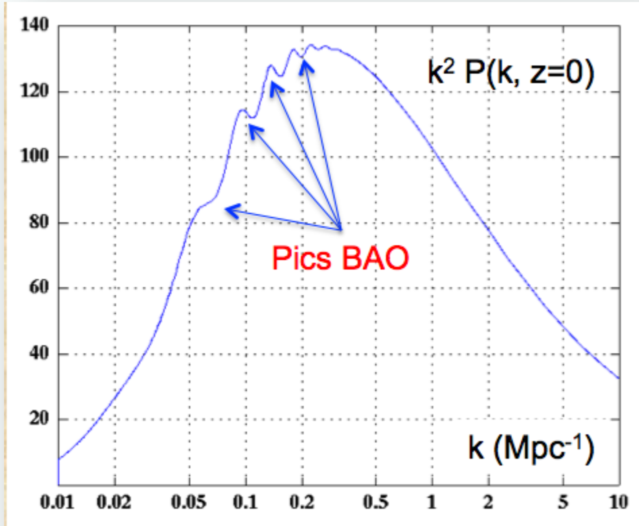
# Angpow

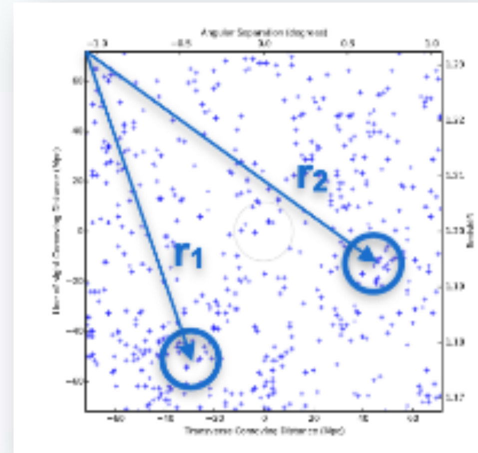
# Du Cartésien au sphérique

J.E Campagne, S. Plaszczynski
J. Neveu

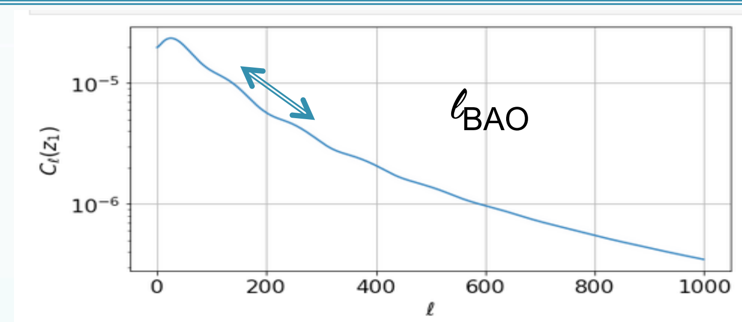
*(Action Dark Energy 19/11/19)*

- ***Angpow: a software for the fast computation of accurate tomographic power spectra*** arXiv:1701.03592 A&A, 602, A72.
- *A direct method to compute the galaxy count angular correlation function including redshift-space distortions* arXiv:1703.02818 ApJ, 845, 28.

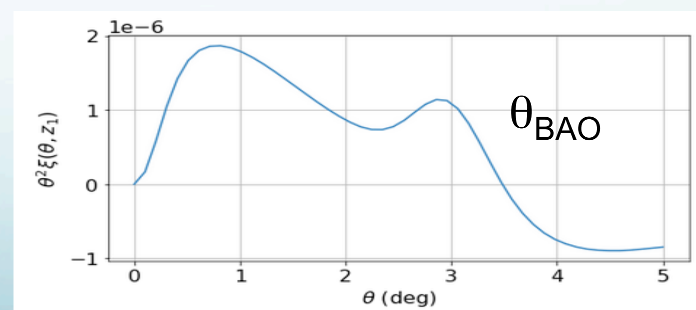


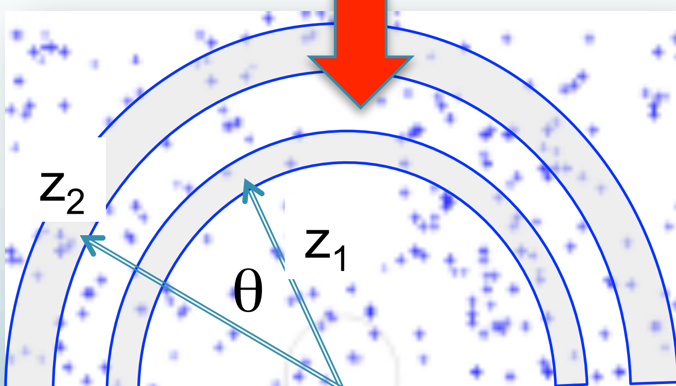




$$\xi(\vec{\mathbf{r}}_1, \vec{\mathbf{r}}_2) = \langle \Delta(\hat{\mathbf{r}}_1, z_1)\Delta(\hat{\mathbf{r}}_2, z_2)\rangle$$




z = 0.90
σ = 0.02

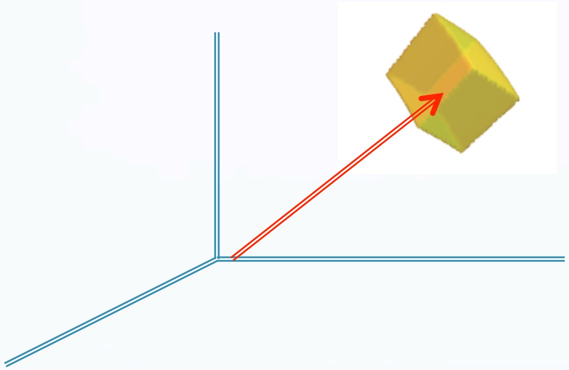



$C_\ell(z_1,z_2)$

$$\xi(\theta, z_1, z_2) = \langle \Delta(z_1, \Omega_1)\Delta(z_2, \Omega_2)\rangle|_{\cos\theta_{12}=\cos\theta}$$

# Galaxy number count



$N(\Omega_r, z)d\Omega_r dz$ # galaxies

$$\Delta(\Omega_r, z) \equiv \frac{N(\Omega_r, z) - \langle N\rangle(z)}{\langle N\rangle(z)} = \frac{\delta\rho}{\rho}(\Omega_r, z) + \frac{\delta Vol}{Vol}(\Omega_r, z)$$

Density Volume

$$\Delta^{(N)}(\mathbf{n}, z, m_*) = D_g(L > \bar{L}_*) + (1+5s)\Phi + \Psi + \frac{1}{\mathcal{H}}\left[\Phi' + \partial_r(\mathbf{V}\cdot\mathbf{n})\right] + \left(\frac{\mathcal{H}'}{\mathcal{H}^2} + \frac{2-5s}{r_S\mathcal{H}} + 5s - f^N_{\rm evo}\right)\left(\Psi + \mathbf{V}\cdot\mathbf{n} + \int_0^{r_S} dr(\Phi' + \Psi')\right) + \frac{2-5s}{2r_S}\int_0^{r_S} dr\left[2 - \frac{r_S - r}{r}\Delta_\Omega\right](\Phi + \Psi) .$$

$$D_g \to bD + (f^N_{\rm evo} - 3)\mathcal{H}V/k - 3\Phi$$

- Green line: Density
- Magenta line: Potential (ou « relativistic corr. »)
- Red line: RSD (Kaiser)
- Red dashed line: Doppler
- Blue line: lensing

Si pas dévolution en ft de la Luminosité:
*$s(z) = f_{evo}(z) = 0$*

Ex. C. Bonvin R. Durrer (2011), Di Dio Montarini Lesgourgues Durrer (2016)

Fourier

$$\Delta(\Omega_r, z) = D(\vec{r}(z), t(z)) - \frac{1}{\mathcal{H}(z)}\partial_r(\vec{V}(\vec{r}(z), t(z)).\hat{r}) - \frac{\alpha(z)}{r(z)}\vec{V}(\vec{r}(z), t(z)).\hat{r}$$

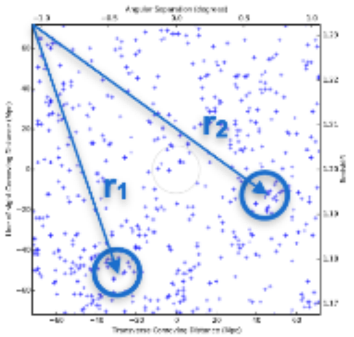
magnification

Densité fluc. | RSD - Kayser | RSD – Doppler like

Kayser | Doppler

$$\Delta(\vec{k}, z) = F_{in}(\vec{k})T_D(k, z)\left[1 - f_z(z)\mu_k^2 + i\frac{\mathcal{H}(z)\alpha(z)f_z(z)}{kr(z)}\mu_k\right]$$

$$\mu_k = \hat{k}.\hat{r} = \cos\theta_k$$



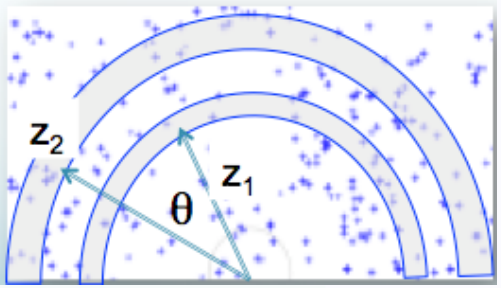

$$\xi(\vec{r}_1, \vec{r}_2) = \langle\Delta(\hat{r}_1, z_1)\Delta(\hat{r}_2, z_2)\rangle$$

$$\xi(\vec{r}_1, \vec{r}_2) = \frac{1}{(2\pi)^{2/3}}\int d\vec{k}\, e^{-i\vec{k}.(\vec{r}_2-\vec{r}_1)}P_{\rm in}(k)T_D(k, z_1)T_D(k, z_2)$$
$$\times\left\{1 - f_z(z_1)(\hat{k}.\hat{r}_1)^2 + i\frac{g(z_1)}{kr(z_1)}(\hat{k}.\hat{r}_1)\right\}\left\{1 - f_z(z_2)(\hat{k}.\hat{r}_2)^2 - i\frac{g(z_2)}{kr(z_2)}(\hat{k}.\hat{r}_2)\right\}$$

Legendre polynomial $\mu_{k1}$ & $\mu_{k2}$, moment studies

Ex. Montanari, F. & Durrer, R. 2012, Phys. Rev. D, 86, 063503

# Spherical Harmonic - Bessel




$$\Delta(\mathbf{n}, z) = \sum_{\ell=0}^{\infty} a_{\ell m}(z) Y_{\ell m}(\mathbf{n})$$

Sph. Bessel

$$a_{\ell m}(z) = i^{\ell} \sqrt{\frac{2}{\pi}} \int d\mathbf{k}\, \Delta(\mathbf{k}, z) j_{\ell}(kr(z)) Y^{*}_{\ell m}(\Omega_k)$$

$$(\mathbf{k}.\mathbf{n})^n \leftrightarrow j_{\ell}^{(n)}(kr)$$

$$a_{\ell m}(z) = i^{\ell} \sqrt{\frac{2}{\pi}} \int d\mathbf{k} \Psi_{in}(\mathbf{k}) D(k, z) \times \left[ j_{\ell}(kr(z)) - f_a(z) j_{\ell}''(kr(z)) - \frac{g(z)}{kr(z)} j_{\ell}'(kr(z)) \right] Y^{*}_{\ell m}(\Omega_k).$$

Density

Kaiser

Doppler-like

+ Magnification...

$$C_{\ell}(z_1, z_2) = \langle a_{\ell m}(z_1) a^{*}_{\ell m}(z_2) \rangle = G(z_1) G(z_2) \frac{2}{\pi} \int \mathrm{d}k\, k^2\, P|_{z=0}(k) \times \left\{ j_{\ell}(kr_1) - f_a(z_1) j_{\ell}''(kr_1) \right\} \left\{ j_{\ell}(kr_2) - f_a(z_2) j_{\ell}''(kr_2) \right\}.$$

... ...

# Angpow

$$C_\ell^{\text{thick}}(z_1, z_2; \sigma_1, \sigma_2) = \frac{2}{\pi} \iint_0^\infty \mathrm{d}z\mathrm{d}z' \, W_1(z; z_1, \sigma_1) W_2(z'; z_2, \sigma_2) \int_0^\infty \mathrm{d}k \, k^2 \, P(k) \Delta_\ell(z, k) \Delta_\ell(z', k)$$

Selection function

$$C_\ell^{\text{obs}}(z_1, z_2) \approx \sum_{p=0}^{N_k - 1} \sum_{i=0}^{N_{z_1}} \sum_{j=0}^{N_{z_2}} w_i w_j W(z_i, z_1) W(z_j, z_2) I_\ell(k_p, k_{p+1}; z_i, z_j).$$

$$\int_{k_p^\ell}^{k_{p+1}^\ell} \mathrm{d}k \; f_\ell(k; z_i) f_\ell(k; z_j)$$

Calcul de $I_\ell$ :

- No Limber approximation
- Clenshaw-Curtis Quadrature
- « 3C-algo » Fast Chebyshev polynomials multiplication
- Intensive use of DCT-I (FFTW)
- C++/OpenMP

https://gitlab.in2p3.fr/campagne/AngPow

# 3C-algo

$$C_\ell^{\text{thick}}(z_1, z_2) \approx \sum_{i=0}^{N_{z_1}-1} \sum_{j=0}^{N_{z_2}-1} w_i w_j W_1(z_i, z_1) W_2(z_j, z_2) \widehat{P}_\ell(r_i, r_j)$$

$$\widehat{P}_\ell(z_i, z_j) \approx \sum_{p=0}^{N_k-1} I_\ell(k_p^\ell, k_{p+1}^\ell; z_i, z_j)$$

$$I_\ell(k_p^\ell, k_{p+1}^\ell; r_i, r_j) = \int_{k_p^\ell}^{k_{p+1}^\ell} \mathrm{d}k \, f_\ell(k; z_i) f_\ell(k; z_j)$$

Clenshaw-Curtis quadrature

$$I \approx \sum_{k=0}^{N_{cc}} w_k f(x_k) g_(x_k) = \sum_{k=0}^{N_{cc}} w_k h(x_k)$$

$$x_k = \cos k\pi / N_{cc}$$

$$(k = 0, \ldots, N_{cc}).$$

But for highly oscillatory functions Ncc could be very large

# 3C-algo

$$f_N(x) = \frac{a_0}{2} + \sum_{k=1}^{N-1} a_k T_k(x).$$

Chebyshev polynomials

$$T_k(\cos x) \; = \; \cos(kx).$$

$$\mathbf{a}^{(N)} = \frac{2}{N} \mathbf{C}_N^I \mathbf{f}^{(N)}$$

$$t_\mu^{(N)} \equiv \cos(\mu\pi/N); \; \mu = 0, \ldots, N$$

$$\mathbf{a}^{(N)} \equiv (a_0, \ldots, a_{N-1}, 0)^T$$

$$\mathbf{f}^{(N)} = (f(t_\mu^{(N)}))^T$$

Discrete Cosine Transform of type I (ie. FFTW)

# If 2 functions f & g

f sampling

$$\mathbf{a}^{(N)} = \frac{2}{N}\mathbf{C}_N^I \mathbf{f}^{(N)}$$

g sampling

$$\mathbf{b}^{(M)} = \frac{2}{M}\mathbf{C}_M^I \mathbf{g}^{(M)}$$

Ncc ~ N+M

$$\begin{aligned} \tilde{\mathbf{a}}^{(N_{cc})} &= (\mathbf{a}^{(N)}, 0, \ldots, 0), \\ \tilde{\mathbf{b}}^{(N_{cc})} &= (\mathbf{b}^{(M)}, 0, \ldots, 0). \end{aligned}$$

DCT-I inverse

$$(\mathbf{C}_P^I)^{-1} = (2/P)\mathbf{C}_P^I)$$

Element wise product

$$\mathbf{h}^{(N_{cc})} = (\mathbf{C}_{N_{cc}}^I \, \tilde{\mathbf{a}}^{(N_{cc})}) \odot (\mathbf{C}_{N_{cc}}^I \, \tilde{\mathbf{b}}^{(N_{cc})})$$

h = fxg sampling

$$\frac{2}{N_{cc}}(1, 0, -1/3, 0, -1/15, \ldots)$$

Clenshaw-Curtis wieights

DCT-I

$$\sum_{k=0}^{N_{cc}} w_k h(x_k)$$

# Un peu de détails implémentation

C++ classes

Integrand dans angpow_integrand.h (IntegrandBase)

PowerSpecFile dans angpow_powerspec.h (PowerSpecBase)

$$f_\ell(z,k) = \sqrt{\frac{2}{\pi}} k \sqrt{P(k,z)}\, \widetilde{\Delta}_\ell(z,k)$$
$$= \sqrt{\frac{2}{\pi}} k \sqrt{P(k,z)} \{\delta_D + \delta_{rsd} + \delta_{lens}\}$$

$$\delta_D(k,z) = b\, j_\ell(kr(z))$$
$$\delta_{rsd}(k,z) = f_z(z)\, j''_\ell(kr(z))$$

En fait le biais peut être b(k,z)

Ex. From C. Bonvin arxiv:1409.2224 (2014) Eq38

$$\delta_{lens}(k,z) = \ell(\ell+1)\frac{3(2-5s)}{2}\frac{\Omega_m}{r\sqrt{P(k,z)}\,(d_H k)^2}\int_0^r dr' \left(\frac{r'-r}{r'}\right)(1+z')\sqrt{P(k,z')}\, j_\ell(kr')$$

Lensing n'était pas dans notre papier à A&A

Ici Limber approx.

# Un peu de détails implémentation

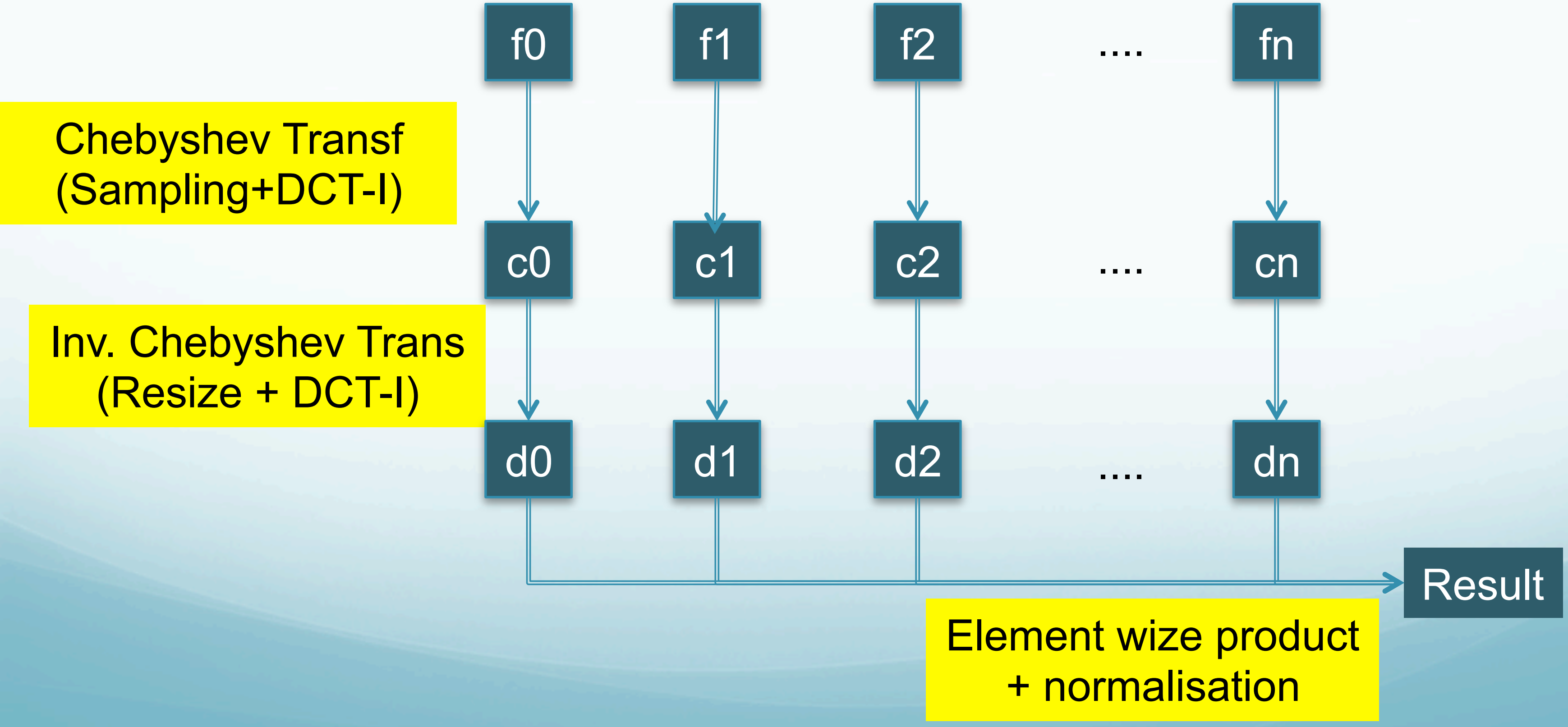
$$C_\ell^{\mathrm{obs}}(z_1, z_2) \approx \sum_{p=0}^{N_k-1} \sum_{i=0}^{N_{z_1}} \sum_{j=0}^{N_{z_2}} \underline{w_i w_j W(z_i, z_1) W(z_j, z_2)} I_\ell(k_p, k_{p+1}; z_i, z_j).$$

1) Les petits w sont des poids de quadratures 1 dim : ex. trapèze, gauss-legendre,...

2) Les grands W sont des fonctions « fenêtres/sélections » voir les classes RadSelectBase et celles dans angpow_radial.h (Dirac, Gauss, Top-Hat et une version en lisant un fichier externe)

# Outlook

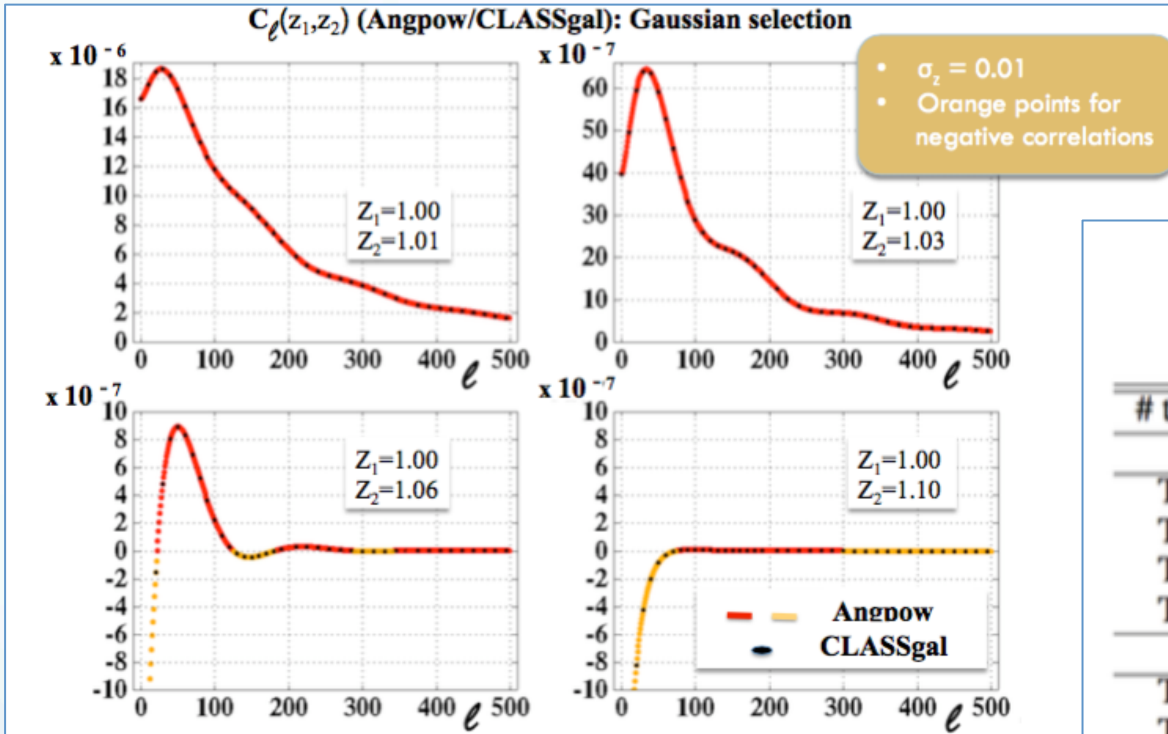
- In fact the 3C integration algorithm may be used elsewhere and I have generalized the decomposition (CheAlgo, CheFunc C++ classes) in case some is interested.



Thanks

# Test Angpow ↔ CLASSgal

Di Dio et al. (2016)



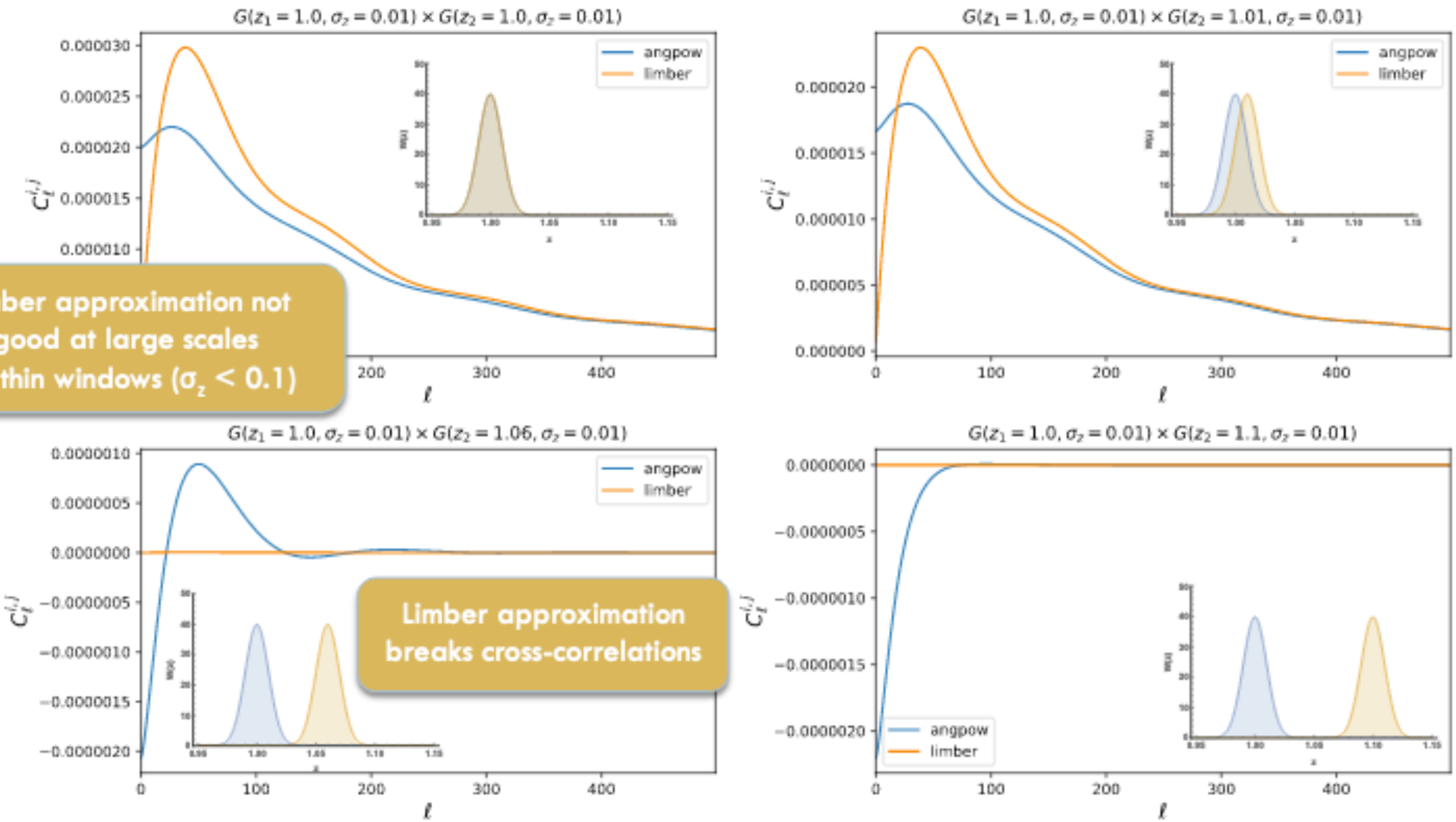

**Computation times in second**

| # threads | 1 | 2 | 4 | 8 | 16 |
|---|---|---|---|---|---|
| Linux/icpc | | | | | |
| Test 1 | 0.38 | 0.21 | 0.13 | 0.09 | 0.08 |
| Test 2 | 0.76 | 0.41 | 0.23 | 0.15 | 0.11 |
| Test 3 | 3.72 | 1.96 | 1.05 | 0.64 | 0.44 |
| Test 4 | 9.97 | 5.25 | 2.79 | 1.60 | 1.01 |
| Linux/gcc | | | | | |
| Test 1 | 0.56 | 0.30 | 0.17 | 0.12 | 0.09 |
| Test 2 | 1.14 | 0.60 | 0.33 | 0.20 | 0.14 |
| Test 3 | 5.01 | 2.59 | 1.38 | 0.81 | 0.50 |
| Test 4 | 13.80 | 7.07 | 3.71 | 2.12 | 1.27 |

CPU time ≈ O(1s) allows further developments with CAMEL (S.P) Campow
Interface with CCL (DESC) (J.N)

# Testing Limber approximation

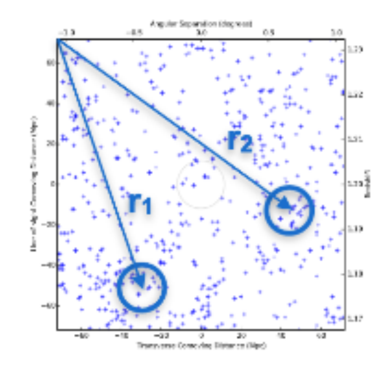
Two Gaussian windows $z_1$=1.0 and $z_2$=1.0, 1.01, 1.06, 1.10 ($\sigma_z$ = 0.01)




Limber approximation not good at large scales for thin windows ($\sigma_z$ < 0.1)

Limber approximation breaks cross-correlations

# Fourier (Cartesian)




Kaiser

Doppler-like

$$\Delta(\mathbf{k}, z) = \Psi_{in}(\mathbf{k}) D(k, z) \left(1 + f_a(z)(\hat{\mathbf{k}}.\,\mathbf{n})^2 - i\,\frac{\alpha(z) f_a(z)\mathcal{H}(z)}{kr(z)}(\hat{\mathbf{k}}.\,\mathbf{n})\right)$$

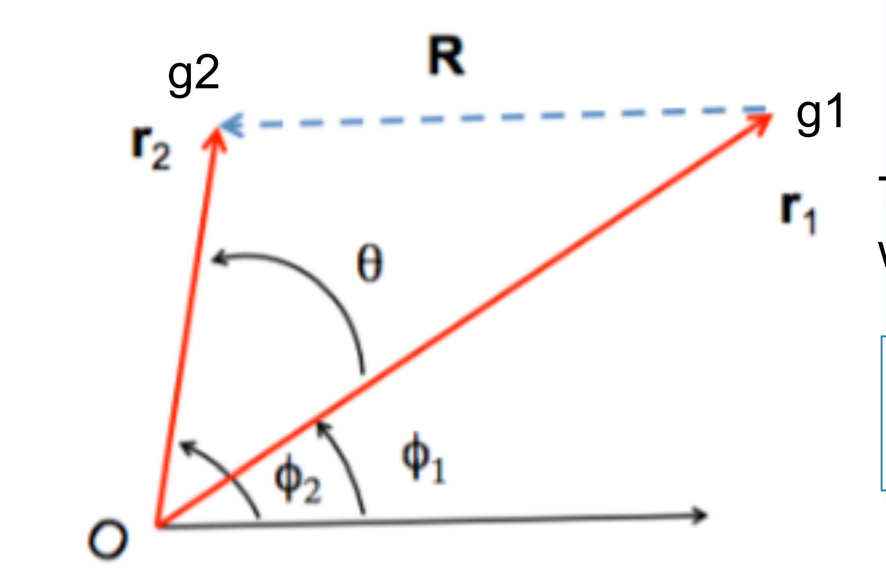
$$\langle \Psi_{\rm in}(\mathbf{k})\Psi_{\rm in}(\mathbf{k}')\rangle = P_{\rm in}(k)\delta(\mathbf{k} - \mathbf{k}')$$

$$P|_{z=0}(k) G(z_1) G(z_2) \approx P_{\rm in}(k) D(k, z_1) D(k, z_2).$$

$$\begin{aligned}
\xi(\mathbf{n}_1, \mathbf{n}_2, z_1, z_2) &= G(z_1)G(z_2)\int \frac{\mathrm{d}\mathbf{k}}{(2\pi)^{2/3}}\, e^{-i\,(kR)\,\hat{\mathbf{k}}.\mathbf{R}}\, P|_{z=0}(k) \\
&\times \left\{1 + \frac{f_a(z_1)}{3} + \frac{2 f_a(z_1)}{3} P_2(\hat{\mathbf{k}}.\,\mathbf{n}_1) - i\,\frac{\alpha(z_1) f_a(z_1)\mathcal{H}(z_1)}{kr(z_1)} P_1(\hat{\mathbf{k}}.\,\mathbf{n}_1)\right\} \\
&\times \left\{1 + \frac{f_a(z_2)}{3} + \frac{2 f_a(z_2)}{3} P_2(\hat{\mathbf{k}}.\,\mathbf{n}_2) + i\,\frac{\alpha(z_2) f_a(2)\mathcal{H}(z_2)}{kr(z_2)} P_1(\hat{\mathbf{k}}.\,\mathbf{n}_2)\right\}.
\end{aligned}$$

Here set b(z)=1 for simplicity

# Closed form (1)




Tripolar spherical harmonics using invariance wrt rotation of the triangle $\{0, g_1, g_2\}$.

$$R(r_1, r_2, \theta) = \sqrt{r_1^2 + r_2^2 - 2 r_1 r_2 \cos\theta}$$

$$\phi_1 = \arcsin(r_2 / R \sin\theta)$$

$$\phi_2 = \phi_1 + \theta = \arcsin(r_1 / R \sin\theta)$$

Flat universe

$$\xi(z_1, z_2, \phi_1, \phi_2) = G(z_1) G(z_2) \times \sum_{m_1 m_2 = 0,1,2} [a_{m_1 m_2} \cos(m_1 \phi_1) \cos(m_2 \phi_2) + b_{m_1 m_2} \sin(m_1 \phi_1) \sin(m_2 \phi_2)]$$

$\xi(\theta, z_1, z_2)$

(Matsubara et al. 2000; Szapudi 2004; Papai & Szapudi 2008; Bertacca et al. 2012; Montanari & Durrer 2012)

# Closed form (1)

$$a_{00} = \left(1 + \frac{2f}{3} + \frac{2f^2}{15}\right)\xi_0^2(x) - \left(\frac{f}{3} + \frac{2f^2}{21}\right)\xi_2^2(x) + \frac{3f^2}{140}\xi_4^2(x)$$

$$a_{02} = a_{20} = \left(\frac{-f}{2} - \frac{3f^2}{14}\right)\xi_2^2(x) + \frac{f^2}{28}\xi_4^2(x)$$

$$a_{22} = \frac{f^2}{15}\xi_0^2(x) - \frac{f^2}{21}\xi_2^2(x) + \frac{19f^2}{140}\xi_4^2(x)$$

$$b_{22} = \frac{f^2}{15}\xi_0^2(x) - \frac{f^2}{21}\xi_2^2(x) - \frac{4f^2}{35}\xi_4^2(x);$$

e.g. here only Kaiser terms
But Doppler-like terms available

$$\xi_l^m(x) = \int \frac{dk}{2\pi^2} k^m j_l(xk) P_0(k)$$

# Closed form (2)

arXiv:1703.02818 ApJ, 845, 28.

$$\xi(\theta, z_1, z_2) = G(z_1)G(z_2)\frac{1}{2\pi^2}\int \mathrm{d}k\, k^2\, P|_{z=0}(k)$$
$$\times \left\{ A(x_1, x_2, \theta) - f_a(z_2)\frac{\partial^2 A}{\partial x_2^2} - f_a(z_1)\frac{\partial^2 A}{\partial x_1^2} + f_a(z_1)f_a(z_2)\frac{\partial^4 A}{\partial x_1^2 \partial x_2^2} \right\}$$

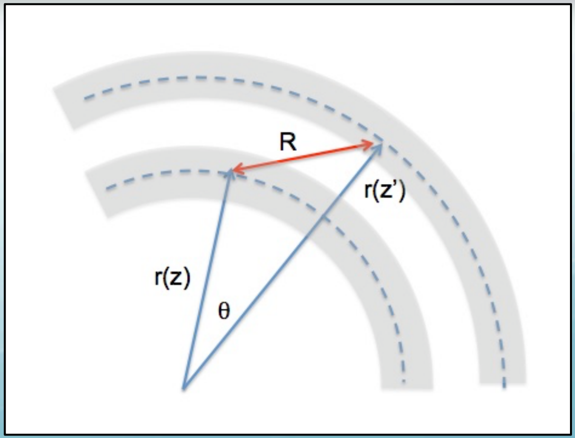
$x_i = k\, r(z_i)$

dens$^2$ — Dens-Kaiser — Kaiser$^2$

$$A(x_1, x_2, \theta) = \sum_{\ell=0}^{\infty}(2\ell+1)j_\ell(x_1)j_\ell(x_2)P_\ell(\cos\theta) = \mathtt{sinc}\,[R(x_1, x_2, \theta)]$$

Addition theorem

$$R^2(r_1, r_2, \theta) = r_1^2 + r_2^2 - 2r_1 r_2 \cos\theta$$
$$= (r_2 - r_1)^2 + 4r_1 r_2 \sin^2(\theta/2)$$



# Closed form (2)

$$\frac{\partial^2 A}{\partial x_1^2} = -\frac{1}{3}j_0(kR) + \frac{1}{6}j_2(kR) + \frac{1}{2}j_2(kR)\cos 2\phi_1$$

$$\left(\frac{\partial R}{\partial x_1}\right)^2\left(\frac{\partial R}{\partial x_2}\right)^2 = \frac{1}{4}\left(1+\cos 2\phi_1\right)\left(1+\cos 2\phi_2\right)$$

$$(kR)\left\{\left(\frac{\partial R}{\partial x_1}\right)^2\frac{\partial^2 R}{\partial x_2^2} + 4\frac{\partial R}{\partial x_1}\frac{\partial R}{\partial x_2}\frac{\partial^2 R}{\partial x_1\partial x_2} + \left(\frac{\partial R}{\partial x_2}\right)^2\frac{\partial^2 R}{\partial x_1^2}\right\} = \frac{1}{2}\left(1-\cos 2\phi_1\cos 2\phi_2 + 2\sin 2\phi_1\sin 2\phi_2\right)$$

$$(kR)^2\left\{2\left(\frac{\partial^2 R}{\partial x_1\partial x_2}\right)^2 + 2\frac{\partial R}{\partial x_1}\frac{\partial^3 R}{\partial x_1\partial x_2^2} + \frac{\partial^2 R}{\partial x_1^2}\frac{\partial^2 R}{\partial x_2^2} + 2\frac{\partial R}{\partial x_2}\frac{\partial^3 R}{\partial x_1^2\partial x_2}\right\} = -\frac{1}{4}\Big(1+3(\cos 2\phi_1+\cos 2\phi_2) - 7\cos 2\phi_1\cos 2\phi_2 + 8\sin 2\phi_1\sin 2\phi_2\Big)$$

$$= -(kR)^3\frac{\partial^4 R}{\partial x_1^2\partial x_2^2}$$

$$\mathrm{sinc}^{(4)}(x) = \frac{1}{5}j_0(x) - \frac{4}{7}j_2(x) + \frac{8}{35}j_4(x)$$

$$\frac{\mathrm{sinc}^{(3)}(x)}{x} = \frac{1}{5}j_0(x) + \frac{1}{7}j_2(x) - \frac{2}{35}j_4(x)$$

$$-\frac{\mathrm{sinc}^{(1)}(x)}{x^3} + \frac{\mathrm{sinc}^{(2)}(x)}{x^2} = \frac{1}{15}j_0(x) + \frac{2}{21}j_2(x) + \frac{1}{35}j_4(x). \quad (30)$$

The closed forms (1) & (2) agree point-to-point

# Extensions (2)

V.n terms (e.g. Doppler)

$$j'_\ell \Longleftrightarrow \frac{\partial}{\partial x_i} \quad \text{A}$$

Lensing

(Bonvin & Durrer 2011; Montanari & Durrer 2015)

$$\Delta_{\text{lens}}(\mathbf{n}, z) = -\frac{1}{r(z)} \int_0^{r(z)} dr' \left( \frac{r(z) - r'}{r'} \right) \Delta_\Omega \left[ (\Psi + \Phi)(r'\mathbf{n}, \tau_0 - r') \right]$$

Laplacian op.

$$a_{\ell m}^{\text{lens}}(z) = i^\ell \sqrt{\frac{2}{\pi}} \int \mathrm{d}\mathbf{k}\, Y^*_{\ell m}(\hat{\mathbf{k}})\, \ell(\ell+1) \times \frac{2}{r(z)} \int_0^{r(z)} dr' \left( \frac{r(z) - r'}{r'} \right) \Psi(\mathbf{k}, z') j_\ell(kr')$$

$$\Psi = \Phi$$

# Extensions (2)

$$\Delta_\theta P_\ell(\cos\theta) = -\ell(\ell+1)P_\ell(\cos\theta),$$

$$S_{\text{lens-cross}} = \sum_{\ell=0}^{\infty}(2\ell+1)P_\ell(\cos\theta)j_\ell^{(n)}(x)\ell(\ell+1)j_\ell(x') = -\left(\frac{\partial^n}{\partial x^n}\Delta_\theta\right)A(\theta,x,x')$$

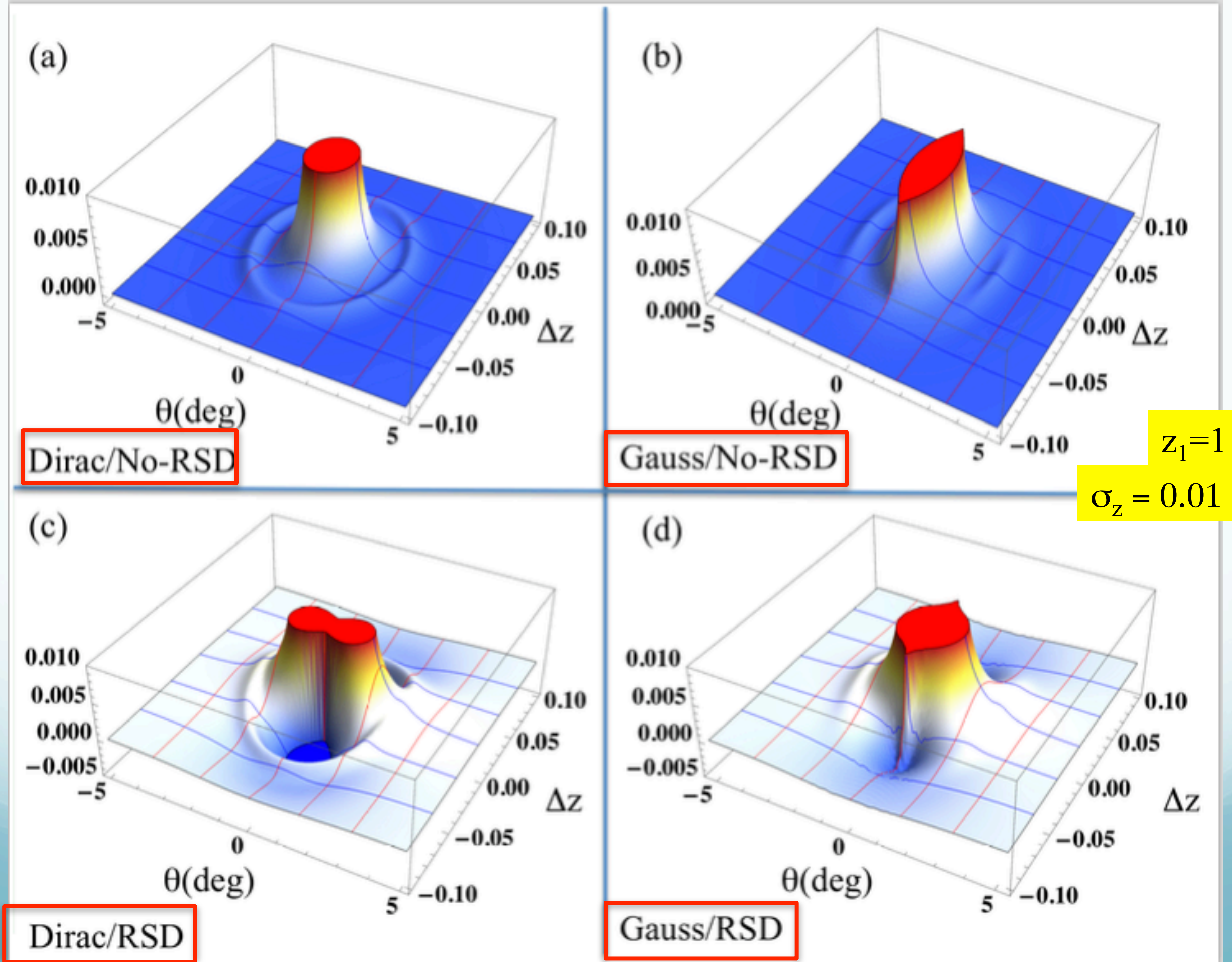
$$S_{\text{lens-lens}} = \sum_{\ell=0}^{\infty}(2\ell+1)P_\ell(\cos\theta)\ell^2(\ell+1)^2 j_\ell(x)j_\ell(x') = \left(\Delta_\theta^2\right)A(\theta,x,x')$$

Numerical integration over thick shells

$$\bar{\xi}(\theta,z_1,z_2) = \iint \mathrm{d}z\,\mathrm{d}z'\,W(z,z_1,\sigma_1)W(z',z_2,\sigma_2)\,\xi(\theta,z,z')$$

$$\xi(\theta,z,z') = \int \mathrm{d}k\,k^2\,P|_{z=0}(k)\,f(k;\theta,z,z')$$

Angpow-like 3C algorithm
Chebyshev Clenshaw Curtis


(a)
0.010
0.005
0.000
−5
0
θ(deg)
5
0.10
0.05
0.00
Δz
−0.05
−0.10
Dirac/No-RSD
(b)
Gauss/No-RSD
(c)
−0.005
Dirac/RSD
(d)
Gauss/RSD
$z_1$=1
$\sigma_z$ = 0.01