Present and future constraints on general theories of Dark Energy

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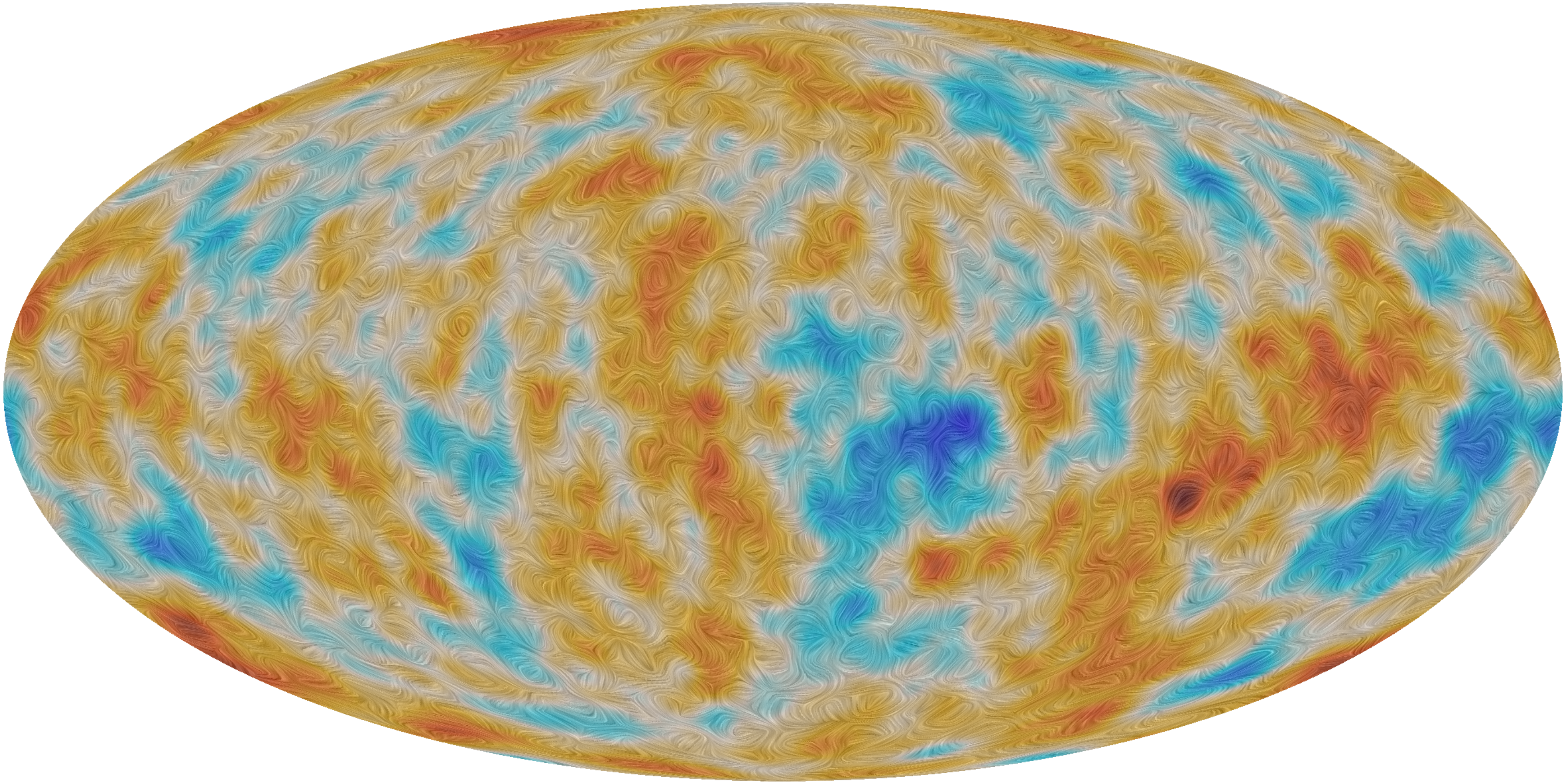
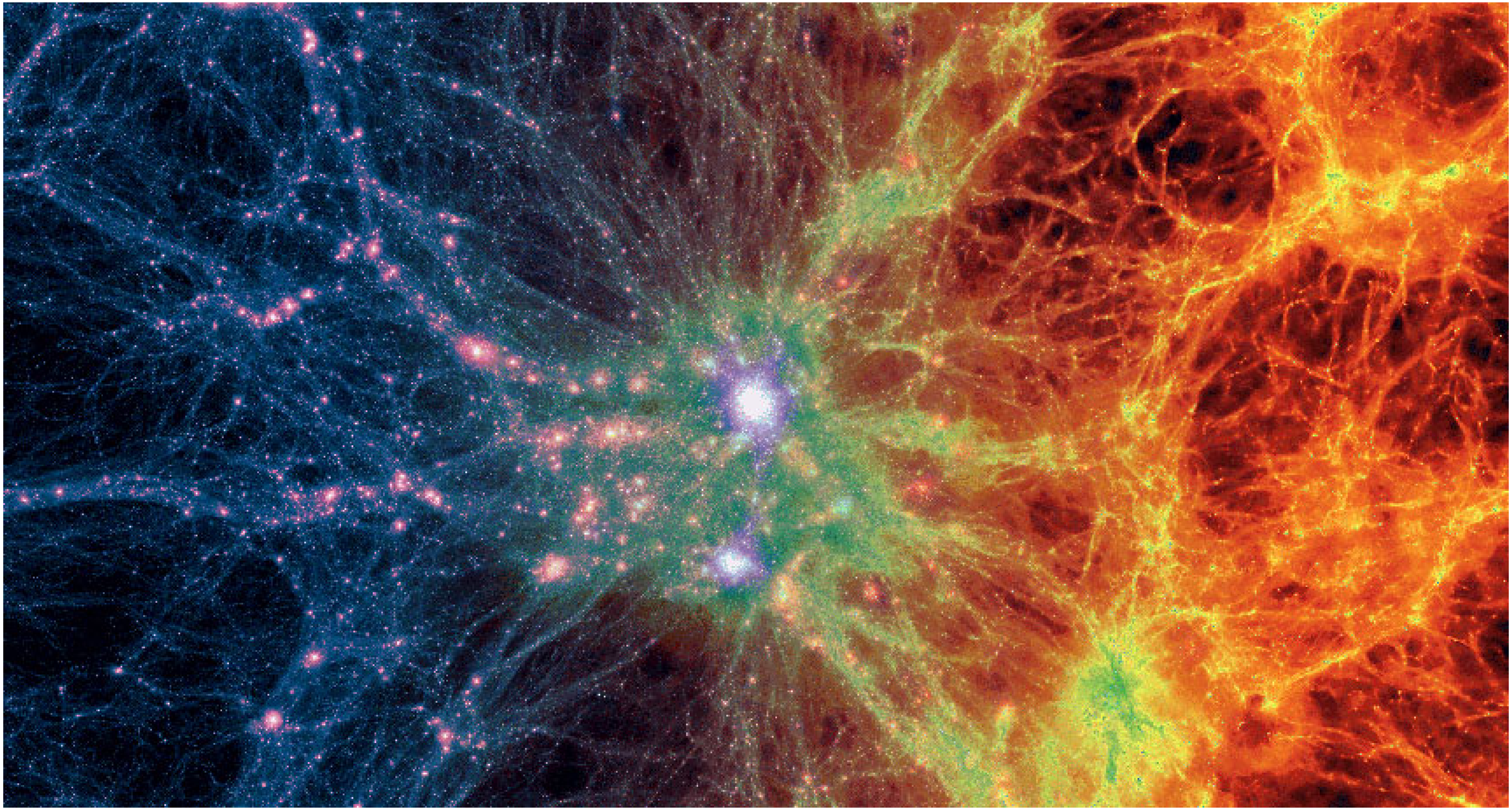
Cosmic Microwave Background



Planck 2015 CMB+polarization.
https://www.cosmos.esa.int/web/planck/picture-gallery

Large Scale Structure



Illustris Simulation: www.nature.com/articles/nature13316

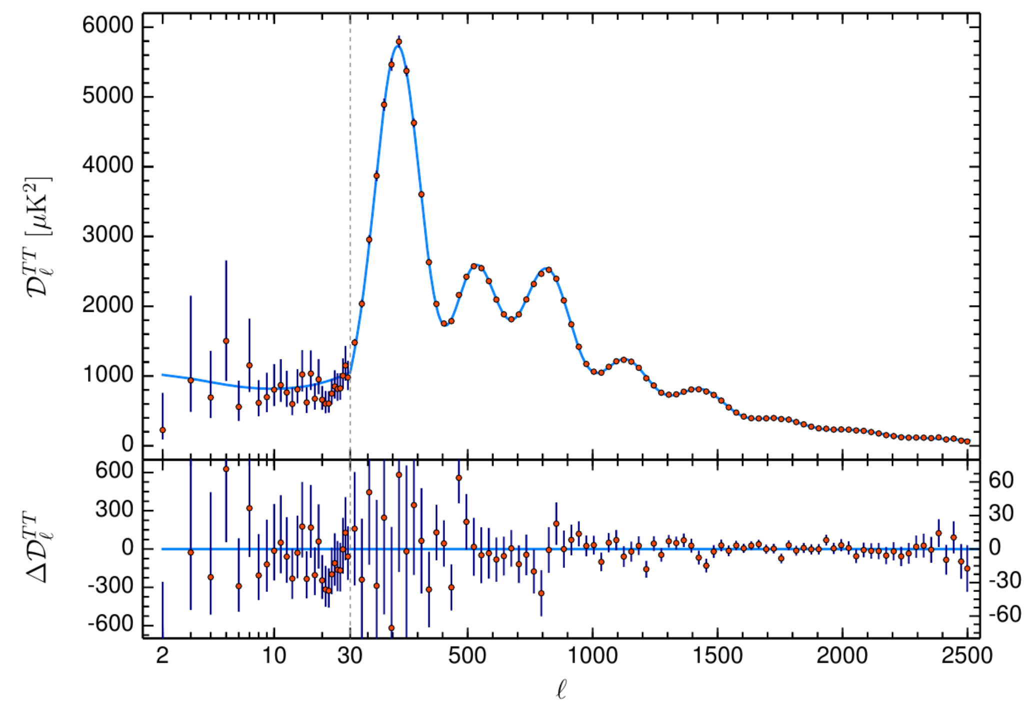
The standard ΛCDM Model

- ΛCDM is still best fit to observations.
- Impressive agreement with data.
- CMB + BAO + WL agree well.

- However some questions remain:

- Cosmological constant problem.

  >60 orders of magnitude wrong (Zeldovich 1967, Weinberg 1989).



$$G_{\mu\nu} + g_{\mu\nu}\Lambda = 8\pi G T_{\mu\nu}$$

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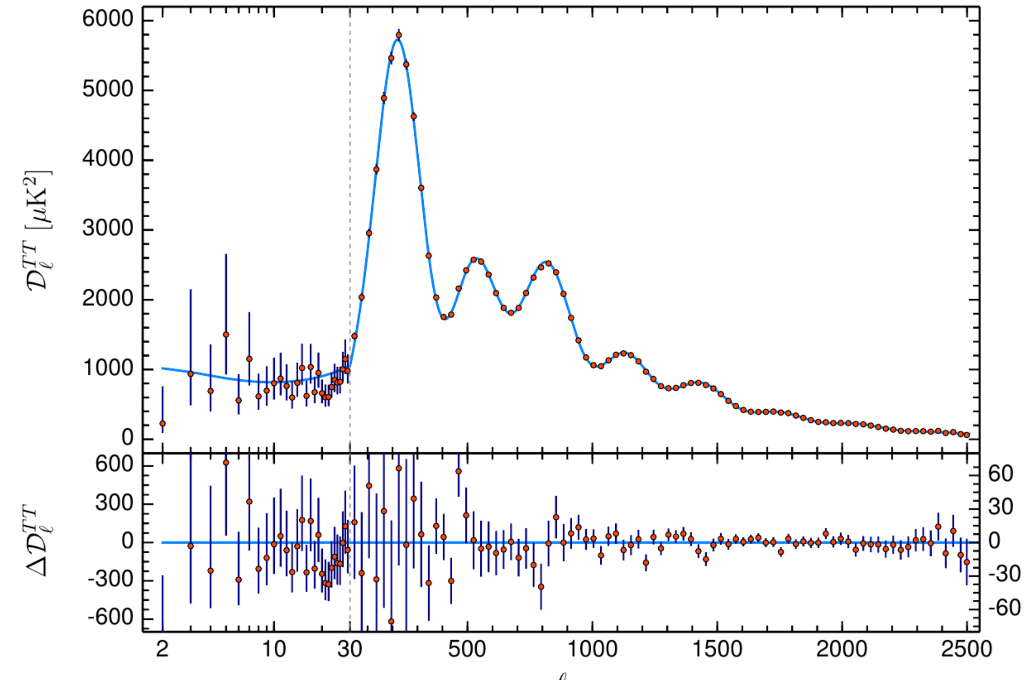
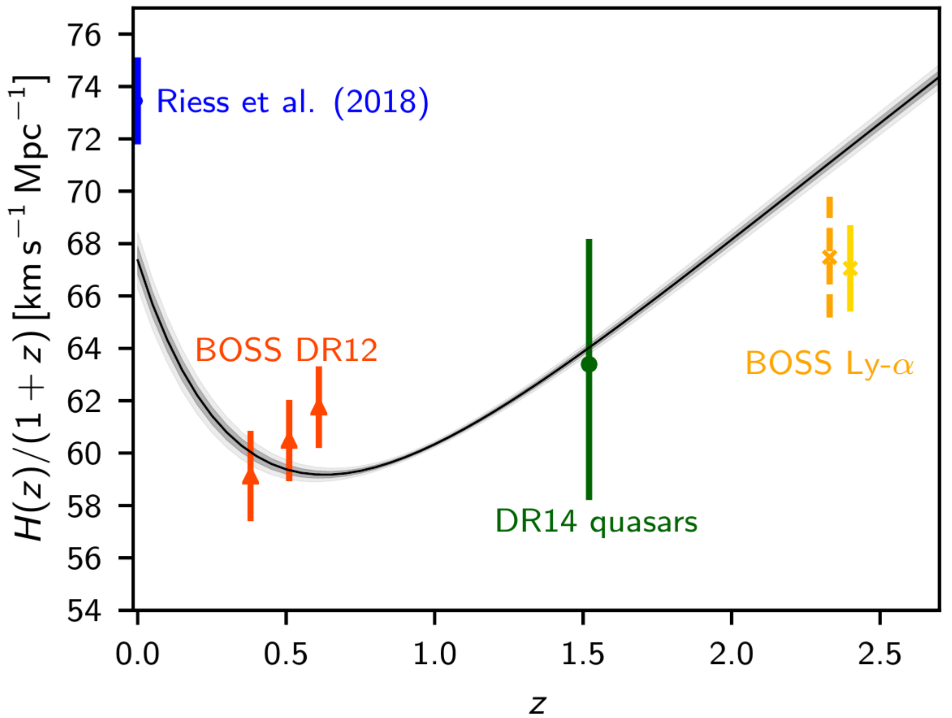
- H0 **tension**
- σ8: amplitude of matter fluctuations.





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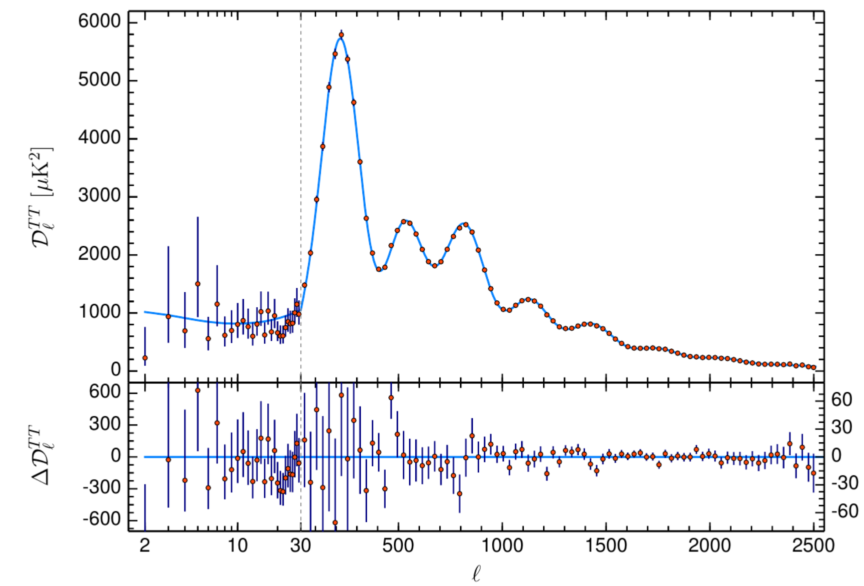
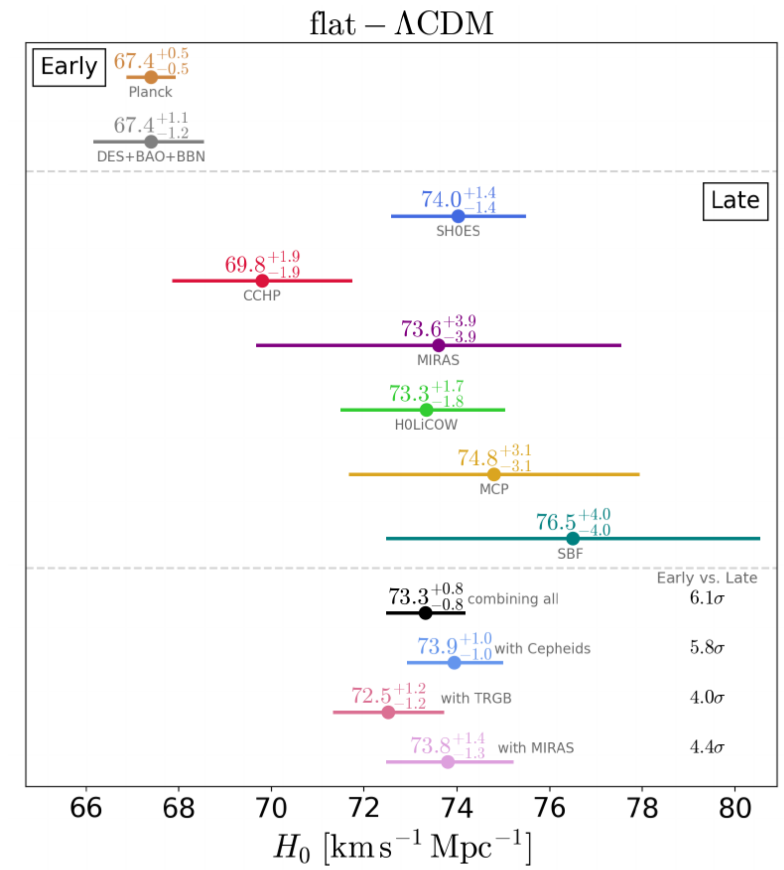
- Growth of structure at nonlinear scales, still unconstrained.

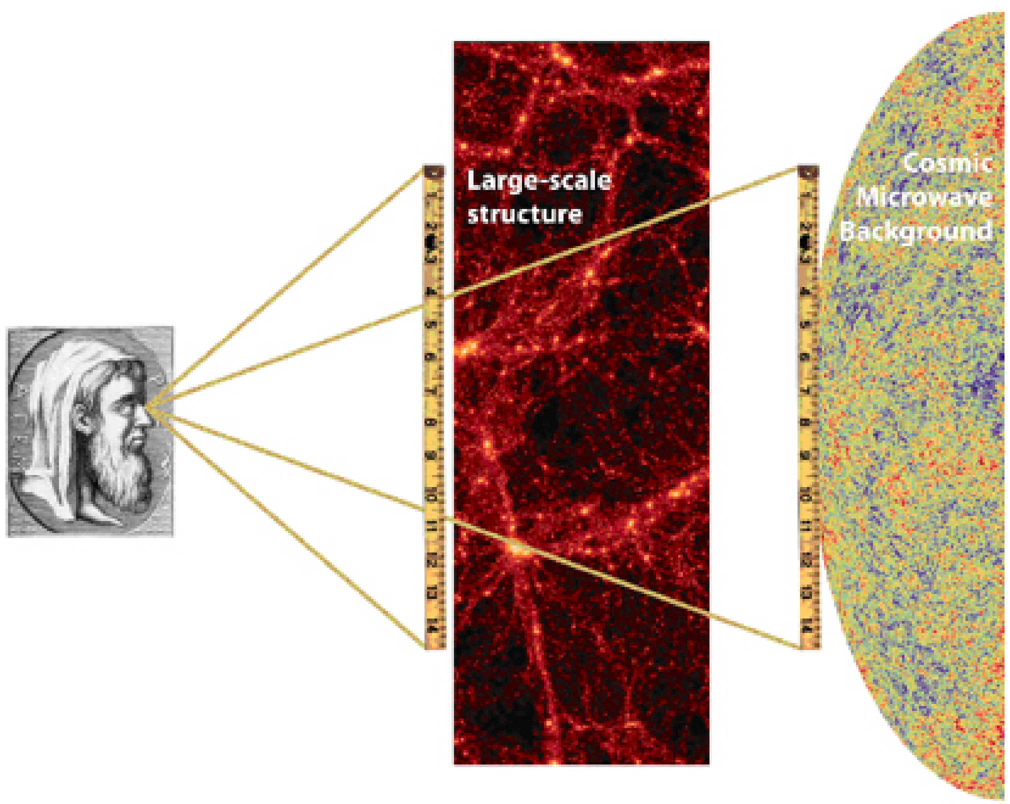




Planck 2018 results: arxiv:1807.06209

Galaxy Clustering and Weak Lensing

- Future data coming in Large Scale Structure (LSS):
- *Euclid, WFIRST, SKA* : GCsp, WL and GCph
- *LSST*: GCph+WL+SNIa (expansion and velocities).
- *DESI*: GCsp



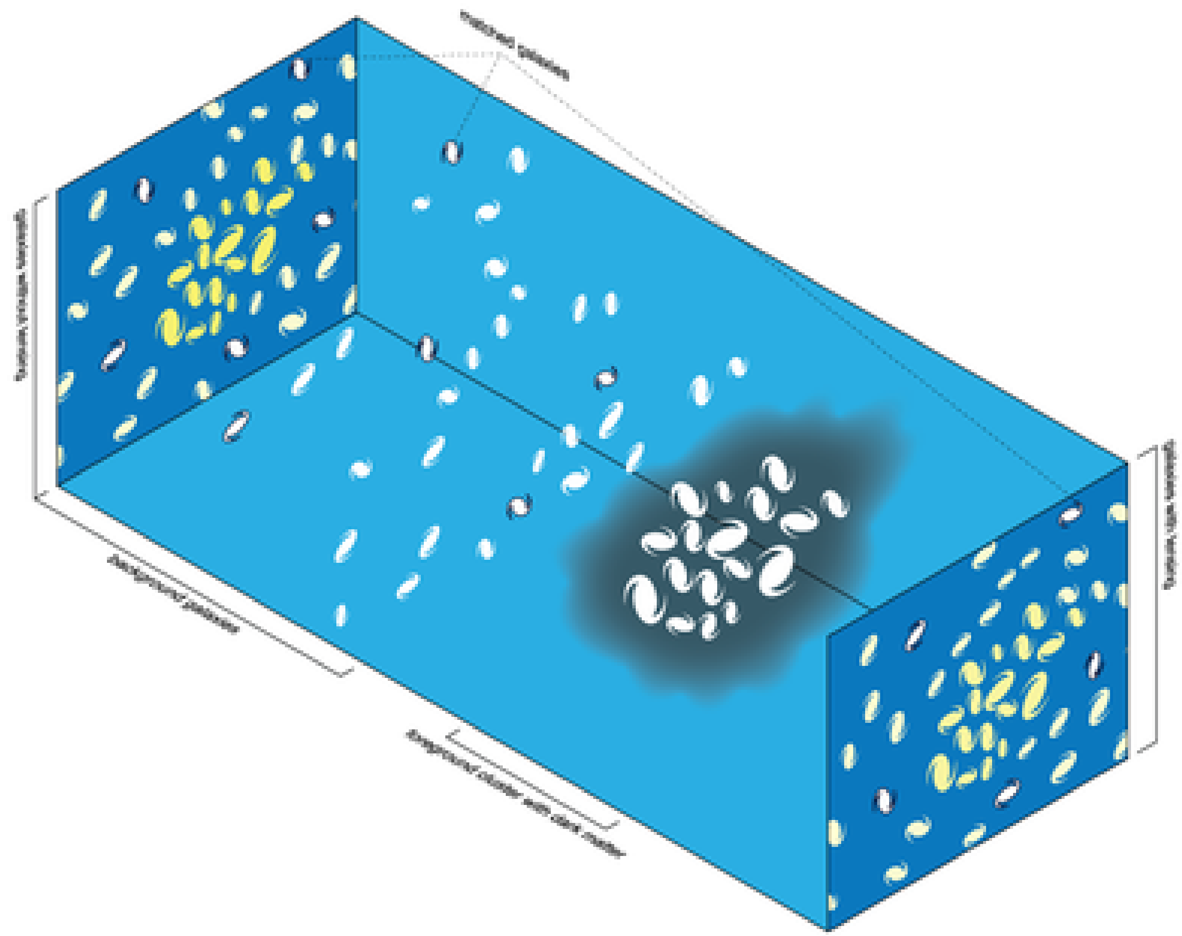


Image credit: http://sci.esa.int/euclid/46681-baryonic-acoustic-oscillations-bao/
Image credit en.wikipedia.org/wiki/Weak_gravitational_lensing

Dark Energy

- Dark Energy: Any dynamical cause for the accelerated expansion:

$$H^2(z) = H_0 \left(\Omega_c (1+z)^3 + \Omega_b (1+z)^3 + \Omega_r (1+z)^4 + \Omega_{DE} \exp \left[\int_0^z \mathrm{d}\tilde{z} \, \frac{3(1+w_{DE}(\tilde{z}))}{1+\tilde{z}} \right] \right)$$

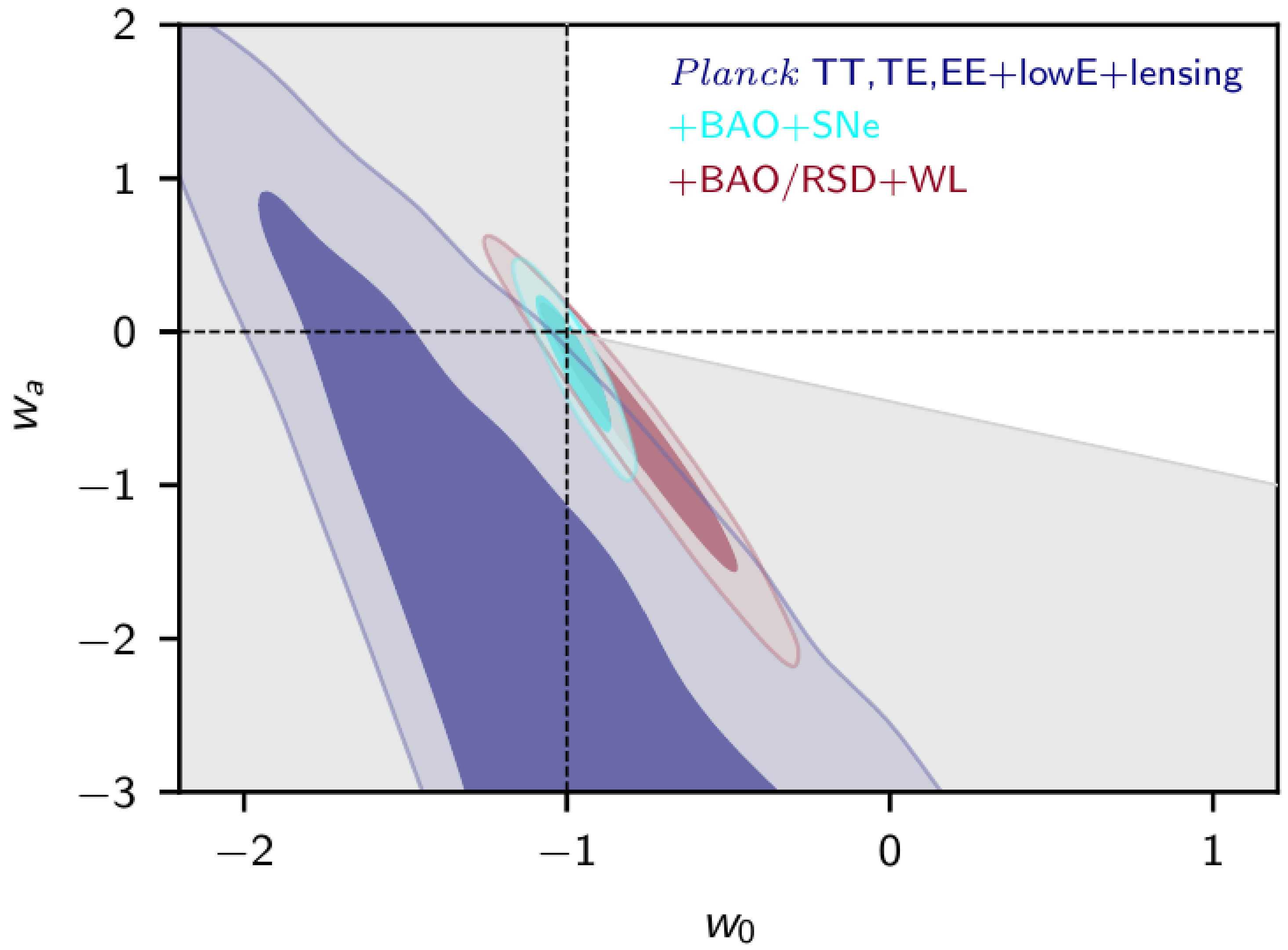
$$w \equiv \frac{p}{\rho} \qquad \Omega_i(t) = \frac{\rho_i(t)}{\rho_{cr}(t)}$$

- Cosmological constant: $w_{DE}(z) = -1$
- To test with observations, the most common parametrization is:

$$w(a) = w_0 + (1-a) w_a$$

Dark Energy

- Current bounds from CMB and LSS alone are still large
- O(1) for w0 combining Planck+GC+WL+SNIa



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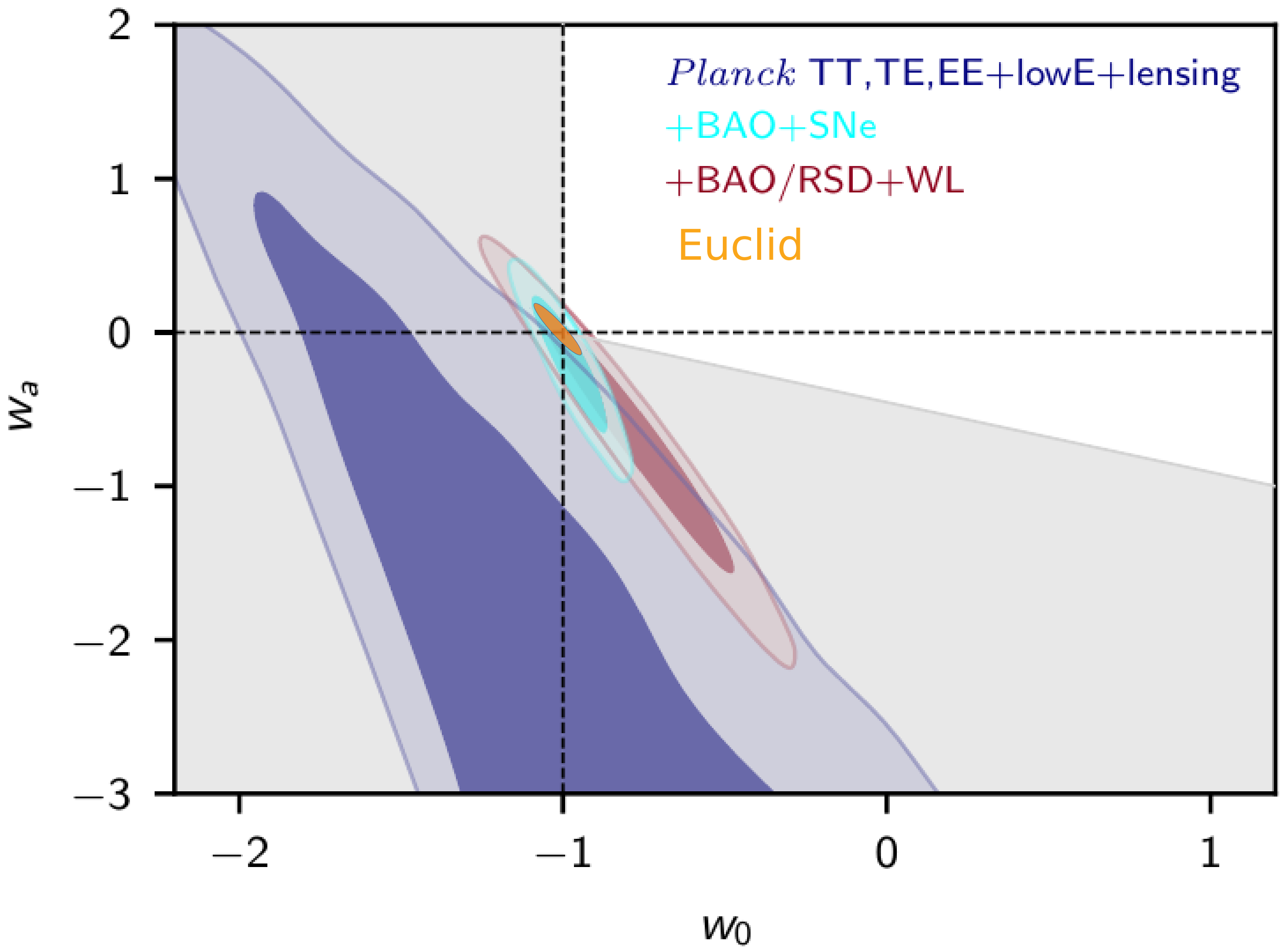
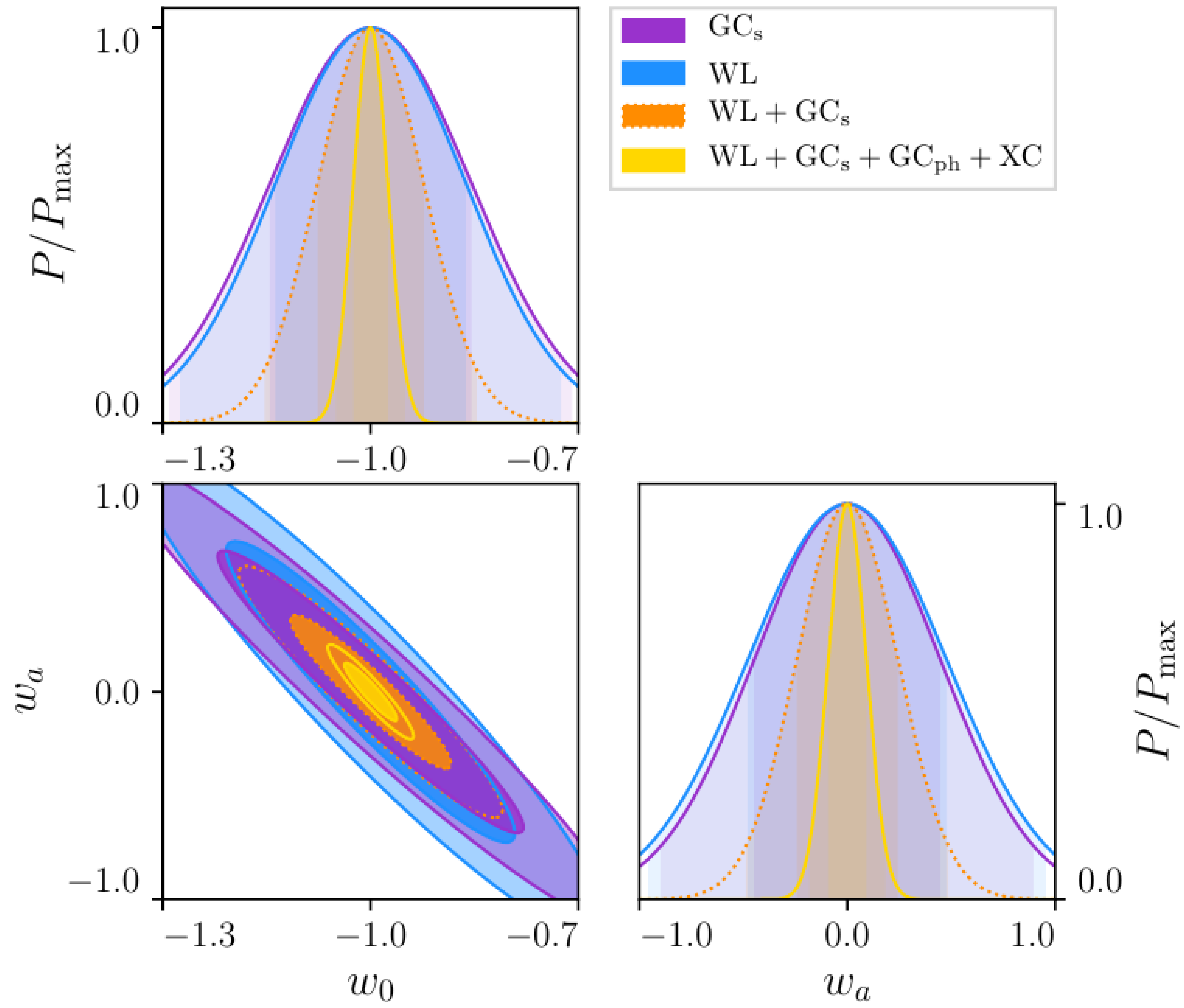


Euclid IST:Forecasts

- Main **goal** of future LSS surveys:
- Bring w0-wa errors, down to percent level.

- IST:F forecasts on Euclid, recently published.
- 10 codes, 3 years.

- GCsp+WL+GCph+XC
- 2% on Euclid alone, for w0waCDM flat.



Euclid preparation: VII. Forecast validation for Euclid cosmological probes. arXiv:1910.09273

Dark Energy and Modified Gravity

GR (LCDM) $\longrightarrow$ $$S = \frac{1}{16\pi G}\int \mathrm{d}x^4 \sqrt{-g}\,(R - 2\Lambda + \mathcal{L}_m)$$

Variety of models with a single extra DoF:

f(R)

K-essence

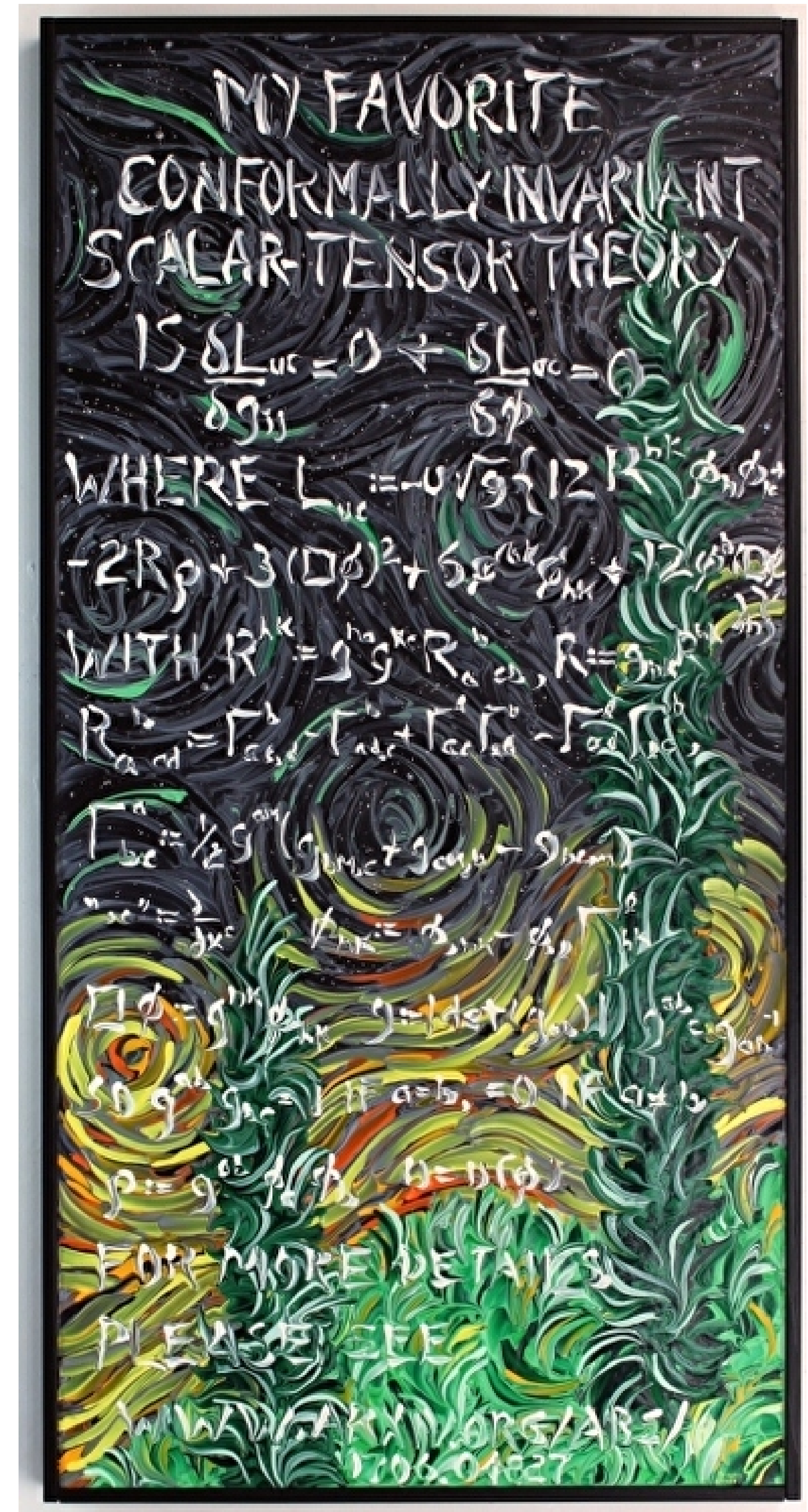
$$\mathcal{S} = \int \mathrm{d}^4x\,\sqrt{-g}\left[\frac{1}{2}F(\varphi, R) - \frac{1}{2}K(\varphi)g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - U(\varphi)\right]$$
$$-\int \mathrm{d}^4x\,\sqrt{-g}\mathcal{L}^i_m(g_{\mu\nu}, \Psi^i_m \zeta^i(\varphi)) \quad ,$$

Quintessence

DM-DE coupling,
Neutrino coupling

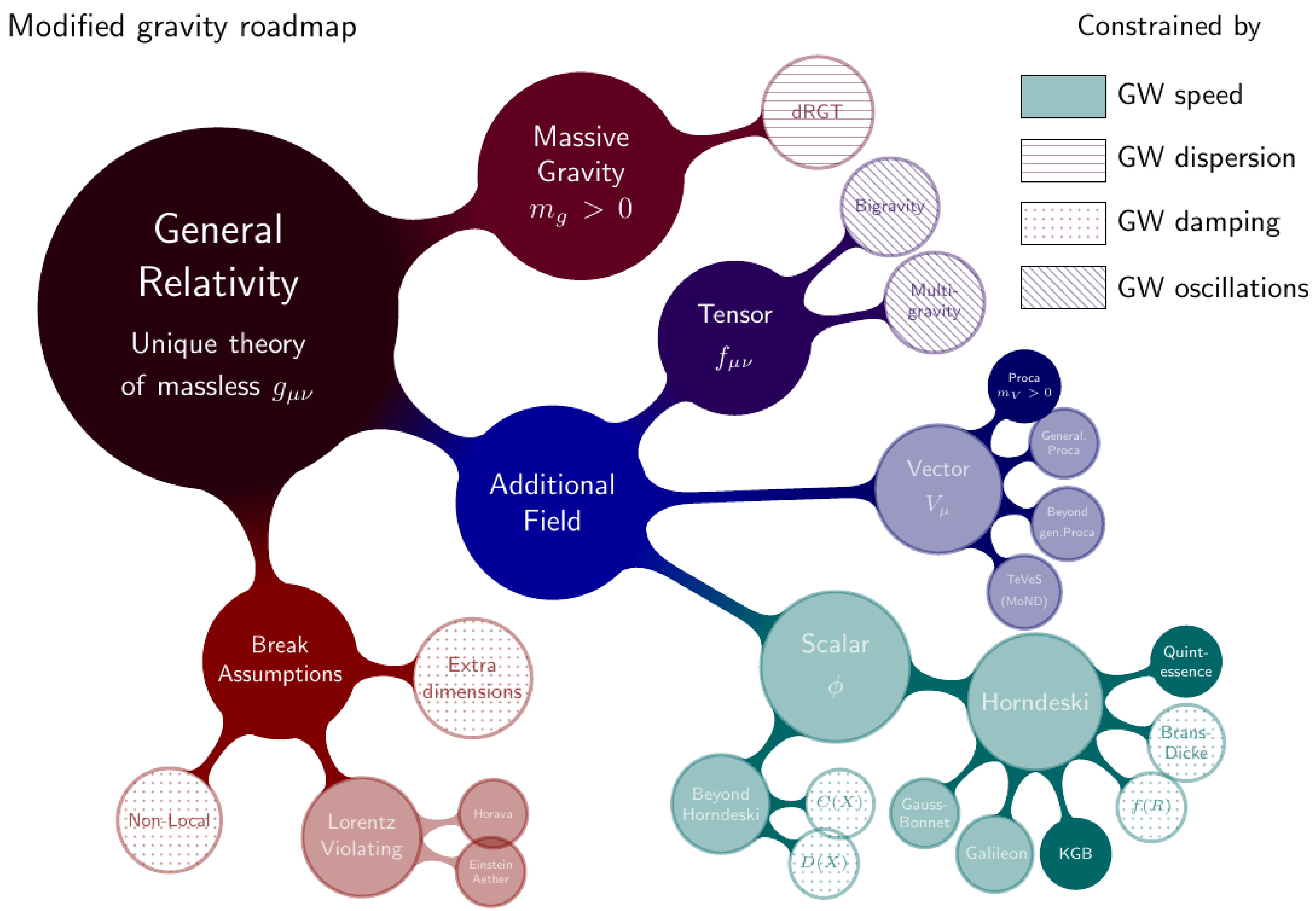
*With many more terms we get the most general stable theory with one extra scalar field and second order eqns. of motion: Horndeski Theory (1974)

Gregory Horndeski



https://horndeskicontemporary.com

Other assumptions can be broken



Taken from: Ezquiaga and Zumalacarregui, arXiv: 1807.09241

Phenomenological Description of MG

• Linearized scalar metric perturbations, two independent scalar potentials:

$$ds^2 = a^2(\tau)\left(-(1+2\Psi)d\tau^2 + (B_{,i} + S_i)d\tau dx^i\right.$$
$$\left.-(-2\Phi\delta_{ij} + 2E_{,ij} + F_{i,j} + F_{j,i} + h_{ij})dx^i dx^j\right)$$

• Modified Gravity (MG) “useful” definition: clustering of DE component, presence of anisotropic stress and/or fifth forces.

$$-k^2(\Phi(a,k) + \Psi(a,k)) \equiv 8\pi G a^2 \Sigma(a,k)\rho(a)\delta(a,k)$$
$$-k^2\Psi(a,k) \equiv 4\pi G a^2 \mu(a,k)\rho(a)\Delta(a,k)$$
$$\eta(a,k) \equiv \Phi(a,k)/\Psi(a,k) \quad .$$

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Lensing

$$-k^2(\Phi(a,k) + \Psi(a,k)) \equiv 8\pi G a^2 \Sigma(a,k)\rho(a)\delta(a,k)$$

Clustering

$$-k^2\Psi(a,k) \equiv 4\pi G a^2 \mu(a,k)\rho(a)\Delta(a,k)$$
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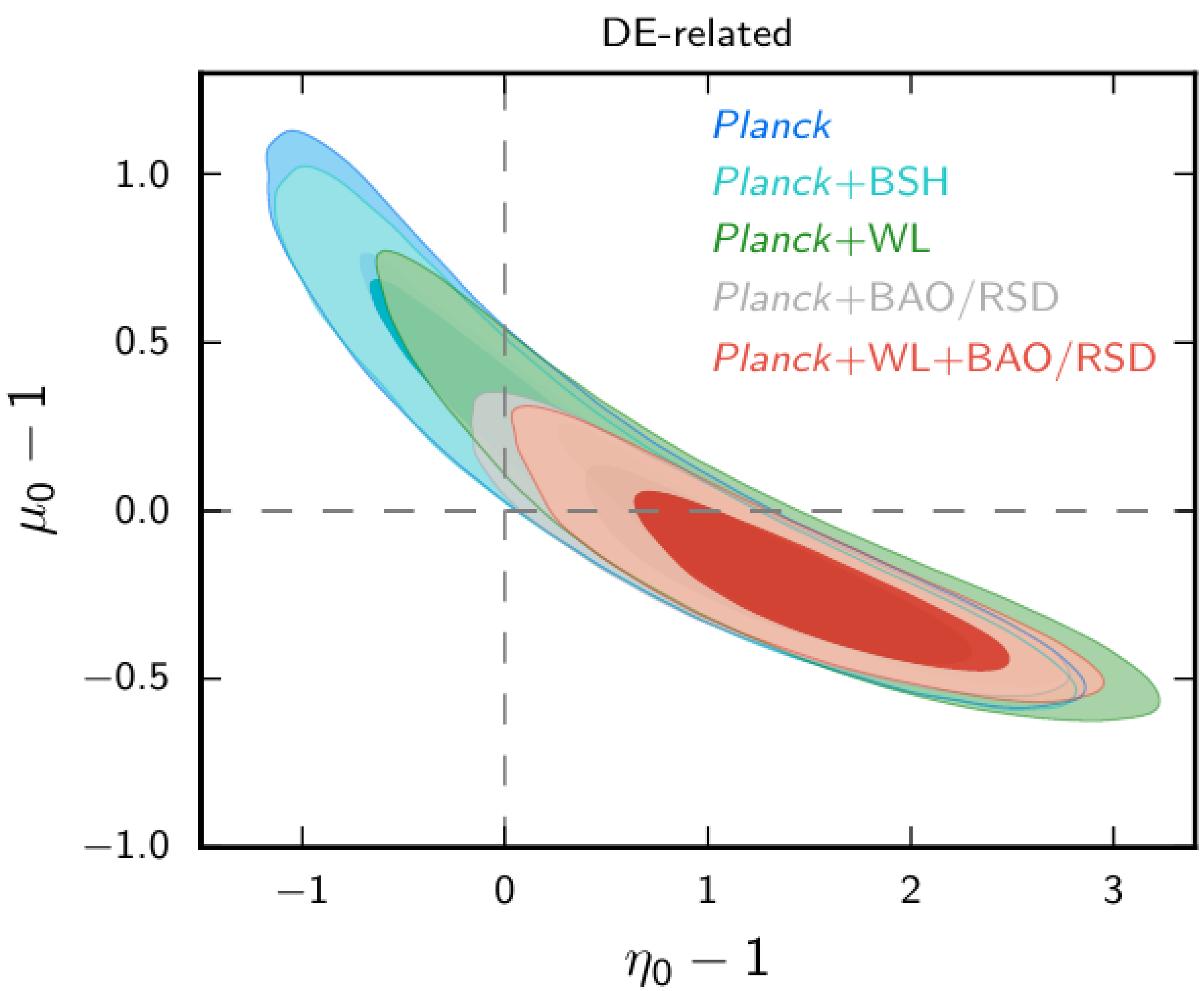
Phenomenological parameters

- Late-time parametrization:



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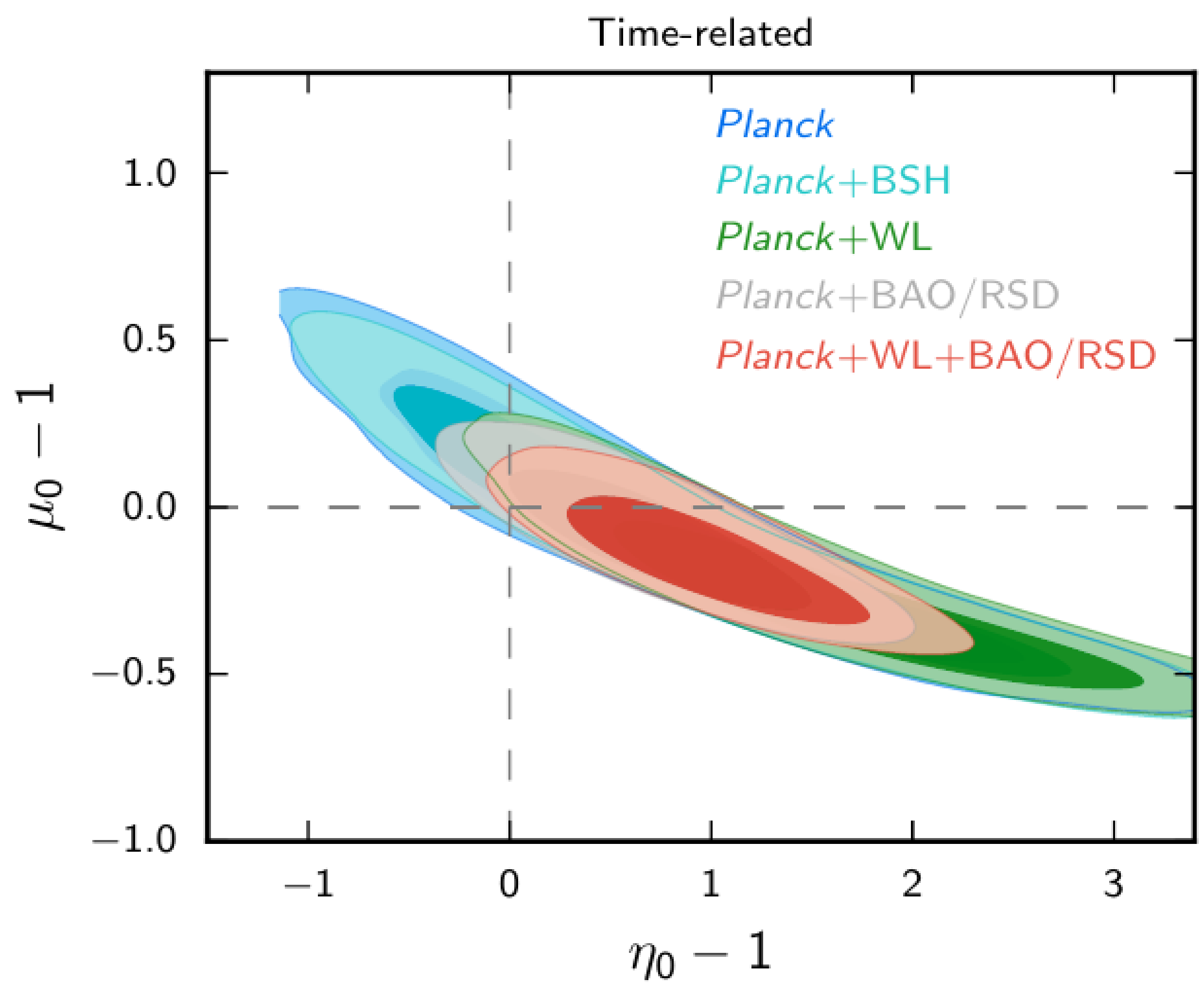


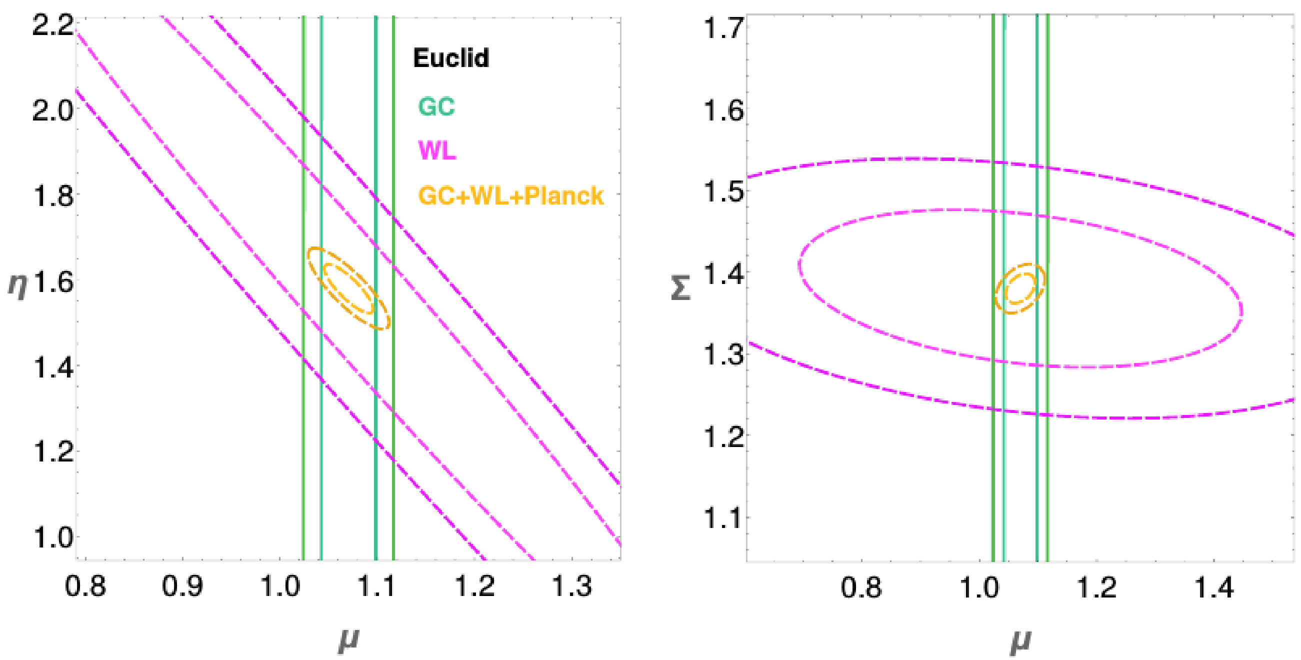
- Constraints are roughly ~O(1).
- Depend on parametrization and inclusion of scale-depence.

Planck 2015 results. XIV. Dark energy and modified gravity. arXiv: 1502.01590

The problem of parametrization

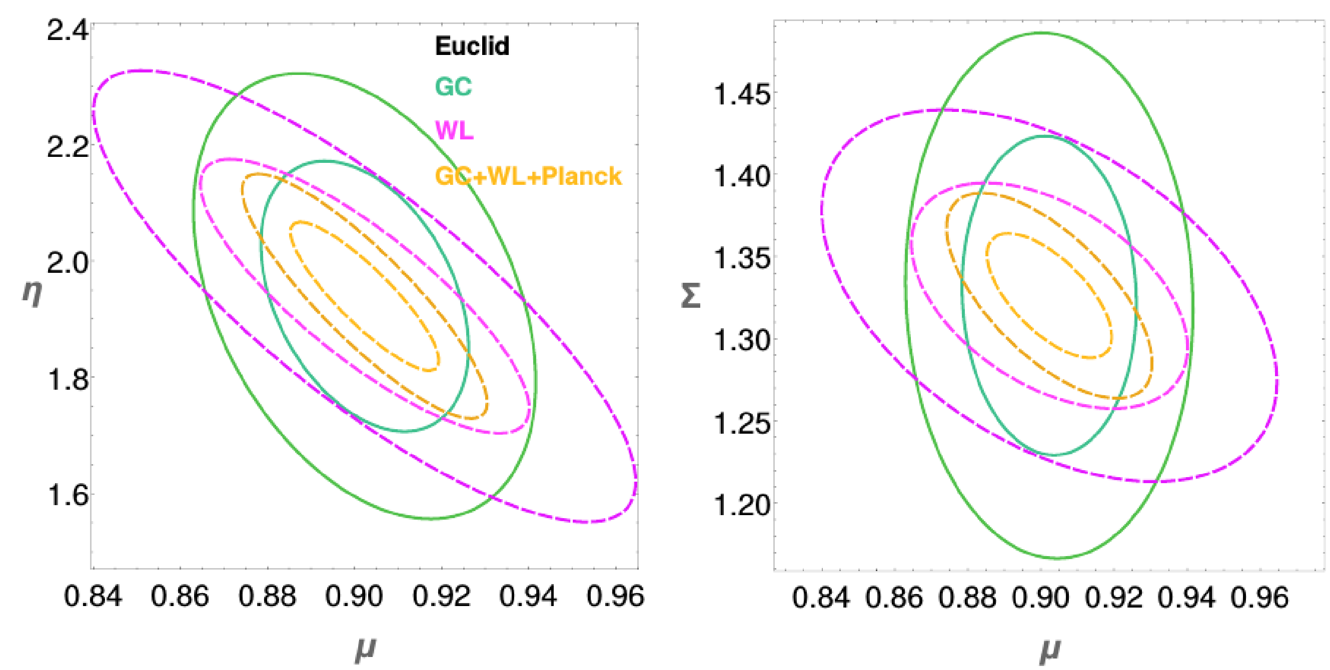
- Late-time parametrization:

$$\mu(a,k) \equiv 1 + E_{11}\Omega_{\rm DE}(a)$$
$$\eta(a,k) \equiv 1 + E_{22}\Omega_{\rm DE}(a)$$



- Early-time parametrization:

$$\mu(a,k) \equiv 1 + E_{11} + E_{12}(1-a)$$
$$\eta(a,k) \equiv 1 + E_{21} + E_{22}(1-a)$$



- Results are strongly parametrization-dependent.

The problem of parametrization

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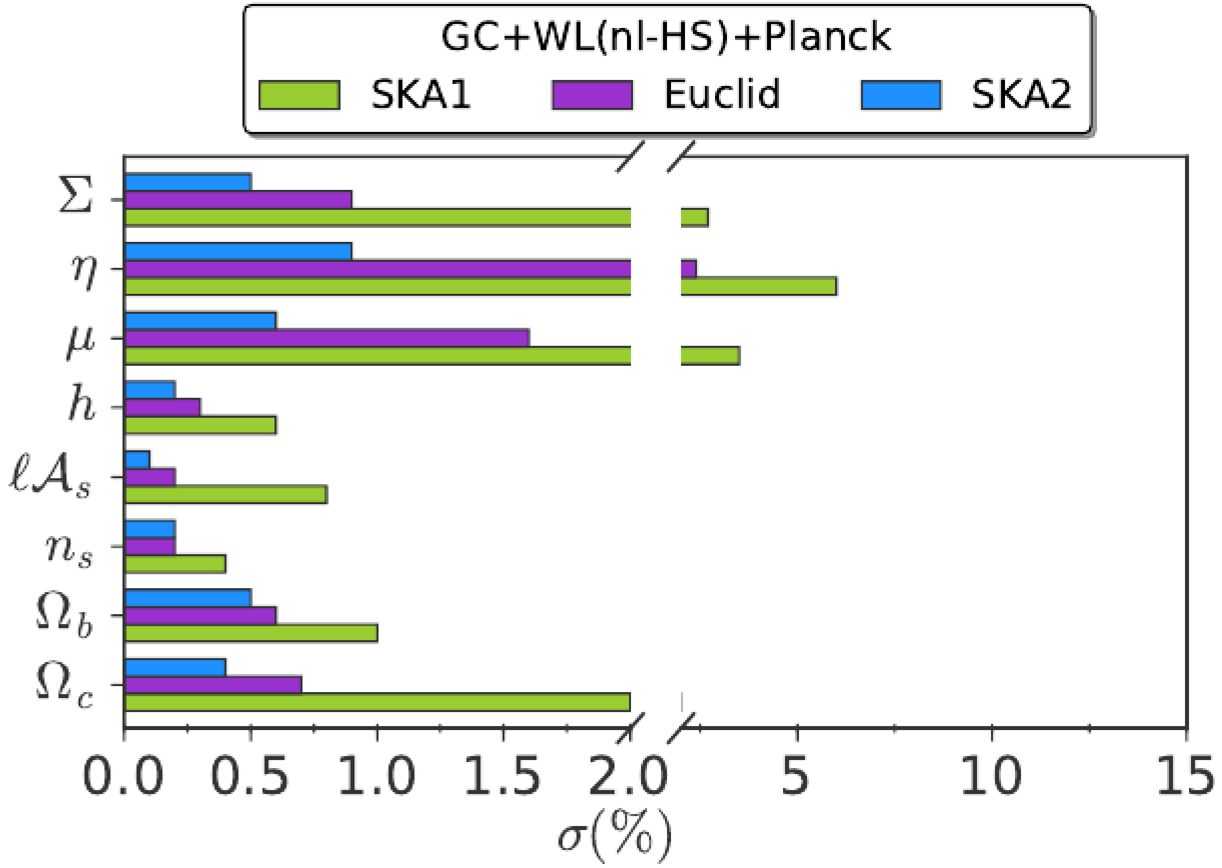

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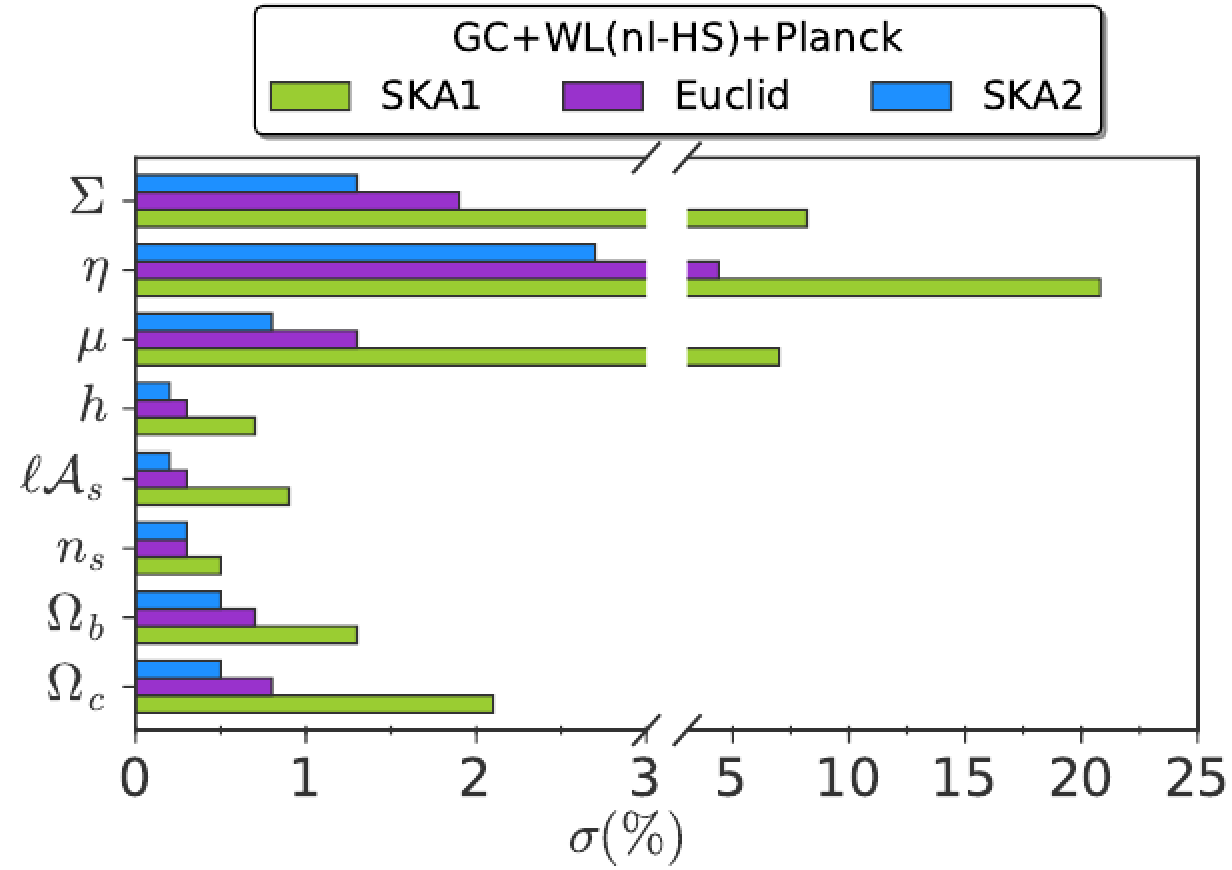
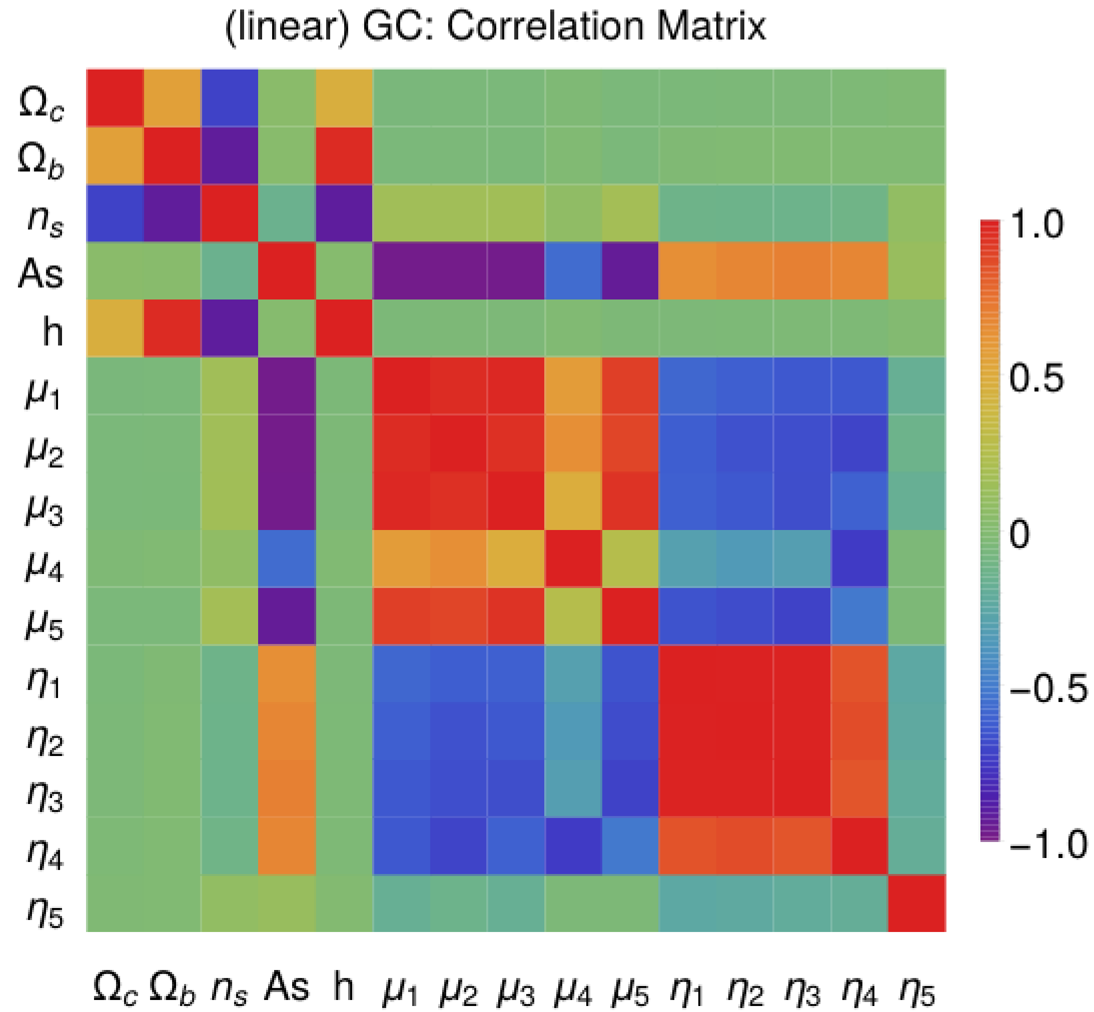
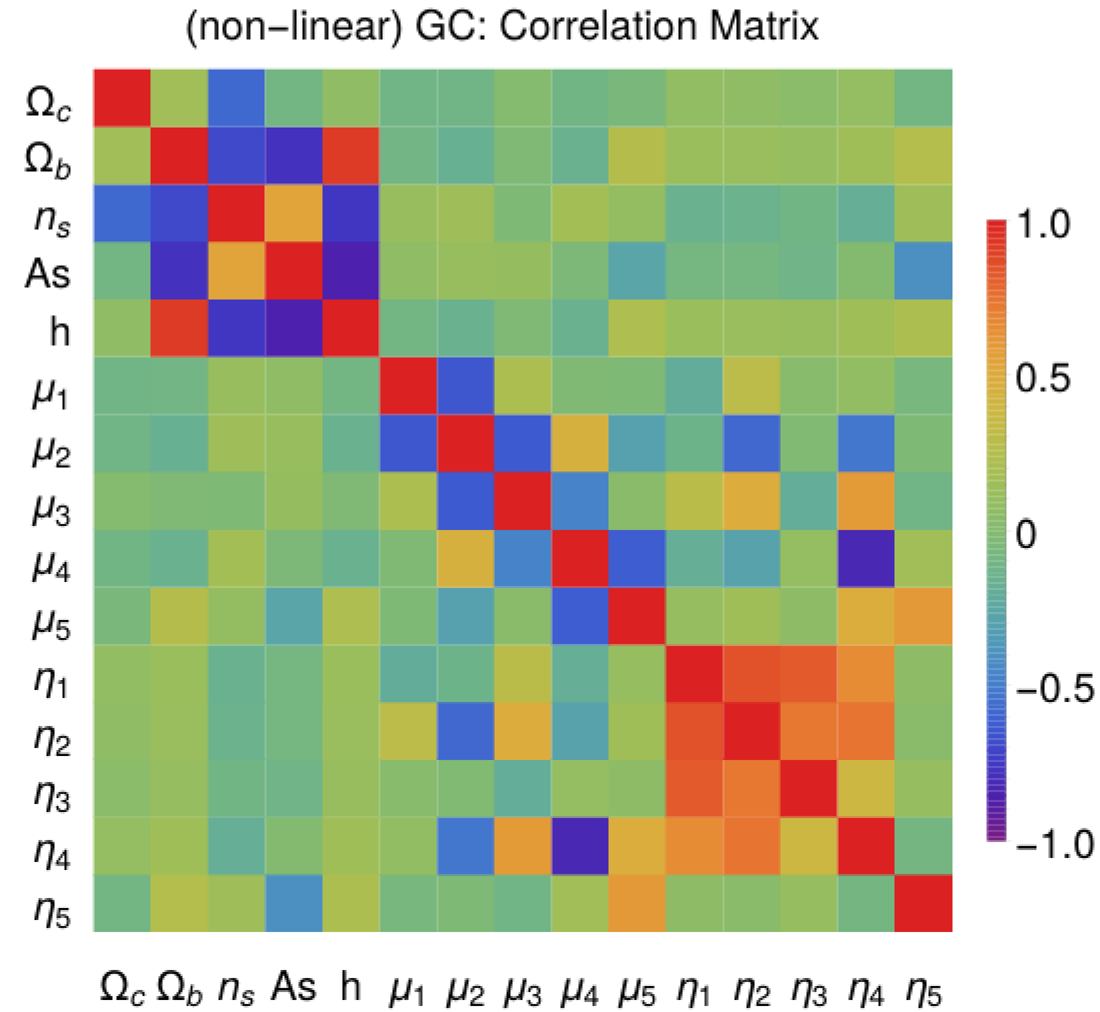


- Late-time parametrization better constrained by future LSS (degeneracy breaking).

Euclid forecasts for Modified Gravity

- Results are parametrization dependent → Redshift binning (discretization) of MG parameters → Correlated errors
- Non-linearities: reduce correlations. FoC goes down 65->32.

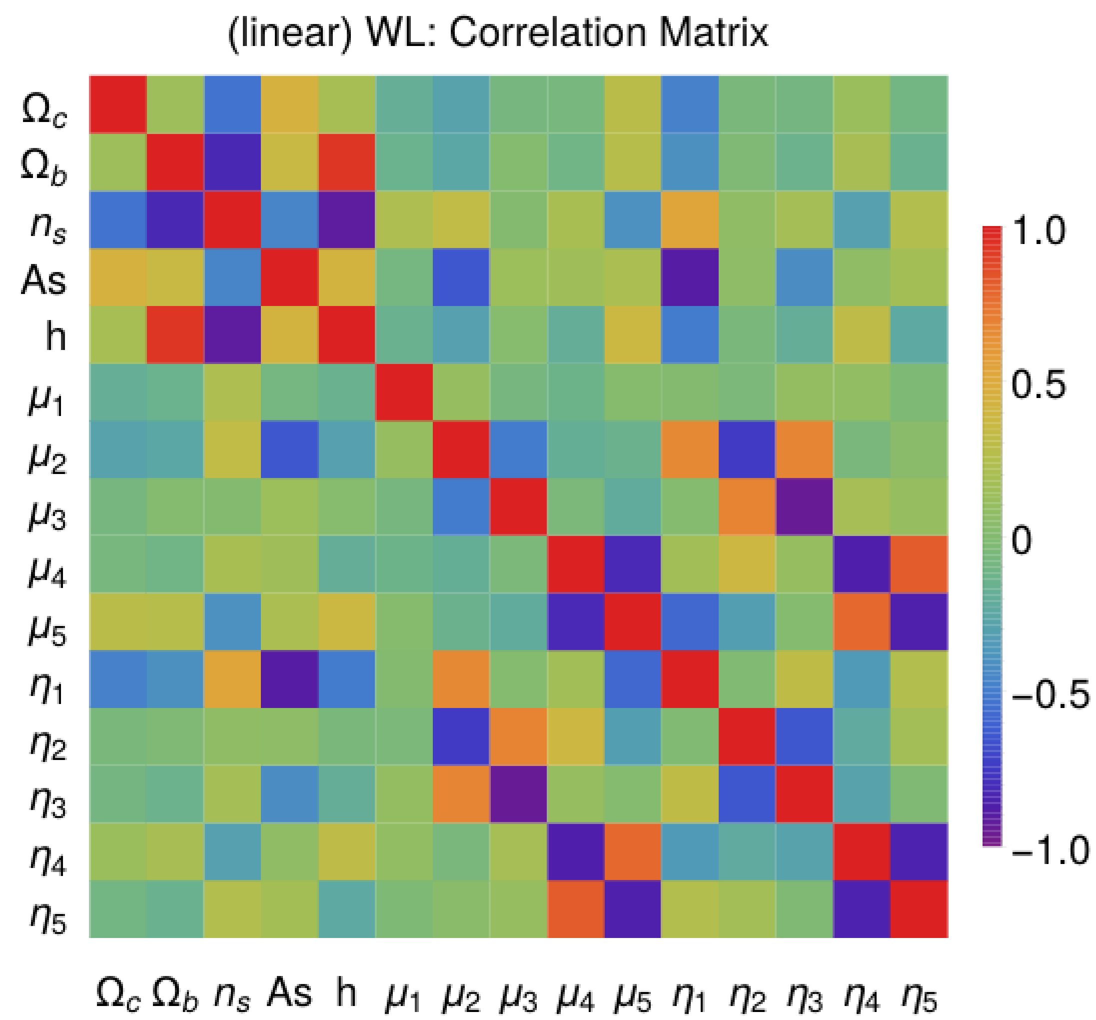


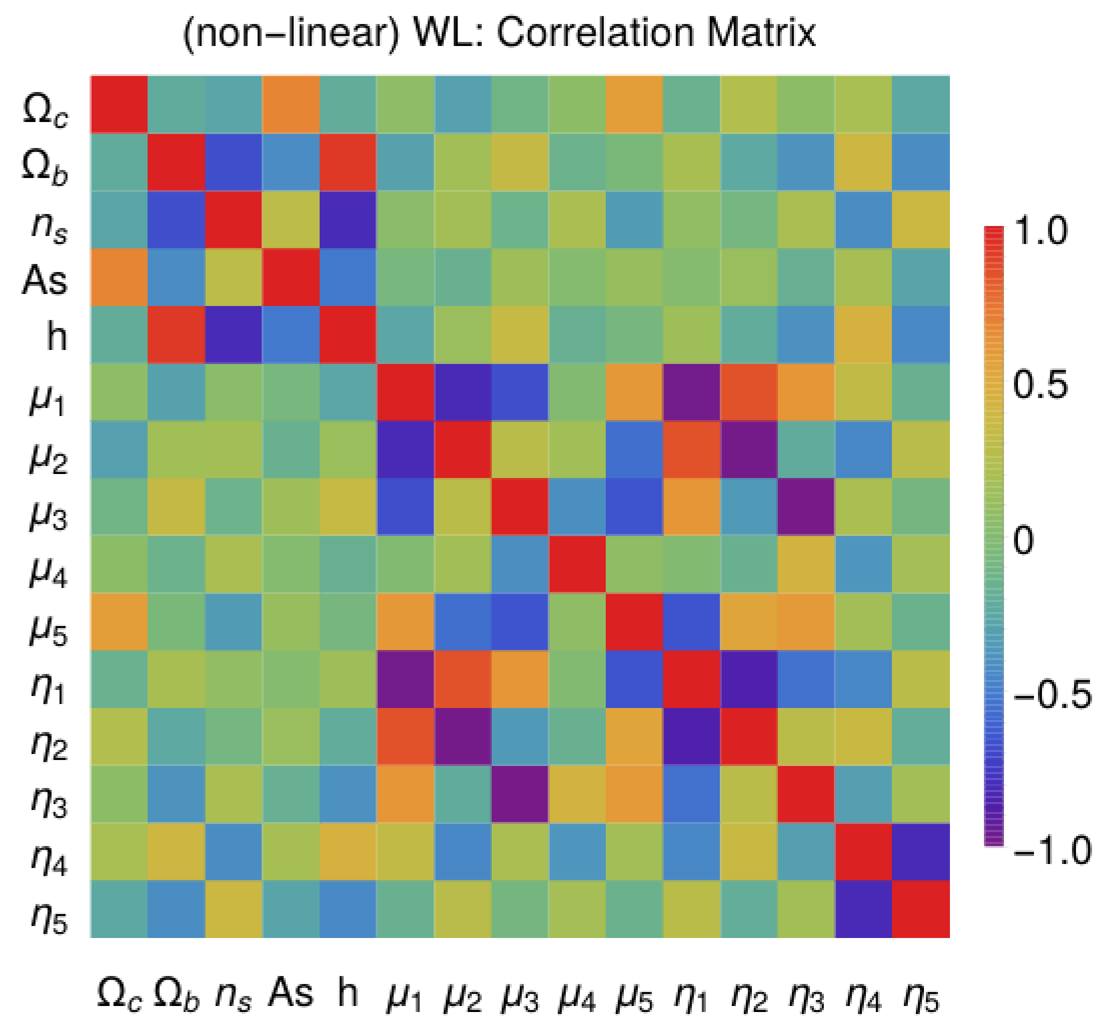


Casas, Martinelli, Pettorino, Kunz; arXiv: 1703.01271, Phys.Dark Univ. 18, 73-104

Euclid forecasts for Modified Gravity

- Results are parametrization dependent → Redshift binning (discretization) of MG parameters → Correlated errors
- FoC goes down 65→32, also in WL, not visible by eye.





Euclid forecasts for Modified Gravity

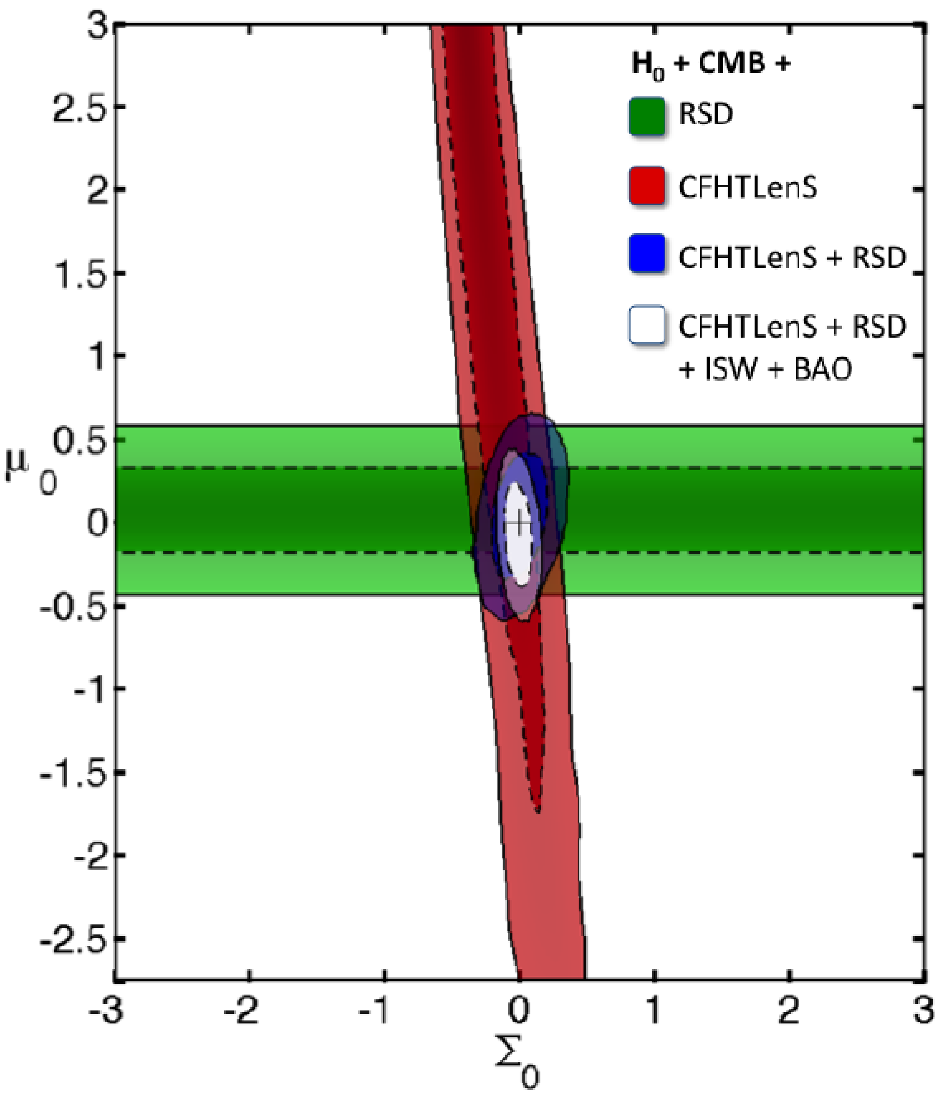
- Results are parametrization dependent → Redshift binning (discretization) of MG parameters → Correlated errors
- Using non-linear scales, improves constraints by 1 order of magnitude.
- P(Z)CA analysis shows that only 2 independent mu,eta bins can be measured.

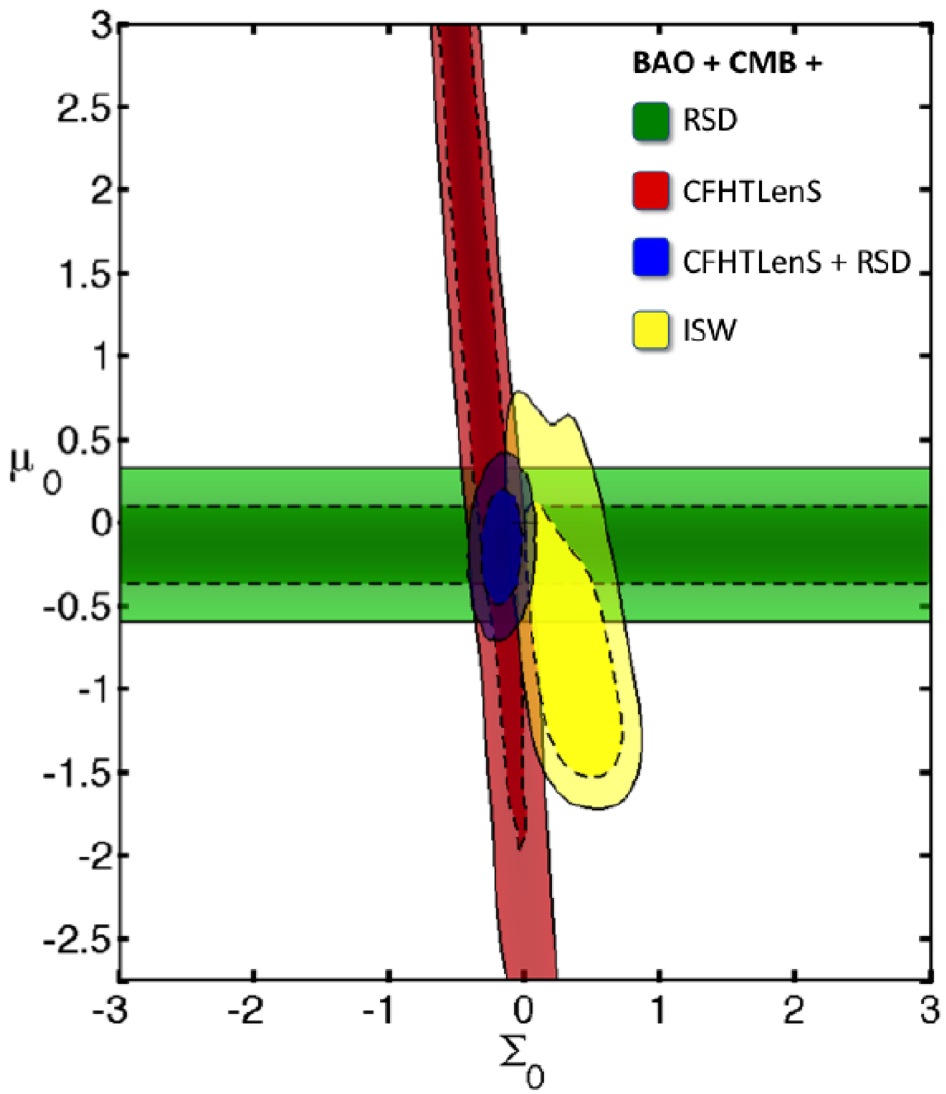
Euclid (Redbook)	$\ell\mathcal{A}_s$	μ_1	μ_2	μ_3	μ_4	μ_5	η_1	η_2	η_3	η_4	η_5	MG FoM
Fiducial	3.057	1.108	1.027	0.973	0.952	0.962	1.135	1.160	1.219	1.226	1.164	relative
GC+WL (lin)	11.3%	5.8%	10%	19.2%	282%	469%	7.9%	9.6%	16.1%	276%	2520%	12
GC+WL+*Planck* (lin)	1.1%	3.4%	4.8%	7.8%	9.3%	13.1%	6.2%	7.7%	9.1%	12.7%	23.6%	27
GC+WL (nl-HS)	0.8%	2.2%	3.3%	8.2%	24.8%	34.1%	3.6%	5.1%	8.1%	25.4%	812%	24
GC+WL+*Planck* (nl-HS)	0.3%	1.8%	2.5%	5.8%	7.8%	10.3%	3.2%	4.1%	5.9%	9.6%	19.5%	33
GC+WL+*Planck* (nl-Halofit)	0.4%	2.0%	2.4%	5.1%	7.4%	10.2%	3.5%	4.1%	5.8%	9.2%	18.9%	33

Casas, Martinelli, Pettorino, Kunz; arXiv: 1703.01271, Phys.Dark Univ. 18, 73-104

Phenomenological Parametrization

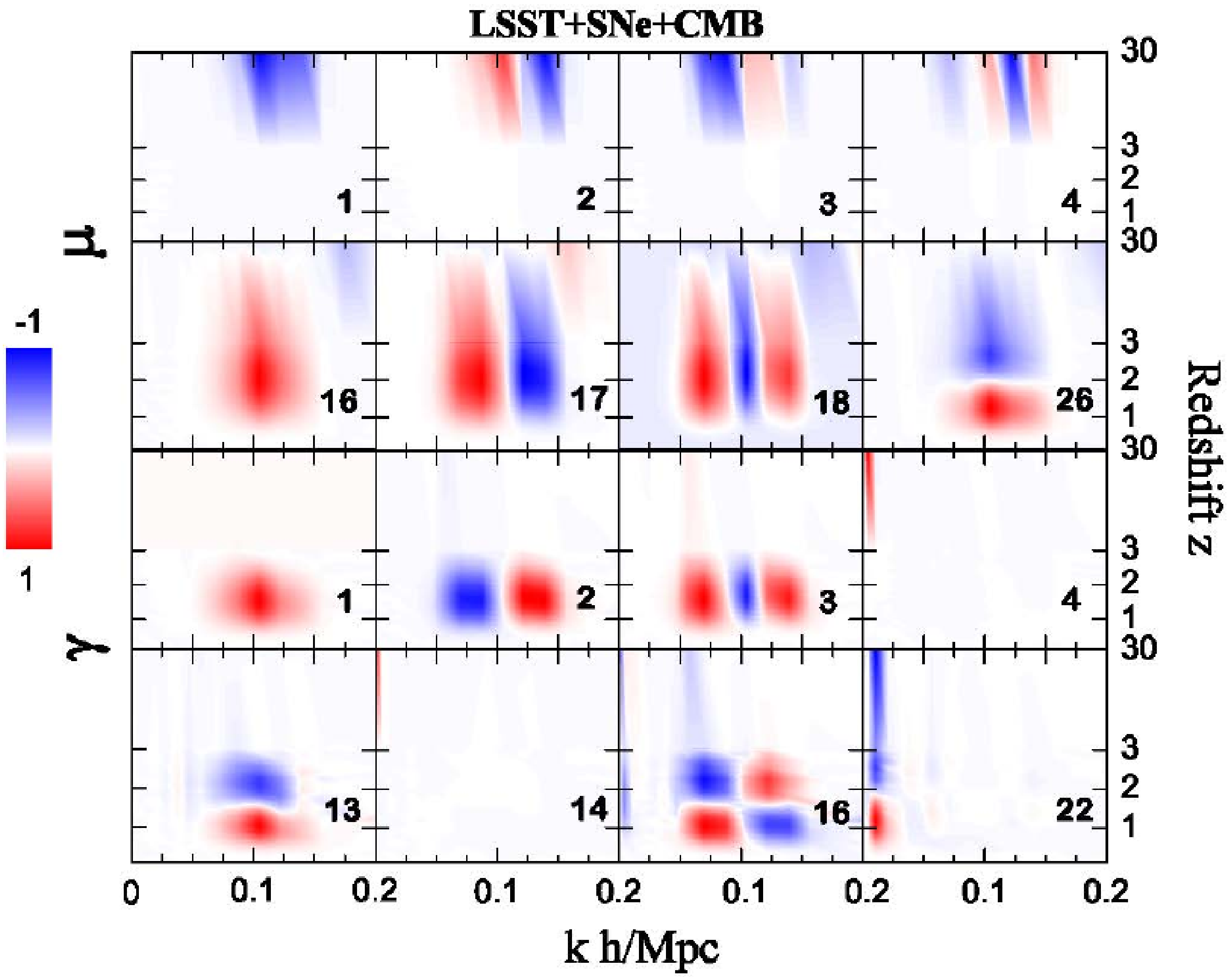
- Analysis of similar parametrization had been done in *CFHTLenS: Testing the Laws of Gravity with Tomographic Weak Lensing and Redshift Space Distortions*, Simpson et al, 2012, arXiv: 1212.3339
- We find the same degeneracy directions.





Phenomenological Parametrization

- Analysis using PCA also performed previously by Hojatti et al, arXiv: 1111.3960.
- They include scale dependence in mu and Sigma.
- Results difficult to interpret due to large number of eigenmodes and PCA choice.



Effective Field Theory of DE

- **History of the Universe:** Invoking accelerated expansion twice.

- **In expanding Universe:** time translation invariance is broken.
- EFT started in the context of inflation[1].
- Add 1 scalar Degree of Freedom:

$$\phi(t,\vec{x}) = \bar{\phi}(t) + \cancel{\delta\phi(t,\vec{x})}$$

- Unitary gauge: Perturbations of field absorbed in the metric. Const. time hypersurfaces coincide with const. field hypersurfaces.

- Construct EFT with operators invariant under time-diffeomorphisms:

$$n_\mu = -\frac{\partial_\mu \phi}{\sqrt{-(\partial_\mu \phi)^2}} \to -\frac{\delta^0_\mu}{\sqrt{-g^{00}}} \qquad K_{\mu\nu} = h_\mu{}^\sigma \nabla_\sigma n_\nu \qquad g^{00} \qquad R$$

[1] Creminelli, Vernizzi, Senatore, several papers around 2007~2010.

Effective Field Theory of DE

The action reads[1]:

$$S = \frac{1}{2}\int d^4x \quad \sqrt{-g} \quad \Big[M_{\rm pl}^2 \mathfrak{f}(t) R - 2\Lambda(t) - 2c(t) g^{00}$$

$$+M_2^4(t)(\delta g^{00})^2 - \bar{m}_1^3(t)\,\delta g^{00}\delta K - \bar{M}_2^2(t)\,\delta K^2$$

$$-\bar{M}_3^2(t)\,\delta K_\mu{}^\nu \delta K^\mu{}_\nu + \mu_1^2(t)\delta g^{00}\delta R + m_2^2(t) h^{\mu\nu}\partial_\mu g^{00}\partial_\nu g^{00}$$

$$+\ldots\Big] + S_m[g_{\mu\nu}, \chi_m]\,,$$

- g^{00} appears because of the unitary gauge.
- K is the extrinsic curvature of the constant time hypersurfaces.
- **First line:** functions that affect the background and linear perturbations.
- **Second and third lines:** functions that affect only the linear perturbations.

[1] Following notation by the latest review: Frusciante, Perenon (2019). arXiv:1907.03150

Effective Field Theory of DE

- Can also capture GLPV theories (Beyond Horndeski).
- Also Lorentz-violating theories like Hořava.
- With additional operators also recover DHOST.

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- Standard EFT resides on the Jordan frame, universal coupling between matter fields and metric.
- However some extensions also to species coupled explicitly to metric in different ways, have been developed[1].

[1] Gleyzes, D'Amico, Tsujikawa and respective collaborators.

Effective Field Theory of DE

	$\mathfrak{f}$	Λ	c	M_2^4	$\bar{m}_1^3$	$\bar{M}_2^2$	$\bar{M}_3^2$	μ_1^2	m_2^2
ΛCDM	1	const.	–	–	–	–	–	–	–
Quintessence [85, 86, 87]	1/✓	✓	✓	–	–	–	–	–	–
K-essence [88]	1/✓	✓	✓	✓	–	–	–	–	–
Brans-Dicke [89, 90]	✓	✓	✓	–	–	–	–	–	–
$f(R)$ [91, 92]	✓	✓	–	–	–	–	–	–	–
Kinetic braiding [93]	1	✓	✓	✓	✓	–	–	–	–
DGP [94]	✓	✓	✓	♯	✓	–	–	–	–
$f(G)$-Gauss-Bonnet [95]	✓	✓	✓	✓	✓	✓	♯	♯	–
Galileons [96, 97]	✓	✓	✓	✓	✓	✓	♯	♯	–
Horndeski [31, 33]	✓	✓	✓	✓	✓	✓	♯	♯	–
GLPV [34, 75]	✓	✓	✓	✓	✓	✓	♯	✓	–
low-energy Hořava [46]	✓	✓	✓	✓	–	✓	✓	–	✓

Table 1: Examples of well known DE/MG models described by the EFT action (5): ✓indicates that the EFT function is present, ♯ means the corresponding EFT function is related to other EFT functions, – indicates the EFT function is not present.

Horndeski:

$$\bar{M}_2^2 = -\bar{M}_3^2 = 2\mu_1^2$$

$$m_2^2 = 0$$

Table from: EFT, a review. Frusciante, Perenon (2019), arXiv:1907.03150

Effective Field Theory of DE

$$\mathfrak{f},\ \Lambda,\ c$$

- Modified Friedmann equations contain these three functions and H.
- By specifyng two out of those three functions, one can obtain the rest (2 eqns).

- Usually: Fix H to w0waCDM, assume some form for "f" and then obtain Lambda and "c" through the Friedmann equations.

- To map one particular theory to EFT: Start from the covariant action and then transform to unitary gauge and compare terms[1].
- Simple example with Quintessence:

$$\mathcal{L}_{\mathcal{Q}} \sim -\frac{1}{2}(\partial\phi)^2 - V(\phi) \xrightarrow[\text{unitary gauge}]{} -\frac{1}{2}\dot{\bar{\phi}}^2 g^{00} - V(\bar{\phi})$$

$$c(t) = \frac{1}{2}\dot{\bar{\phi}}^2\,, \qquad \Lambda(t) = V(\bar{\phi})$$

[1] This mapping procedure is cumbersome for more complicated theories, in those cases better to do ADM decomposition.

The alpha basis

- The alpha-basis is a redefinition of EFT functions, that allows for a more physically-motivated description of the cosmological properties of the theory. Developed by Bellini and Sawicki in 2014.

$$
\begin{aligned}
S &= \frac{1}{(2\pi)^3}\int d^3k dt\, a^3\, \frac{M^2}{2}\Big\{(1+\alpha_H)\,\delta N\delta_1\tilde{\mathcal{R}} + 2H\alpha_B\delta N\delta\tilde{K} \\
&+ \delta\tilde{K}^{\mu}_{\nu}\delta\tilde{K}^{\nu}_{\mu} - (\alpha_B^{GLPV}+1)(\delta\tilde{K})^2 + \left(\alpha_K + \alpha_{K_2}\frac{k^2}{a^2}\right)H^2(\delta N)^2 \\
&+ (1+\alpha_T)\delta_2(\tilde{\mathcal{R}}\delta(\sqrt{h}))\Big\}\,,
\end{aligned}
$$

- Despite this physical picture, there can be stability issues with any model, such as Ghosts, Gradient instabilities or Tachyonic instabilities.

Effective Field Theory of DE

$$\alpha_B(t) = -\frac{M_{\rm pl}^2 \dot{\mathfrak{f}} + \bar{m}_1^3}{H M^2}\,, \quad \alpha_T(t) = \frac{\bar{M}_3^2}{M^2} \equiv c_t^2 - 1\,, \quad \alpha_K(t) = \frac{2c + 4M_2^4}{H^2 M^2}\,,$$

$$\alpha_H(t) = \frac{2\mu_1^2 + \bar{M}_3^2}{M^2}\,,$$

$$\alpha_M = \frac{1}{H}\frac{d\ln M^2}{d\ln t} \qquad M^2(t) = M_{\rm pl}^2 \mathfrak{f} - \bar{M}_3^2$$

- alpha_H: beyond Horndeski, vanishes in the case of Horndeski we showed above.
- alpha_M: related to variation of the Planck mass (i.e. Newton's G). Modifies growth of structures, introduces anisotropic stress and changes friction term of GW's.
- alpha_B: Braiding. Appears among others for non-minimal coupling to gravity. Impacts clustering properties of DE.
- alpha_K: Is a purely kinetic term. It affects speed of propagation of DE fields. Also suppresses sound speed of DE.
- alpha_T: Affects speed of GW, strongly constrained after GW170817.

Einstein-Boltzmann codes for EFT

- Huge code comparison among Einstein-Boltzmann solvers. Also covering non-EFT models, such as Non-Local.
- Agreement better than 0.5% for most scales, except at low-ell, where also precision settings from CAMB and CLASS influence results.

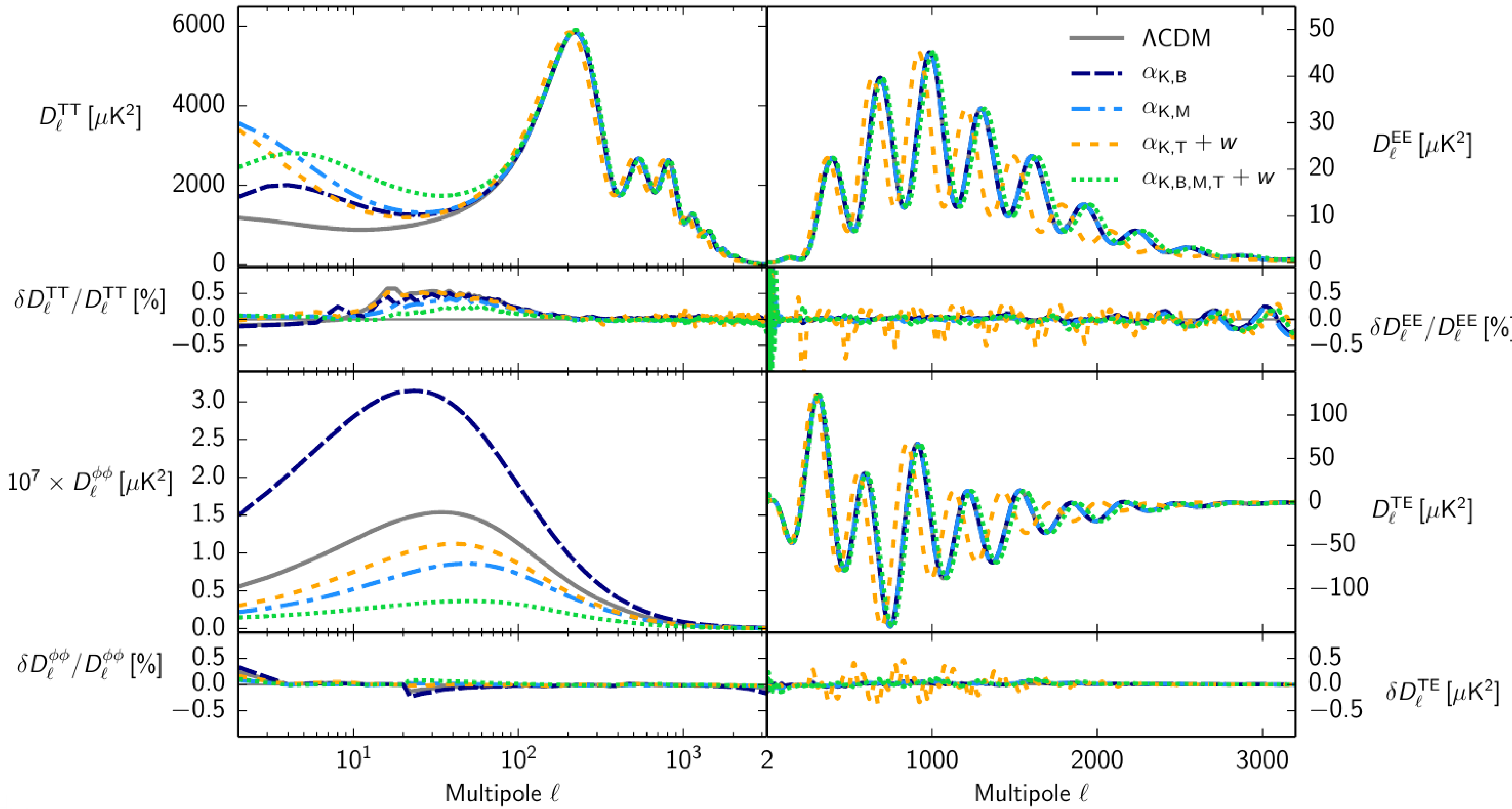


Figure taken from: A comparison of Einstein-Boltzmann solvers for testing General Relativity. arXiv: 1709.09135.

Einstein-Boltzmann codes for EFT

- Main codes: EFTCAMB and Hi_CLASS. Contain designer, mapping, full and alpha-basis approaches.

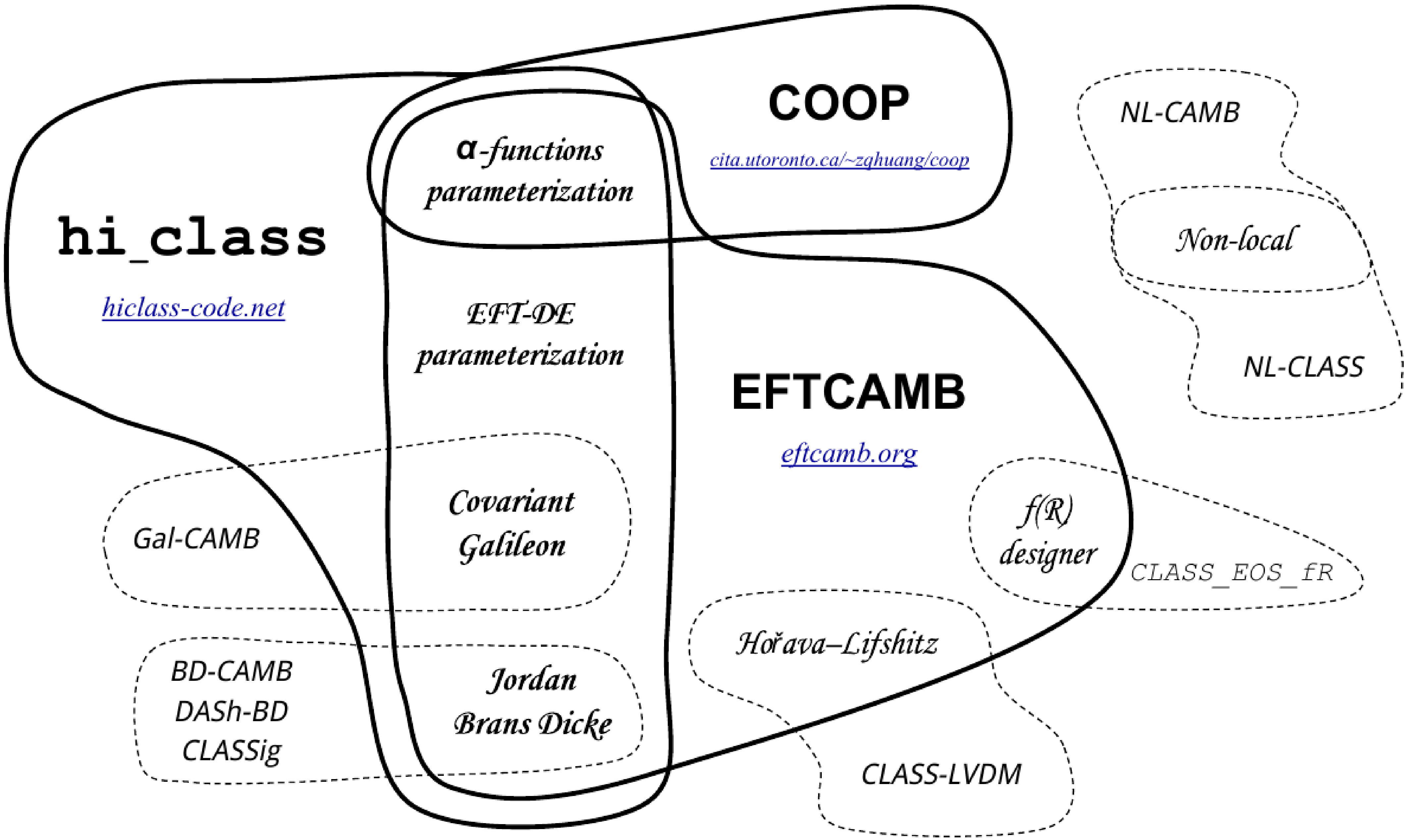


Figure taken from: A comparison of Einstein-Boltzmann solvers for testing General Relativity. arXiv: 1709.09135.

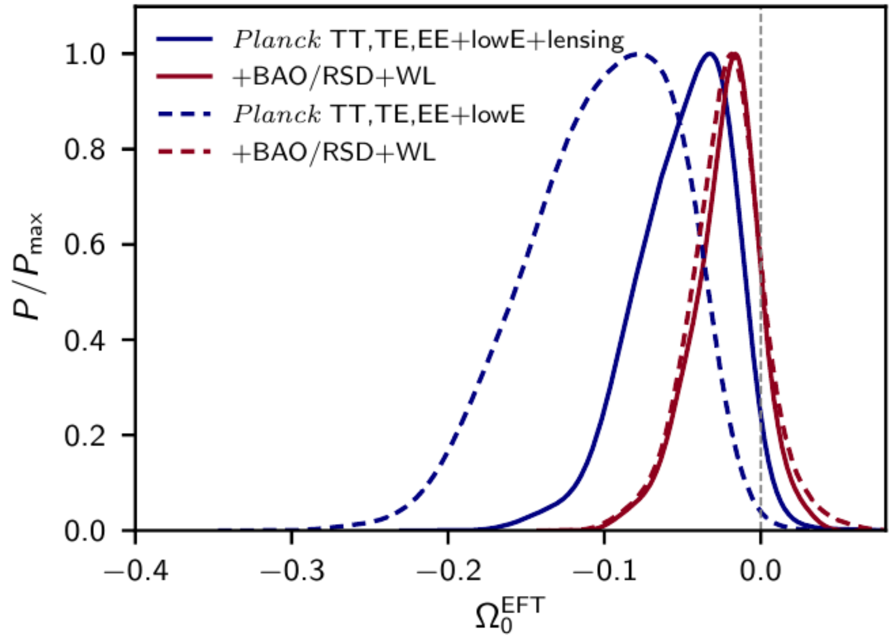
EFT constraints from Planck

Planck 2015/2018 considered a simple case where:

$$\alpha_{\mathrm{K}} \text{ is fixed}$$
$$\alpha_{\mathrm{M}} = -\alpha_{\mathrm{B}}$$
$$\alpha_{\mathrm{T}} = \alpha_{\mathrm{H}} = 0$$

$$1 + \Omega(a) \equiv \mathfrak{f}$$

$$\alpha_{\mathrm{M}} = \alpha_{\mathrm{M}0} a^{\beta} \longrightarrow \Omega(a) = \exp\left\{\frac{\alpha_{\mathrm{M}0}}{\beta} a^{\beta}\right\} - 1$$



Parameter	With CMB lensing: *Planck*	With CMB lensing: *Planck* +BAO/RSD+WL	Without CMB lensing: *Planck*	Without CMB lensing: *Planck* +BAO/RSD+WL
Ω_0^{EFT}	$-0.049^{+0.037}_{-0.024}$ $(1.6\,\sigma)$	$-0.019^{+0.024}_{-0.019}$ $(0.8\,\sigma)$	$-0.101^{+0.059}_{-0.038}$ $(2.1\,\sigma)$	-0.021 ± 0.025 $(0.9\,\sigma)$
α_{M0}	$-0.040^{+0.041}_{-0.016}$	$-0.015^{+0.019}_{-0.017}$	$-0.075^{+0.073}_{-0.028}$	$-0.014^{+0.017}_{-0.014}$
β	$0.72^{+0.38}_{-0.14}$	$0.66^{+0.44}_{-0.21}$	$0.66^{+0.38}_{-0.16}$	$0.62^{+0.45}_{-0.24}$
τ	$0.0489^{+0.0083}_{-0.0072}$	$0.0549^{+0.0096}_{-0.011}$	0.0497 ± 0.0082	0.0528 ± 0.0086
H_0 [km s^{-1}Mpc^{-1}]	68.19 ± 0.67	68.22 ± 0.46	68.30 ± 0.71	68.16 ± 0.46
σ_8	0.8198 ± 0.0074	0.8151 ± 0.0067	$0.845^{+0.013}_{-0.015}$	$0.8164^{+0.0087}_{-0.010}$
S_8	0.826 ± 0.013	0.8205 ± 0.0098	0.849 ± 0.017	0.823 ± 0.011
$\Delta\chi^2$	-4.3	-2.1	-9.7	-2.9

Planck 2018 results. VI. Cosmological parameters. arXiv: 1807.06209

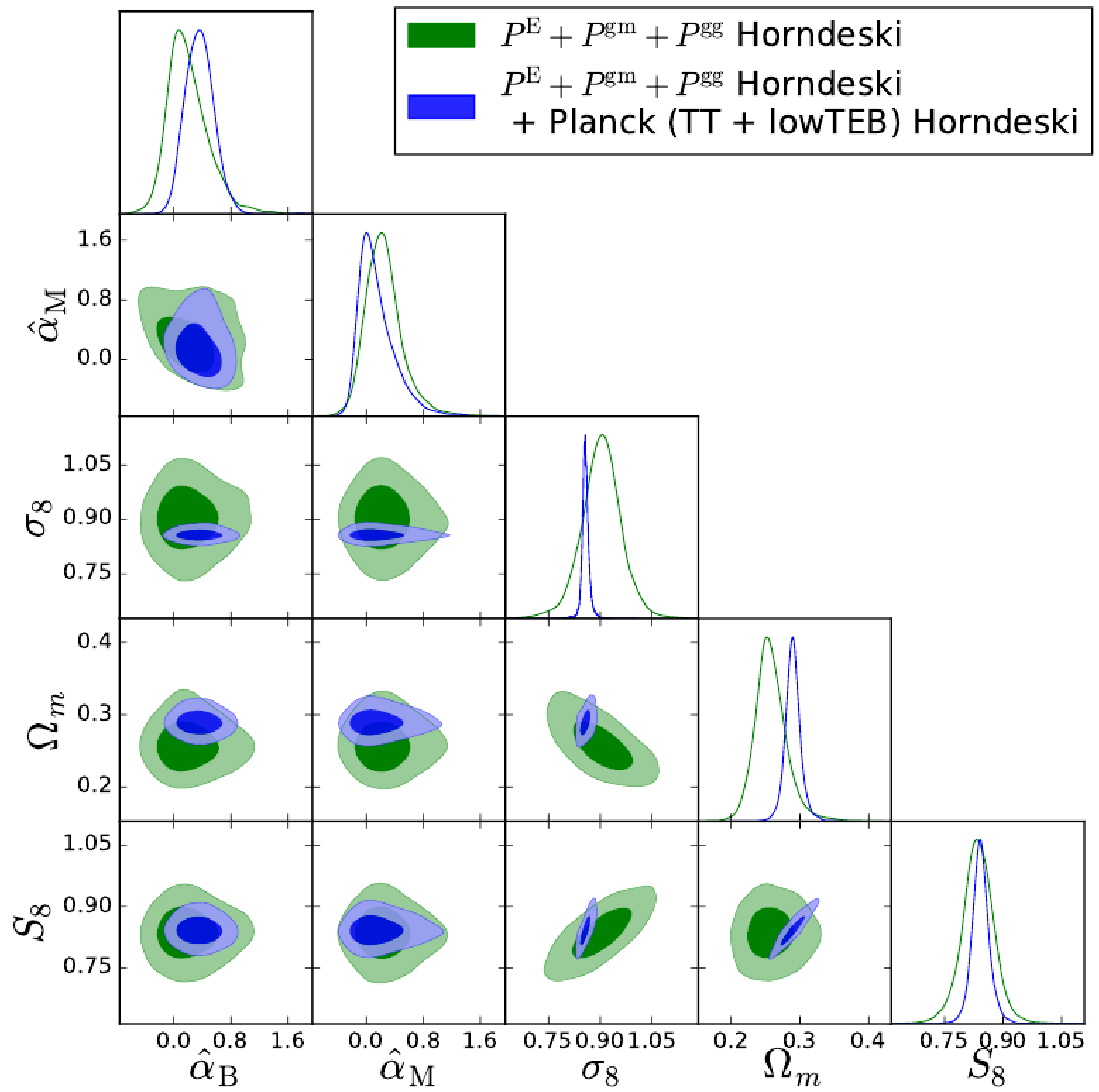
EFT constraints KiDS+GAMA

- KiDS: Kilo-Degree Survey, 450 sq. deg.
- GAMA: Galaxy and Mass Assembly survey

- Cosmic shear, galaxy-galaxy lensing and galaxy-clustering.

- Constraints on alphas ~0.4
- Addition of Planck improves by roughly a factor 2.

$$\alpha_i(\tau) = \hat{\alpha}_i\,\Omega_{\rm DE}(\tau)$$



Spurio Mancini et al, KiDS+GAMA: Constraints on Horndeski gravity from combined large-scale structure probes, arXiv1901.03686

GW170817 events

- GW170817+EM counterpart: Almost simultaneous arrival of photons and gravitons.
- Strong constraint on speed of gravitational waves.
- alpha_T function constrained to be basically zero.
- Remaining Horndeski:

$$\ddot{h}_{ij}^T + (3+\alpha_M)H\dot{h}_{ij}^T + (1+\alpha_T)\frac{k^2}{a^2}h_{ij}^T = 0$$

$$-3\cdot 10^{-15} \leq c_g/c - 1 \leq 7\cdot 10^{-16}$$

$$c_g^2 \ = \ 1 \ + \ \alpha_T$$

$$\mathcal{S}_{sH} = \int d^4x\sqrt{-g}\left[\mathcal{K}(\phi,X) + G_3(\phi,X)\Box\phi + G_4(\phi)R\right]$$

Notice yet another notation

- Corresponding EFT:

$$\mathcal{S} = \int d^4x\sqrt{-g}\Big\{\frac{m_0^2}{2}\left[1+\Omega(a)\right]R + \Lambda(a) - c(a)\,a^2\delta g^{00} + m_0^2H_0^2\frac{\gamma_1(a)}{2}\left(a^2\delta g^{00}\right)^2 - m_0^2H_0\frac{\gamma_2(a)}{2}\,a^2\delta g^{00}\,\delta K\Big\}$$

N.Frusciante, S.Peirone, SC, N.Lima, (2018), arXiv: 1810.10521. The road ahead of Horndeski.

EFTCAMB basis

$$\gamma_1 = \frac{M_2^4}{m_0^2 H_0^2}, \quad \gamma_2 = \frac{\bar{M}_1^3}{m_0^2 H_0}, \quad \gamma_3 = \frac{\bar{M}_2^2}{m_0^2},$$
$$\gamma_4 = \frac{\bar{M}_3^2}{m_0^2}, \quad \gamma_5 = \frac{\hat{M}^2}{m_0^2}, \quad \gamma_6 = \frac{m_2^2}{m_0^2}.$$

Recent studies of decay of GW and instabilities, have even constrained further the allowed EFT operators. See: Filippo Vernizzi and collaborators, recently.

Horndeski after GW170817 in EFT

- We consider two models with a w0wa background:
- M1a/b
- M2a/b
- gamma_1 and alpha_K are related.
- alpha_K does not have much effect on LSS.
- gamma_1 doesn't enter Quasistatic limit expressions.

$$\gamma_i^0 \ (i = 1, 2) \text{ are constants}$$

- **M1a**:

$$\Omega(a) = \Omega_0 a^{s_0}\,, \qquad \gamma_i(a) = 0\,,$$

- **M1b**:

$$\Omega(a) = \Omega_0 a^{s_0}\,, \qquad \gamma_i(a) = \gamma_i^0 a^{s_i}\,,$$

- **M2a**:

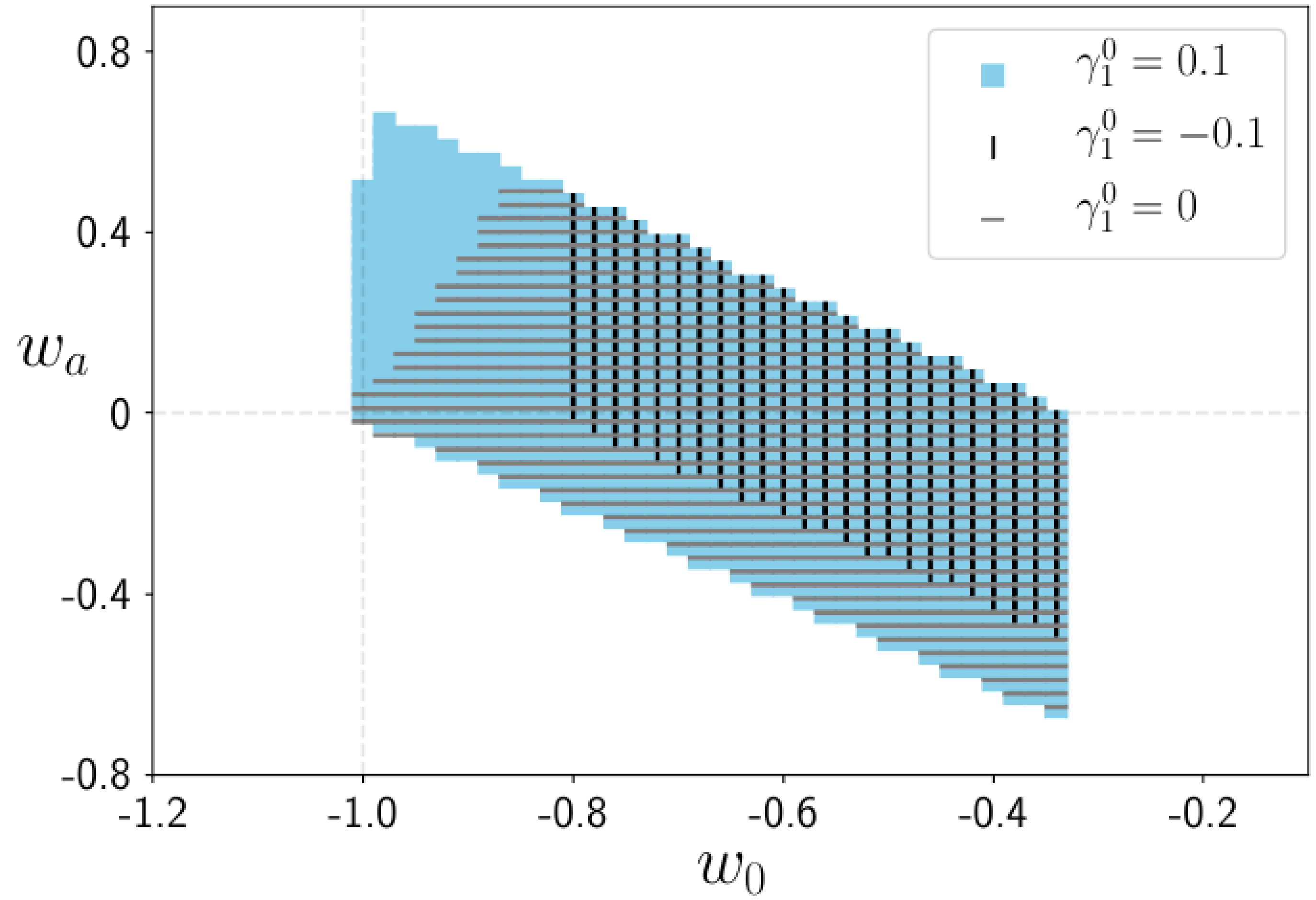
$$\Omega(a) = \Omega_0 a^{-3(1+w_0+w_a)} e^{-3w_a(1-a)}$$
$$\gamma_i(a) = 0,$$

- **M2b**:

$$\Omega(a) = \Omega_0 a^{-3(1+w_0+w_a)} e^{-3w_a(1-a)}\,,$$
$$\gamma_i(a) = \gamma_i^0 a^{-3(1+w_0+w_a)} e^{-3w_a(1-a)}\,,$$

Horndeski after GW170817 in EFT

- However:
- gamma_1 is very important for stability conditions.
- Ghost stability issues analogous for alpha_K (see Kreisch,Komatsu (2017) and Bellini et al (2016))
- Changing gamma_1, changes stable CPL space from blue to vertical-dashed to horizontal-dashed.



N.Frusciante, S.Peirone, SC, N.Lima, (2018), arXiv: 1810.10521. The road ahead of Horndeski.

Horndeski after GW170817 in EFT

- MCMC Likelihood analysis with:
- Planck TT+lensing (2015)
- BAO BOSS DR12
- JLA
- KiDS

Model	$10^9\,A_s$	n_s	Ω_m	H_0	Σm_ν
ΛCDM	$2.11^{+0.12}_{-0.12}$	$0.969^{+0.009}_{-0.009}$	$0.297^{+0.013}_{-0.013}$	$68.7^{+1.1}_{-1.0}$	
ΛCDM$+\nu$	$2.22^{+0.23}_{-0.19}$	$0.974^{+0.012}_{-0.011}$	$0.300^{+0.015}_{-0.014}$	$68.4^{+1.2}_{-1.2}$	< 0.288
M1a	$2.21^{+0.21}_{-0.21}$	$0.974^{+0.012}_{-0.012}$	$0.295^{+0.017}_{-0.016}$	$68.7^{+1.8}_{-1.7}$	
M1a $+\nu$	$2.29^{+0.25}_{-0.22}$	$0.976^{+0.013}_{-0.013}$	$0.298^{+0.017}_{-0.018}$	$68.4^{+1.8}_{-1.6}$	< 0.281
M1b	$2.19^{+0.24}_{-0.23}$	$0.973^{+0.013}_{-0.012}$	$0.293^{+0.017}_{-0.017}$	$68.9^{+1.8}_{-1.8}$	
M1b $+\nu$	$2.28^{+0.25}_{-0.25}$	$0.975^{+0.013}_{-0.015}$	$0.295^{+0.018}_{-0.016}$	$68.8^{+1.8}_{-1.7}$	< 0.347
M2a	$2.27^{+0.21}_{-0.20}$	$0.972^{+0.010}_{-0.010}$	$0.302^{+0.015}_{-0.014}$	$68.1^{+1.3}_{-1.4}$	
M2a $+\nu$	$2.35^{+0.24}_{-0.22}$	$0.975^{+0.011}_{-0.011}$	$0.303^{+0.016}_{-0.014}$	$67.9^{+1.3}_{-1.4}$	< 0.236
M2b	$2.20^{+0.28}_{-0.26}$	$0.968^{+0.013}_{-0.013}$	$0.300^{+0.016}_{-0.016}$	$68.6^{+1.8}_{-1.6}$	
M2b $+\nu$	$2.30^{+0.29}_{-0.29}$	$0.970^{+0.014}_{-0.014}$	$0.304^{+0.017}_{-0.017}$	$68.5^{+1.7}_{-1.6}$	< 0.543

Model	w_0	w_a	Ω_0	s_0	γ_1^0	s_1	γ_2^0	s_2
M1a	$-1.04^{+0.14}_{-0.16}$	$0.22^{+0.46}_{-0.39}$	$-0.07^{+0.17}_{-0.18}$	> 0.435				
M1a $+\nu$	$-1.02^{+0.13}_{-0.18}$	$0.12^{+0.49}_{-0.37}$	$-0.04^{+0.15}_{-0.21}$	> 0.240				
M1b	$-1.07^{+0.15}_{-0.16}$	$0.30^{+0.47}_{-0.42}$	$0.03^{+0.31}_{-0.25}$	> 0.215	> 0.217	$--$	$-0.9^{+1.3}_{-2.0}$	> 0.330
M1b $+\nu$	$-1.08^{+0.16}_{-0.15}$	$0.24^{+0.49}_{-0.48}$	$0.01^{+0.33}_{-0.33}$	> 0.296	> 0.103	$--$	$-1.9^{+2.3}_{-5.0}$	> 0.147
M2a	$-0.946^{+0.090}_{-0.060}$	$-0.098^{+0.25}_{-0.28}$	$0.018^{+0.032}_{-0.019}$					
M2a $+\nu$	$-0.950^{+0.087}_{-0.056}$	$-0.11^{+0.23}_{-0.30}$	$0.019^{+0.037}_{-0.020}$					
M2b	$-0.94^{+0.15}_{-0.13}$	$-0.31^{+0.48}_{-0.63}$	$0.047^{+0.068}_{-0.051}$		> 0.295		$-0.23^{+0.26}_{-0.32}$	
M2b $+\nu$	$-0.93^{+0.15}_{-0.14}$	$-0.61^{+0.66}_{-0.66}$	$0.080^{+0.099}_{-0.081}$		> 0.151		$-0.36^{+0.36}_{-0.47}$	

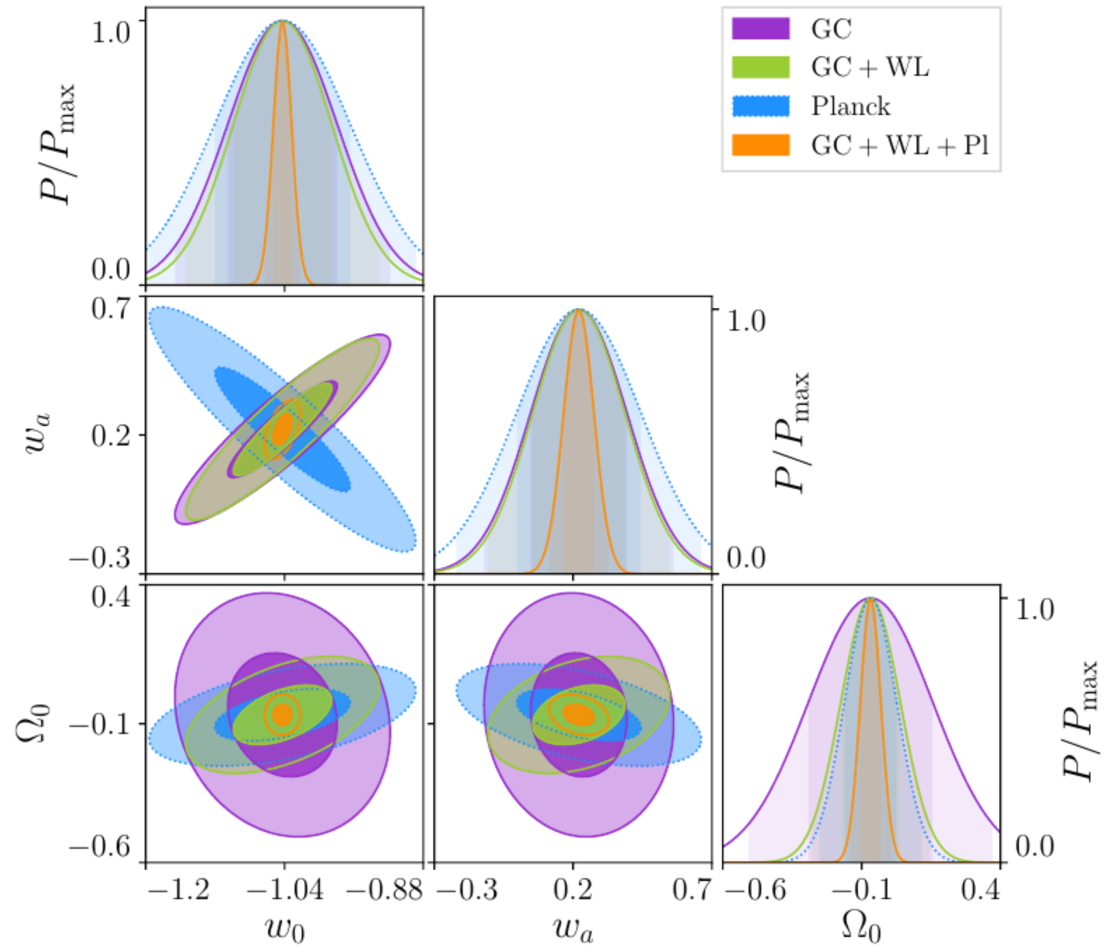
N.Frusciante, S.Peirone, SC, N.Lima, (2018), arXiv: 1810.10521. The road ahead of Horndeski.

Horndeski after GW170817 in EFT

- Forecasts for:
- DESI-ELG: GCspec
- SKA2: WL
- Only linear scales
- No-cross correlation
- Shear-shear Cls:

$$C_{ij}(\ell) = \frac{9}{4}\int_0^\infty \mathrm{d}z \frac{W_i(z)W_j(z)H^3(z)\Omega_m^2(z)}{(1+z)^4}\Sigma^2(k,z)P_m$$

Model	$2\sigma(w_0)$	$2\sigma(w_a)$	$2\sigma(\Omega_0)$	$2\sigma(s_0)$	$2\sigma(\gamma_2^0)$	$2\sigma(s_2)$
M1a	2.0%	50%	110%	68%	–	–
M1b	2.2%	40%	128%	96%	240%	136%
M2a	1.9%	44%	22%	–	–	–
M2b	2.6%	18%	48%	–	40%	–



Horndeski after GW170817 in EFT

- Forecasts for:
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Model	$2\sigma(w_0)$	$2\sigma(w_a)$	$2\sigma(\Omega_0)$	$2\sigma(s_0)$	$2\sigma(\gamma_2^0)$	$2\sigma(s_2)$
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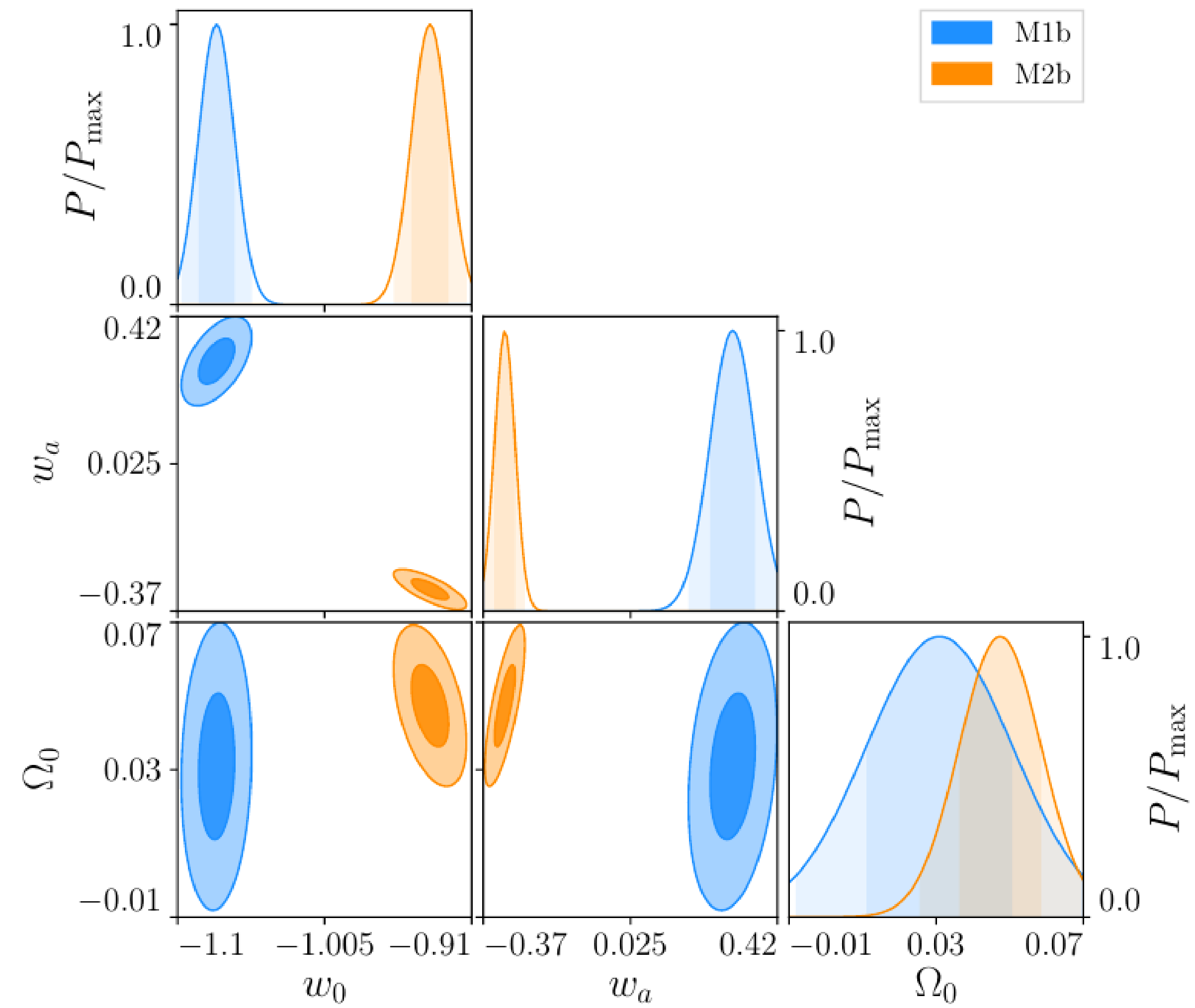
M2b	2.6%	18%	48%	–	40%	–



N.Frusciante, S.Peirone, SC, N.Lima, (2018), arXiv: 1810.10521. The road ahead of Horndeski.

The Quasistatic Limit

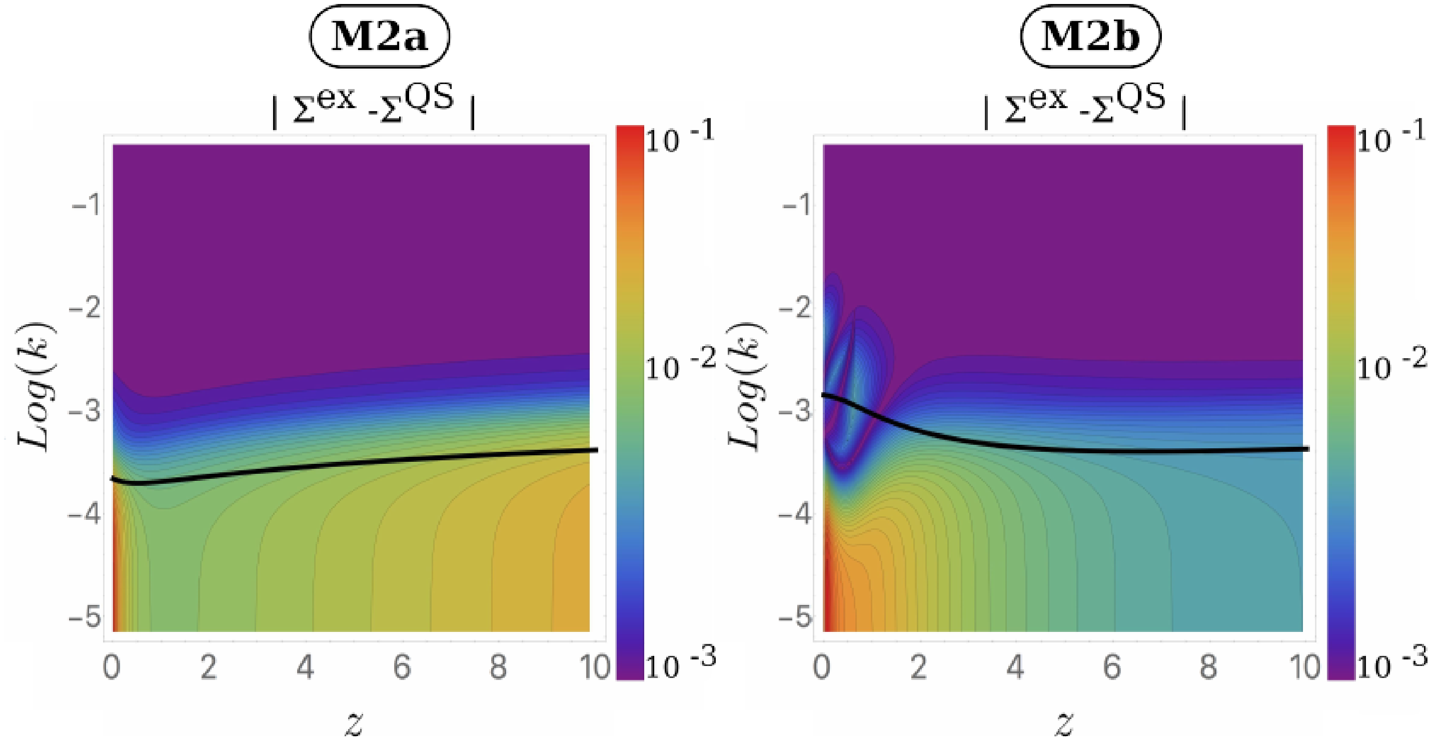
- The Quasistatic Limit:

$$\mu(a,k) = \frac{1}{1+\Omega}\,\frac{1+M^2\frac{a^2}{k^2}}{g_1+M^2\frac{a^2}{k^2}}\,,$$

$$\Sigma(a,k) = \frac{1}{2(1+\Omega)}\,\frac{1+g_2+M^2\frac{a^2}{k^2}}{g_1+M^2\frac{a^2}{k^2}}$$

- g1 and g2 are functions of the EFT operators.

- Computing the exact solution with EFTCAMB and comparing, we get a subpercent agreement for M1a/b and a subpercent agreement below the sound horizon scale for M2a/b.



N.Frusciante, S.Peirone, SC, N.Lima, (2018), arXiv: 1810.10521. The road ahead of Horndeski.

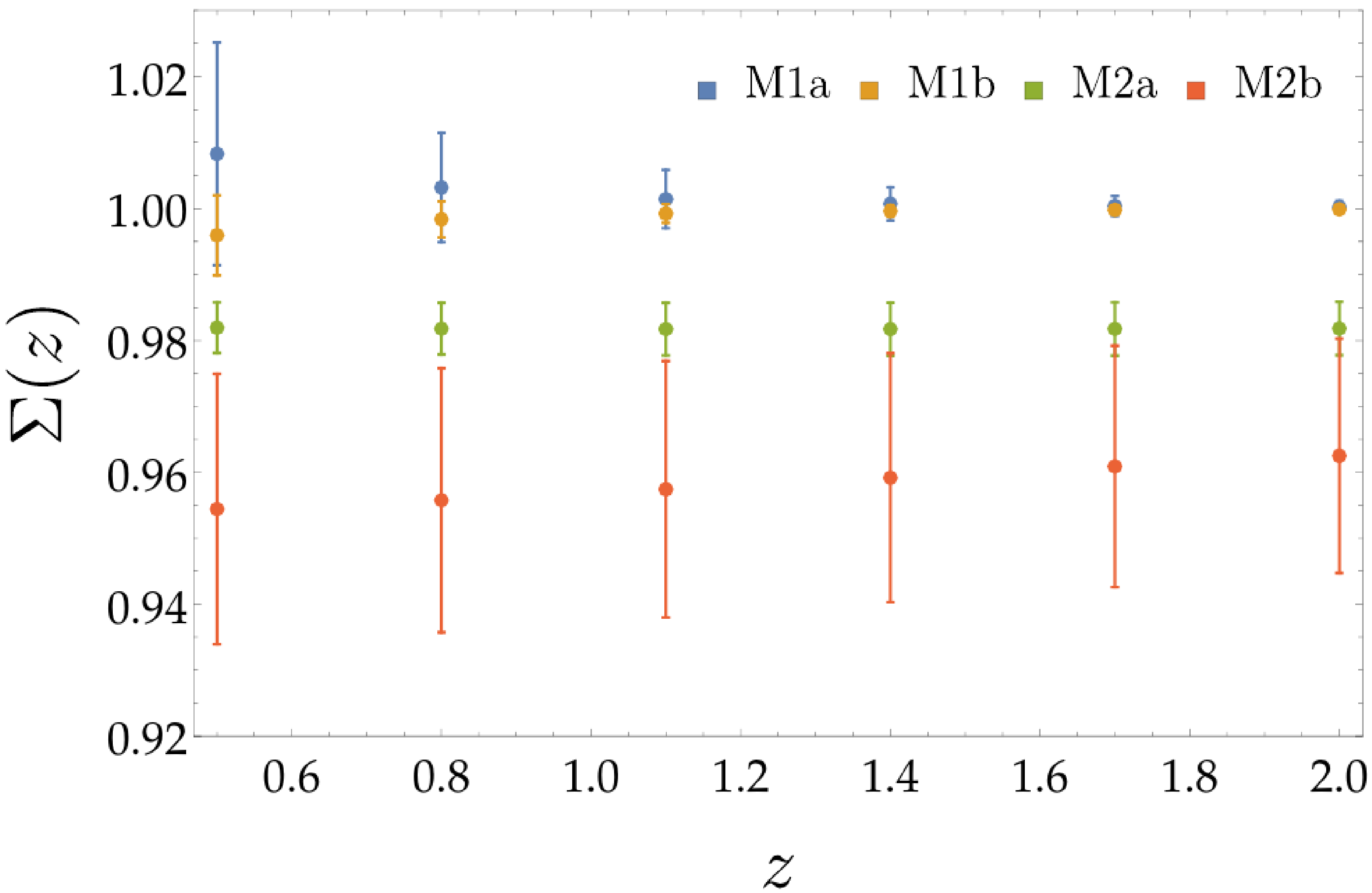
The Quasistatic Limit

$$\tilde{\mathbf{F}} = \mathbf{J}^T \mathbf{F} \mathbf{J} \,,$$

- We can then use a Jacobian to go to the phenomenological Sigma and mu parameters.

$$\mathbf{J} \equiv J_{ij} = \frac{\partial p_i}{\partial \tilde{q}_j}$$

- The deviations from GR are up to 4% for k=0.01.
- The error bars on Sigma are small in M1a/b since there is almost no cosmology dependence in the QS limit expressions.



- Other works on Horndeski after GW170817: Kreisch, Komatsu, Noller, Baker and others.

Linear-de form eq.(B.2)				
Basis	EFT functions	Constraints	Dataset	Ref.
α-basis	α_M	$\alpha_M > -1.6$ (*)	CMB + H_0 prior 95.4% C.L.	[70]
α-basis	α_M, α_B α_K, $c_t^2 = 1$	$\hat{\alpha}_M = 0.25^{+0.19}_{-0.29}$ $\hat{\alpha}_B = 0.20^{+0.20}_{-0.33}$ $\hat{\alpha}_K = 0$ (fixed)	KiDS+GAMA 95% C.L.	[208]
α-basis	α_M, α_B α_K, $c_t^2 = 1$	$c_M = 0.20^{+1.15}_{-0.82}$ $c_B = 0.63^{+0.83}_{-0.62}$ $c_K = 0.1$ (fixed)	CMB+BAO +RSD+mPk 95%C.L.	[209]
α-basis	α_M, α_B, α_T, α_K	$-1.36 < c_M < -0.06$ $0.19 < c_B < 2.30$ $-0.90 < c_T < -0.41$ $c_K = 10$(fixed)	CMB+BAO +RSD+PK 95% C.L.	[199]
α-basis	$(\alpha_M, \alpha_B) \times S(\frac{k}{k_V})$, $\alpha_K \times S(\frac{k}{k_V})$, $c_t^2 = 1$, see eq. (71)	$\hat{a}_K = 0.056$ (fixed) $\sigma(\hat{a}_M) = 0.065$ $\sigma(\hat{a}_B) = 0.049$	CMB+GC+CS (forecasts)	[216]
α-basis	$(\alpha_M, \alpha_B) \times S(\frac{k}{k_V})$, $\alpha_K \times S(\frac{k}{k_V})$, $c_t^2 = 1$, see eq. (71)	$\hat{a}_K = 0.01$ (fixed) $\hat{a}_M = 126\%$ $\hat{a}_B = 41\%$	CS: 3DWL linear (forecasts)	[219]
α-basis	$(\alpha_M, \alpha_B) \times S(\frac{k}{k_V})$, $\alpha_K \times S(\frac{k}{k_V})$, $c_t^2 = 1$, see eq. (71)	$\hat{a}_K = 0.01$ (fixed) $\hat{a}_M = 158\%$ $\hat{a}_B = 54\%$	CS: tomography linear (forecasts)	[219]
α-basis	$(\alpha_M, \alpha_B) \times S(\frac{k}{k_V})$, $(\alpha_K, \alpha_T) \times S(\frac{k}{k_V})$, see eq. (71)	$\sigma(c_M) = 0.056$ $\sigma(c_B) = 0.123$ $\sigma(c_K) = 3.1$ $\sigma(c_T) = 0.146$	S4+LSST +SKA1-IM+DESI (forecasts)	[141]
α-basis	α_M, α_B, β_γ^2 (see eq.(72))	$\sigma(\alpha_{M,0}) = 0.0146$ $\sigma(\alpha_{B,0}) = 0.0030$ $\sigma(\beta_\gamma^2) = 0.00135$	GC+WL ISW-Galaxy (forecasts)	[217]
α-basis	α_M, α_B, α_K, α_H, $c_t^2 = 1$	$-0.75 < \hat{\alpha}_M < 3.75$ (*) $0.2 < \hat{\alpha}_B < 3$ (*) $0.382 < \hat{\alpha}_H < 2.457$ α_K (fixed)	CMB+BAO+RSD 95% C.L.	[140]

Table from: EFT, a review. Frusciante, Perenon (2019), arXiv:1907.03150

Basis	EFT functions	Constraints	Data sets	Ref.
Constant form				
α-basis	M^2, α_B, α_T	$\sigma(\tilde{M}_0) = 0.006$ $\sigma(\alpha_0^B) = 0.02$ $\sigma(\alpha_0^T) = 0.001$	CMB-S4+DESI (forecasts)	[65]
`EFTCAMB` basis	Ω, γ_2, γ_3	$\sigma(\Omega_0) = 0.01$ $\sigma(\gamma_2^{(0)}) = 0.05$ $\sigma(\gamma_3^{(0)}) = 0.003$	CMB-S4+DESI (forecasts)	[65]
early/late transition form eq. (B.10)				
α-basis	M^2, α_B, α_T	$\sigma(\tilde{M}_{early}) = 0.05$ $\sigma(\tilde{M}_{late}) = 0.007$ $\sigma(\alpha^B_{early}) = 0.04$ $\sigma(\alpha^B_{late}) = 0.08$ $\sigma(\alpha^T_{early}) = 0.02$ $\sigma(\alpha^T_{late}) = 0.002$	CMB-S4+DESI (forecasts)	[65]
`EFTCAMB` basis	Ω	$\sigma(\Omega_{early}) = 0.03$ $\sigma(\Omega_{late}) = 0.02$	CMB-S4+DESI (forecasts)	[65]
Scaling-a form eq. (B.5)				
α-basis	$\alpha_M = -\alpha_B$	$\alpha_{M0} = -0.015^{+0.019}_{-0.017}$ $\beta = 0.66^{+0.44}_{-0.21}$	Planck18+WL +BAO/RSD 68%C.L.	[1]
	α_M, α_B, α_K, $c_t^2 = 1$	$c_M = 0.27^{+0.54}_{-0.26}$ $c_B = 0.48^{+0.83}_{-0.46}$ $c_K = 0.1$ (fixed)	CMB+BAO +RSD+mPk 95%C.L.	[209]
`EFTCAMB` basis	Ω	$\Omega_0^{EFT} < 0.061$	Planck13+WP +BAO+Lensing 95%C.L.	[68]
	Ω, CPL	$\Omega_0 = -0.07^{+0.17}_{-0.18}$ $s_0 > 0.435$	CMB+BAO +SNIa+WL 95% C.L.	[130]
		$2\sigma(\Omega_0) = 110\%$ $2\sigma(s_0) = 68\%$	GC+WL+CMB (forecasts)	[130]
	Ω, γ_1, γ_2 $c_t^2 = 1$, CPL	$\Omega_0 = 0.03^{+0.31}_{-0.25}$ $s_0 > 0.215$ $\gamma_1^0 > 0.217$ $\gamma_2^0 = -0.9^{+1.3}_{-2.0}$ $s_2 > 0.330$	CMB+BAO +SNIa+WL 95% C.L.	[130]
		$2\sigma(\Omega_0) = 128\%$ $2\sigma(s_0) = 96\%$ $2\sigma(\gamma_2^0) = 240\%$ $2\sigma(s_2^0) = 136\%$ $\gamma_1^0 = 5$, $s_1 = 1.4$ (fixed)	GC+WL+CMB (forecasts)	[130]

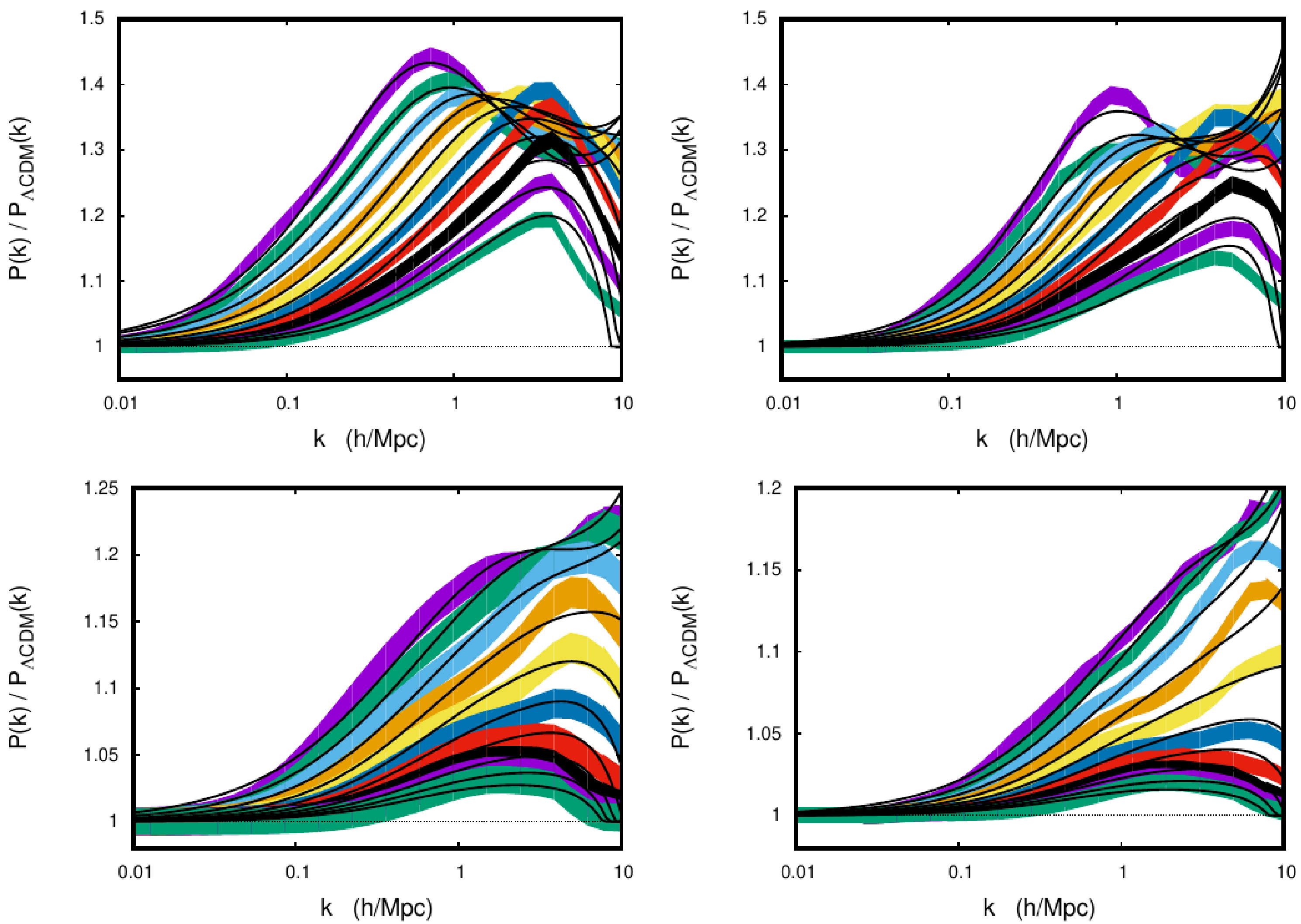
Table from: EFT, a review. Frusciante, Perenon (2019), arXiv:1907.03150

Basis	EFT functions	Constraints	Data sets	Ref.
e-fold form eq. (B.7)				
α-basis	$\alpha_B = -2\alpha_M$	$0.055 < \mu < 0.145$ (*) $a_t = 0.5, \tau = 1.5$ (fixed)	RSD (DESI) (forecast) 68% C.L.	[186]
α-basis	$\alpha_B = -2\alpha_M$ CPL	$-0.07995 < c_M < 0.0$ $0.2615 < a_t < 1.0$ $0.8304 < \tau < 2.19$	CMB+BAO +RSD+SNIa 95% C.L.	[201]
de-1 form eq. (B.3)				
[79]	μ, μ_3, ϵ_4	$p_1 = -0.28^{+0.17}_{-0.20}$ $p_3 = 0.04 \pm 0.17$ $p_4 = -0.030^{+0.068}_{-0.035}$	CMB (Planck,WMAP) 68% C.L.	[106]
[79]	μ, μ_3, ϵ_4	$p_1 = 0.10^{+0.58}_{-0.37}$ $p_3 = 0.13^{+0.28}_{-0.40}$ $p_4 = -0.18^{+0.28}_{-0.13}$ $p_3^1 = 0.41^{+0.39}_{-0.91}$ $p_4^1 = 0.03^{+0.18}_{-0.11}$	CMB (Planck,WMAP) 68% C.L.	[106]
[79]	μ_1, μ_2^2, μ_3, ϵ_4	$p_{10} = -0.000^{+0.002}_{-0.002}$ $p_{11} = -0.127^{+0.095}_{-0.096}$ $p_{20} = 1.697^{+2.933}_{-2.157}$ $p_{21} = -0.926^{+5.852}_{-5.990}$ $p_{30} = 1.022^{+0.930}_{-0.806}$ $p_{31} = -1.447^{+1.510}_{-1.812}$	$f\sigma_8 + f + \sigma_8 + \dot{G}_N$ 95% C.L.	[168]
de-density form eq. (B.6)				
`EFTCAMB` basis	Ω, CPL	$\Omega_0 = -0.018^{+0.032}_{-0.019}$	CMB+BAO +SNIa+WL 95% C.L.	[130]
`EFTCAMB` basis	Ω, CPL	$2\sigma(\Omega_0) = 22\%$	GC+WL+CMB (forecasts)	[130]
`EFTCAMB` basis	Ω, γ_1, γ_2 $c_t^2 = 1$, CPL	$\Omega_0 = 0.047^{+0.068}_{-0.051}$ $\gamma_1^0 > 0.295$ $\gamma_2^0 = -0.23^{+0.26}_{-0.32}$	CMB+BAO +SNIa+WL 95% C.L.	[130]
`EFTCAMB` basis	Ω, γ_1, γ_2 $c_t^2 = 1$, CPL	$2\sigma(\Omega_0) = 48\%$ $2\sigma(\gamma_2^0) = 40\%$ $\gamma_1^0 = 4.4$ (fixed)	GC+WL+CMB (forecasts)	[130]

Table from: EFT, a review. Frusciante, Perenon (2019), arXiv:1907.03150

What to do about non-linear scales? f(R) example

- To compare data with observations we need the matter power spectrum at non-linear scales.
- For WL: at least k~10h/Mpc at 1% precision ?!
- Perturbation theory, up to: k~0.35.
- Not enough ressources to run infinite N-body simulations.
- Emulator approach.

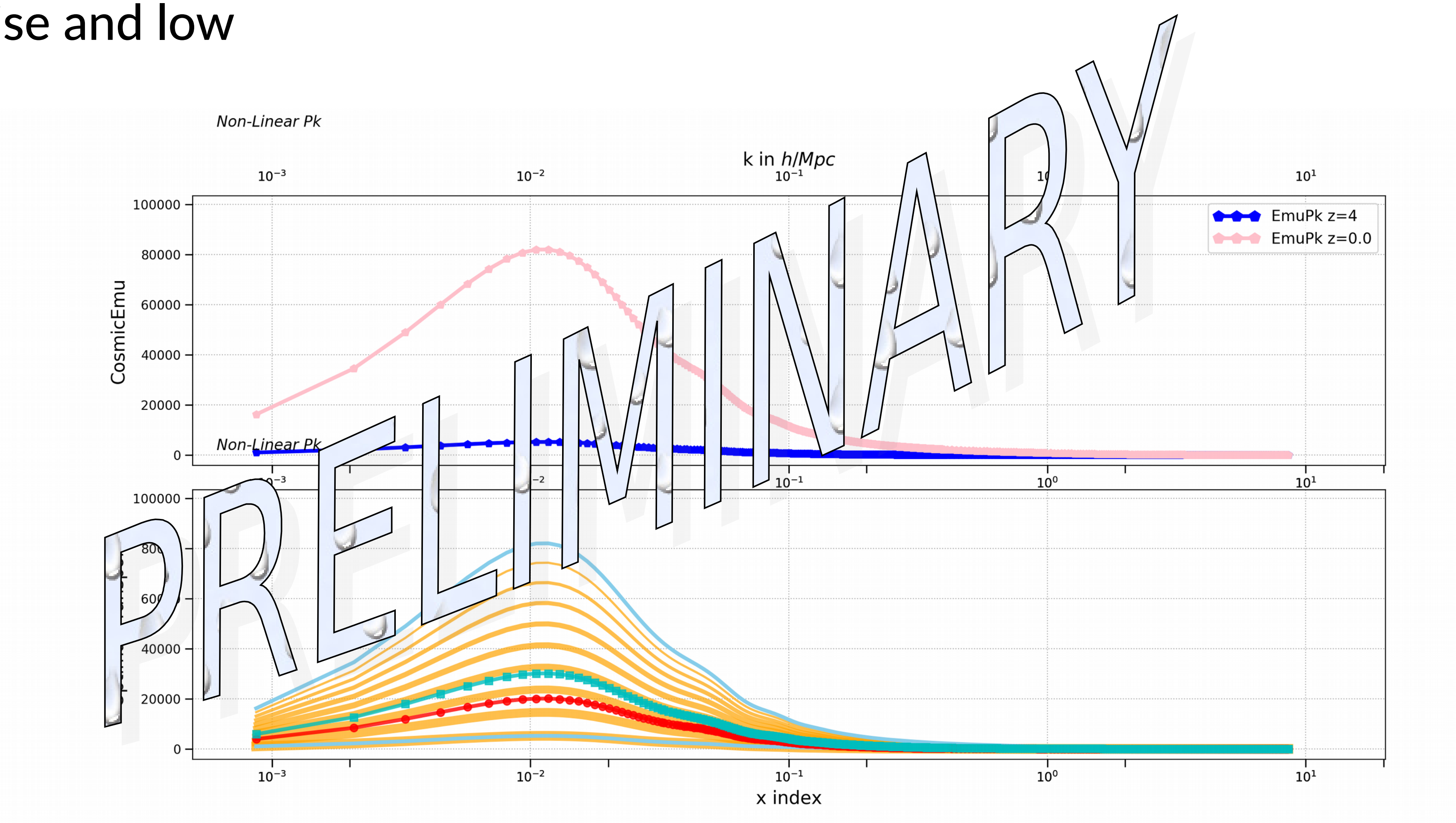


Emulators for the non-linear matter power spectrum beyond ΛCDM

- Winther, SC, Baldi, Koyama, Li, Lombriser, Zhao, arXiv:1903.08798

What to do about non-linear scales? DE example

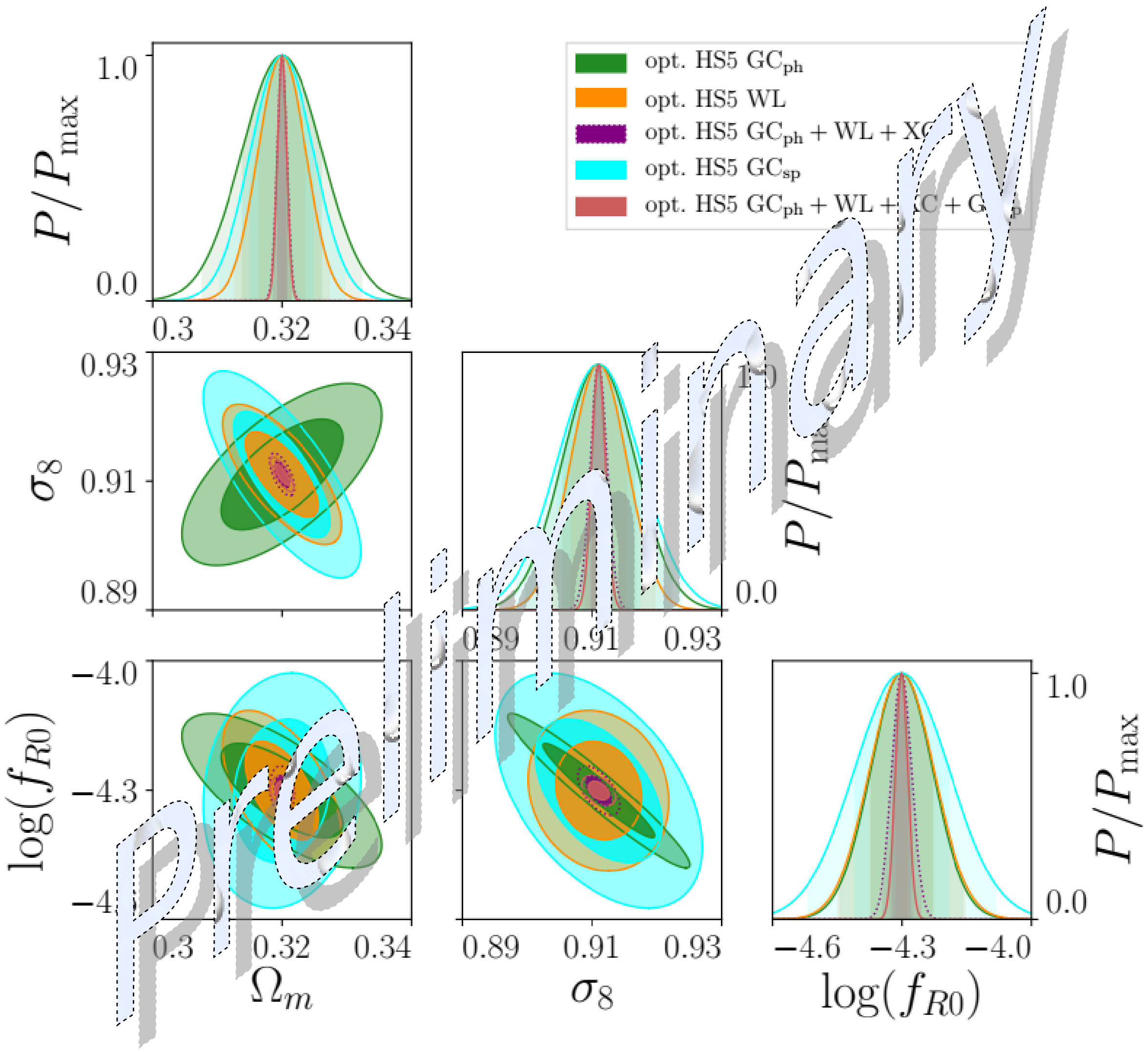
- Machine Learning approach.
- Good in regime with noise and low number of realizations.



SC, Schmitz, Pettorino, Starck (in preparation)

What to do about non-linear scales?
f(R) example

- Non-linear scales crucial to constrain models of MG/DE with Euclid and future surveys.
- Improvements in constraints range from factor 2 to 10, depending on settings.
- Cross-correlations are very important.
- Need to study galaxy bias in MG.



Euclid, TWG, WP6. In progress, very preliminary.

Conclusions

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Thanks!

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- EFT is a useful tool to study remaining sectors of the theory in an agnostic way.

Thanks!

Conclusions

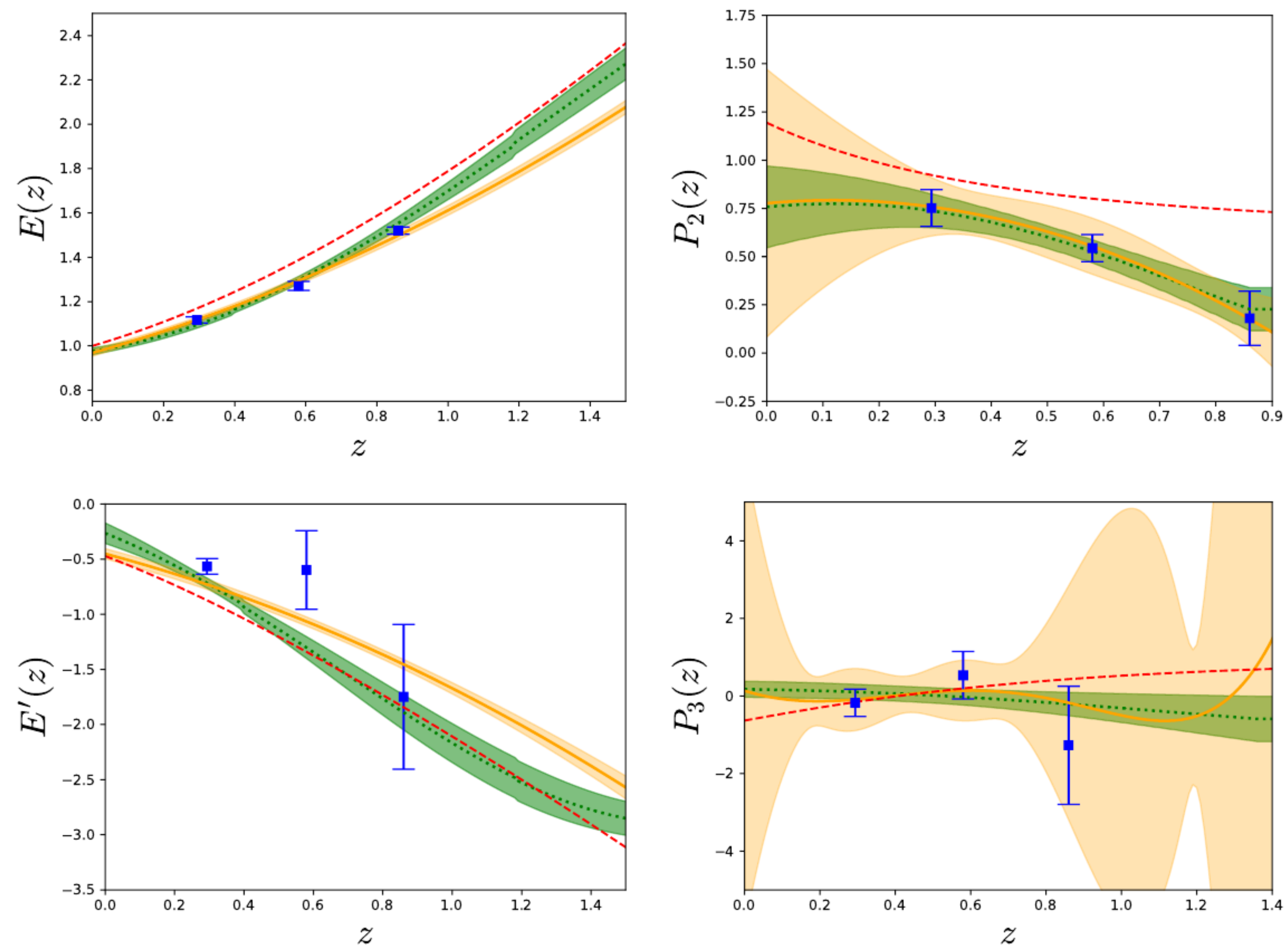
- GR and LCDM still passing multiple tests.
- Many of the early models of a scalar field have been ruled out.
- With future LSS surveys, we will have 2% constraints on w0-wa and 5-10% constraints on effective MG parameters.
- EFT is a useful tool to study remaining sectors of the theory in an agnostic way.
- However, hard to constrain all parameters at once without the curse of parametrization.
- We need to understand better nonlinearities and wait for more GW data.

Thanks!

Backup

Measuring the gravitational slip

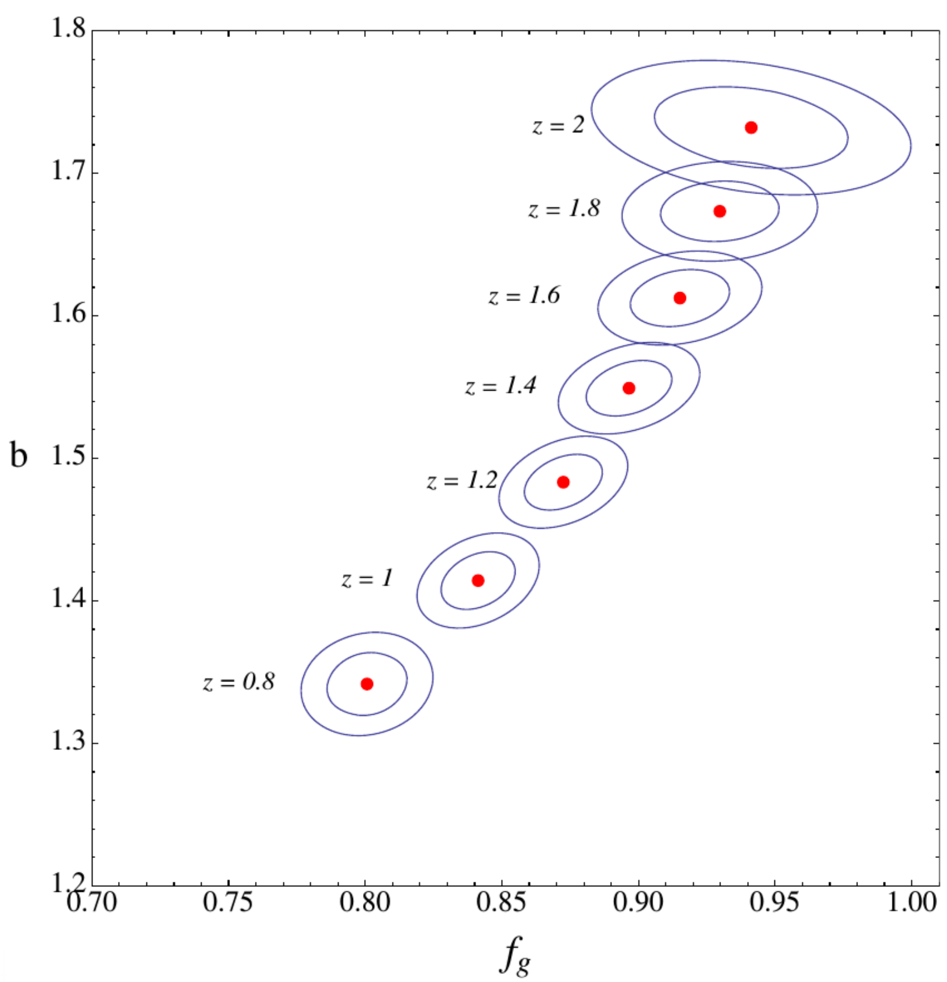
- The ratio of the scalar gravitational potentials can be constrained in a model-independent way.
- Using "Machine Learning" to obtain diferentiable functions from data.
- P3~RSD, H/H0=E, P2~Eg
- No assumptions about Dark Matter, primordial power spectrum or galaxy bias.
- Future Constraints: ~10-20% on η.

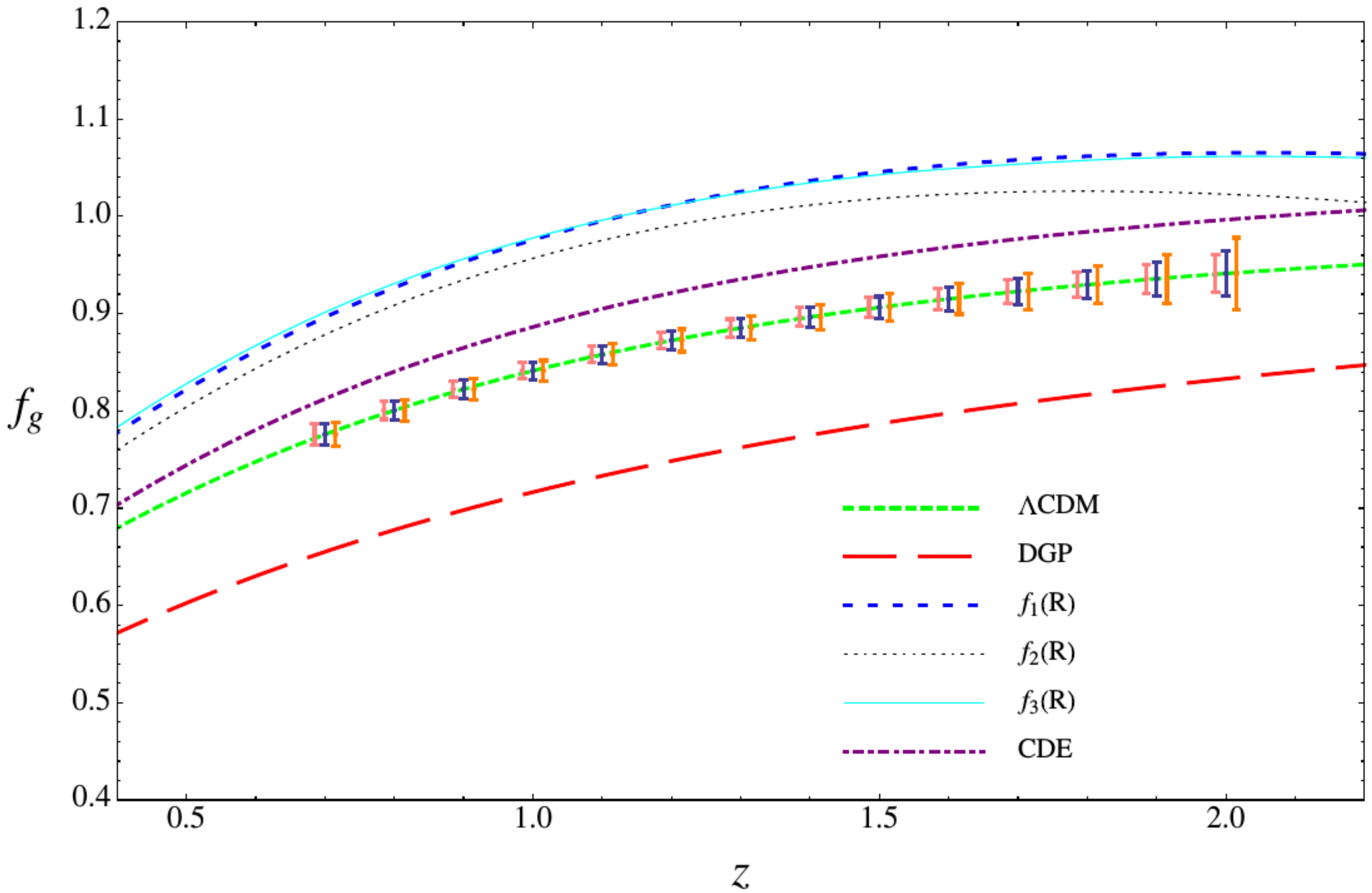


$$\eta_{obs} \equiv \frac{3P_2(1+z)^3}{2E^2\left(P_3 + 2 + \frac{E'}{E}\right)} - 1 = \eta$$

Euclid forecasts

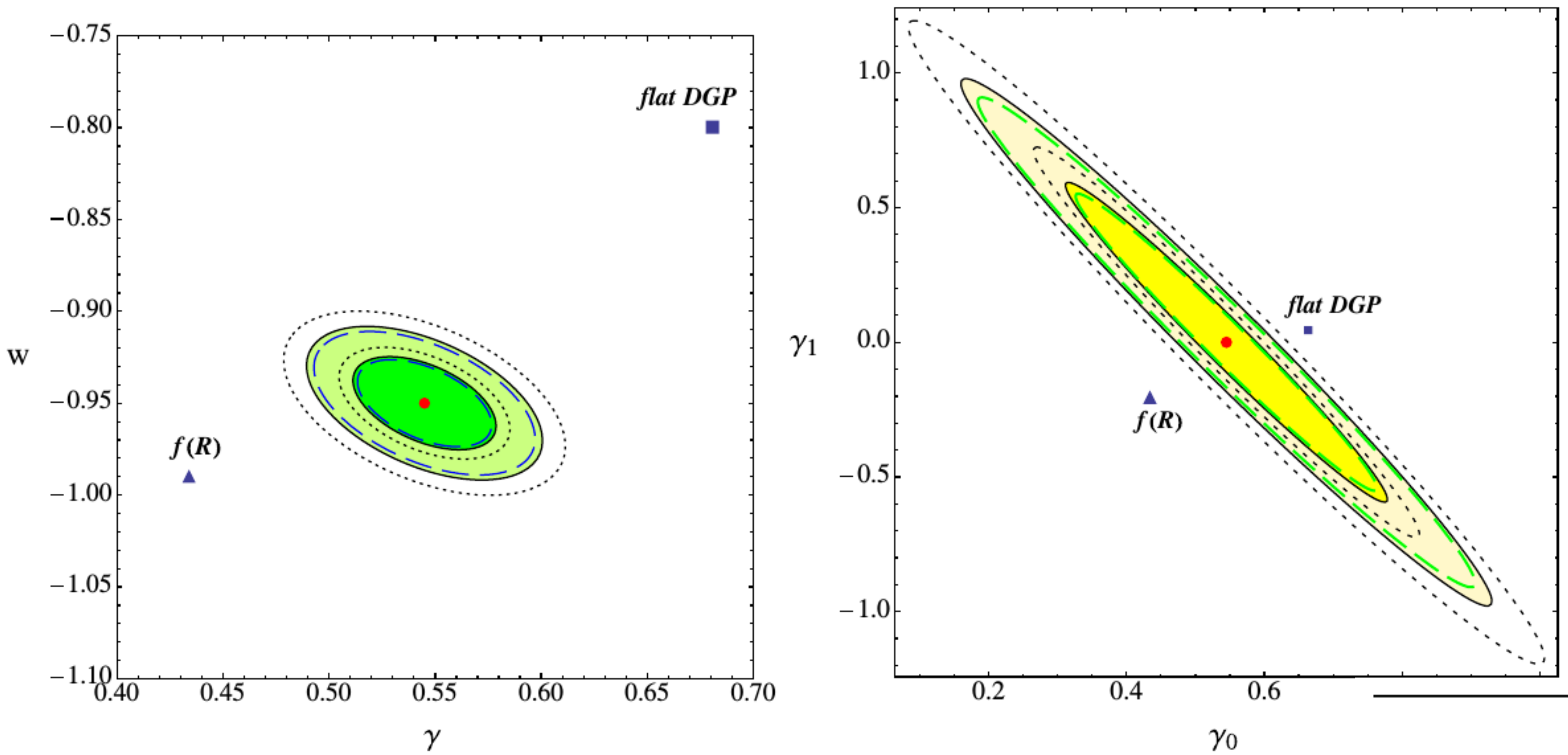
- Simple scenario: Euclid can measure fsigma8 (growth rate times amplitude of perturbations) at the 2-3% level, with marginalization over bias.
- Redshift bins >10, useful to rule out several MG theories.





Euclid forecasts for Dark Energy

- In simplest case: FoM for w0-wa ~ 400. If w=-1, gives a Bayes factor that favors strongly a cosmological constant.
- For phantom Dark Energy (w < -1), we need less precision to strongly rule out a cosmological constant.



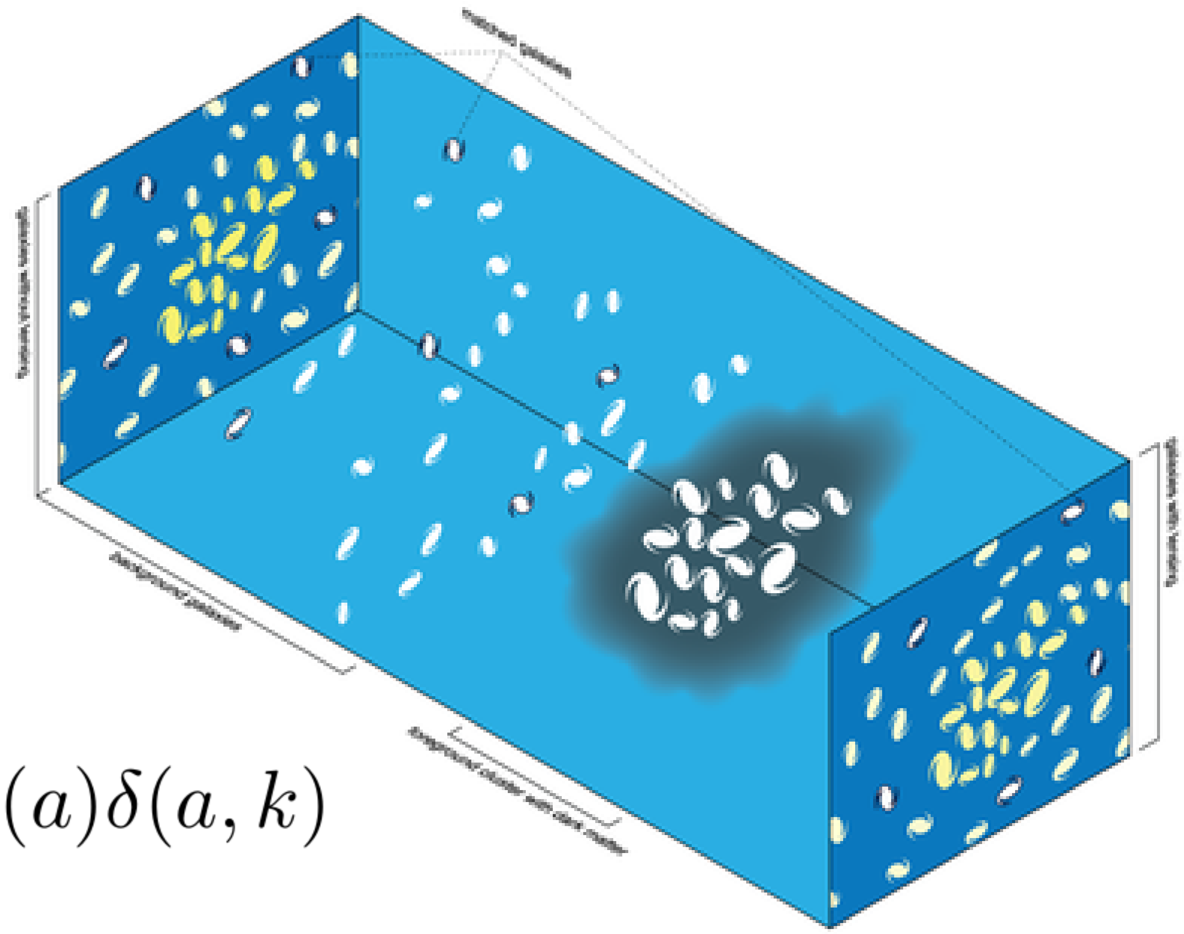
	Case	σ_γ	σ_w
$b=\sqrt{1+z}$	Ref.	0.02	0.017
With	Opt.	0.02	0.016
Ω_k fixed	Pess.	0.026	0.02

Here we have fixed Ω_k to its fiducial value, $\Omega_k = 0$

Weak Lensing

- Under the influence of matter-energy, galaxies align and get distorted in a coherent way.

- The correlation function of cosmic shear: valuable information about matter content and expansion.



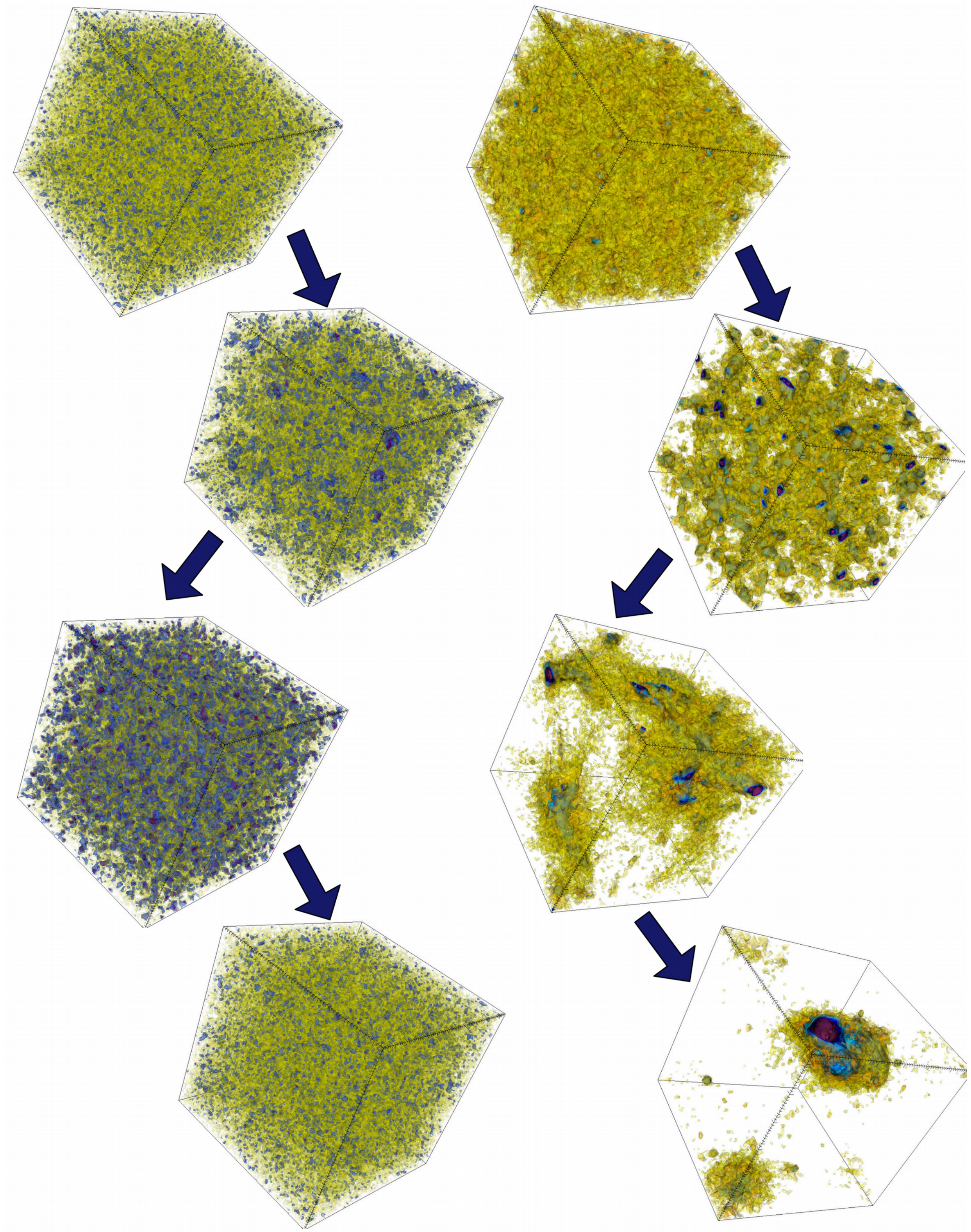
$$-k^2(\Phi(a,k)+\Psi(a,k)) \equiv 8\pi G a^2 \Sigma(a,k)\rho(a)\delta(a,k)$$

$$C_{ij}(\ell) = \frac{9}{4}\int_0^\infty \mathrm{d}z\, \frac{W_i(z)W_j(z)H^3(z)\Omega_m^2(z)}{(1+z)^4}\Sigma^2(\ell/r(z),z)\, P_m(\ell/r(z))$$

Image credit en.wikipedia.org/wiki/Weak_gravitational_lensing

Growing Neutrino Quintessence

- Coupling between Neutrino and Quintessence Dark Energy.
- Connects mass of Neutrino (few eV) to energy scale of Dark Energy.
- Oscillating Neutrino Structures.
- Very non-linear problem, relativistic N-body simulations.



Casas, Pettorino, Wetterich, arXiv: 1608.02358, PRD94, 103518