Faculté de physique et ingénierie
Université de Strasbourg

Characterization of the Singlet-Triplet anticrossing in Germanium Nanowire Double Quantum Dots

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Condensed Matter Theory and Quantum Computing Group

University of Basel
Department of Physics

Double Quantum Dots (DQD)

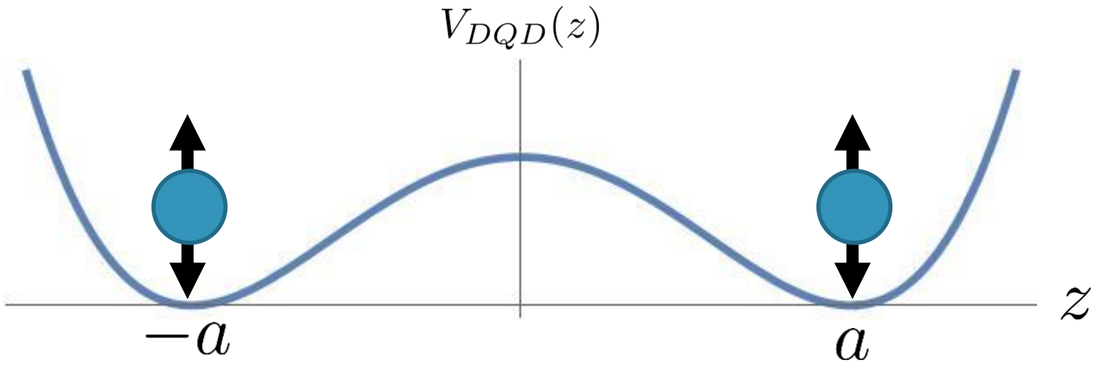
- Electron confined by a potential – “Artificial Atom”.

- Quantum dots (QD) as qubits :
  - Electronic spin $|\downarrow\rangle = |0\rangle \;\; |\uparrow\rangle = |1\rangle$

- Double quantum dot (DQD) : coupling 2 QD – “Artificial Molecule”
  - 2 electrons
  - Possible entanglement
  - Crucial for universal quantum computers

Our Model – DQD in a Ge Nanowire (Ge NW)

- Confining potential :



$$V_{DQD}(z) = \frac{m\omega_0^2}{2}\left(\frac{1}{4a^2}\left(z^2 - a^2\right)^2\right)$$

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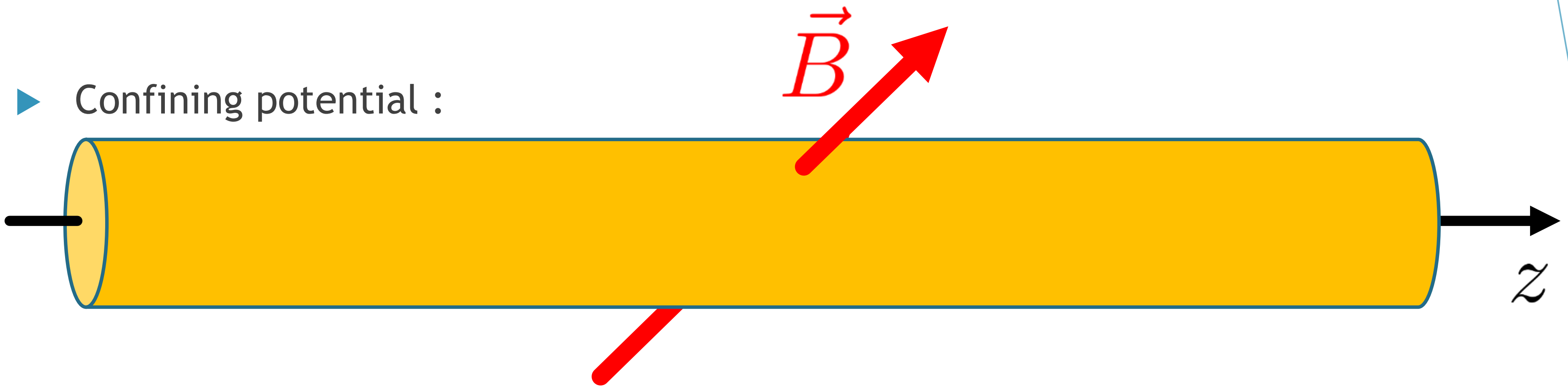

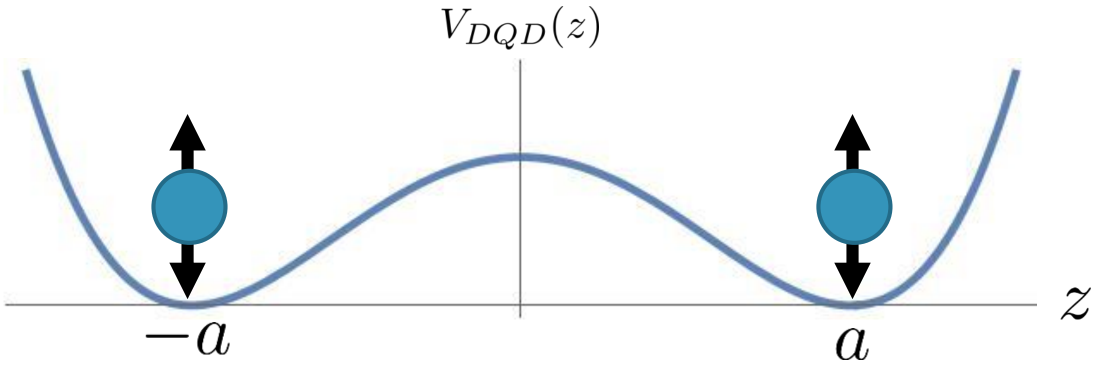

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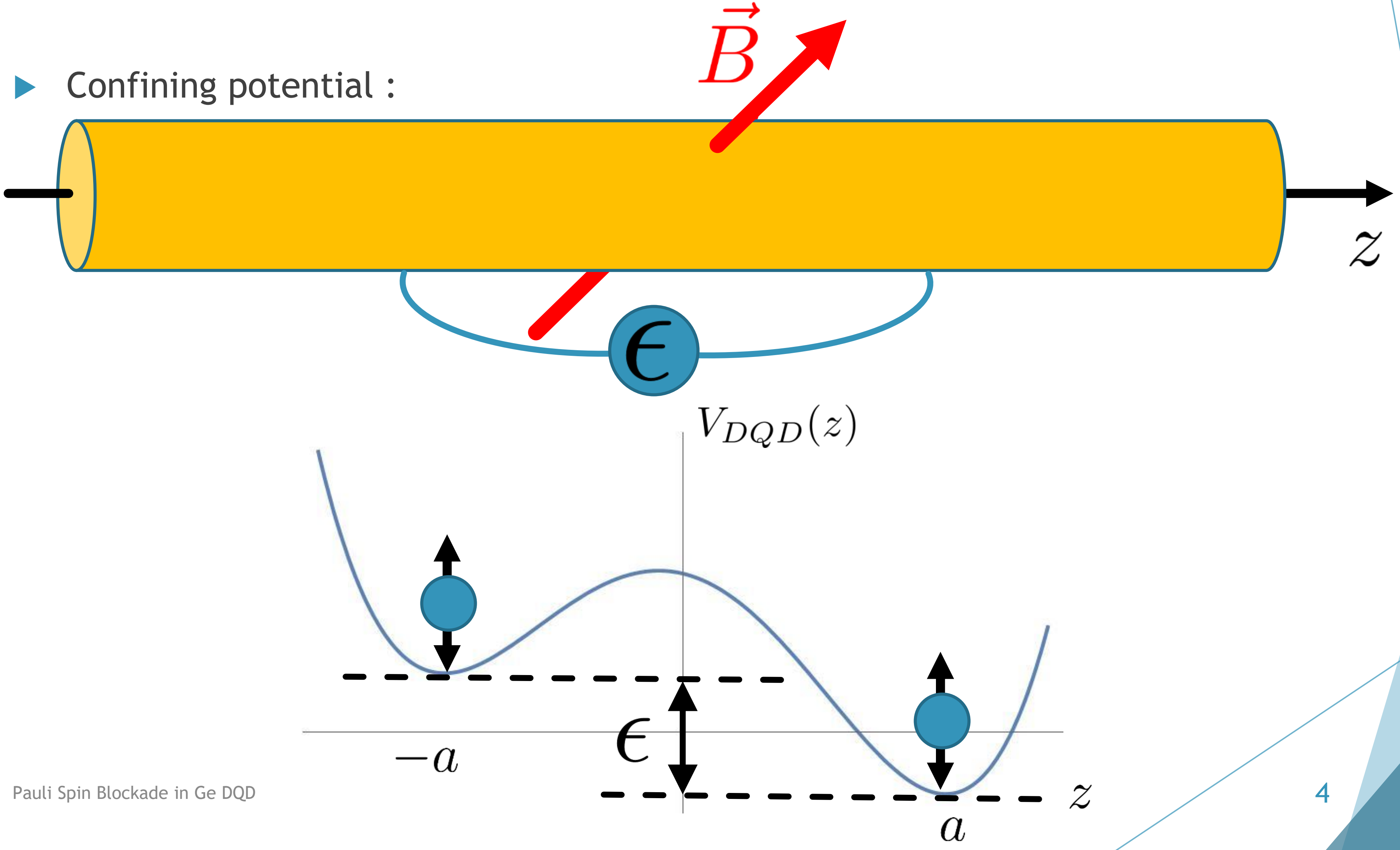

Basis States

State	Configuration		S_{tot}
$\|(2,0)S\rangle$	↑↓		0
$\|(0,2)S\rangle$		↑↓	0
$\|(1,1)S\rangle$	↑↓	↑↓	0
$\|T_0\rangle$			0
$\|T_-\rangle$	↓	↓	-1
$\|T_+\rangle$	↑	↑	$+1$

Model Hamiltonian

$$H = \overbrace{H_{\mathrm{orb}} + H_{\mathrm{Z}} + eV_{bias}}^{H_0} + H_{\mathrm{SO}}$$

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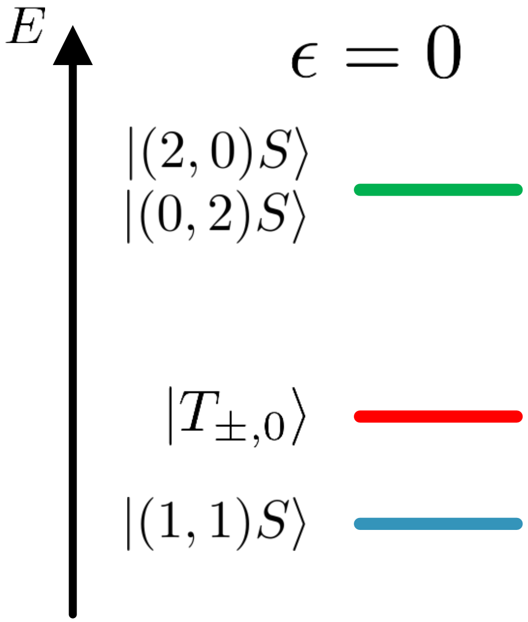
$$H_Z = -g\mu_e \vec{B} \cdot \vec{S}_{tot}$$

Energy Levels

- When $B = 0$, depends on detuning ϵ



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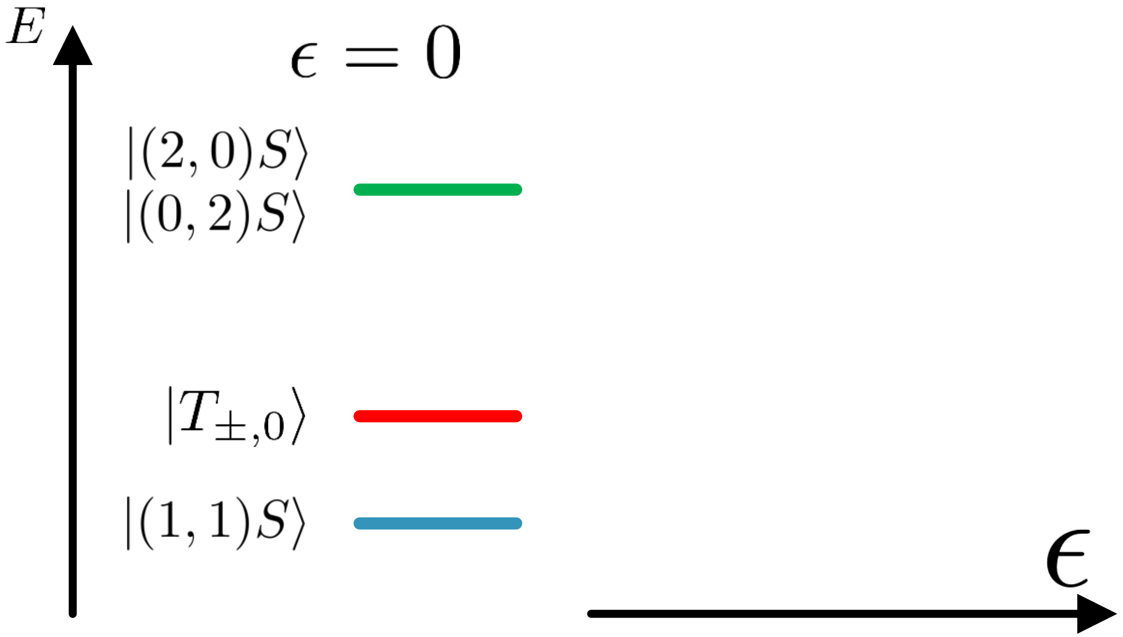

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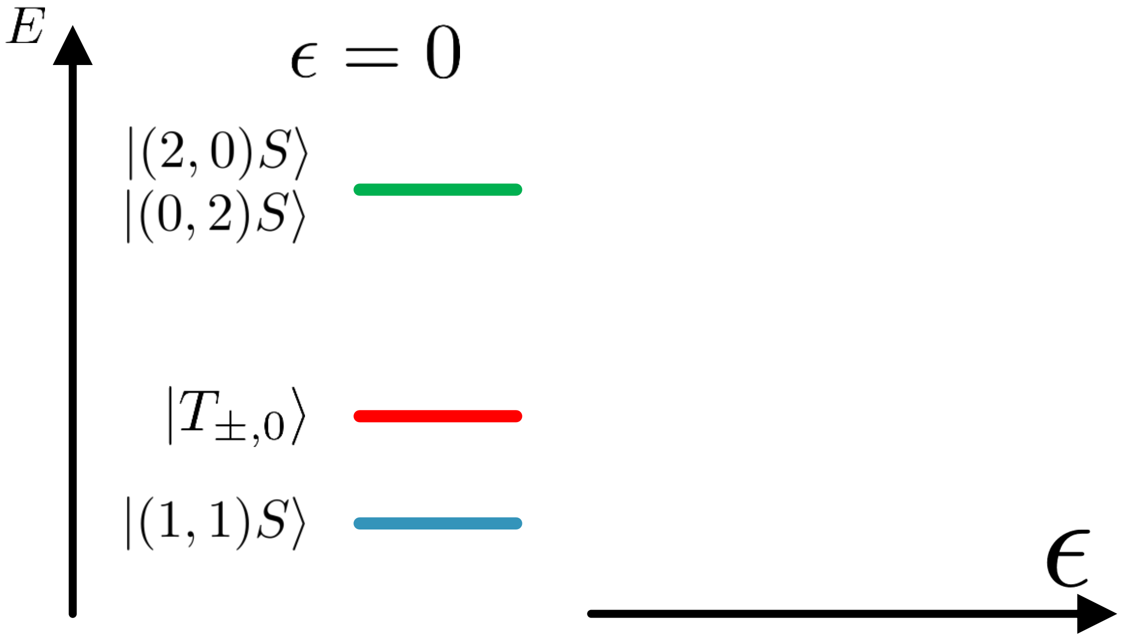

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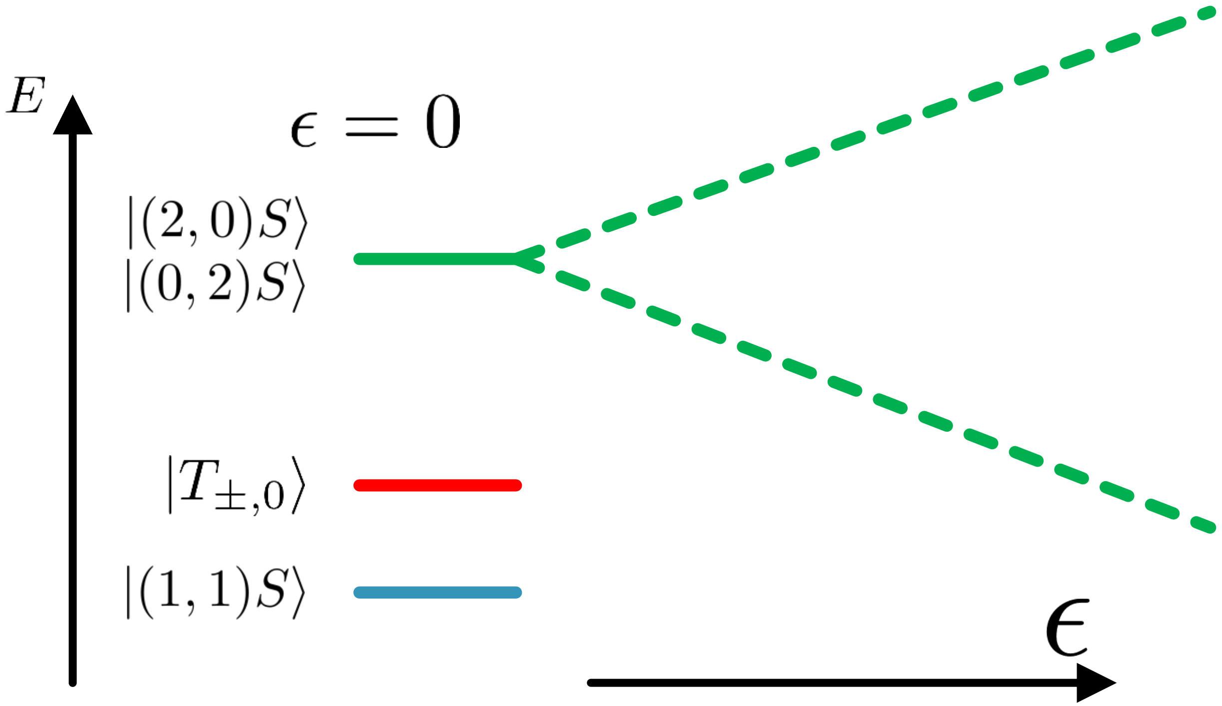

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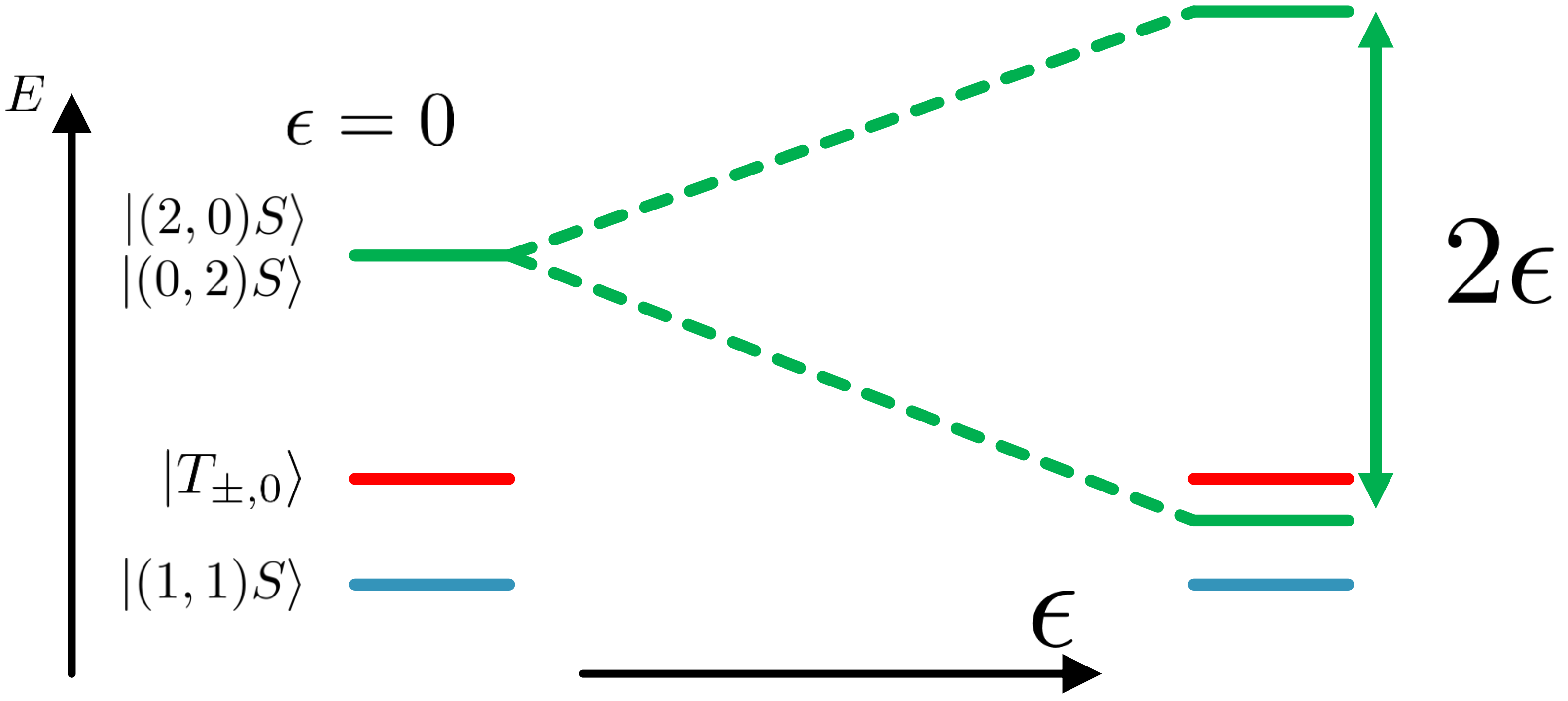

Energy Levels

- $|\epsilon|$ big (comparing to other energies)

$$|S^{+}\rangle = \sin\psi|(1,1)S\rangle - \cos\psi|(0,2)S\rangle$$
$$|S^{-}\rangle = \cos\psi|(1,1)S\rangle + \sin\psi|(0,2)S\rangle$$

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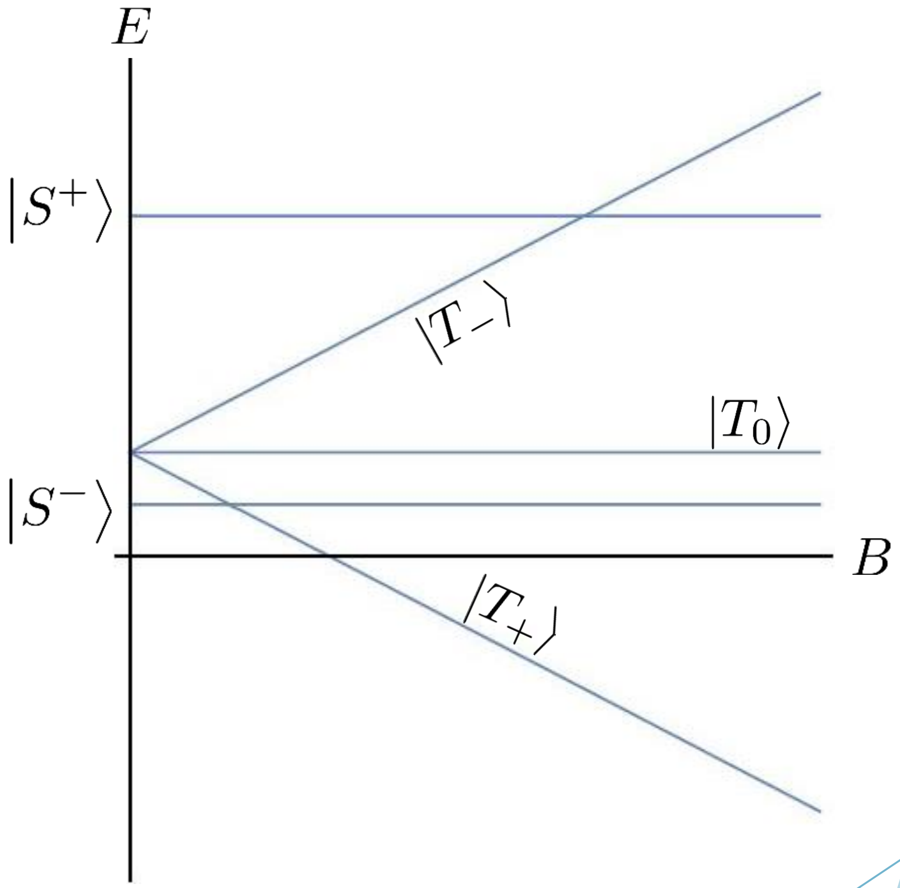

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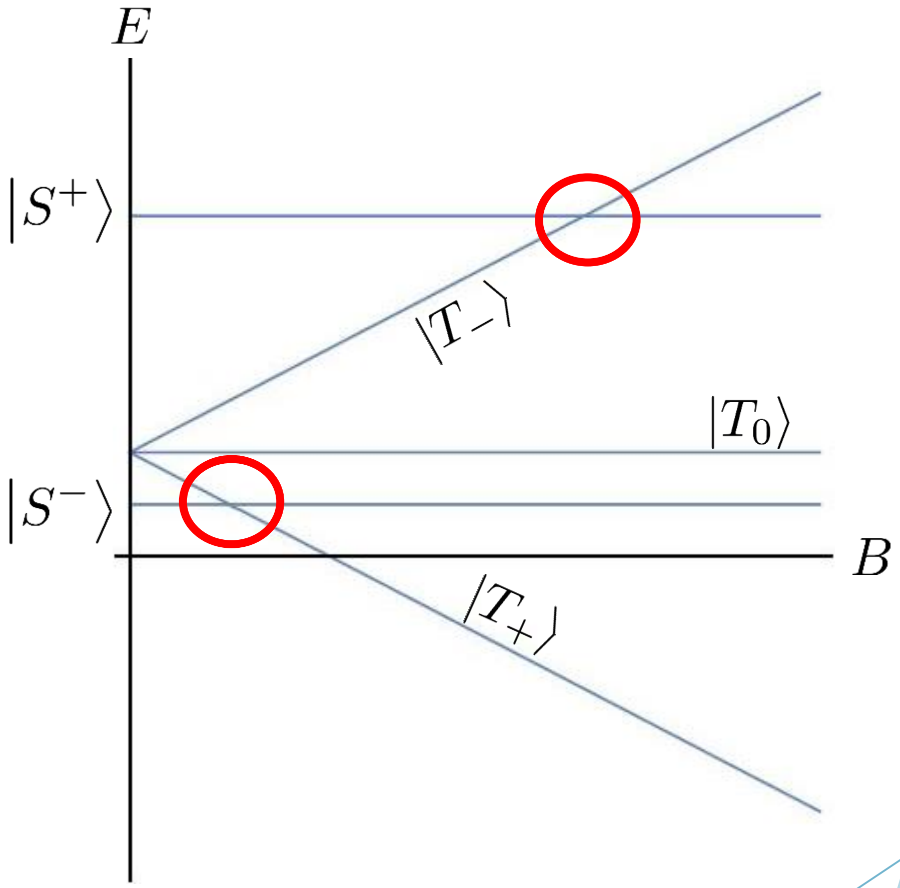
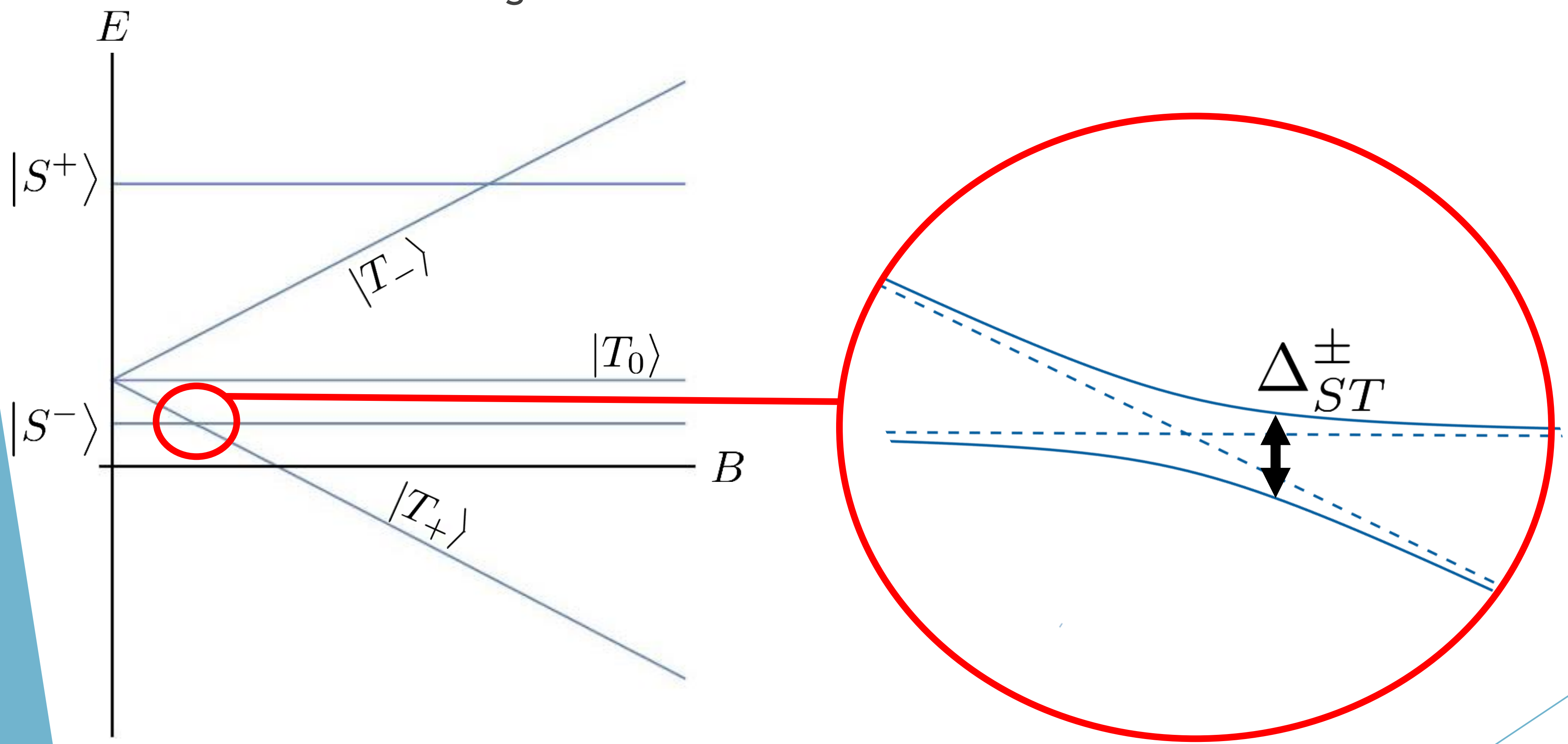


Splitting width minimalization

- At the anticrossing :



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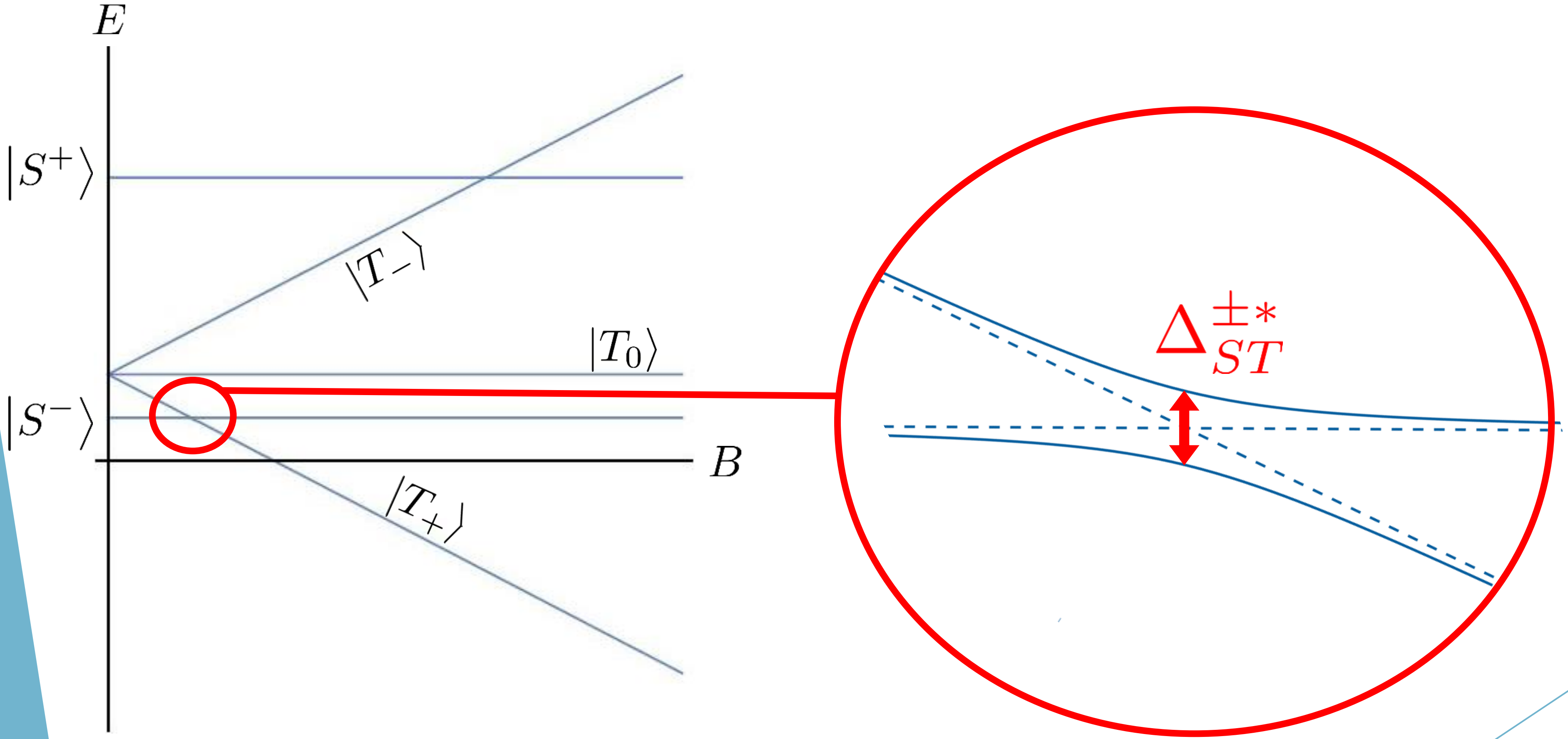
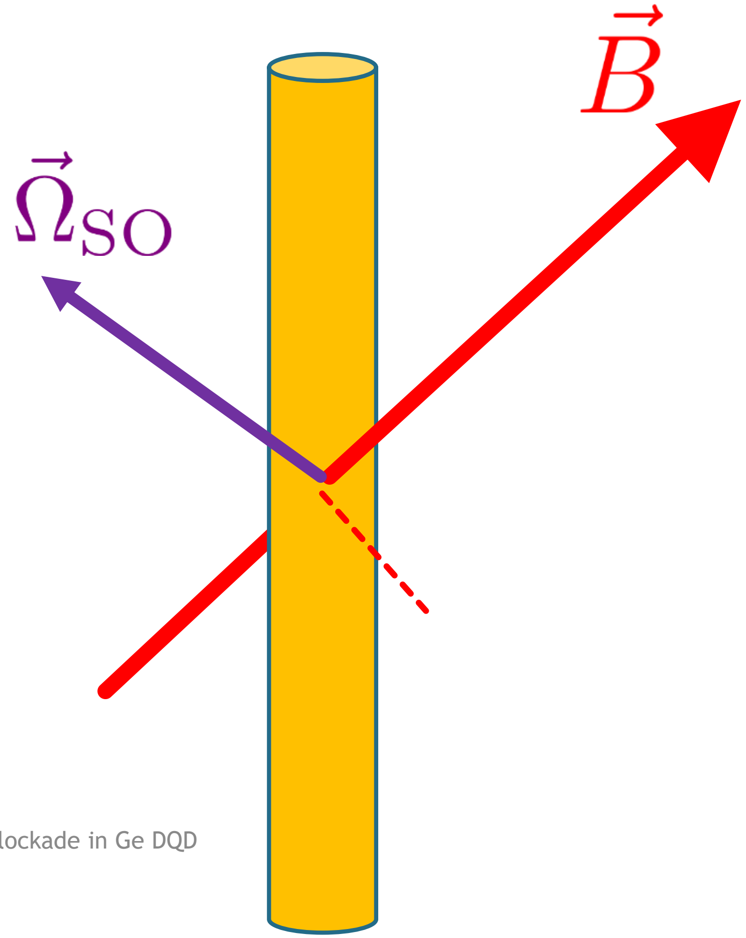


Spin-Orbit Coupling

$$H_{SO} = k_z \vec{\Omega}_{\mathrm{SO}} \cdot \underline{\vec{\sigma}}$$



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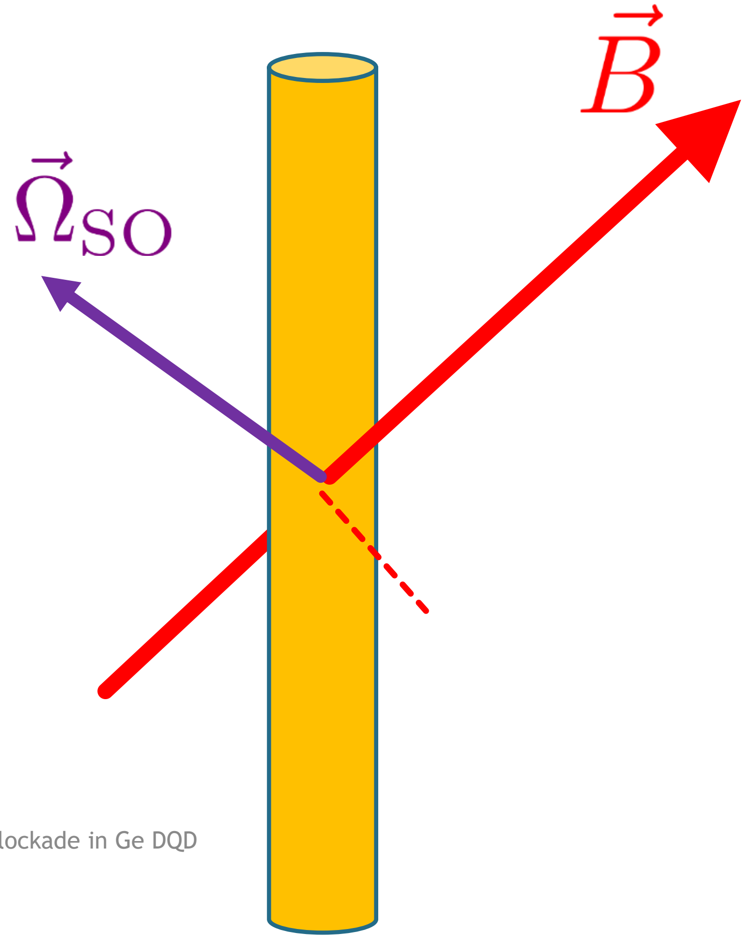

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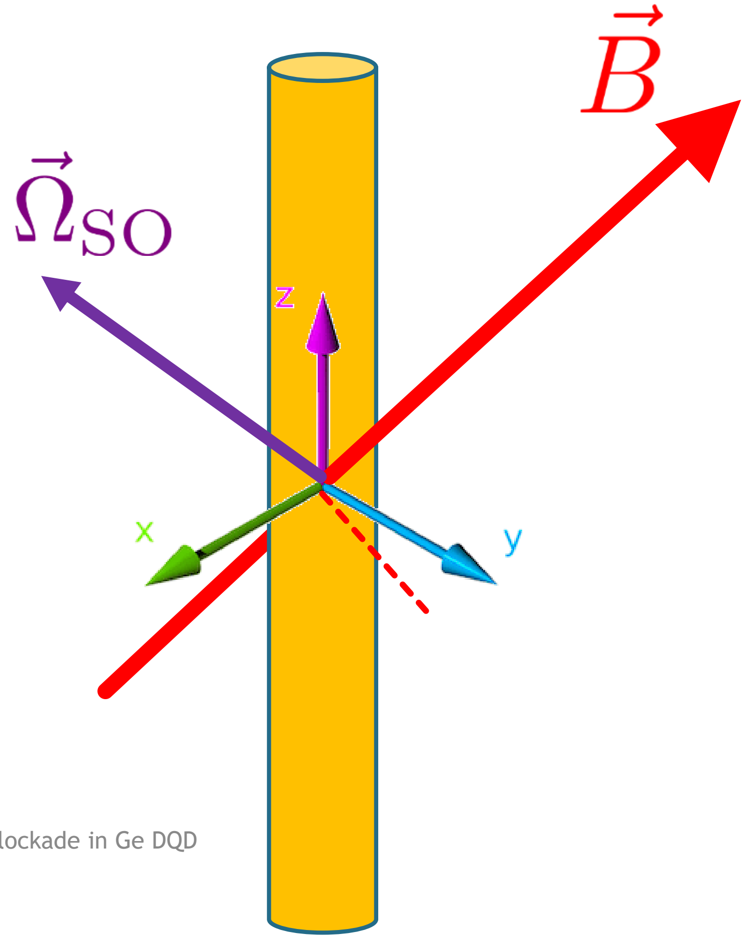

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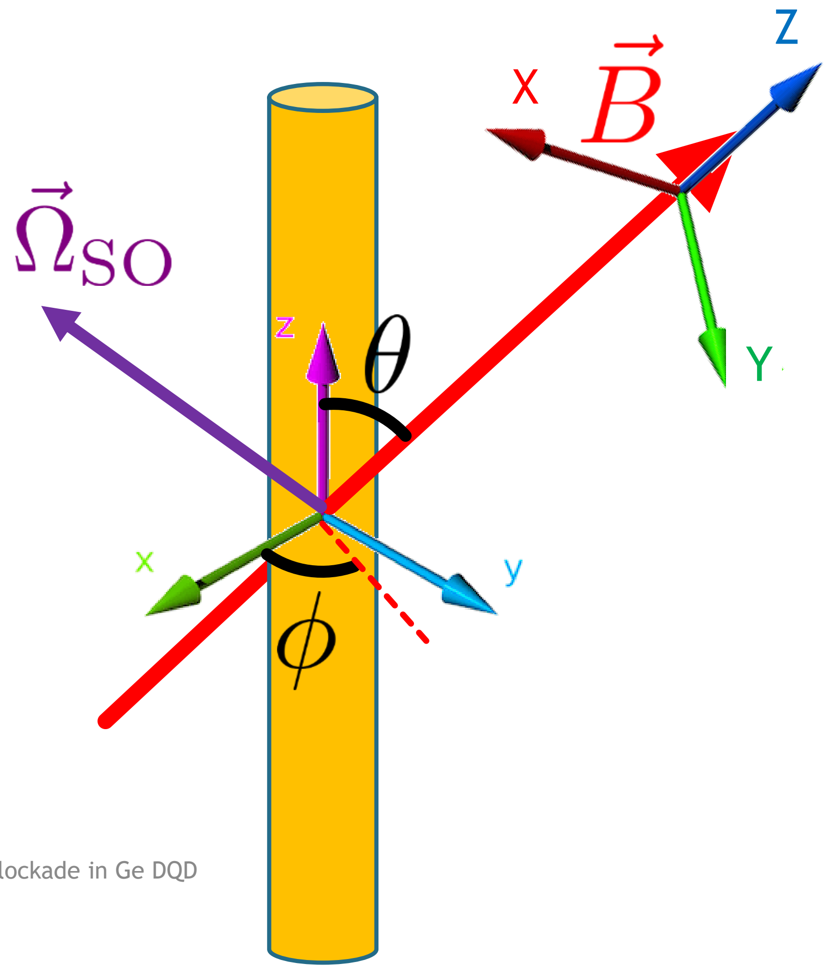

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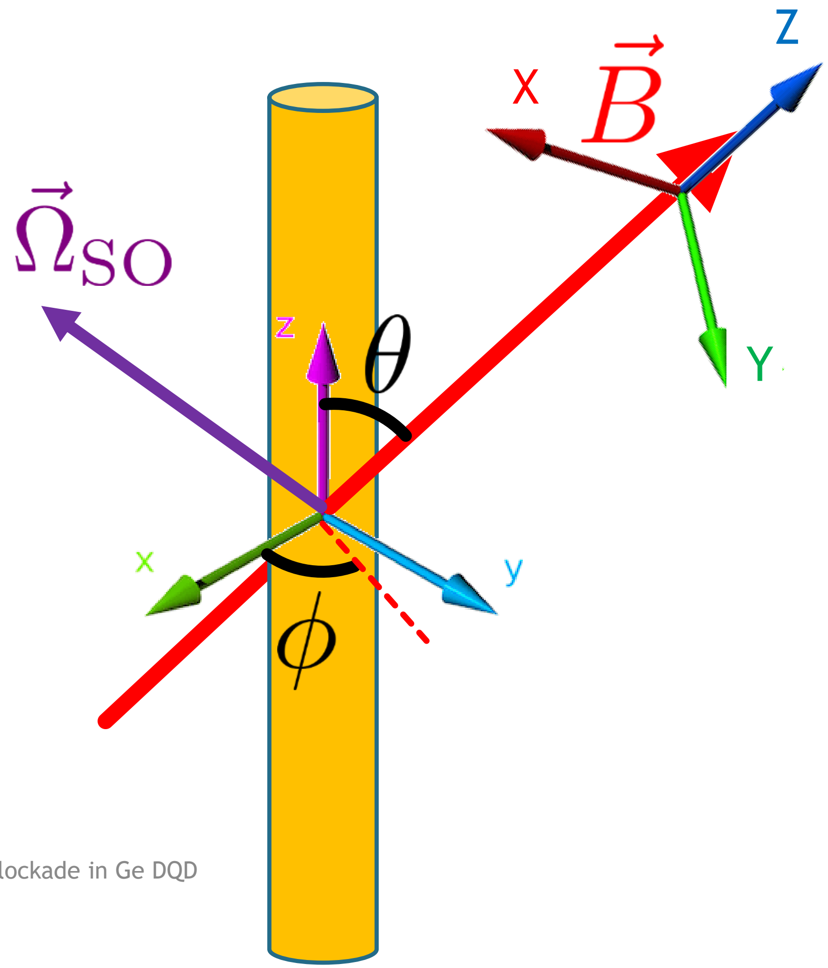

$$\underline{\sigma_X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\underline{\sigma_Y} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

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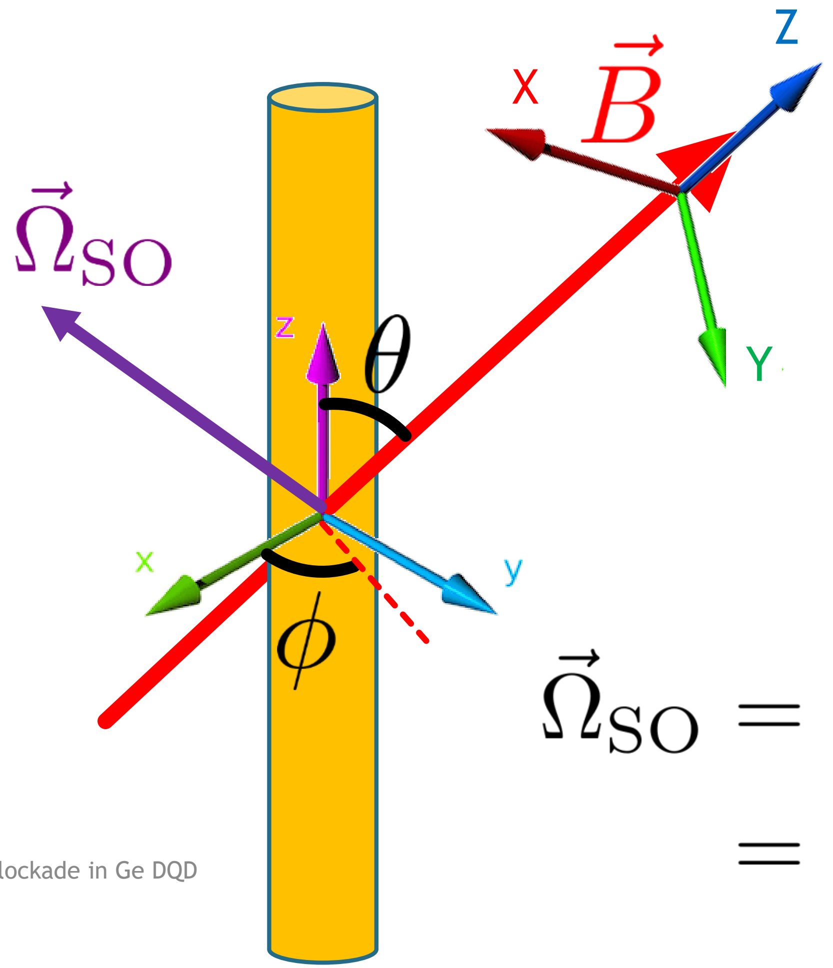

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$$\vec{\Omega}_{\mathrm{SO}} = (\alpha_X, \alpha_Y, \beta_Z)_{\vec{B}} = (\alpha_x, \alpha_y, \beta_z)_{\mathrm{NW}}$$

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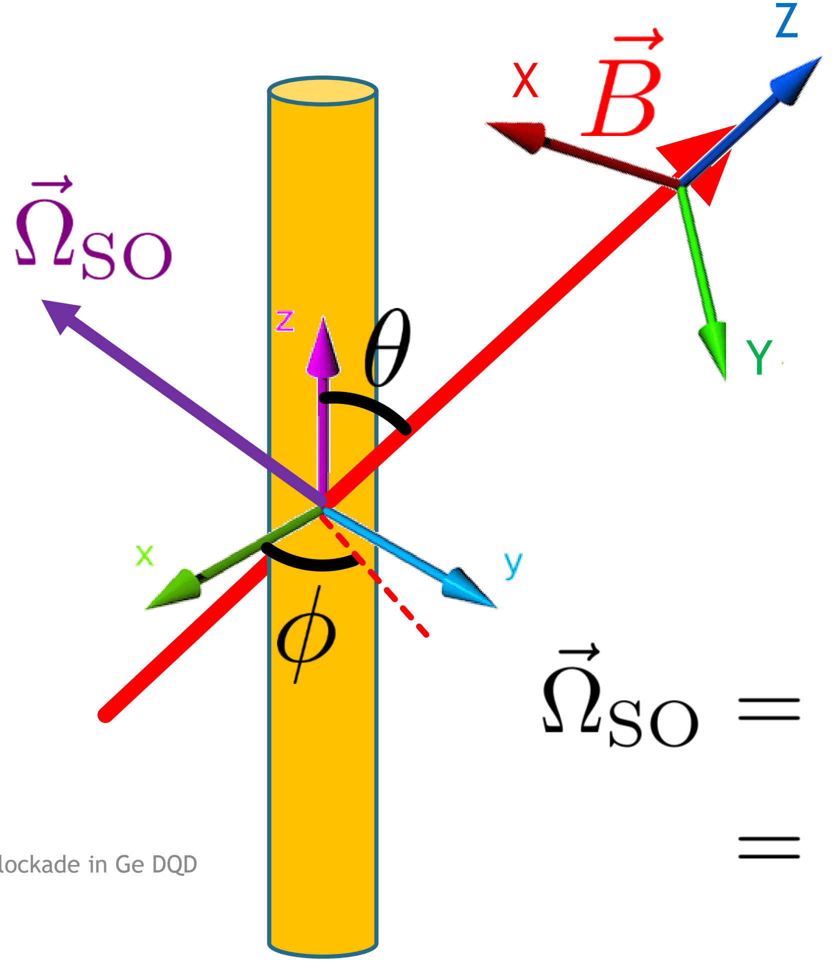
$$H_{SO} = k_z \vec{\Omega}_{SO} \cdot \underline{\vec{\sigma}} = \alpha_X k_z \underline{\sigma_X} + \alpha_Y k_z \underline{\sigma_Y} + \beta_Z k_z \underline{\sigma_Z}$$



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Effective Hamiltonian near the anticrossing

- Basis : $\{|S^{\pm}\rangle, |T_{\mp}\rangle\}$ $\quad$ $H^{\pm}_{\text{SO}} = \langle S^{\pm}| H_{\text{SO}} |T_{\mp}\rangle$

$$H^{\pm}_{\text{cross}} = \begin{pmatrix} E_{S^{\pm}} & H^{\pm}_{\text{SO}} \\ \left(H^{\pm}_{\text{SO}}\right)^{*} & E_{T_{\mp}} \end{pmatrix}$$

Anti-crossing width

- Basis : $\{|S^{\pm}\rangle, |T_{\mp}\rangle\}$

$$H_{\text{SO}}^{\pm} = \langle S^{\pm}| H_{\text{SO}} |T_{\mp}\rangle$$

$$H_{\text{cross}}^{\pm} = \begin{pmatrix} E_0 - \frac{\Delta E}{2} & H_{\text{SO}}^{\pm} \\ \left(H_{\text{SO}}^{\pm}\right)^* & E_0 + \frac{\Delta E}{2} \end{pmatrix}$$

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$$\Delta^{\pm *}_{ST} = 2|H^{\pm}_{\mathrm{SO}}|$$

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$$\langle (0,2)S| H_{\mathrm{SO}} |T_{\pm}\rangle = \langle \phi_L| k_z |\phi_R\rangle (\alpha_X + i\alpha_Y)$$

$$\langle (1,1)S| H_{\mathrm{SO}} |T_{\pm,0}\rangle = 0$$

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$$H_{\mathrm{SO}}^{+} = -\cos\psi \langle \phi_L| k_z |\phi_R\rangle \left[(\cos\theta\cos\phi \mp i\sin\phi)\alpha_x + (\cos\theta\sin\phi \pm i\cos\phi)\alpha_y - \beta_z\sin\theta\right]$$
$$H_{\mathrm{SO}}^{-} = \sin\psi \langle \phi_L| k_z |\phi_R\rangle \left[(\cos\theta\cos\phi \mp i\sin\phi)\alpha_x + (\cos\theta\sin\phi \pm i\cos\phi)\alpha_y - \beta_z\sin\theta\right]$$

Results

- Splitting width : $\Delta_{ST}^{+*}(\vec{B})$

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Results

$$\left(\vec{\Omega}_{\text{SO}}\right)_i = \frac{\hbar^2}{m^*\lambda_i} \quad B = 5\text{T} \quad m^* = 0.28m_e$$

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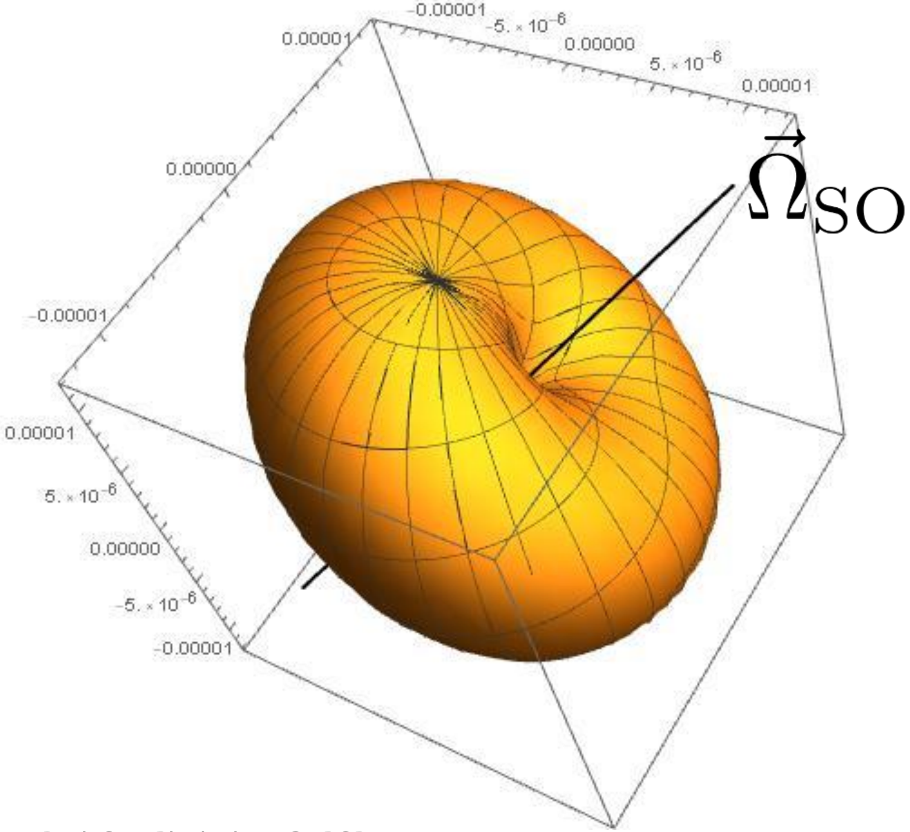
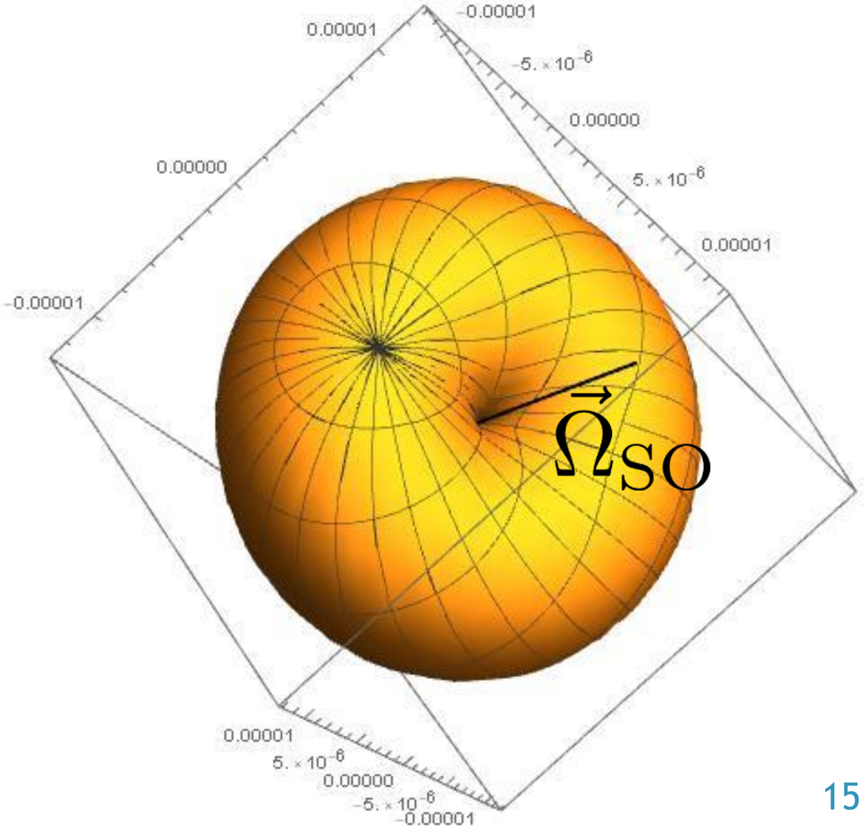
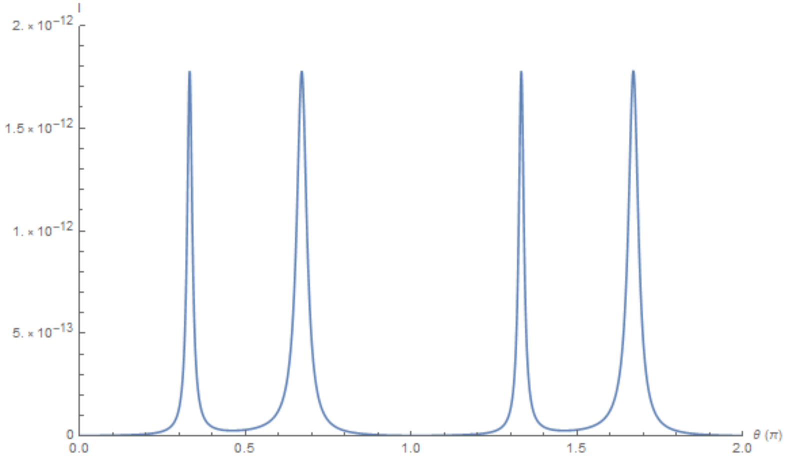




Further Results & Possible uses

- Probability of the S-T spin flip (Rabi oscillations)
- Intensity of the current through NW



- Spin-flip used in a CNOT double qubit gate
- S-O coupling is a source of decoherence

Bibliography

- Loss, Daniel & P. DiVincenzo, David. (1997). **Quantum Computation with Quantum Dots.** Physical Review A. 57. 10.1103/PhysRevA.57.120.
- Burkard, Guido & Loss, Daniel & P DiVincenzo, David. (1998). **Coupled quantum dots as quantum gates.** Physical Review B. 59. 2070-2078. 10.1103/PhysRevB.59.2070.
- Stepanenko, Dimitrije & Rudner, Mark & I. Halperin, Bertrand & Loss, Daniel. (2011). **Singlet-triplet splitting in double quantum dots due to spin orbit and hyperfine interactions.** Physical Review B. 85. 10.1103/PhysRevB.85.075416.
- Rančić, Marko. (2016). **Electrical control and coherence of spin qubits in indium gallium arsenide and silicon quantum dots.**
- Kloeffel, Christoph & Loss, Daniel. (2012). **Prospects for Spin-Based Quantum Computing in Quantum Dots**. Annual Review of Condensed Matter Physics. 4. 10.1146/annurev-conmatphys-030212-184248.
- Wiel, Wilfred & De Franceschi, S & M. Elzerman, J & Fujisawa, Toshimasa & Tarucha, Seigo & P. Kouwenhoven, L. (2002). **Electron transport through double quantum dots.** Reviews of Modern Physics. 75. 10.1103/RevModPhys.75.1.
- H Kobe, D & C -T Wen, E. (1999). **Gauge invariance in quantum mechanics: Charged harmonic oscillator in an electromagnetic field.** Journal of Physics A: Mathematical and General. 15. 787. 10.1088/0305-4470/15/3/018.

Bibliography

- Liu, Zhi-Hai & Entin-Wohlman, O & Aharony, A & Q. You, J. (2018). **Control of the Two-Electron Exchange Interaction in a Nanowire Double Quantum Dot**.
- Fabian, Jaroslav & Matos-Abiague, Alex & Ertler, Christian & Stano, Peter & Zutic, Igor. (2007). **Semiconductor Spintronics.** Acta Physica Slovaca. 57. 10.2478/v10155-010-0086-8.
- Golovach, Vitaly & Khaetskii, Alexander & Loss, Daniel. (2007). **Spin relaxation at the singlet-triplet crossing in a quantum dot.** Physical Review B. 77. 10.1103/PhysRevB.77.045328.
- Chan, Guo Xuan & Wang, Xin. (2018). **On the validity of microscopic calculations of double-quantum-dot spin qubits based on Fock-Darwin states.** Science China Physics, Mechanics & Astronomy. 61. 10.1007/s11433-017-9145-6.

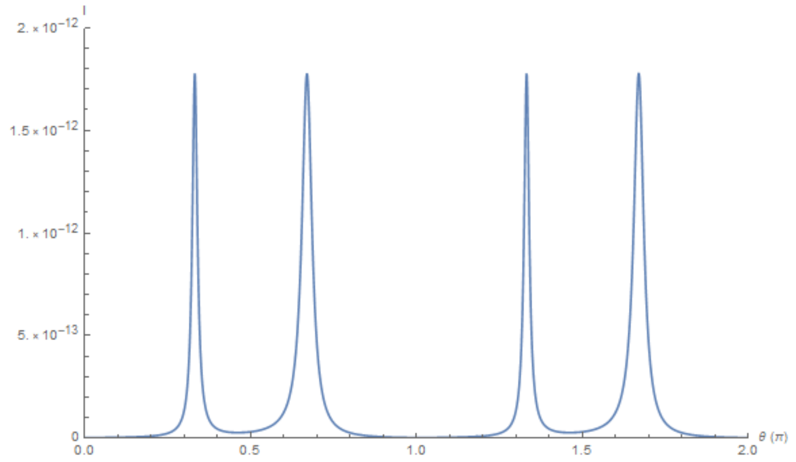
- **Many-body theory**, J.Polonyi
- Michael A. Nielsen and Isaac L. Chuang. 2011. **Quantum Computation and Quantum Information: 10th Anniversary Edition (10th ed.).** Cambridge University Press, New York, NY, USA.
- Marc Fox. 2011. **Quantum Optics : an introduction.** OUP Oxford, UK.
- Claude Cohen-Tannoudji, Bernard Diu et Franck Laloë, 2017. **Mécanique Quantique vol.3.** EDP-Sciences.

Thank you !

Questions ?

Further Results & Possible uses

- Probability of the S-T spin flip (Rabi oscillations)
- Intensity of the current through NW



- Spin-flip used in a CNOT double qubit gate
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Appendix

Quantum Computing

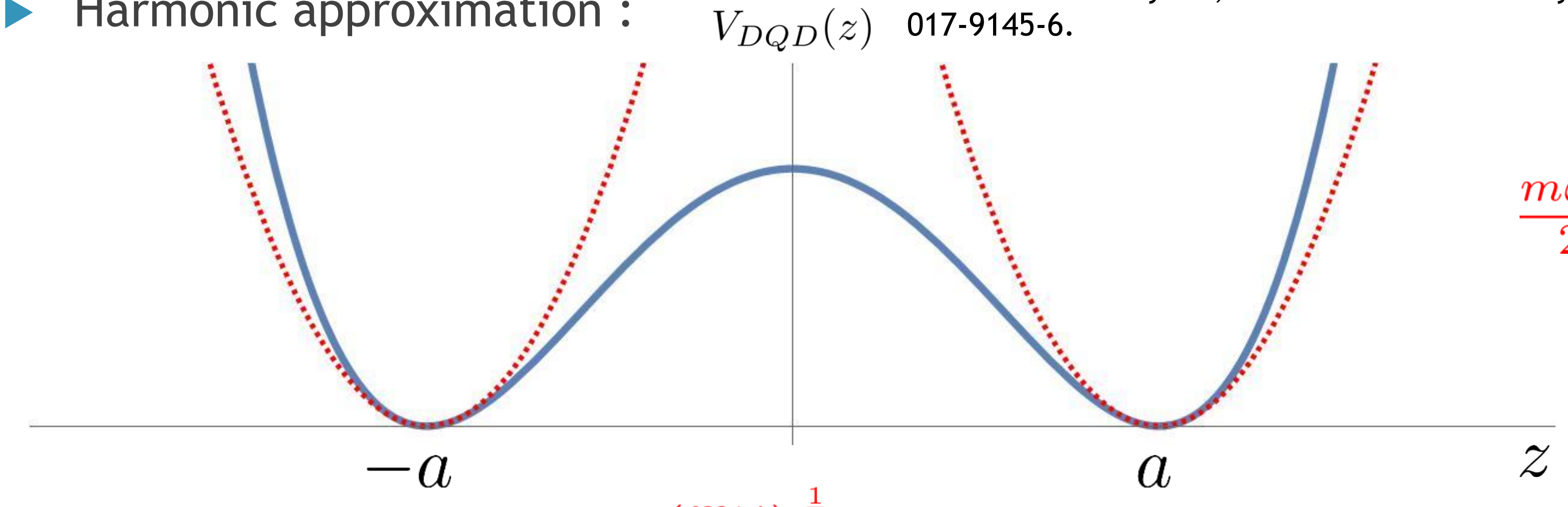
- Digital bits : 0 or 1
- Quantum bits (Qubits) : Superposition of $|0\rangle$ and $|1\rangle$

Number of bits	Classical bits	Qubits
1	0 or 1	$c_0 \|0\rangle + c_1 \|1\rangle$
2	00, 01, 10 or 11	$c_{00} \|00\rangle + c_{01} \|01\rangle + c_{10} \|10\rangle + c_{11} \|11\rangle$
N	1 of 2^N N-uplets	All of the 2^N c_n's

Molecular Orbit Approximation

Chan & Al.(2018). **On the validity of microscopic calculations of double-quantum-dot spin qubits based on Fock-Darwin states.** Science China Physics, Mechanics & Astronomy. 61. 10.1007/s11433-017-9145-6.

- Harmonic approximation :



$$\frac{m\omega_0^2}{2}(z \pm a)^2$$

$$\varphi(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-m\omega x^2/2\hbar}$$

$$\varphi_L(z) = \varphi(z+a) \qquad \varphi_R(x) = \varphi(z-a)$$

- Hund-Mulliken approach :

$$\Phi_{\mathrm{L,R}} = \frac{1}{\sqrt{1-2Sg+g^2}}\left(\varphi_{L,R} - g\varphi_{R,L}\right)$$

$$S = \int dz \varphi_R^*(z)\varphi_L(z) \qquad g = \left(1-\sqrt{1-S^2}\right)/S$$

Total Wave Functions

<table>
<tr><th>State</th><th colspan="2">Configuration</th><th>Orbital Part</th><th>Spin Part</th><th>S_{tot}</th></tr>
<tr><td>$|(2,0)S\rangle$</td><td>↑↓</td><td></td><td>$\Psi^d_{\mathrm{L}}(z_1, z_2) = \Phi_{\mathrm{L}}(z_1)\Phi_{\mathrm{L}}(z_2)$</td><td rowspan="3">$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$</td><td rowspan="4">0</td></tr>
<tr><td>$|(0,2)S\rangle$</td><td></td><td>↑↓</td><td>$\Psi^d_{\mathrm{R}}(z_1, z_2) = \Phi_{\mathrm{R}}(z_1)\Phi_{\mathrm{R}}(z_2)$</td></tr>
<tr><td>$|(1,1)S\rangle$</td><td rowspan="2">↕</td><td rowspan="2">↕</td><td>$\Psi^s_{+}(z_1, z_2) = \frac{1}{\sqrt{2}}(\Phi_{\mathrm{L}}(z_1)\Phi_{\mathrm{R}}(z_2) + \Phi_{\mathrm{L}}(z_2)\Phi_{\mathrm{R}}(z_1))$</td></tr>
<tr><td>$|T_0\rangle$</td><td rowspan="3">$\Psi^s_{-}(z_1, z_2) = \frac{1}{\sqrt{2}}(\Phi_{\mathrm{L}}(z_1)\Phi_{\mathrm{R}}(z_2) - \Phi_{\mathrm{L}}(z_2)\Phi_{\mathrm{R}}(z_1))$</td><td>$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$</td></tr>
<tr><td>$|T_-\rangle$</td><td>↓</td><td>↓</td><td>$|\downarrow\downarrow\rangle$</td><td>$-1$</td></tr>
<tr><td>$|T_+\rangle$</td><td>↑</td><td>↑</td><td>$|\uparrow\uparrow\rangle$</td><td>$+1$</td></tr>
</table>

Hamiltonian Matrix

- Basis : $\{|(2,0)S\rangle, |(0,2)S\rangle, |(1,1)S\rangle, |T_+\rangle, |T_0\rangle, |T_-\rangle\}$

$$H_0 = \begin{pmatrix} H_{\mathrm{SS}} & 0 \\ 0 & H_{\mathrm{TT}} \end{pmatrix}$$

$$H = H_{\mathrm{orb}} + H_{\mathrm{Z}}$$

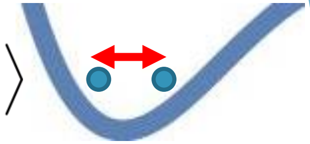
Hamiltonian Matrix

$$H_{\text{orb}} = \sum_{i=1,2} h_i + C$$

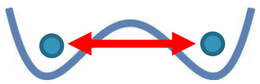
- Basis : $\{|(2,0)S\rangle\,, |(0,2)S\rangle\,, |(1,1)S\rangle\,, |T_+\rangle\,, |T_0\rangle\,, |T_-\rangle\}$

$$H_{\text{SS}} = \begin{pmatrix} U+\epsilon & X & -\sqrt{2}t \\ X & U-\epsilon & -\sqrt{2}t \\ -\sqrt{2}t & -\sqrt{2}t & V_+ \end{pmatrix}$$

$$U = \langle \Psi^d_{\text{L,R}} | C | \Psi^d_{\text{L,R}} \rangle$$

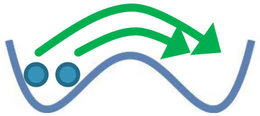


$$V_\pm = \langle \Psi^s_\pm | C | \Psi^s_\pm \rangle$$

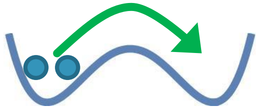


$$H_{\text{TT}} = \begin{pmatrix} V_- + g\mu_e B & 0 & 0 \\ 0 & V_- & 0 \\ 0 & 0 & V_- - g\mu_e B \end{pmatrix}$$

$$X = \langle \Psi^d_{\text{L,R}} | \, C \, | \Psi^d_{\text{R,L}} \rangle$$



$$t = \langle \Phi_{\text{L,R}} \, | h^0_{1,2} | \, \Phi_{\text{R,L}} \rangle - \frac{1}{\sqrt{2}} \langle \Psi^{\text{s}}_+ | C | \Psi^{\text{d}}_{\text{L,R}} \rangle$$



Mixing angle

$$U + \varepsilon \gg U - \varepsilon, V_+ \gg t \gg X$$

$$H_{\mathrm{SS}} = \begin{pmatrix} U+\epsilon & X & -\sqrt{2}t \\ X & U-\epsilon & -\sqrt{2}t \\ -\sqrt{2}t & -\sqrt{2}t & V_+ \end{pmatrix}$$

$$H_{\mathrm{SS}} = \begin{pmatrix} \epsilon & 0 & 0 \\ 0 & U & -\sqrt{2}t \\ 0 & -\sqrt{2}t & V_+ \end{pmatrix}$$

$$\cos 2\psi = \frac{U - V_+}{\sqrt{(U - V_+)^2 + 8t^2}}$$

$$\sin 2\psi = \frac{2\sqrt{2}t}{\sqrt{(U - V_+)^2 + 8t^2}}$$

$$H_{\mathrm{SS}} = \begin{pmatrix} \epsilon & 0 & 0 \\ 0 & E^+ & 0 \\ 0 & 0 & E^- \end{pmatrix}$$

$$|S^+\rangle = \sin\psi|(1,1)S\rangle - \cos\psi|(0,2)S\rangle$$
$$|S^-\rangle = \cos\psi|(1,1)S\rangle + \sin\psi|(0,2)S\rangle$$

Rabi Oscillation

- Basis : $\{|S^{\pm}\rangle, |T_{\mp}\rangle\}$

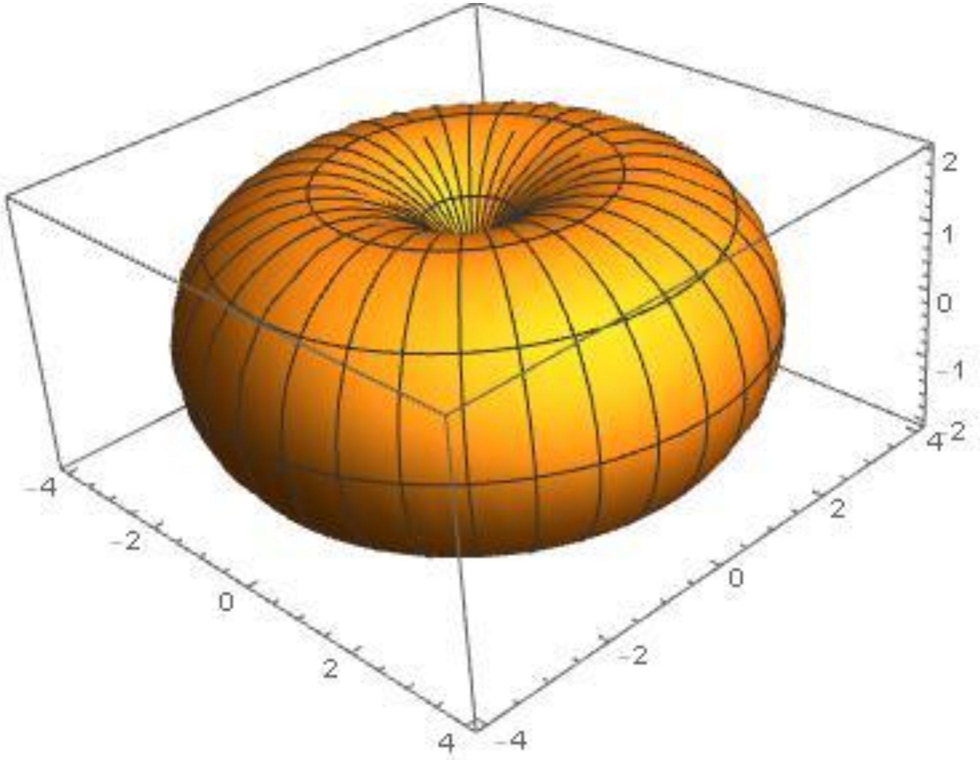
$$H^{\pm}_{\mathrm{SO}} = \langle S^{\pm}| H_{\mathrm{SO}} |T_{\mp}\rangle$$

$$H^{\pm}_{\mathrm{cross}} = \begin{pmatrix} E_0 - \frac{\Delta E}{2} & H^{\pm}_{\mathrm{SO}} \\ \left(H^{\pm}_{\mathrm{SO}}\right)^* & E_0 + \frac{\Delta E}{2} \end{pmatrix}$$

$$\Delta E = \pm g(\vec{B})\mu_e B + V_T - V_S$$

g-factor

- Depends on orientation of $\vec{B}$:



$$g = \vec{\mathcal{G}}\frac{\vec{B}}{B}$$

$$\mathcal{G} = (4, 4, 0.4)$$

Rabi Oscillation

- Basis : $\{|S^{\pm}\rangle, |T_{\mp}\rangle\}$

$$H_{\rm SO}^{\pm} = \langle S^{\pm}| H_{\rm SO} |T_{\mp}\rangle$$

$$H_{\rm cross}^{\pm} = \begin{pmatrix} E_0 - \frac{\Delta E}{2} & H_{\rm SO}^{\pm} \\ (H_{\rm SO}^{\pm})^* & E_0 + \frac{\Delta E}{2} \end{pmatrix}$$

$$\Delta E = \pm g(\vec{B})\mu_e B + V_T - V_S$$

$$\Delta_{ST}^{\pm *} = 2|H_{\rm SO}^{\pm}|$$

Rabi Oscillation

- Basis : $\{|S^{\pm}\rangle, |T_{\mp}\rangle\}$

$$H_{\mathrm{SO}}^{\pm} = \langle S^{\pm}| H_{\mathrm{SO}} |T_{\mp}\rangle$$

$$H_{\mathrm{cross}}^{\pm} = \begin{pmatrix} E_0 - \frac{\Delta E}{2} & H_{\mathrm{SO}}^{\pm} \\ \left(H_{\mathrm{SO}}^{\pm}\right)^* & E_0 + \frac{\Delta E}{2} \end{pmatrix}$$

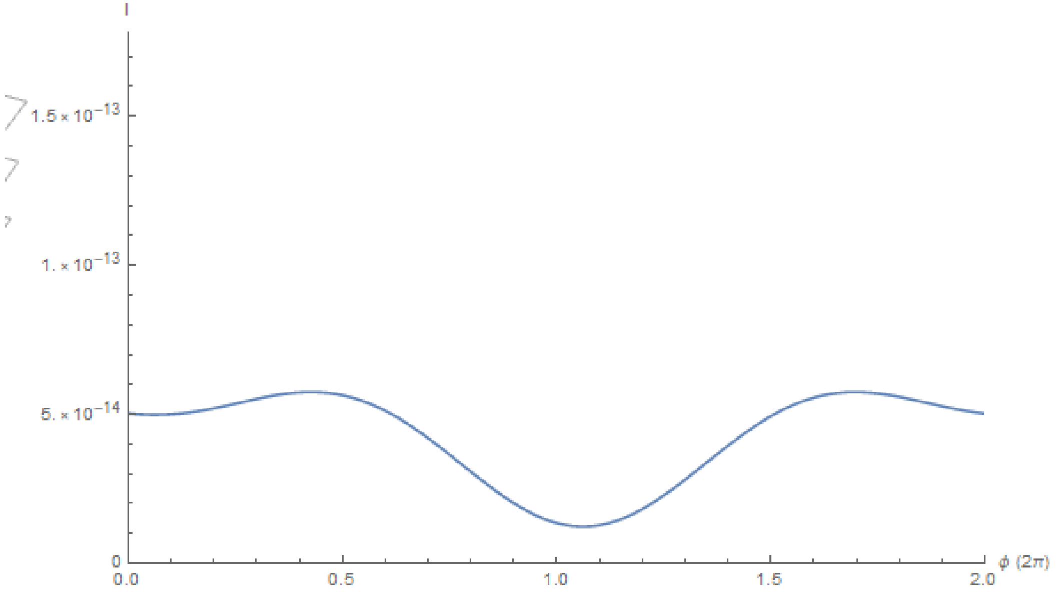
$$\Delta E = \pm g(\vec{B})\mu_e B + V_T - V_S \qquad \Delta_{ST}^{\pm *} = 2|H_{\mathrm{SO}}^{\pm}|$$

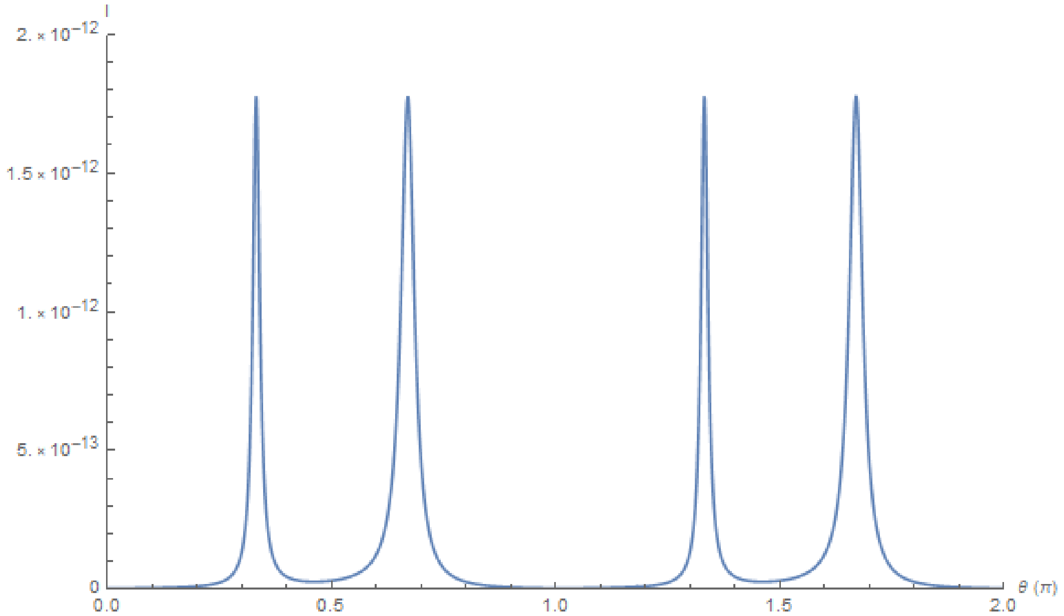
$$A_{\mathrm{Rabi}}^{\pm} = |P_{S^{\pm} \to T_{\mp}}|_{max} = \frac{\left(\Delta_{ST}^{\pm *}\right)^2}{\left(\Delta_{ST}^{\pm *}\right)^2 + \Delta E^2}$$

Results

- Intensity:

$$\lambda_x = 5 \times 10^{-18} m, \lambda_y = 2.5 \times 10^{-17} m, \lambda_z = 1.3 \times 10^{-17} m$$





$$\frac{\Gamma e \Delta_{ST}(\alpha \mathrm{x}, \alpha \mathrm{y}, \beta, \theta, \phi)^2}{3\Delta_{ST}(\alpha \mathrm{x}, \alpha \mathrm{y}, \beta, \theta, \phi)^2 + \Delta \mathrm{E}(\mathrm{gx}, \mathrm{gy}, \mathrm{gz}, \theta, \phi)^2 + \left(\frac{\Gamma \mathrm{hb}}{2}\right)^2}$$

Wilfred & Al (2002). **Electron transport through double quantum dots.** Reviews of Modern Physics. 75. 10.1103/RevModPhys.75.1.

Two-level system

- Basis : $\{|S^{\pm}\rangle, |T_{\mp}\rangle\}$

$$H^{\pm}_{\mathrm{SO}} = \langle S^{\pm}| H_{\mathrm{SO}} |T_{\mp}\rangle$$

$$H^{\pm}_{\mathrm{cross}} = \begin{pmatrix} E_0 - W & H^{\pm}_{\mathrm{SO}} \\ \left(H^{\pm}_{\mathrm{SO}}\right)^* & E_0 + W \end{pmatrix}$$

$$W = \frac{E_{T_{\mp}} - E_{S^{\pm}}}{2} = \frac{1}{2}(\pm g\mu_e B + V_T - V_S) = \frac{\Delta E}{2}$$

$$\Delta^{\pm}_{ST}(\vec{B}) = 2\sqrt{|H^{\pm}_{\mathrm{SO}}(\vec{B})|^2 + \Delta E(\vec{B})^2}$$

$$A^{\pm}_{\mathrm{Rabi}} = \left|P_{S^{\pm}\to T_{\mp}}\right| = \frac{\left(\Delta^{\pm *}_{ST}\right)^2}{\left(\Delta^{\pm}_{ST}\right)^2}$$