LagSHT

J.E Campagne

(Action Dark Energy 19/11/19)

Features

- LagSHT performs Laguerre Spherical Harmonic transform over different set of 2D-sphere mappings with user choice of generalized Laguerre parameter α and spin (eg. 0 for scalar, 2 for polarization).
- Code available at https://gitlab.in2p3.fr/campagne/LagSHT
- Deliverable accepted for DESC 3DDC
- Original work by J. McEwen & B. Leistedt (IEEE VOL. 60, NO. 12 (2012))

Basics spin-0

$$K_{lmn}(r,\Omega;\tau) \equiv Y_{l,m}(\Omega) \times \mathcal{K}_n(r,\tau)$$ 3D ortho-basis

$$\mathcal{K}_n(r,\tau) = \tau^{-3/2}\sqrt{\frac{n!}{(n+\alpha)!}}e^{-r/2\tau}\left(\frac{r}{\tau}\right)^{\frac{\alpha}{2}-1}L_n^{(\alpha)}(r/\tau)$$ Gen. Lag. Func

$$f(r,\Omega) = \sum_{n=0}^{\infty}\sum_{l=0}^{\infty}\sum_{m=-l}^{l} f_{lmn}\, K_{lmn}(r,\Omega;\tau)$$ Synthesis

$$f_{lmn} = \int_{\mathrm{B}^3} dr d\Omega\, r^2\, f(r,\Omega)\, K^*_{lmn}(r,\Omega;\tau)$$ Analysis

Basics (cont'ed)

3D-Discretization (fijk)* and also in Spherical-Laguerre space (flmn)

$$\textbf{Analysis :} \quad f_{ijk} \xrightarrow{SHT} a_{lmk} = a_{lm}(r_k) \xrightarrow{Lag.Trans} f_{lmn}$$

$$\textbf{Synthesis :} \quad f_{lmn} \xrightarrow{Inv.Lag.Trans} a_{lmk} = a_{lm}(r_k) \xrightarrow{Inv.SHT} f_{ijk}$$

Exact Gauss-Laguerre quadrature

$$f_{lmn} \propto \sum_{k=0}^{N-1} w_k \; a_{lmk} \; e^{-r_k/2} L_n^{(\alpha)}(r_k) r_k^{1-\frac{\alpha}{2}} \qquad n \in \{0, \ldots, N-1\}$$

By product

Alm on each sub-shells → Tomography

New coefficients

i,j: index 2D-sphere, k: radial index

Basics (cont'ed)

$$\textbf{Analysis :} \quad f_{ijk} \xrightarrow{SHT} a_{lmk} = a_{lm}(r_k) \xrightarrow{Lag.Trans} f_{lmn}$$

$$\textbf{Synthesis :} \quad f_{lmn} \xrightarrow{Inv.Lag.Trans} a_{lmk} = a_{lm}(r_k) \xrightarrow{Inv.SHT} f_{ijk}$$

- SHT on each sub-shells performed via libsharps (ECP/ Fejer1 sum rule, Gauss, Healpix, user-defined). Tomographic analysis for free
- Radial part: Laguerre Transform quadrature: new own-made fast & accurate implementation with Martin Reinecke expertise.
- Spin 0 & 2 available

Installation

- Mac OSX/Linux with gcc OpenMP compliant
- Uses OpenBLASS on Linux if installed, on Mac uses native framework accelerator
- Libsharp
- *FFTW for the Laguerre to Bessel transform passage (experimental)*

Exemple

- Lmax = 1024, N shells = 128 (Gauss mapping) Nphi = 2048:
  - Analysis or Synthesis in 13-15sec on Mac OSX i7 dominated by the SHT (Laguerre part < 1sec)
  - Max abs. err. 2 10^{-11}

$$C_\ell(k) = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} |a_{\ell m k}|^2$$

Classical Analysis

$$C_\ell(n) = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} |f_{\ell m n}|^2$$

New type of analysis no yet exploited. Who want to try ?

$$C_l(n,n') = \frac{1}{2l+1} \sum_m f_{lmn} f^*_{lmn'} = \mathbb{C}_l$$

$$F_{\alpha\beta} \propto \sum_l \frac{2l+1}{2} \mathrm{Tr}\left[\mathbb{C}_l^{-1} \frac{\partial \mathbb{C}_l}{\partial \Theta_\alpha} \mathbb{C}_l^{-1} \frac{\partial \mathbb{C}_l}{\partial \Theta_\beta}\right]$$

$$\Theta = (\Omega_m, \Omega_b, w_0, w_a, \dots)$$