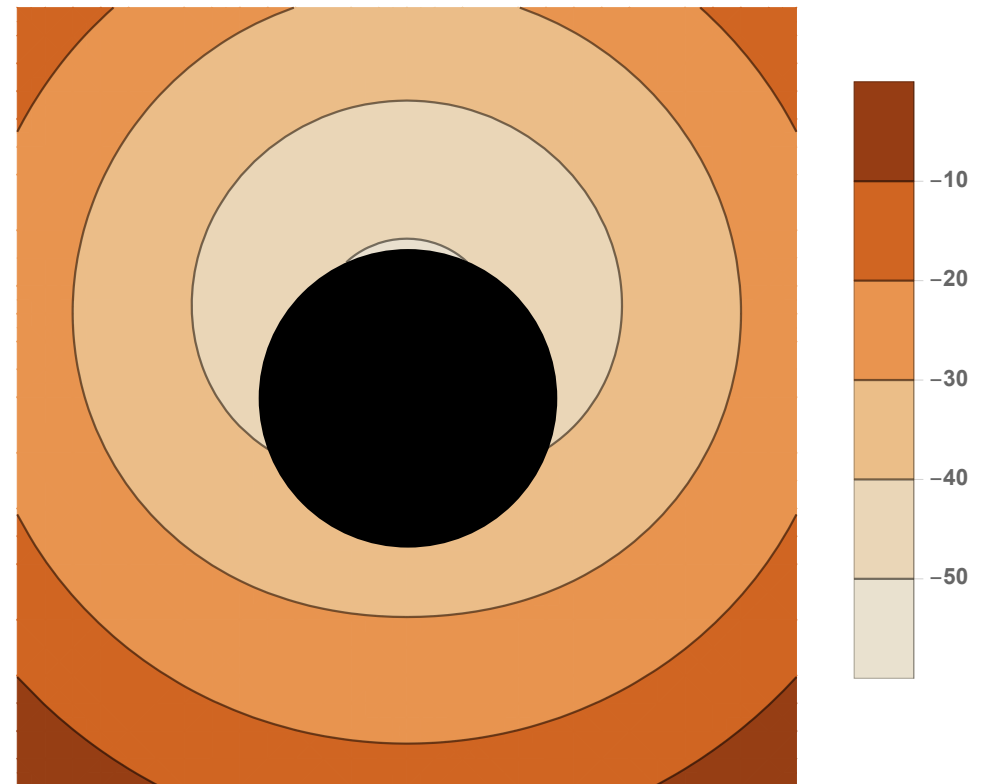
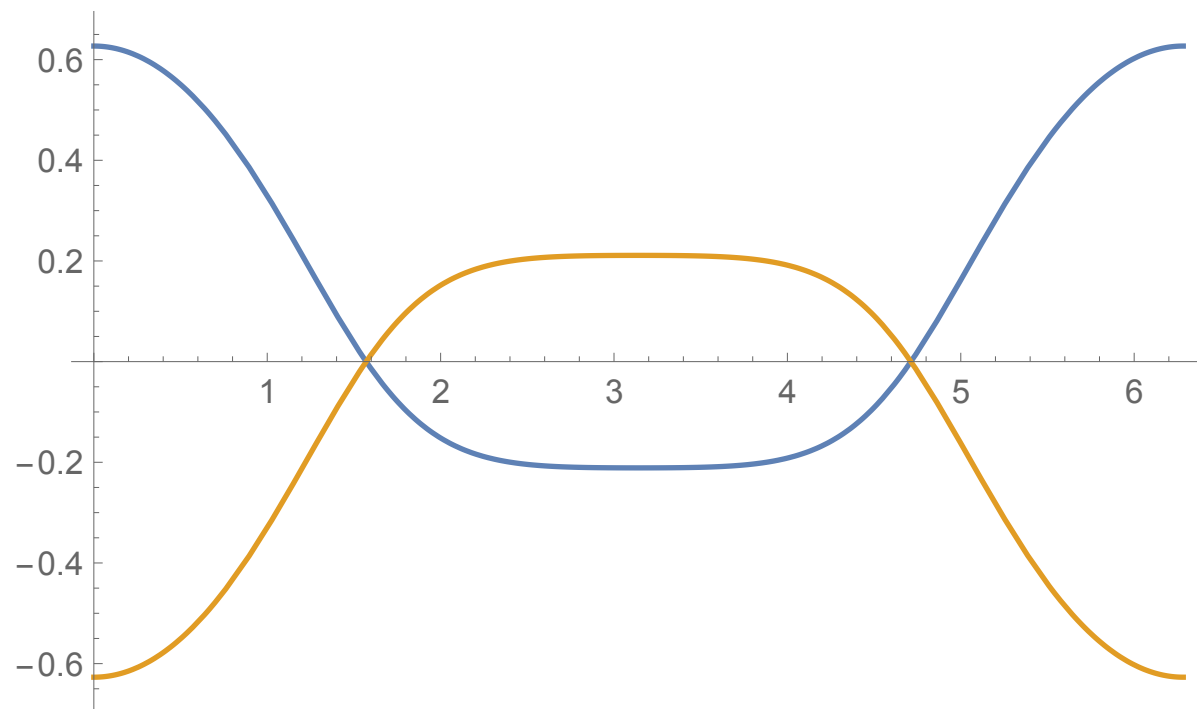
# Rotating Black Holes

[Charmousis, MC, Gregory, Stergioulas]

$$S = -E t + \cancel{I_z} \varphi + S_r(r) + S_\theta(\theta)$$

**Carter 1968**

$$S_\theta = \pm \int \frac{\sqrt{\Theta}}{\Delta_\theta} d\theta \quad \Theta = a^2 m^2 \sin^2 \theta (\Delta_\theta - \eta^2)$$



# Rotating Black Holes

[Charmousis, MC, Langlois, Noui]

$$S = -E t + \cancel{I_z} \varphi + S_r(r) + S_\theta(\theta)$$

**Carter 1968**

## Perturbations

**GR:**

$$\psi(r, \theta) e^{-i\omega t + im\varphi}$$

$$\mathcal{O}(\omega, m) \psi = 0$$

$$\mathcal{O}(\omega, m) = \mathcal{O}_r(\omega, m) + \mathcal{O}_\theta(\omega, m) \longrightarrow \psi(r, \theta) = \psi_r(r) \psi_\theta(\theta)$$

**DHOST:**

$$\mathcal{O}(\omega, m) \psi = \mathcal{T}$$

$$\left\{ \begin{array}{l} \delta G_{\mu\nu} = \delta T_{\mu\nu} \equiv \frac{1}{2} \Xi \bar{\phi}_\mu \bar{\phi}_\nu \delta X \\ \bar{\nabla}_\mu \left( \Xi \bar{\phi}^\mu \delta X \right) = 0 \end{array} \right.$$



# Rotating Black Holes

[Charmousis, MC, Langlois, Noui]

$$S = -E t + \cancel{I_z} \varphi + S_r(r) + S_\theta(\theta)$$

**Carter 1968**

## Perturbations

**DHOST:**

$$\bar{\nabla}_\mu \left( \Xi \bar{\phi}^\mu \delta X \right) = 0$$

$$\chi = \sum_m \int d\omega \chi_{m,\omega}(r, \theta) e^{-i\omega t + im\varphi}$$

$$\chi_{m,\omega}(r, \theta) = \frac{C_{m,\omega}(\theta)}{\sqrt{R(r)}} \exp \left[ i\varepsilon \left( -\omega I(r) - \omega \sin^2 \theta J(r) + mK(r) \right) \right]$$

$$I(r) \equiv - \int dr \frac{(r^2 + a^2)^2}{\Delta(r) \sqrt{R(r)}}$$

$$J(r) \equiv \int dr \frac{a^2}{\sqrt{R(r)}},$$

$$K(r) \equiv - \int dr \frac{2Mar}{\Delta(r) \sqrt{R(r)}}$$

# Conclusions

$$L = K + G_3 \Box \phi + G \cdot R + \cancel{A_1 [\phi_{\mu\nu} \phi^{\mu\nu} - (\Box \phi)^2]} + \cancel{A_3 (\Box \phi) \phi^\mu \phi_{\mu\nu} \phi^\nu} \\ + f(G, A_1, A_3) \phi^\mu \phi_{\mu\rho} \phi^{\rho\nu} \phi_\nu + \cancel{g(G, A_1, A_3) (\phi^\mu \phi_{\mu\nu} \phi^\nu)^2}$$

- 3 free functions (  $\alpha_K$   $\alpha_B$   $\alpha_M$   $\beta_1$  )

- Screening is OK (hopefully)

- Self-acceleration is OK

**Thanks!**

- Distinctive signatures in LSS

- QNM & Ringdown