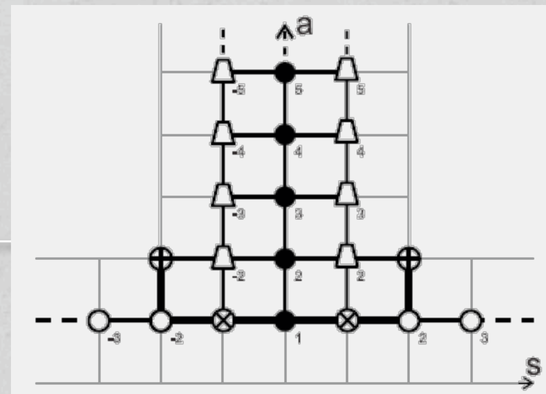
Nikolay Gromov

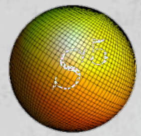
Based on works with V.Kazakov, P.Vieira & A.Kozak

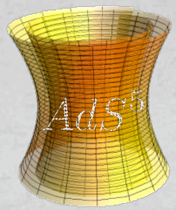


Integrability for the exact spectrum of AdS/ CFT

Facets of Integrability, CEA Saclay and ENS Paris
2009

AdS/CFT correspondence

AdS/CFT duality: 

$\times$ 

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String tension $T = \frac{\sqrt{\lambda}}{4\pi} \equiv g$ `t Hooft coupling $\lambda = g_{YM}^2 N$

Anomalous dimensions = spectrum of 2D integrable field theories

•S-matrix is well defined

•S-matrix factorizes – “only” 256 components

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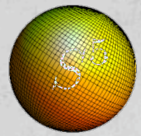
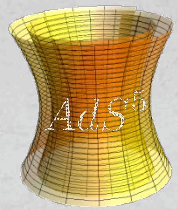
AdS/CFT duality: 

$\times$ 

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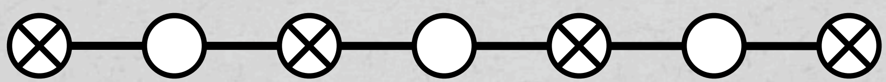

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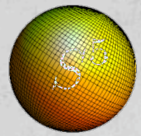
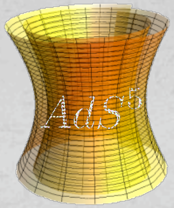
AdS/CFT duality: 

$\times$ 

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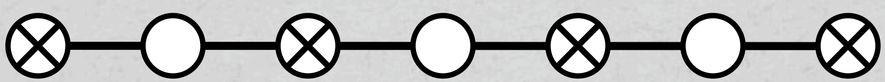

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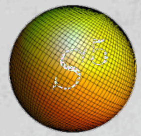
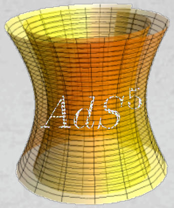
AdS/CFT duality: 

$\times$ 

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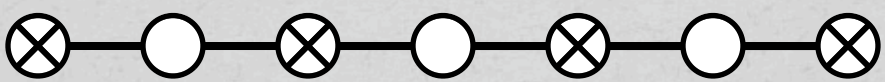

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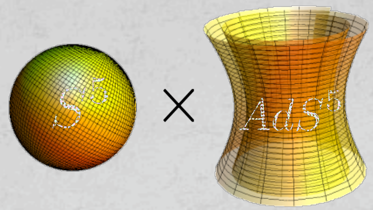
AdS/CFT duality: 

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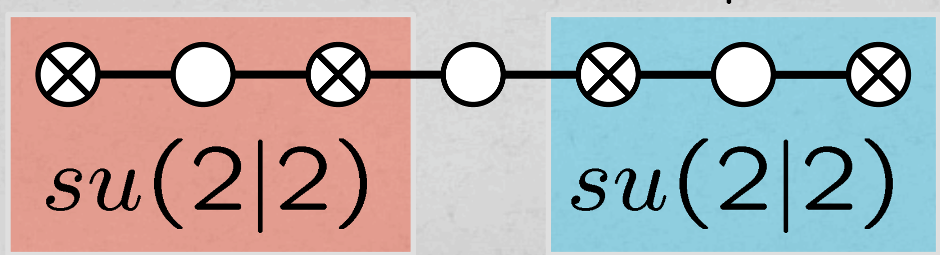

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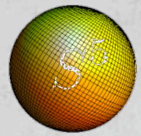
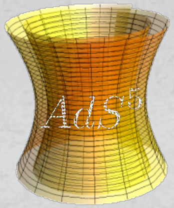
AdS/CFT duality:  $\times$  $S = \frac{T}{2}\int \partial_\mu \vec{u}\cdot\partial^\mu \vec{u}\;\; d\sigma d\tau \; + fermions$

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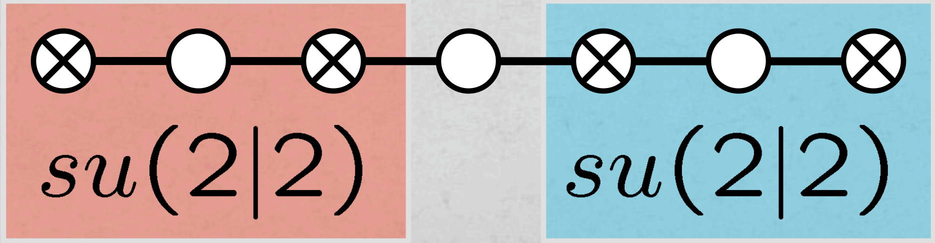

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Integrability in N=4 SYM

N=4 Super Yang-Mills: $S = \frac{1}{g_{YM}^2} \int d^4x \; \mathrm{tr} \left\{ \frac{1}{2} F_{\mu\nu}^2 + (D_\mu \Phi_i)^2 - \frac{1}{2} [\Phi_i, \Phi_j]^2 + \text{fermions} \right\}$

$$\mathcal{O}_i(x) = \mathrm{tr}\Phi_1\Phi_2\Phi_1\Phi_1\Phi_1\Phi_2\Phi_2\Phi_1\Phi_1\Phi_1$$

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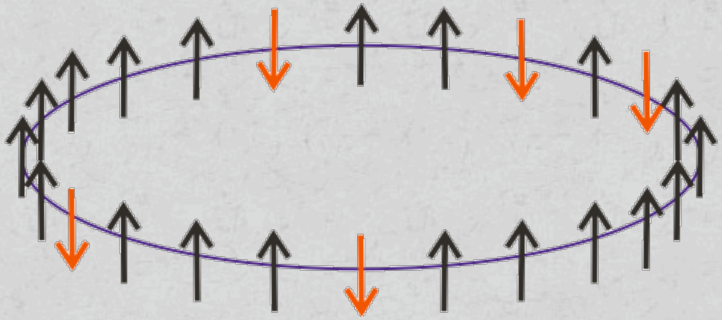

$$\mathcal{O}_i^{\mathrm{ren}} = Z_{ij}(\Lambda)\mathcal{O}_j^{\mathrm{bare}} \qquad \Gamma = Z^{-1}\frac{dZ}{d\log\Lambda}$$

– Mixing matrix – integrable Hamiltonian

[Minahan, Zarembo 2002&2008]

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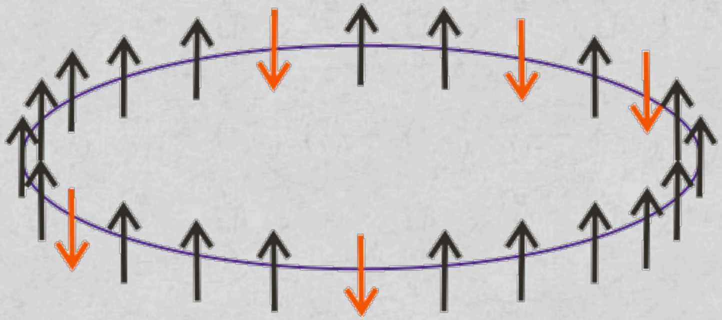

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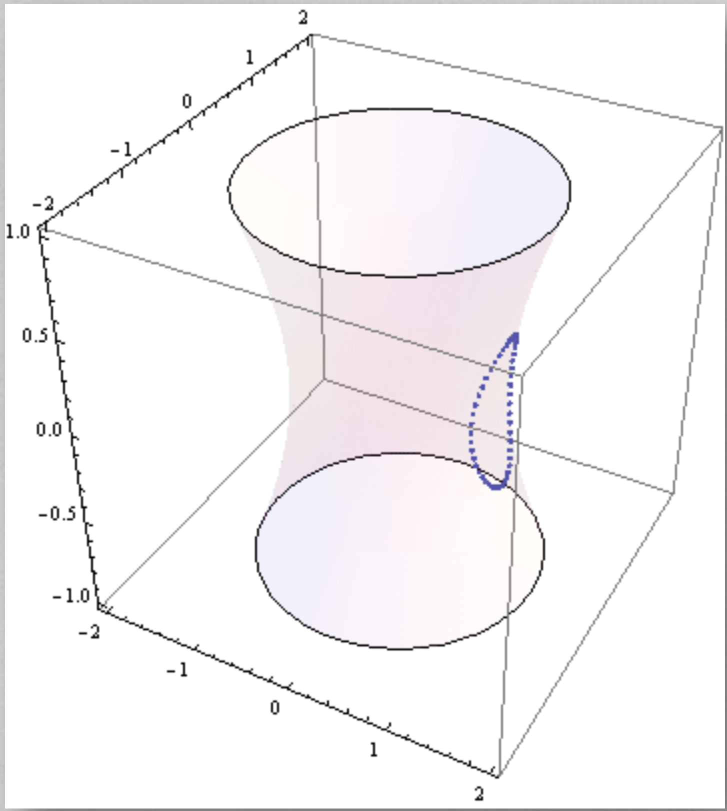
At one loop:

$$\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = -\prod_{k=1}^{M} \frac{u_j - u_k + i}{u_j - u_k - i}$$

$$\gamma = \sum_{k=1}^{M} \frac{g}{u_k^2 + 1/4}$$

Clasical integrability



Motion of the string in AdS:

$$\partial^2 X_a + (\partial X_b \partial X^b) X_a = 0$$

Infinitely many
Integrals of motion:

$$Q(z) \equiv \mathrm{tr}\left[\vec{\mathrm{P}}\mathrm{exp}\left(\int_0^1 \frac{J_1 + zJ_0}{1-z^2} d\sigma\right)\right]$$

Integrable field theory

- No particle creation/anihilation
- Factorizability of S-matrix

- Yang-Baxter equation

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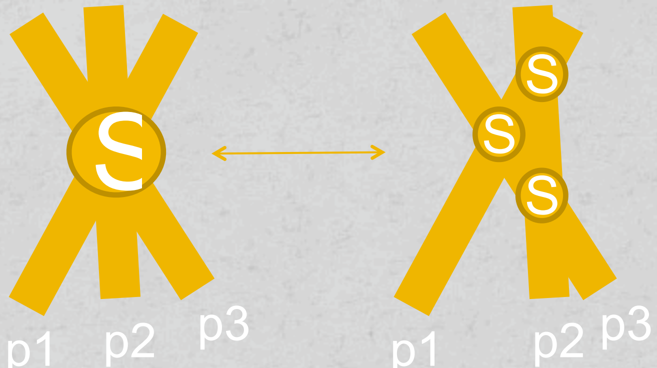

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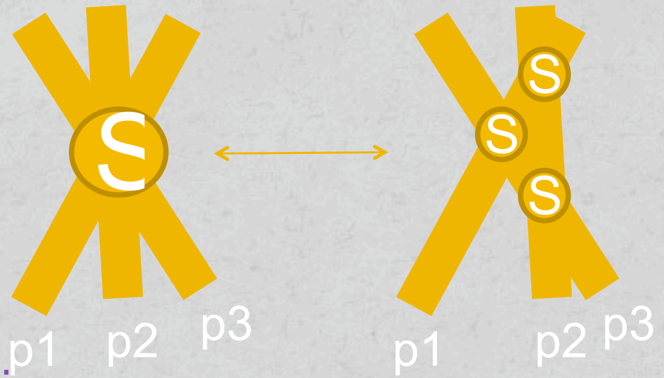

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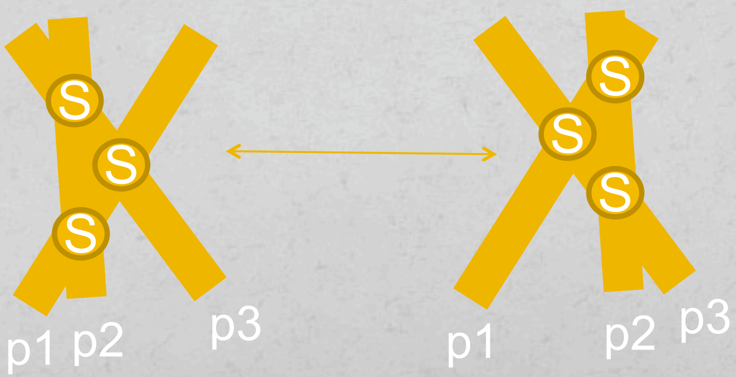
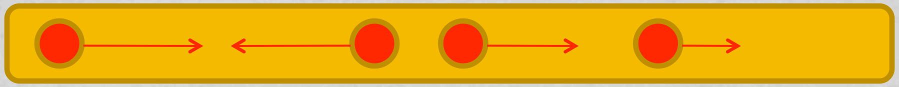


Asymptotic Spectrum

- For spectral density we need finite volume



$$\Psi(x_1+L, x_2, \ldots) = e^{ip_1 L} S(p_1, p_2) \ldots S(p_1, p_n) \Psi(x_1, x_2, \ldots)$$

- From periodicity of the wave function

$$ip_i L = 2\pi i n_i + \sum_j \log S(p_i, p_j)$$

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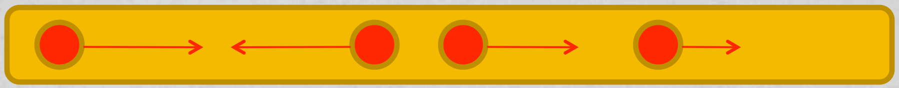

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All loop Bethe equations

$$1=\prod_{j=1}^{K_2}\frac{u_{1,k}-u_{2,j}+\frac{i}{2}}{u_{1,k}-u_{2,j}-\frac{i}{2}}\prod_{j=1}^{K_4}\frac{1-1/x_{1,k}x^+_{4,j}}{1-1/x_{1,k}x^-_{4,j}},$$

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Beisert, Staudacher;
Beisert, Hernandez, Lopez;
Beisert,Eden,Staudacher

All loop Bethe equations

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$$x + \frac{1}{x} = \frac{u}{g} \;, \quad x^{\pm} + \frac{1}{x^{\pm}} = \frac{u \pm i/2}{g}$$

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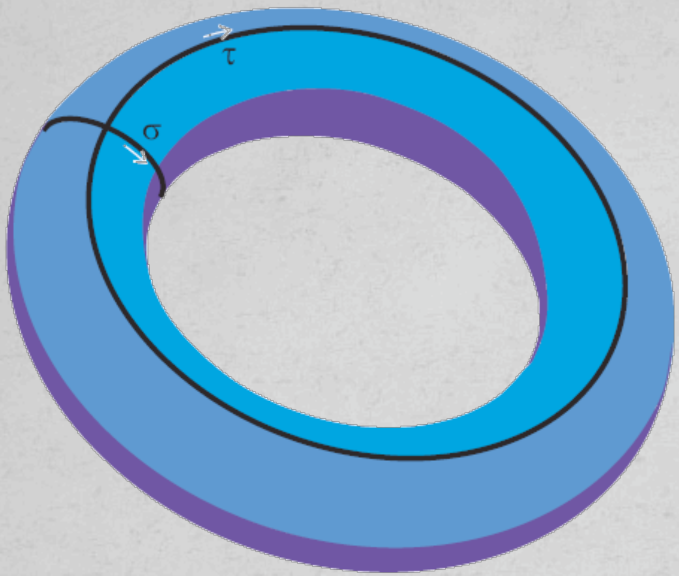
Vacuum

…,Matsubara, Zamolodchikov,…



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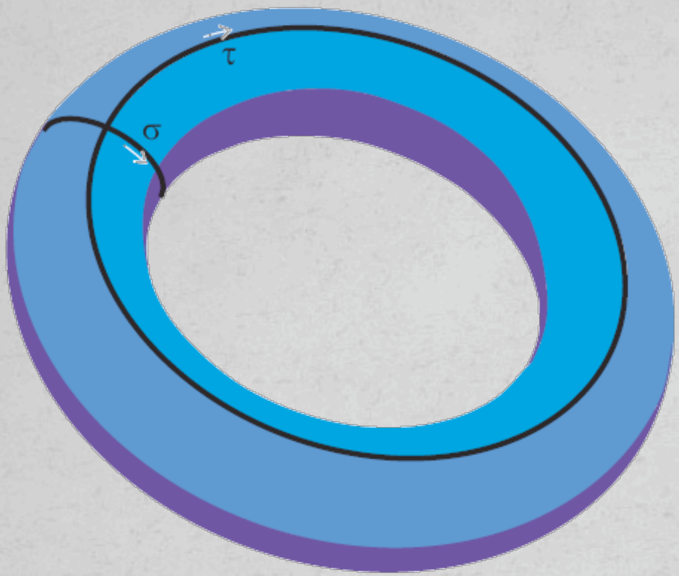

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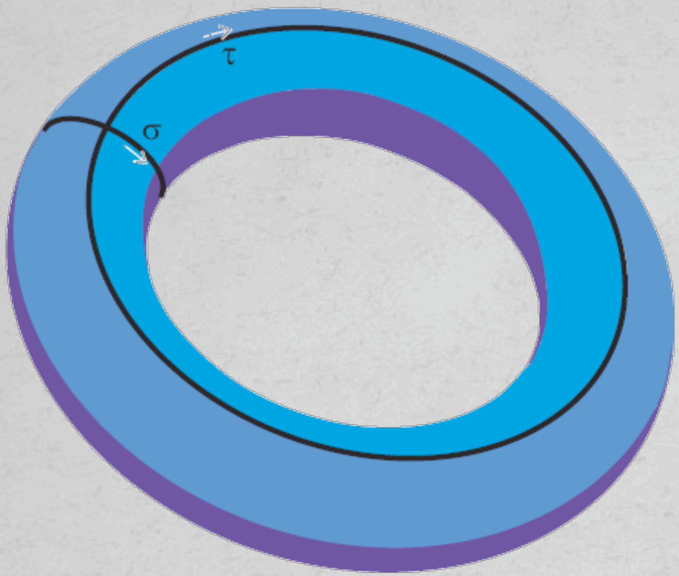

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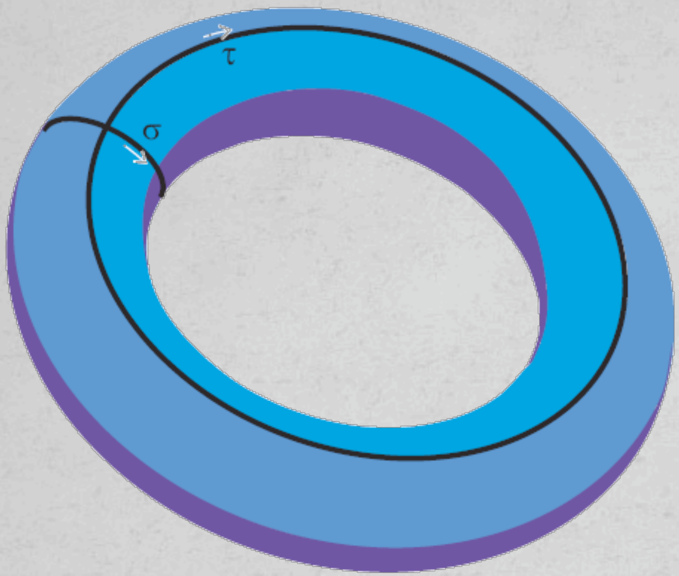

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I.e. from the asymptotical spectrum (infinite R) we can compute the Ground state energy for ANY finite volume!

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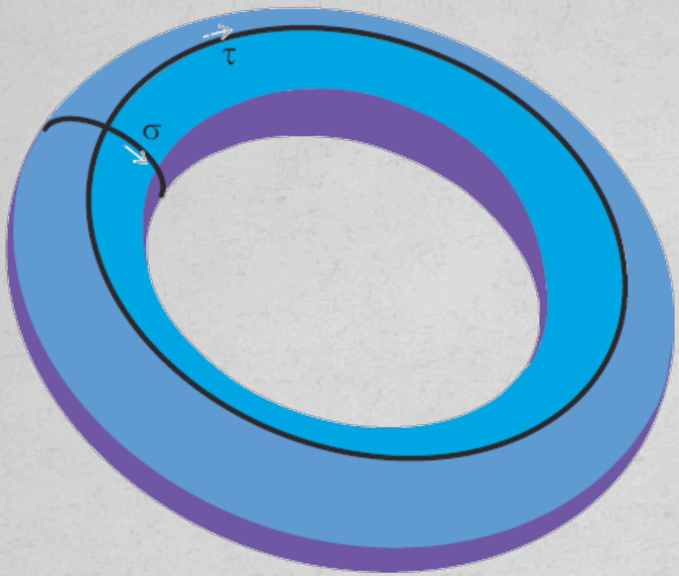

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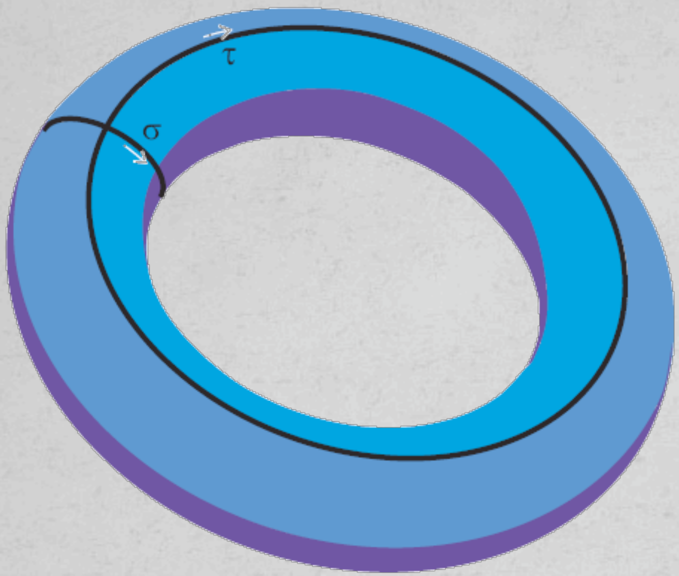

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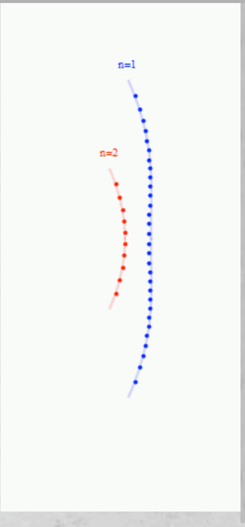
In our case the wick rotated theory is different – "mirror" theory

Ambjorn, Janik, Kristjansen; Arutynov, Frolov

AdS/CFT bound states

$$\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k=1 (k\neq j)}^{J} \frac{u_j - u_k + i}{u_j - u_k - i}$$

Till Bargheer,
Niklas Beisert, N. G.



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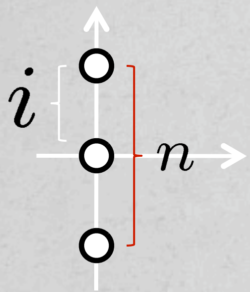

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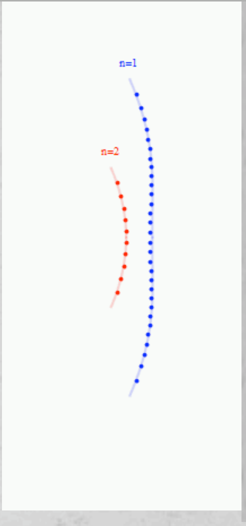

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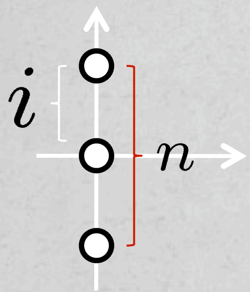

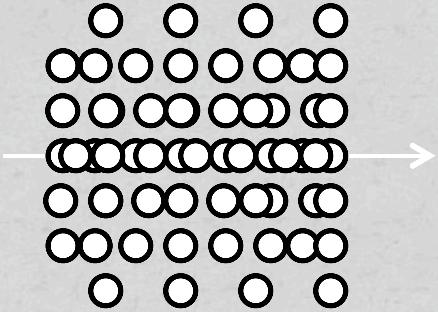

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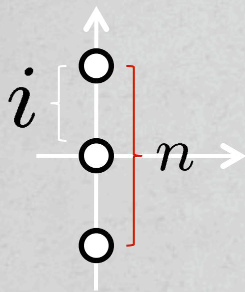

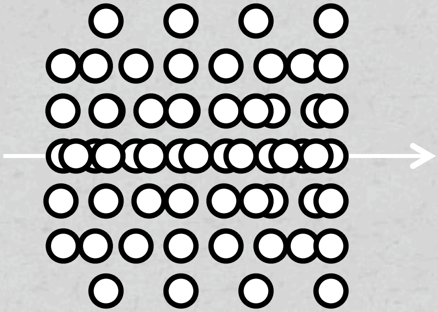

+

$$Y_1(u)$$

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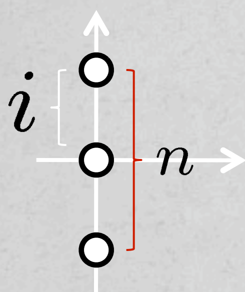

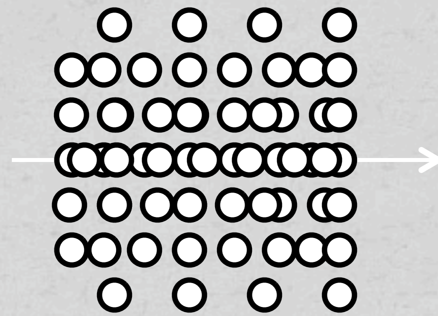

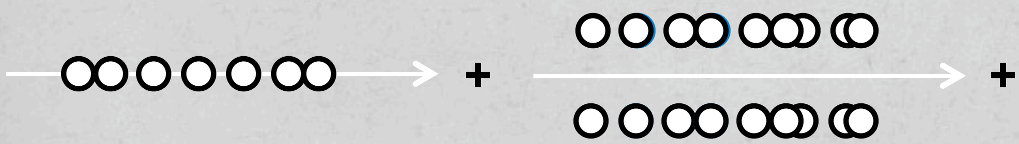

$Y_1(u)$ $Y_2(u)$

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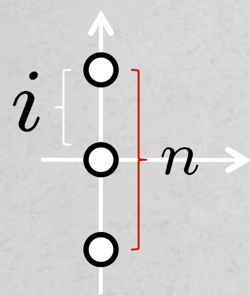

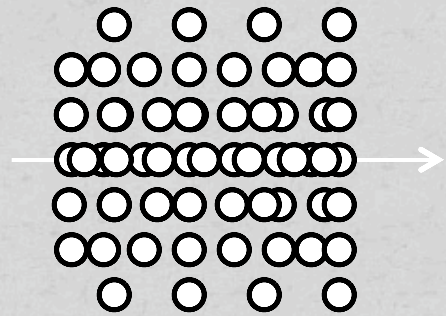

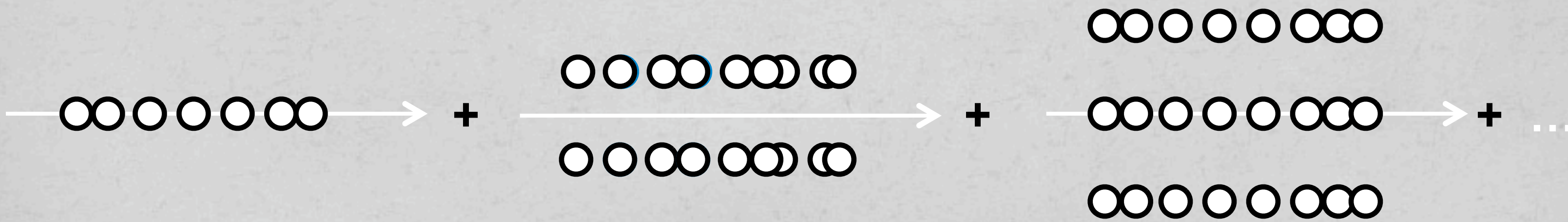

$Y_1(u)$ $Y_2(u)$ $Y_3(u)$

All loop Bethe equations

$$1=\prod_{j=1}^{K_2}\frac{u_{1,k}-u_{2,j}+\frac{i}{2}}{u_{1,k}-u_{2,j}-\frac{i}{2}}\prod_{j=1}^{K_4}\frac{1-1/x_{1,k}x_{4,j}^{+}}{1-1/x_{1,k}x_{4,j}^{-}},$$

$$1=\prod_{j\neq k}^{K_2}\frac{u_{2,k}-u_{2,j}-i}{u_{2,k}-u_{2,j}+i}\prod_{j=1}^{K_3}\frac{u_{2,k}-u_{3,j}+\frac{i}{2}}{u_{2,k}-u_{3,j}-\frac{i}{2}}\prod_{j=1}^{K_1}\frac{u_{2,k}-u_{1,j}+\frac{i}{2}}{u_{2,k}-u_{1,j}-\frac{i}{2}},$$

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$$1=\left(\frac{x_{4,k}^{-}}{x_{4,k}^{+}}\right)^{L}\prod_{j\neq k}^{K_4}\frac{u_{4,k}-u_{4,j}+i}{u_{4,k}-u_{4,j}-i}\prod_{j}^{K_4}\left(\frac{1-1/x_{4,k}^{+}x_{4,j}^{-}}{1-1/x_{4,k}^{-}x_{4,j}^{+}}\right)\sigma^2(x_{4,k},x_{4,j})$$

$$\times\prod_{j=1}^{K_1}\frac{1-1/x_{4,k}^{-}x_{1,j}}{1-1/x_{4,k}^{+}x_{1,j}}\prod_{j=1}^{K_3}\frac{x_{4,k}^{-}-x_{3,j}}{x_{4,k}^{+}-x_{3,j}}\prod_{j=1}^{K_5}\frac{x_{4,k}^{-}-x_{5,j}}{x_{4,k}^{+}-x_{5,j}}\prod_{j=1}^{K_7}\frac{1-1/x_{4,k}^{-}x_{7,j}}{1-1/x_{4,k}^{+}x_{7,j}},$$

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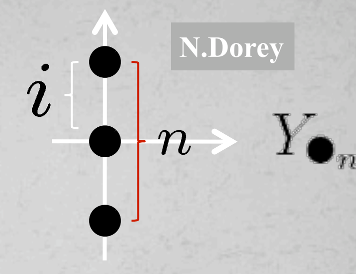
All loop Bethe equations



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All loop Bethe equations

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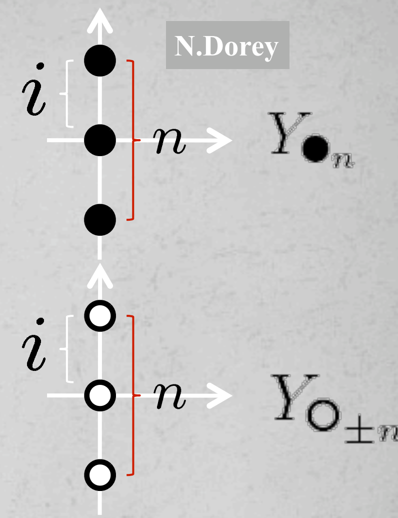

All loop Bethe equations

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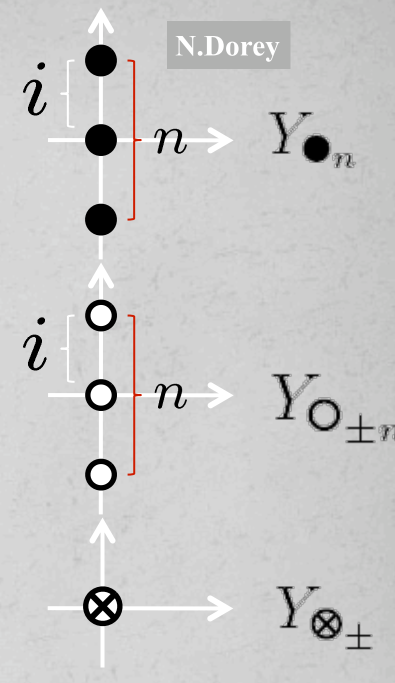

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$$\times\prod_{j=1}^{K_1}\frac{1-1/x_{4,k}^{-}x_{1,j}}{1-1/x_{4,k}^{+}x_{1,j}}\prod_{j=1}^{K_3}\frac{x_{4,k}^{-}-x_{3,j}}{x_{4,k}^{+}-x_{3,j}}\prod_{j=1}^{K_5}\frac{x_{4,k}^{-}-x_{5,j}}{x_{4,k}^{+}-x_{5,j}}\prod_{j=1}^{K_7}\frac{1-1/x_{4,k}^{-}x_{7,j}}{1-1/x_{4,k}^{+}x_{7,j}},$$

$$1=\prod_{j=1}^{K_6}\frac{u_{5,k}-u_{6,j}+\frac{i}{2}}{u_{5,k}-u_{6,j}-\frac{i}{2}}\prod_{j=1}^{K_4}\frac{x_{5,k}-x_{4,j}^{+}}{x_{5,k}-x_{4,j}^{-}},$$

$$1=\prod_{j\neq k}^{K_6}\frac{u_{6,k}-u_{6,j}-i}{u_{6,k}-u_{6,j}+i}\prod_{j=1}^{K_5}\frac{u_{6,k}-u_{5,j}+\frac{i}{2}}{u_{6,k}-u_{5,j}-\frac{i}{2}}\prod_{j=1}^{K_7}\frac{u_{6,k}-u_{7,j}+\frac{i}{2}}{u_{6,k}-u_{7,j}-\frac{i}{2}},$$

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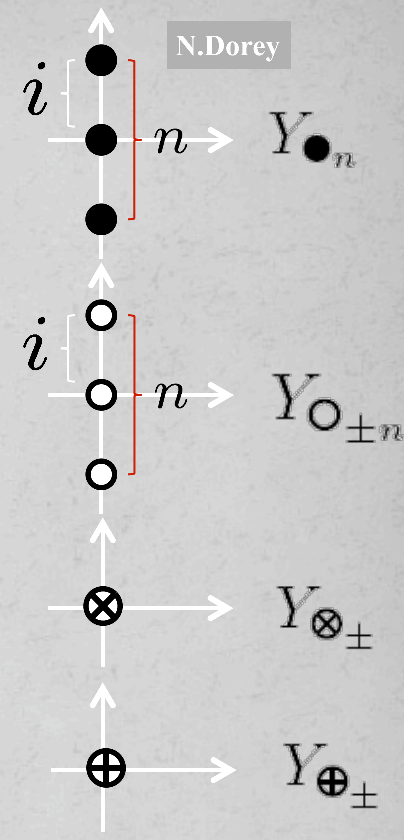

All loop Bethe equations

$$1=\prod_{j=1}^{K_2}\frac{u_{1,k}-u_{2,j}+\frac{i}{2}}{u_{1,k}-u_{2,j}-\frac{i}{2}}\prod_{j=1}^{K_4}\frac{1-1/x_{1,k}x^+_{4,j}}{1-1/x_{1,k}x^-_{4,j}},$$

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$$1=\left(\frac{x^-_{4,k}}{x^+_{4,k}}\right)^L\prod_{j\neq k}^{K_4}\frac{u_{4,k}-u_{4,j}+i}{u_{4,k}-u_{4,j}-i}\prod_{j}^{K_4}\left(\frac{1-1/x^+_{4,k}x^-_{4,j}}{1-1/x^-_{4,k}x^+_{4,j}}\right)\sigma^2(x_{4,k},x_{4,j})$$

$$\times\prod_{j=1}^{K_1}\frac{1-1/x^-_{4,k}x_{1,j}}{1-1/x^+_{4,k}x_{1,j}}\prod_{j=1}^{K_3}\frac{x^-_{4,k}-x_{3,j}}{x^+_{4,k}-x_{3,j}}\prod_{j=1}^{K_5}\frac{x^-_{4,k}-x_{5,j}}{x^+_{4,k}-x_{5,j}}\prod_{j=1}^{K_7}\frac{1-1/x^-_{4,k}x_{7,j}}{1-1/x^+_{4,k}x_{7,j}},$$

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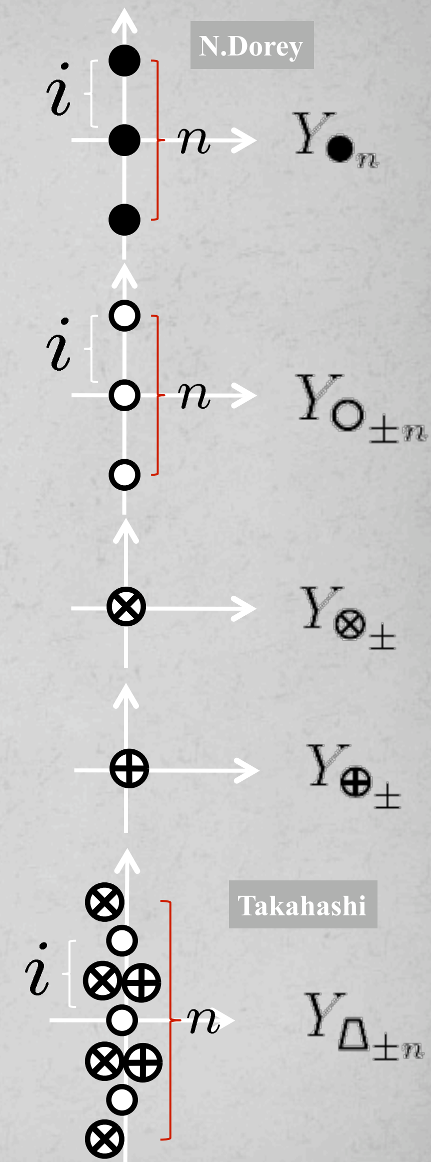
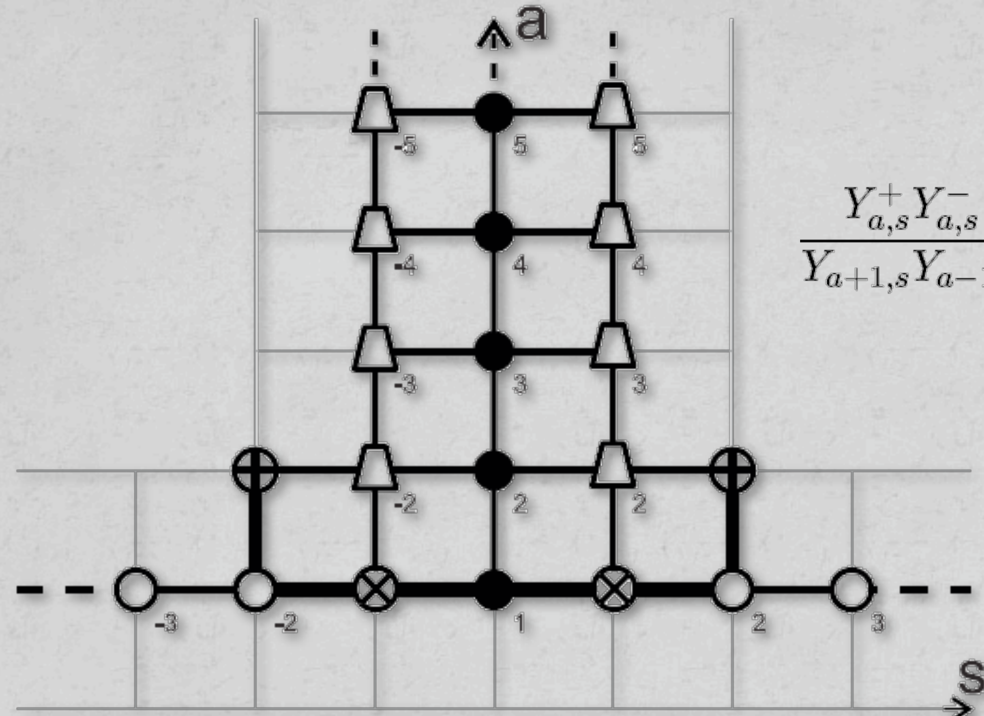


AdS/CFT Y-system



N.G., Kazakov, Vieira

$$\frac{Y_{a,s}^{+} Y_{a,s}^{-}}{Y_{a+1,s} Y_{a-1,s}} = \frac{(1+Y_{a,s+1})(1+Y_{a,s-1})}{(1+Y_{a+1,s})(1+Y_{a-1,s})}$$

$$E = \sum_{a=1}^{\infty} \int_{-\infty}^{\infty} \frac{du}{2\pi i} \frac{\partial \epsilon_a}{\partial u} \log\left(1 + Y_{a,0}(u)\right)$$

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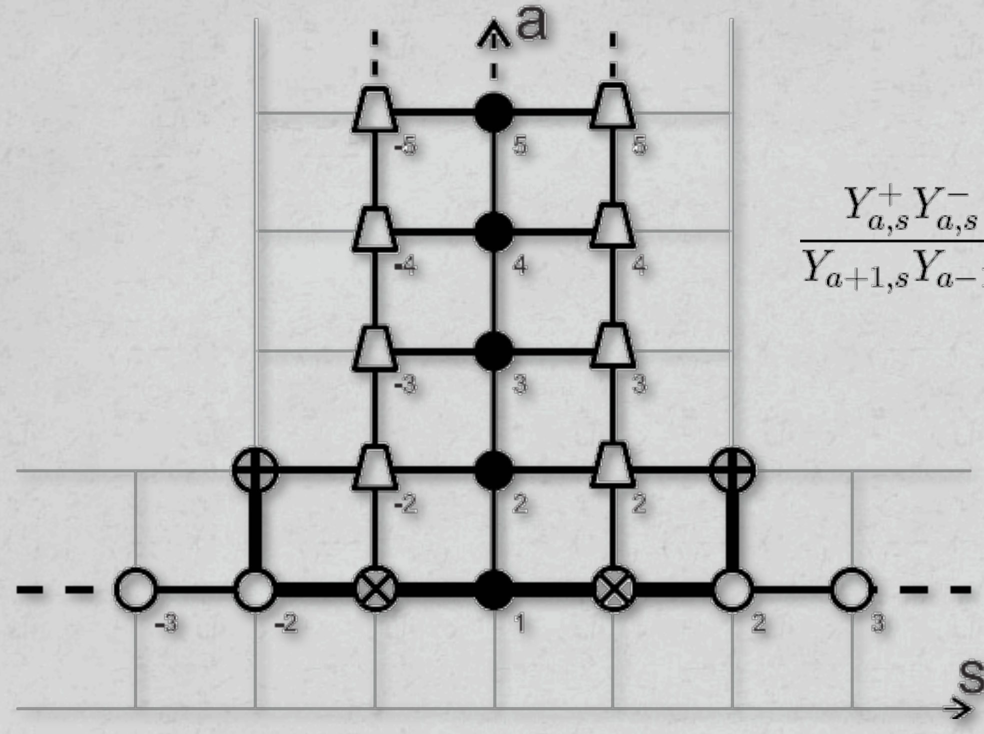

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…Bazhanov, Lukyanov, Zamolodchikov, P.Dorey, Totteo…

…Destri de Vega, Bytsko ,Teschner….

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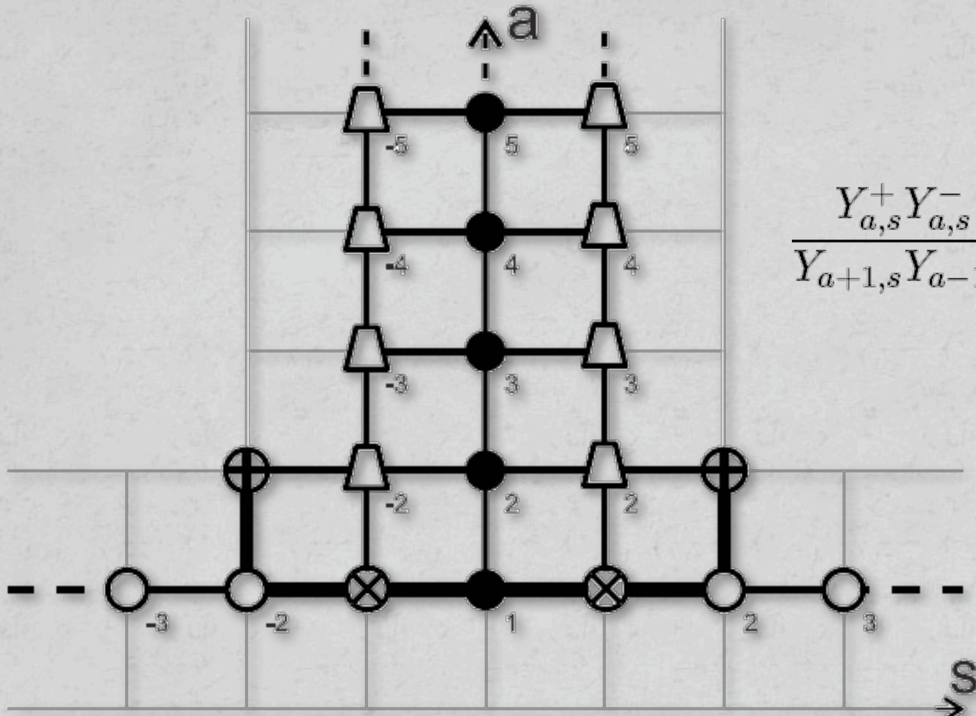

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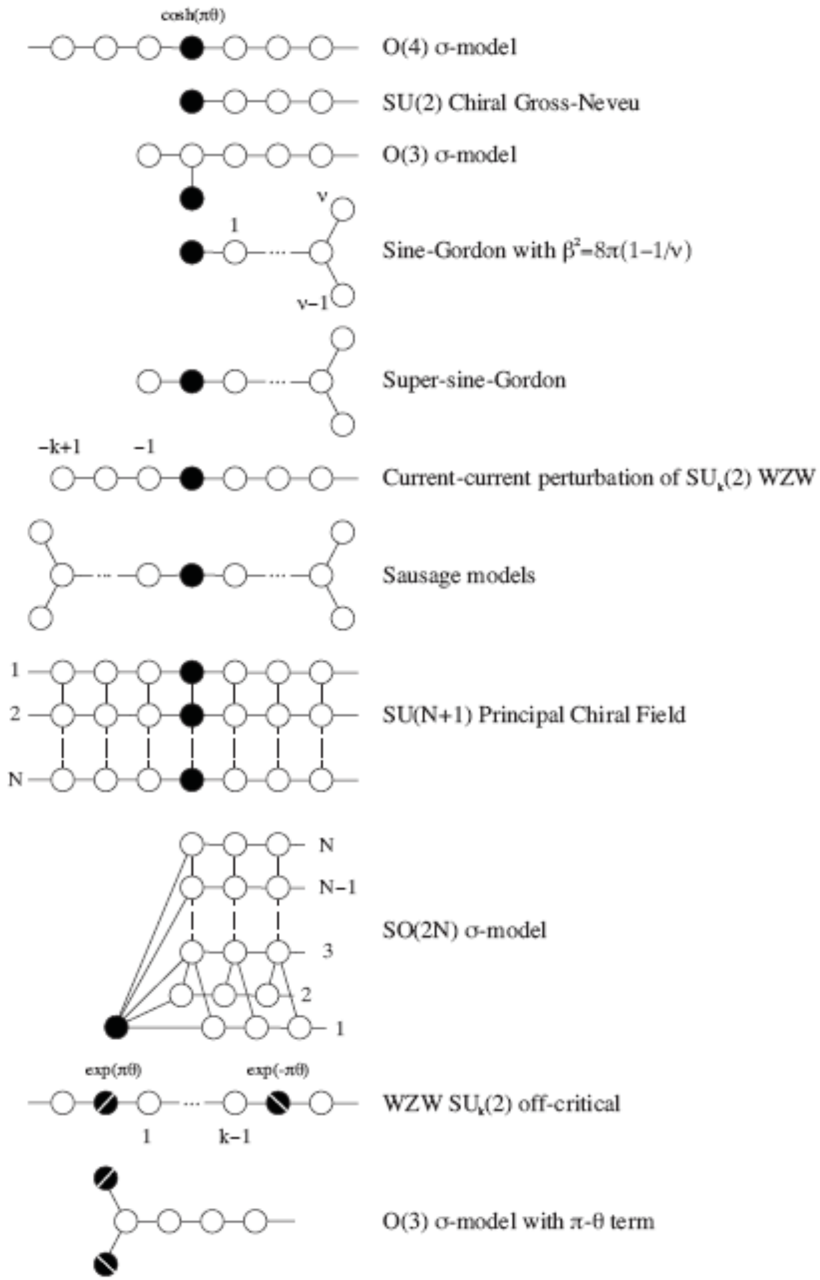

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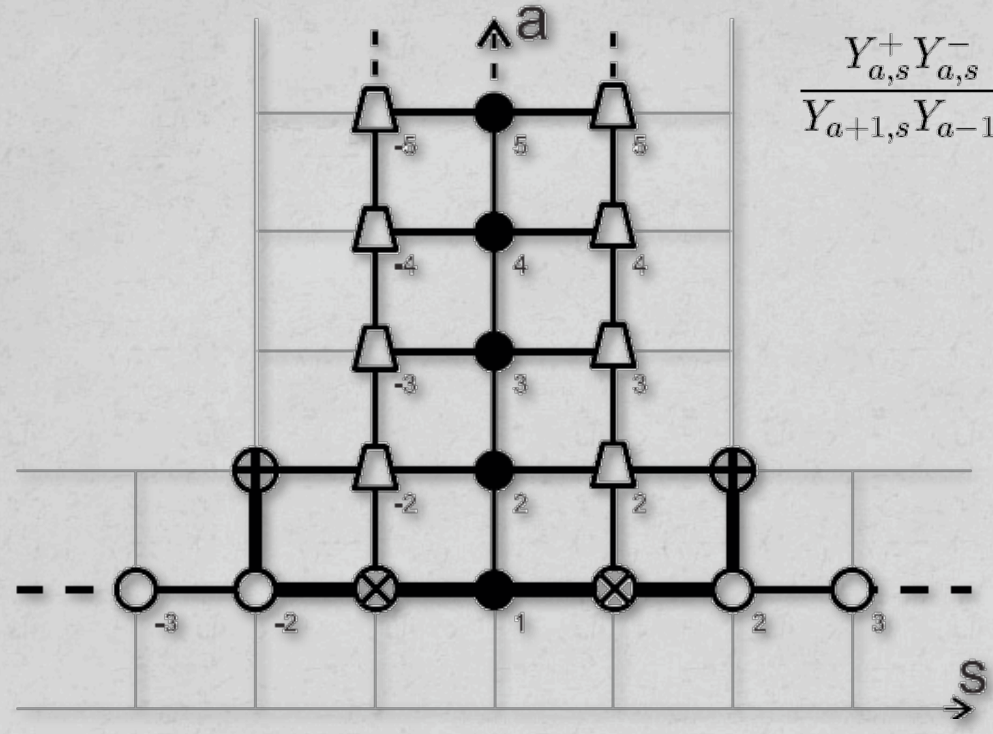
Tests of the proposal

Large volume

N.G., Kazakov, Vieira



$$\frac{Y_{a,s}^{+}Y_{a,s}^{-}}{Y_{a+1,s}Y_{a-1,s}} = \frac{(1+Y_{a,s+1})(1+Y_{a,s-1})}{(1+Y_{a+1,s})(1+Y_{a-1,s})}$$

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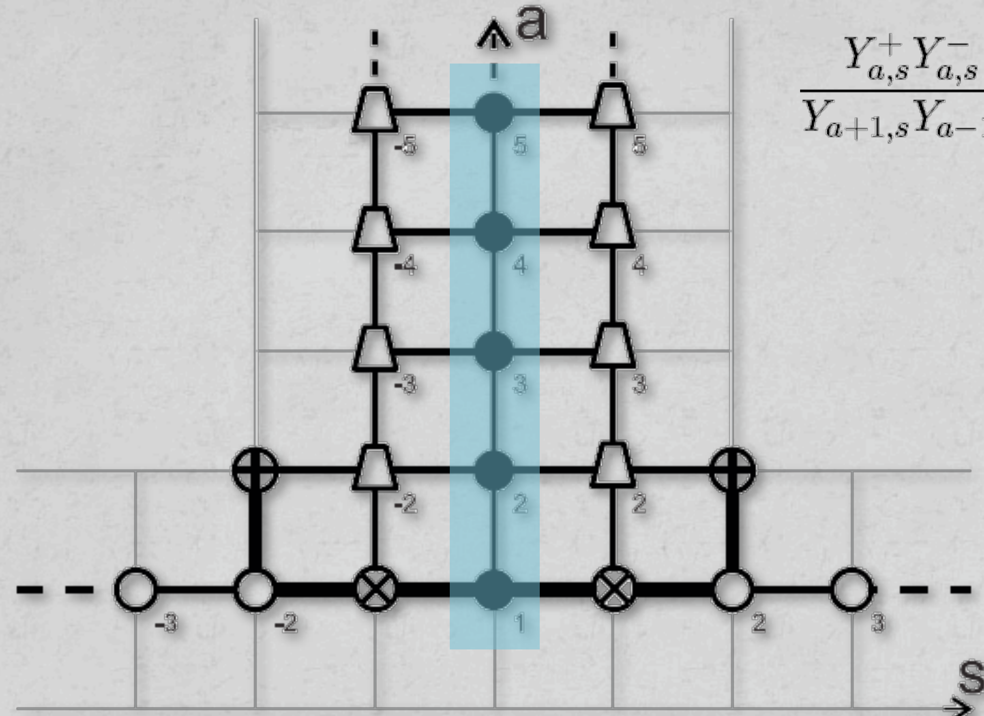

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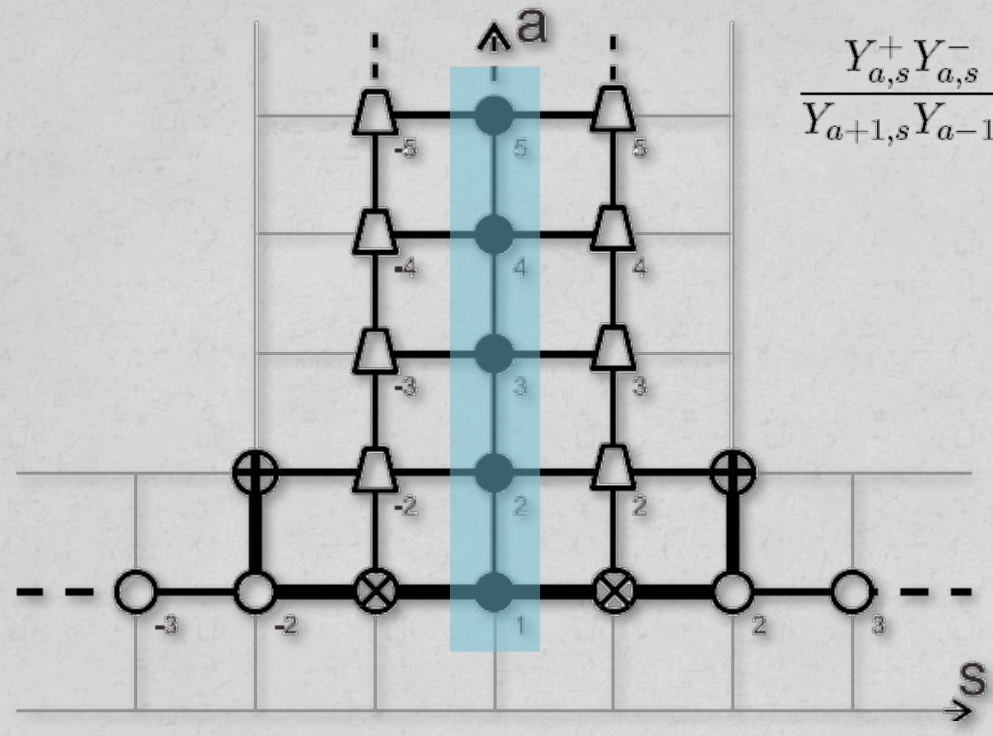

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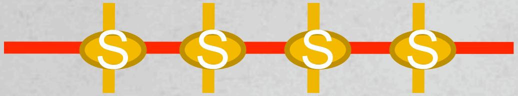
Use Hirota equation:

$$T_{a,s}^{+}T_{a,s}^{-} = T_{a+1,s}T_{a-1,s} + T_{a,s+1}T_{a,s-1}\,,$$

$$\text{where } Y_{a,s} = \frac{T_{a,s+1}T_{a,s-1}}{T_{a+1,s}T_{a-1,s}}\,.$$

Hirota – large L

$$\hat{T}_{rep}(u|\theta_1,\theta_2,\ldots) = \mathrm{Tr}_{rep}(S(u,\theta_1)S(u,\theta_2)\ldots)$$



Generating functional

$$\mathcal{W} = \left[1 - \frac{Q_1^- B^{+(+)} R^{-(+)}}{Q_1^+ B^{+(-)} R^{-(-)}} D\right]\left[1 - \frac{Q_1^- Q_2^{++} R^{-(+)}}{Q_1^+ Q_2 R^{-(-)}} D\right]^{-1} \times$$
$$\times \left[1 - \frac{Q_2^{--} Q_3^+ R^{-(+)}}{Q_2 Q_3^- R^{-(-)}} D\right]^{-1}\left[1 - \frac{Q_3^+}{Q_3^-} D\right] \;,\; D = e^{-i\partial_u}$$

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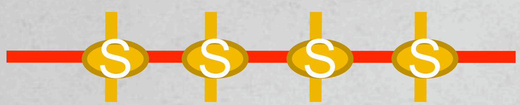
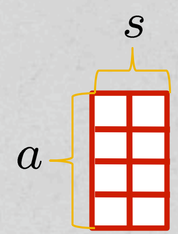


The eigevalues solve Hirota!

for rep =



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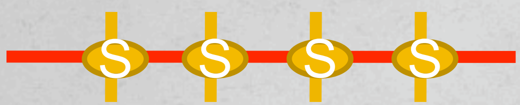

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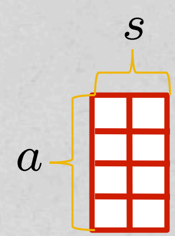

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$$\mathcal{W} = \sum_{s=0}^{\infty} T_{1,s}^{[1-s]} D^s$$

$$\mathcal{W}^{-1} = \sum_{a=0}^{\infty} (-1)^a T_{a,1}^{[1-a]} D^a$$

Beisert

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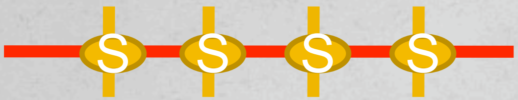

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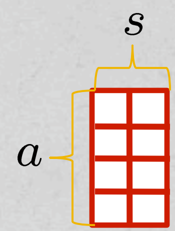

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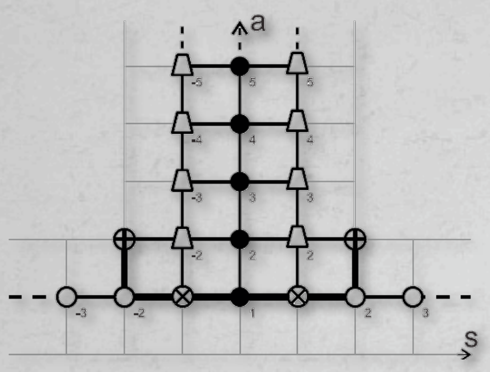
Beisert

The ABA equations follows then from

$$Y_{1,0}(u_j) = -1$$

Strong coupling



$$Y_{as}(z+\tfrac{i}{4g})Y_{as}(z-\tfrac{i}{4g}) = \frac{(1+Y_{a,s+1}(z))(1+Y_{a,s-1}(z))}{(1+1/Y_{a+1,s}(z))(1+1/Y_{a-1,s}(z))}$$

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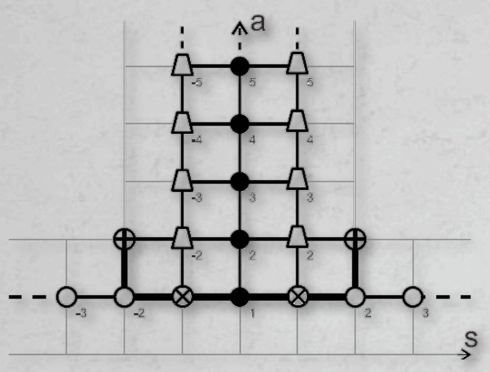

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We can rewrite it in terms of T's:

$$Y_{a,s}=\frac{T_{a,s+1}T_{a,s-1}}{T_{a+1,s}T_{a-1,s}}$$

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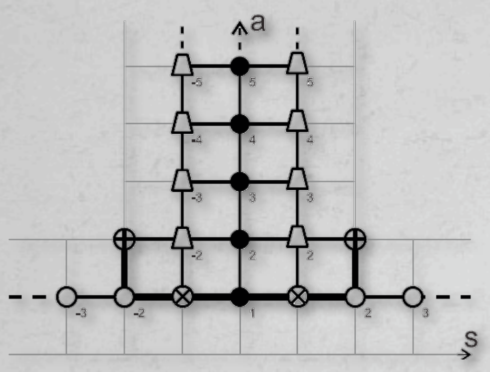

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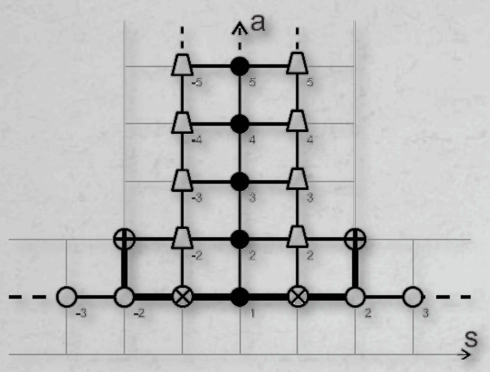

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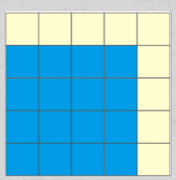

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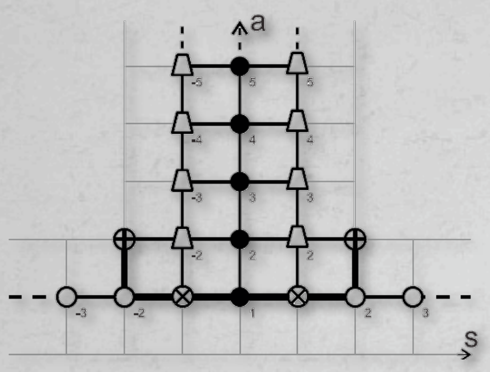

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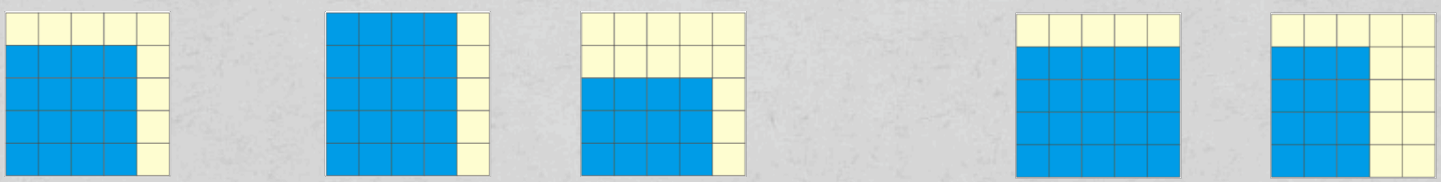

Strong coupling

- For GL(n) characters the general expression are known

$$s_\lambda(y) = \frac{\det(y_i^{\lambda_j+4-j})_{1\leq i,j\leq 4}}{\det(y_i^{4-j})_{1\leq i,j\leq 4}}$$

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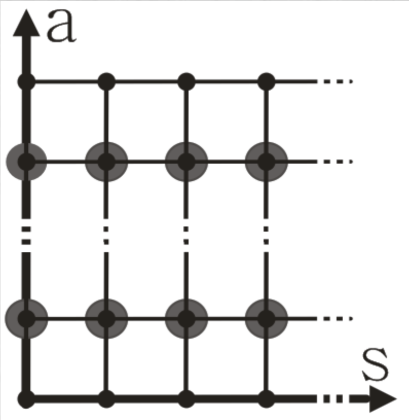

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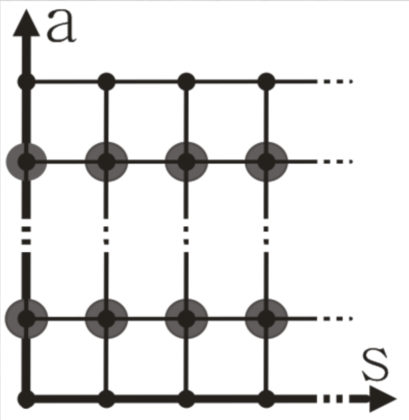

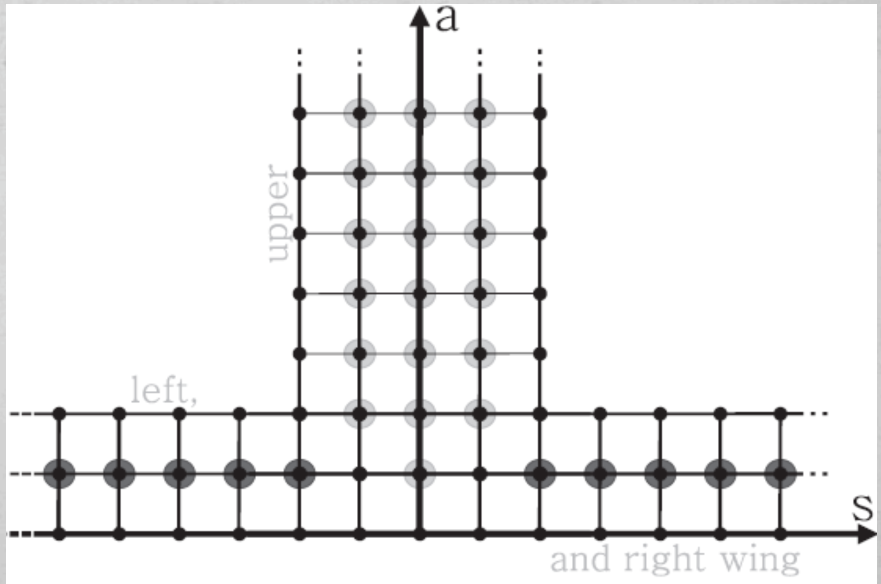

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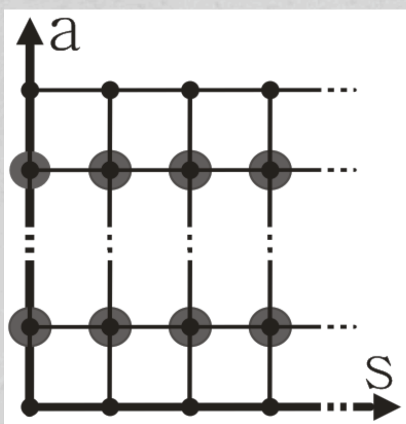

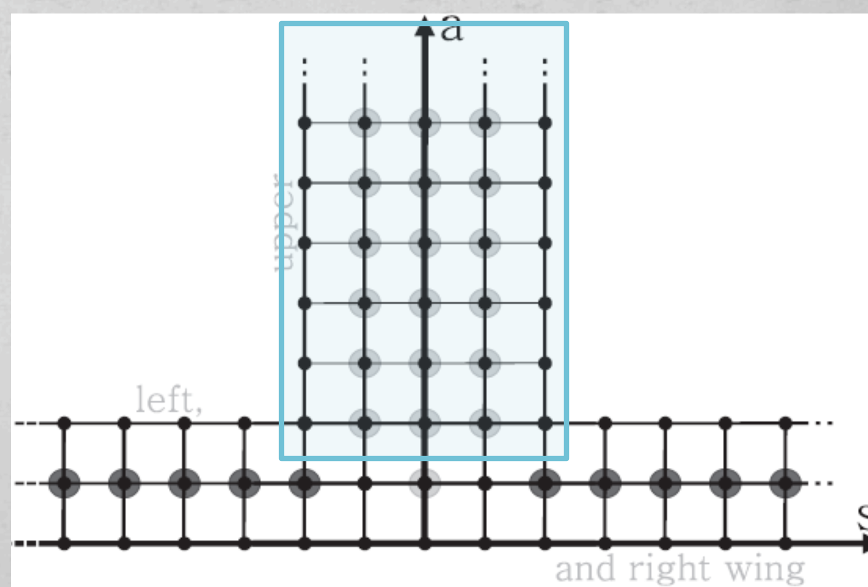

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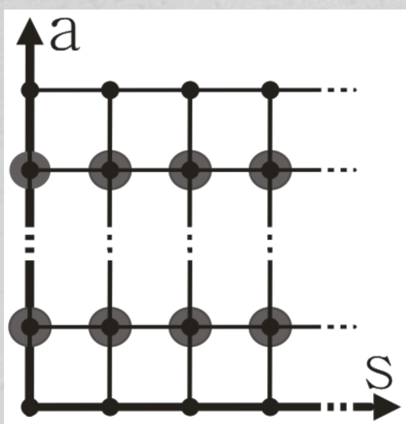

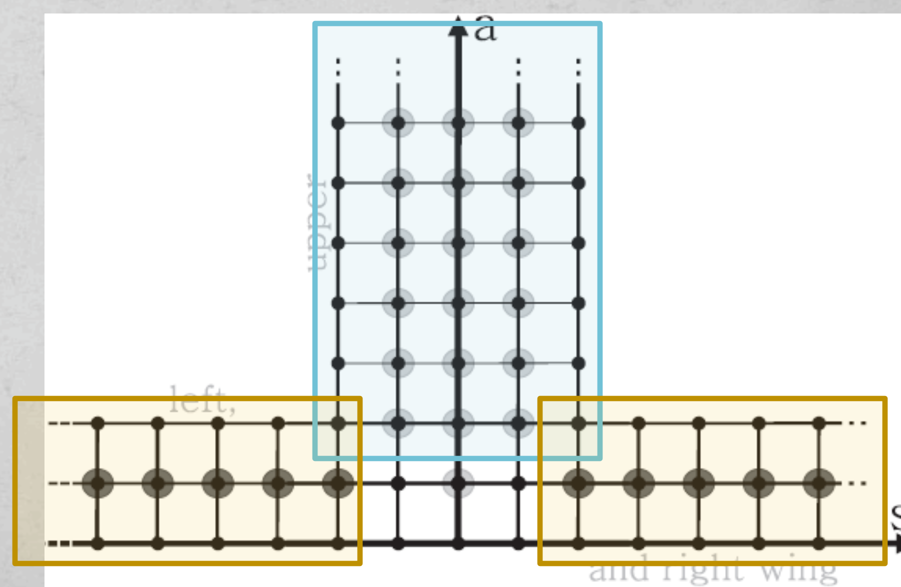

Boundary conditions and solution

N.G.

Solving Hirota equation with the boundary conditions:

$$Y_{a,0} \simeq Y_{a,0}^{L=\infty} \simeq (\Delta f^2)^a$$

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For $Y_{1,0}$ we get:

$$\begin{aligned}
&\left(\Delta\left(f^2\Delta\left(\left(\Delta^2-4\Delta-1\right)\bar{f}+2\Delta\bar{f}^2+2\right)+\left(\Delta^2-4\Delta-1\right)\bar{f}+f\left(\Delta^3\left(\bar{f}-2\right)\bar{f}+\Delta^2\left(-4\bar{f}^2+4\bar{f}+1\right)-\Delta\left(\bar{f}^2-6\bar{f}+4\right)-1\right)+2\Delta\bar{f}^2+2\right)^2\right)/\\
&\quad\left(\left(\Delta-1\right)^2\left(-f^2\Delta^5\bar{f}^2\left(\bar{f}+f-2\right)^2+\Delta\left(2f\left(\bar{f}-2\right)+\left(\bar{f}-4\right)\bar{f}+f^2\right)+\right.\right.\\
&\qquad f\Delta^4\bar{f}\left(f^3\bar{f}\left(\bar{f}^2-4\bar{f}+6\right)-2f^2\left(2\bar{f}^3-8\bar{f}^2+8\bar{f}+1\right)+2f\left(3\bar{f}^3-8\bar{f}^2+4\bar{f}+2\right)-2\left(\bar{f}-2\right)\bar{f}\right)+\\
&\qquad \Delta^3\left(f^4\left(\bar{f}-4\right)\bar{f}+2f^3\bar{f}\left(\bar{f}^2-6\bar{f}+8\right)+f^2\left(\bar{f}^4-12\bar{f}^3+28\bar{f}^2-16\bar{f}-1\right)-2f\bar{f}\left(2\bar{f}^3-8\bar{f}^2+8\bar{f}+1\right)-\bar{f}^2\right)+\\
&\qquad \left.\left.\Delta^2\left(2f^3\left(\bar{f}-2\right)+2f^2\left(2\bar{f}^2-6\bar{f}+3\right)+2f\bar{f}\left(\bar{f}^2-6\bar{f}+8\right)+\bar{f}^2\left(\bar{f}^2-4\bar{f}+6\right)+f^4\right)+1\right)\right)
\end{aligned}$$

Strong coupling solution

$$E = \sum_{i=1}^{M} \epsilon(u_{4,i}) + \sum_{a=1}^{\infty} \int_{-\infty}^{\infty} \frac{du}{2\pi i} \frac{\partial \epsilon_a}{\partial u} \log\left(1 + Y_{a,0}(u)\right)$$

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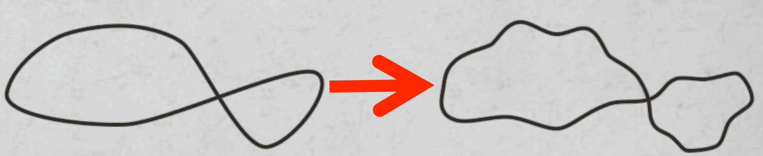
Frolov, Tseytlin

$$\delta E_{1-\text{loop}}=\frac{1}{2}\sum_{n=-\infty}^{\infty}\sum_{p=1}^{16}(-1)^{F_p}\mathcal{E}_n^p$$



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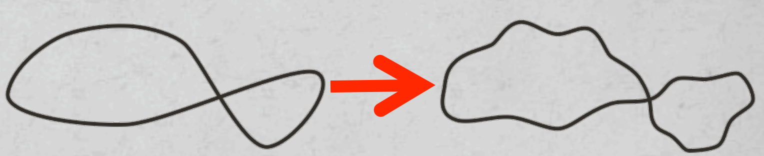

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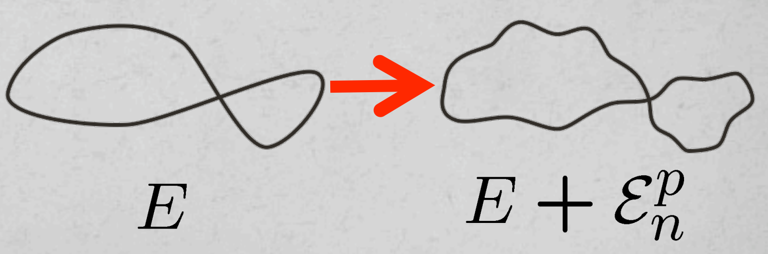

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4 S5 2+1+1 AdS5

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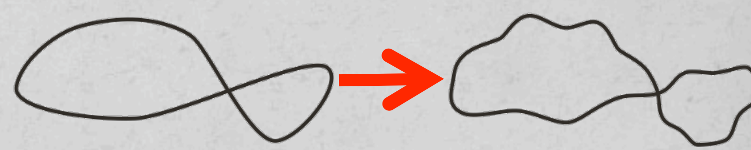

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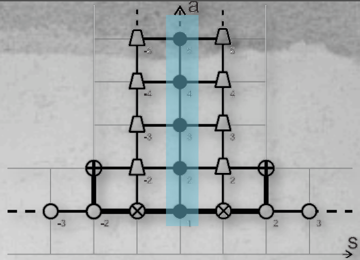
Konishi operator



The simplest operator

$$\mathcal{O} = tr(ZZWW) - tr(ZWZW) \qquad\qquad g \to 0$$

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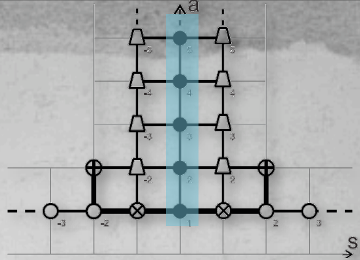

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$$Y_{a,0} = g^8 \left(3\,2^7\,\frac{3a^3+12au^2-4a}{(a^2+4u^2)^2}\right)^2 \frac{1}{y_a(u)y_{-a}(u)}$$

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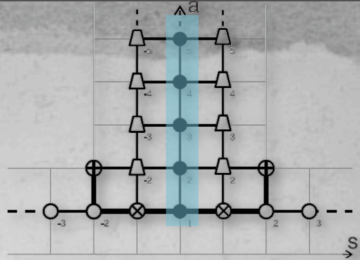

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$$(324 + 864\zeta_3 - 1440\zeta_5) g^8$$

In agreement with perturbation theory!!
4-loops!

Kotikov, Lipatov, Rej, Staudacher and Velizhanin
Sieg, Torrielli; Janik, Bojnok,;
N.G., Kazakov, Vieira

Integral form of Y-system for exitations

$$\log Y_n(u) = \int K_{nm}(u,v) \log(1 + Y_m(v))dv$$

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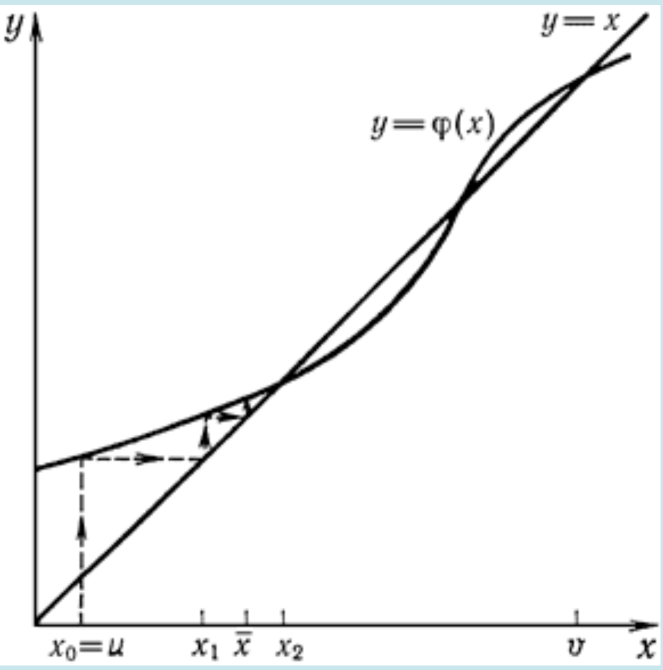

Konishi – from weak to strong

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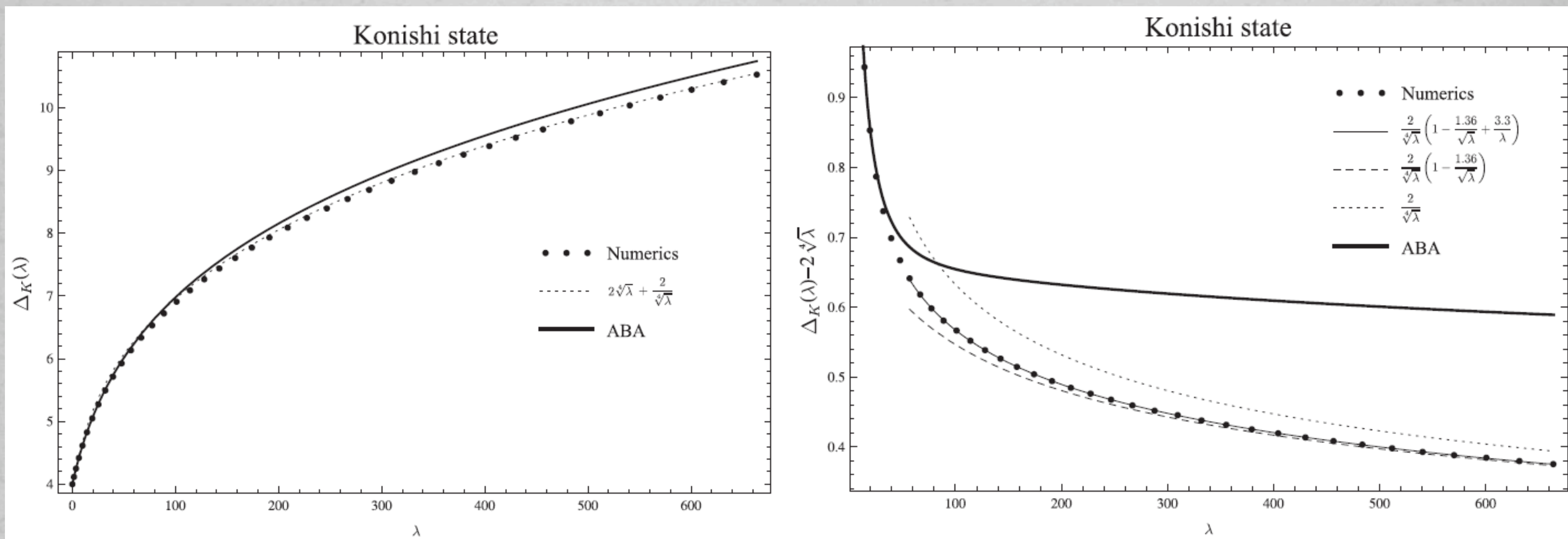

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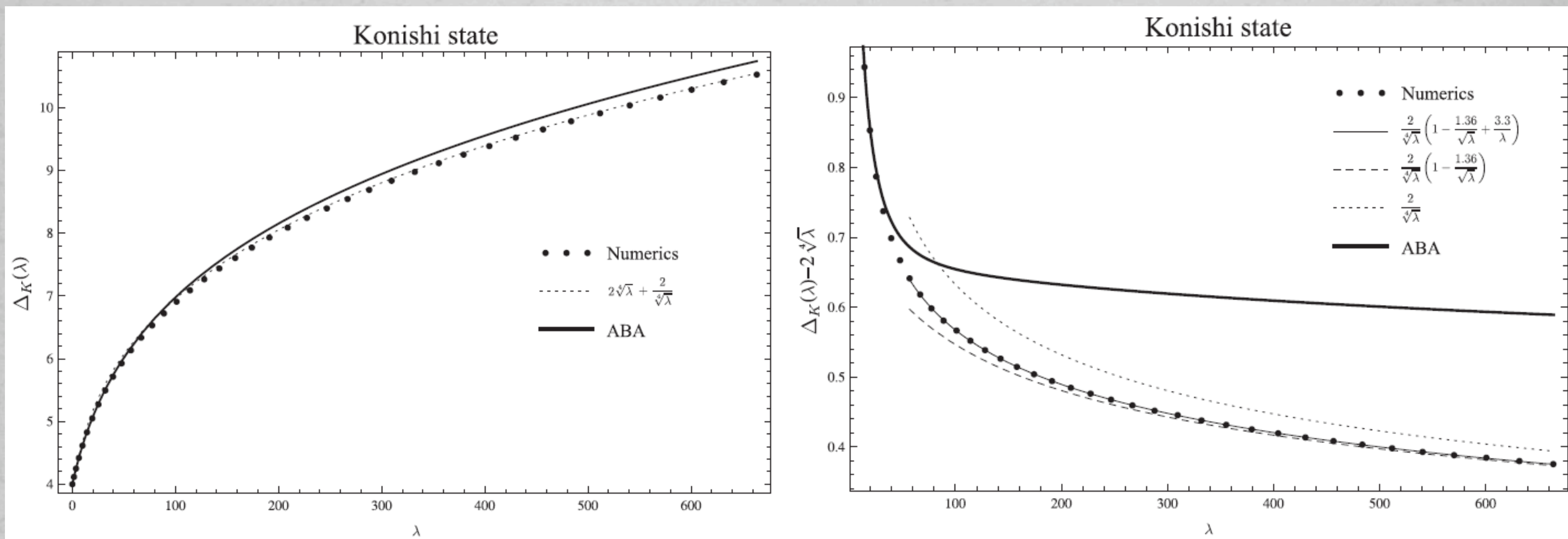

Fit:

$$\Delta_K = 2\lambda^{1/4}\left(1.0002 + \frac{0.994}{\lambda^{1/2}} - \frac{1.30}{\lambda} + \frac{3.1}{\lambda^{3/2}} + \dots\right)$$

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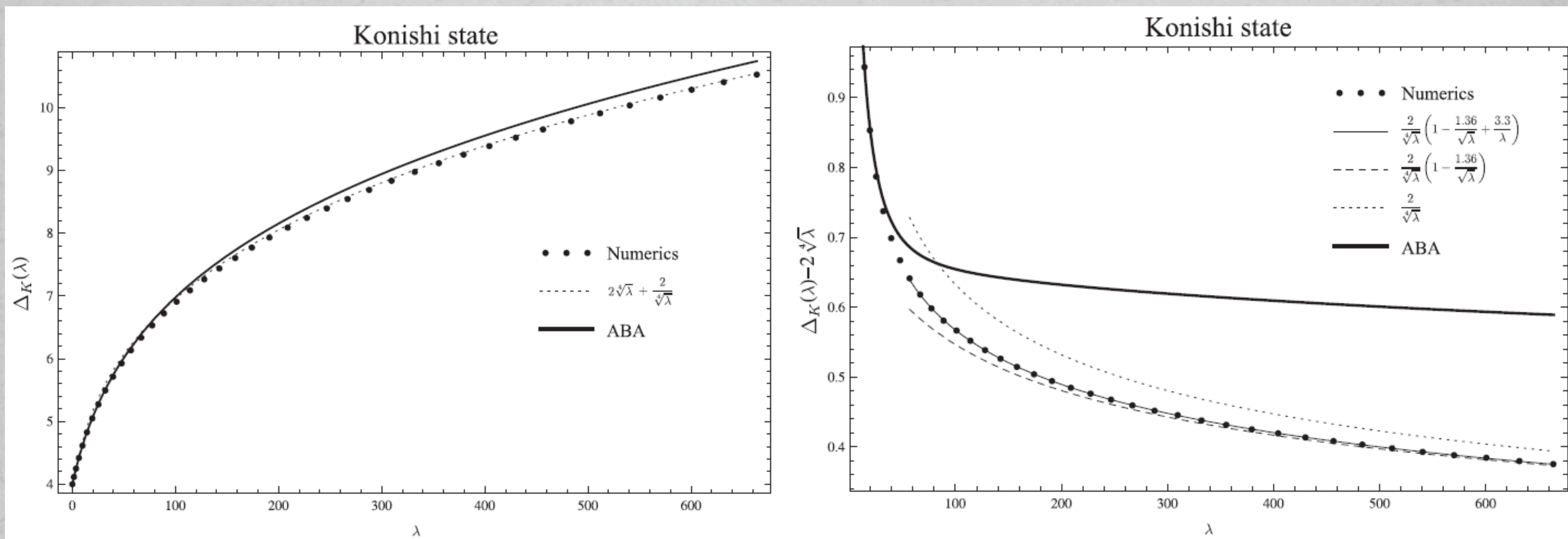

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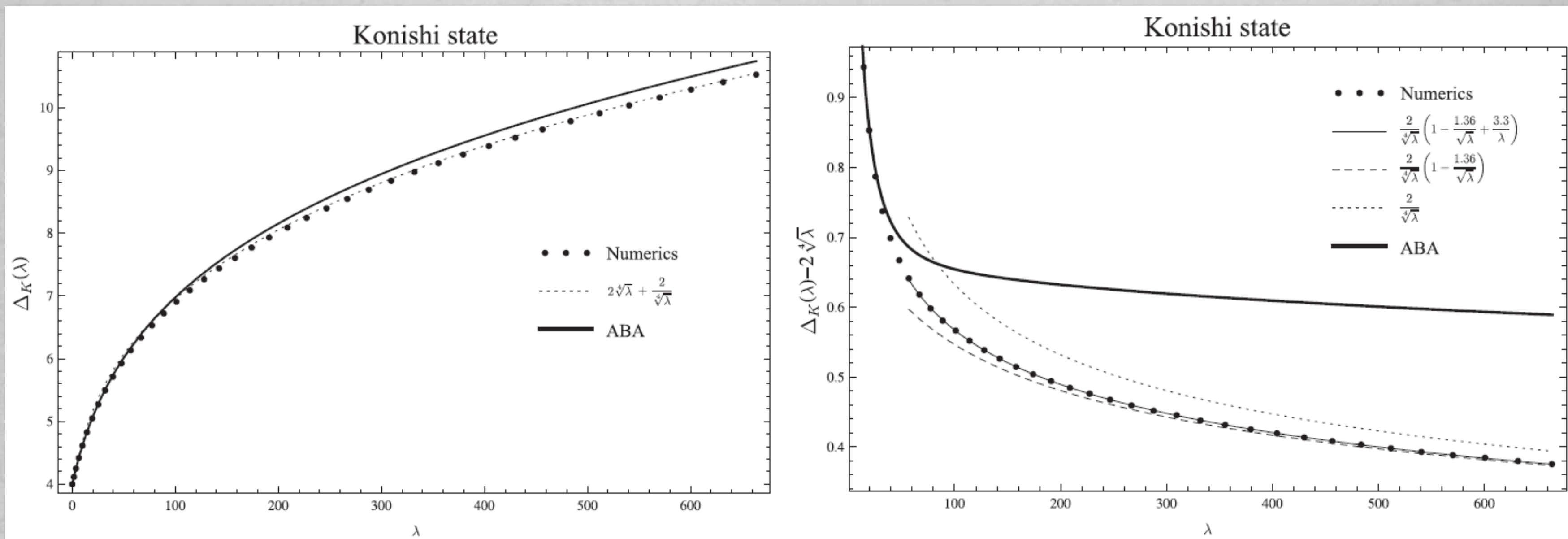

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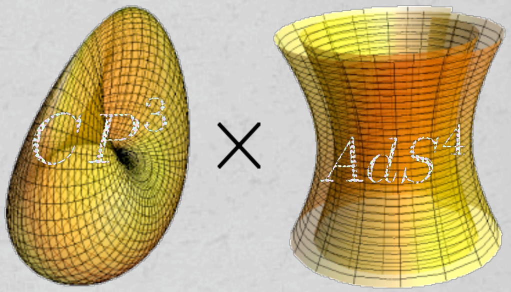

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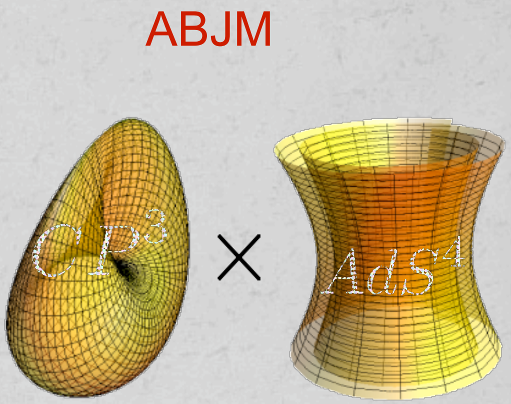
N.G., Kazakov, Vieira



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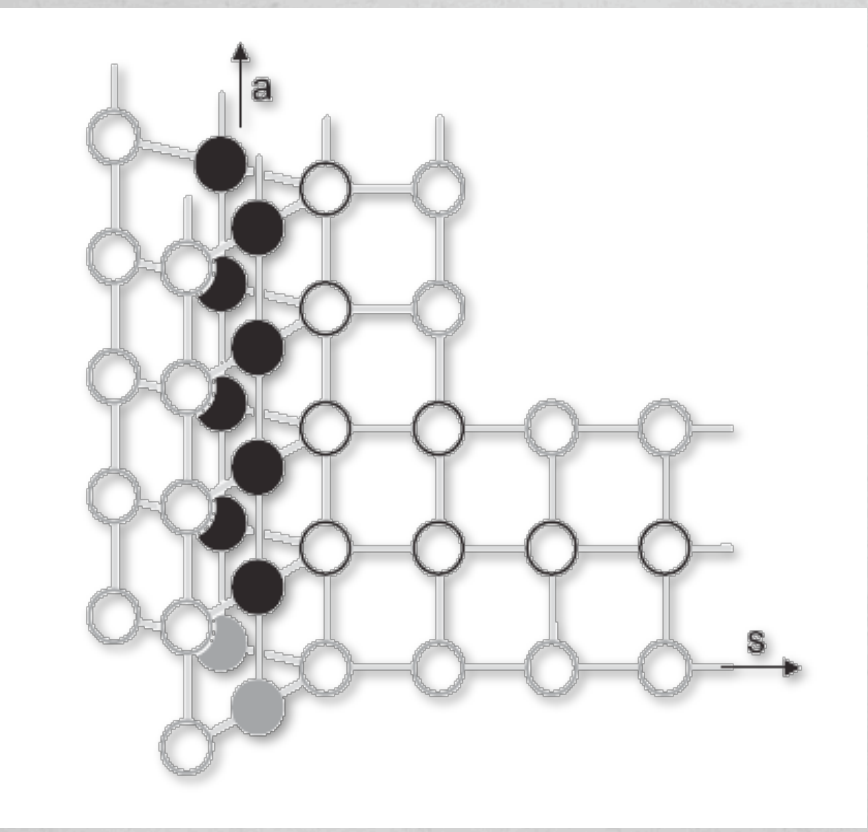

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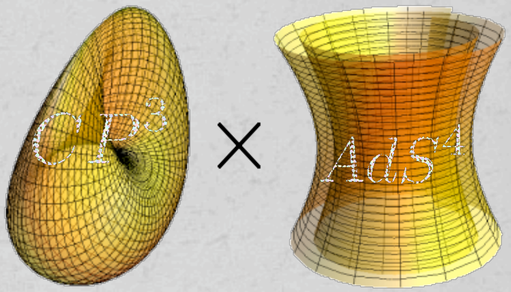

0901.3753

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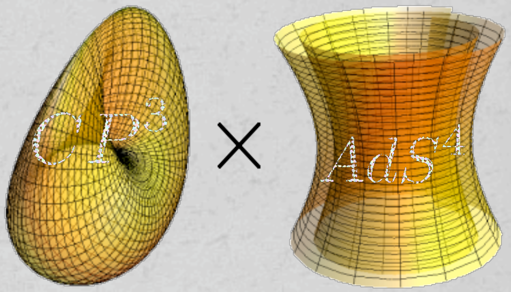
ABJM



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0908.2463

J.A. Minahan, O.Ohlsson Sax, C. Sieg

Conclutions

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- YY? Hidden structures in the perturbation theory from the gauge side