

erc



# Dark Energy and the Swampland



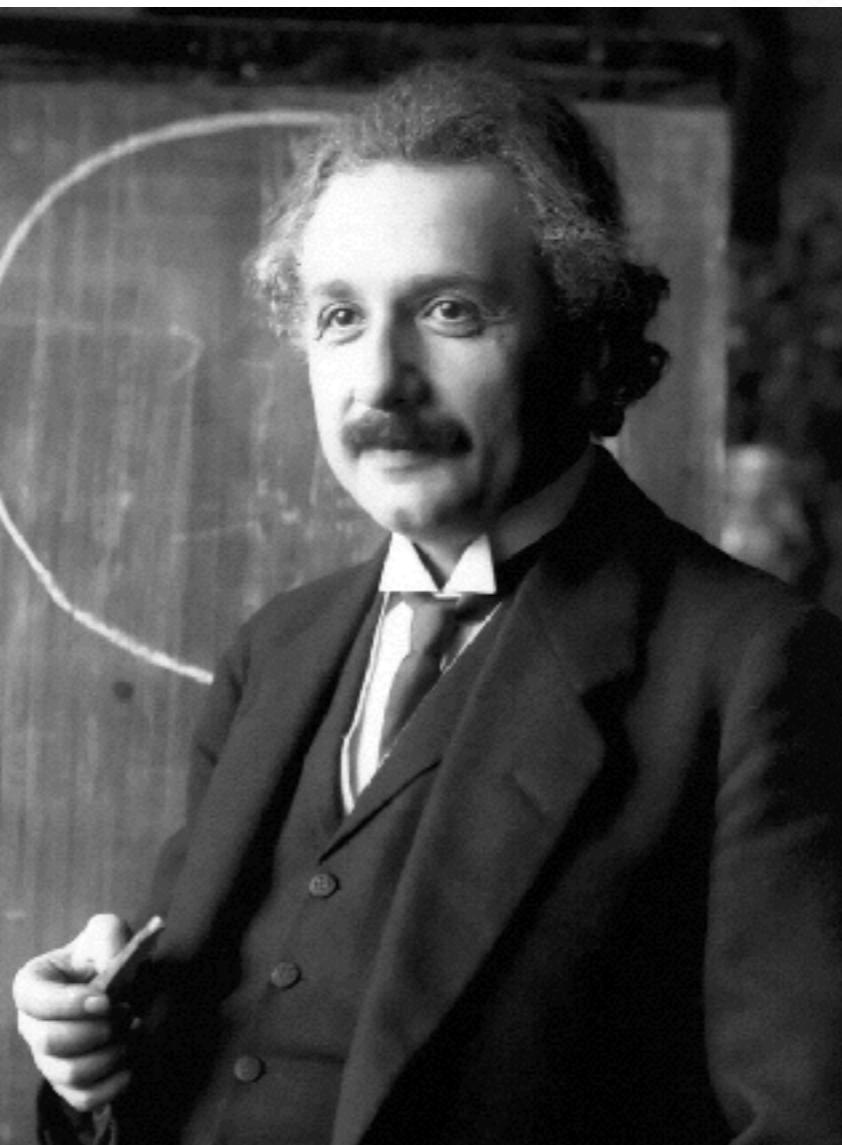
Lavinia Heisenberg  
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20th Nov 2019, Colloque national Action Dark Energy 2019, Paris/France

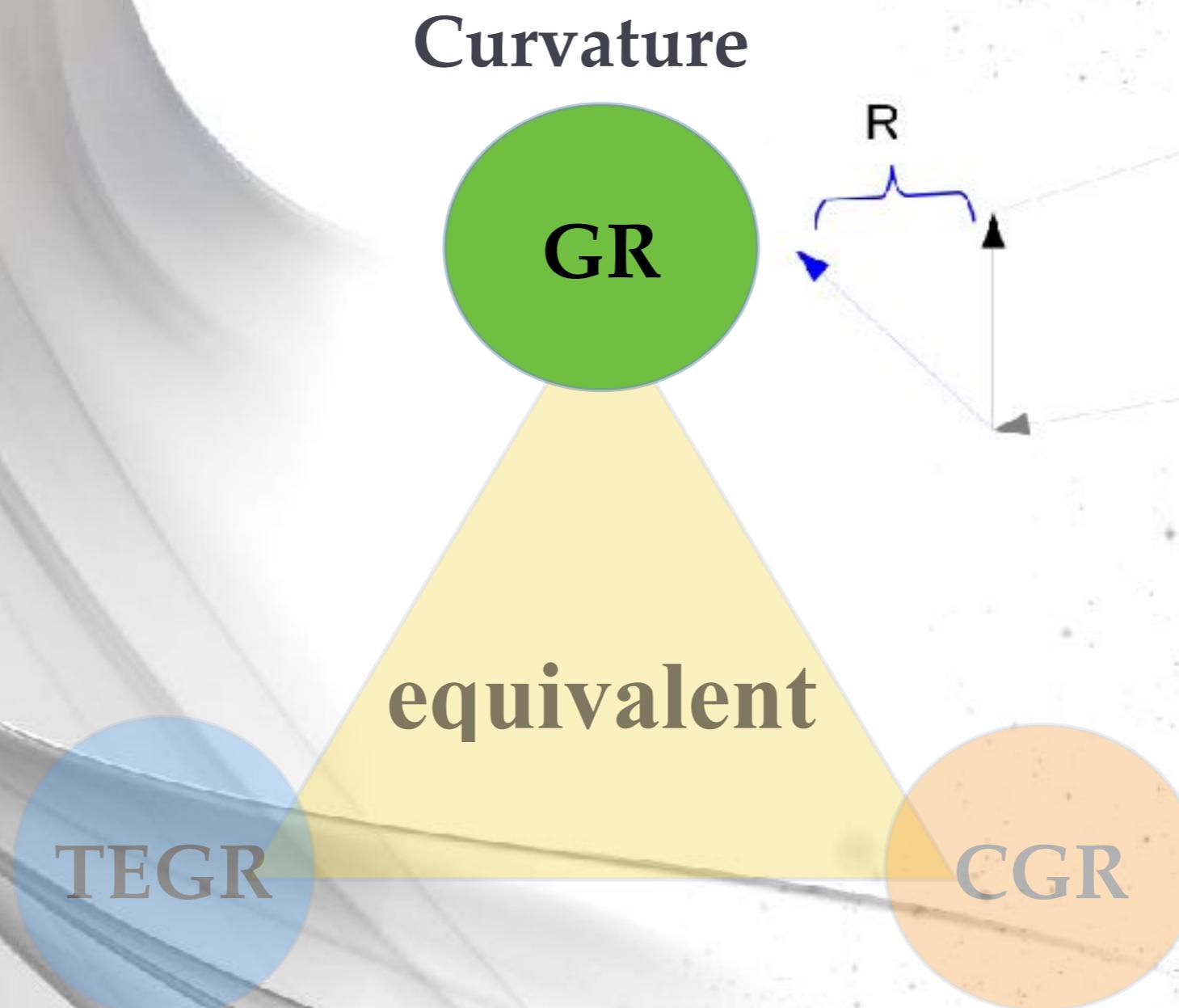
# General Relativity



Albert Einstein

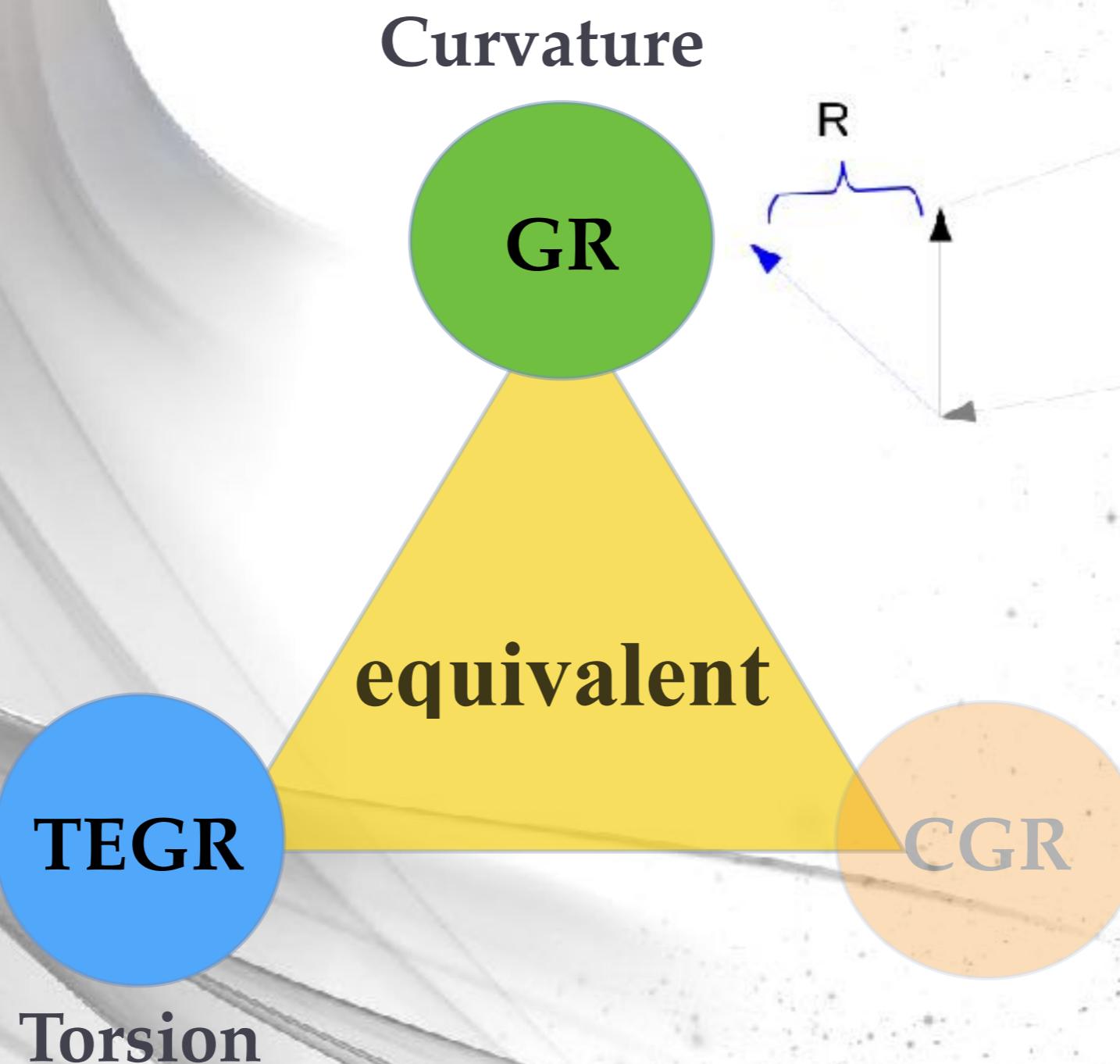
# Geometrical Trinity of General Relativity

L.H & J.Beltran, T.Koivisto  
Universe 5 (2019) 7, 173,  
arXiv:1903.06830



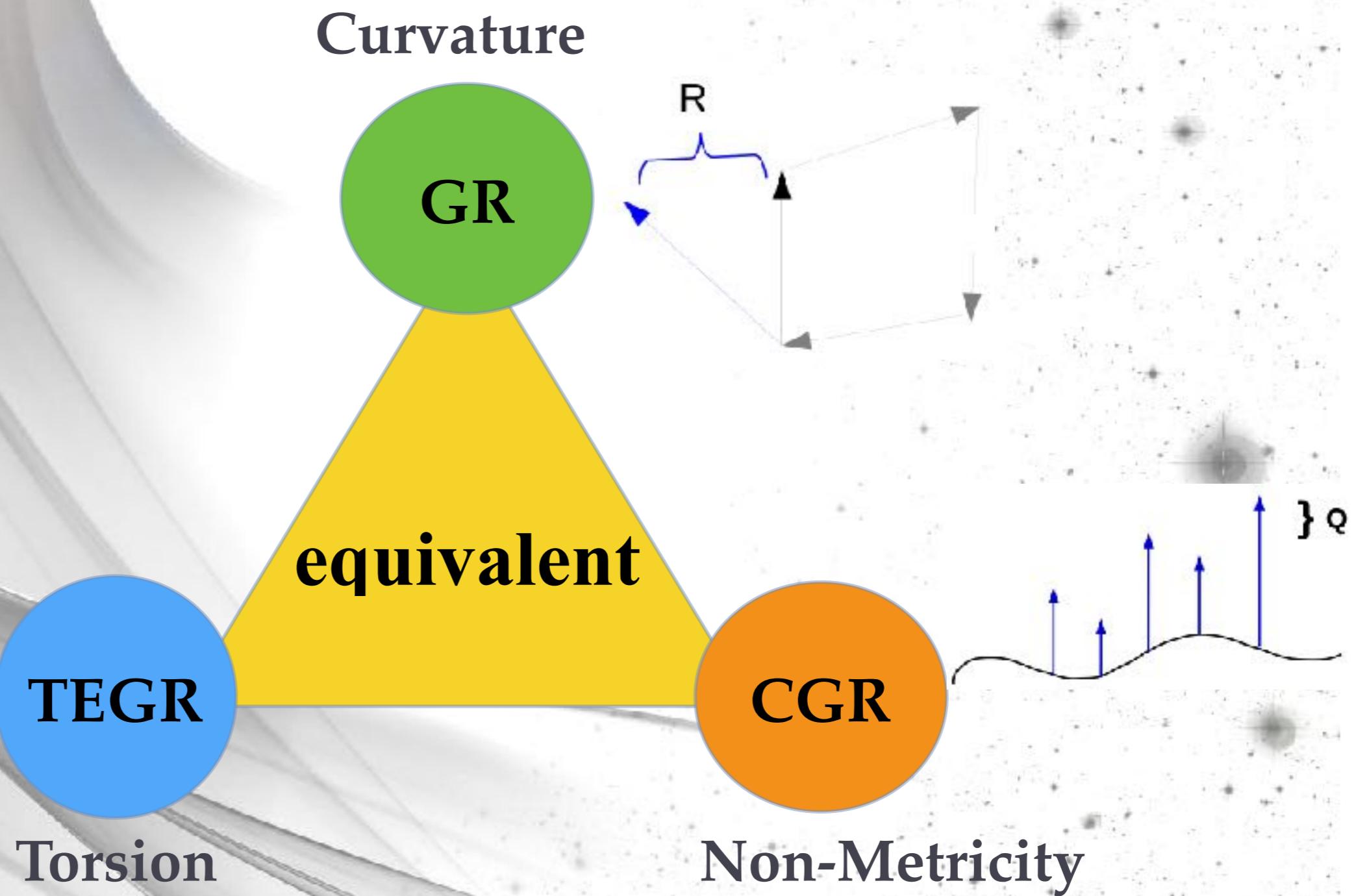
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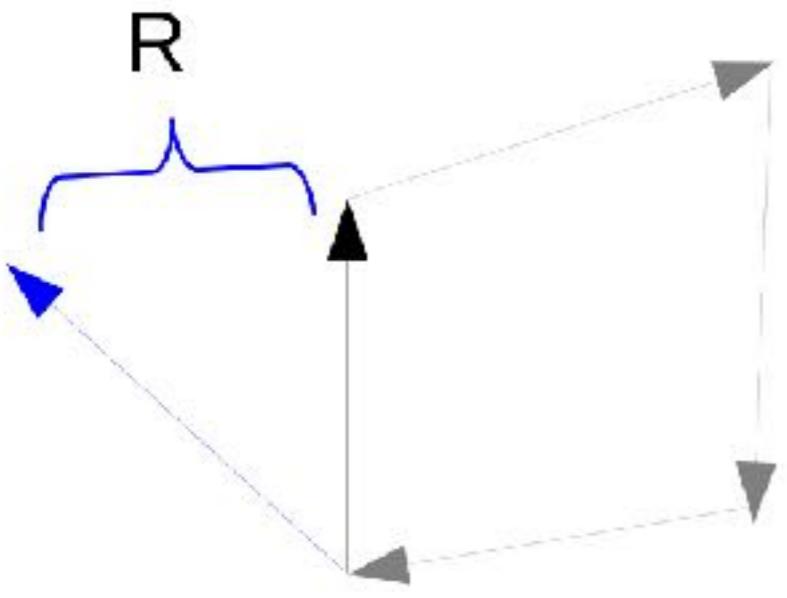


# Geometrical Trinity of General Relativity

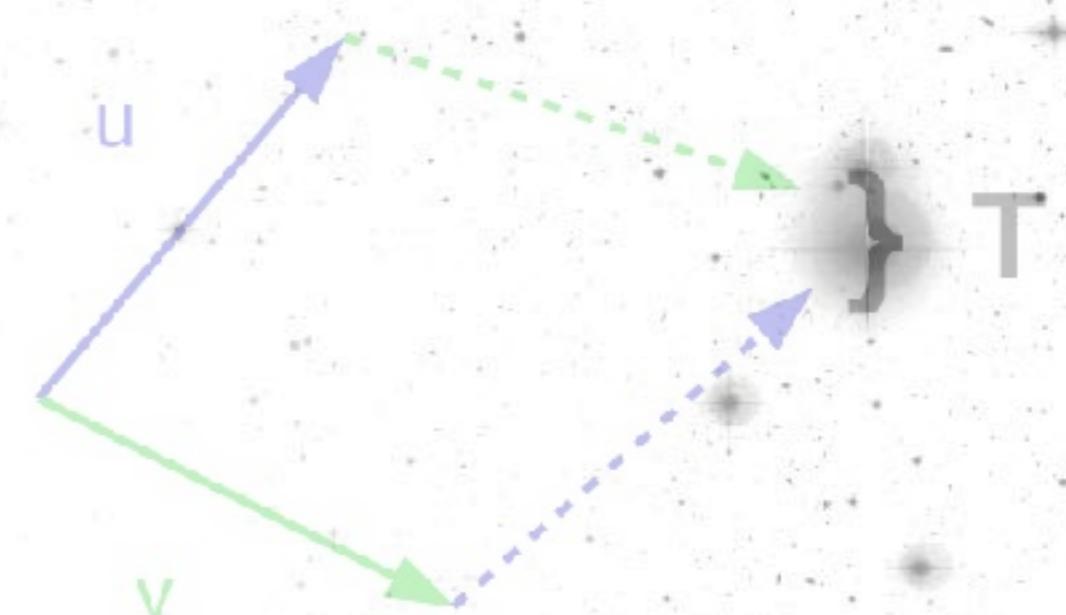
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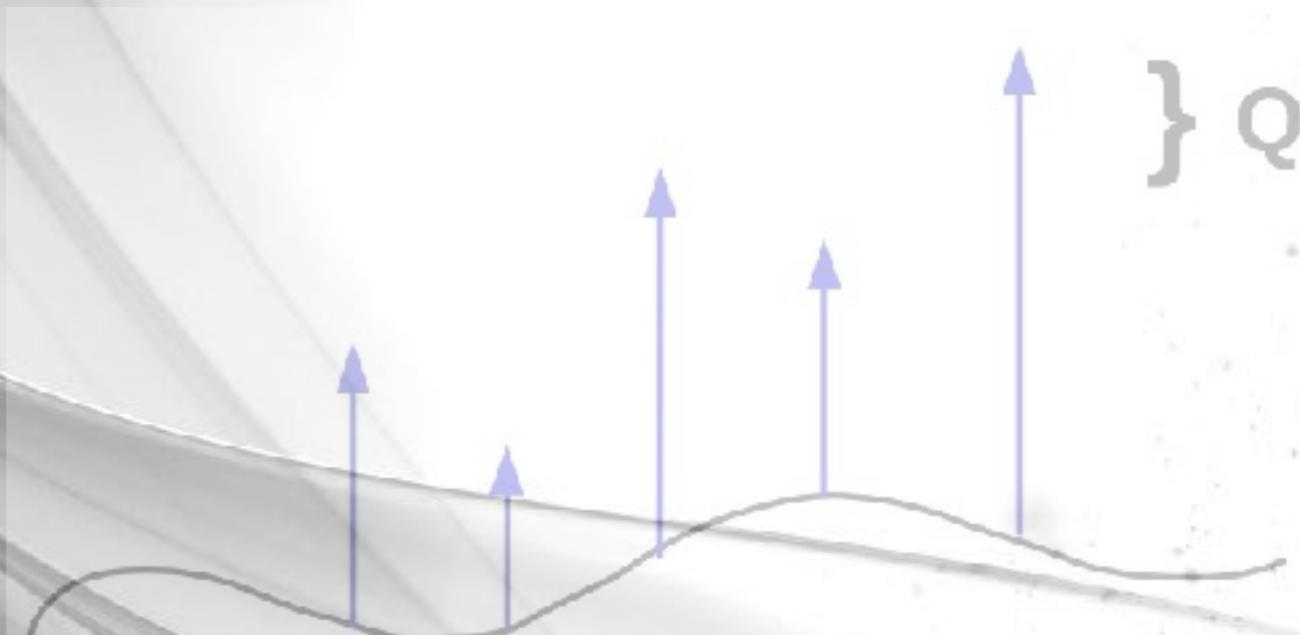
# Geometrical objects



- Curvature:  $R_{\beta\mu\nu}^{\alpha} \neq 0$

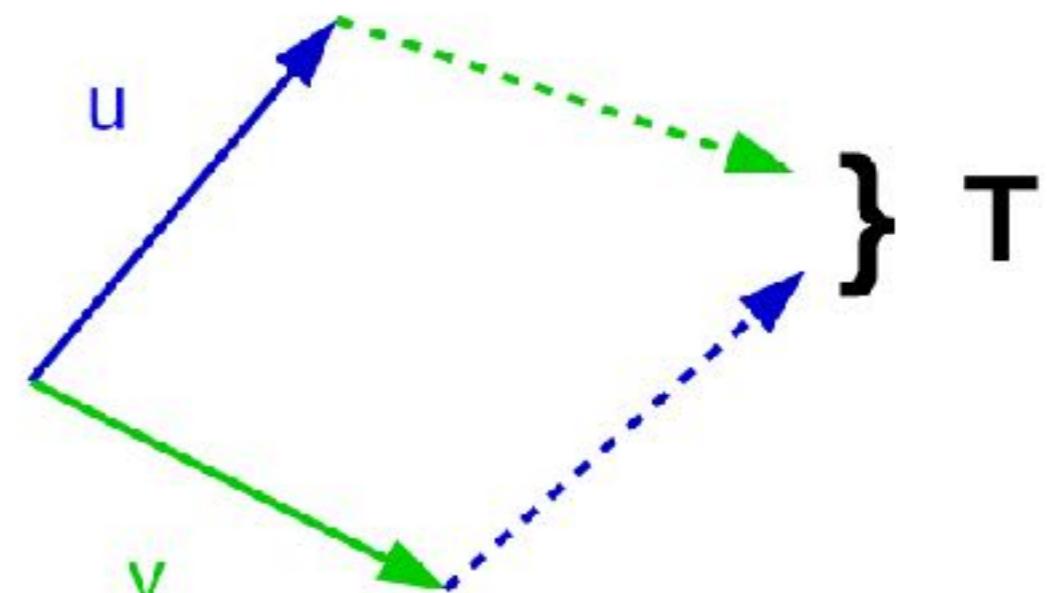
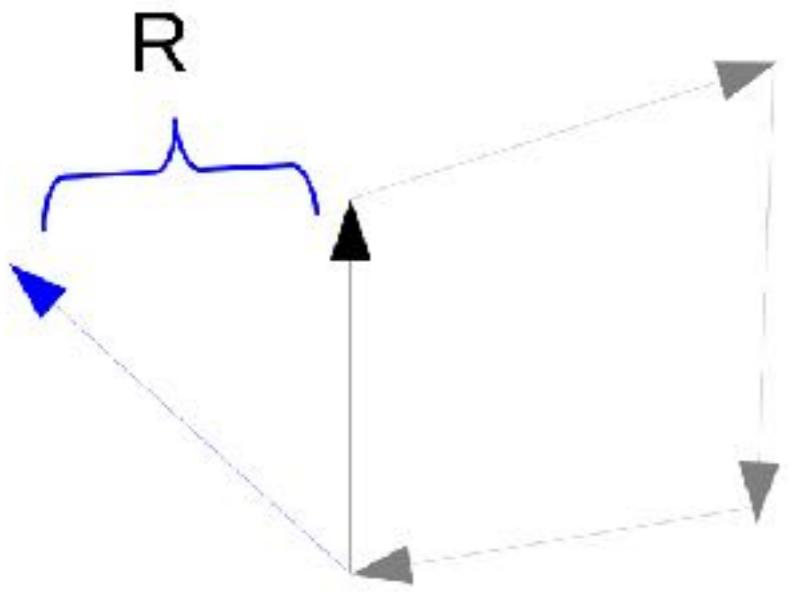


- Torsion:  $T_{\mu\nu}^{\alpha} \neq 0$



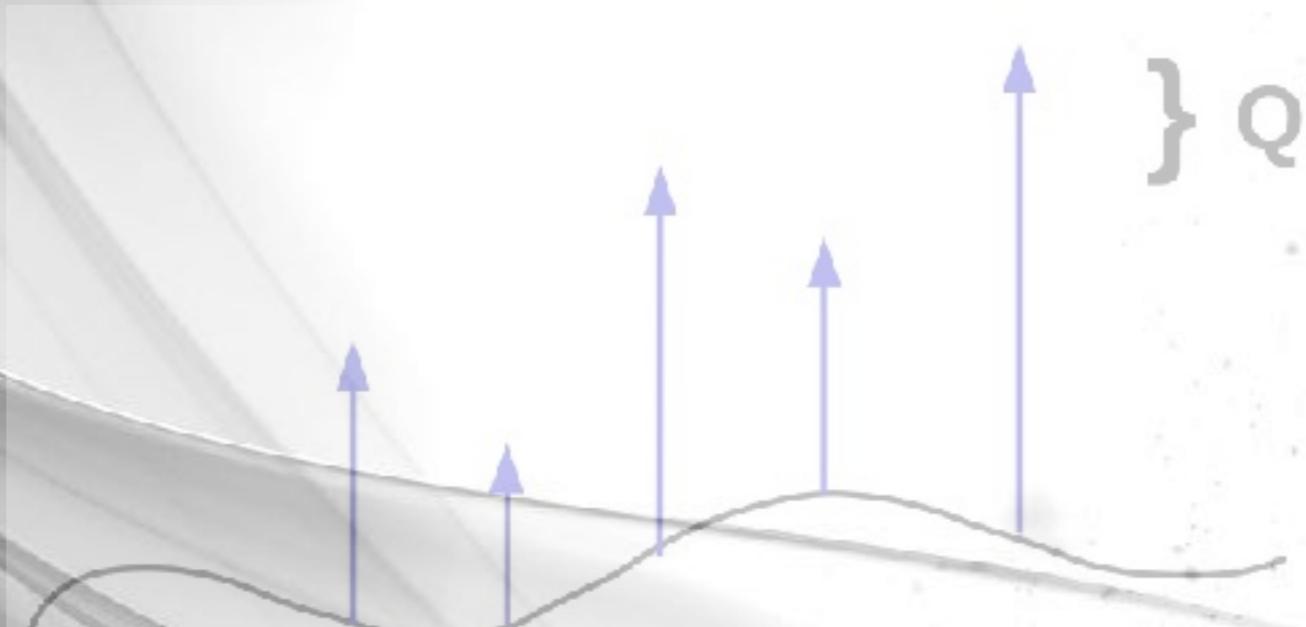
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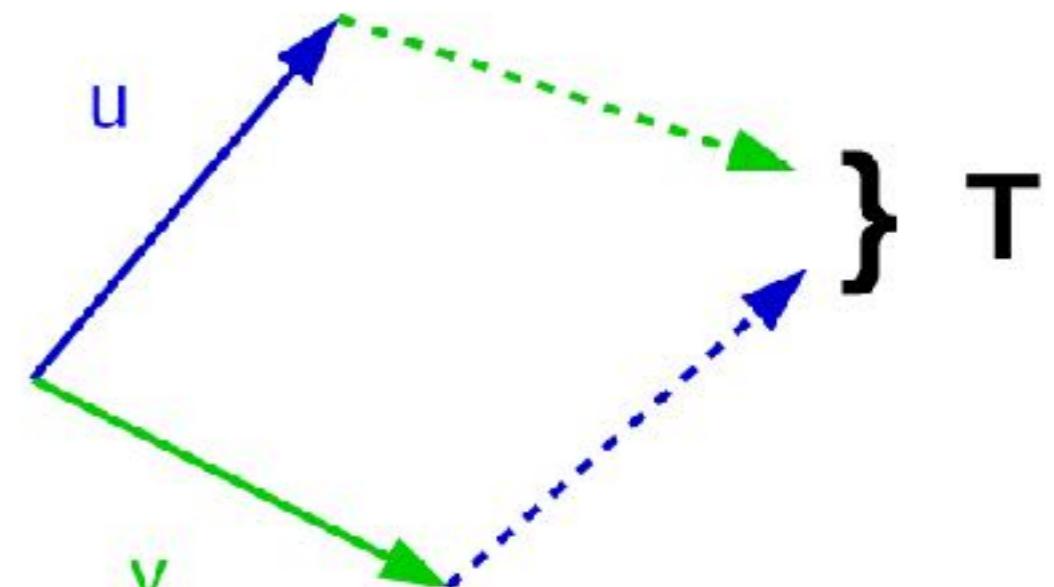
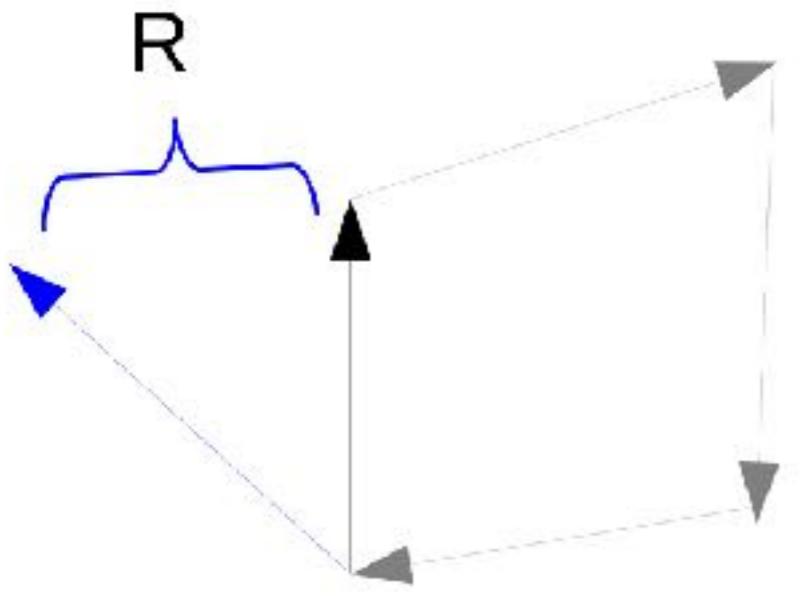
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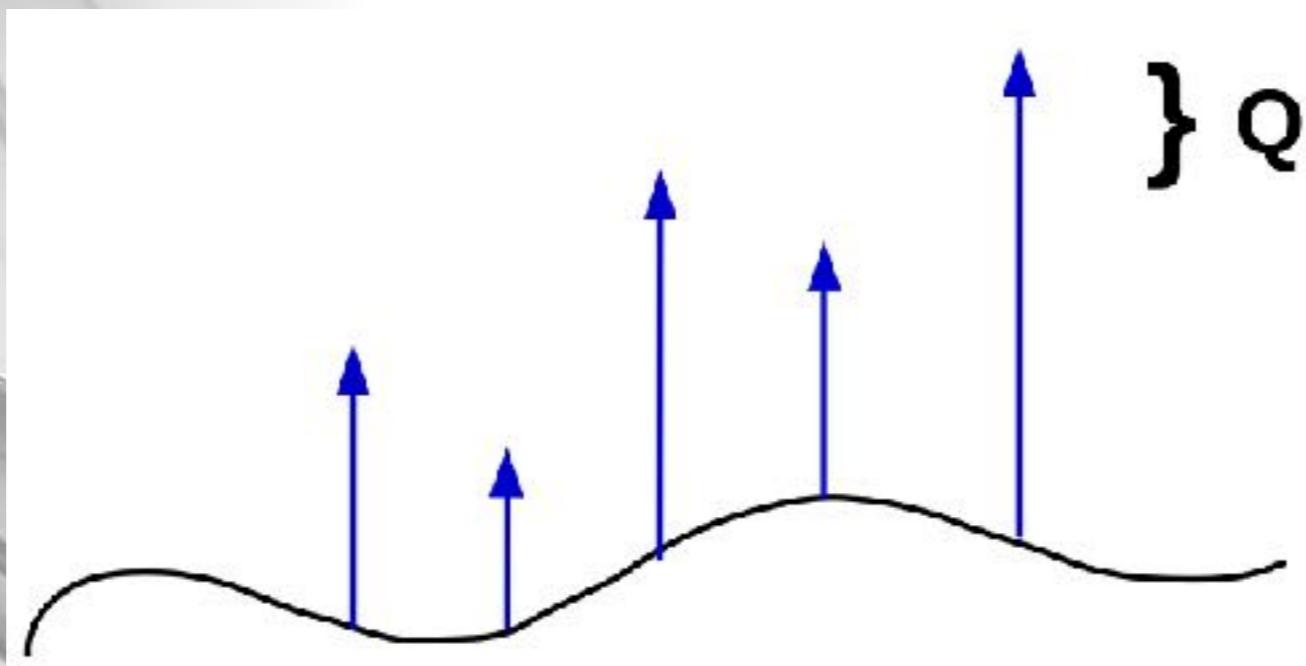


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- Curvature:



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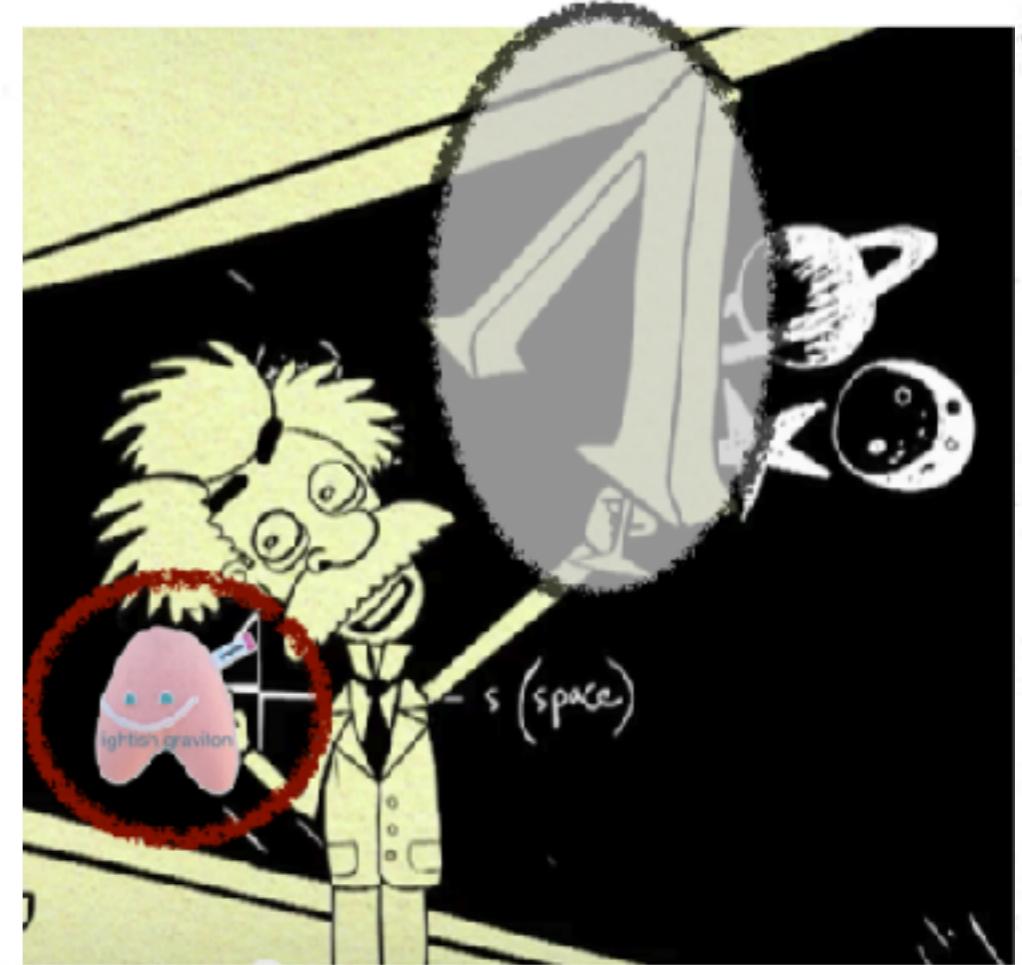
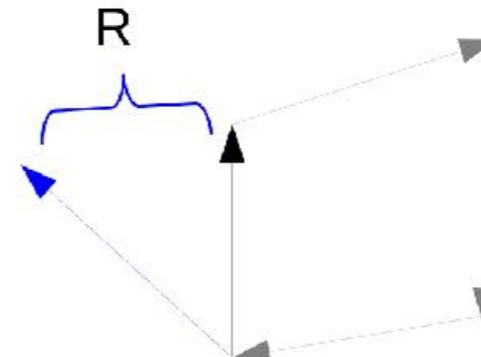
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GR

# General Relativity (Curvature)

(pseudo-) Riemannian manifold

$g_{\mu\nu}$



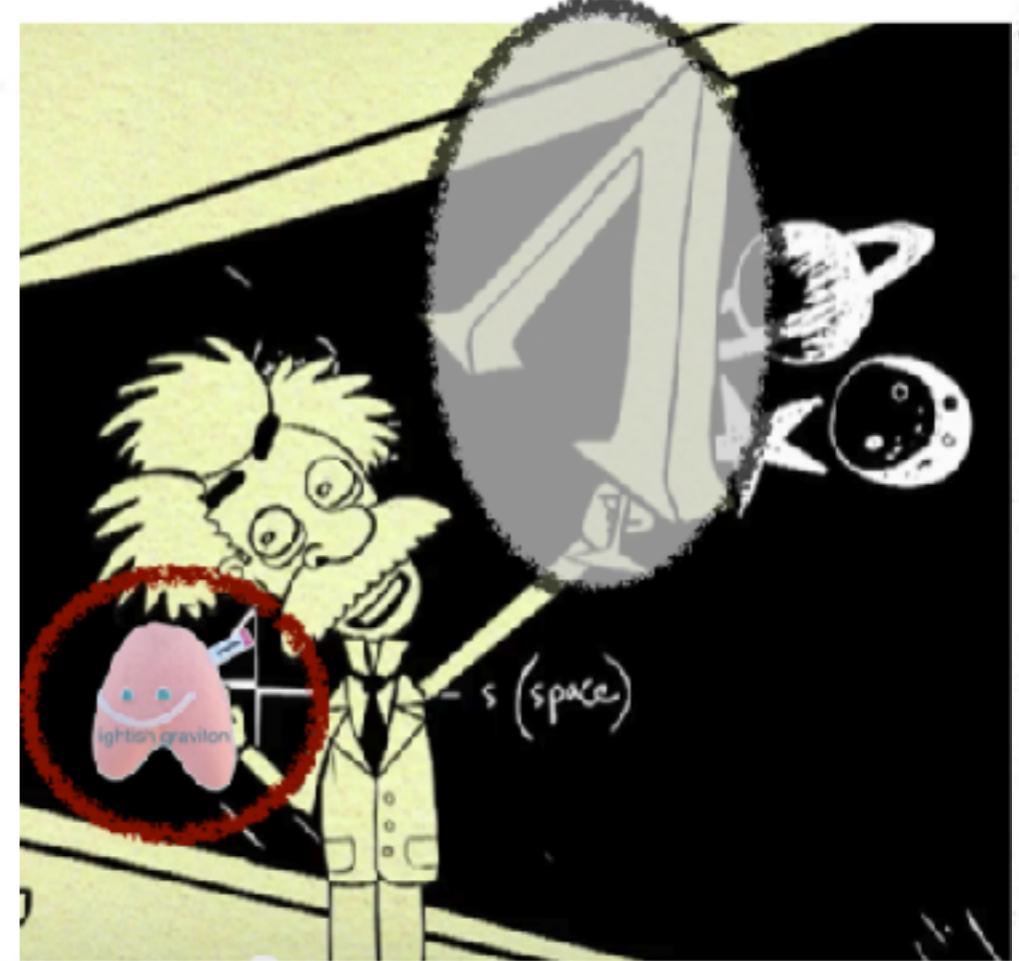
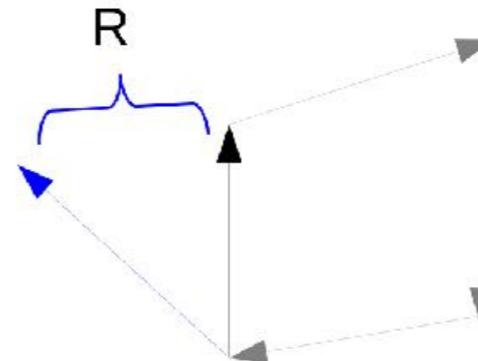
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$$\Gamma_{\mu\nu}^{\alpha} = \{^{\alpha}_{\mu\nu}\}$$



GR

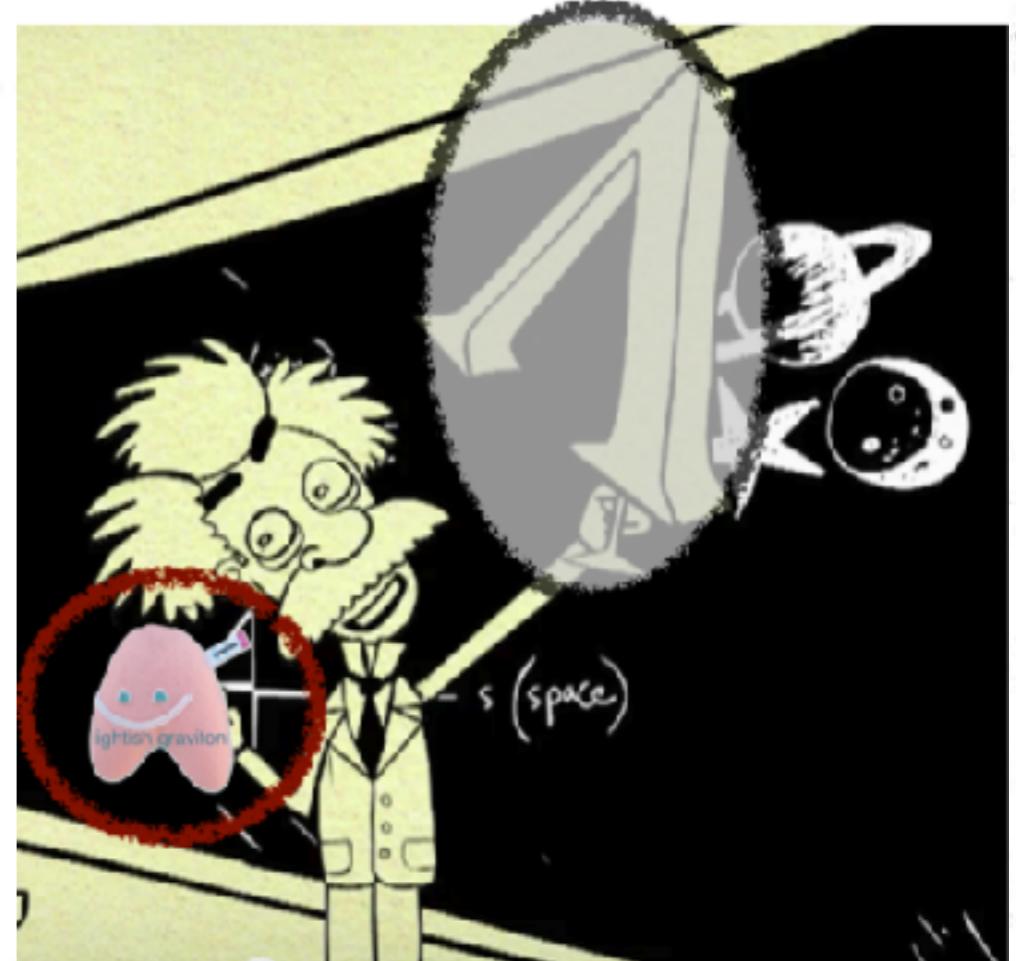
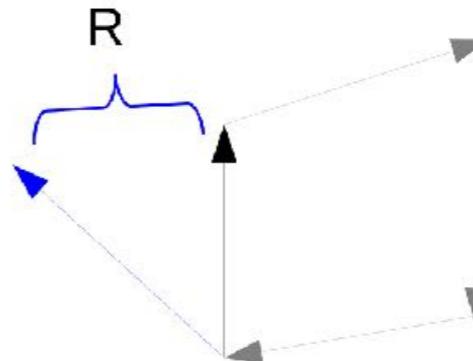
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GR

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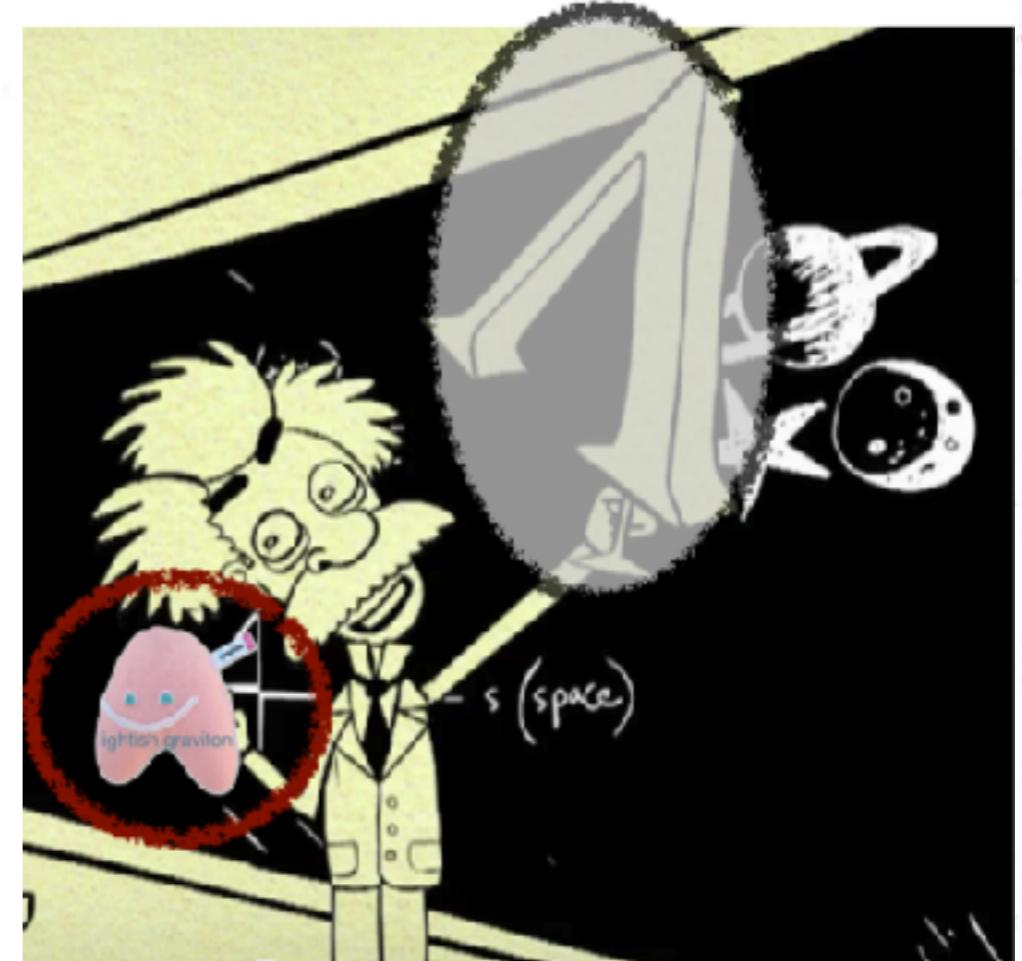
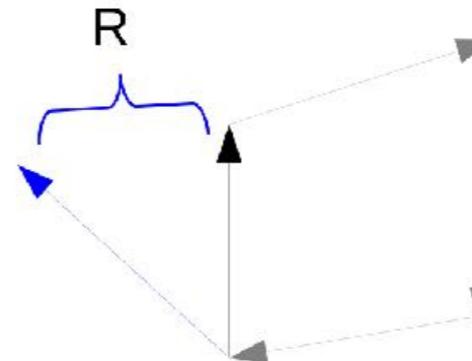
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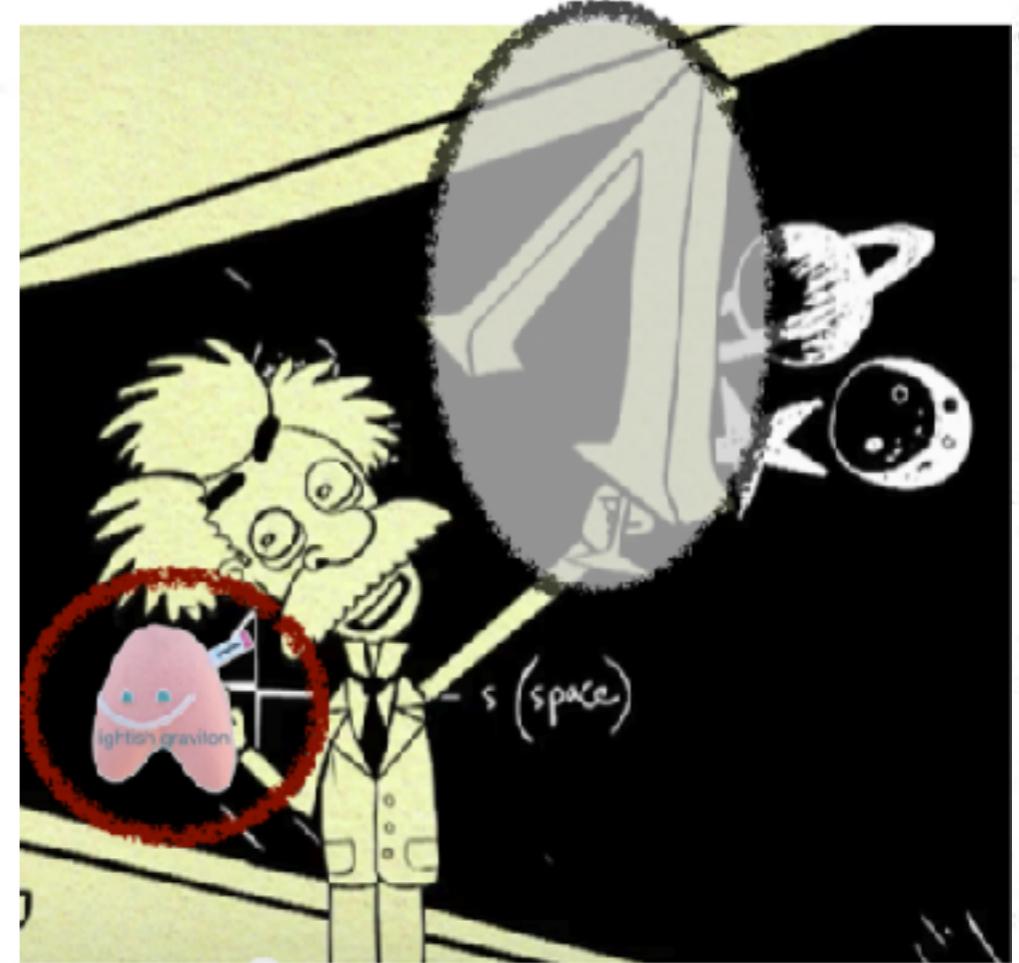
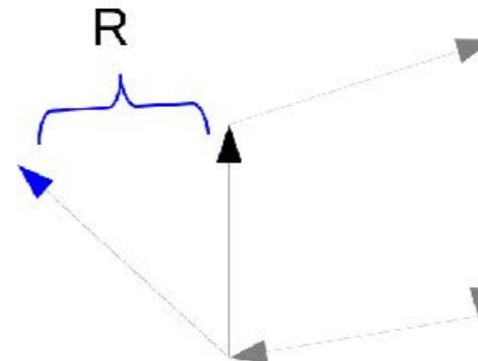
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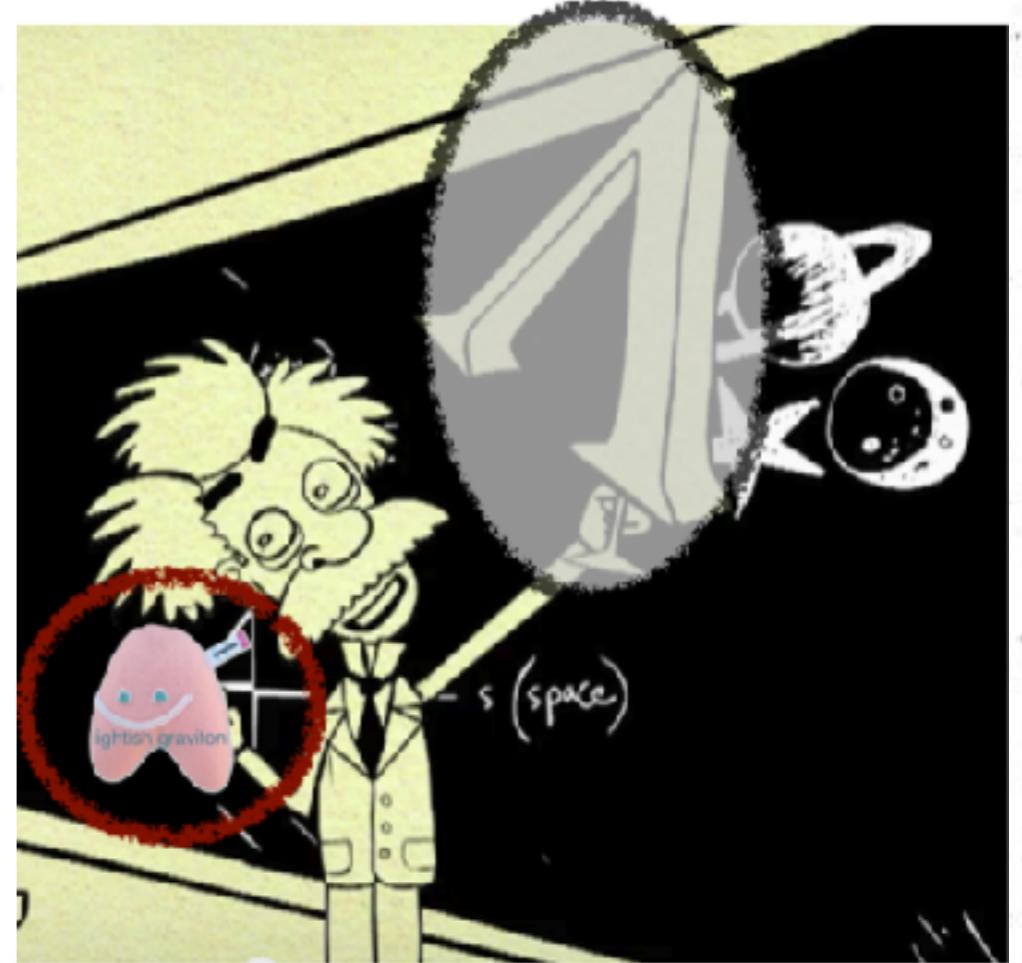
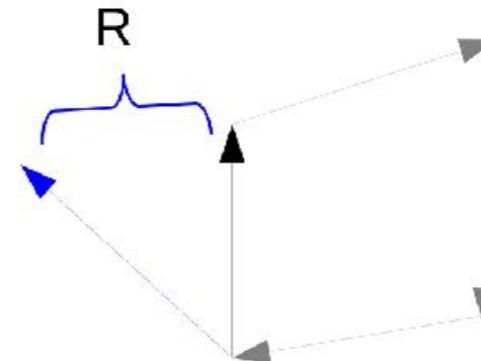


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$$R_{\beta\mu\nu}^{\alpha} = \partial_{\mu} \{_{\nu\beta}^{\alpha}\} - \partial_{\nu} \{_{\mu\beta}^{\alpha}\}$$

$$+ \{_{\mu\rho}^{\alpha}\} \{_{\nu\beta}^{\rho}\} - \{_{\nu\rho}^{\alpha}\} \{_{\mu\beta}^{\rho}\}$$

→ Einstein-Hilbert action



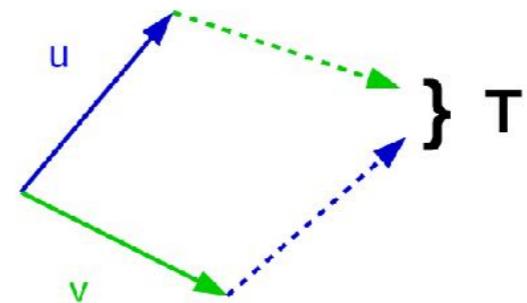
$$\mathcal{L}_g = \sqrt{-g} R \supset \partial^2 g$$

# TEGR (Torsion)

TEGR

a manifold based on torsion

$$\Gamma^\alpha_{\mu\nu}$$



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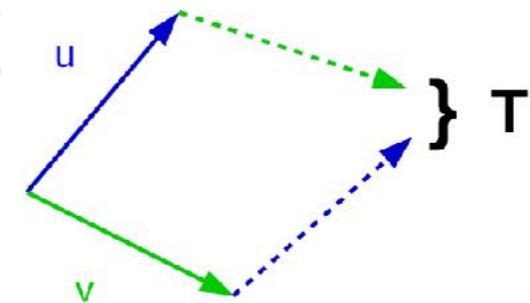
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$\Gamma_{\mu\nu}^\alpha$

$$\Gamma_{\mu\nu}^\alpha = \{\mu\nu\}^\alpha + K_{\mu\nu}^\alpha(T)$$

$$K_{\mu\nu}^\alpha = \frac{1}{2} T_{\mu\nu}^\alpha + T_{(\mu}{}^{\alpha}{}_{\nu)}$$

contorsion tensor



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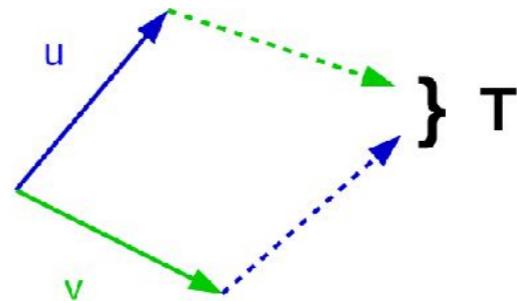
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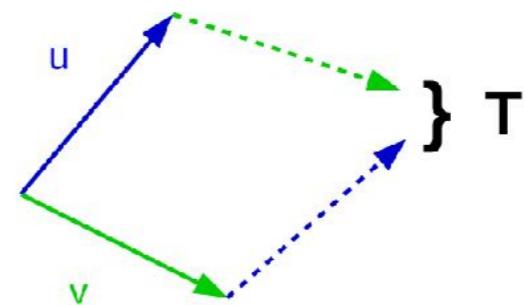
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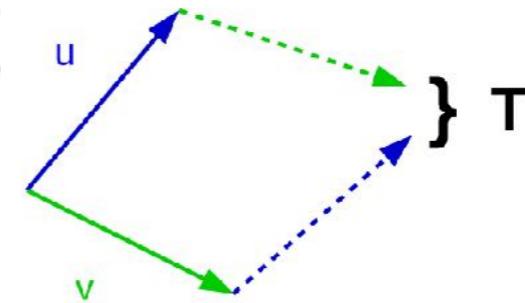
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contorsion tensor

$$\mathcal{S} = \frac{1}{2} \sqrt{-g} \left( c_1 T_\alpha{}^{\mu\nu} T^\alpha{}_{\mu\nu} + c_2 T_\alpha{}^{\mu\nu} T_\mu{}^\alpha{}_\nu + c_3 T_\mu T^\mu \right)$$

$$+ \lambda_\alpha^{\beta\mu\nu} R_{\beta\mu\nu}^\alpha + \tilde{\lambda}_\alpha^{\mu\nu} Q_\alpha{}^{\mu\nu}$$

$$c_1 = \frac{1}{4}, \quad c_2 = \frac{1}{2}, \quad c_3 = -1$$

→ equivalent to GR!

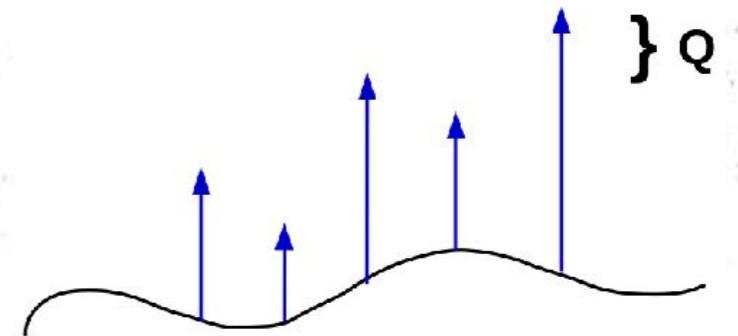
CGR

# CGR (Non-metricity)

L.H & J.Beltran, T.Koivisto  
PRD98 (2018) 4, 044048,  
arXiv:1803.10185

a manifold based on non-metricity

$g_{\mu\nu}, \Gamma_{\mu\nu}^\alpha$



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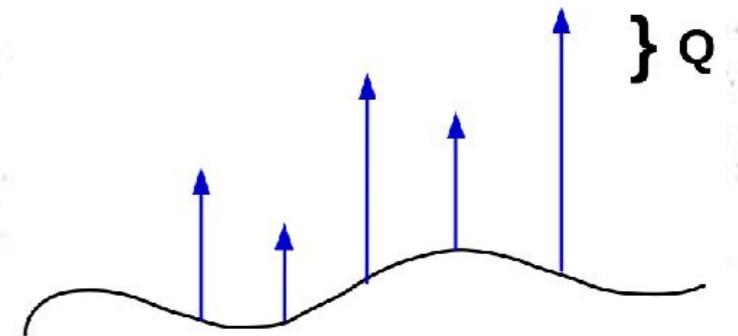
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disformation tensor

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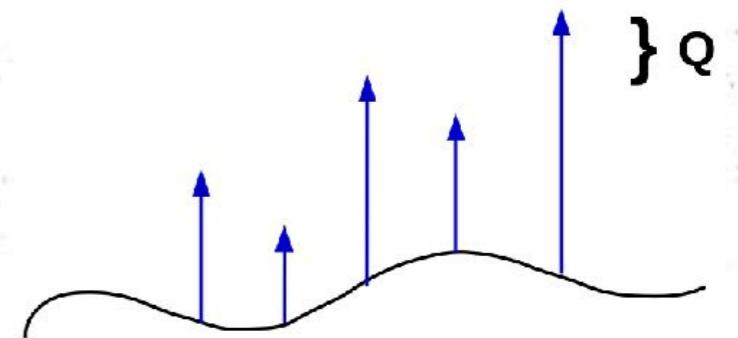
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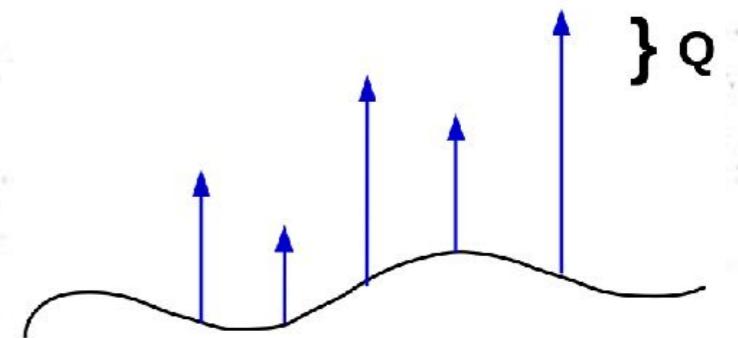
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$$\nabla_\alpha g_{\mu\nu} = Q_{\alpha\mu\nu}$$



Non-metricity

CGR

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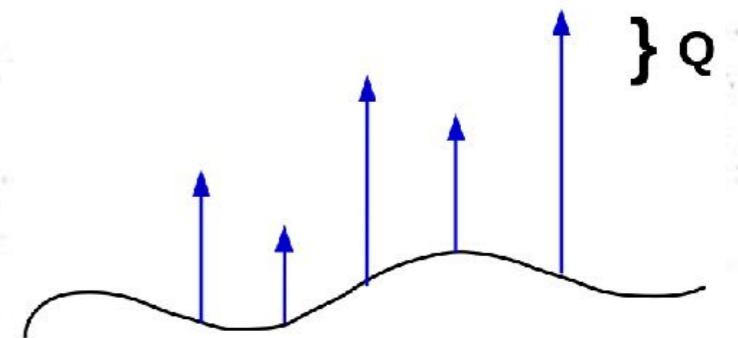
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$$\mathcal{S} = \int d^4x \sqrt{-g} \sum_{i=1}^5 c_i Q_i^2 + \lambda_\alpha{}^{\beta\mu\nu} R^\alpha{}_{\beta\mu\nu} + \tilde{\lambda}_\alpha{}^{\mu\nu} T^\alpha{}_{\mu\nu}$$

$$c_1 = -c_3 = -\frac{1}{2}c_2 = -\frac{1}{2}c_5 = -\frac{1}{4} \text{ and } c_4 = 0 \rightarrow \text{equivalent to GR!}$$

CGR

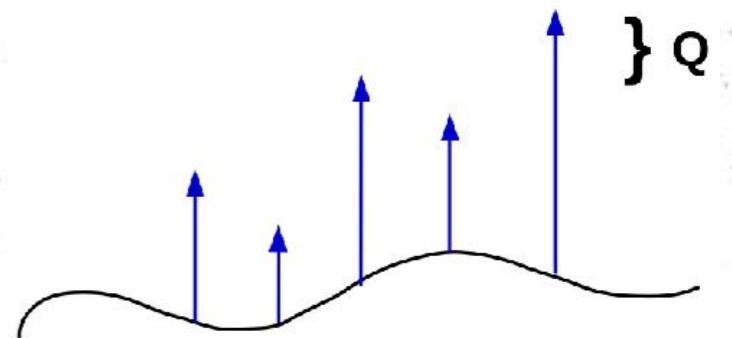
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- Curvature:  $R_{\beta\mu\nu}^\alpha = 0$
  - Torsion:  $T_{\mu\nu}^\alpha = 0$
- } huge consequences!

the connection can be set to zero by means of a gauge choice

$$\boxed{\Gamma_{\mu\nu}^\alpha = 0}$$

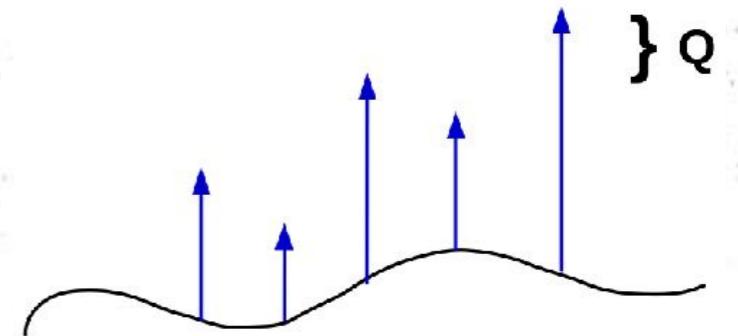
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$$\Gamma_{\mu\nu}^\alpha = \{\mu\nu\}^\alpha + L_{\mu\nu}^\alpha(Q) = 0$$



- No need for Gibbons-Hawking-York boundary term for a well-defined variational principle
- Just partial derivatives → easier for calculation purposes
- All points are equivalent!
- New tool to compute the Gravitational Energy?
- Improved and unambiguous entropy of Black Holes?
- New tool to canonically quantize General Relativity?

# Modified gravity (geometrical perspective)

- Promote the scalars to general functions

$$\begin{aligned}\int \sqrt{-q} \mathcal{R} &\rightarrow \int \sqrt{-q} f(\mathcal{R}) \\ \int \sqrt{-q} \mathring{\mathbb{T}} &\rightarrow \int \sqrt{-q} f(\mathring{\mathbb{T}}) \\ \int \sqrt{-q} \mathring{\mathcal{Q}} &\rightarrow \int \sqrt{-q} f(\mathring{\mathcal{Q}})\end{aligned}$$

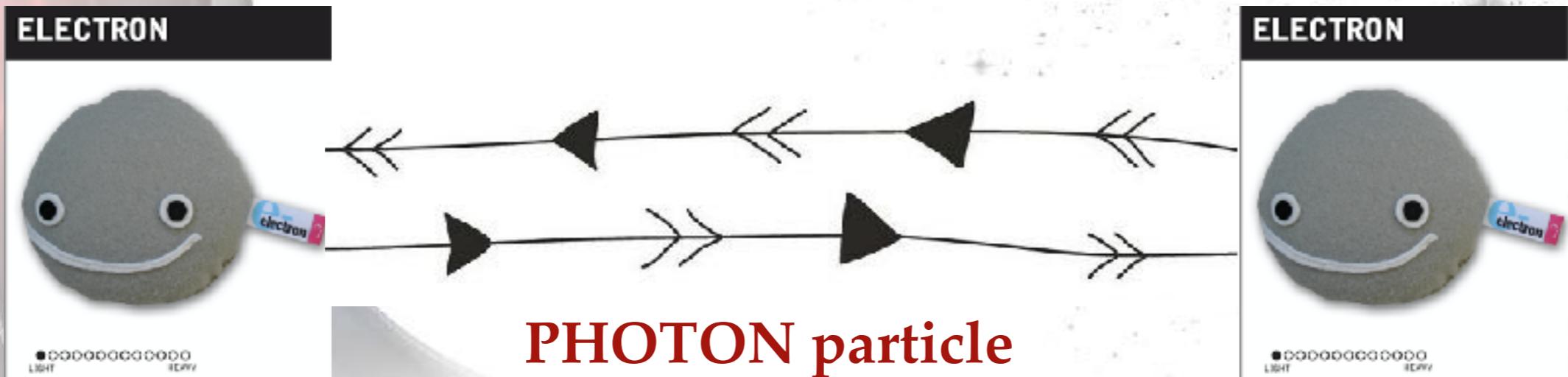
- Other consistent quadratic actions?

$a_i$   $b_i$   $c_i$

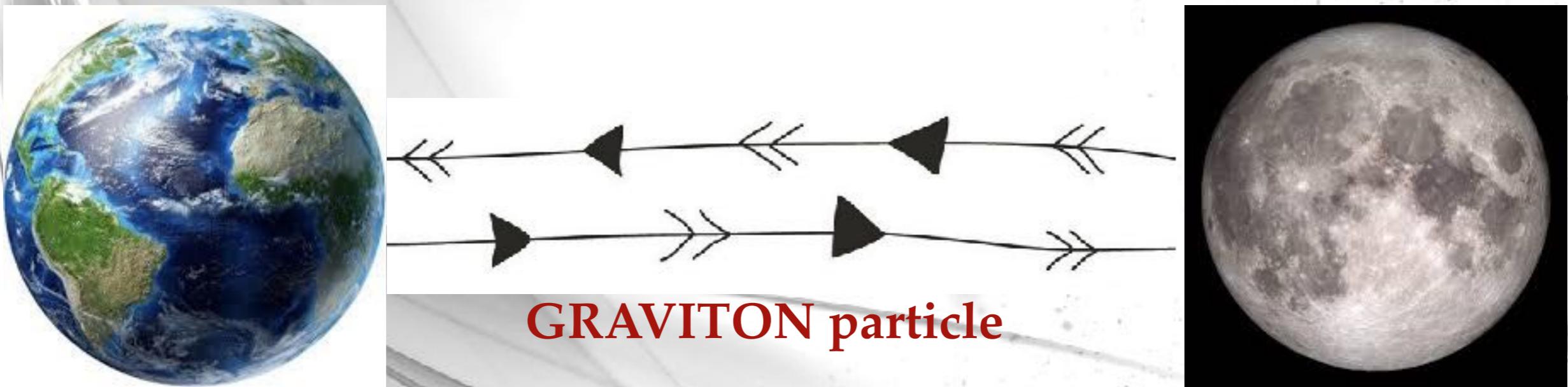
$$\mathcal{S} = \int d^4x \sqrt{-g} \left( \sum_{i=1}^5 a_i Q_i^2 + \sum_{i=1}^3 b_i T_i^2 + \sum_{i=1}^3 c_i Q_i T_i \right) + \lambda_\alpha{}^{\beta\mu\nu} R^\alpha{}_{\beta\mu\nu}$$

# Particle Physics Perspective

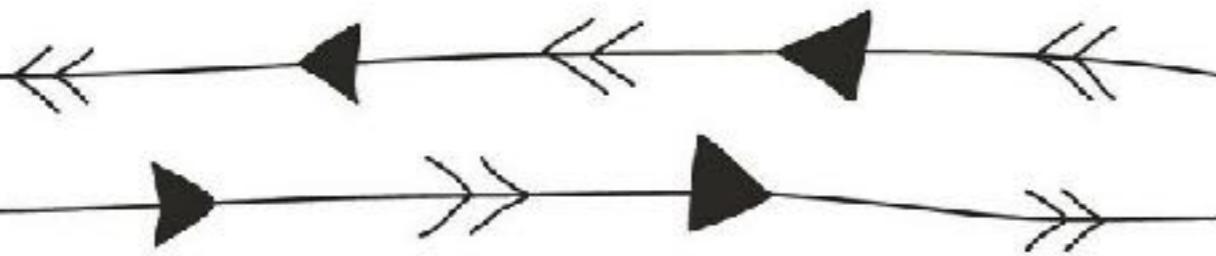
- Electromagnetic interactions



- Gravitational interactions



# Particle Physics Perspective



**GRAVITON particle**



**What is the most natural candidate for  
the GRAVITON particle?**

## Facts that we know about gravity:

### Gravity

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This leaves massless spin-2 field as a natural candidate!

→ General Relativity

# A model of the Universe

$$G_{\mu\nu} = \frac{\mathcal{T}_{\mu\nu}}{M_{\text{Pl}}^2}$$

The equations of General Relativity are very difficult to solve.

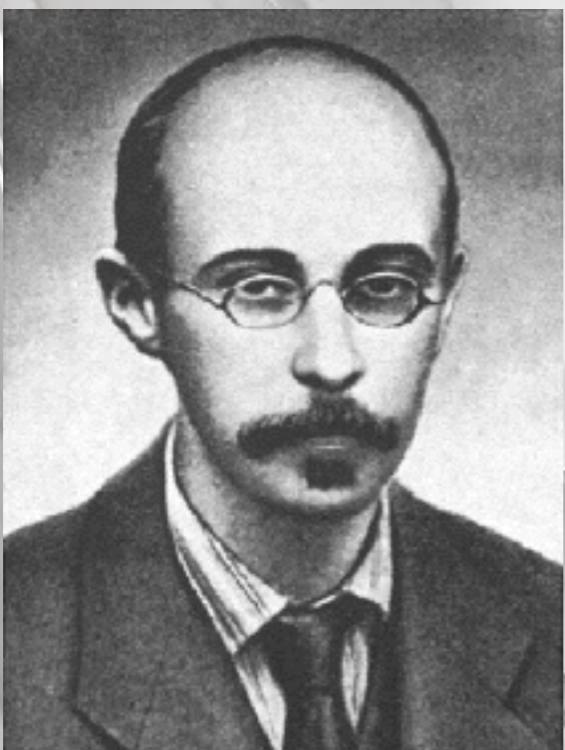
Make simplified assumptions!

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Make simplified assumptions!



homogeneous and  
isotropic universe!

spherically symmetric  
for every observer  
(FLRW model)



Alexander Friedmann

Georges Lemaître

# Budget of the Universe

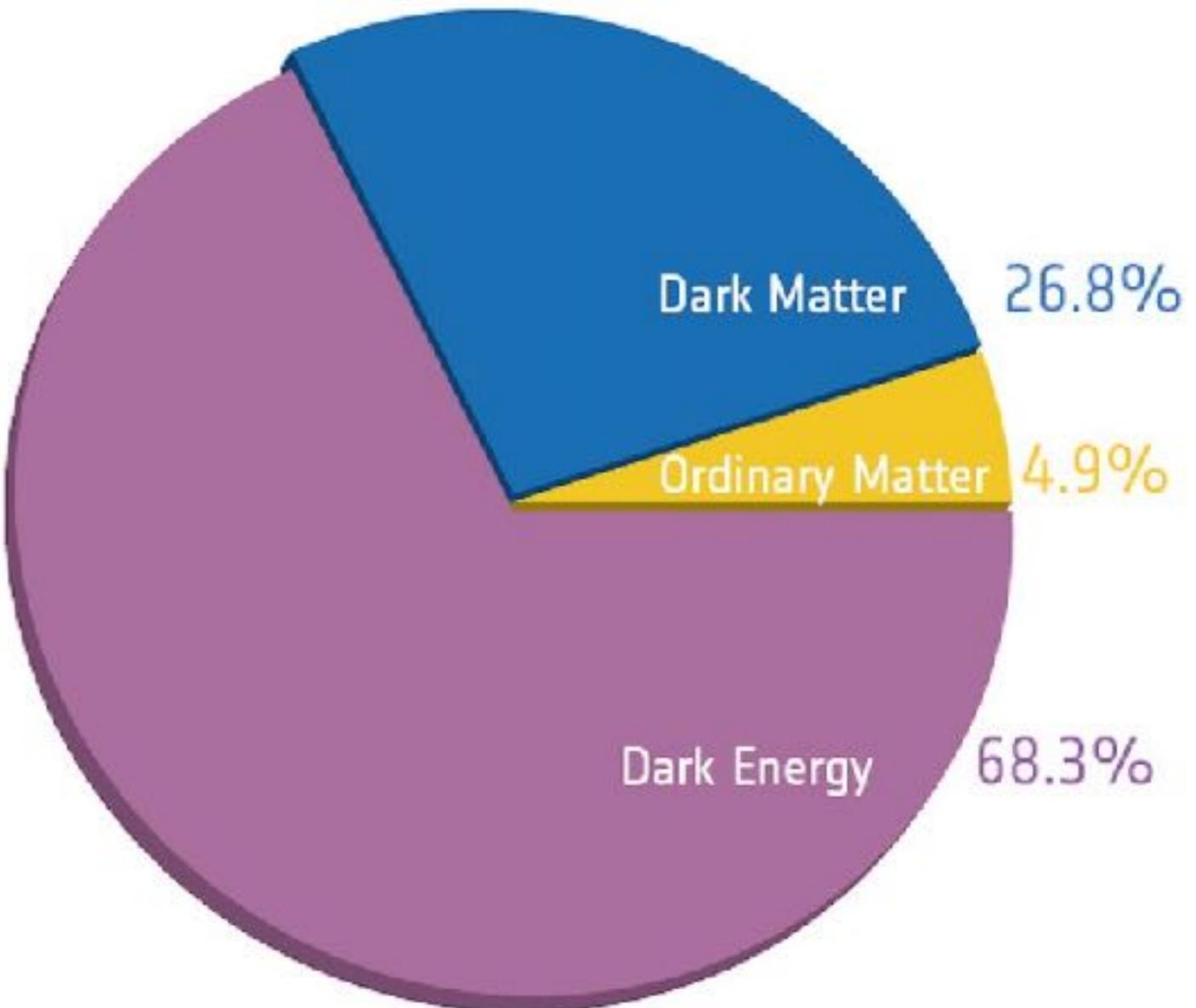
GR + hom. & iso.

Observations indicate

$$G_{\mu\nu} = \frac{\mathcal{T}_{\mu\nu}}{M_{\text{Pl}}^2}$$

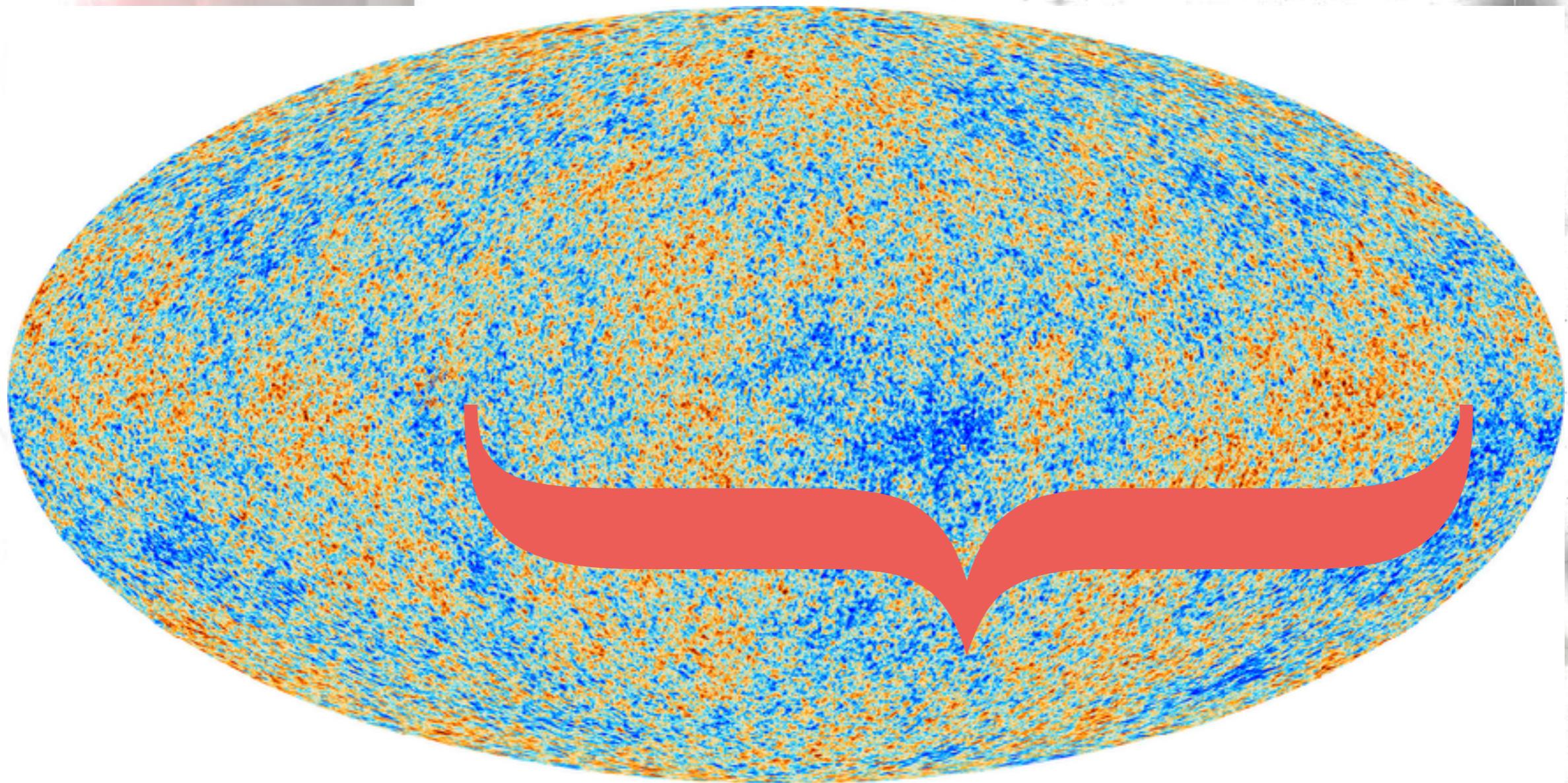
DM

DE

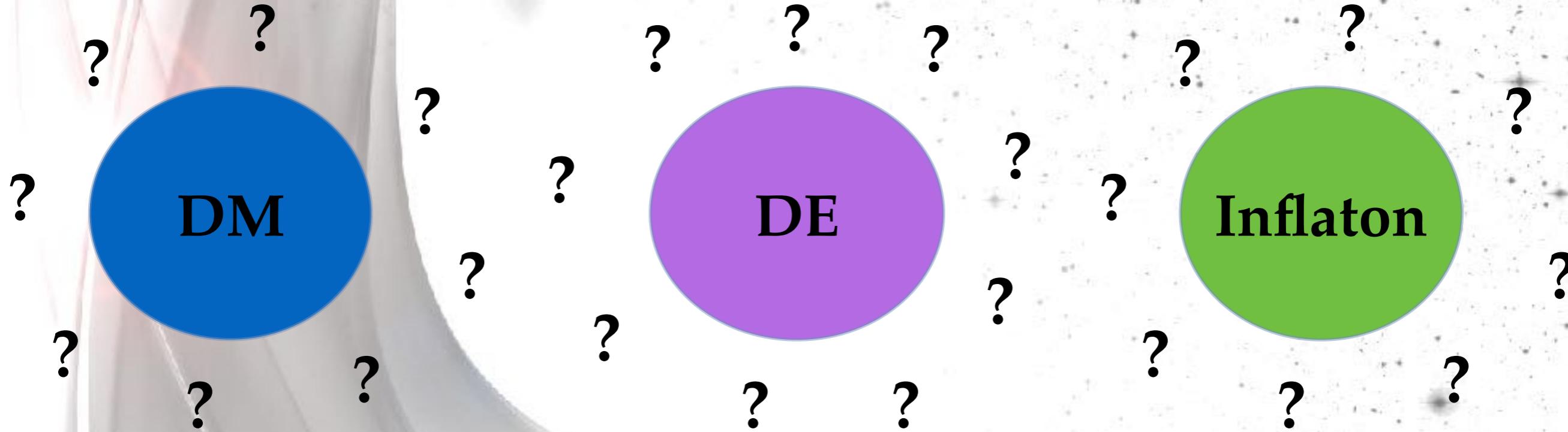


# A model of the Universe

Residual heat: cosmic microwaves



**INFLATON: early phase of accelerated expansion!**

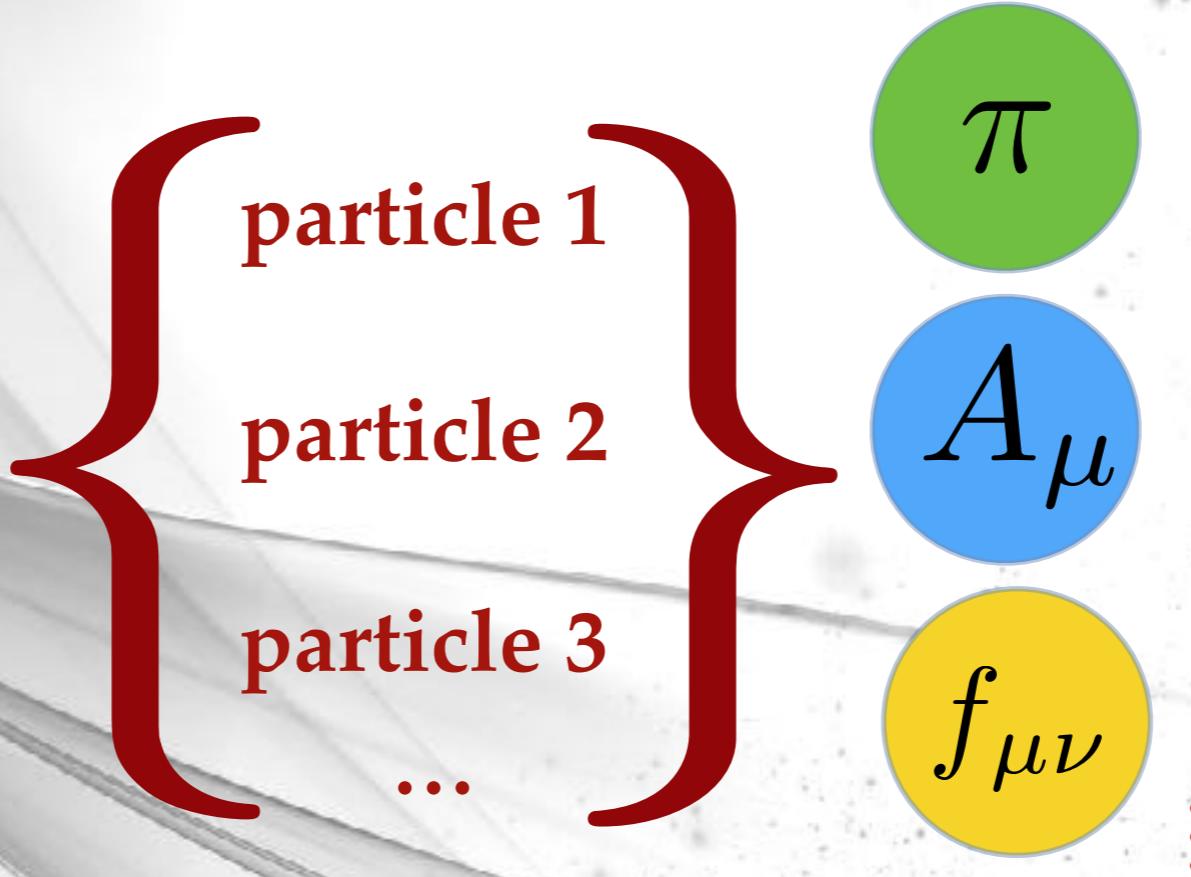
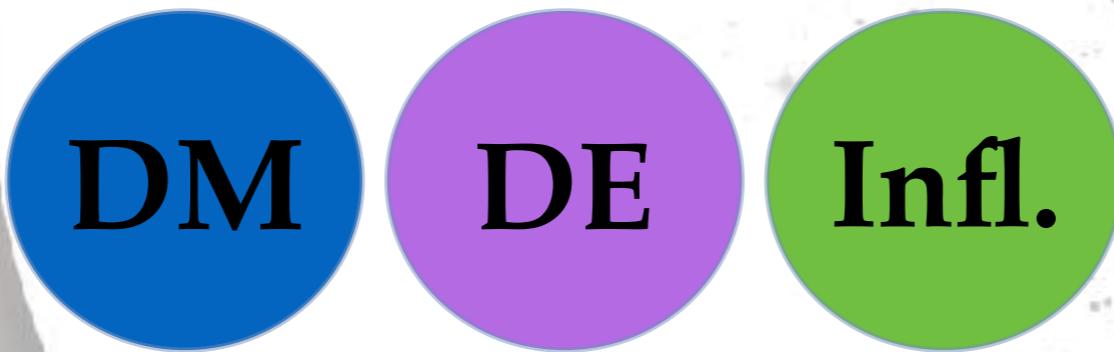


What is the origin of DM?

What causes these two accelerated expansion phases of the universe?

Is DE just a cosmological constant or a field evolving in time like the inflaton?

# Effective Field Theories



$\pi$

# Scalar Field (Galileons)

- second order equations of motion
- Lorentz invariant and local
- shift and galileon symmetry  $\pi \rightarrow \pi + c + x_\mu b^\mu$

$\pi$

# Scalar Field (Galileons)

- second order equations of motion
- Lorentz invariant and local
- shift and galileon symmetry  $\pi \rightarrow \pi + c + x_\mu b^\mu$

## Galileon interactions

$$\mathcal{L}_2 = (\partial\pi)^2$$

$$\mathcal{L}_3 = (\partial\pi)^2[\Pi]$$

$$\mathcal{L}_4 = (\partial\pi)^2([\Pi]^2 - [\Pi^2])$$

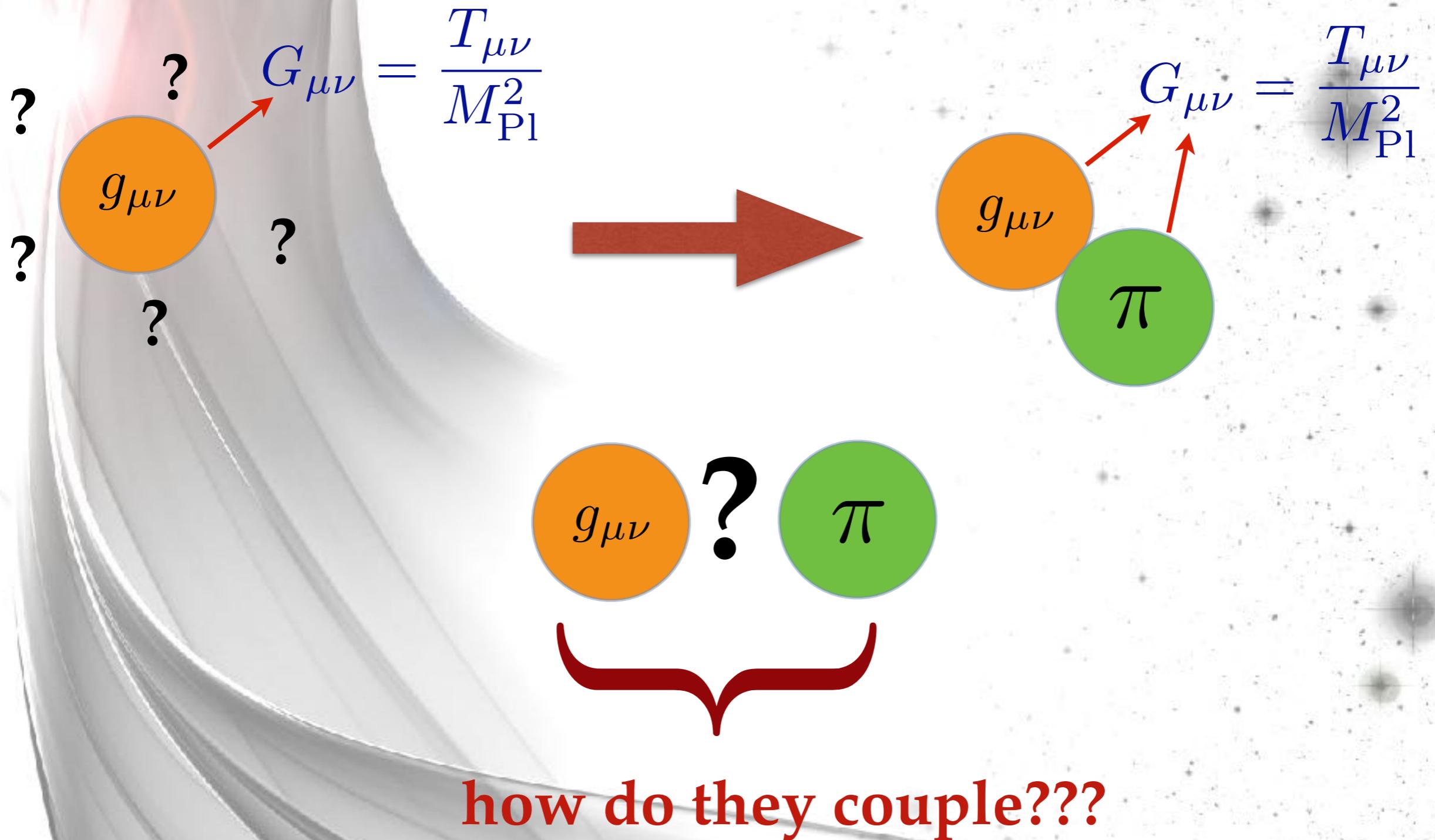
$$\mathcal{L}_5 = (\partial\pi)^2([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3])$$

Nicolis, Rattazzi, Trincherini  
Phys.Rev.D79 064036, 2009

$$\Pi_{\mu\nu} = \partial_\mu \partial_\nu \pi$$

$$[\Pi] = \square \pi$$

# Horndeski theory (scalar-tensor theory)



# Scalar-Tensor-interactions

The generalisation to curved space-time yield the rediscovery of Horndeski interactions

G.W.Horndeski Int. J. Theo.Phys. 10, 363-384 (1974)

C.Deffayet, Esposito-Farese,Vikman  
Phys. Rev.D (79), 084003 (2009)

T. Kobayashi, M. Yamaguchi, J. Yokoyama,  
Prog. Theor. Phys. 126 (2011),511-529

$$\mathcal{L}_2 = K(\pi, X)$$

$$\mathcal{L}_3 = G_3(\pi, X)[\Pi]$$

$$\mathcal{L}_4 = G_4(\pi, X)R + G_{4,X}([\Pi]^2 - [\Pi^2])$$

$$\mathcal{L}_5 = G_5(\pi, X)G_{\mu\nu}\Pi^{\mu\nu} - \frac{G_{5,X}}{6}([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^2])$$

$$\Pi_{\mu\nu} = \nabla_\mu \partial_\nu \pi$$

$$X = -\frac{1}{2}(\partial\pi)^2$$

Non-minimal couplings to gravity have to be added to maintain second order equations of motions

# Horndeski theory (scalar-tensor theory)



homogeneous &  
isotropic solutions

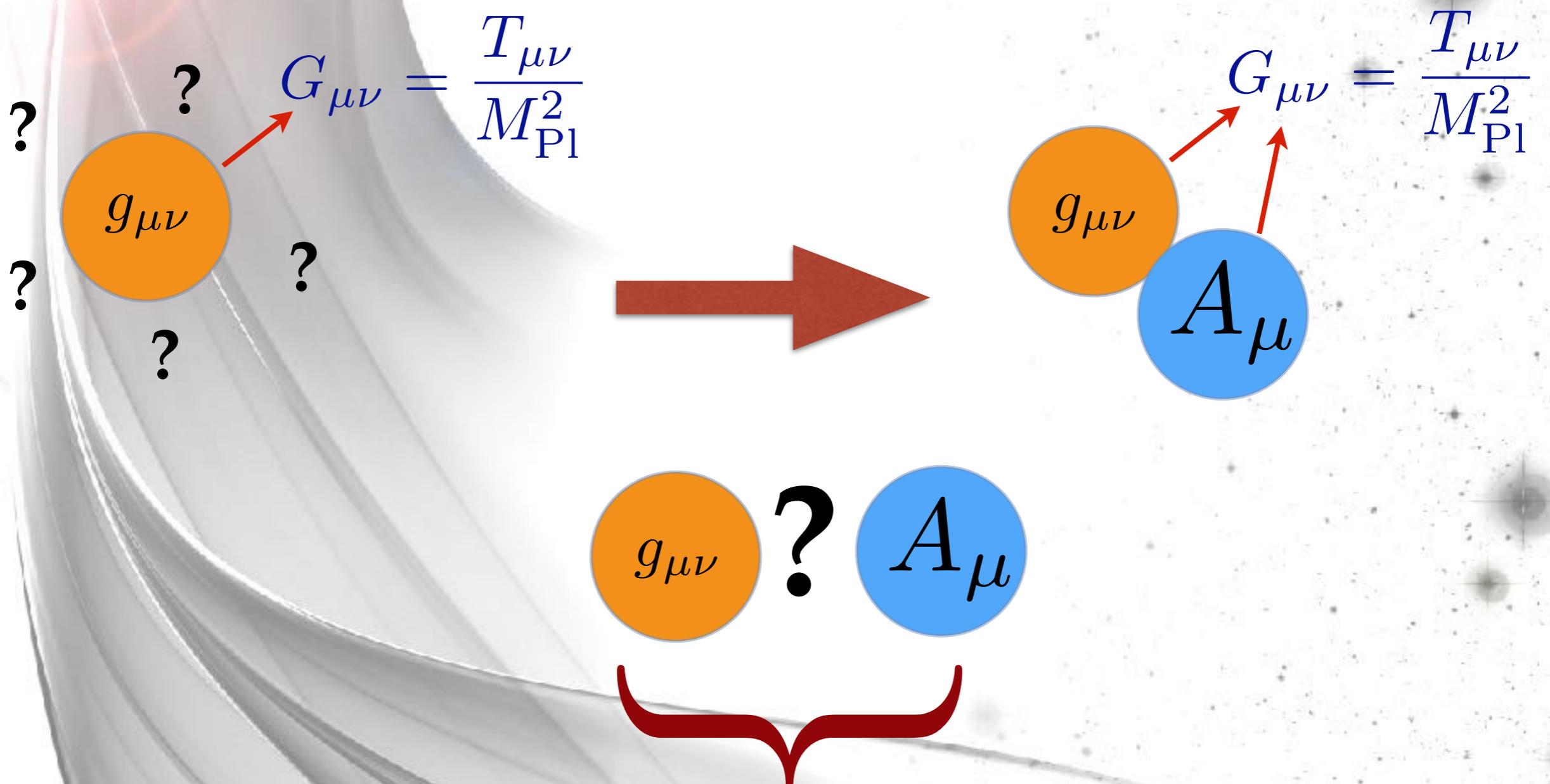
$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j$$

$$\pi = \pi(t)$$

# Scalar-Tensor-interactions

- Horndeski
- Beyond Horndeski
- Beyond Beyond Horndeski
- DHOST

# Vector-Tensor Theories



how do they couple???

$A_\mu$ 

# Vector Field (Generalized Proca)

- second order equations of motion
- Lorentz invariant and local

$A_\mu$ 

# Vector Field (Generalized Proca)

- second order equations of motion
- Lorentz invariant and local

L.H (JCAP 1405,2014,015,  
arXiv:1402.7026)

L.H & J.Beltran,  
Phys.Lett.B757 (2016)  
405-411, arXiv:1602.03410

$$\mathcal{L}_2 = f_2(A_\mu, F_{\mu\nu}, \tilde{F}_{\mu\nu})$$

$$\mathcal{L}_3 = f_3(A^2) \partial \cdot A$$

$$\mathcal{L}_4 = f_4(A^2) [(\partial \cdot A)^2 - \partial_\rho A_\sigma \partial^\sigma A^\rho]$$

$$\begin{aligned} \mathcal{L}_5 = f_5(A^2) & [(\partial \cdot A)^3 - 3(\partial \cdot A)\partial_\rho A_\sigma \partial^\sigma A^\rho \\ & + 2\partial_\rho A_\sigma \partial^\gamma A^\rho \partial^\sigma A_\gamma] + \tilde{f}_5(A^2) \tilde{F}^{\alpha\mu} \tilde{F}^\beta_\mu \partial_\alpha A_\beta \end{aligned}$$

$$\mathcal{L}_6 = f_6(A^2) \tilde{F}^{\alpha\beta} \tilde{F}^{\mu\nu} \partial_\alpha A_\mu \partial_\beta A_\nu$$

# Generalized Proca action

Interactions on curved space-time requires the presence of non-minimal couplings to gravity

L.H & J.Beltran,  
Phys.Lett.B757 (2016)  
405-411, arXiv:1602.03410

L. H., JCAP 1405, 015 (2014),  
arXiv:1402.7026

G.Tasinato JHEP 1404 (2014)067  
arXiv:1402.6450

Allys, Peter, Rodriguez,  
JCAP 1602 (2016) 02, 004

$$\mathcal{L}_2 = G_2(A_\mu, F_{\mu\nu}, \tilde{F}_{\mu\nu})$$

$$\mathcal{L}_3 = G_3(Y) \nabla_\mu A^\mu$$

$$\mathcal{L}_4 = G_4(Y)R + G_{4,Y} [(\nabla_\mu A^\mu)^2 - \nabla_\rho A_\sigma \nabla^\sigma A^\rho]$$

$$\mathcal{L}_5 = G_5(Y)G_{\mu\nu} \nabla^\mu A^\nu - \frac{1}{6}G_{5,Y} [(\nabla \cdot A)^3$$

$$+ 2\nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla^\sigma A_\gamma - 3(\nabla \cdot A) \nabla_\rho A_\sigma \nabla^\sigma A^\rho]$$

$$- \tilde{G}_5(Y) \tilde{F}^{\alpha\mu} \tilde{F}^\beta_\mu \nabla_\alpha A_\beta$$

$$\mathcal{L}_6 = G_6(Y) L^{\mu\nu\alpha\beta} \nabla_\mu A_\nu \nabla_\alpha A_\beta + \frac{G_{6,Y}}{2} \tilde{F}^{\alpha\beta} \tilde{F}^{\mu\nu} \nabla_\alpha A_\mu \nabla_\beta A_\nu$$

$$Y = -\frac{1}{2} A_\mu A^\mu$$

# Cosmology with Vector Fields

Flat Friedmann-Lemaitre-  
Robertson-Walker background

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j$$

Homogeneity & Isotropy

$$A_\mu = (\phi(t), 0, 0, 0)$$

$$\begin{aligned} A_i^0 &= 0 \\ A_i^a &= A(t) \delta_i^a \end{aligned}$$

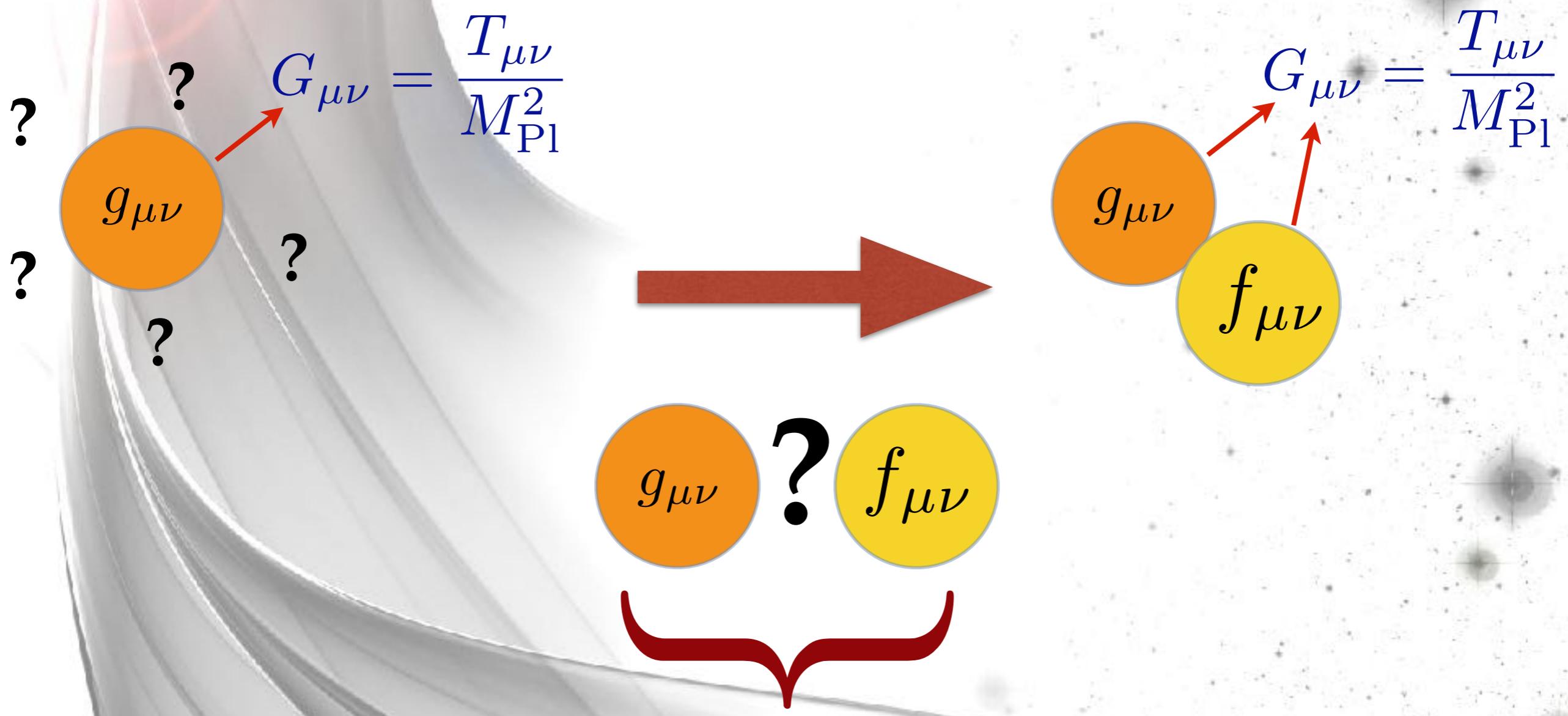
$$A_\mu$$

$$A_\mu^a$$

# Vector-Tensor Theories

- Generalized Proca
- Extended Vector Theories
- Beyond Generalized Proca

# Massive Gravity (tensor-tensor theory)



how do they couple???

# Massive Gravity (**tensor-tensor** theory)

→  $m^2 g^{\mu\nu} f_{\mu\nu}?$

The building block for a consistent theory of massive gravity

$$m^2 \sqrt{g^{-1} f}$$

$$S = \int \frac{M_{\text{Pl}}^2}{2} \sqrt{-g} R + \mathcal{L}_{\text{matter}} + m^2 \mathcal{U}(\sqrt{g^{-1} f})$$

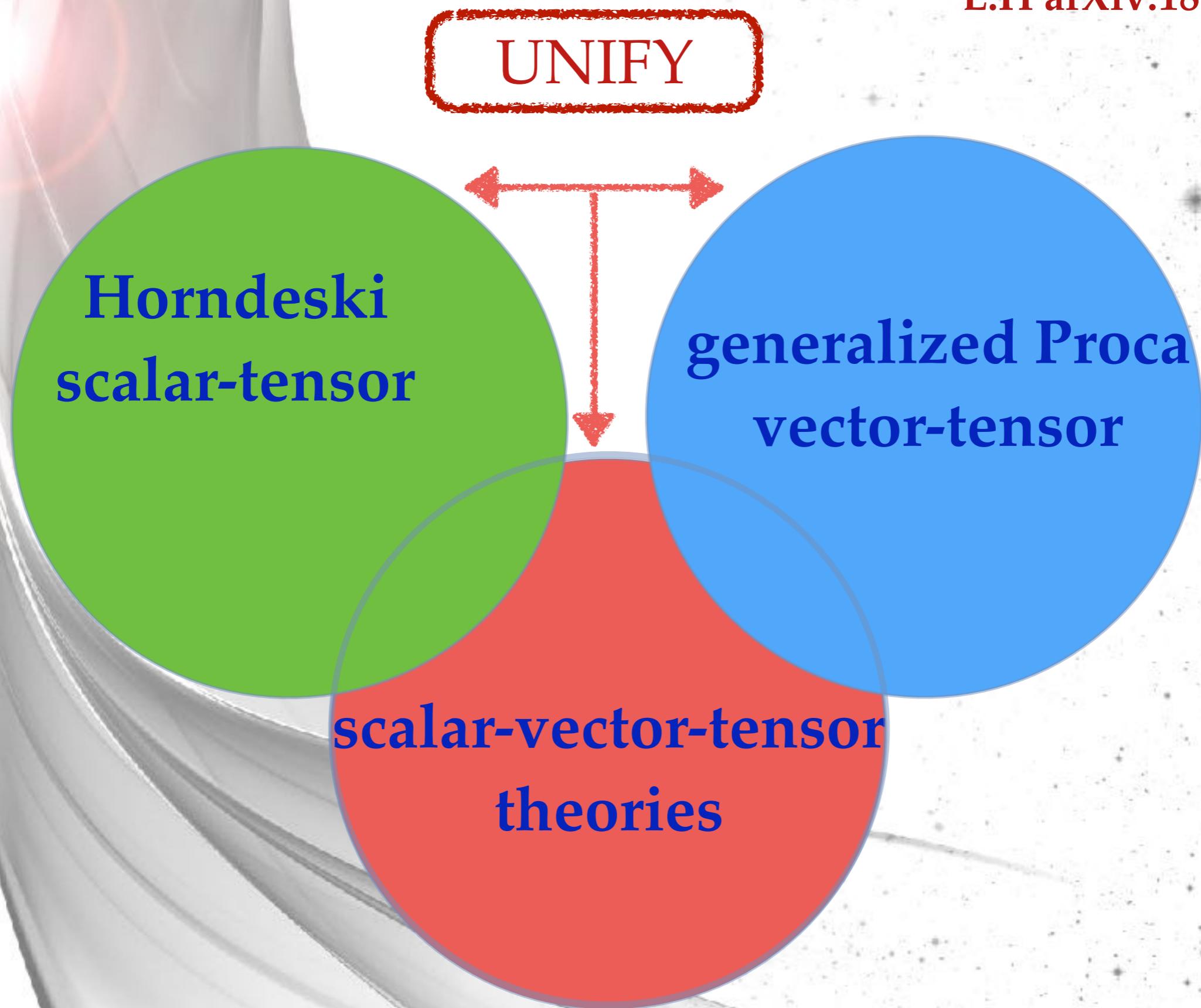
$$\mathcal{S}_{\text{MG}} = \int d^4x \sqrt{-g} \sum_{n=0}^4 \frac{\beta_n}{n!(4-n)!} e_n(\sqrt{g^{-1} f})$$

elementary symmetric polynomials

C. de Rham, G. Gabadaze,  
A.J.Tolley, PRL106 (2011)  
S.F. Hassan, R.A. Rosen  
JHEP1107 (2011)

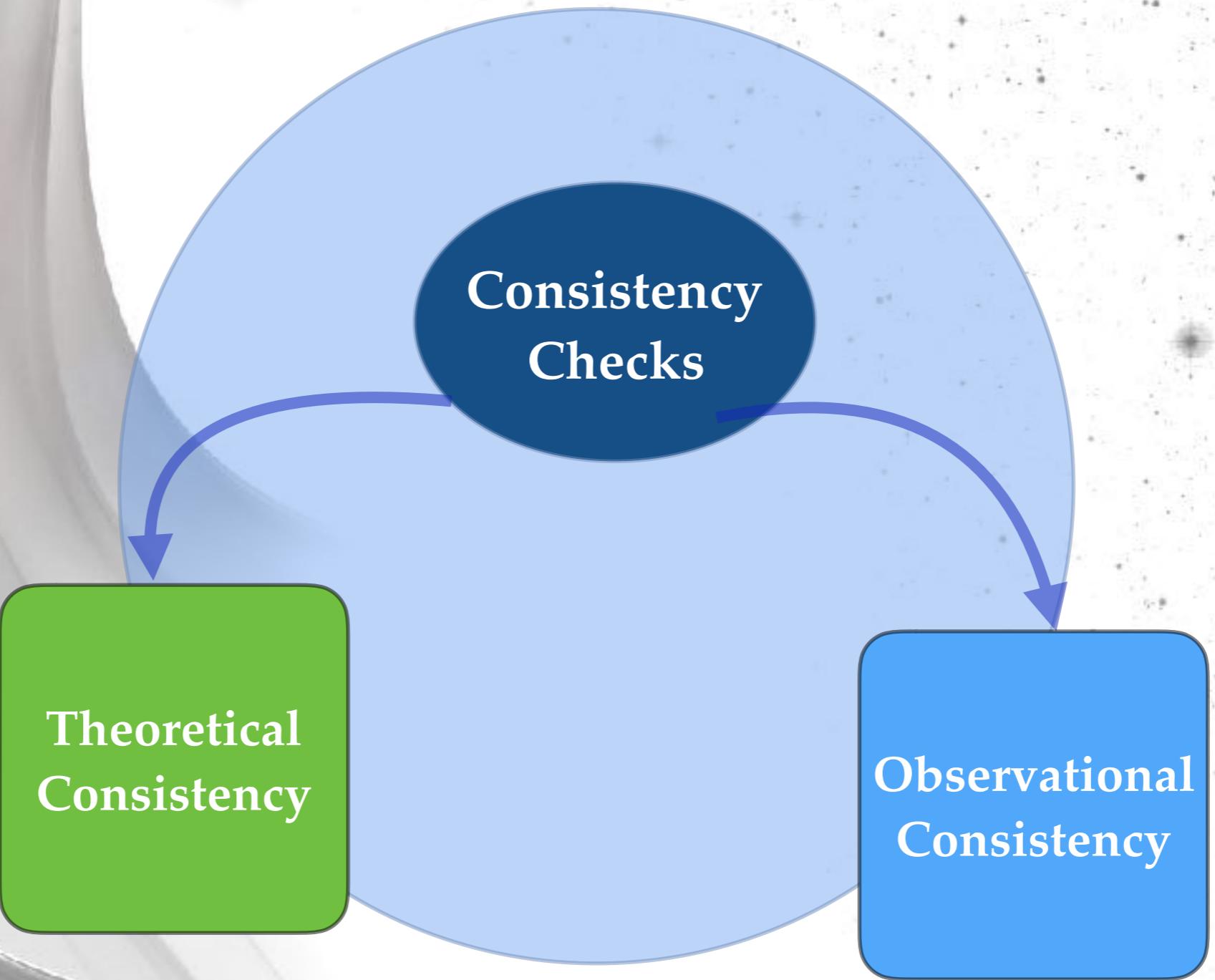
# Scalar-Vector-Tensor Theories

L.H arXiv:1801.01523



# Huge Landscape of Theories





## Consistency Checks

DOF?

Instabilities?

Swampland?

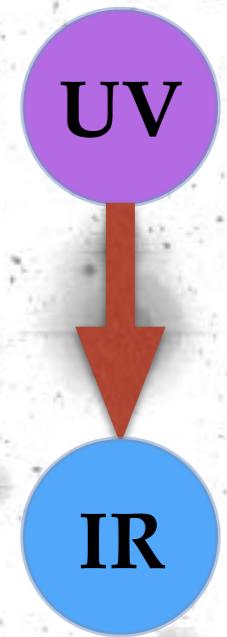
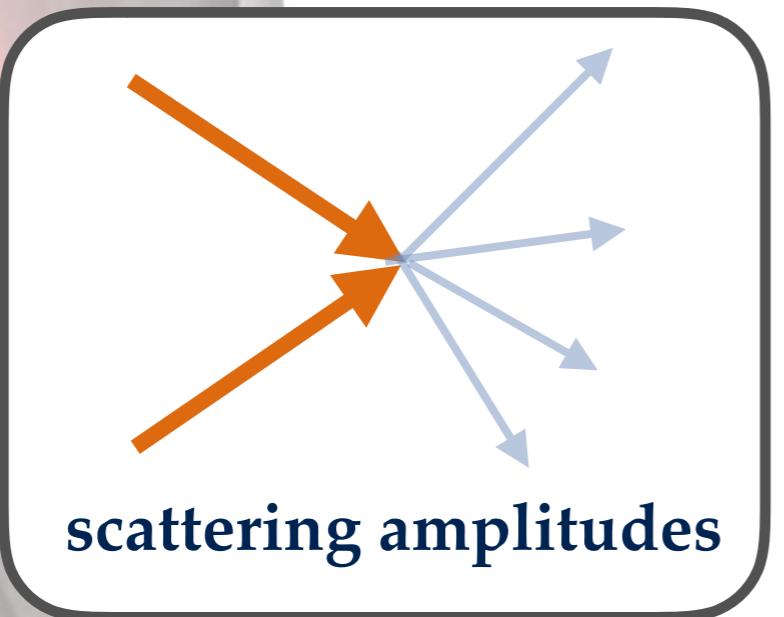
Positivity?

Quantum  
corrections?

Theoretical  
Consistency

Observational  
Consistency

# Theoretical Consistency



Lorentz invariant,  
local, UV completion

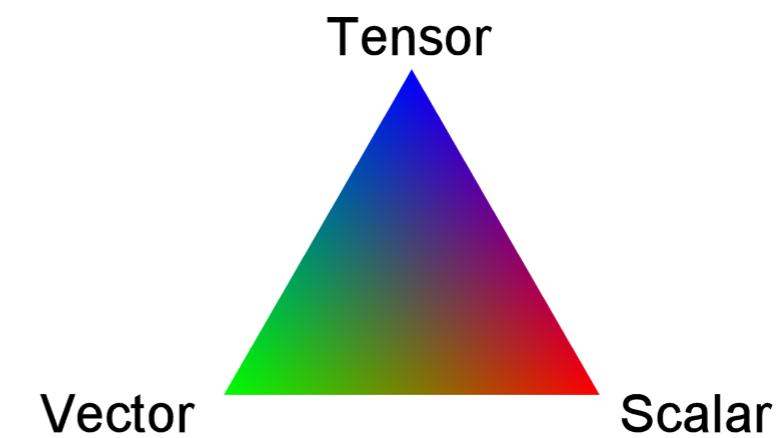
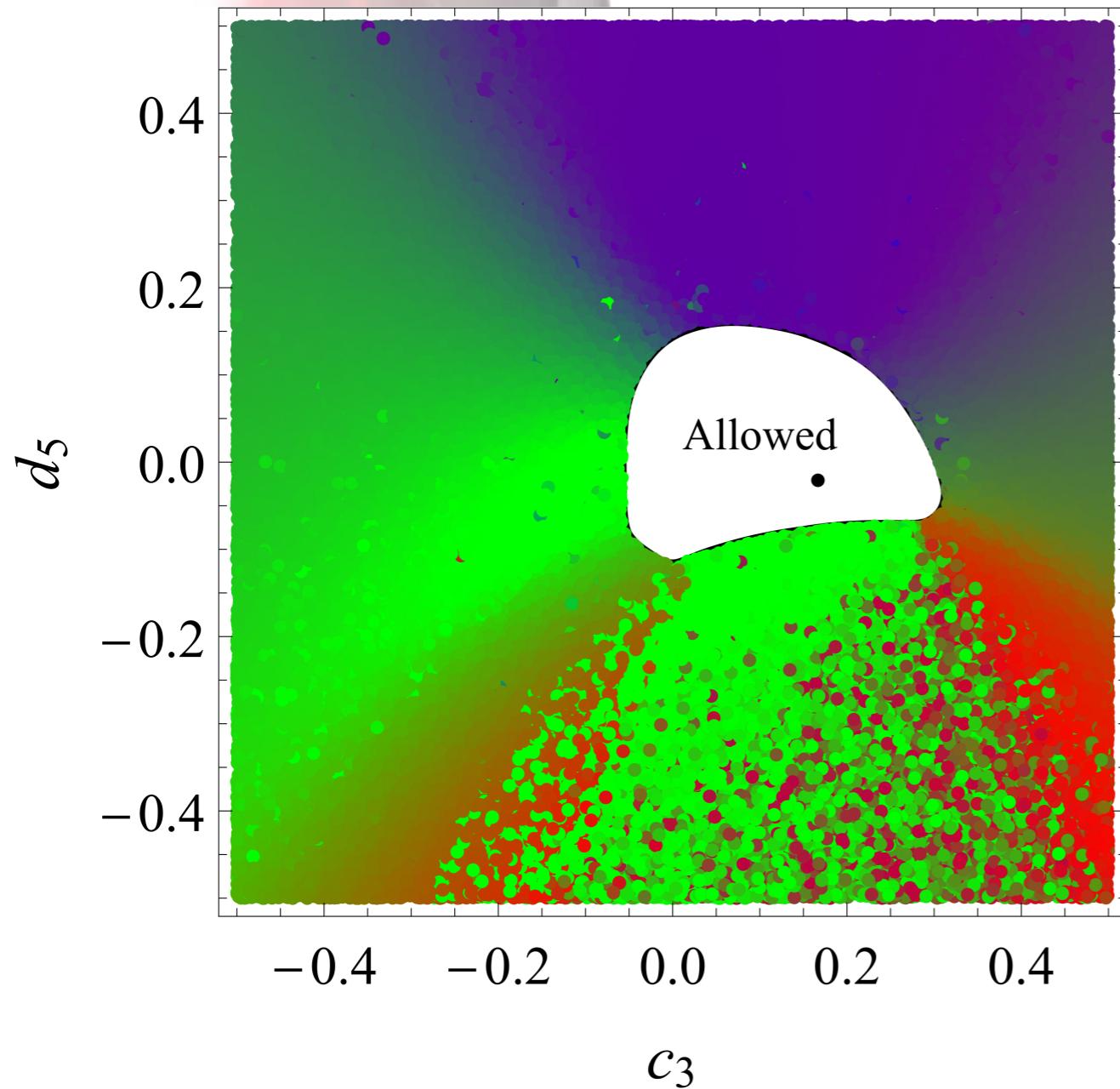


Positivity bounds on the tree  
level scattering amplitudes

The positivity bounds filter out the EFT interactions, that may  
have a local, Lorentz invariant Wilsonian UV completion

# Theoretical Consistency

## Positivity bounds in Massive Gravity



Cheung, Remmen, JHEP 1604, 002,  
2016, arXiv:1601.04068

# Theoretical Consistency

## Effective Field Theories

Landscape

Swampland

- EFTs can be successfully embedded

- EFTs do not have a UV completion

EFTs have to satisfy given conditions in order not to land on the Swampland

- Weak Gravity Conjecture
- Distance Conjecture
- No Global Symmetry
- De Sitter Conjecture ... etc

# Theoretical Consistency

Swampland

- No global symmetries
- An EFT coupled to gravity cannot have global symmetries
- Motivation: BH physics, no-hair theorem
- All global symmetries in string theory are gauged

# Theoretical Consistency

Swampland

- No free parameters
  - An EFT coupled to gravity must have no free parameters.
  - Every parameter entering in the Lagrangian should be viewed as the vacuum expectation value of a field.
- Motivation: M theory in 11D has no free parameter
- Parameters of lower energy EFTs get related to the internal geometry of the compactifications

# Theoretical Consistency

Swampland

- Weak Gravity Conjecture

Arkani-Hamed, Motl, Nicolis, Vafa,  
[arXiv:hep-th/0601001](https://arxiv.org/abs/hep-th/0601001)

→ In a consistent EFT coupled to gravity, gravity must always be the weakest force.

- Motivation: BH should be able to decay

- Applies to charged particles - U(1) gauge theory

$$F_g \leq F_e$$

$$\frac{m^2}{M_P^2 r^2} \leq \frac{q^2}{r^2}$$

$$m/M_P \leq q$$

# Theoretical Consistency

Swampland

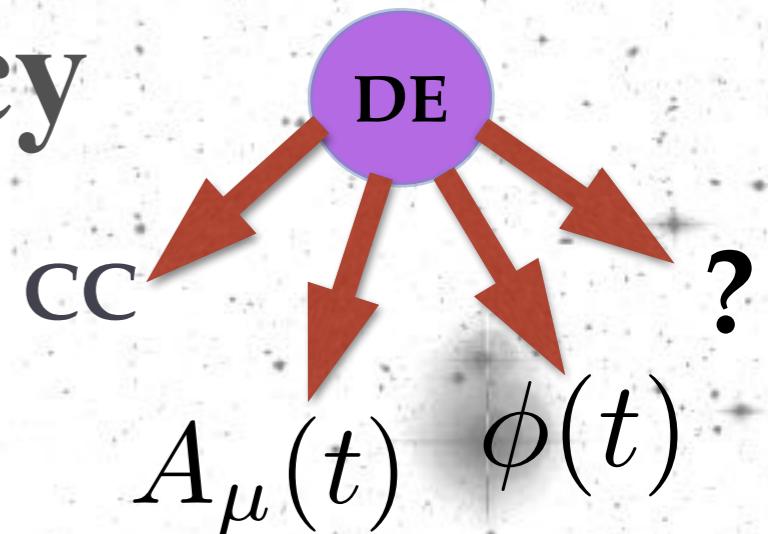
## Distance Conjecture

Ooguri, Rafa, Nucl. Phys. B766  
(2007) 21-33, arXiv:hep-th/0605264

→ In a consistent EFT coupled to gravity, the range traversed by scalar fields in field space is bounded

$$|\Delta\phi| < d \sim \mathcal{O}(1)$$

# Theoretical Consistency



## De Sitter Conjecture

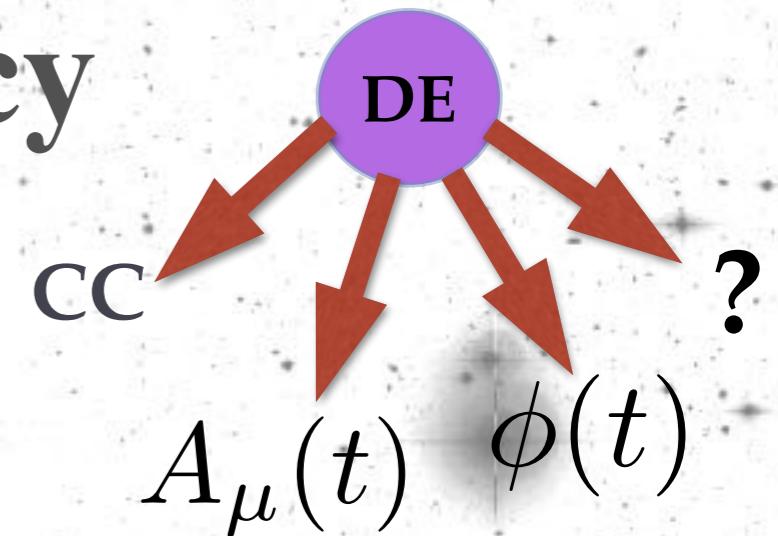
Obied, Ooguri, Spodyneiko  
Vafa, arXiv:1806.08362

→ dS space does not exist as a consistent quantum theory of gravity and it belongs to the swampland.

→ Scalar fields in the EFTs have to satisfy

$$\frac{|\nabla V|}{V} > c \sim \mathcal{O}(1)$$

# Theoretical Consistency



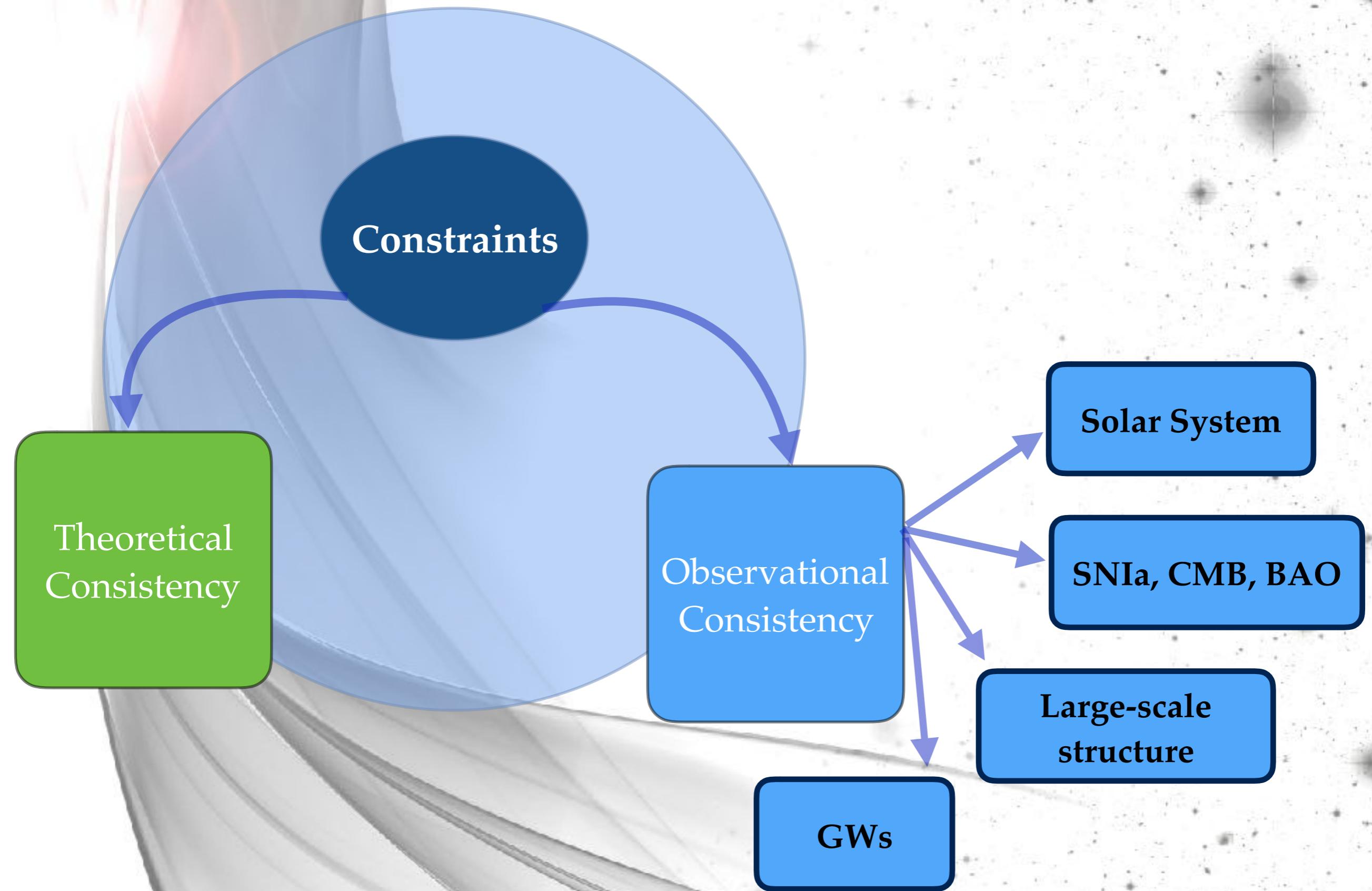
Trans-Planckian  
Censorship Conjecture

Bedroya, Vafa, arXiv:1909.11063

→ A field theory consistent with a quantum theory of gravity does not lead to a cosmological expansion where any perturbation with length scale greater than the Hubble radius trace back to trans-Planckian scales at an earlier time.

$$\frac{a_f}{a_i} < \frac{M_{\text{Pl}}}{H_f}$$

# Constraints!!!

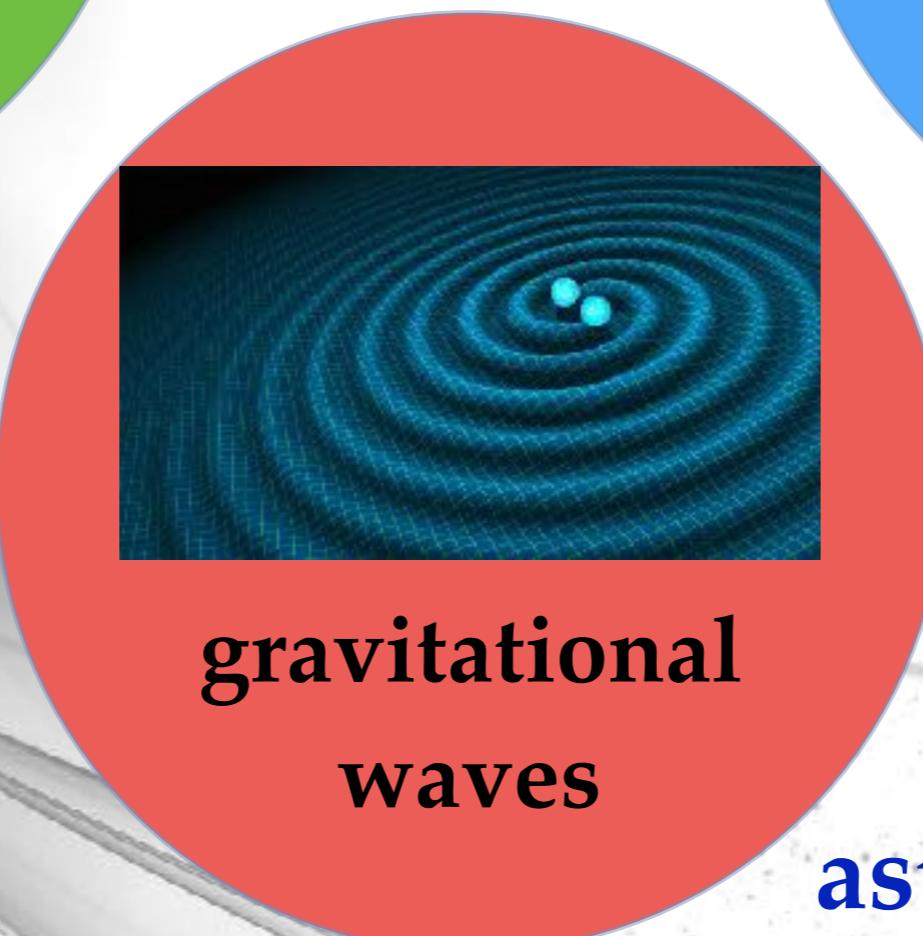
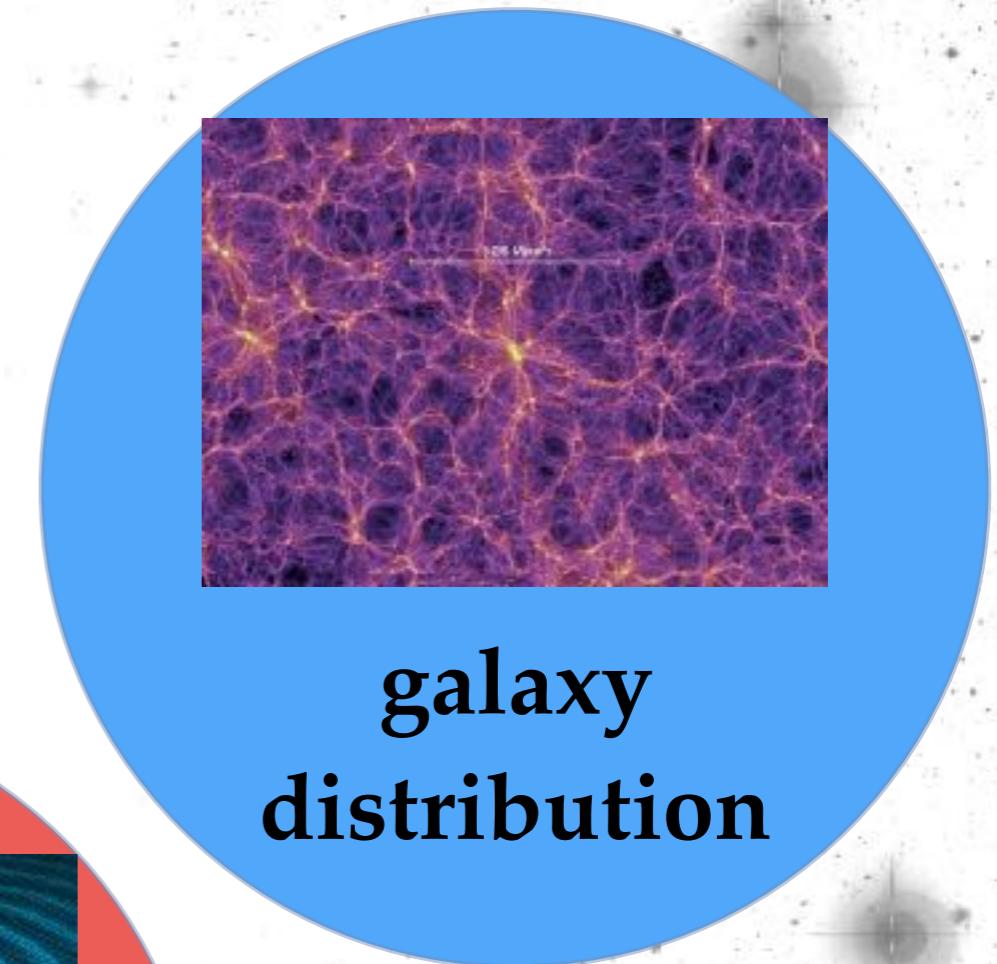


# Cosmological observations

geometrical probes



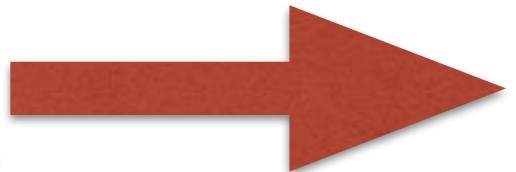
Structure formation probes



astrophysical probes

# Gravitational Waves observations

$$c_{\text{gw}}^2 = 1$$



$$G_{\mu\nu} = \frac{T_{\mu\nu}}{M_{\text{Pl}}^2}$$

$$\begin{array}{c} g_{\mu\nu} \\ \pi \end{array}$$

NO non-minimal coupling!

Horndeski

$$\mathcal{L}_2 = K(\pi, X)$$

$$\mathcal{L}_3 = G_3(\pi, X)[\Pi]$$

$$\mathcal{L}_4 = G_4(\pi, X)R + \cancel{G_{4,X}([\Pi]^2 - [\Pi^2])}$$

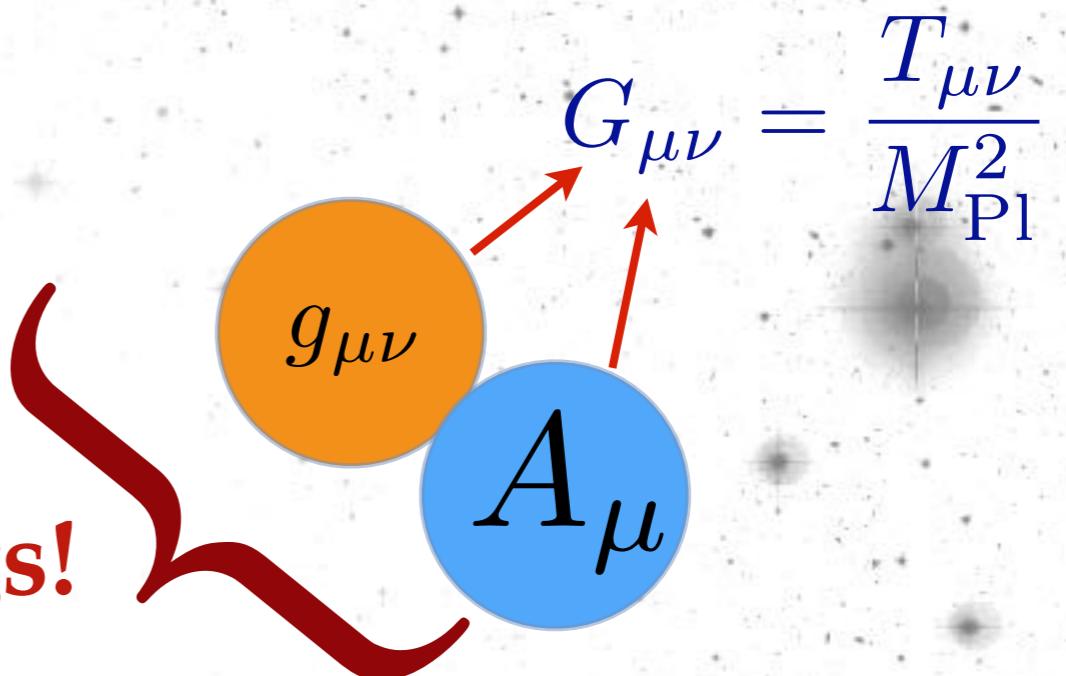
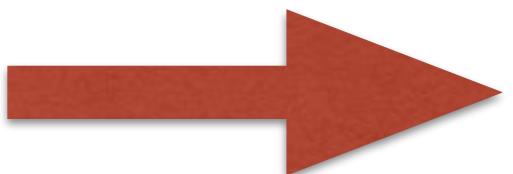
$$\Pi_{\mu\nu} = \nabla_\mu \partial_\nu \pi$$

$$X = -\frac{1}{2}(\partial\pi)^2$$

$$\mathcal{L}_5 = \cancel{G_5(\pi, X)} G_{\mu\nu} \Pi^{\mu\nu} - \frac{G_{5,X}}{6} ([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^2])$$

# Gravitational Waves observations

$$c_{\text{gw}}^2 = 1$$



Some non-minimal couplings!

## Generalized Proca

$$\mathcal{L}_2 = G_2(A_\mu, F_{\mu\nu}, \tilde{F}_{\mu\nu})$$

$$\mathcal{L}_3 = G_3(Y) \nabla_\mu A^\mu$$

$$\mathcal{L}_4 = G_4(Y) R + G_{4,Y} \left[ (\nabla_\mu A^\mu)^2 - \nabla_\rho A_\sigma \nabla^\sigma A^\rho \right]$$

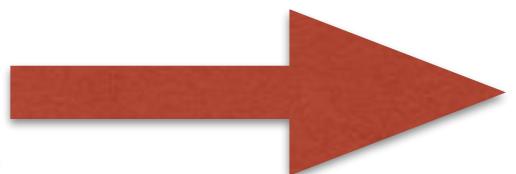
$$\begin{aligned} \mathcal{L}_5 = & G_5(Y) G_{\mu\nu} \nabla^\mu A^\nu - \frac{1}{6} G_{5,Y} \left[ (\nabla_\mu A^\mu)^3 \right. \\ & \left. + 2 \nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla^\sigma A_\gamma - 3 (\nabla_\mu A^\mu) \nabla_\rho A_\sigma \nabla^\sigma A^\rho \right] \end{aligned}$$

$$- \tilde{G}_5(Y) \tilde{F}^{\alpha\mu} \tilde{F}^{\beta}_\mu \nabla_\alpha A_\beta$$

$$\mathcal{L}_6 = G_6(Y) L^{\mu\nu\alpha\beta} \nabla_\mu A_\nu \nabla_\alpha A_\beta + \frac{G_{6,Y}}{2} \tilde{F}^{\alpha\beta} \tilde{F}^{\mu\nu} \nabla_\alpha A_\mu \nabla_\beta A_\nu$$

# Gravitational Waves observations

$$c_{\text{gw}}^2 = 1$$



$$G_{\mu\nu} = \frac{T_{\mu\nu}}{M_{\text{Pl}}^2}$$
$$g_{\mu\nu}$$
$$f_{\mu\nu}$$
$$G_{\mu\nu}$$

## Massive Gravity

$$m^2 \sqrt{g^{-1} f}$$

$$m < 10^{-22} eV$$

compared to the other existing  
cosmological bounds not so relevant

$$m < 10^{-32} eV$$

# Combine theoretical and observational constraints!

## Dark Energy in the Swampland



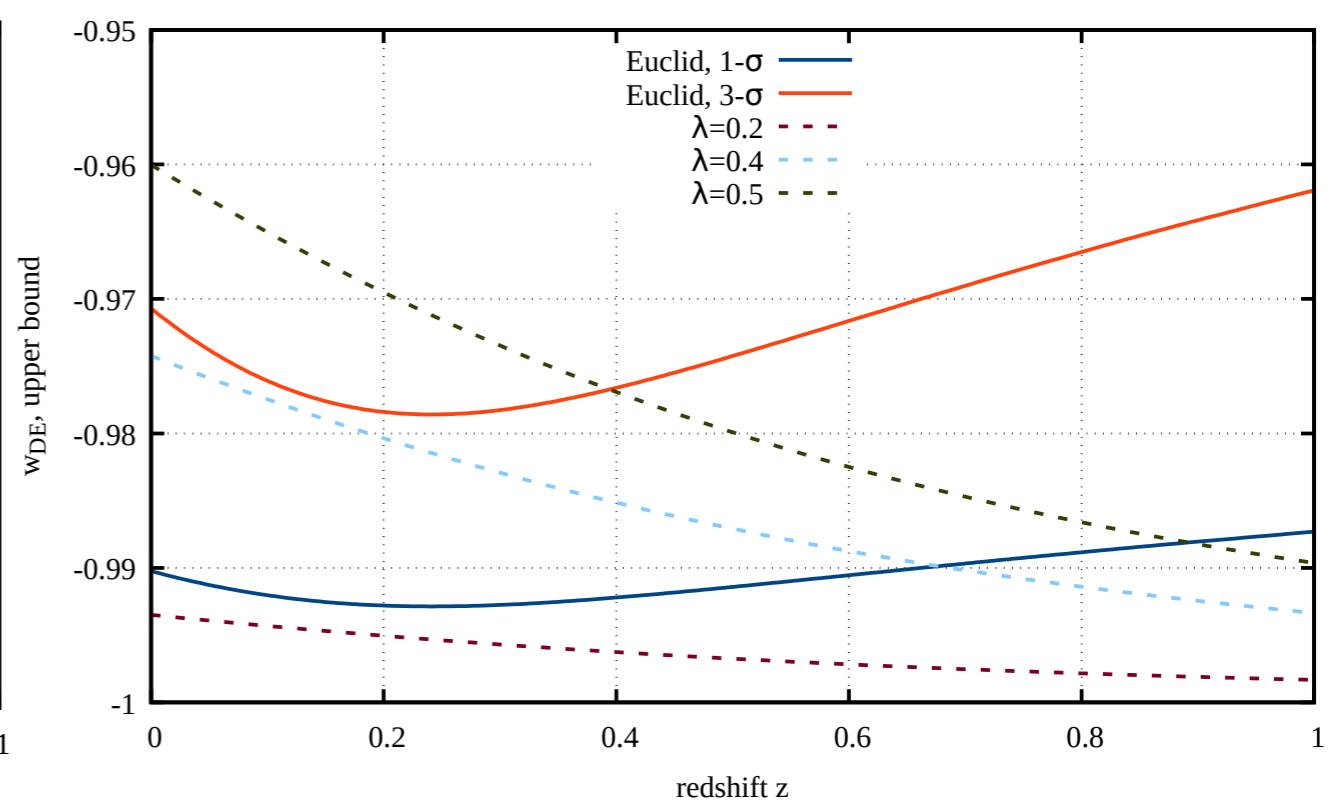
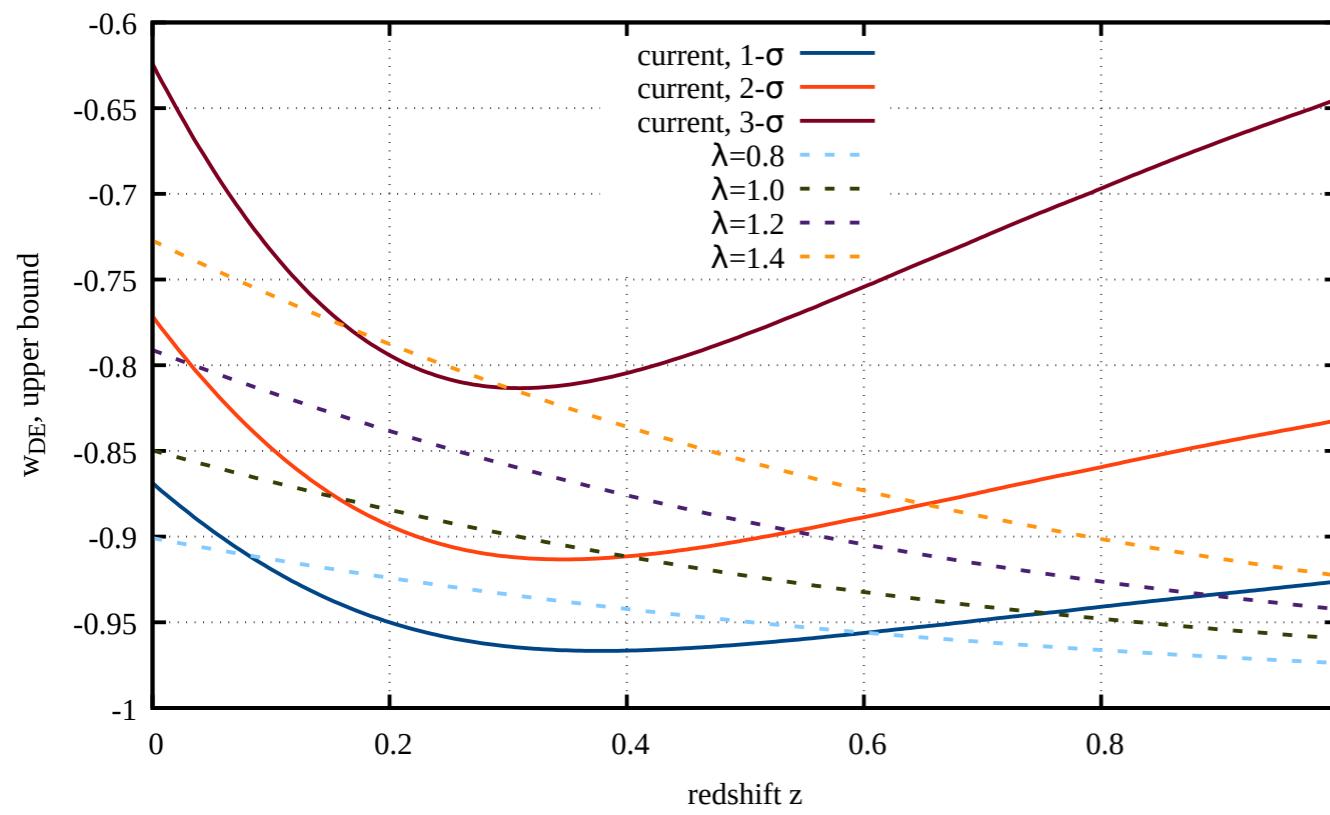
Ex: Quintessence Model

$$\phi(t)$$

Agrawal, Obied, Steinhardt,  
Vafa, arXiv:1806.09718

- De Sitter Conjecture  $\frac{|\nabla V|}{V} > c \sim \mathcal{O}(1)$
- SNIa, BAO, CMB, H0 measurements

L.H & Bartelmann,  
Brandenberger, Refregier,  
arXiv:1808.02877



# Combine theoretical and observational constraints!

## Dark Energy in the Swampland



Ex: Horndeski Model

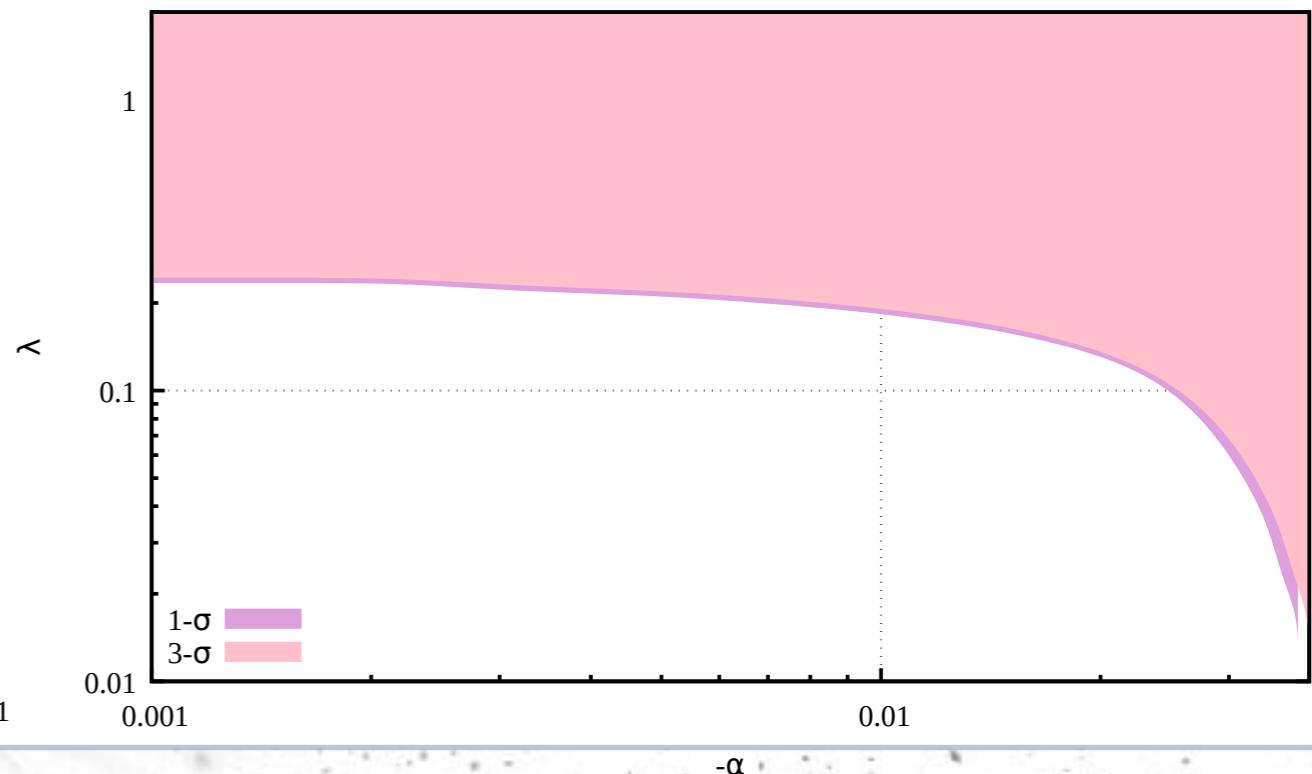
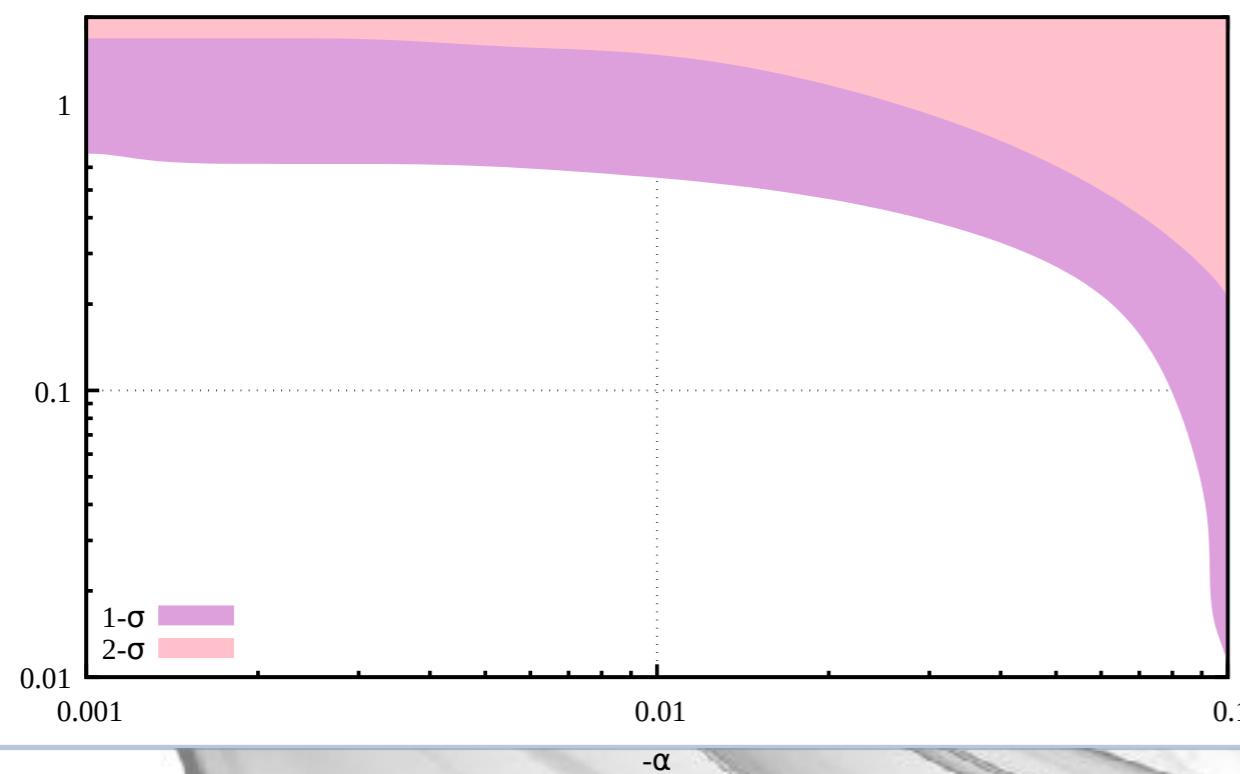
- De Sitter Conjecture  $\frac{|\nabla V|}{V} > c \sim \mathcal{O}(1)$
- SNIa, BAO, CMB, H0 measurements

$$G_2(\pi, X) = (1 - 6\alpha^2)f(\pi)X - V(\pi) ,$$

$$G_3(\pi, X) = \alpha_3 X ,$$

$$G_4(\pi, X) = \frac{M_{\text{Pl}}}{2}f(\pi) + \alpha_4 X^2 ,$$

L.H & Bartelmann,  
Brandenberger, Refregier,  
arXiv:1902.03939



# All you ever wanted to know about modified gravity, but were afraid to ask.

arXiv:1807.01725

A systematic approach to generalisations of  
General Relativity and their cosmological  
implications

Lavinia Heisenberg

*Institute for Theoretical Studies, ETH Zurich, Clausiusstrasse 47, 8092 Zurich, Switzerland.*

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## Abstract

A century ago, Einstein formulated his elegant and elaborate theory of General Relativity, which has so far withstood a multitude of empirical tests with remarkable success. Notwithstanding the triumphs of Einstein's theory, the tenacious challenges of modern cosmology and of particle physics have motivated the exploration of further generalised theories of spacetime. Even though Einstein's interpretation of gravity in terms of the curvature of spacetime is commonly adopted, the assignment of geometrical concepts to gravity is ambiguous because General Relativity allows three entirely different, but equivalent approaches of which Einstein's interpretation is only one. From a field-theoretical perspective, however, the construction of a consistent theory for a Lorentz-invariant massless spin-2 particle uniquely leads to General Relativity. Keeping Lorentz invariance then implies that any modification of General Relativity will inevitably introduce additional propagating degrees of freedom into the gravity sector. Adopting this perspective, we will review the recent progress in constructing consistent field theories of gravity based on additional scalar, vector and tensor fields. Within this conceptual framework, we will discuss theories with Galileons, with Lagrange densities as constructed by Horndeski and beyond, extended to DHOST interactions, or containing generalized Proca fields and extensions thereof, or several Proca fields, as well as bigravity theories and scalar-vector-tensor theories. We will review the motivation of their inception, different formulations, and essential results obtained within these classes of theories together with their empirical viability.

*Keywords:* Modified Gravity, Massive Gravity, Scalar-Tensor theories, Generalized Proca, Multi-Proca, Scalar-Vector-Tensor theories, Cosmology

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