Unaccounted QED corrections in $R(D^{(*)})$?

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collaboration with **Stefan de Boer, Ivan Nisandzic**

GDR-InF workshop:
QED corrections to (semi)leptonic *B* decays,
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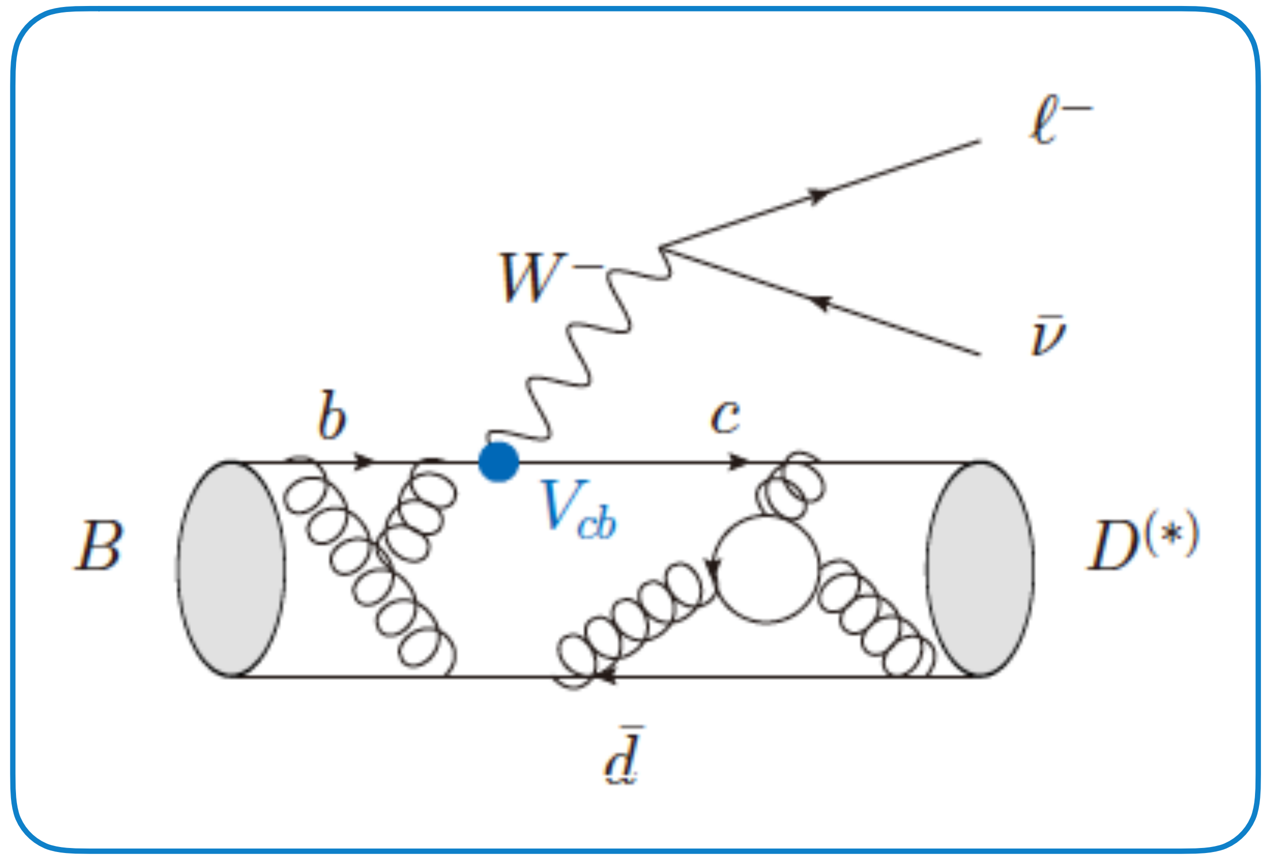
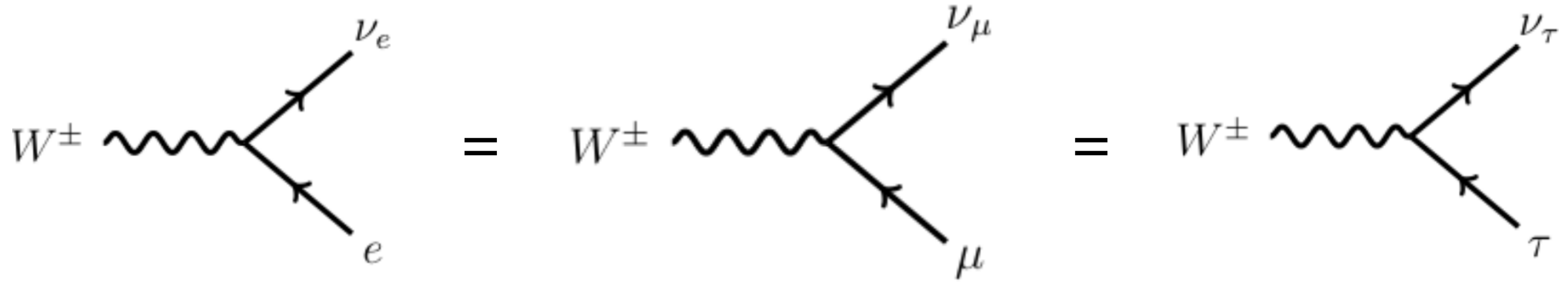
Semileptonic B decay



- Semileptonic B-meson decays induced by $b \to c l \nu$ play an important role for **testing the Standard Model at low energy**: $|V_{cb}|$ and **Lepton Flavor Universality (LFU)**



- Lepton flavor universality is violated by **only tau lepton mass** which leads to **small phase space and scalar form factors $f_0(q^2)$ and $A_0(q^2)$**

- **Light lepton universalities** in kaon, pion and τ decays **have been checked precisely**

$K^+ \to \pi^0 \ell^+ \nu(\gamma)$ $\quad r_{\mu e}(K^+) = 0.998(9)$

$K_L \to \pi^- \ell^+ \nu(\gamma)$ $\quad r_{\mu e}(K_L) = 1.003(5)$

$\pi^+ \to \ell^+ \nu(\gamma)$ $\quad r_{\mu e}(\pi^+) = 1.0042(33)$

$\tau^+ \to \ell^+ \nu \bar{\nu}(\gamma)$ $\quad r_{\mu e}(\tau^+) = 1.000(4)$

$$r_{\mu e}^{\rm SM} = (g_{W\mu\bar{\nu}}/g_{We\bar{\nu}})^2 = 1$$

[Rainer Wanke, KAON 2007; Cristina Lazzeroni, IoP Nuclear and Particle Divisional Conference, 2011]

$R(D)$ and $R(D^*)$

- Heavy lepton flavour universality can be measured by $R(D)$ and $R(D^*)$

$$R(D^{(*)+}) = \frac{\mathcal{B}(\bar{B}^0 \to D^{(*)+}\tau^-\bar{\nu})}{\mathcal{B}(\bar{B}^0 \to D^{(*)+}\ell^-\bar{\nu})}$$

$$R(D^{(*)0}) = \frac{\mathcal{B}(B^- \to D^{(*)0}\tau^-\bar{\nu})}{\mathcal{B}(B^- \to D^{(*)0}\ell^-\bar{\nu})}$$

BaBar: $\ell = e + \mu$

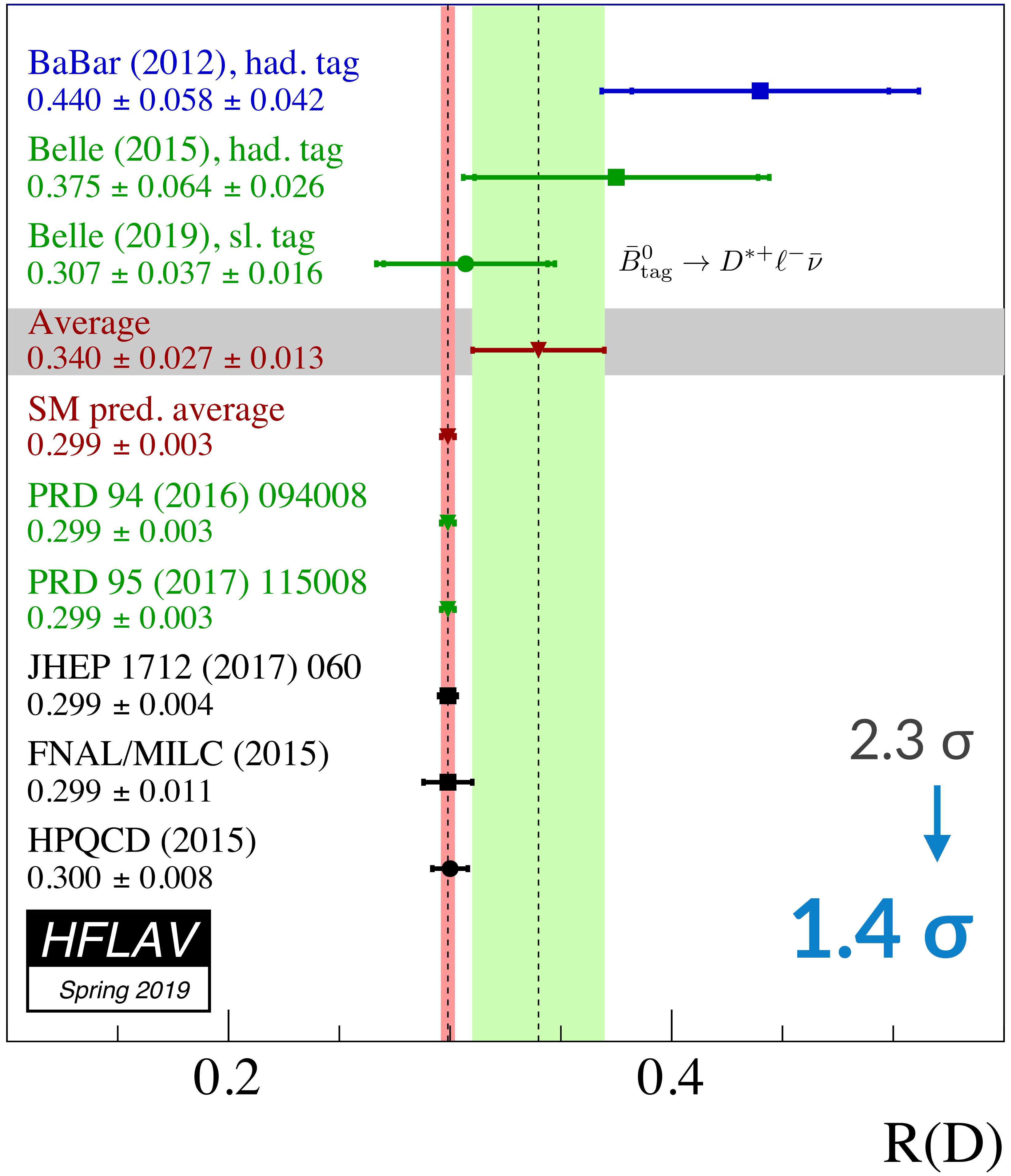
Belle: $\ell = e + \mu$

LHCb: $\ell = \mu$

- **Theoretically clean:** dominant hadronic uncertainty is largely canceled in the ratios
- CKM dependence ($|V_{cb}|$) is also totally canceled
- We separately define ***R*(*D*$^+$)** and ***R*(*D*^{0})** to distinguish different QED corrections in neutral and charged *B* decays

Status of $R(D)$ and $R(D^*)$

[HFLAV averages Spring 2019]



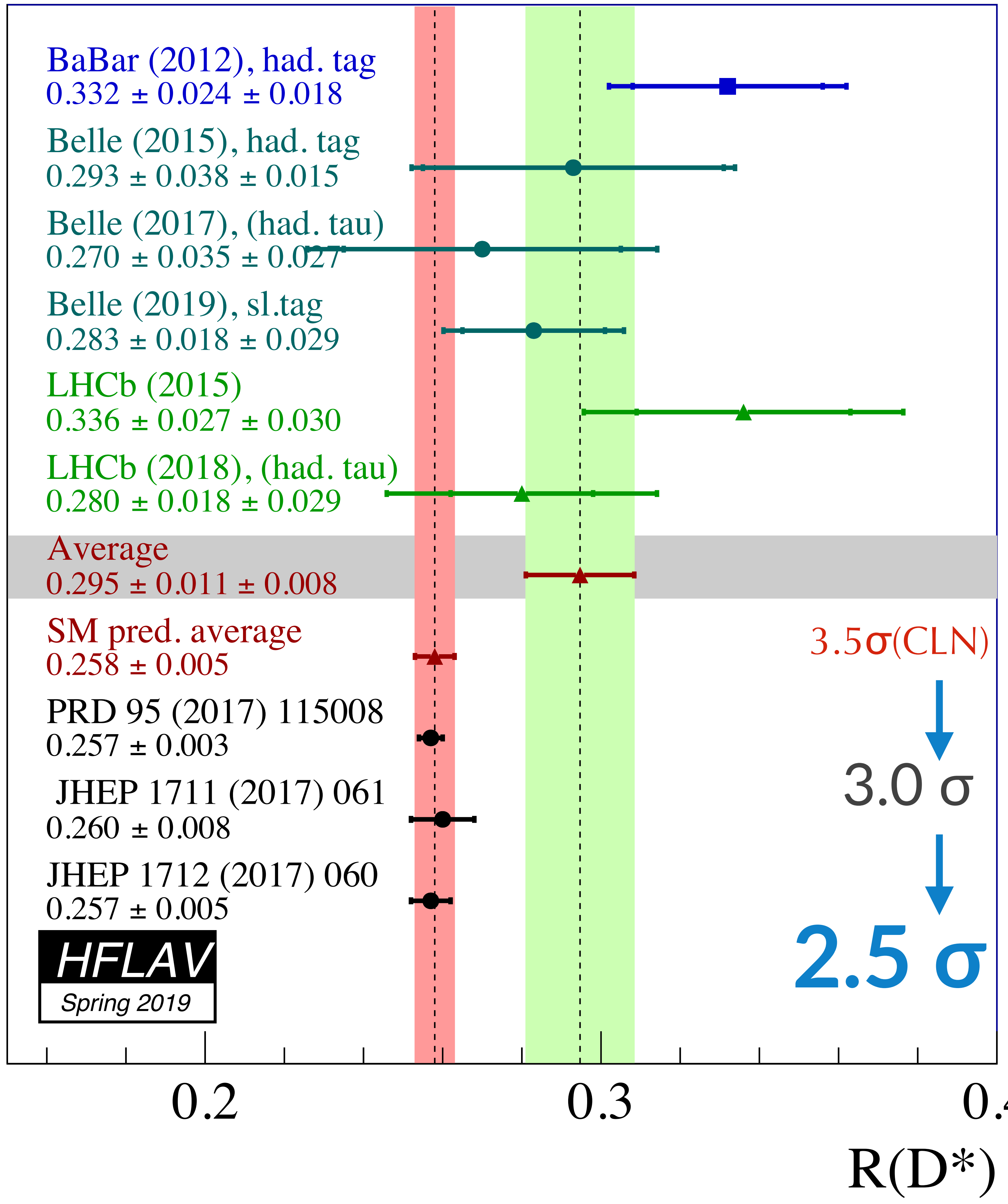
$D^{0,+}$ ↔

$D^{0,+}$ ↔

$D^{0,+}$ ↔

Belle New result 1904.08794

After Belle 2019



$D^{*0,+}$

$D^{*0,+}$

D^{*+}

$D^{*0,+}$

D^{*+}

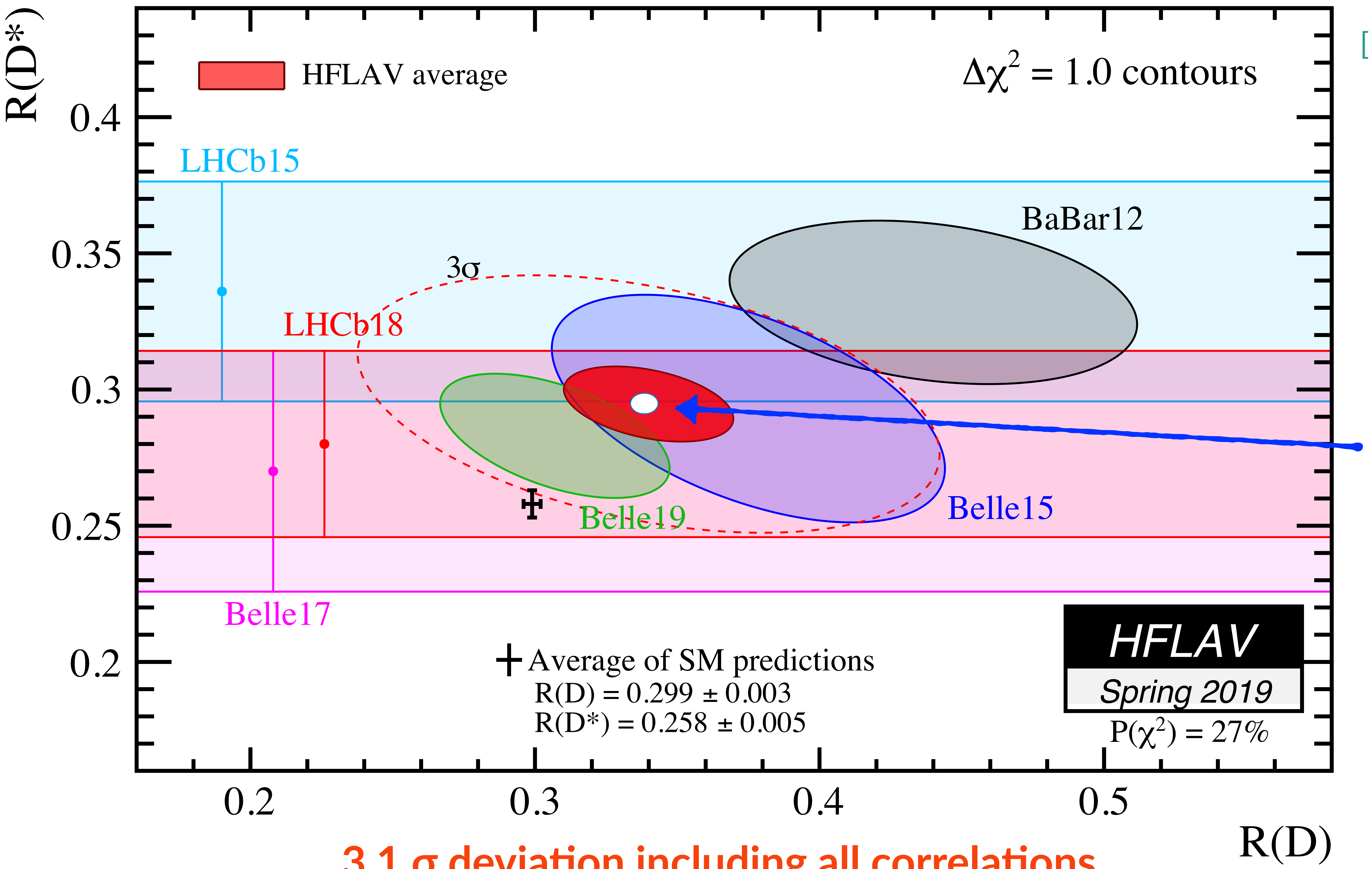
D^{*+}

S.Fajfer et al. (2012)
0.252 ± 0.003

BGL parametrization

After Belle 2019

LHCb can not measure D^{*0} precisely [$D^{*0} \to D^0\gamma$, $D^0\pi^0 (\to\gamma\gamma)$]



[HFLAV averages Spring 2019]

[Belle-II sensitivity, 50 ab-1]

$\Delta R(D)$: 3%

$\Delta R(D^*)$: 2%

[Belle, 1901.06380]

3.1 σ deviation including all correlations

Related observables

- $R(\Lambda_c) = \mathrm{Br}(\Lambda_b \to \Lambda_c \tau \nu)/\mathrm{Br}(\Lambda_b \to \Lambda_c \ell \nu)$ @ LHCb [Blanke, Crivellin, TK, Moscati, Nierste, Nisandzic, 1905.08253]

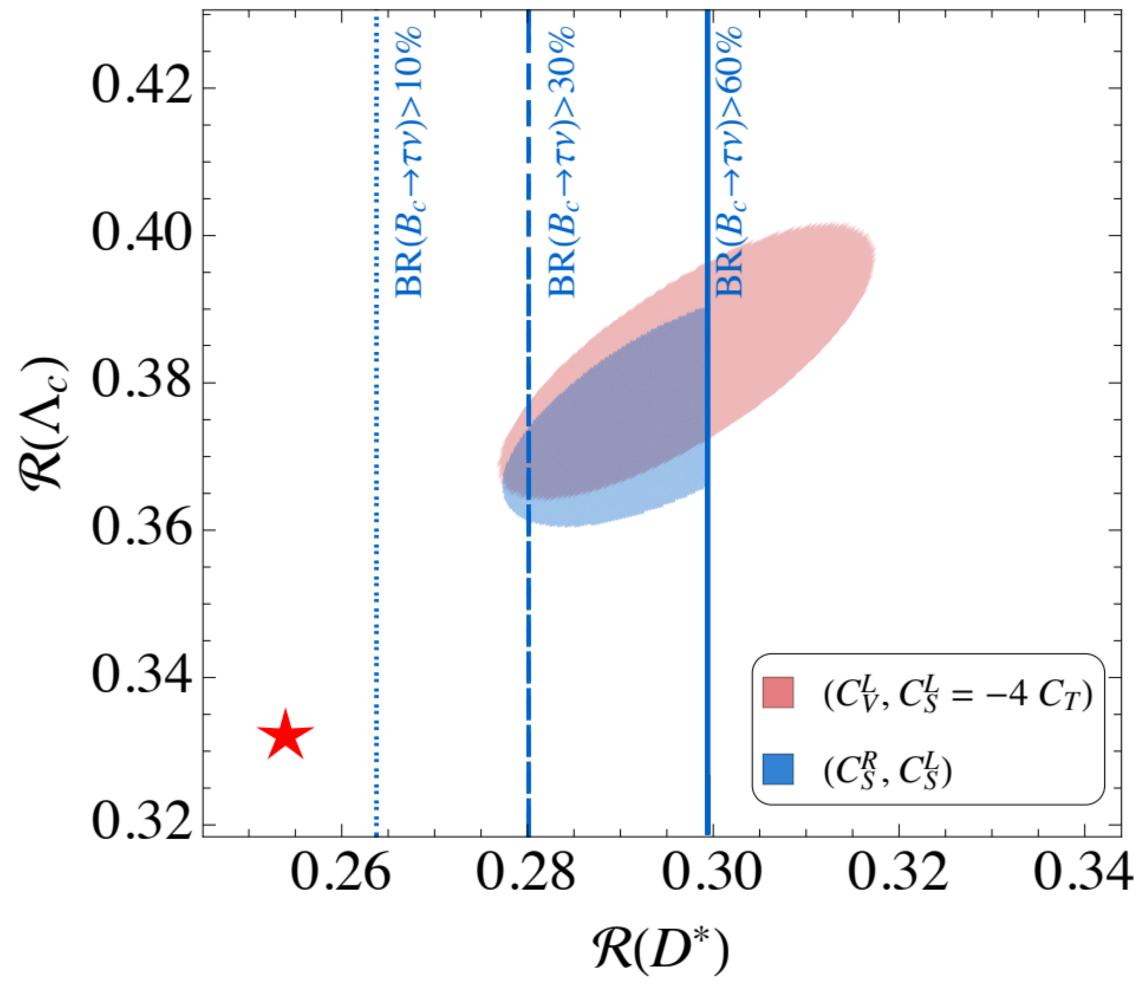
Motivated NP models

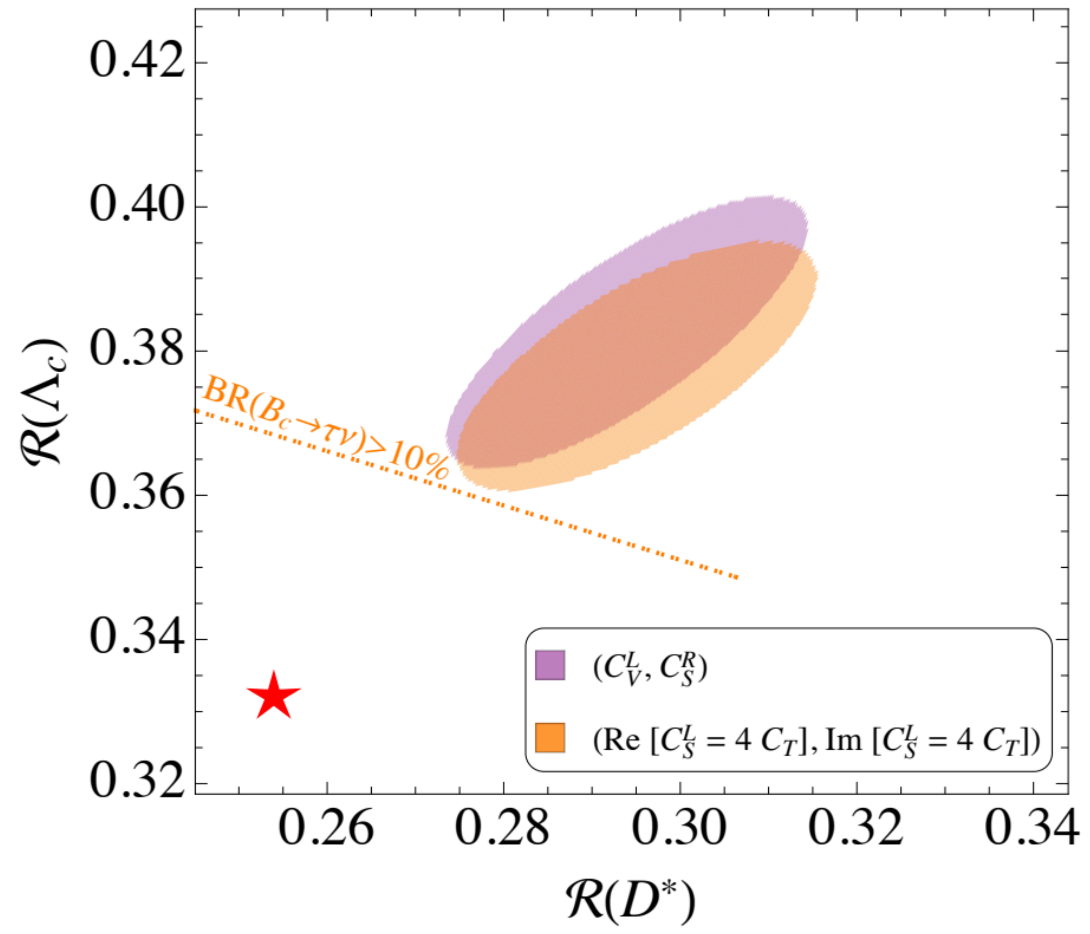
Scalar Leptoquark S1: SU(2) singlet

Generalized Charged Higgs

Vector Leptoquark U1: SU(2) singlet

Sclar Leptoquark R2: SU(2) doublet





- Sum rule for $R(\Lambda_c)$ prediction in general NP [Blanke, Crivellin, de Boer, TK, Moscati, Nierste, Nisandzic, PRD]

$$\frac{R(\Lambda_c)}{R(\Lambda_c)_{\mathrm{SM}}} \simeq 0.26 \frac{R(D)}{R(D)_{\mathrm{SM}}} + 0.74 \frac{R(D^*)}{R(D^*)_{\mathrm{SM}}}$$

crosscheck of $R(D^{(*)})$ anomaly is possible

→ $$R(\Lambda_c) = 0.38 \pm 0.01_{R(D^{(*)})} \pm 0.01_{\mathrm{FF}}$$ → There is no data yet, but soon?

- $R(J/\Psi) = \mathrm{Br}(B_c \to J/\Psi \tau \nu)/\mathrm{Br}(B_c \to J/\Psi \ell \nu)$ @ LHCb : same-direction deviation (but poorly known form factors)

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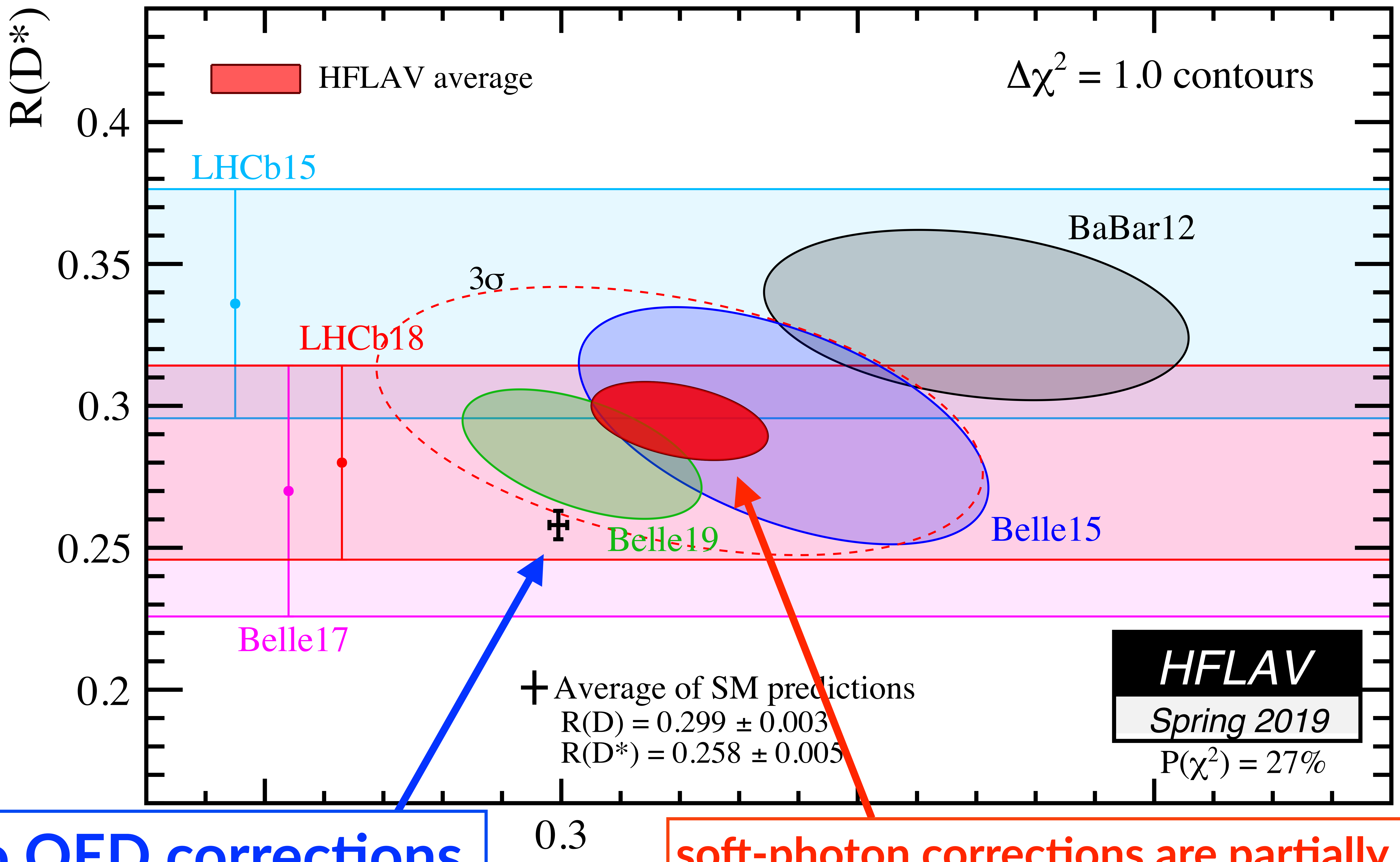


[HFLAV averages Spring 2019]

BaBar:
PHOTOS **version 2.13**

Belle:
PHOTOS **version 2.02**

LHCb:
PHOTOS **version 3.56**

No QED corrections

Error = QCD

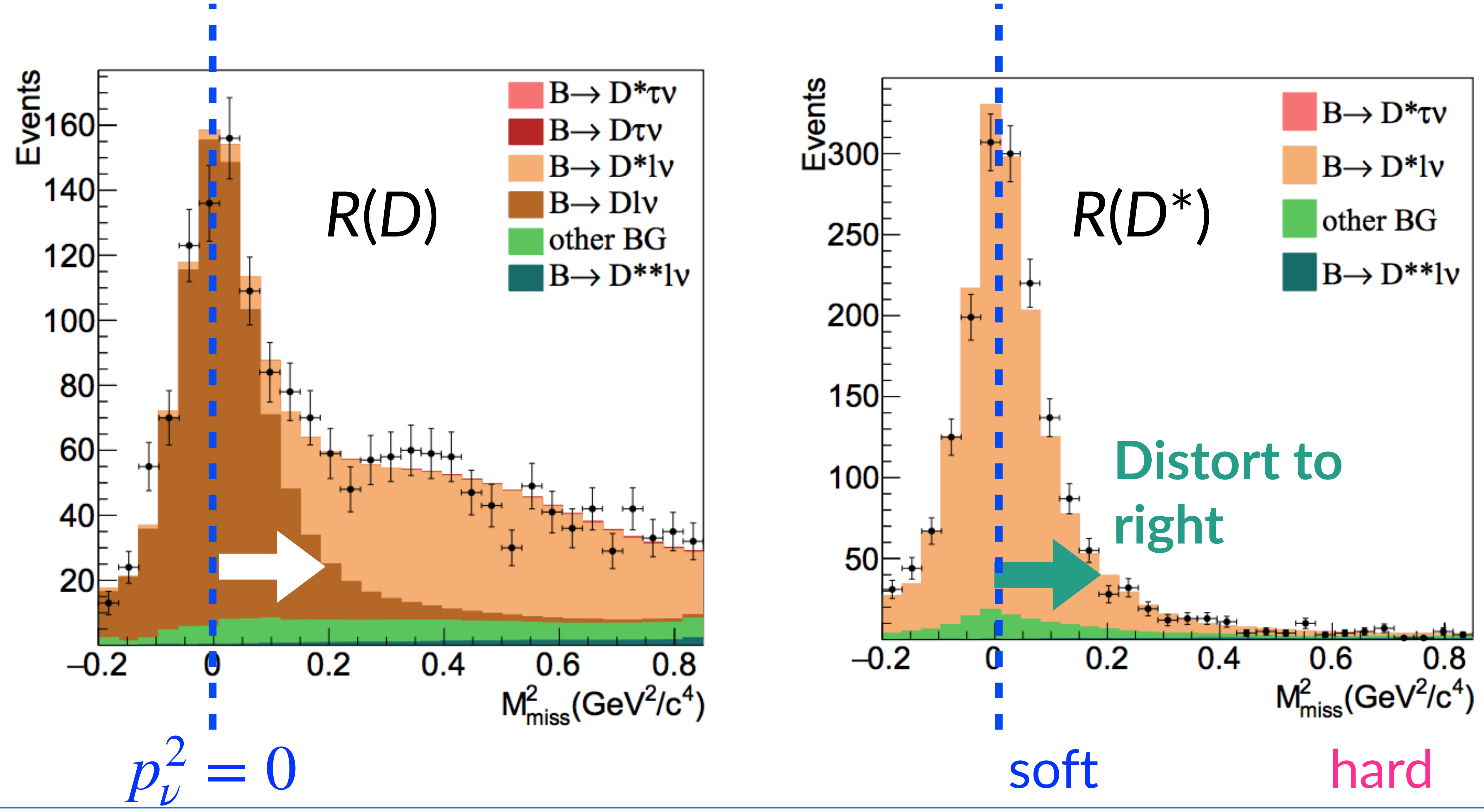
soft-photon corrections are partially subtracted by PHOTOS Monte-Carlo simulation

Photon emissions in data

- The experiments have **not explicitly utilized** the photon cut $\boldsymbol{E_{max}}$ for event selections for B semileptonic decay
- The photon radiation **distorts** the missing mass squared distribution **to the positive direction**

$$M^2_{\rm miss} \equiv \left(p_{e^+e^-} - p_{B_{\rm tag}} - p_D - p_\ell\right)^2 = (p_\nu + p_\gamma)^2 = 2E_\nu E_\gamma\left(1 - \cos\theta_{\nu\gamma}\right) > 0 \qquad E_{\nu_\ell} = 0.5 - 2\,\mathrm{GeV}$$

Missing mass squared distributions of selected events@ Belle



Soft radiation distorts the center of the shape of signal

Hard radiation distorts only the tail

[Belle, PRD92 (2015) no.7, 072014]

Short-distance QED corrections

Talk by Florian Bernlochner

- A famous correction for the semi-leptonic decays: **short-distance leading-log EW+QED correction**

[Sirlin '82, Atwood, Marciano '90]

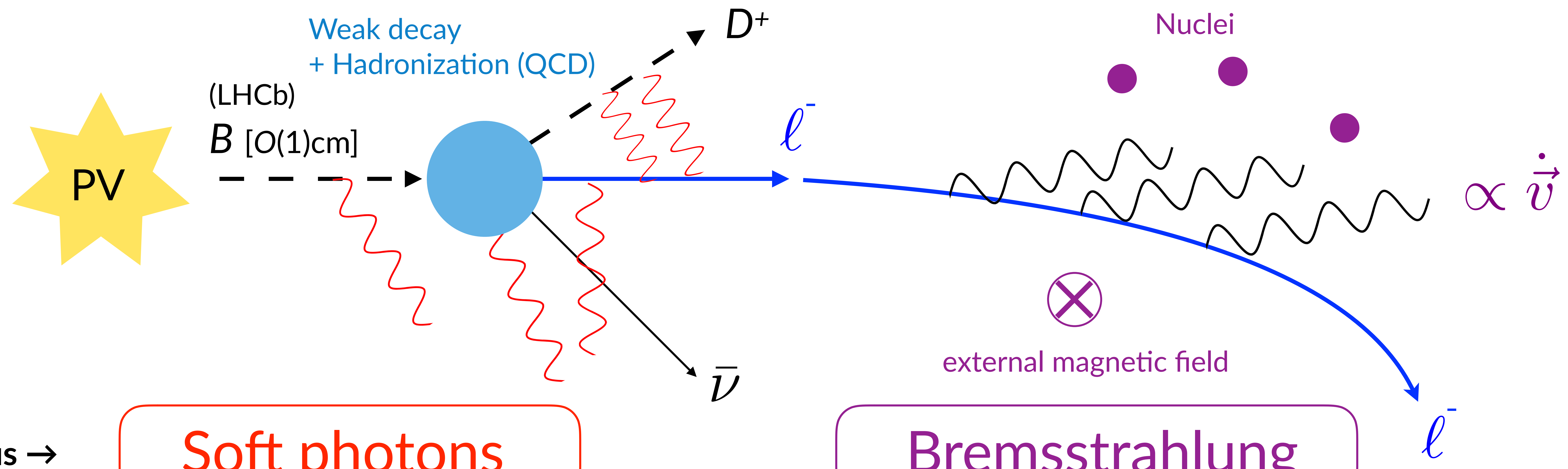
$$\mathcal{M}(b \to c\ell\nu) = \eta_{\rm EW}\mathcal{M}_0(b \to c\ell\nu)$$

$$\eta_{\rm EW} = 1 + \frac{3\alpha}{4\pi}(1 + \frac{1}{3})\log\frac{m_Z}{m_B} = 1.0066$$

Log resummation is negligible

- Leading Log from m_W **to** m_B e.g., photon-W box diagrams + Log m_Z/m_W comes from Z-W box
- QED corrections to muon lifetime are subtracted; G_F in $\mathcal{M}_0$ is defined from the QED corrected muon decay
- It can apply arbitrary semileptonic decay (note that $+1/3$ and m_B are changed properly)
- **It does not include coulomb correction**
- It is lepton flavour universal correction, so that **it is dropped in $R(D)$ and $R(D^*)$**

Long-distance QED corrections



our focus →

Soft photons

Physics **in vacuum**

Invisible photon energy

Sum of invisible emissions and loop corrections gives a total quantum corrections
= **Long-distance QED corrections**

Bremsstrahlung

Physics **in detector**

Emitted photon energy that corresponds to dropped lepton energy

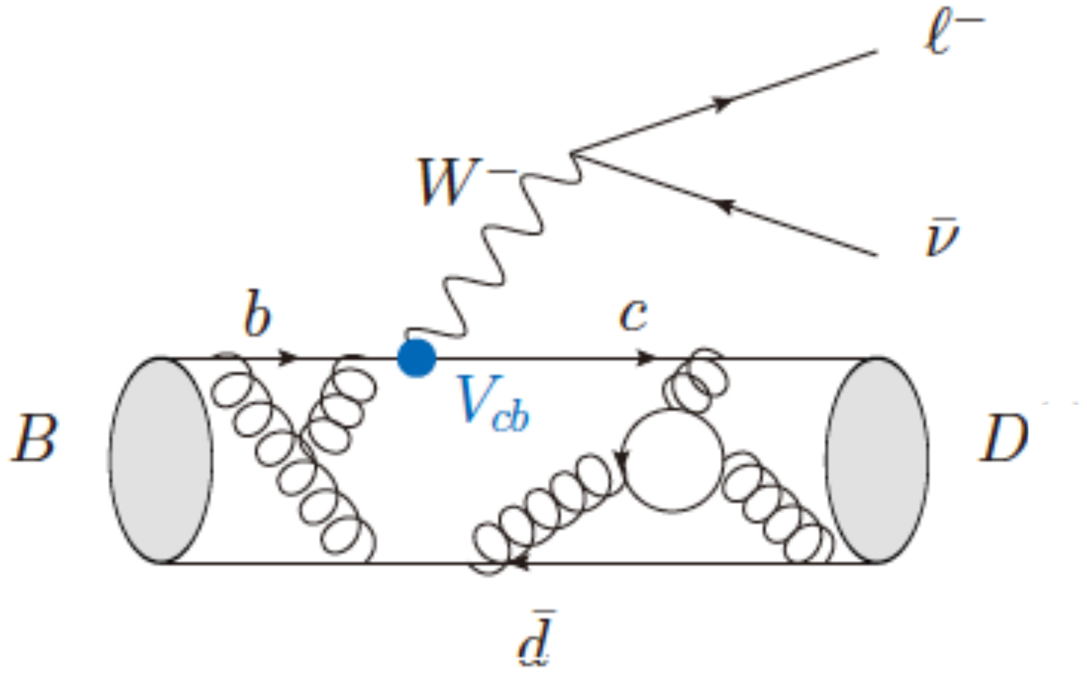
Small bremsstrahlung in μ^- and τ^-

Multi scales in $B \to D\ell\nu$

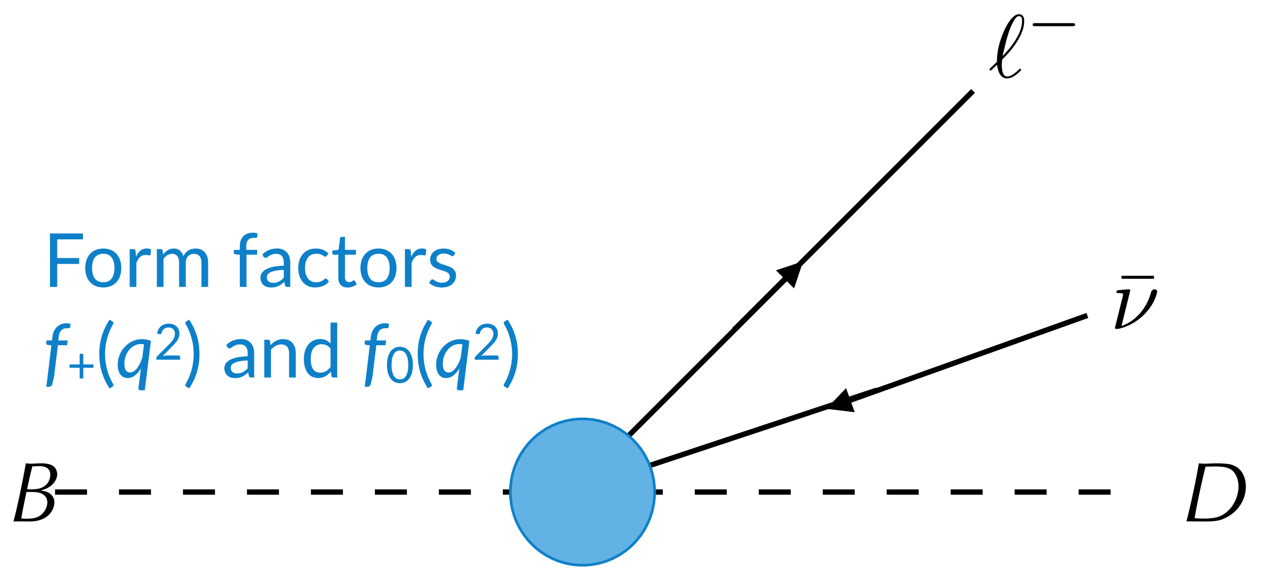
				SM, SMEFT	
$\mathcal{O}(100)$ GeV	m_t, m_W	Weak scale	*W*, *Z*, *t* (,*h*) decoupled	- - - - - - - - -	EW, QED corrections are lepton universal [Sirlin '82, Atwood, Marciano '90]
				Weak Hamiltonian	
~ 5 GeV	m_B, m_b	bottom scale	*b* decoupled	- - - - - - - - -	
~ 2 GeV	$\sqrt{m_b m_c}$, m_D, m_τ	*D*, *τ* scales		HQET $\mathcal{O}(1), \mathcal{O}(\alpha_s), \mathcal{O}(\Lambda_{\text{QCD}}/m_{b,c})$, $\mathcal{O}(\Lambda^2_{\text{QCD}}/m_c^2)$	[Bernlochner, Ligeti, Papucci, Robinson '17] [Jung, Straub '18]
$\mathcal{O}(100)$ MeV	Λ_{QCD}, m_μ	QCD scale	soft quark decoupled	- - - - - - - - -	
$\mathcal{O}(10)$ MeV	E_{max}	invisible photon		our focus: scalar QED	QED correction breaks LFU
0.5 MeV	m_e				

Soft-photon corrections

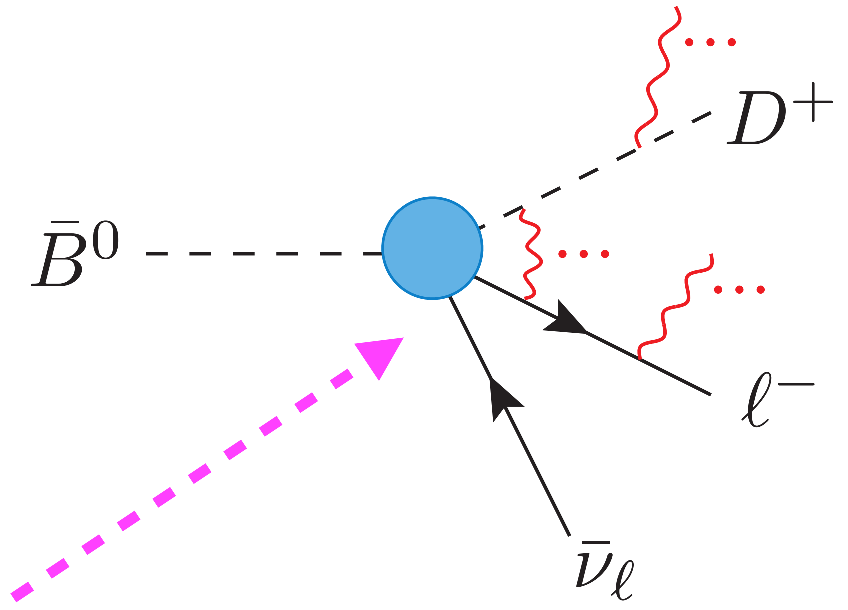
- At large distance ($\mu \lesssim \Lambda_{\rm QCD}$), the QED interactions of the charged mesons are well described by **the scalar QED**



$\mu \lesssim \Lambda_{\rm QCD}$

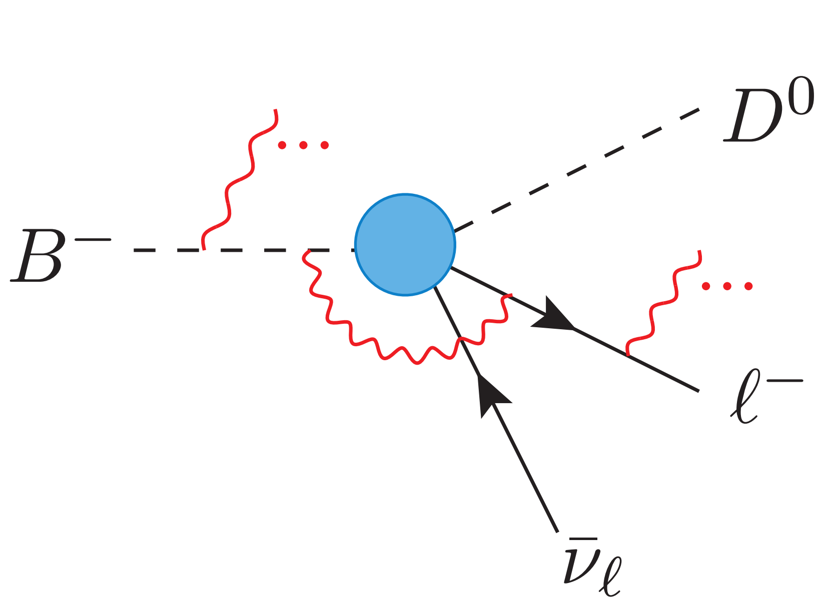


- We have to distinguish neutral and charged-*B* modes



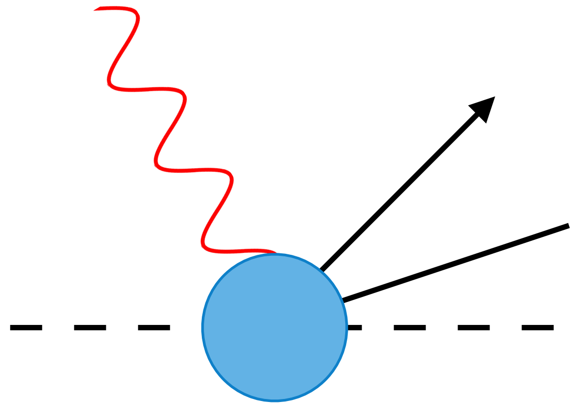
Coulomb pole $\frac{\pi\alpha}{v_{\rm rel}}$

$\bar{B}^0 \to D^+ \ell \bar{\nu}$ $[R(D^+)]$



$B^- \to D^0 \ell \bar{\nu}$ $[R(D^0)]$

- ***BDℓνγ* vertex** (Inner-Bremsstrahlung)



is also included

Scalar QED

- Hadronic matrix element of *B* to *D* transition (P →P'):

$$\langle D\left|\bar{c}\gamma_\mu b\right|B\rangle = f_+\left(q^2\right)\left(p_B + p_D\right)_\mu + f_-\left(q^2\right)q_\mu$$

$$q_\mu \equiv \left(p_B - p_D\right)_\mu = \left(p_\ell + p_\nu + p_{\gamma s}\right)$$

$$f_-(q^2) = \frac{m_B^2 - m_D^2}{q^2}\left[f_0(q^2) - f_+(q^2)\right]$$

$f_0(q^2)$: scalar form factor

with $f_+(0) = f_0(0)$

- *B* to *D* semi-leptonic decay amplitude:

$$\begin{aligned}\mathcal{A} &= \langle D\left|\mathcal{H}_{\text{eff}}\right|B\rangle \\ &= \sqrt{2}G_F V_{cb}\left[\overline{u}\left(p_\ell\right)\gamma^\mu P_L v\left(p_\nu\right)\right]\left[f_+\left(q^2\right)\left(p_B + p_D\right)_\mu + f_-\left(q^2\right)q_\mu\right]\end{aligned}$$

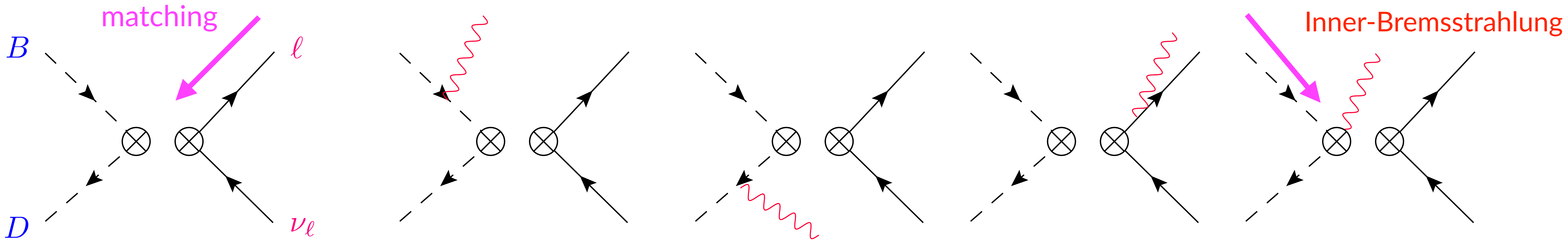
Scalar QED

- Corresponding effective scalar QED Lagrangian is

$$\mathcal{L}_{\rm eff} = -\sqrt{2} G_F V_{cb} \left[\phi_D^* i \left(\partial_\mu \phi_B\right) \left(f_+ + f_-\right) - i \left(\partial_\mu \phi_D\right)^* \phi_B \left(f_+ - f_-\right)\right] \left(\bar{\ell} \gamma^\mu P_L \nu\right)$$

U(1)em Gauge symmetry requires $\partial_\mu \to D_\mu = \partial_\mu + ieQ_\phi A_\mu$

$$\mathcal{L}_{\rm eff} = -\sqrt{2} G_F V_{cb} \left[\phi_D^* i \left(D_\mu \phi_B\right) \left(f_+ + f_-\right) - i \left(D_\mu \phi_D\right)^* \phi_B \left(f_+ - f_-\right)\right] \left(\bar{\ell} \gamma^\mu P_L \nu\right)$$

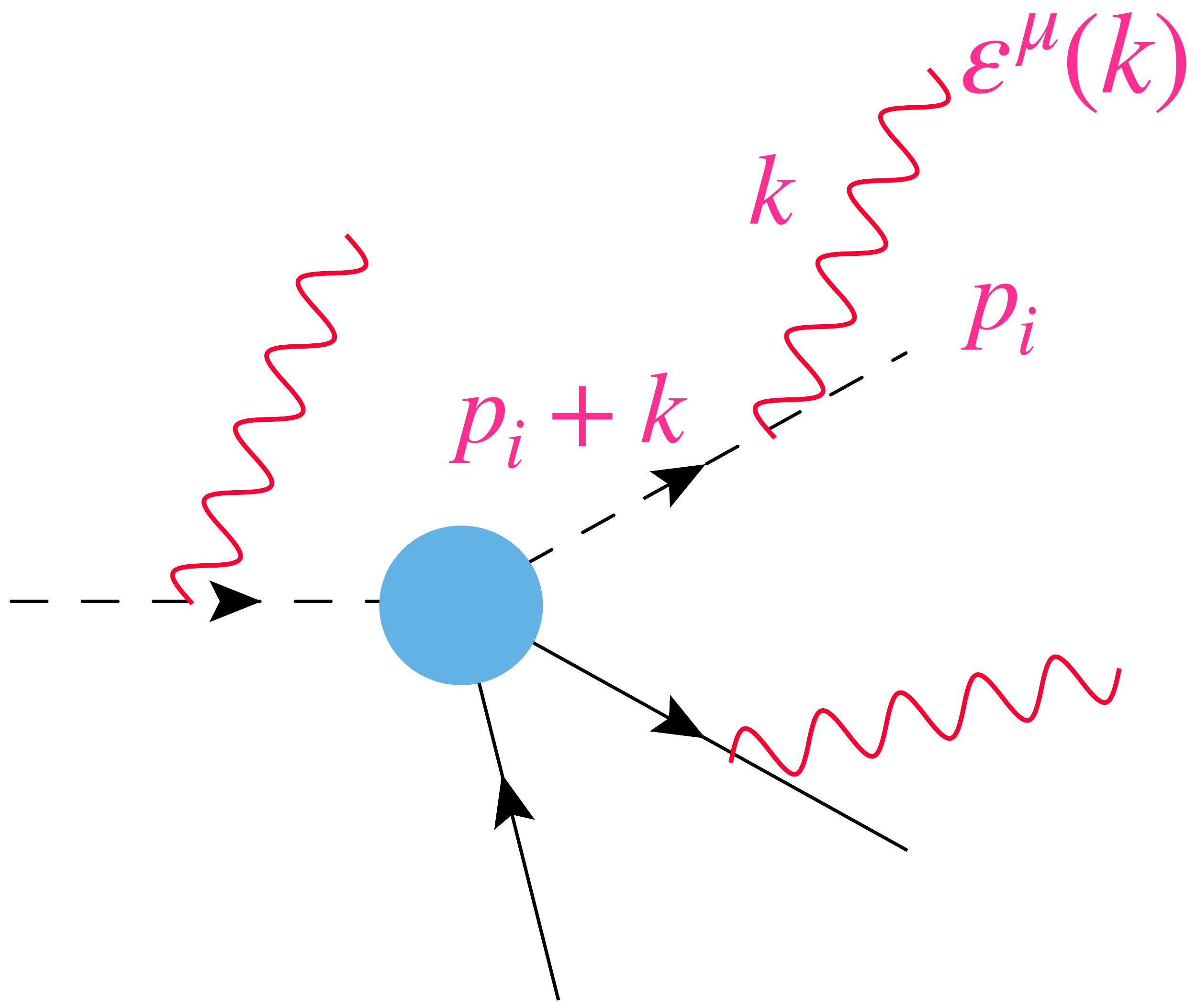


- This inner-bremsstrahlung matching reproduces the soft-photon limit of more general results by Kubis, Muller, Gasser, Schmid '07, Bernlochner, Schonherr '10 **Talk by Florian Bernlochner**

1 page summary of soft-photon corrections

- **E_{max} is the maximum total energy of undetected soft photons in the rest frame of the *B*-meson**: E_{max} = 20~30 MeV
- **The soft-photon approximation** is used for analytic evaluation: we keep $O(\ln E_{max})$ and $O(E^0_{max})$ and drop $O(E_{max})$, which is **valid only $l = \tau$ and μ.** In this framework, electron mode suffers from collinear singularity $\ln(m_e)$, $(\ln(m_e))^2$
- Real soft emissions = $O(\ln E_{max})$ + $O(E^0_{max})$
    - **$O(\ln E_{max})$ terms are resumed: arbitrary number of soft photon emissions**
    - Finite terms [$O(E^0_{max})$] are numerically comparable to $O(\ln E_{max})$
- Virtual corrections = $O(E^0_{max})$ and **μ-dependent** : $\mu \lesssim \Lambda_{\rm QCD}$ would correspond to the matching scale onto the scalar QED
    - We separate k_l=0 contribution and the rest part (k_l is loop momentum)
    - **Coulomb pole $(\alpha/v_{\rm rel})$ exits only in $R(D^+)$ case, and we resumed them** (=Sommerfeld enhancement)
- Both of contributions depend on lepton kinematics → **source of LFU violation**

Real emissions



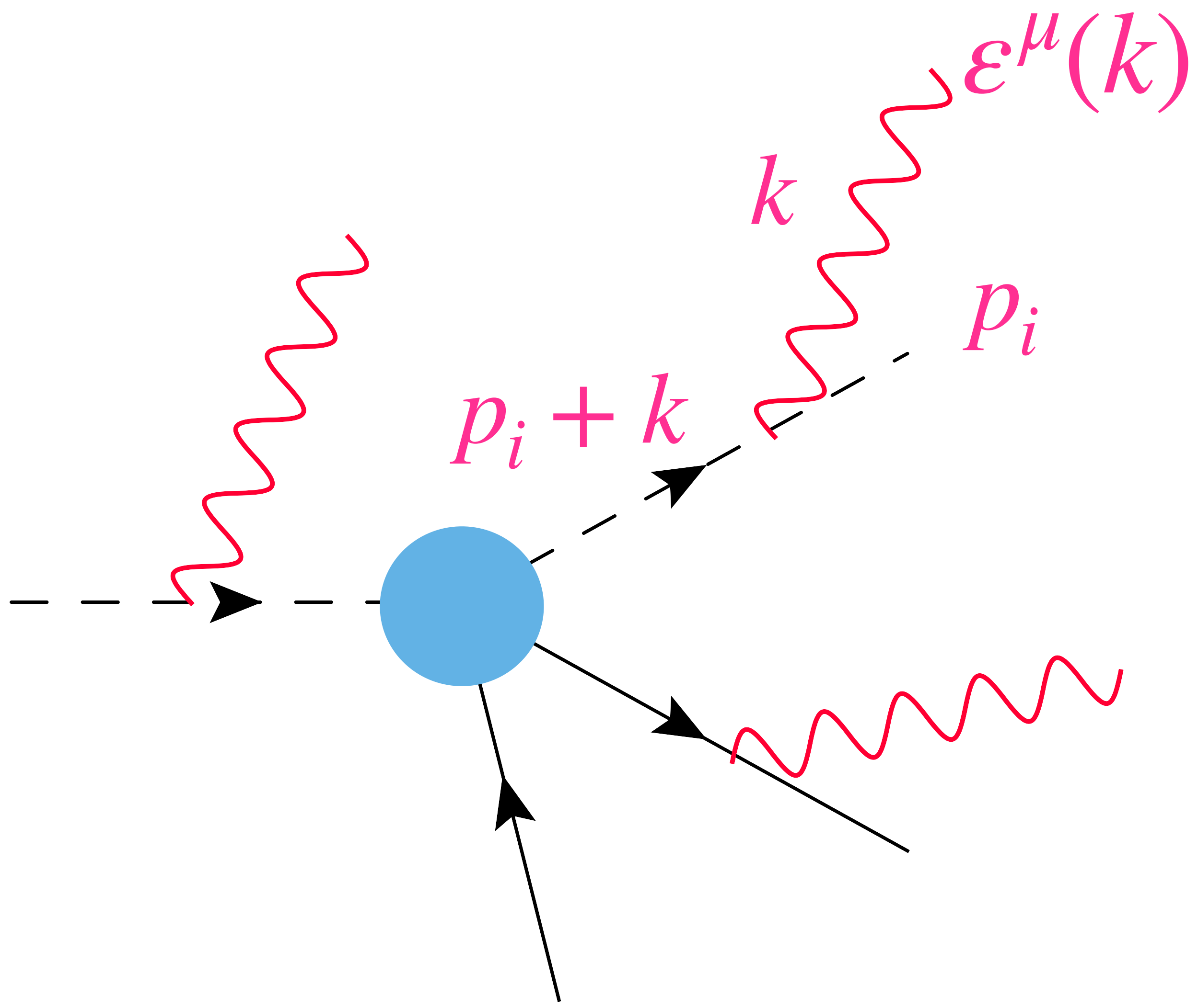
Scalar
$$\frac{i}{(p_i+k)^2 - m_i^2}(-i)Q_i e\,(2p_i + k)\cdot\varepsilon(k) = Q_i e \frac{p_i \cdot \epsilon(k)}{p_i \cdot k}$$

Fermion
$$\overline{u}\,(p_i)\,(-i)Q_i e \gamma_\mu \frac{i\,(\not{p}_i + \not{k} + m_i)}{(p_i+k)^2 - m_i^2}\varepsilon^\mu(k) = \overline{u}\,(p_i)\,Q_i e \frac{2p_{i,\mu} - \not{k}\gamma_\mu}{2p_i \cdot k}\varepsilon^\mu(k)$$
$$= \overline{u}\,(p_i)\,Q_i e \frac{p_i \cdot \varepsilon(k)}{p_i \cdot k} + \mathcal{O}(k^0)$$

Vector
$$\varepsilon^\sigma\,(p_i)\,\varepsilon^\mu(k)(-i)Q_i e \left[g_{\sigma\rho}\,(-2p_i - k)_\mu + g_{\rho\mu}\,(2k + p_i)_\sigma + g_{\mu\sigma}\,(p_i - k)_\rho\right]$$
$$\times \frac{-i}{(p_i+k)^2 - m_i^2}\left[g^{\nu\rho} - \frac{(p_i+k)^\nu\,(p_i+k)^\rho}{m_i^2}\right]$$
$$= \varepsilon^\sigma\,(p_i)\,\varepsilon^\mu(k)\frac{Q_i e}{2p_i \cdot k}\left[2p_{i,\mu}g_{\nu\sigma} - 2k_\sigma g_{\mu\nu} + 2k_\nu g_{\mu\sigma}\right]$$
$$= \varepsilon_\nu\,(p_i)\,Q_i e \frac{p_i \cdot \varepsilon(k)}{p_i \cdot k} + \varepsilon^\sigma\,(p_i)\,\mathcal{O}(k^0)_{\sigma\nu}$$

- $\mathcal{O}(k^0)$ **terms are discarded in the soft-photon approximation:** $\int^{E_\gamma} \mathcal{O}(k^0)dk = \mathcal{O}(E_\gamma/M)$ suppressed
  = eikonal approximation

Soft emissions



- After dropping $\mathcal{O}(k^0)$ terms (soft-photon approximation), the single soft emissions can be given by a simple spin-independent amplitude:

$$\mathcal{M}_{\text{soft emission}} = \sum_{i=\text{all}} Q_i e \frac{p_i \cdot \varepsilon(k)}{p_i \cdot k} \mathcal{M}_0$$

i also runs initial-state radiation

$\mathcal{M}_0$ no-emission amplitude

- Soft-emitted decay width is given by an integral of **square of the sum: interference is encoded**

$$\Gamma_{\text{soft emission}} = \int dk \left[\sum_{i=\text{all}} Q_i e \frac{p_i \cdot \varepsilon(k)}{p_i \cdot k} \right]^2 \Gamma_0$$

Γ_0 no-emission decay width

PHOTOS (**v<2.07(single), 2.13 (multiple)**) is based on the integral of **squared sum:**

$$\Gamma_{\text{soft emission}} = \int dk \sum_{i=\text{all}} \left[Q_i e \frac{p_i \cdot \varepsilon(k)}{p_i \cdot k} \right]^2 \Gamma_0$$

Integral → Next slide

Soft emission integral

- This k dependence (**square of the sum**) can be integrated out without any approximations

$$\Gamma_{\text{soft emission}} = \int dk \left[\sum_{i=\text{all}} Q_i e \frac{p_i \cdot \varepsilon(k)}{p_i \cdot k} \right]^2 \Gamma_0$$

$$= e^2 \int^{E_{\max}} \frac{d^3\mathbf{k}}{(2\pi)^3 E_\gamma} \sum_{i,j=\text{all}} Q_i Q_j \frac{-p_i \cdot p_j}{(p_i \cdot k)(p_j \cdot k)}$$

Photon mass IR cutoff →

$$= -\frac{\alpha}{2\pi} \int_{m_\gamma}^{E_{\max}} dE_\gamma \int_{-1}^{1} d\cos\theta \sqrt{E_\gamma^2 - m_\gamma^2} \sum_{i,j} Q_i Q_j \frac{p_i \cdot p_j}{(p_i \cdot k)(p_j \cdot k)}$$

$$= \frac{\alpha}{\pi} \sum_{i,j} Q_i Q_j \left(2b_{ij} \ln \frac{m_\gamma}{2E_{\max}} + F_{ij} \right)$$

$$|\mathbf{k}|^2 = E_\gamma^2 - m_\gamma^2$$

$$d^3\mathbf{k} = d\Omega |\mathbf{k}|^2 d|\mathbf{k}|$$

$$= 2\pi \int_{-1}^{1} d\cos\theta \left(E_\gamma^2 - m_\gamma^2\right) d|\mathbf{k}|$$

$$= 2\pi \int_{-1}^{1} d\cos\theta \sqrt{E_\gamma^2 - m_\gamma^2} E_\gamma dE_\gamma$$

Feynman integral is required:

$$\frac{1}{(p_i \cdot k)(p_j \cdot k)} = \int_0^1 dz \frac{1}{[z(p_i \cdot k) + (1-z)(p_j \cdot k)]^2}$$

$$= \int_0^1 dz \frac{1}{[(zp_i + (1-z)p_j) \cdot k]^2}$$

This relative angle is $\cos\theta$

- F_{ij} is m_γ, $E_{\max}$ independent terms (definition)

Details of b_{ij}, F_{ij} → Next slide

Soft emission corrections

- Single soft-photon emission gives the following correction: [Isidori '08, de Boer, TK, Nisandzic, '18]

$$\Gamma_{\text{soft emission}} = \frac{\alpha}{\pi}\sum_{i,j} Q_i Q_j \left(2b_{ij}\ln\frac{m_\gamma}{2E_{\max}} + F_{ij}\right)\Gamma_0$$

Emission from a single particle

$$b_{ii} = \frac{1}{2}$$ soft

$$F_{ii} = \frac{1}{2\beta_{0i}}\ln\left(\frac{1+\beta_{0i}}{1-\beta_{0i}}\right)$$ collinear

Interference

$$b_{ij} = \frac{1}{4\beta_{ij}}\ln\left(\frac{1+\beta_{ij}}{1-\beta_{ij}}\right)$$ soft + collinear

$$F_{ij} = \frac{1}{2}\frac{m_i m_j}{\sqrt{1-\beta_{ij}^2}}\int_0^1 dz \frac{E(z)}{P(z)\left[E(z)^2 - P(z)^2\right]}\log\left[\frac{E(z)+P(z)}{E(z)-P(z)}\right]$$ collinear

where

$$E(z) = zE_i + (1-z)E_j$$

$$P(z) = \sqrt{E(z)^2 - z^2 m_i^2 - (1-z)^2 m_j^2 - 2z(1-z)\frac{m_i m_j}{\sqrt{1-\beta_{ij}^2}}}$$

$\beta_{0,i}$ is velocity of particle i on the B-rest frame

$\beta_{i,j}$ is relative velocity between particle i and j

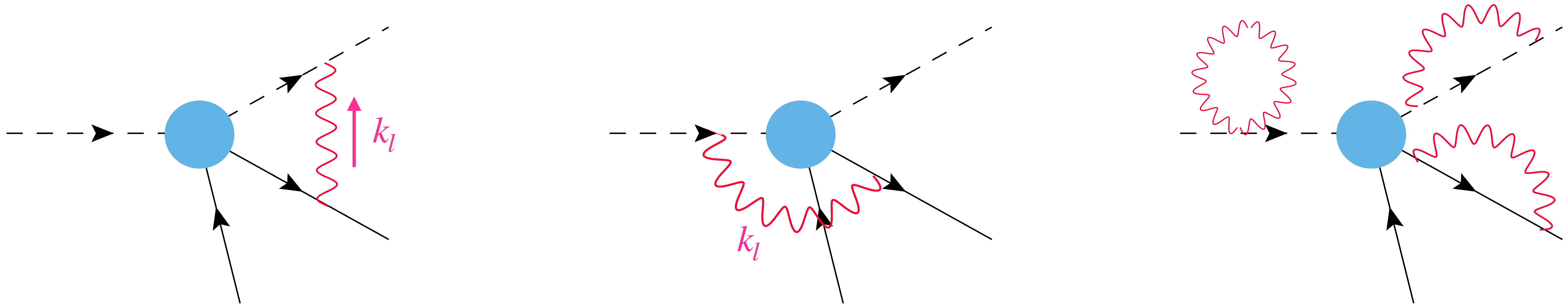
$0 < \beta < 1$

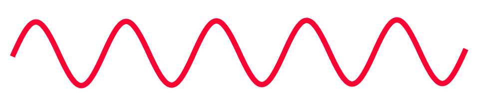
Collider singularity arises from $\beta \to 1$

Velocities and energies are related under kinematics of the decay channel

Virtual correction

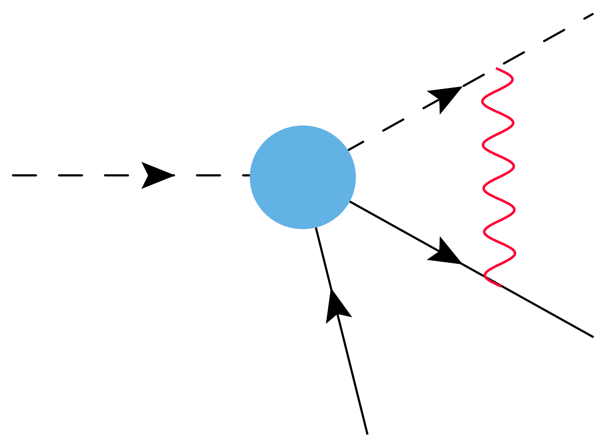
- Vertex corrections and self-energy corrections are required for the IR-pole cancelation





$$\frac{-ig_{\mu\nu}}{k_l^2 - m_\gamma^2}$$

k_l : Loop momentum

- Coulomb pole $\frac{\pi\alpha}{v_{\mathrm{rel}}}$ appears in only space-like photon exchange  = **neutral B-meson channel**

Vertex corrections

- Scalar-fermion-photon vertex correction: one can derive the analytic formula

$$\Gamma_{\text{virtual}} = \frac{\alpha}{\pi}\,\mathrm{Re}\frac{4\pi^2}{i}\int\frac{d^dk_l}{(2\pi)^d}\overline{u}(p_i)\left(2p_{i,\mu}+\gamma_\mu \not{k}_l\right) f^\nu_{0,+}\left((q+k_l)^2\right)\gamma_\nu P_L v \frac{(2p_j-k_l)^\mu}{(k_l^2+2p_i\cdot k_l)(k_l^2-2p_j\cdot k_l)\left(k_l^2-m_\gamma^2\right)} \times \mathcal{M}_0\int_{\text{phase space}} d\Phi$$

Im is automatically dropped in tree-loop + loop-tree interference

$f^\nu_{0,+}(q^2)$, $q_\mu \equiv (p_B - p_D)_\mu$
Our procedure

Tree part

- We decompose this integral into two parts: $I = I_{\text{low}k_l} + I_{\text{high}k_l}$

$$I_{\text{low}k_l} = (p_i\cdot p_j)\,\mathrm{Re}\frac{16\pi^2}{i}\int\frac{d^dk_l}{(2\pi)^d}\frac{1}{(k_l^2+2p_i\cdot k_l)(k_l^2-2p_j\cdot k_l)\left(k_l^2-m_\gamma^2\right)}$$

IR divergence ($1/m_\gamma$ pole), UV finite, **Coulomb pole ($1/v_{\text{rel}}$ pole)**, **spin-independent**

$$I_{\text{high}k_l} = \mathrm{Re}\frac{4\pi^2}{i}\int\frac{d^dk_l}{(2\pi)^d}\frac{-k_l^2-2p_i\cdot k_l+2\not{p}_j\not{k}_l}{(k_l^2+2p_i\cdot k_l)(k_l^2-2p_j\cdot k_l)\,k_l^2}$$

IR finite, **UV divergence**, **spin-dependent**

$\rightarrow \overline{MS}$ renormalization: UV pole is replaced by renormalization scale (matching scale) $\log\mu$

Vertex corrections

- $I_{\mathrm{low}k_l}$ is equivalent to one of the Passarino-Veltman scalar integral C_0

$$
\begin{aligned}
I_{\mathrm{low}k_l} &= (p_i \cdot p_j)\,\mathrm{Re}\,\frac{16\pi^2}{i}\int \frac{d^d k_l}{(2\pi)^d}\frac{1}{(k_l^2 + 2p_i\cdot k_l)\,(k_l^2 - 2p_j\cdot k_l)\,\left(k_l^2 - m_\gamma^2\right)} \\
&= \frac{1}{2}(s_{ij} - m_i^2 - m_j^2)\,\mathrm{Re}\,C_0(m_i^2, m_j^2, s_{ij}, m_i, m_\gamma(\to 0), m_j)
\end{aligned}
$$

- For the Taylor expansion of C_0 by m_γ, we found that old but famous results 't Hooft, Veltman '79, Batdin, Passarino '99 include typos (or wrong formula). The first correct expansion was derived by Beenakker, Denner '90.

☑ We confirmed Beenakker, Denner formula by using Packege-X and LoopTools, numerically.

Vertex corrections

- Beenakker, Denner C_0 formula gives [Isidori '08, de Boer, TK, Nisandzic, '18]

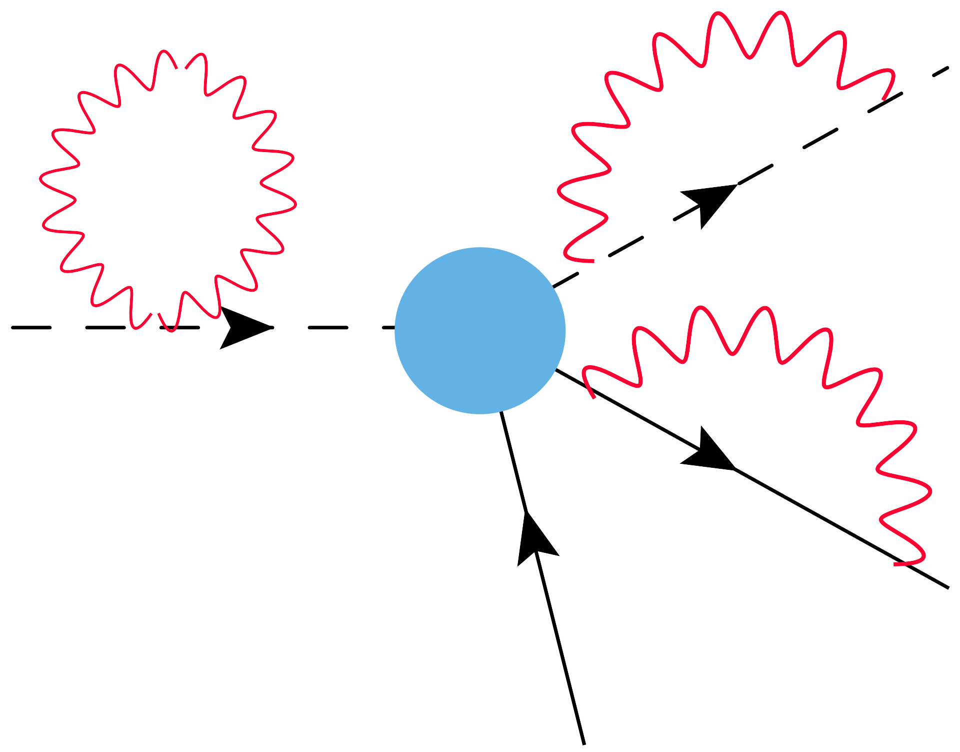
$$
\begin{aligned}
I_{\mathrm{low}k_l} =& \frac{1}{2}(s_{ij} - m_i^2 - m_j^2)\,\mathrm{Re}\,C_0(m_i^2, m_j^2, s_{ij}, m_i, m_\gamma(\to 0), m_j) \\
=& -\frac{1}{2\beta_{ij}} \left\{ -\frac{1}{2} \ln\left(\frac{1+\beta_{ij}}{1-\beta_{ij}}\right) \ln\left(\frac{m_i m_j}{m_\gamma^2}\right) + \pi^2 - \frac{1}{8}\ln^2\left(\frac{1+\beta_{ij}}{1-\beta_{ij}}\right) \right. \\
& \left. + \frac{1}{2}\ln^2\left(\frac{m_i}{m_j}\right) - \frac{1}{2}\ln^2\left(\frac{\Delta_i + \Delta_{ij}\beta_{ij}}{\Delta_j + \Delta_{ij}\beta_{ij}}\right) - \mathrm{Li}_2\left(\frac{2\Delta_{ij}\beta_{ij}}{\Delta_i + \Delta_{ij}\beta_{ij}}\right) - \mathrm{Li}_2\left(\frac{2\Delta_{ij}\beta_{ij}}{\Delta_j + \Delta_{ij}\beta_{ij}}\right) \right\}
\end{aligned}
$$

with $\Delta_{ij} = \dfrac{s_{ij} - m_i^2 - m_j^2}{2s_{ij}}, \quad \Delta_{i,j} = \dfrac{s_{ij} + m_{i,j}^2 - m_{j,i}^2}{2s_{ij}}$

- Soft + collinear singularities in the interference of soft-emissions are analytically canceled out
- One can observe the Coulomb term with correct velocity coefficients. For the time-like photon exchange, Li_2 gives additional $-\pi^2$, and the Coulomb pole is canceled out.

Self-energy corrections

- Self-energy contributes to the wave functions, which include IR divergence and UV divergence
- UV divergence and the finite corrections are subtracted in the on-shell renormalization scheme



$$\Gamma_{\text{self energy}} = \sum_i \frac{\alpha}{\pi} Q_i^2 \ln\left(\frac{m_i}{m_\gamma}\right) \Gamma_0$$

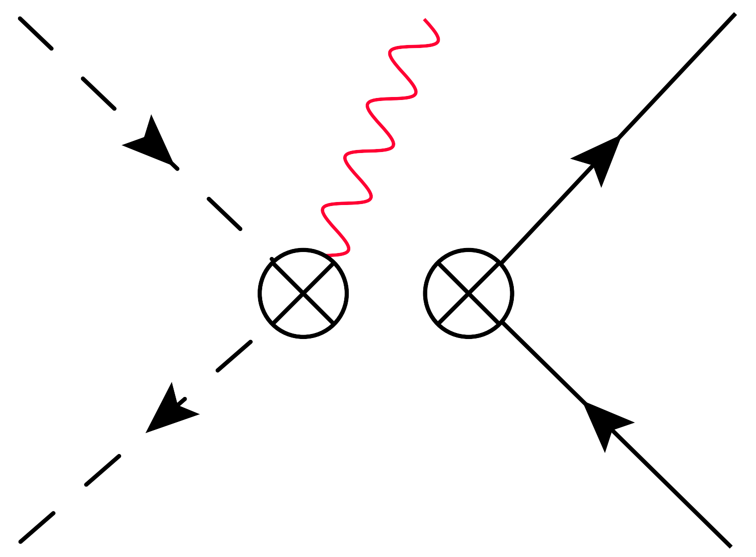
- Soft singularities in an emission from a single particle are analytically canceled out

☑ All soft singularities ($m_\gamma \to 0$) cancel between |single soft-emissions|^2 and the virtual corrections: **We have checked analytically**

Inner-bremsstrahlung virtual corrections

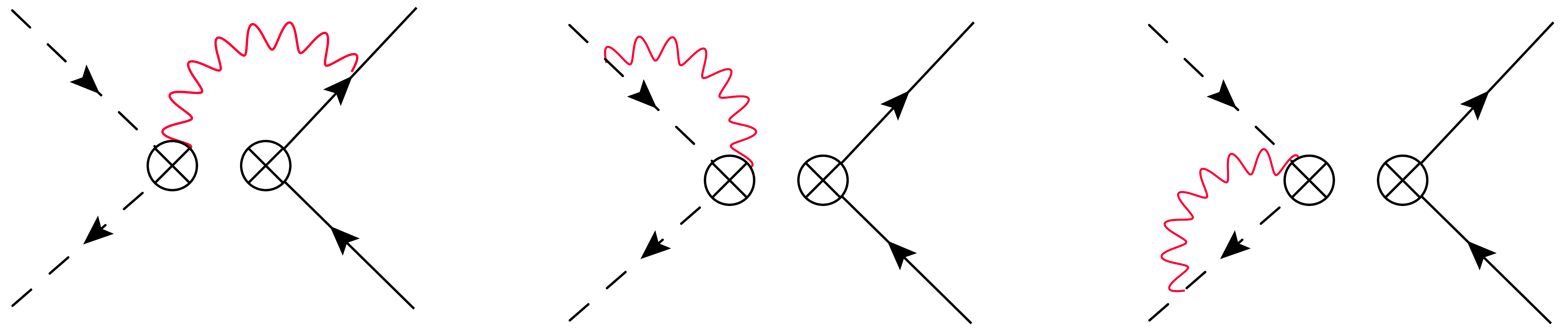
- The soft-emissions from the inner-bremsstrahlung does not induce IR pole, and it is $\mathcal{O}(k^0)$ which is suppressed by $\mathcal{O}(E_\gamma/M)$, and it is discarded in the soft-photon approximation
- The nonzero contribution comes from the virtual correction from the inner-bremsstrahlung, which is IR finite but has UV divergence. We subtract the UV pole by $\overline{MS}$ renormalization

inner-bremsstrahlung emission



vanish in the soft-photon limit

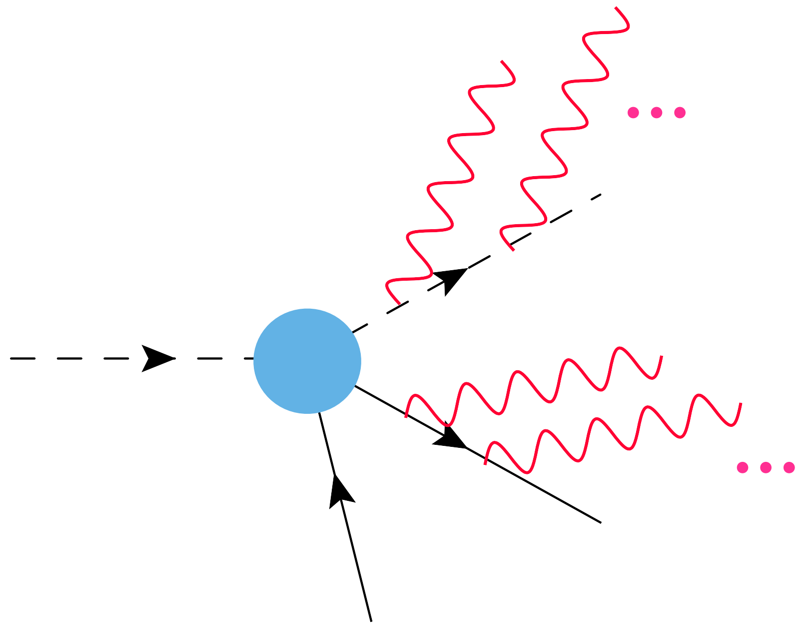
Virtual correction from the inner-bremsstrahlung



renormalization scale (matching scale) $\log \mu$ dependence is introduced

Resummation

- We resumed two potentially large contributions:
  - 1, $(\alpha \ln E_{\max})^n$ from arbitrary number of soft-photon emissions



$\Gamma \to \Omega_B \Gamma$

e.g., neutral B-meson channel

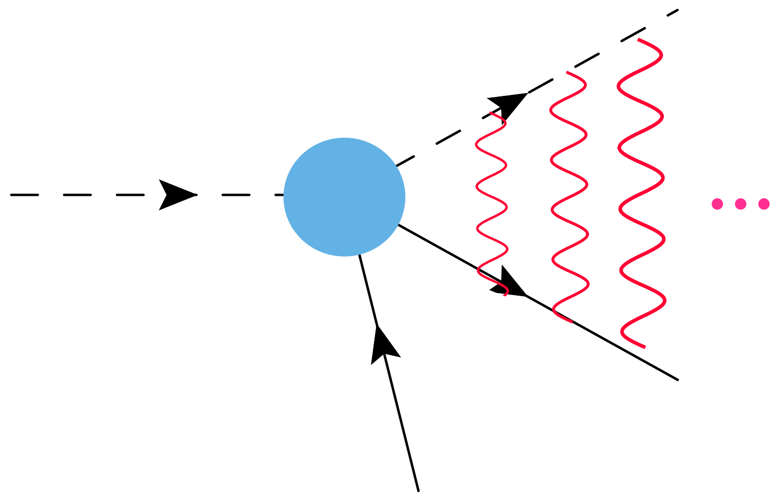
$$\Omega_B = \left(\frac{2E_{\max}}{\sqrt{m_i m_j}}\right)^{-\frac{2\alpha}{\pi}\sum_{ij} Q_i Q_j b_{ij}} = \left(\frac{2E_{\max}}{\sqrt{m_D m_\ell}}\right)^{-\frac{2\alpha}{\pi}(1-2b_{D\ell})}$$

with

$$b_{D\ell} = \frac{1}{4\beta_{D\ell}} \ln\left(\frac{1+\beta_{D\ell}}{1-\beta_{D\ell}}\right)$$

We have checked $\mathcal{O}(\alpha^2)$ [double emission] is consistent with the expansion of Ω_B

  - 2, $(\pi\alpha/\beta_{D\ell})^n$ from the photon ladder [cf. scalar-fermion bound state Hryczuk '11]



$\Gamma \to \Omega_C \Gamma$ Only neutral B-meson channel

$$\Omega_C = \Pi_{\{0<i<j\}} \frac{2\pi\alpha Q_i Q_j}{\beta_{ij}} \frac{1}{e^{\frac{2\pi\alpha_{ij} Q_j}{\beta_{ij}}} - 1} = \frac{2\pi\alpha}{\beta_{D\ell}} \frac{1}{1 - e^{\frac{-2\pi\alpha}{\beta_{D\ell}}}}$$

Important in τ channel
$\beta_{D\tau} = \mathcal{O}(0.5)$

Corresponds to the Sommerfeld enhancement factor [Sommerfeld '31]

$E_{\max}$ dependence

- In τ mode, $E_{\max}$ dependence is suppressed by τ non-relativistic velocity

$$\Omega_B = \left(\frac{2E_{\max}}{\sqrt{m_i m_j}}\right)^{-\frac{2\alpha}{\pi}(1-2b_{ij})} = \left(\frac{2E_{\max}}{\sqrt{m_i m_j}}\right)^{-\frac{2\alpha}{\pi}\left(1-\frac{1}{2\beta_{ij}}\ln\frac{1+\beta_{ij}}{1-\beta_{ij}}\right)}$$

Relativistic region

$$= 1+\frac{\alpha}{\pi}\left(-2+\ln\frac{2}{\epsilon}\right)\ln\left(\frac{2E_{\max}}{\sqrt{m_i m_j}}\right)+\mathcal{O}(\alpha\epsilon\ln\epsilon) \qquad \beta_{ij}=1-\epsilon,\ \epsilon\ll 1,\ \epsilon=\mathcal{O}\left(\frac{m_\ell^2}{E_\ell^2}\right)$$

Collinear singularity : $\ln m_\ell$

Non-rela region

$$= 1+\frac{2\alpha}{3\pi}\beta_{ij}^2\ln\left(\frac{2E_{\max}}{\sqrt{m_i m_j}}\right)+\mathcal{O}(\alpha\beta_{ij}^4) \qquad \beta_{ij}^2 \ll 1$$

Suppressed by the non-real velocity

Result

[de Boer, TK, Nisandzic, '18]

$$\frac{d^2\Gamma}{dq^2 ds_{D\ell}} = \frac{d^2\Gamma_0}{dq^2 ds_{D\ell}} \Omega_B^{D^+} \Omega_C \left[1 + \frac{\alpha}{\pi}(F_D + F_\ell - 2F_{D\ell} - 2H_{D\ell})\right] + \frac{\alpha}{\pi} \frac{d^2\tilde{\Gamma}^{D^+}}{dq^2 ds_{D\ell}},$$

with $\Omega_B^{D^+} = \left(\frac{2E_{\max}}{\sqrt{m_D m_\ell}}\right)^{-\frac{2\alpha}{\pi}(1-2b_{D\ell})}, \quad \Omega_B^{D^0} = \left(\frac{2E_{\max}}{\sqrt{m_B m_\ell}}\right)^{-\frac{2\alpha}{\pi}(1-2b_{B\ell})}, \quad \Omega_C = -\frac{2\pi\alpha}{\beta_{D\ell}} \frac{1}{e^{-\frac{2\pi\alpha}{\beta_{D\ell}}} - 1}$

$$F_i = \frac{1}{2\beta_{Bi}} \ln \frac{1+\beta_{Bi}}{1-\beta_{Bi}},$$

$$F_{D\ell} = \frac{1}{2} \frac{m_D m_\ell}{\sqrt{1-\beta_{D\ell}^2}} \int_0^1 dz \frac{E(z)}{P(z)\,[E(z)^2 - P(z)^2]} \ln \frac{E(z)+P(z)}{E(z)-P(z)},$$

$$F_{B\ell} = \frac{1}{4\beta_{B\ell}} \left\{ \mathrm{Li}_2\left(\frac{1-\beta_{B\ell}}{2}\right) - \mathrm{Li}_2\left(\frac{1+\beta_{B\ell}}{2}\right) + 4\mathrm{Li}_2(\beta_{B\ell}) - \mathrm{Li}_2(\beta_{B\ell}^2) + \ln 2 \ln \frac{1+\beta_{B\ell}}{1-\beta_{B\ell}} + \frac{1}{2}\ln^2(1-\beta_{B\ell}) - \frac{1}{2}\ln^2(1+\beta_{B\ell}) \right\},$$

$$H_{ij} = -\frac{1}{2\beta_{ij}} \left\{ \frac{1}{2}\ln^2 \frac{m_i}{m_j} - \frac{1}{8}\ln^2 \frac{1+\beta_{ij}}{1-\beta_{ij}} - \frac{1}{2}\ln^2 \left|\frac{\Delta_{ij}^i + \Delta_{ij}\beta_{ij}}{\Delta_{ij}^j + \Delta_{ij}\beta_{ij}}\right| - \mathrm{Li}_2\left(\frac{2\Delta_{ij}\beta_{ij}}{\Delta_{ij}^i + \Delta_{ij}\beta_{ij}}\right) - \mathrm{Li}_2\left(\frac{2\Delta_{ij}\beta_{ij}}{\Delta_{ij}^j + \Delta_{ij}\beta_{ij}}\right) \right\}$$
$$+ \frac{1}{4}\ln \frac{m_i m_j}{\mu^2} - \frac{1}{2} - \frac{m_i^2 - m_j^2}{4s_{ij}} \ln \frac{m_i}{m_j} - \frac{1}{4}\Delta_{ij}\beta_{ij} \ln \frac{1+\beta_{ij}}{1-\beta_{ij}} - \frac{\Delta_{ij}}{2} \ln \frac{m_i}{m_j} - \frac{\Delta_{ij}^i}{4\beta_{ij}} \ln \frac{1+\beta_{ij}}{1-\beta_{ij}},$$

where

$$b_{i\ell} = \frac{1}{4\beta_{i\ell}} \ln \frac{1+\beta_{i\ell}}{1-\beta_{i\ell}}, \quad \beta_{D\ell} = \left[1 - \frac{4m_D^2 m_\ell^2}{(s_{D\ell} - m_D^2 - m_\ell^2)^2}\right]^{\frac{1}{2}}, \quad \beta_{B\ell} = \left(1 - \frac{m_\ell^2}{E_\ell^2}\right)^{\frac{1}{2}}, \quad E_\ell = \frac{s_{D\ell} + q^2 - m_D^2}{2m_B},$$

$$\Delta_{ij} = \frac{s_{ij} - m_i^2 - m_j^2}{2s_{ij}}, \quad \Delta_{ij}^{i,j} = \frac{s_{ij} + m_{i,j}^2 - m_{j,i}^2}{2s_{ij}},$$

Result cont. [de Boer, TK, Nisandzic, '18]

$$\frac{d^2\Gamma}{dq^2 ds_{D\ell}} = \frac{d^2\Gamma_0}{dq^2 ds_{D\ell}} \Omega_B^{D^+} \Omega_C \left[1 + \frac{\alpha}{\pi}\left(F_D + F_\ell - 2F_{D\ell} - 2H_{D\ell}\right)\right] + \frac{\alpha}{\pi}\frac{d^2\tilde{\Gamma}^{D^+}}{dq^2 ds_{D\ell}},$$

and the non-factorizable contributions are

$$\begin{aligned}
\frac{\alpha}{\pi}\frac{d^2\tilde{\Gamma}^{D^+}}{dq^2 ds_{D\ell}} =& \frac{G_F^2 |V_{cb}|^2}{512\pi^3 m_B^3}\frac{\alpha}{\pi} \\
&\times \Bigg[\Big\{\left[f_+(q^2)\right]^2 \left[m_B^2\left(4m_D^2 - m_\ell^2\right) + m_D^2\left(m_\ell^2 - q^2 - 4s_{D\ell}\right) + q^2 s_{D\ell}\right] \\
&\quad + \left[f_-(q^2)\right]^2 \left[-m_B^2 m_\ell^2 + m_D^2\left(m_\ell^2 - q^2\right) + q^2 s_{D\ell}\right] + 2f_+(q^2) f_-(q^2)\left[-m_B^2 m_\ell^2 - m_D^2\left(m_\ell^2 - q^2\right) + q^2 s_{D\ell}\right]\Big\} \\
&\times \frac{8m_\ell^2}{s_{D\ell}\Delta_{D\ell}}\left[\frac{\Delta_{D\ell}}{2}\ln\frac{m_\ell}{m_D} - \frac{\Delta_{D\ell}^D}{4\beta_D k}\ln\frac{1+\beta_{D\ell}}{1-\beta_{D\ell}}\right] \\
&+ \Big\{Q_\ell\left\{Q_B\left[f_+(q^2) + f_-(q^2)\right] + Q_D\left[f_+(q^2) - f_-(q^2)\right]\right\}\left(3\ln\frac{m_\ell^2}{\mu^2} - 4\right) \\
&\quad \times 2m_\ell^2\left[f_+(q^2)\left(2m_B^2 + m_\ell^2 - q^2 - 2s_{D\ell}\right) + f_-(q^2)\left(q^2 - m_\ell^2\right)\right] \\
&+ Q_D^2\left[f_+(q^2) - f_-(q^2)\right]\left(\frac{3}{2}\ln\frac{m_D^2}{\mu^2} - \frac{7}{2}\right) \\
&\quad \times \left\{-2f_+(q^2)\left[m_B^2\left(2m_D^2 + m_\ell^2 - 2s_{D\ell}\right) + 2s_{D\ell}\left(-m_D^2 + q^2 + s_{D\ell}\right) + m_\ell^4 - m_\ell^2\left(q^2 + 3s_{D\ell}\right)\right]\right. \\
&\quad \left. + 2f_-(q^2)\, m_\ell^2\left(m_B^2 + m_\ell^2 - q^2 - s_{D\ell}\right)\right\} \\
&+ Q_B^2\left[f_+(q^2) + f_-(q^2)\right]\left(\frac{3}{2}\ln\frac{m_B^2}{\mu^2} - \frac{7}{2}\right) \\
&\quad \times \left\{2f_+(q^2)\left\{m_B^2\left(-2m_D^2 + m_\ell^2 + 2s_{D\ell}\right) + s_{D\ell}\left[2m_D^2 + m_\ell^2 - 2\left(q^2 + s_{D\ell}\right)\right]\right\} + 2f_-(q^2)\, m_\ell^2\left(m_B^2 - s_{D\ell}\right)\right\}\Big\}
\end{aligned}$$

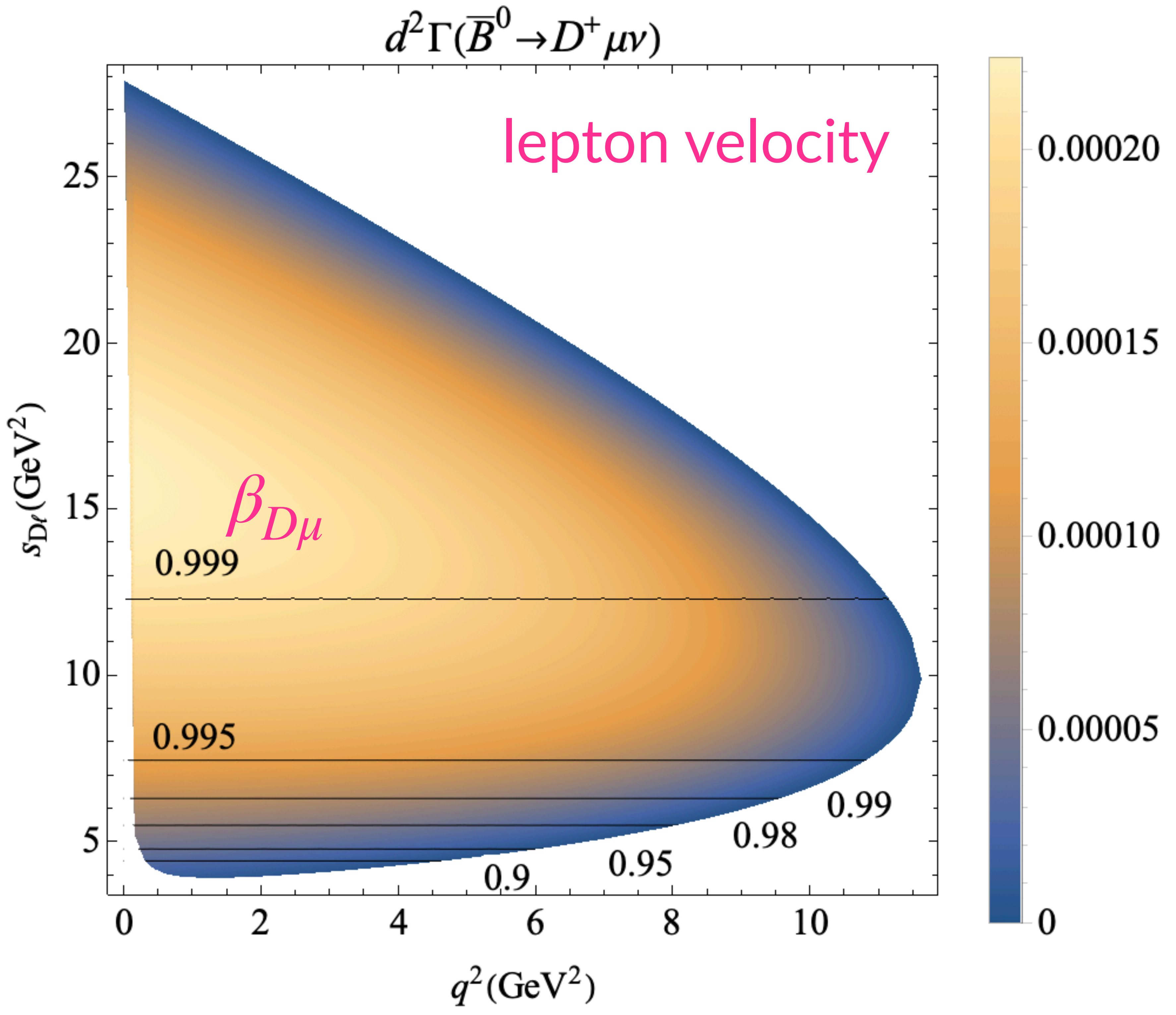
Contents

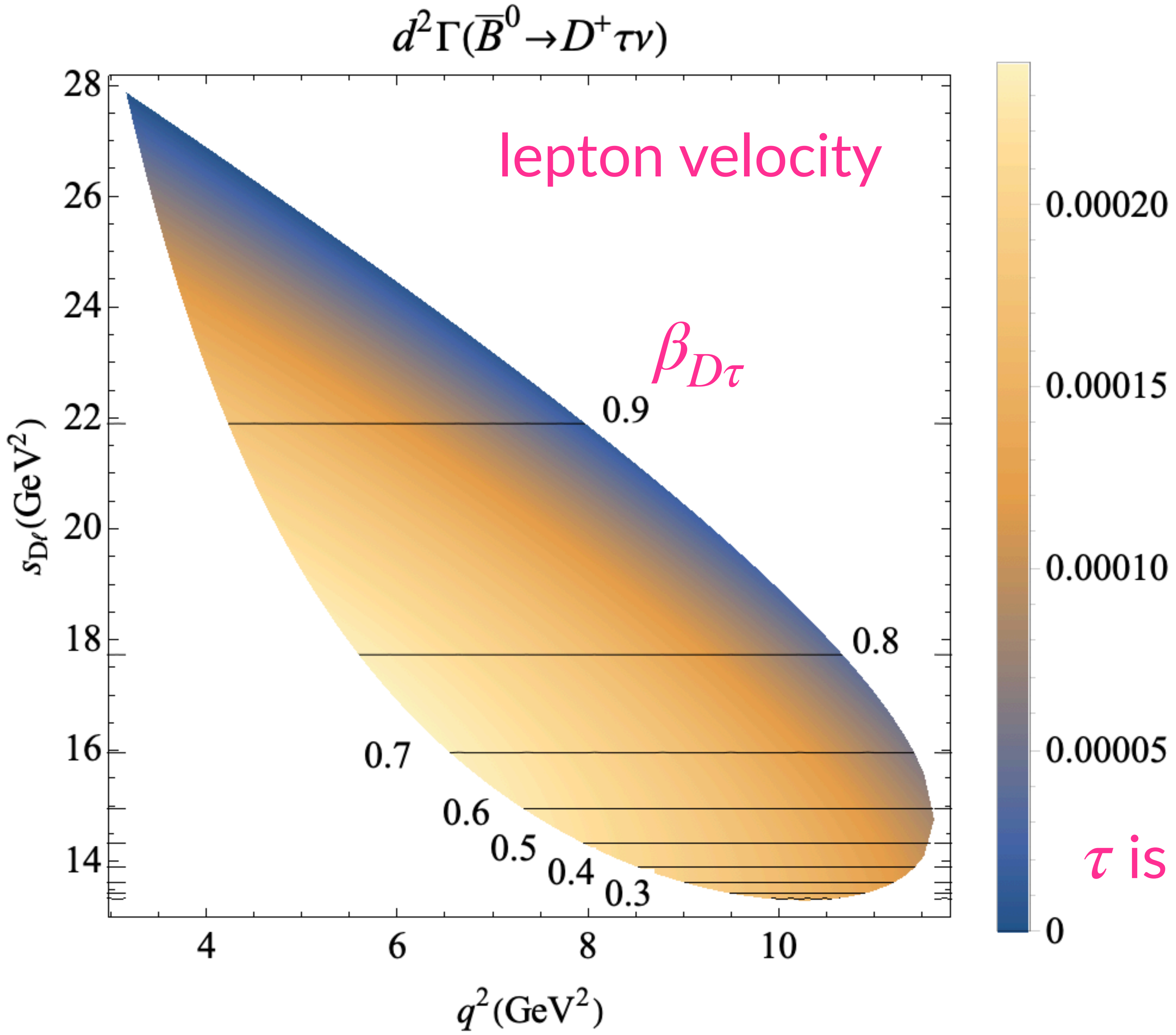
Dalitz phase space

- We obtain analytic long-distance QED corrections to $\Gamma(B^{0,-}\to D^{+,0}\mu\nu)$ and $\Gamma(B^{0,-}\to D^{+,0}\tau\nu)$ as a function of **three** independent parameters: $\boldsymbol{E_{max}}$ and **2 Dalitz variables,** $q^2=(p_B-p_D)^2$, $s_{D\ell}=(p_D+p_\ell)^2$

Double differential decay distributions without QED corrections

μ is relativistic



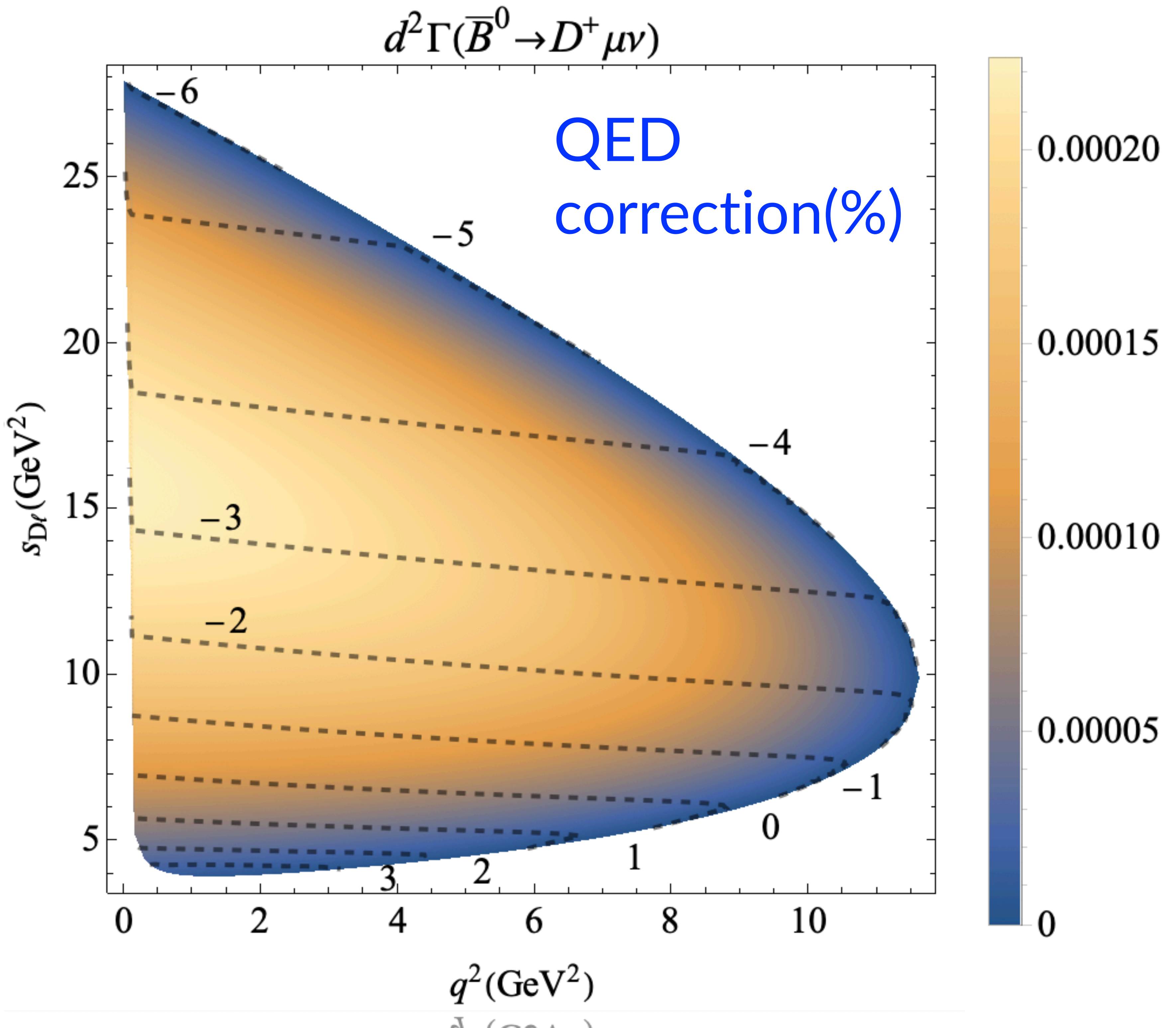


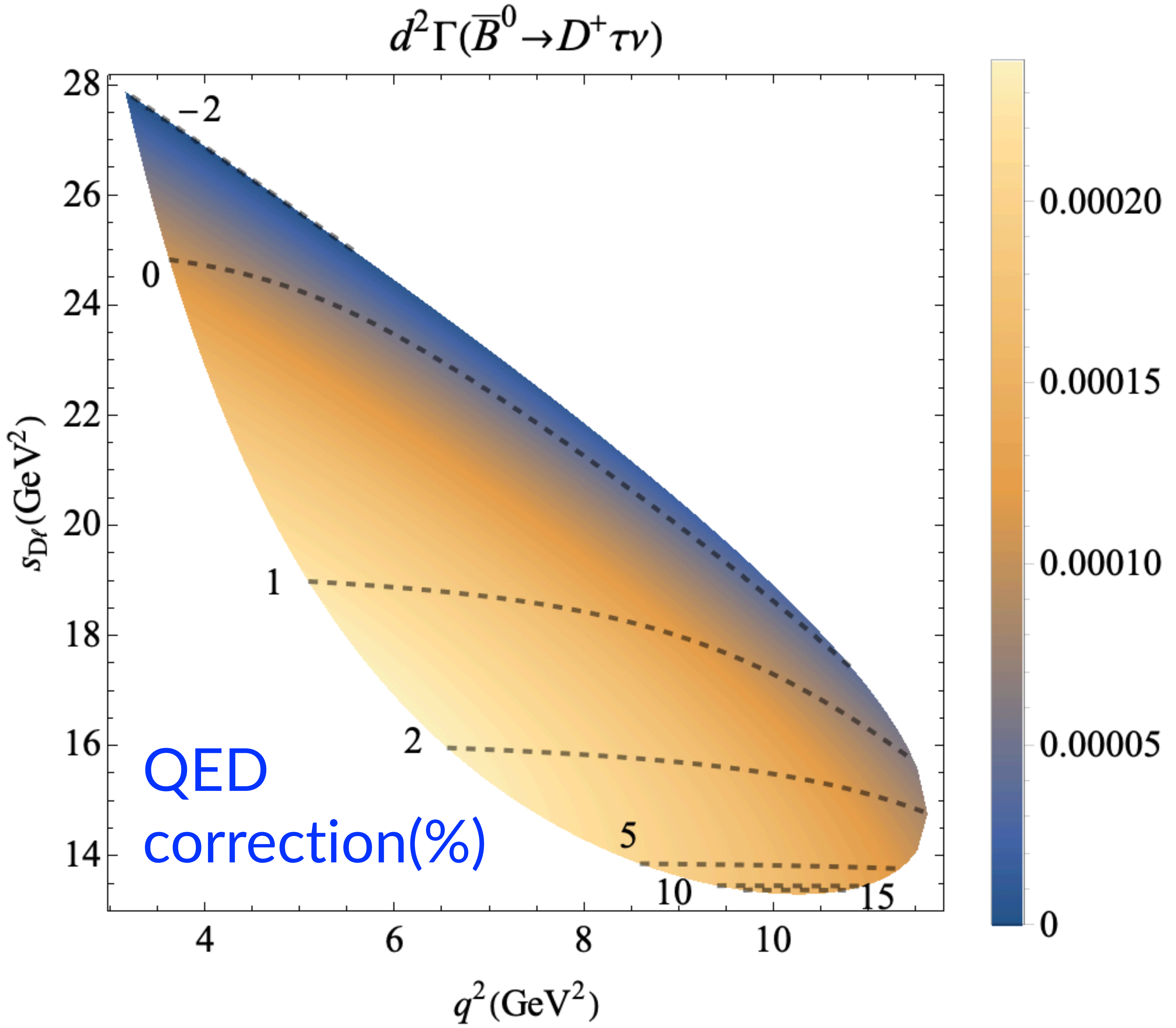
τ is non-relativistic

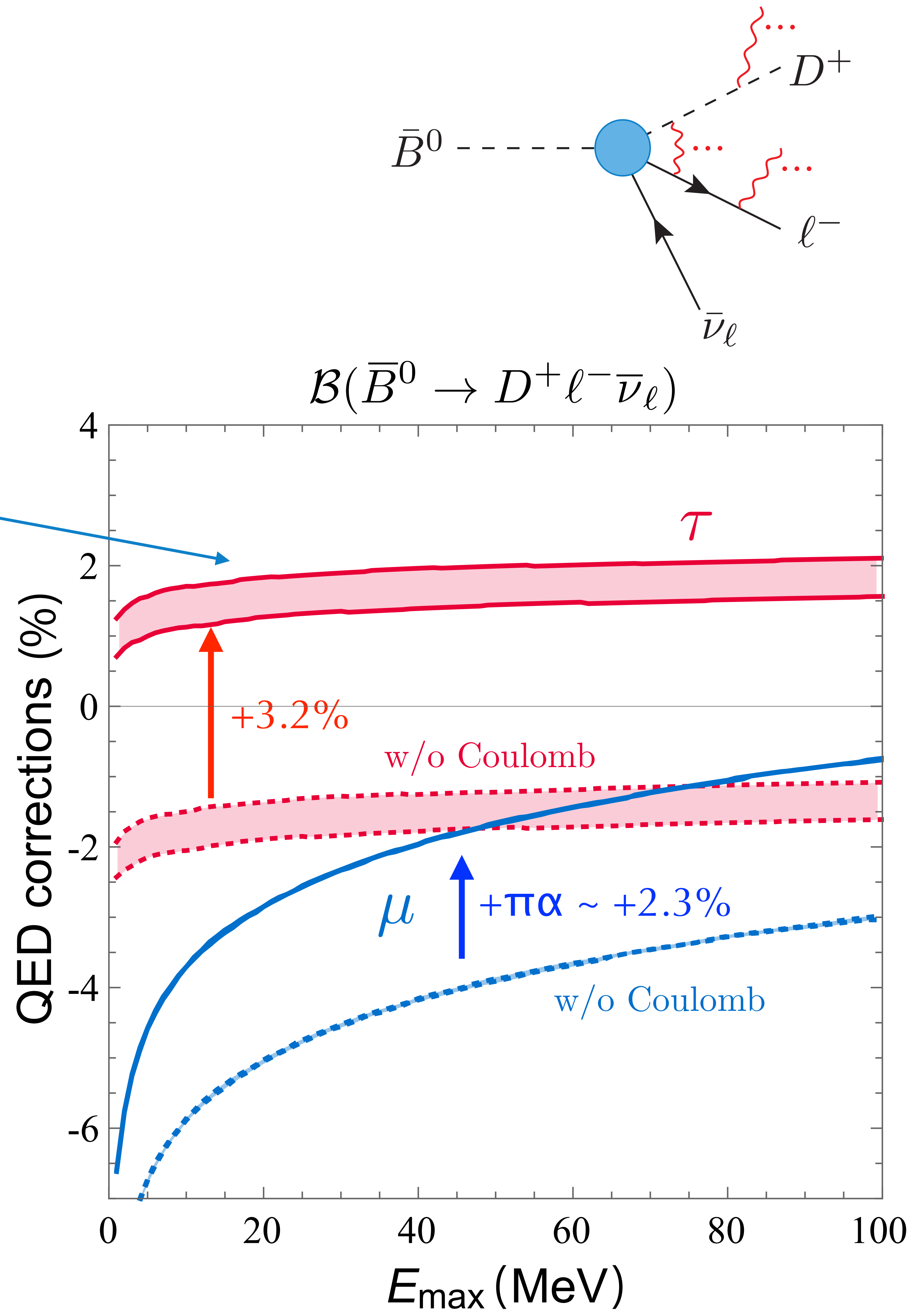
Dalitz phase space

- We obtain analytic long-distance QED corrections to $\Gamma(B^{0,-}\to D^{+,0}\mu\nu)$ and $\Gamma(B^{0,-}\to D^{+,0}\tau\nu)$ as a function of **three** independent parameters: $\boldsymbol{E_{\max}}$ and **2 Dalitz variables,** $q^2 = (p_B - p_D)^2$, $s_{D\ell} = (p_D + p_\ell)^2$

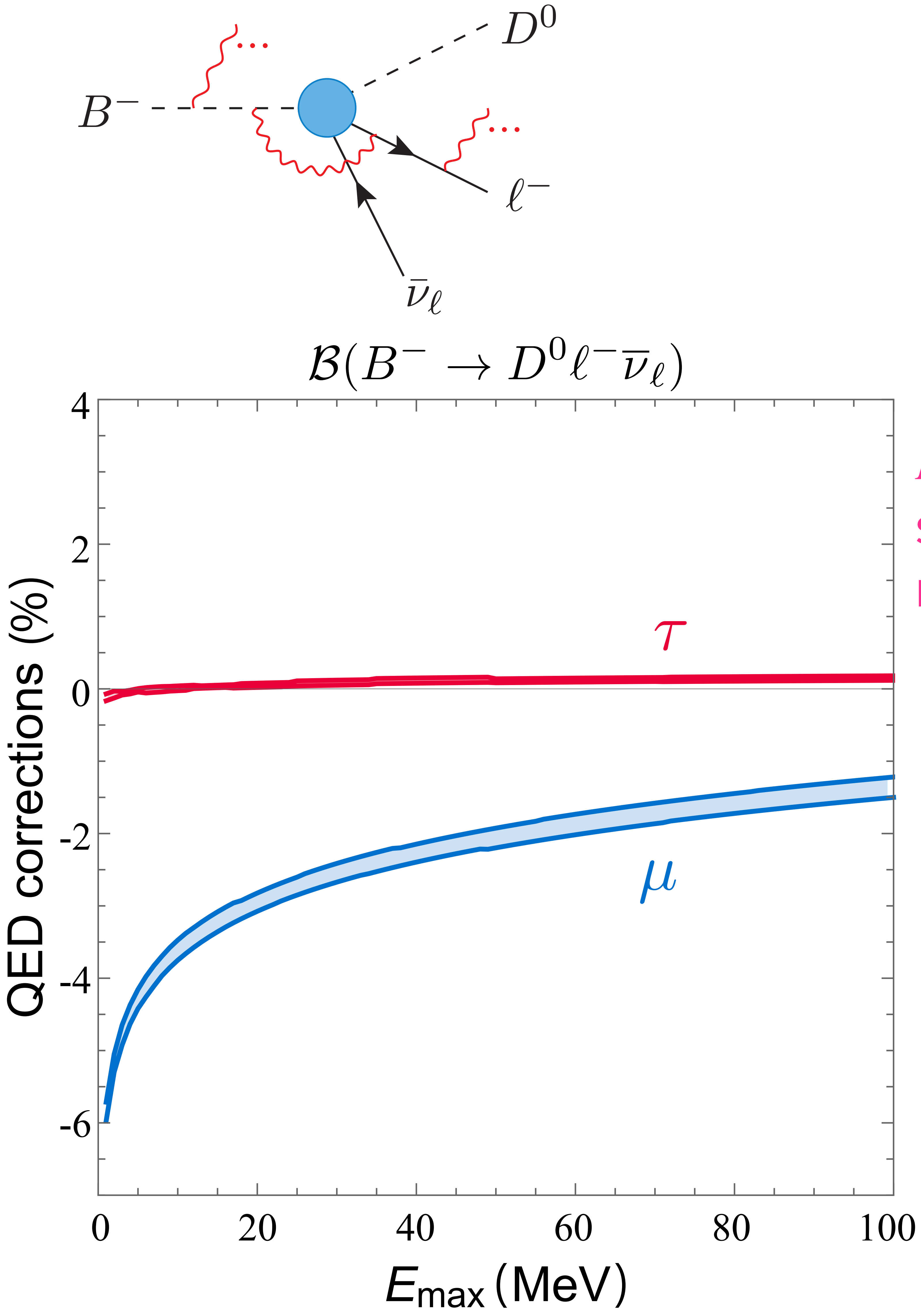
Long-distance QED corrections (%) @ $E_{\max}$= 20 MeV





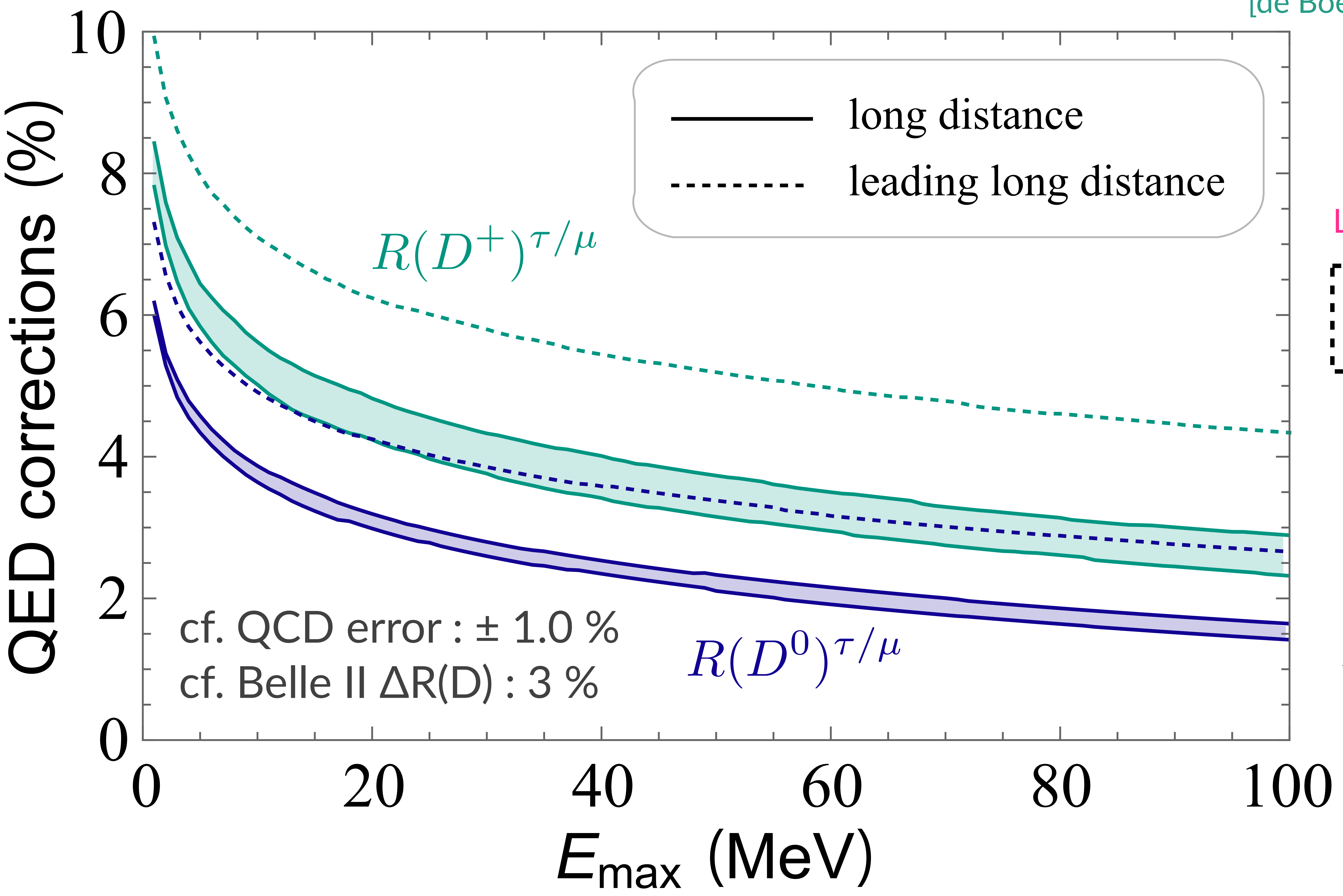


renormalization scale
100MeV < μ < 1GeV



E_{max} dependence is suppressed by the non-rela velocity

[de Boer, TK, Nisandzic, '18]



Leading long distance

Real emissions + $I_{\mathrm{low}k_l}$ + self energy

= μ-independent, spin-independent

renormalization scale

100MeV < μ < 1GeV

We conclude that the QED corrections to $R(D^+)$ and $R(D^0)$ are different at 1-1.5%

$B^0 \to D^+ \ell^- \nu$

Naive size of QED corrections ~ O(α/π) ~ 0.2%

QED corrections (%)

$B^0 \to D^+ \ell^- \nu$	Ω_B (E_{max}=20MeV)	Ω_C	F_D	F_ℓ	$F_{D\ell}$	$H_{D\ell}$ (k_l=0)	$H_{D\ell}$ (k_l>0) (μ=200MeV)	IB-loop (μ=200MeV)	TOTAL
Br(τ)	-0.72	3.22	0.26	0.27	-0.65	-0.64	-0.10	-0.10	1.41
Br(μ)	-3.14	2.31	0.28	0.73	-3.10	-1.40	1.25	0.24	-2.84
Br(e)	Our formalism suffers from large $\ln m_e/M$, $(\ln m_e/M)^2$								
R(D^+) τ/μ	**2.50**	**0.89**	-0.02	-0.45	**2.52**	**0.77**	**-1.33**	-0.34	**4.38**

$$\approx \alpha \log\left(\frac{E_{max}^2}{m_D m_\mu}\right)$$

$$\approx \frac{\alpha}{\pi} \frac{\log^2(1-v_{rel})}{4 v_{rel}}$$

$$\approx \frac{\alpha}{2\pi} \log^2 \frac{m_\tau}{m_\mu}$$

ΩB: log(E_{max}) contributions from full real emissions
ΩC: Coulomb correction
FD: finite terms [=$O(E^0_{max})$] of real emission from D^+
Fℓ: finite terms of real emission from ℓ
FDℓ: finite terms of interference between real emissions from D^+ and ℓ
HDℓ: loop correction between D^+ and ℓ
IB-loop: loop correction containing Inner-Bremsstrahlung vertex

$B^- \to D^0 \ell^- \nu$

Naive size of QED corrections ~ $O(\alpha/\pi)$ ~ 0.2%

QED corrections (%)

$B^- \to D^0 \ell^- \nu$	Ω_B (E_{max}= 20MeV)	Ω_C	F_B	F_ℓ	$F_{B\ell}$	$H_{B\ell}$ (k_l=0)	$H_{B\ell}$ (k_l>0) (μ=200MeV)	IB-loop (μ=200MeV)	TOTAL
Br(τ)	-0.22	-	0.23	0.27	-0.53	0.03	-0.09	0.37	0.03
Br(μ)	-2.93	-	0.23	0.74	-2.79	0.35	1.11	0.19	-2.98
Br(e)	Our formalism suffers from large $\ln m_e/M$, $(\ln m_e/M)^2$								
R(D^0) τ/μ	**2.79**	-	0.00	-0.46	**2.32**	-0.31	**-1.19**	0.18	**3.11**

$$\approx \alpha \log\left(\frac{E_{max}^2}{m_B m_\mu}\right)$$

$$\approx \frac{\alpha}{\pi} \frac{\log^2(1-v_{rel})}{4 v_{rel}}$$

$$\approx \frac{\alpha}{2\pi} \log^2 \frac{m_\tau}{m_\mu}$$

ΩB: log(E_{max}) contributions from full real emissions
FB: finite terms [=$O(E^0_{max})$] of real emission from B^-
Fℓ: finite terms of real emission from ℓ
FBℓ: finite terms of interference between real emissions from B^- and ℓ
HBℓ: loop correction between B^- and ℓ
IB-loop: loop correction containing Inner-Bremsstrahlung vertex

PHOTOS MC simulation

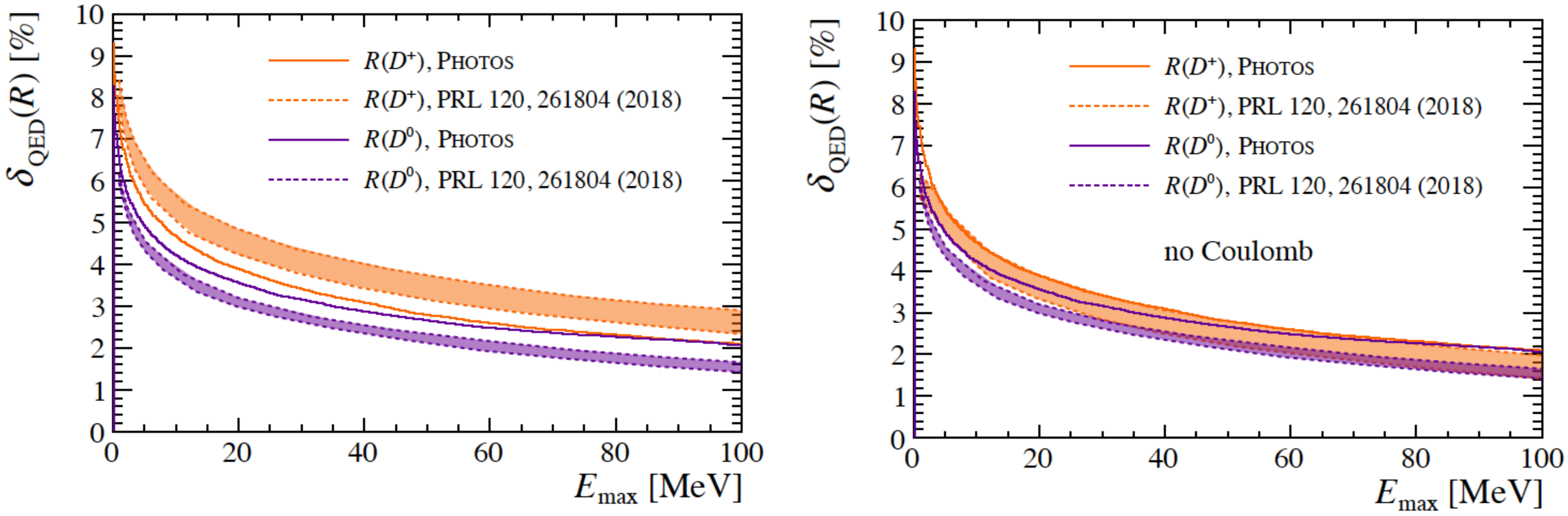
[Barberio, Eijk, Was, ’91; Barberio, Was, ‘94; Davidson, Przedzinski, Was ’16]

- PHOTOS Monte-Carlo generator can simulate modifications of the kinematic variables **induced by final-state photon radiations (not initial-state one)** →**Talk by Zbigniew Was**
- PHOTOS is utilized in Belle (v2.02) /BaBar (v2.13)/LHCb (v3.56) for B semileptonic decay search
- For general decay processes, PHOTOS can simulate final-state radiation in the leading-logarithmic collinear approximation
  - **All** virtual corrections **including Coulomb pole** are **not covered** in PHOTOS
  - **Quantum interference in emissions** are **not covered** in PHOTOS (**< ver. 2.07(single), 2.13 (multiple)**)
  - LHCb analysis *does* **include the final-state radiation interference**

Crosscheck by PHOTOS

- Part of LHCb colleagues have checked the soft-photon correction by PHOTOS v.3.56

[Calí, Klaver, Rotondo, Sciascia, 1905.02702]

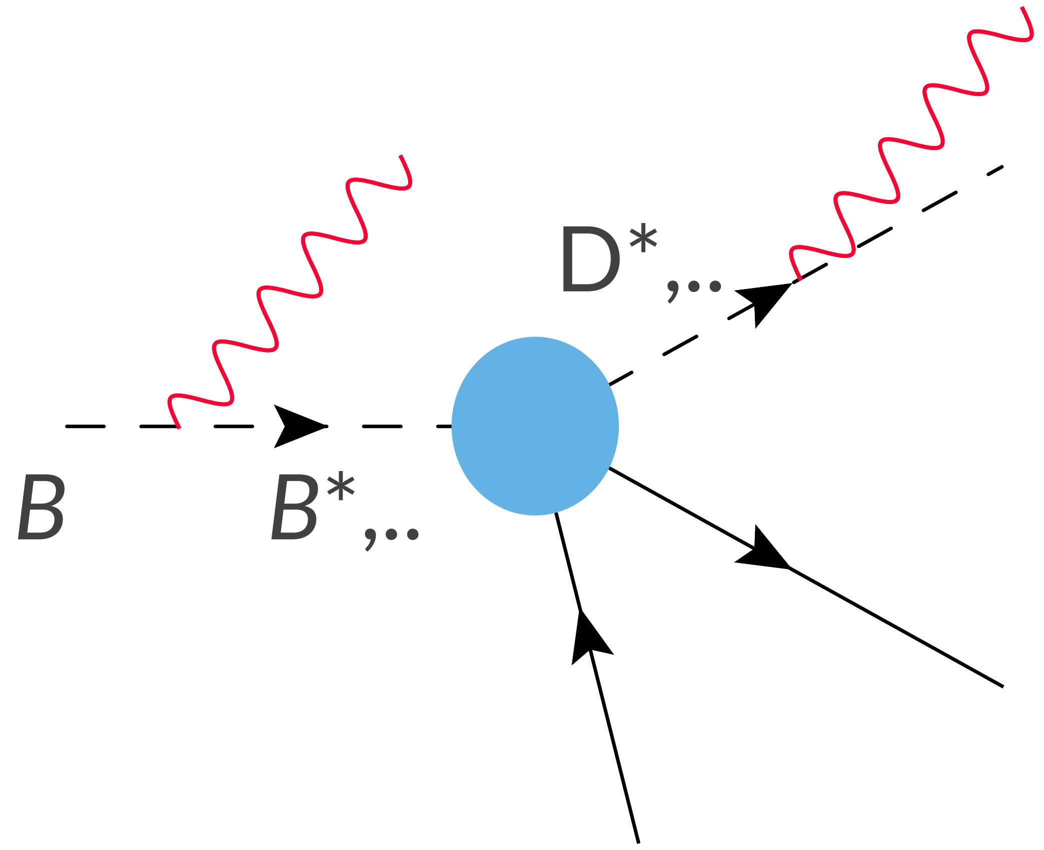


- Leading LFU-violating contribution is reproduced by PHOTOS → Talk by Barbara Sciascia

- The small gap comes from **virtual (Coulomb) correction** which is absent in PHOTOS

Missing contributions

- Structure-dependent radiations [Becirevic, Kosnik ’10, Bernlochner, Schonherr ’10] **Talk by Nazario Tantalo and Florian Bernlochner**



Depend on model for excited states

No radiation in the soft-photon region

Hard radiation could be dominated by the structure-dependent one [e.g., $B_s \to \mu^+\mu^-\gamma$ Dettori, Guadagnoli, Reboud ’17]

- $\mathcal{O}(k^0)$ **contributions** (suppressed by $\mathcal{O}(E_\gamma/M)$) from real emissions

- Soft lepton with soft (and collinear) photon which live in **four-body phase space integral**

- Electron suffers from collinear singularity $\ln m_e/M$, $(\ln m_e/M)^2$. Is collinear cutoff required?

Conclusions

- We analytically evaluated soft-photon corrections to $B \to D\tau\nu$ and $B \to D\mu\nu$ using the soft-photon approx.
- Soft-photon corrections depend on lepton's kinematics: **mass** and **velocity** and hence **can violate lepton flavor universality**, which is **larger than the QCD uncertainty** of form factors
- PHOTOS v.3.56 numerically reproduces the LFU-violating QED corrections to $R(D^{0,+})_{\rm SM}$

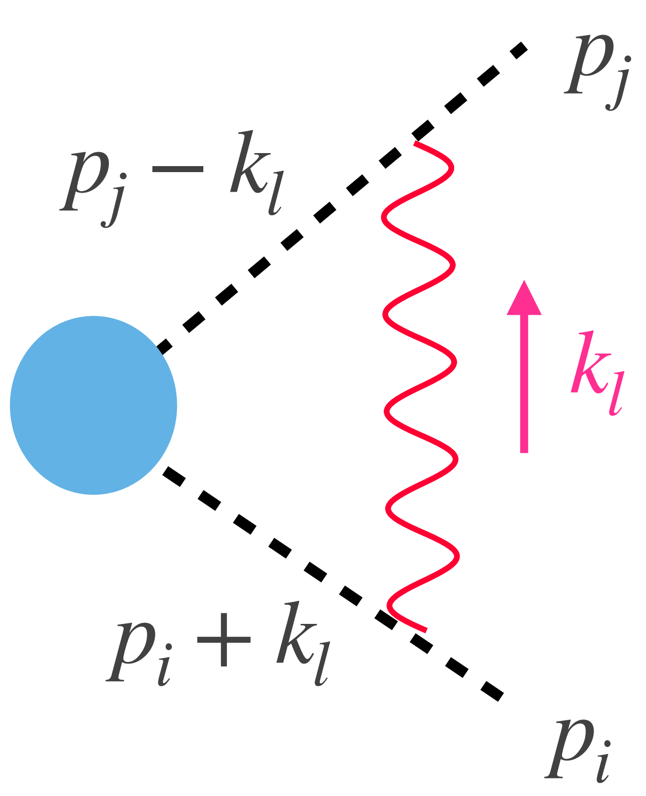
Outlook

- Beyond soft-photon approximation (electron mode, 4-body phase space)
- Soft-photon corrections to $B \to D^*\ell\nu$ [$R(D^*)$], exclusive $|V_{cb}|$?

Back up

Weinberg virtual corrections

- Weinberg integral for spin-independent contribution: **IR and UV divergence** [Weinberg '65]



Neglecting k_l^2 in the denominator

$$\int_\lambda^\Lambda dk_l^4 \frac{-i}{(2\pi)^4} \frac{e^2 Q_i Q_j \left(p_i \cdot p_j\right)}{k_l^2 \left(2p_i \cdot k_l\right)\left(-2p_j \cdot k_l\right)} = -\frac{\alpha Q_i Q_j}{8\pi} \frac{1}{\beta_{ij}} \ln \frac{1+\beta_{ij}}{1-\beta_{ij}} \ln \frac{\lambda}{\Lambda}$$

Our integral for spin-independent contribution: IR divergence but **UV finite**

$$\int \frac{d^d k_l}{(2\pi)^d} \frac{e^2 Q_i Q_j \left(p_i \cdot p_j\right)}{\left(k_l^2 + 2p_i \cdot k_l\right)\left(k_l^2 - 2p_j \cdot k_l\right)\left(k_l^2 - m_\gamma^2\right)} \supset -\frac{\alpha Q_i Q_j}{8\pi} \frac{1}{\beta_{ij}} \ln \frac{1+\beta_{ij}}{1-\beta_{ij}} \ln \frac{m_\gamma}{\sqrt{m_i m_j}}$$

Passarino-Veltman scalar integral C_0

Related observables

- Polarization observables can distinguish between single LQ models

[Blanke, Crivellin, TK, Moscati, Nierste, Nisandzic, 1905.08253]

[Belle, 1901.06380]

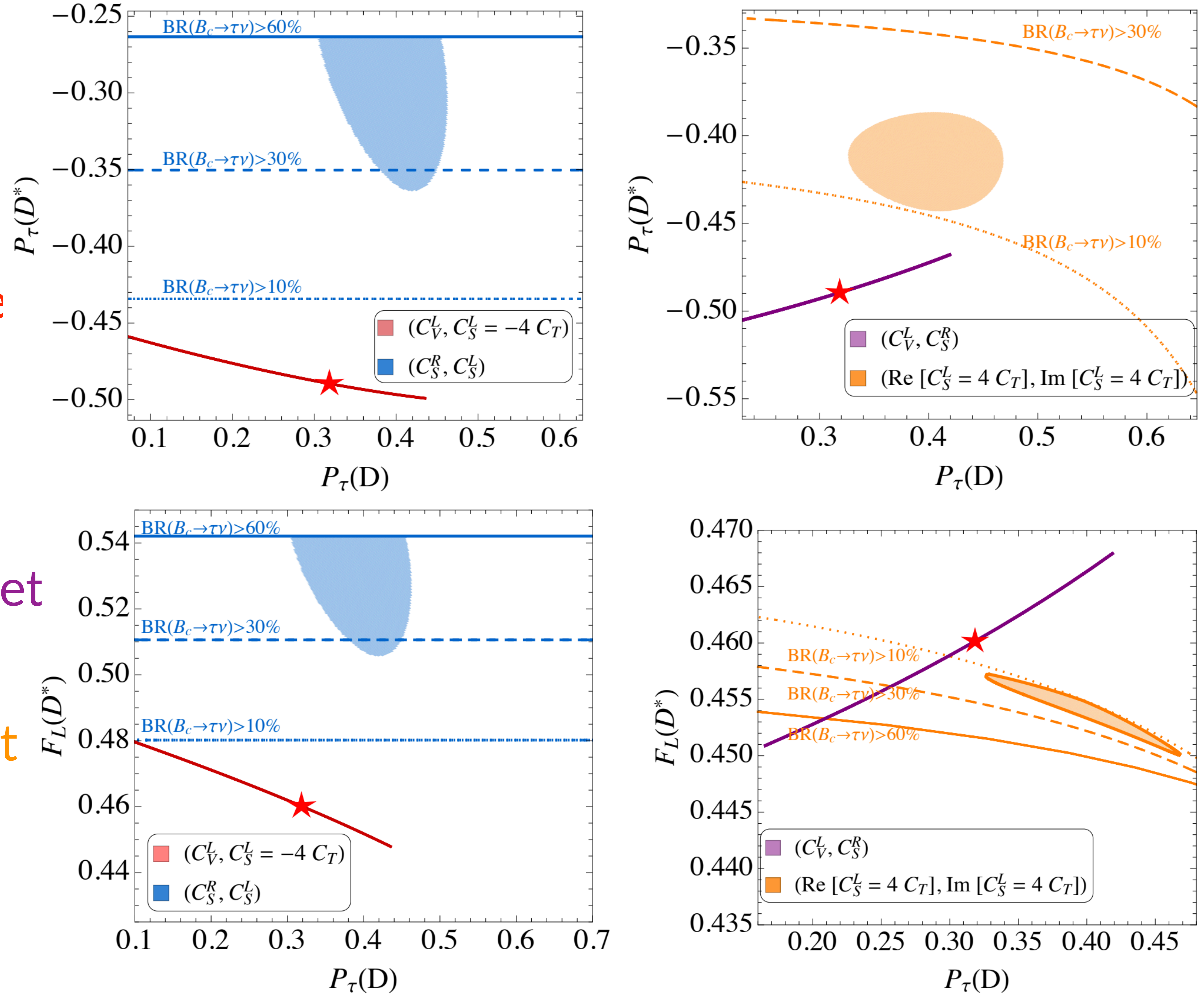
Motivated NP models

Scalar Leptoquark S1: SU(2) single

Generalized Charged Higgs

Vector Leptoquark U1: SU(2) singlet

Sclar Leptoquark R2: SU(2) doublet



$P\tau(D^{(*)})$: τ polarization asymmetry

$\Delta P\tau(D)$: 3%

$\Delta P\tau(D^*)$: 0.07

[Belle-II sensitivity, 50 ab-1]

$F_L(D^*)$: D^* longitudinal polarization ratio

$\Delta F_L(D^*)$: 0.04

[Belle-II sensitivity, 50 ab-1]