QED corrections and systematic uncertainties from the viewpoint of a Monte Carlo

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1. High precision multi-bilion samples of B, τ, K ... decays are (will be soon) collected.
2. Challenge: experimental precison way better than what theory can provide.
3. It is essential to separate description into segments and keep quality.
4. Even, if for the time and some work segments, it may look unjustified.
5. Work segments definition will vary with precision : · perturbative expansion · phase space · detector response · production process · resonance decays and bremsstrahlung in decays · beamstrahlung, PDFs, parton showers, bremsstrahlung isr-fsr-ifi · how to use the programs.
6. My talk is about one of these segments; is it bound to be a pedagogical failure?
7. I apologize for too many details on some segments and too little on the others.

QED corrections and systematic uncertainties from the viewpoint of a Monte Carlo

bremsstrahlung in decays

1. Is it useful to isolate such simulation segment, is it feasible?
2. How reliable it can be? 5%, 1%, 0.1%? and what these % actually mean?
3. QED is a QFT: predictions may rely on Phase space $\times$ matrix element.
4. Scalar QED has limitations, non-pointlike mesons require electromagnetic form-factors.
5. Sub-topics: · perturbative expansion · incoming resonance spin state · approximating theory · infrared cancellations · manifolds triangulation, CW complexes · algorithms design, · numerical stability · numerical tests precision and comparisons with other programs · how to use the programs · benchmarks.
6. My 35 years with mathematical, perturbative, Monte Carlo, numerical issues.
7. It all started in Marseille and Lozére in fact by an accident.

Web page: http://photospp.web.cern.ch/photospp/

1) Why?

Bremsstrahlung in decays

• Physics goal: clear observables like CP sensitive elements of quark mixing matrix e.g. V_{ub}, W mass, or Higgs decay branching ratios.

• In every decay and in every event bremsstrahlung is unavoidable. It follows rules and, is *rather* process independent. (Beware of photons which are messengers from internal states of strong interaction.)

• Bremmstrahlung represent simple corrections for 'theoretical observables' ...
... which grow in importance and complexity with experimental details and precision.

• It complicates detector responses: μ/e with collinear photon gives different detector response than lepton alone.

• Complication: ignore for the observable design $\rightarrow$ include in realistic simulation $\rightarrow$ sometimes redefine observables or prepare QED corrected data.

• Example from Belle, Phys. Lett. B621 (2005) 28: V_{ub} signature; lepton spectrum in semileptonic B decays π. In general, large background of K's does not contribute, its endpoint is lower than for π but bremmstrahlung push it up... See slide 67 and other talks.

2) How?

Quest for precision

- **For the 5% physics precison we may need 1% level precision for matrix elements and 0.1 % technical precision.**

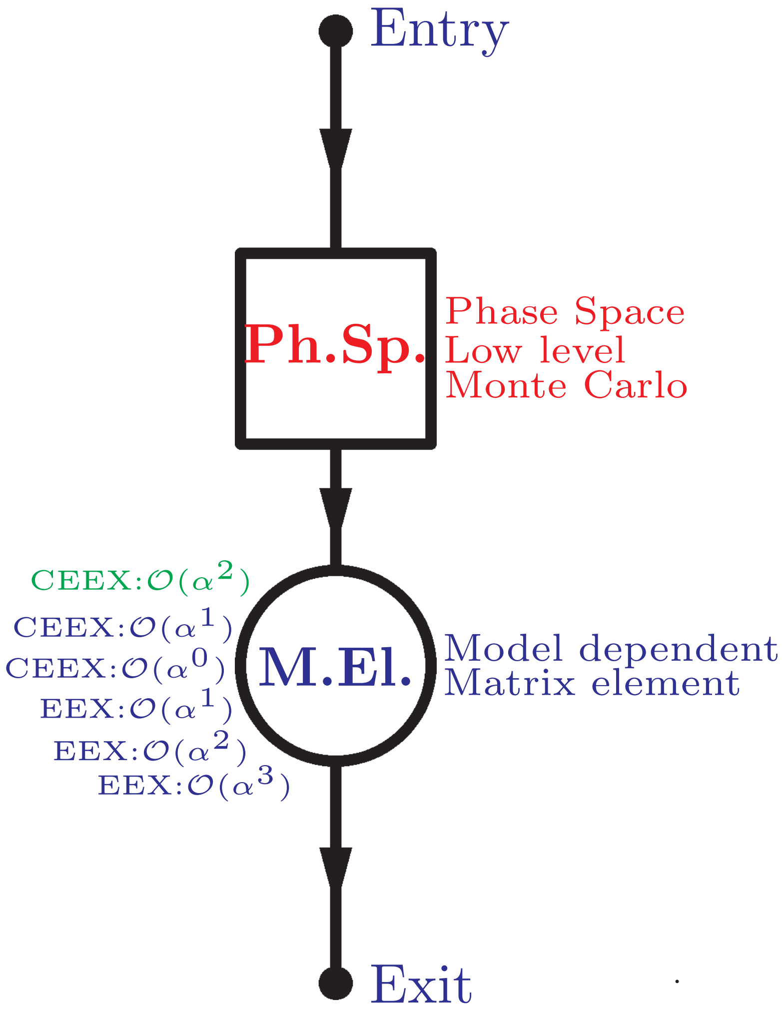
Why it is so and what does it mean?

- Only when Monte Carlo has 0.1 % technical precison simulation say at 0.3% statistical error level is meaningful and useful to discuss matrix elements at 1 % precision. Technical precision: verify if results follow design to that level.
- 1 % precision level matrix elements mean precision of matrix element implementation, good example is **infrared cut off**; below this value soft phothons are integrated over and combined with virtual corrections.
- That limitation required years to overcome.
- The 5% precision of matrix elements evaluated from pert. series convergence, may be not the final word. ME foundations may be ambiguous.

- Precision steps: 0.05% $\to$ 0.1% $\to$ 0.3% (or better 0.01% $\to$ 0.03% $\to$ 0.1%)

3) If QFT we can do: *phase space* × *matrix element design*

Textbook principle “matrix element × full phase space”



- Phase-space Monte Carlo module producing “raw events”.
- Library of “models” provides input for tests of physics approximations
- **Phase-space generation must be treated with the great care. Any simplification lead sooner or later to massive progress slow down.**
- Extra benefits: statistically correlated samples useful for tracking small effect,
- convenient for Machine Learning applications.

- The fully differential distribution from MUSTRAAL Nucl.Phys. B202 (1982) 63 (used also in KORALZ Comput.Phys.Commun. 66 (1991) 276 for single FSR photon mode) $e^+(p^+)e^-(p^-) \to \mu^+(q^+)\mu^-(q^-)\gamma(k)$ reads:

$$X_f = \frac{Q'^2\alpha(1-\Delta)}{4\pi^2 s}s^2 \quad \left\{ \frac{1}{(k'_+ k'_-)} \left[\frac{d\sigma_B}{d\Omega}(s,t,u') + \frac{d\sigma_B}{d\Omega}(s,t',u) \right] \right\}$$

- Here:

$$\begin{aligned} s &= 2p_+ \cdot p_-, \quad & s' &= 2q_+ \cdot q_-, \\ t &= 2p_+ \cdot q_+, \quad & t' &= 2p_+ \cdot q_-, \\ u &= 2p_+ \cdot q_-, \quad & u' &= 2_- \cdot q_+, \\ k'_\pm &= q_\pm \cdot k, \quad & x_k &= 2E_\gamma/\sqrt{s} \end{aligned}$$

- The Δ term is responsable for final state mass dependent terms, p_+, p_-, q_+, q_-, k denote four-momenta of incoming positron, electron beams, outcoming muons and bremsstrahlung photon.

- The above formula, where one can clearly see separation into emision factor and born like factor is valuable because:
  - phase space matrix element factors separation
  - it is fully differential
  - appropriate phase space parametrization of old school geometrical approach.
- YFS exponentiation as perturbation expansion re-ordering, universality of dominant terms
- lots of other results, I will refer to some of those later
- The most important was to assure, proper matching of phase space and matrix elements
- in context of approximations used for higer orders too.

3) No! Phase-space first

Goal: use n-body phase space as source of random numbers for n+1

After some meanders in 80's PHOTOS use orthodox Lorentz-invariant phase space (*Lips*). **It must be presented.**

$$dLips_{n+1}(P) =$$
$$\frac{d^3k_1}{2k_1^0(2\pi)^3} \cdots \frac{d^3k_n}{2k_n^0(2\pi)^3} \frac{d^3q}{2q^0(2\pi)^3}(2\pi)^4\delta^4\Big(P - \sum_1^n k_i - q\Big)$$
$$= \quad d^4p\delta^4(P-p-q)\frac{d^3q}{2q^0(2\pi)^3}\frac{d^3k_1}{2k_1^0(2\pi)^3} \cdots \frac{d^3k_n}{2k_n^0(2\pi)^3}(2\pi)^4\delta^4\Big(p - \sum_1^n k_i\Big)$$
$$= \quad d^4p\delta^4(P-p-q)\frac{d^3q}{2q^0(2\pi)^3}dLips_n(p \to k_1...k_n).$$

Integration variables, the four-vector p, compensated with $\delta^4\big(p - \sum_1^n k_i\big)$, and another integration variable M_1 compensated with $\delta\big(p^2 - M_1^2\big)$ are introduced.

Exact Phase Space Fparametrization

$$dLips_{n+1}(P \to k_1...k_n, k_{n+1}) = dLips_n^{+1\ tangent} \times W_n^{n+1},$$

$$dLips_n^{+1\ tangent} = dk_\gamma d\cos\theta d\phi \times dLips_n(P \to \bar{k}_1...\bar{k}_n),$$
$$\{k_1, \dots, k_{n+1}\} = \mathbf{T}\big(k_\gamma, \theta, \phi, \{\bar{k}_1, \dots, \bar{k}_n\}\big). \quad (1)$$

1. One can verify that if $dLips_n(P)$ was exact, then this formula lead to exact parametrization of $dLips_{n+1}(P)$
2. Practical implementation: Take the configurations from n-body phase space.
3. Turn it back into some coordinate variables.
4. construct new kinematical configuration from all variables.
5. **Forget about temporary** $k_\gamma \theta \phi$. **From now on, only weight and four vectors count.**
6. A lot depend on $\mathbf{T}$. Options depend on matrix element: must tangent at singularities. Simultaneous use of several $\mathbf{T}$ is possible and necessary/convenient if more than one charge is present in final state.

Phase Space: (main formula)

If we choose

$$G_n \ : \ M^2_{2\dots n}, \theta_1, \phi_1, M^2_{3\dots n}, \theta_2, \phi_2, \dots, \theta_{n-1}, \phi_{n-1} \ \rightarrow \ \bar{k}_1 \dots \bar{k}_n \tag{2}$$

and

$$G_{n+1} \ : \ k_\gamma, \theta, \phi, M^2_{2\dots n}, \theta_1, \phi_1, M^2_{3\dots n}, \theta_2, \phi_2, \dots, \theta_{n-1}, \phi_{n-1} \ \rightarrow \ k_1 \dots k_n, k_{n+1} \tag{3}$$

then

$$\mathbf{T} = G_{n+1}(k_\gamma, \theta, \phi, G_n^{-1}(\bar{k}_1, \dots, \bar{k}_n)). \tag{4}$$

The ratio of the Jacobians form the phase space weight W_n^{n+1} for the transformation. Such solution is universal and valid for any choice of G's. However, G_{n+1} and G_n has to match matrix element, otherwise algorithm will be inefficient (factor 10^{10} ...).

In case of PHOTOS G_n's

$$W_n^{n+1} = k_\gamma \frac{1}{2(2\pi)^3} \times \frac{\lambda^{1/2}(1, m_1^2/M^2_{1\dots n}, M^2_{2\dots n}/M^2_{1\dots n})}{\lambda^{1/2}(1, m_1^2/M^2, M^2_{2\dots n}/M^2)}, \tag{5}$$

Phase Space: (multiply iterated)

By iteration, we can generalize formula (1) and add l particles:

$$dLips_{n+l}(P \to k_1...k_n, k_{n+1}...k_{n+l}) = \frac{1}{l!} \prod_{i=1}^{l} \Big[dk_{\gamma_i} d\cos\theta_{\gamma_i} d\phi_{\gamma_i} W_{n+i-1}^{n+i} \Big]$$
$$\times dLips_n(P \to \bar{k}_1...\bar{k}_n), \qquad (6)$$
$$\{k_1, \dots, k_{n+l}\} = \mathbf{T}\big(k_{\gamma_l}, \theta_{\gamma_l}, \phi_{\gamma_l}, \mathbf{T}\big(\dots, \mathbf{T}\big(k_{\gamma_1}, \theta_{\gamma_1}, \phi_{\gamma_1}, \{\bar{k}_1, \dots, \bar{k}_n\}\big) \dots\big).$$

Note that variables $k_{\gamma_m}, \theta_{\gamma_m}, \phi_{\gamma_m}$ are used at a time of the $m-$th step of iteration only, and are not needed elsewhere in construction of the physical phase space; the same is true for invariants and angles $M^2_{2\dots n}, \theta_1, \phi_1, \dots, \theta_{n-1}, \phi_{n-1} \;\to\; \bar{k}_1 \dots \bar{k}_n$ of (2,3), which are also redefined at each step of the iteration. Also intermediate steps require explicit construction of temporary $\bar{k}'_1 \dots \bar{k}'_n \dots \bar{k}'_{n+m}$, statistical factor $\frac{1}{l!}$ added.

We have got **exact distribution** of **weighted** events over $n + l$ body phase space.

Phase Space: Summary.

• We use n-body phase-space as source of random numbers for the sub-set of variables used in n+l phase spaces.

• The number of added particles l, is phase space parameter too.

— original phase space events were stripped off from matrix elements and phase space jacobians, effectively used as random numbers.

— that is why n+l phase space could be covered without any approximations.

• Any new patrix element could be introduced ...

• **... but if to a good approximation all followed factorization structure, one could obtain efficient algorithm for the replacement of n-body decay with n+l body configuration of l extra photons.**

$\mathcal{P}$ase $\mathcal{S}$pace: Trivial link to evolution equations used in benchmarks

- if c=0, $\lambda^{1/2}(a,b,c) := \sqrt{a^2+b^2+c^2-2ab-2ac-2bc} = |a-b|$

- If eg. in 3 body phase space invariant mass M of pair is used and third particle is masless then its energy k in rest frame: $k = \frac{S-M^2}{2\sqrt{S}}$.

- That is all what is needed from phase space jacobians for picture of structure functions. Rest belong to matrix elements.

- Side remark: This simplification is **not** used in PHOTOS, but it provides good way for getting benchmarks and can be used to show that PHOTOS sums up those terms which contribute to leading logarithms (of bremsstrahlung) to all orders.

Phase Space Formula: multi-channels.

Often MC algorithm has to be split into branches. In the most general case, when n different parametrizations of the phase space with different orderings of particles are in use, the cross section can be written as follows:

$$d\Gamma_X = \sum_{\lambda=1}^{n} \int_0^1 \prod_{i=1}^{m} dx_i \; P_\lambda \left[\sum_{\delta=1}^{n} P_\delta J_\delta^{-1}(q_1(\lambda, x_i), ... q_k(\lambda, x_i)) \right]^{-1} \times |M|^2.$$

In the above formula the four-momenta $q_i(\lambda, x_i)$ are calculated from the random numbers x_i according to the parametrization of the phase space of type λ. The Jacobians J_δ have to be calculated for all parametrizations of the phase space at the point q_i; P_λ denotes the probability of choosing the parametrization of type λ in the generation, λ thus takes[a] a role of an additional discrete variable in the generation. Numerical values of probabilities P_λ do not affect the final distributions, but only the efficiency of the generation.

[a]But not δ.

Phase Space case of complex singularity structure

- Several G_{n+1} can be used simultaneously (branching of the generation algorithm).
- Each G_{n+1} can be used presample distinct singularities.
- The price: W_n^{n+1} is more complicated but all remain exact.
- HOWEVER: We have observed that while matching Jacobians for the two branches related to collinear singularity of photons along direction of l^+ and l^+ (in Z decay) approximation must be used if more than one photon is present in final state. Otherwise solution become inconsistent. Phase space is not iterative, whereas matrix element for multiphoton state is obtained by iteration.
- AVOID INOCONSISTENCY: in expanding manifold curvature: must be the same for phase space and Matrix Element. Phase space is manifold, Matrix element squared – bilinear form on it. Truncation of perturbative expansion or iterative solutions mean truncation in powers of Ricci tensor, this has to be consistent. Multichannel phase space is not iterative, single branch algorithm we explained before is that is OK for expansion and exact phasese space remain. I have learned that hard way.

Phase Space: (multiply iterated)

We have generalized formula phase space formula to case of l particles added:

$$dLips_{n+l}(P \to k_1...k_n, k_{n+1}...k_{n+l}) = \frac{1}{l!} \prod_{i=1}^{l} \Big[dk_{\gamma_i} d\cos\theta_{\gamma_i} d\phi_{\gamma_i} W^{n+i}_{n+i-1} \Big]$$
$$\times dLips_n(P \to \bar{k}_1...\bar{k}_n), \qquad (7)$$
$$\{k_1, \dots, k_{n+l}\} = \mathbf{T}\Big(k_{\gamma_l}, \theta_{\gamma_l}, \phi_{\gamma_l}, \mathbf{T}\Big(\dots, \mathbf{T}\Big(k_{\gamma_1}, \theta_{\gamma_1}, \phi_{\gamma_1}, \{\bar{k}_1, \dots, \bar{k}_n\}\Big) \dots\Big).$$

Now we have to start talking about matrix elements: Our relation between n and n+l body phase space is motivated by cancellation of infrared singulatrites. It provides kind of triangulation. Measure defining distance between points from manifolds of distinct no. of particles. Such phase space points are close if they differ by presence of soft photons only.

Experimental user attention necessary. Can 1 GeV photon be ignored or only 0.1 MeV one.

We will move now from **exact distribution** of **weighted** events over $n + l$ body phase space to case where l is parameter too, but all remain exact!

Crude Ddistribution and crude matrix element

If we add arbitrary factors $f(k_{\gamma_i}, \theta_{\gamma_i}, \phi_{\gamma_i})$ and sum over l we obtain:

$$\sum_{l=0} \exp(-F)\frac{1}{l!}\prod_{i=1}^{l} f(k_{\gamma_i}, \theta_{\gamma_i}, \phi_{\gamma_i}) dLips_{n+l}(P \to k_1...k_n, k_{n+1}...k_{n+l}) =$$
$$\sum_{l=0} \exp(-F)\frac{1}{l!}\prod_{i=1}^{l} \Big[f(k_{\gamma_i}, \theta_{\gamma_i}, \phi_{\gamma_i}) dk_{\gamma_i} d\cos\theta_{\gamma_i} d\phi_{\gamma_i} W^{n+i}_{n+i-1} \Big] \times$$
$$dLips_n(P \to \bar{k}_1...\bar{k}_n), \tag{8}$$
$$\{k_1, \ldots, k_{n+l}\} = \mathbf{T}\Big(k_{\gamma_l}, \theta_{\gamma_l}, \phi_{\gamma_l}, \mathbf{T}\Big(\ldots, \mathbf{T}\Big(k_{\gamma_1}, \theta_{\gamma_1}, \phi_{\gamma_1}, \{\bar{k}_1, \ldots, \bar{k}_n\}\Big)\ldots\Big),$$
$$F = \int_{k_{min}}^{k_{max}} dk_\gamma d\cos\theta_\gamma d\phi_\gamma f(k_\gamma, \theta_\gamma, \phi_\gamma). \leftarrow \text{KLN good start}$$

- The olive parts of rhs. alone, give crude distribution over tangent space (orthogonal set of variables k_i, θ_i, ϕ_i). We restrict phase space by k_{min} (typically 10^{-6} but not by k_{max}.

• Factors f must be integrable over tangent space. Regulators of singularities necessary.

• We requested

$$\sigma_{tangent} = 1 =$$

$$\sum_{l=0} \exp(-F)\frac{1}{l!}\prod_{i=1}^{l}\Big[f(k_{\gamma_i},\theta_{\gamma_i},\phi_{\gamma_i})dk_{\gamma_i}d\cos\theta_{\gamma_i}d\phi_{\gamma_i}\Big]$$

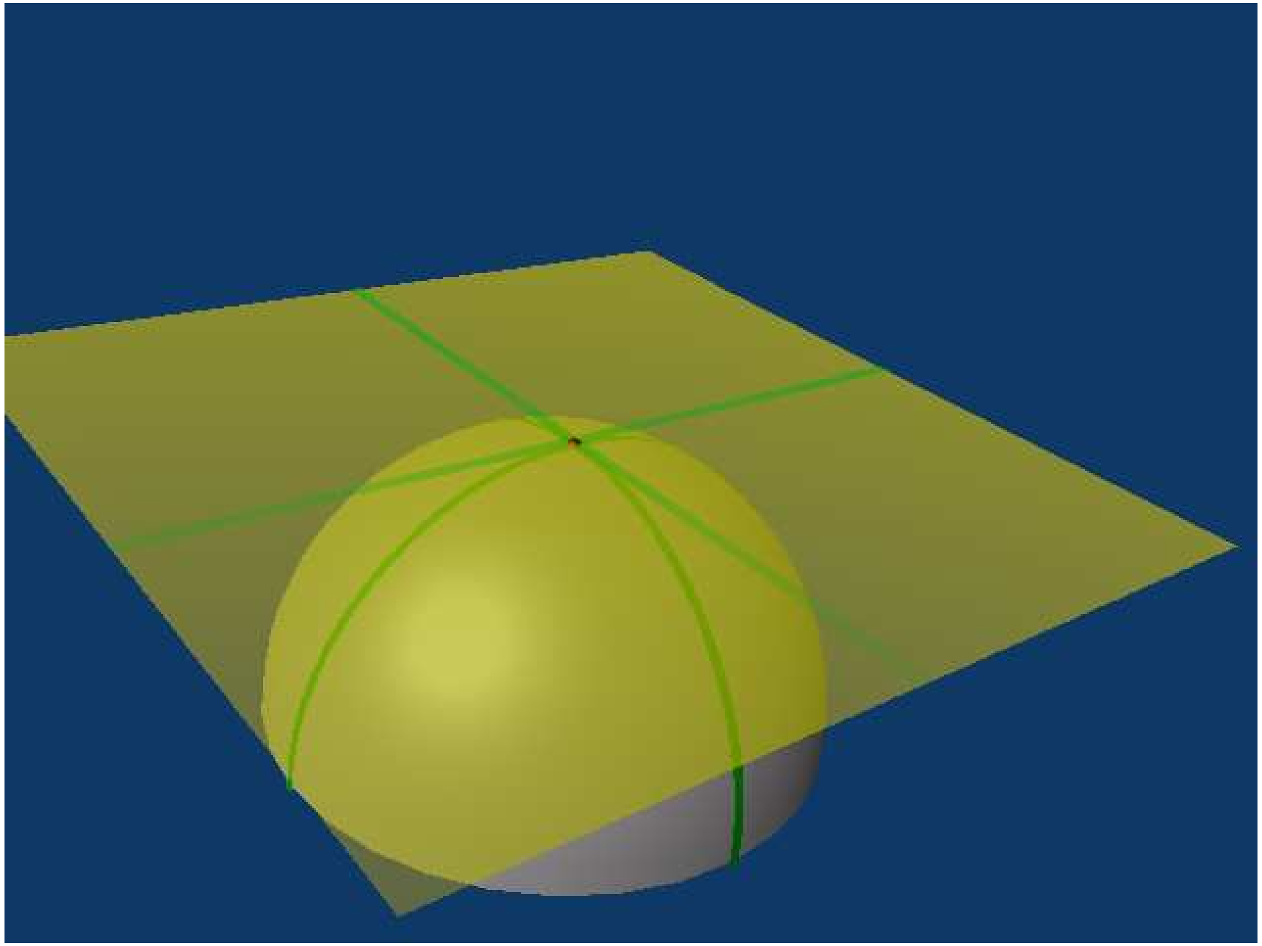
Because of Kinoshita Lee Nauenberg theorem we know that exact calculation of the cross section give result of similar property. We get another element of PHOTOS basis.

• For precise predictions, real emission and virtual corrections need to be calculated and their factorization properties analyzed.

• Choice of f must be synchronized so correcting weights for matrix elements do not fluctuate wildely.

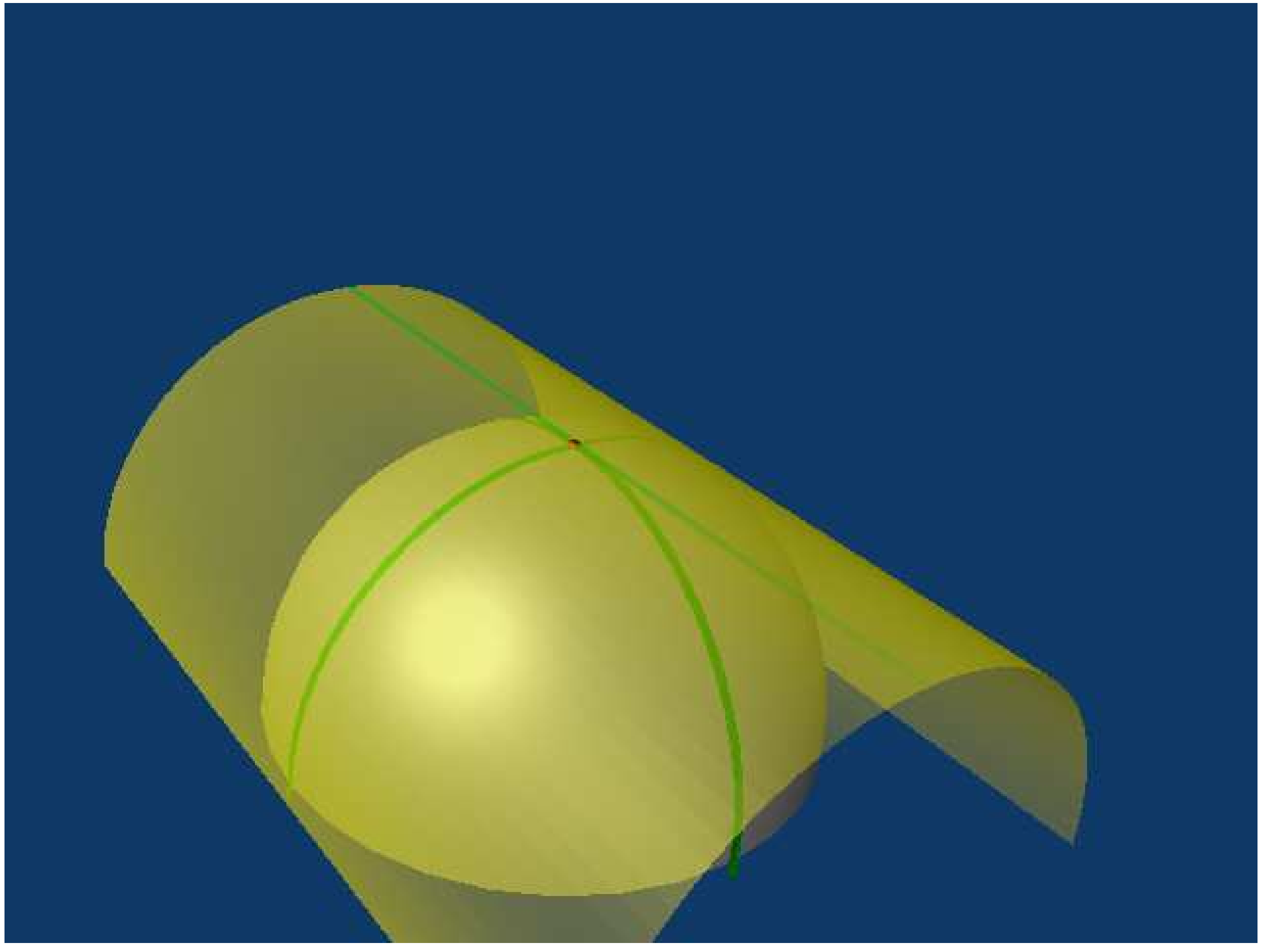
Heuristic CW complexes

We define our crude distribution over yellow space (surface=1) (reprersented by sum of: red point, green lines and flat yellow square), then we do projections using transformation T and matrix lements.



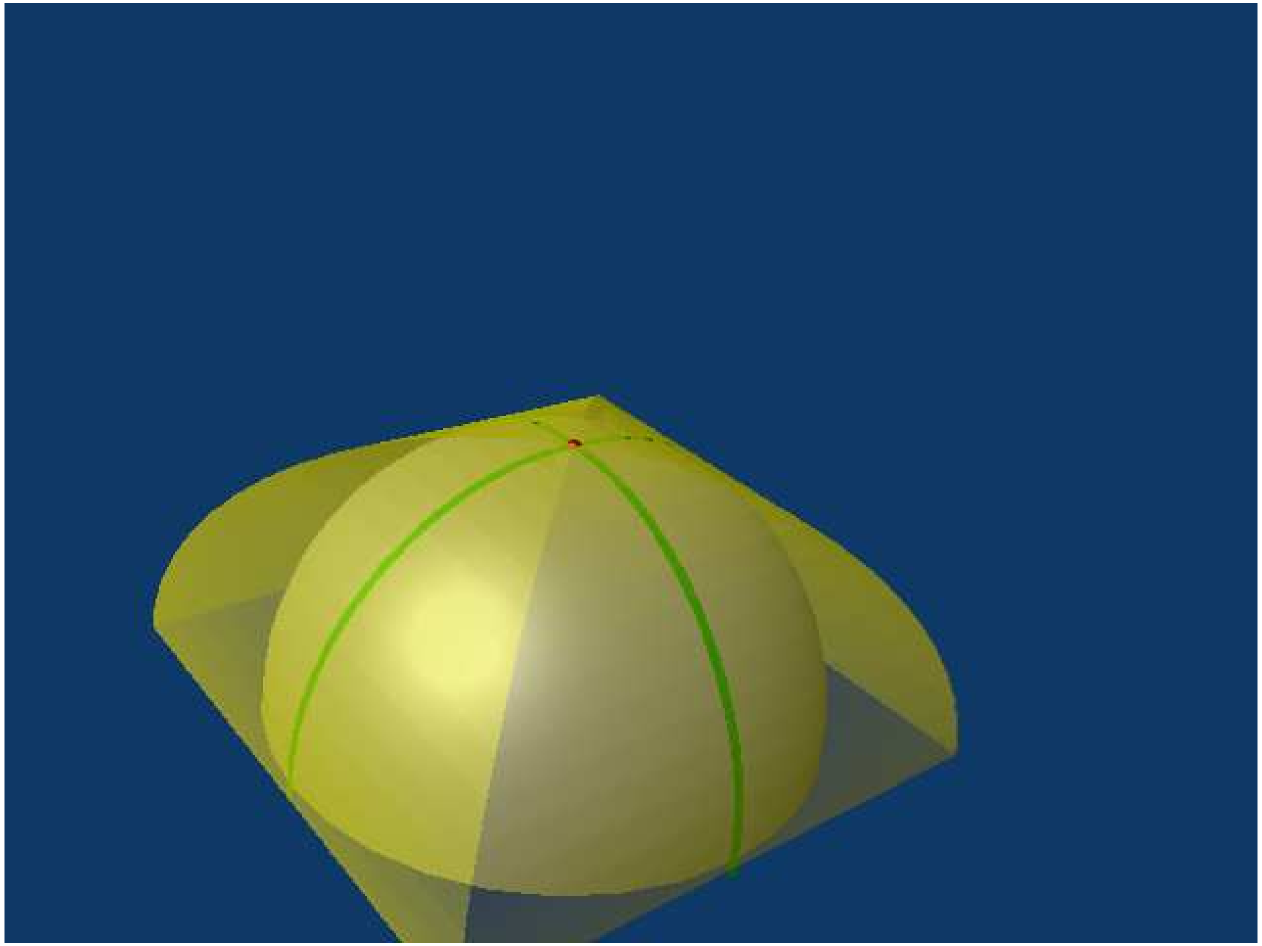
Heuristic CW complexes projection step 1

We project in steps, relative measure of point and lines on cylinder is larger than in previous step, overall measure remain 1.



Heuristic CW complexes projection step 2

Final distribution does not match the exact one, solely because approximation in matrix elements, phase space is exact.



Crude distribution of quality $\rightarrow$ matrix elements can be introduced

- we defined relation of some n- and $(n+l)$-body ph. space param.
- resulting triangulation (CW-complexes) of phase space parametrizations match structure of singularities.
- CW-complexes for exact space and tangent space match
- to achieve that we used properties of factorization as known in QED, guessed in scalar QED, also tried for QCD?
- infrared singularity being within perturbative domain was a bonus.
- we studied (1982-2008) properties of cross sections and later also spin amplitudes: distinct processes.
- formulas for virtual corrections more often than real emissions could be taken from other people works in unchanged form.
- M.E. content of CEEX-YFS exponentiation is the basis $\rightarrow$ benchmarks too.

Matrix Element (starting point):

- Directly starting from Feynman rules one can calculate spin amplitude for any QED/QCD process.
- The case of $Z \to l^+l^-\gamma$ is the backbone of PHOTOS design
- single photon amplitude(momentum k_1 polarization e_1 fermion spinors $u(p)$ and $v(q)$ dropped):

$$I = \not{J}\left[\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{q\cdot e_1}{q\cdot k_1}\right)\right] - \left[\frac{1}{2}\frac{\not{e}_1\not{k}_1}{p\cdot k_1}\right]\not{J} + \not{J}\left[\frac{1}{2}\frac{\not{e}_1\not{k}_1}{q\cdot k_1}\right]$$

these three gauge invariant parts appear in other processes too

Universal pre-property for factorizations of any sorts... must be deciphered

- The fully differential distribution from MUSTRAAL (used also in KORALZ for single photon mode) reads:

$$X_f = \frac{Q'^2\alpha(1-\Delta)}{4\pi^2 s} s^2 \quad \left\{ \frac{1}{(k'_+ k'_-)} \left[\frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s,t,u') + \frac{\mathrm{d}\sigma_B}{\mathrm{d}\Omega}(s,t',u) \right] \right\}$$

- Here:

$$\begin{aligned} s &= 2p_+ \cdot p_-, \quad & s' &= 2q_+ \cdot q_-, \\ t &= 2p_+ \cdot q_+, \quad & t' &= 2p_+ \cdot q_-, \\ u &= 2p_+ \cdot q_-, \quad & u' &= 2_- \cdot q_+, \\ k'_\pm &= q_\pm \cdot k, \quad & x_k &= 2E_\gamma/\sqrt{s} \end{aligned}$$

- The Δ term is responsable for final state mass dependent terms, p_+, p_-, q_+, q_-, k denote four-momenta of incoming positron, electron beams, outcoming muons and bremsstrahlung photon.

- after trivial manipulation it can be written as:

$$X_f = \frac{Q'^2\alpha(1-\Delta)}{4\pi^2 s} s^2 \Bigg\{ \frac{1}{(k'_+ + k'_-)} \frac{1}{k'_-} \Big[\frac{d\sigma_B}{d\Omega}(s,t,u') + \frac{d\sigma_B}{d\Omega}(s,t',u) \Big]$$

$$+ \frac{1}{(k'_+ + k'_-)} \frac{1}{k'_+} \Big[\frac{d\sigma_B}{d\Omega}(s,t,u') + \frac{d\sigma_B}{d\Omega}(s,t',u) \Big] \Bigg\}$$

- In PHOTOS the following expression is used in universal application (AP adj.):

$$X_f^{PHOTOS} = \frac{Q'^2\alpha(1-\Delta)}{4\pi^2 s} s^2 \Bigg\{$$

$$\frac{1}{k'_+ + k'_-} \frac{1}{k'_-} \Big[(1 + (1-x_k)^2) \frac{d\sigma_B}{d\Omega}\Big(s, \frac{s(1-\cos\Theta_+)}{2}, \frac{s(1+\cos\Theta_+)}{2}\Big) \Big] \frac{(1+\beta\cos\Theta_\gamma)}{2}$$

$$+ \frac{1}{k'_+ + k'_-} \frac{1}{k'_+} \Big[(1 + (1-x_k)^2) \frac{d\sigma_B}{d\Omega}\Big(s, \frac{s(1-\cos\Theta_-)}{2}, \frac{s(1+\cos\Theta_-)}{2}\Big) \Big] \frac{(1-\beta\cos\Theta_\gamma)}{2} \Bigg\}$$

$$\text{where}: \Theta_+ = \angle(p_+, q_+),\ \Theta_- = \angle(p_-, q_-)$$

$\Theta_\gamma = \angle(\gamma, \mu^-)$ are defined in (μ^+, μ^-)-pair rest frame

The matrix element weight

- weight for exact matrix element is easy to implement $WT = X_f / X_f^{PHOTOS}$
- also factor $\Gamma^{total}/\Gamma^{Born} = 1 + \frac{3}{4}\frac{\alpha}{\pi}$ defines first order weight, it depends on virtual corrections if non leading mass terms are kept.
- $WT = \frac{X_f}{X_f^{PHOTOS}} \frac{\Gamma^{Born}}{\Gamma^{total}}$

The differences of X_f and X_f^{PHOTOS} are important

- Without process dependent weight PHOTOS is universal and can be combined with any generator rather easily.
- Photos weight becomes process independent at a cost of the approximation.
- Virtual corrections for decays like $K^+ \to \pi^+\pi^- l^+ \nu_l$ more complicated than overall constant.

Matrix Element (anything in common?):

- We have seen nice properties of matrix element squared which were factorizing into Born-like distribution and photon factor.
- It was shown many years ago by Ronald Kleiss that such property of distributions does not hold beyond first order!
- Dead end? Not really, just complex weights[a]
- single photon amplitude again:

$$I = \not{J}\left[\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{q\cdot e_1}{q\cdot k_1}\right)\right] - \left[\frac{1}{2}\frac{\not{e}_1\not{k}_1}{p\cdot k_1}\right]\not{J} + \not{J}\left[\frac{1}{2}\frac{\not{e}_1\not{k}_1}{q\cdot k_1}\right]$$

three gauge invariant parts, first is eikonal, other for collinear configuration along q and p

We look for these parts in higher order amplitudes

[a] Also: samples at different level of spophistication can be correlated up to NLO level. That is enough for most of experimental techniques, precision of correlated programs can be higher.

Matrix Element (double emission):

- The structure of exact spin amplitude for single emission looks promising.
- How does it translate to distributions?
- Does it extend to other processes, intercations? Scalar QED QCD as well?
- Does it extent to higher orders?
- Can one decipher anything without enforcing some phase space conditions?
- To identify the building blocks we have used gauge invariance, and we have used also segments localized at lower order.
- For tree diagrams gauge invariance mean in practice that replacement $k \to e$ set experession to zero
- Virtual corrections add complication because of regularization schemes, we will skip that now.

Exact Matrix Element: $e^+e^- \to \nu_\mu \bar{\nu}_\mu \gamma\gamma$ explicitly;

- Expressions are valid for any current J,
- For complete amplitude add fermionic fields, eg. $\bar{u}(p)$ and $v(q)$; 1-st/2-nd photon momenta/polarizations are: k_1/k_2 e_1/e_2.

$$I_1^{\{1,2\}} = \frac{1}{2} \not{J} \left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{q\cdot e_1}{q\cdot k_1} \right) \left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{q\cdot e_2}{q\cdot k_2} \right) \qquad eikonal$$

$$I_{2l}^{\{1,2\}} = -\frac{1}{4} \left[\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{q\cdot e_1}{q\cdot k_1} \right) \frac{\not{e}_2 \not{k}_2}{p\cdot k_2} + \left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{q\cdot e_2}{q\cdot k_2} \right) \frac{\not{e}_1 \not{k}_1}{p\cdot k_1} \right] \not{J} \qquad \beta_1$$

$$I_{2r}^{\{1,2\}} = \frac{1}{4} \not{J} \left[\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{q\cdot e_1}{q\cdot k_1} \right) \frac{\not{k}_2 \not{e}_2}{q\cdot k_2} + \left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{q\cdot e_2}{q\cdot k_2} \right) \frac{\not{k}_1 \not{e}_1}{q\cdot k_1} \right] \qquad \beta_1$$

$$I_3^{\{1,2\}} = -\frac{1}{8} \left(\frac{\not{e}_1 \not{k}_1}{p\cdot k_1} \not{J} \frac{\not{k}_2 \not{e}_2}{q\cdot k_2} + \frac{\not{e}_2 \not{k}_2}{p\cdot k_2} \not{J} \frac{\not{k}_1 \not{e}_1}{q\cdot k_1} \right) \qquad start for \beta_2 ...$$

$$I_{4p}^{\{1,2\}} = \frac{1}{8}\frac{1}{p\cdot k_1 + p\cdot k_2 - k_1\cdot k_2}\left(\frac{\not e_1 \not k_1 \not e_2 \not k_2}{p\cdot k_1} + \frac{\not e_2 \not k_2 \not e_1 \not k_1}{p\cdot k_2}\right)\not J$$

$$I_{4q}^{\{1,2\}} = \frac{1}{8}\not J\frac{1}{q\cdot k_1 + q\cdot k_2 - k_1\cdot k_2}\left(\frac{\not k_2 \not e_2 \not k_1 \not e_1}{q\cdot k_1} + \frac{\not k_1 \not e_1 \not k_2 \not e_2}{q\cdot k_2}\right)$$

$$I_{5pA}^{\{1,2\}} = \frac{1}{2}\not J\frac{k_1\cdot k_2}{p\cdot k_1 + p\cdot k_2 - k_1\cdot k_2}\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{k_2\cdot e_1}{k_2\cdot k_1}\right)\left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2}\right)$$

$$I_{5pB}^{\{1,2\}} = -\frac{1}{2}\not J\frac{1}{p\cdot k_1 + p\cdot k_2 - k_1\cdot k_2}\left(\frac{k_1\cdot e_2 k_2\cdot e_1}{k_1\cdot k_2} - e_1\cdot e_2\right)$$

$$I_{5qA}^{\{1,2\}} = \frac{1}{2}\not J\frac{k_1\cdot k_2}{q\cdot k_1 + q\cdot k_2 - k_1\cdot k_2}\left(\frac{q\cdot e_1}{q\cdot k_1} - \frac{k_2\cdot e_1}{k_2\cdot k_1}\right)\left(\frac{q\cdot e_2}{q\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2}\right)$$

$$I_{5qB}^{\{1,2\}} = -\frac{1}{2}\not J\frac{1}{q\cdot k_1 + q\cdot k_2 - k_1\cdot k_2}\left(\frac{k_1\cdot e_2 k_2\cdot e_1}{k_1\cdot k_2} - e_1\cdot e_2\right)$$

$$I_{6B}^{\{1,2\}} = -\frac{1}{4}\frac{k_1\cdot k_2}{p\cdot k_1 + p\cdot k_2 - k_1\cdot k_2}\left[+\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{k_2\cdot e_1}{k_1\cdot k_2}\right)\frac{\not e_2 \not k_2}{p\cdot k_2} + \left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2}\right)\frac{\not e_1 \not k_1}{p\cdot k_1}\right]\not J$$

$$I_{7B}^{\{1,2\}} = -\frac{1}{4} \not{J} \frac{k_1 \cdot k_2}{q \cdot k_1 + q \cdot k_2 - k_1 \cdot k_2} \left[+ \left(\frac{q \cdot e_1}{q \cdot k_1} - \frac{k_2 \cdot e_1}{k_1 \cdot k_2} \right) \frac{\not{k}_2 \not{e}_2}{q \cdot k_2} + \left(\frac{q \cdot e_2}{q \cdot k_2} - \frac{k_1 \cdot e_2}{k_1 \cdot k_2} \right) \frac{\not{k}_1 \not{e}_1}{q \cdot k_1} \right]$$

- for the **exponentiation** we have used **separation** into 3 parts only. It is **crystal clear**, also in case of contributions with t-channel W, was very useful for KKMC,
- for PHOTOS kernel, parts $I_3^{\{1,2\}}$, $I_{4p}^{\{1,2\}}$, $I_{4q}^{\{1,2\}}$ were studied separately as well.
- In fact older works on spin amplitdues wre used E. Richter-Was Z.Phys.C64:227-240,1994, Z.Phys.C61:323-340,1994.
- Clearly visible but not used for PHOTOS further separation of β_2 terms ...
- Presented above properties of spin amplitudes were used for PHOTOS design to make a choice of phase space parametrization and iteration of consecutive emission kernels that respect numerically as much as possible results of second order amplitudes. Also one want to remain consistent with NLO and exponentiation to all orders.

I had to present project foundation

Important to know: homework completed

Somebody around should be always able to return to details

Important when precision thresholds are crossed

−>.<−

Presentation

- PHOTOS (first versions by E. Barberio, B. van Eijk, Z. W., P. Golonka) is available since 1989, to simulate the effect of radiatiative corrections in decays.
- Full events combining complicated tree structure of production and subsequent decays have to be fed into PHOTOS, usually with the help of HEPEVT event record of F77 (beta version working with HepMC event record of C++ exist).
- This is often source of technical difficulties as standard is often overruled. Also numerical precision may be an issue.
- At every event decay branching, PHOTOS intervene. With certain probability extra photon may be added and kinematics of other particles adjusted.
- PHOTOS works on four-momenta double precision four-vectors. This creates some 'fun' with numerical stability.
- I will not talk about those time consuming aspects and my frustrations ...

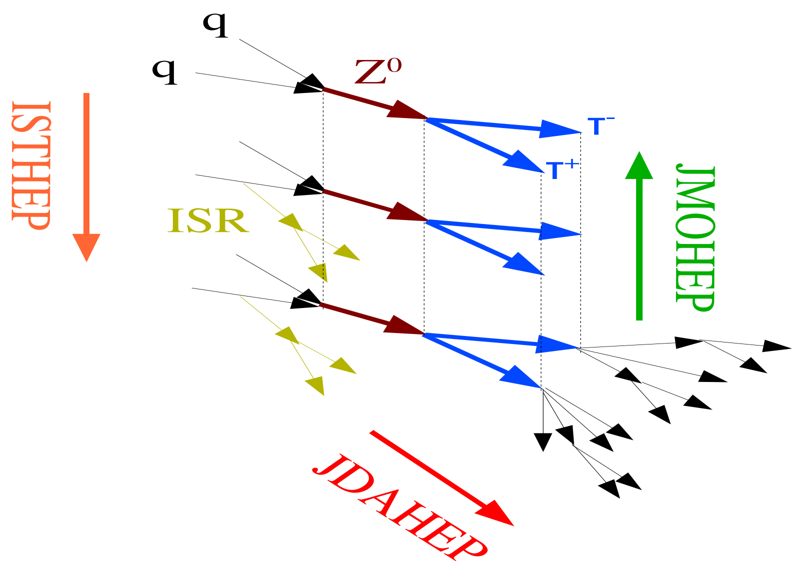
Main References

- E. Barberio and Z. Was, Comput. Phys. Commun. **79**, 291 (1994).

  E. Barberio, B. van Eijk and Z. Was, Comput. Phys. Commun. **66**, 115 (1991).
- Z. Was, Eur. Phys. J. C **44** (2005) 489, useful work for other project matrix elements
- P. Golonka and Z. Was, Eur. Phys. J. C **45** (2006) 97, ibid, C50:53-62,2007 exponentiation and no approximations for matrix elements
- G. Nanava, Z. Was, Eur.Phys.J.C51:569-583,2007 Scalar QED and room for Form factors
- G. Nanava, Z. Was, Q. Xu Eur.Phys.J. C70 (2010) 673 Scalar QED for vector state decay, W decays
- N. Davidson, T. Przedzinski, Z. Was, Comput.Phys.Commun. 199 (2016) 86 C++, HepMC, bremsstrahlung of pairs
- S. Antropov, A. Arbuzov, R. Sadykov, Z. Was, Acta Phys.Polon. B48 (2017) 1469 example of comparison/external tests

Limitations

Problems With Event Record Content



1. Hard process
2. with shower
3. after hadronization
4. Event record overloaded with physics beyond design → gramar problems.
5. That is potential problem for PHOTOS, TAUOLA, MC-TESTER, ...

Precision: complication arise with spin state of eg. Z^o

Preparation of analytic benchmarks

- One of the necessary steps was to verify, that once PHOTOS activated, the lepton spectra will be reproduced as far as the LL corrections to required order.
- Formal solution of QED evolution equation can be written as:

$$D(x,\beta_{ch}) = \delta(1-x)+\beta_{ch}P(x)+\frac{1}{2!}\beta_{ch}^2\{P\times P\}(x)+\frac{1}{3!}\beta_{ch}^3\{P\times P\times P\}(x)+\dots \quad (9)$$

where $P(x) = \delta(1-x)(\ln\varepsilon + 3/4) + \Theta(1-x-\varepsilon)\frac{1}{x}(1+x^2)/(1-x)$
and $\{P \times P\}(x) = \int_0^1 dx_1 \int_0^1 dx_2 \delta(x - x_1 x_2) P(x_1) P(x_2)$.

- In the LL contributing regions, phase space Jacobian's of PHOTOS trivialize (CPC 1994). The solution above is reproduced by PHOTOS in a straightforward manner, for each of the outcoming charged lines.
- But it is only a limit! **PHOTOS treat phase space exactly and covers all corners.**

- In a similar way (simplifying phase space Jacobians and droping parts of ME) one can get convinced that distribution of soft photons is as should be for exlusive exponentiation.

- So far we were discussing building blocks, but how does it work in practice?

- Avalanche of numerical results...

- **Important:** We could see that kernel simplified with respect to NLO is sufficient for sub-permille precision. $\rightarrow$ Much easier to use.

- MC-TESTER by P. Golonka, N. Davidson, T. Przedzinski, Z. Was is used for tests. Idea is to generate histograms of all possible invariant masses which can be constructed from final state momenta.

- One can select events, for example only photons of energy above 1 GeV will be considered as final state.

- On one frame distribution from two program is printed (in logarithmic scale) and their ration (in linear scale).

- No of events of distinct (in the selected way) final states is printed too.

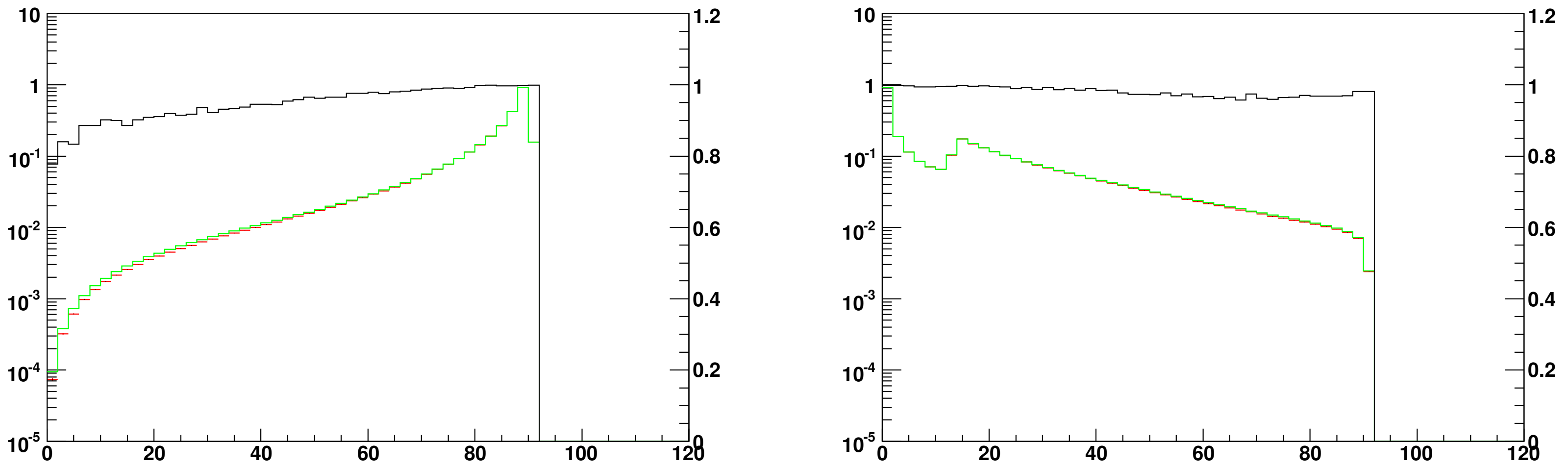


Figure 1: Comparison of standard PHOTOS and KORALZ for single photon emission. In the left frame the invariant mass of the $\mu^+\mu^-$ pair; SDP=0.00534. In the right frame the invariant mass of $\mu^-\gamma$; SDP=0.00296. The histograms produced by the two programs (logarithmic scale) and their ratio (linear scale, black line) are plotted in both frames. The fraction of events with hard photon was 17.4863 $\pm$ 0.0042% for KORALZ and 17.6378 $\pm$ 0.0042% for PHOTOS.

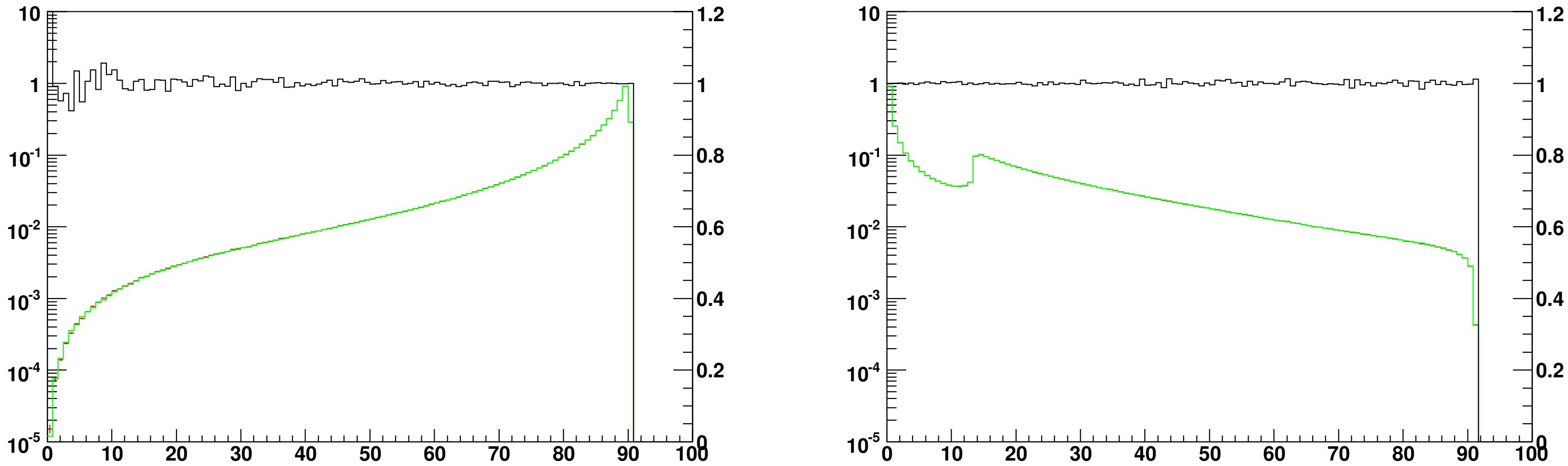


Figure 2: Comparisons of improved PHOTOS and KORALZ for single photon emission. In the left frame the invariant mass of the $\mu^+\mu^-$ pair. In the right frame the invariant mass of $\mu^-\gamma$ pair is shown. In both cases differences between PHOTOS and KORALZ are below statistical error. The fraction of events with hard photon was 17.4890 $\pm$ 0.0042% for KORALZ and 17.4926 $\pm$ 0.0042% for PHOTOS.

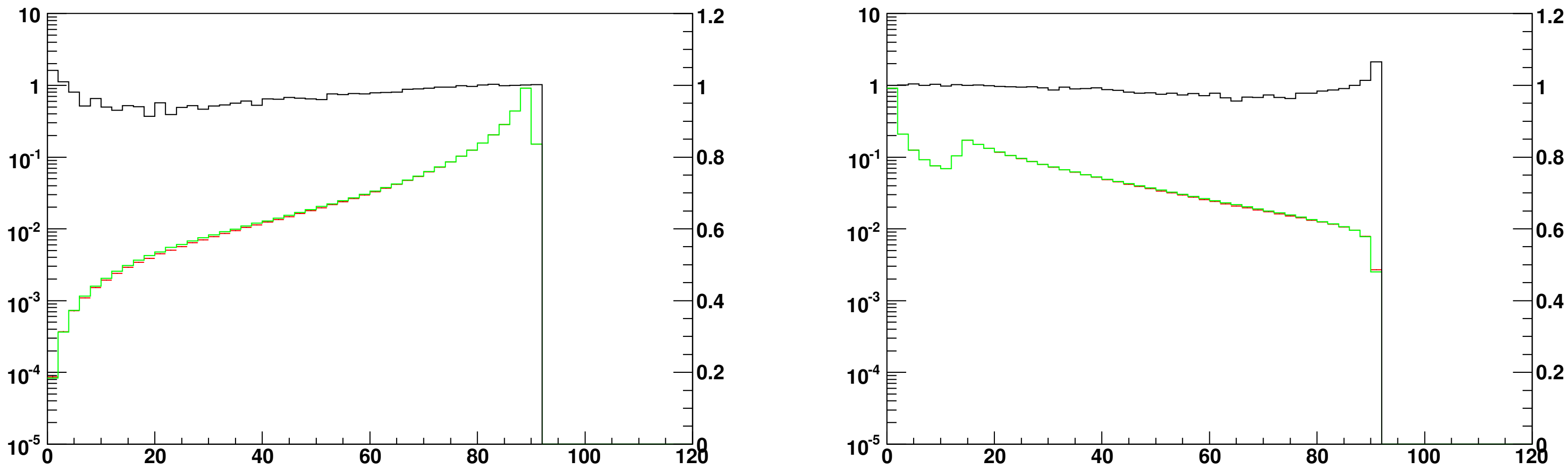


Figure 3: Comparison of standard PHOTOS with multiple photon emission and KKMC with second order matrix element and exponentiation. In the left frame the invariant mass of the $\mu^+\mu^-$ pair; SDP=0.00409. In right frame the invariant mass of the $\mu^-\gamma$ pair; SDP=0.0025. The pattern of differences between PHOTOS and KKMC is similar to the one of Fig 1. The fraction of events with hard photon was 16.0824 $\pm$ 0.0040% for KKMC and 16.1628 $\pm$ 0.0040% for PHOTOS.

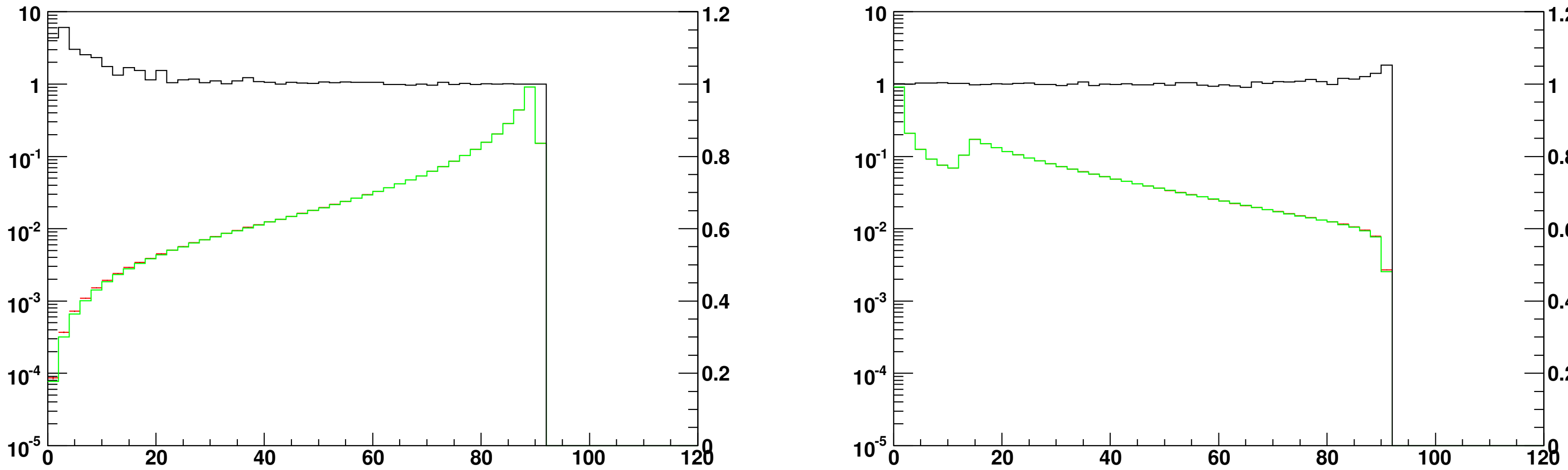


Figure 4: Comparisons of improved PHOTOS with multiple photon emission and KKMC with second order matrix element and exponentiation. In the left frame the invariant mass of the $\mu^+\mu^-$ pair; SDP=0.0000249. In the right frame the invariant mass of the $\mu^-\gamma$ pair; SDP=0.0000203. The fraction of events with hard photon was 16.0824 $\pm$ 0.004% for KKMC and 16.0688 $\pm$ 0.004% for PHOTOS.

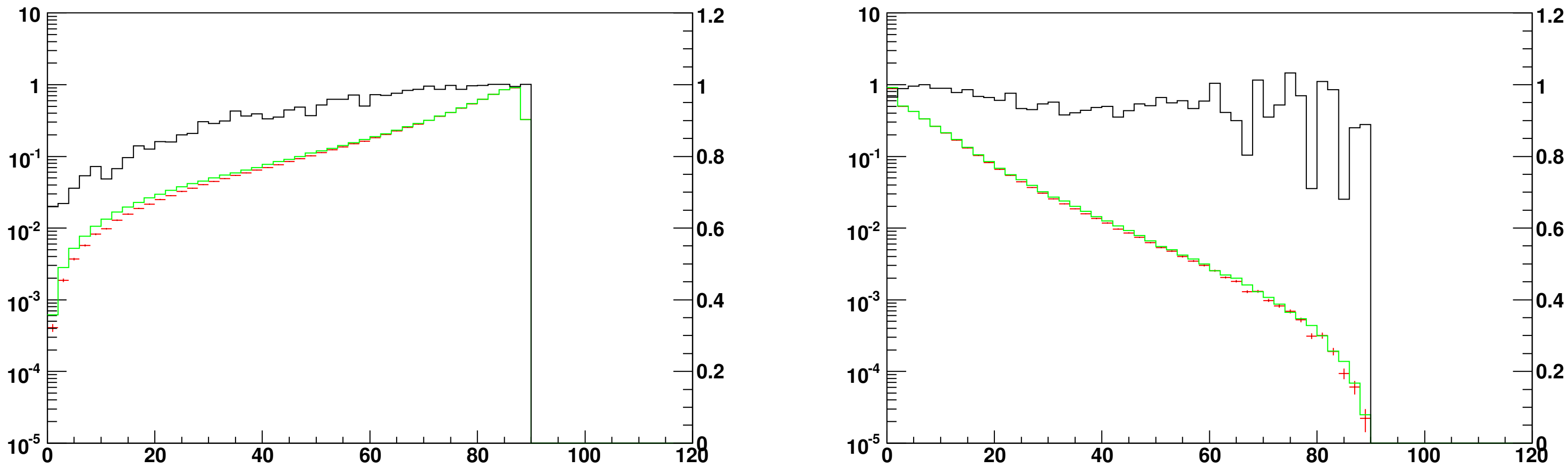


Figure 5: Comparisons of standard PHOTOS with multiple photon emission and KKMC with second order matrix element and exponentiation. In the left frame the invariant mass of the $\mu^+\mu^-$ pair; SDP= 0.00918. In the right frame the invariant mass of the $\gamma\gamma$ pair; SDP=0.00268. The fraction of events with two hard photons was 1.2659 $\pm$ 0.0011% for KKMC and 1.2952 $\pm$ 0.0011% for PHOTOS.

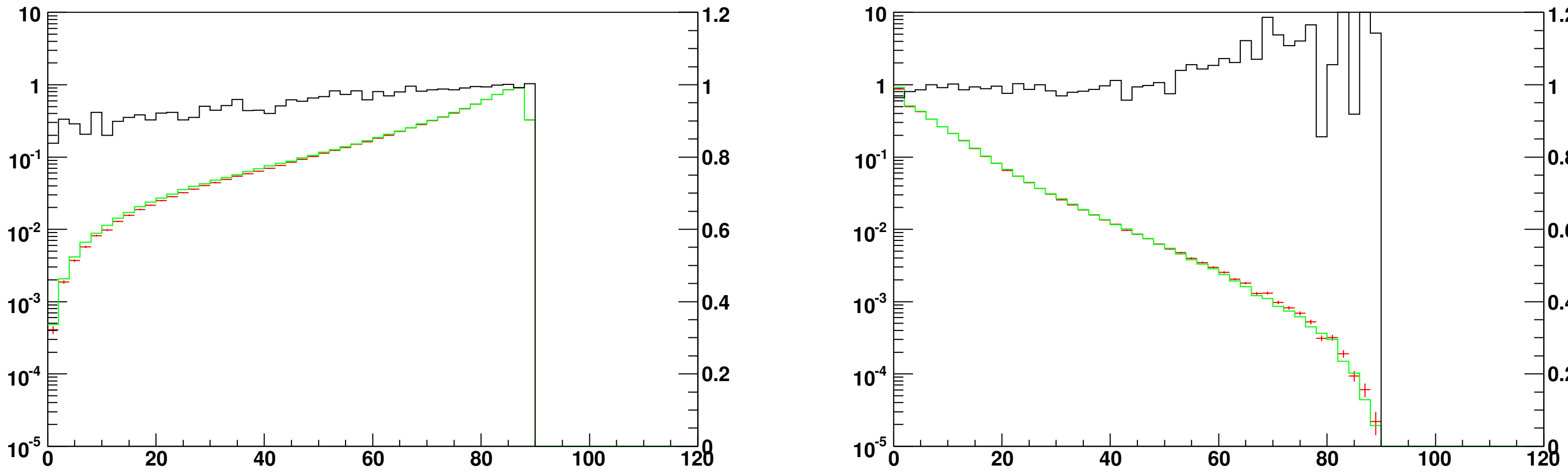


Figure 6: Comparisons of improved PHOTOS with multiple photon emission and KKMC with second order matrix element and exponentiation. In the left frame the invariant mass of the $\mu^+\mu^-$ pair; SDP= 0.00142. In the right frame the invariant mass of the $\gamma\gamma$; SDP=0.00293. The fraction of events with two hard photons was 1.2659 $\pm$ 0.0011% for KKMC and 1.2868 $\pm$ 0.0011% for PHOTOS.

Again Matrix Element for Z decay:

- Our discussion of structure for double emission amplitudes was started from the details of the single photon one
- The same is true for amplitudes of other processes. We have to check if they are indeed similar to these of Z decay.
- Only if the structure match we can expect our discussion of multiemission to apply as well:
-
$$I = I^A \qquad\qquad + I^B \qquad\qquad + I^C$$
-
$$I = \not{J}\left[\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{q\cdot e_1}{q\cdot k_1}\right)\right] - \left[\frac{1}{2}\frac{\not{e}_1 \not{k}_1}{p\cdot k_1}\right]\not{J} + \not{J}\left[\frac{1}{2}\frac{\not{e}_1 \not{k}_1}{q\cdot k_1}\right]$$

three gauge invariant parts, I^A is eikonal; I^B, I^C carry collinear contrib from p and q

QED for $W \to l\nu_l\gamma$: I^A, I^B and non-leading

$$
\begin{aligned}
M^{\sigma}_{\lambda,\lambda_\nu,\lambda_l}(k,Q,p_\nu,p_l) = & \left[\frac{Q_l}{2\,k\cdot p_l} b_\sigma(k,p_l) - \frac{Q_W}{2\,k\cdot Q}\left(b_\sigma(k,p_l)+b_\sigma(k,p_\nu)\right)\right] B^{\lambda}_{\lambda_l,\lambda_\nu}(p_l,Q,p_\nu) \\
& + \frac{Q_l}{2\,k\cdot p_l}\sum_{\rho=\pm} U^{\sigma}_{\lambda_l,\rho}(p_l,m_l,k,0,k,0) B^{\lambda}_{\rho,-\lambda_\nu}(k,Q,p_\nu) \\
& - \frac{Q_W}{2\,k\cdot Q}\sum_{\rho=\pm}\Big(B^{\lambda}_{\lambda_l,-\rho}(p_l,Q,k) U^{\sigma}_{-\rho,-\lambda_\nu}(k,0,k,0,p_\nu,0) \\
& \qquad + U^{\sigma}_{\lambda_l,\rho}(p_l,m_l,k,0,k,0) B^{\lambda}_{\rho,-\lambda_\nu}(k,Q,p_\nu)\Big)\,,
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
B^{\lambda}_{\lambda_1,\lambda_2}(p_1,Q,p_2) & \equiv \frac{g}{2\sqrt{2}}\bar{u}(p_1,\lambda_1)\,\widehat{\epsilon}^{\lambda}_W(Q)(1+\gamma_5)\,v(p_2,\lambda_2)\,, \\
U^{\sigma}_{\lambda_1,\lambda_2}(p_1,m_1,k,0,p_2,m_2) & \equiv \bar{u}(p_1,\lambda_1)\,\widehat{\epsilon}^{\sigma}_\gamma(k)\,u(p_2,\lambda_2)\,, \\
\delta_{\lambda_1\lambda_2} b_\sigma(k,p) & \equiv U^{\sigma}_{\lambda_1,\lambda_2}(p,m,k,0,p,m)\,,
\end{aligned} \tag{11}
$$

Q_l and Q_W are the electric charges of the fermion l and the W boson, respectively, in units of the positron charge, $\epsilon^{\sigma}_{\gamma}(k)$ and $\epsilon^{\lambda}_{W}(Q)$ denote respectively the polarization vectors of the photon and the W boson. An expression of the function $U^{\sigma}_{\lambda_1,\lambda_2}$ in terms of the massels spinors.

Scalar QED: $B \to K\pi$ decays – pure I^A

- The one-loop QED correction to the decay width can be represented as the sum o f the Born contribution with the contributions due to virtual loop diagrams and soft and hard photon emissions.

$$d\Gamma^{\rm Total} = d\Gamma^{\rm Born}\left\{1 + \frac{\alpha}{\pi}\left[\delta^{\rm Soft}(m_\gamma, \omega) + \delta^{\rm Virt}(m_\gamma, \mu_{UV})\right]\right\} + d\Gamma^{\rm Hard}(\omega)$$

- where for **Neutral meson decay channels**, hard photon contribution:

$$d\Gamma^{\rm Hard} = |A^{\rm Born}|^2 4\pi\alpha \left(q_1 \frac{k_1.\epsilon}{k_1.k_\gamma} - q_2 \frac{k_2.\epsilon}{k_2.k_\gamma}\right)^2 dLips_3(P \to k_1, k_2, k_\gamma)$$

- for **Charged meson decay channels**, hard photon contribution:

$$d\Gamma^{\rm Hard} = |A^{\rm Born}|^2 4\pi\alpha \left(q_1 \frac{k_1.\epsilon}{k_1.k_\gamma} - q \frac{P.\epsilon}{P.k_\gamma}\right)^2 dLips_3(P \to k_1, k_2, k_\gamma)$$

Scalar QED for $\gamma^* \to \pi^+\pi^-\gamma$: I^A and non-leading

- This case is different, because of spin structure. One can not make spin of initial state out of internal spin of outgoing particles.

$$H^\mu = \frac{e^2 F_{2\pi}(p^2)}{p^2} \left\{ (q_1 + k - q_2)^\mu \frac{q_1 \cdot \epsilon^*}{q_1 \cdot k} + (q_2 + k - q_1)^\mu \frac{q_2 \cdot \epsilon^*}{q_2 \cdot k} - 2\epsilon^{*\mu} \right\}$$

- As in case of Z decay one can separate spin amplitude into gauge invariant parts ($C = \frac{e^2 F_{2\pi}(p^2)}{p^2}$):

$$H_I^\mu = C \, (q_1 - q_2)^\mu \left(\frac{q_1 \cdot \epsilon^*}{q_1 \cdot k} - \frac{q_2 \cdot \epsilon^*}{q_2 \cdot k} \right), H_{II}^\mu = C \, \left(k^\mu \left(\frac{q_1 \cdot \epsilon^*}{q_1 \cdot k} + \frac{q_2 \cdot \epsilon^*}{q_2 \cdot k} \right) - 2\epsilon^{*\mu} \right), \tag{12}$$

- This can be improved with the following change:

$$H_{I'}^\mu = C \, \left((q_1 - q_2)^\mu + k^\mu \frac{q_2 \cdot k - q_1 \cdot k}{q_2 \cdot k + q_1 \cdot k} \right) \left(\frac{q_1 \cdot \epsilon^*}{q_1 \cdot k} - \frac{q_2 \cdot \epsilon^*}{q_2 \cdot k} \right), \tag{13}$$

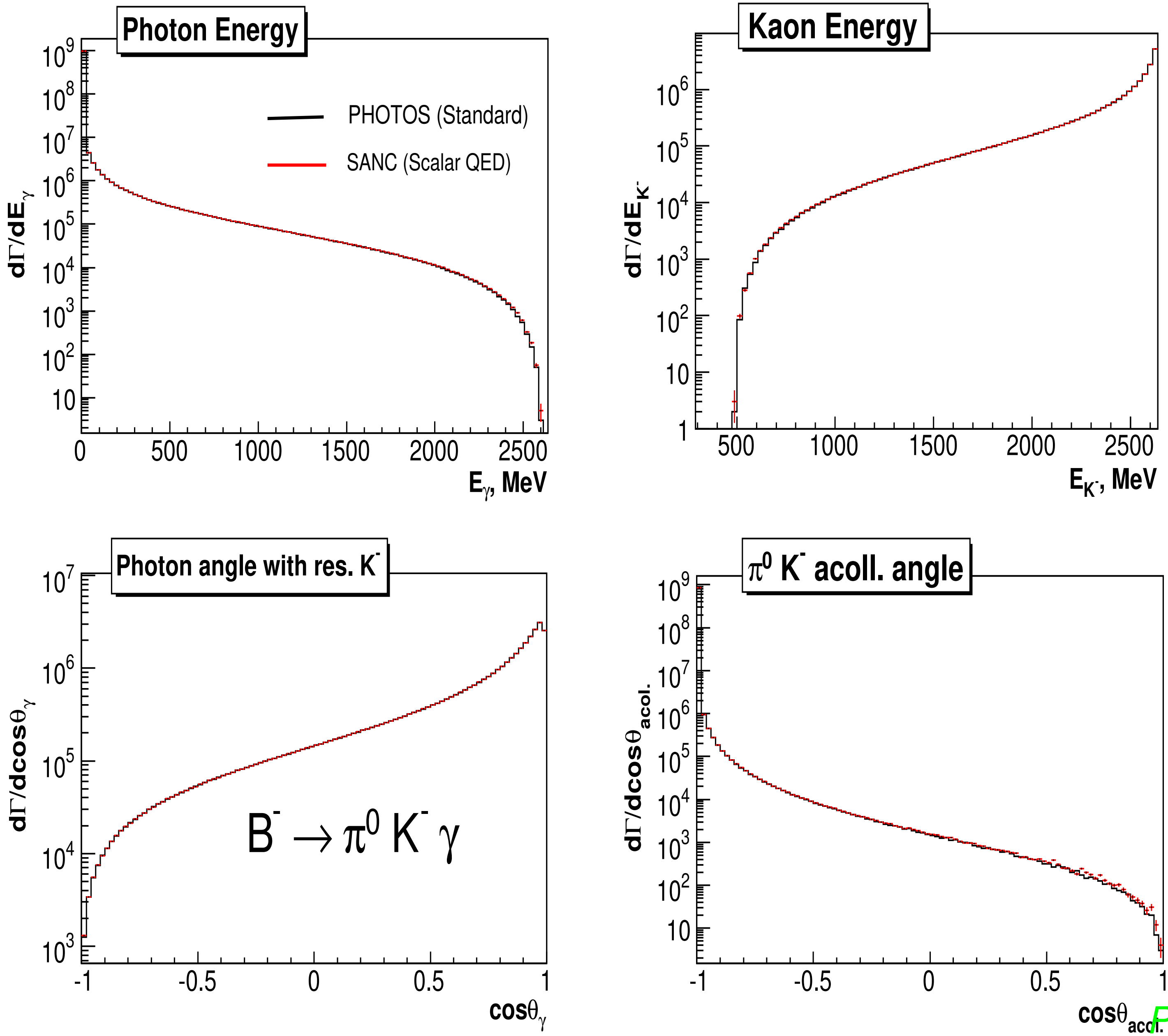
$$H_{II'}^\mu = C \, \left(\frac{k^\mu}{q_2 \cdot k + q_1 \cdot k} (q_1 \cdot \epsilon^* + q_2 \cdot \epsilon^*) - \epsilon^{*\mu} \right). \tag{14}$$

- In the second case non-eikonal term is free of collinear logarithm, but is non trivial and contributes 0.2 % to total rate, thus can be numerically studied!

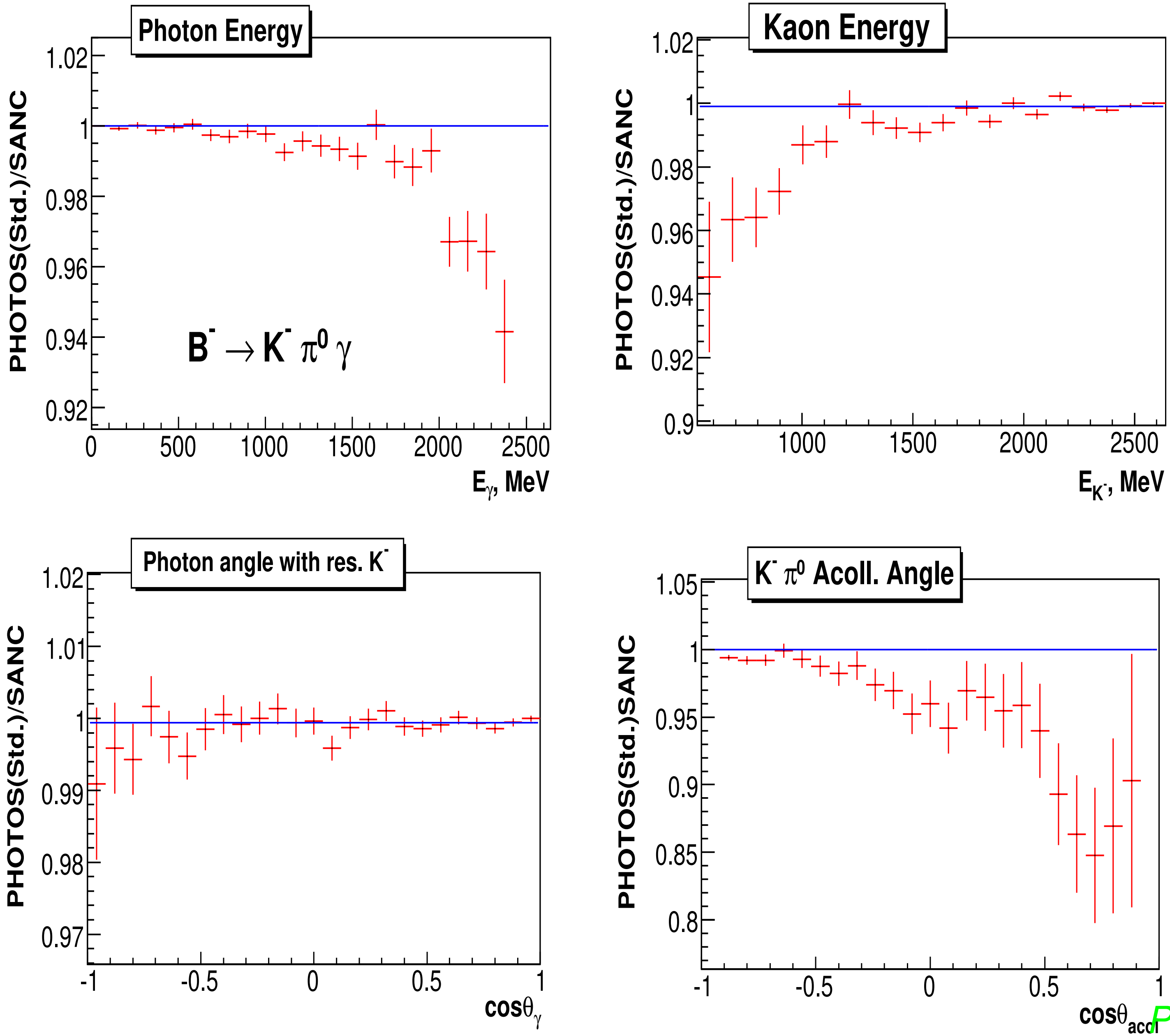
Matrix Element (anything useful seen?):

- We have seen that in all cases terms I^A, I^B, I^C appear
- These are the only ones which carry soft or collinear contributions
- That is why universal weight of Photos could be defined.
- That is also why the soultion defined from Z amplitudes work for other processes as well
- part of NLO proof.
- Tests confirm that NLO complete kernel (process dependent) is not necessary even for sub-permille precision level $\rightarrow$ good for users.

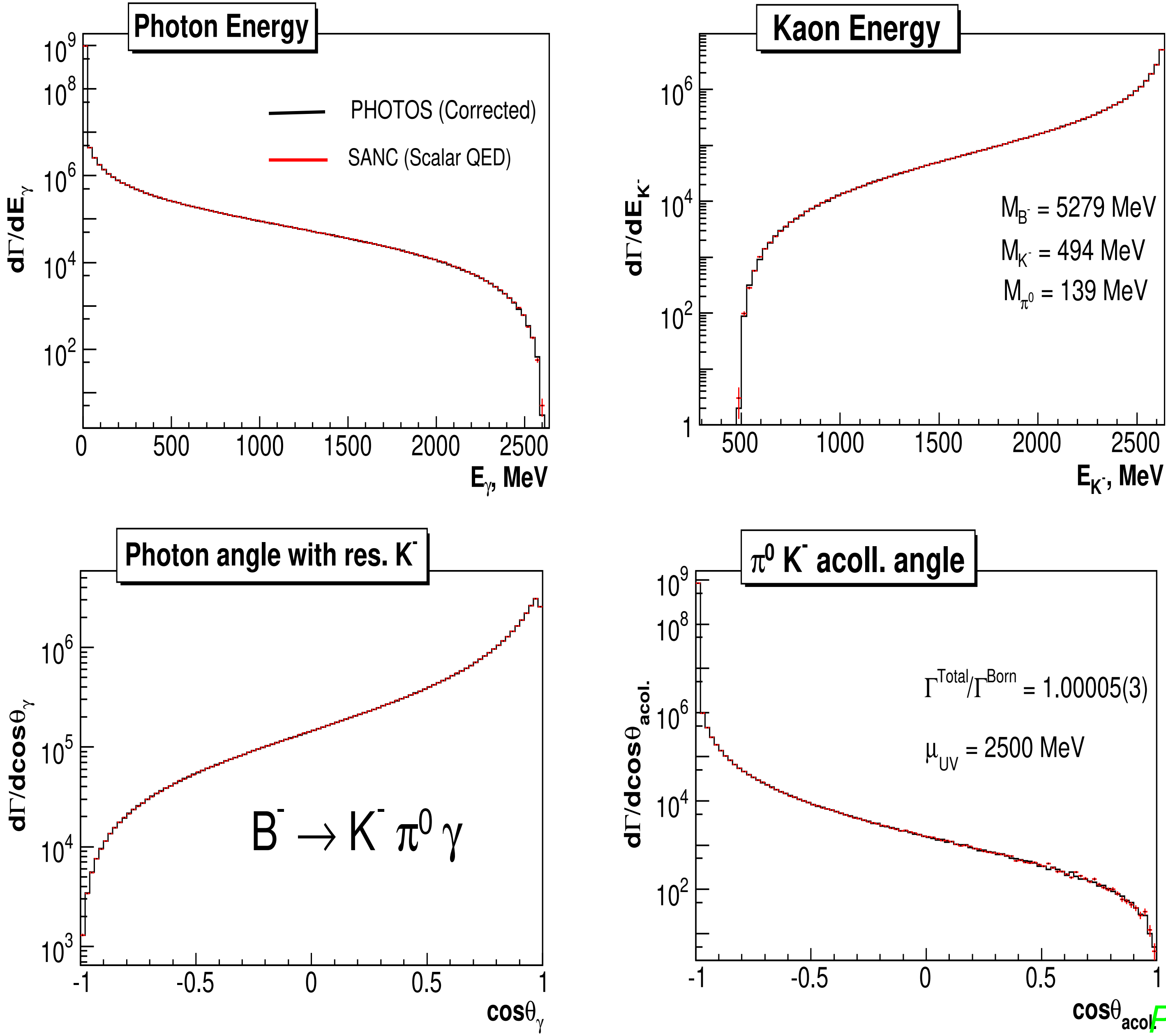
$B^- \rightarrow \pi^0 K^-$: *standard PHOTOS* looks good, but ...



$B^- \to \pi^0 K^-$: standard PHOTOS not perfect

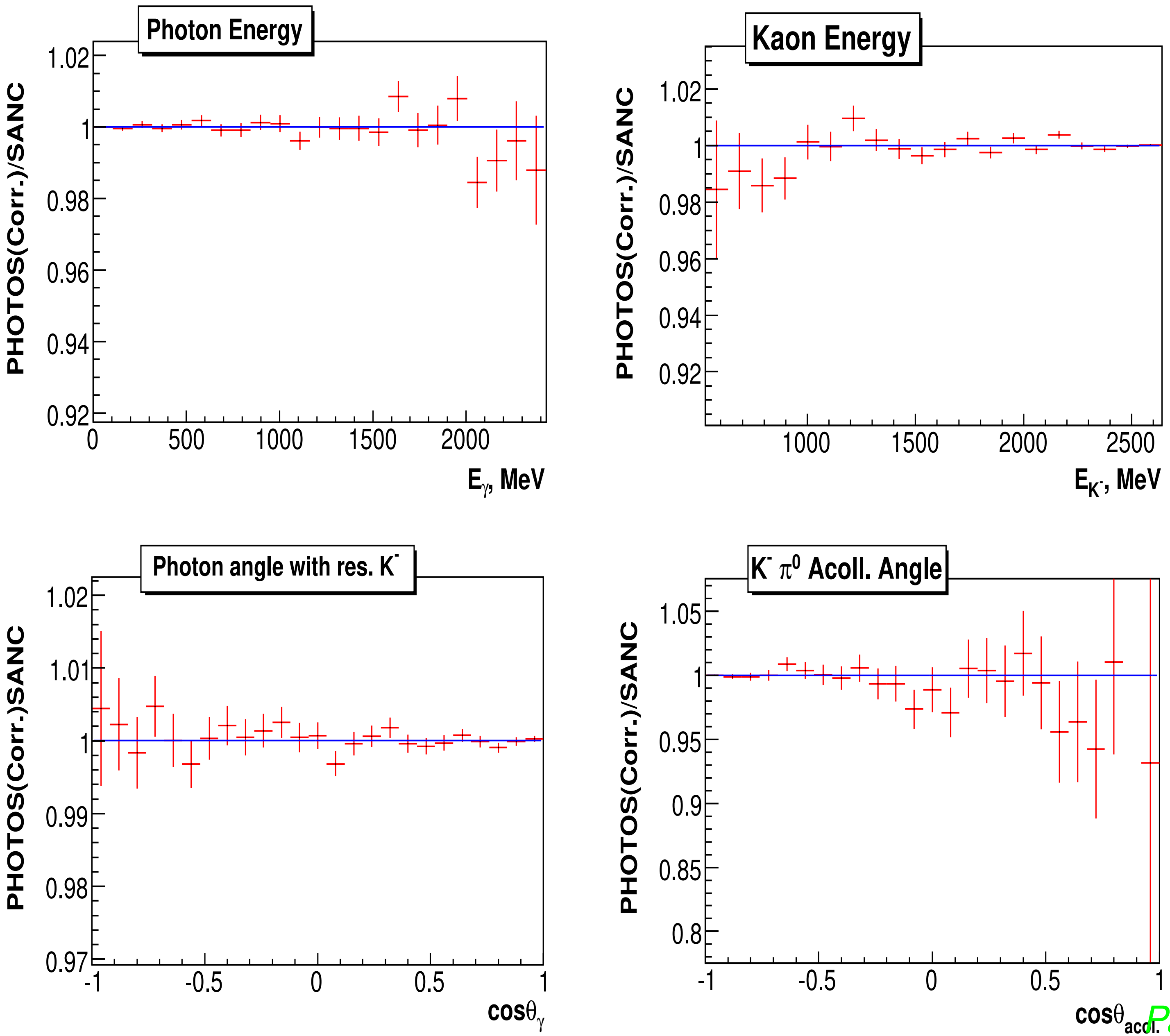


$B^- \to \pi^0 K^-$: NLO improved PHOTOS Looks good



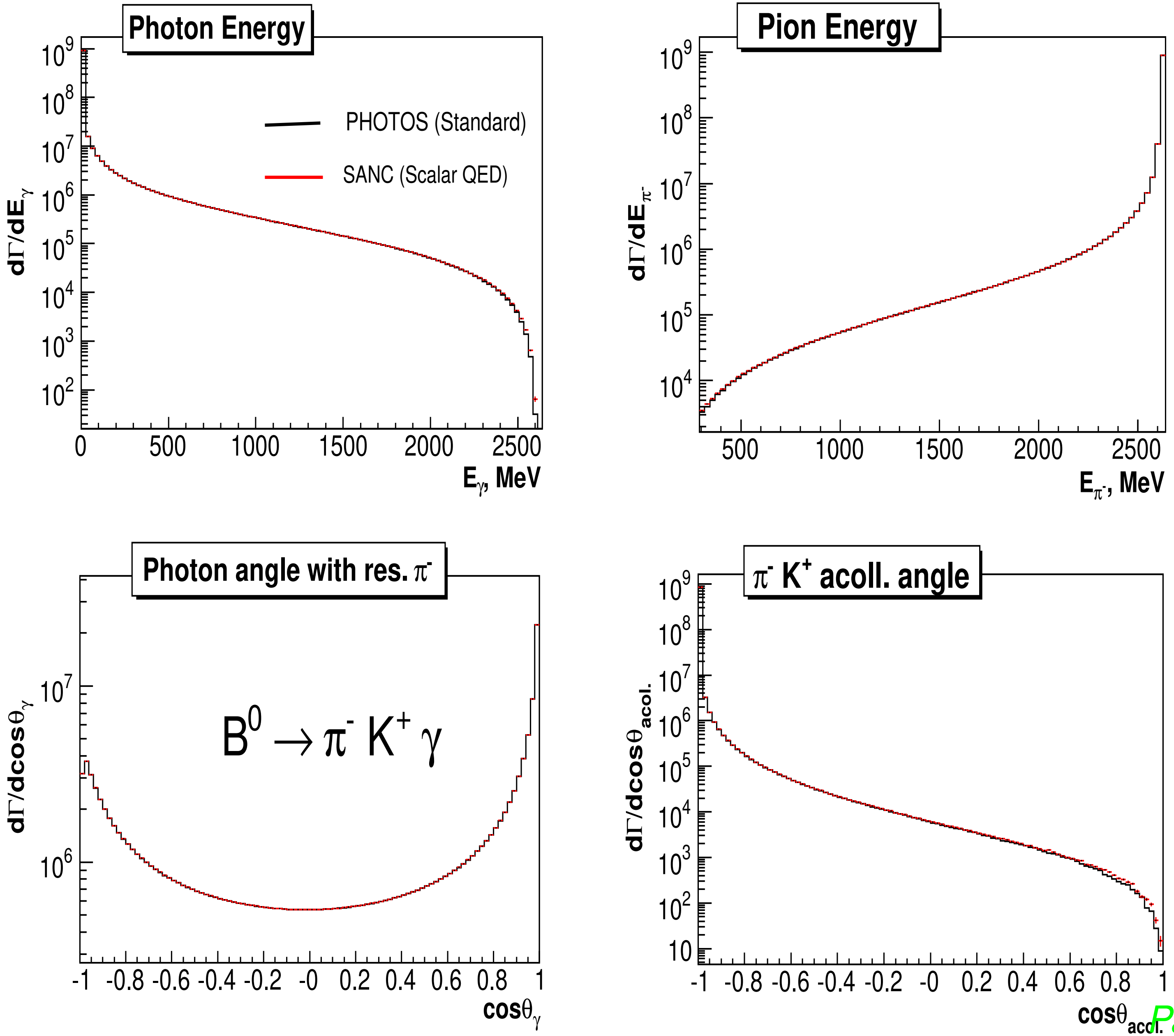
4) Other processes single photon ME , from questionable models but precisely.

$B^- \to \pi^0 K^-$: NLO improved PHOTOS ... and is good.

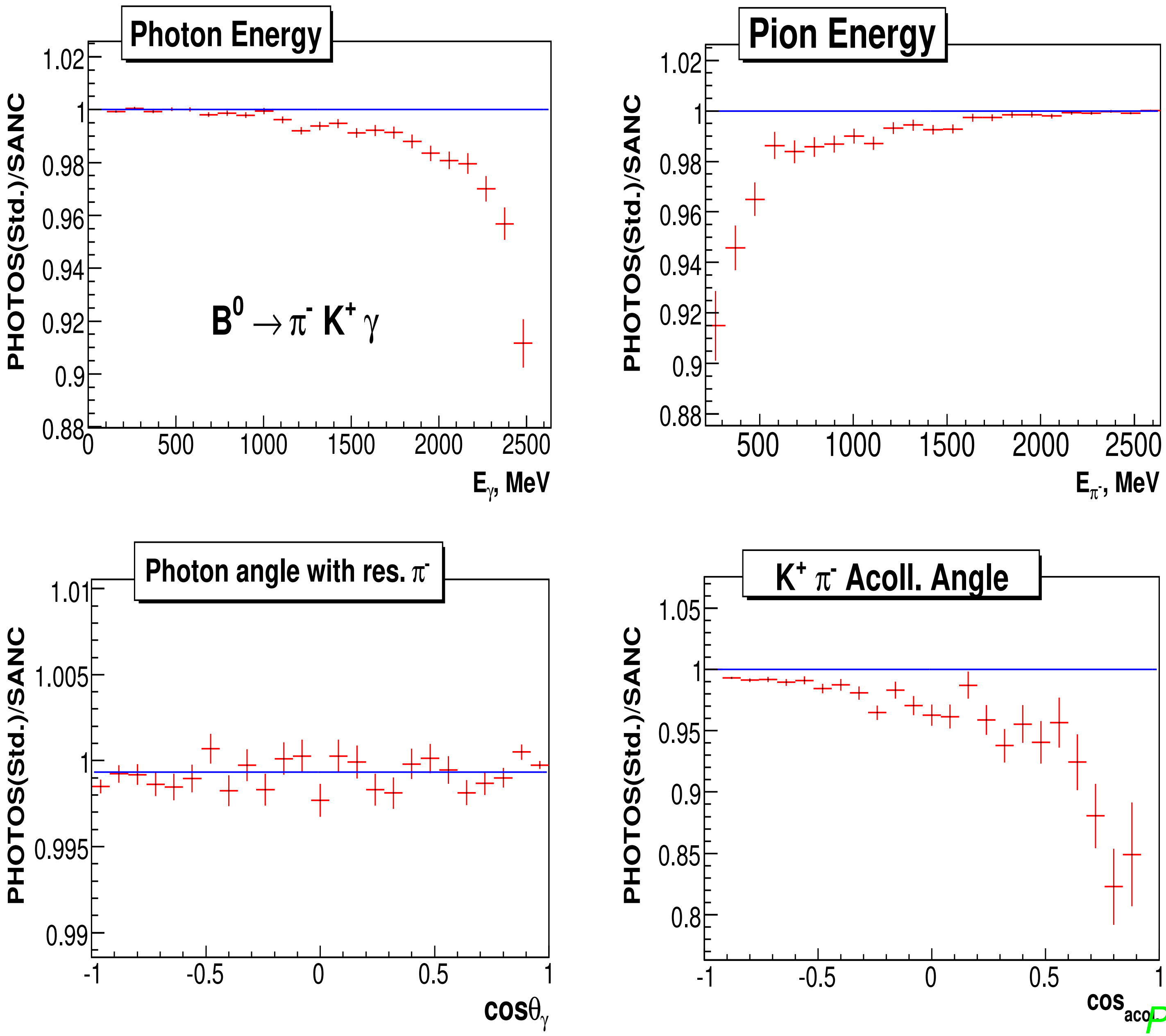


4) Other processes single photon ME , from questionable models but precisely.

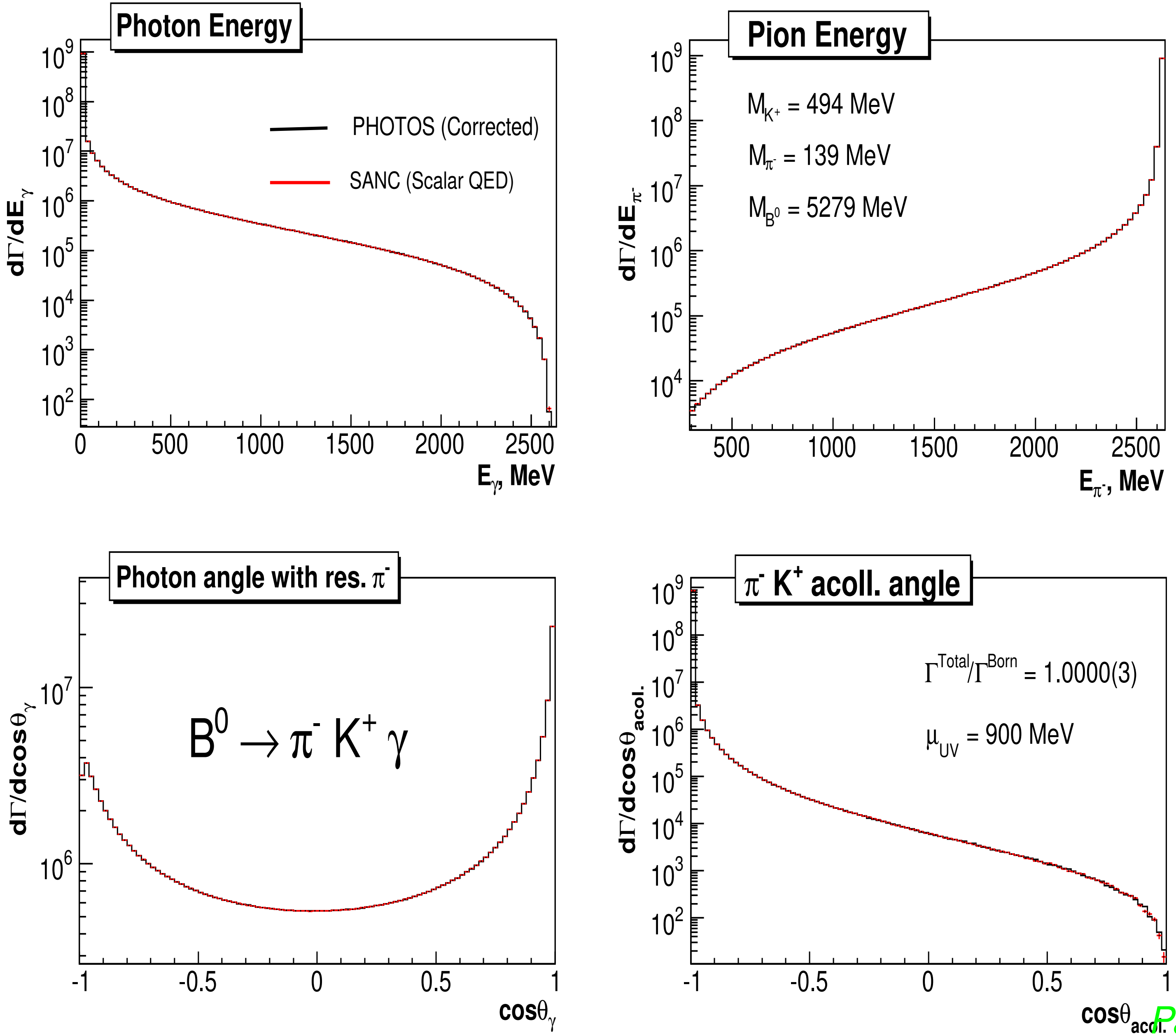
$B^0 \to \pi^- K^+$: *standard PHOTOS* Looks good ...



$B^0 \to \pi^- K^+$: standard PHOTOS ... but not perfect.

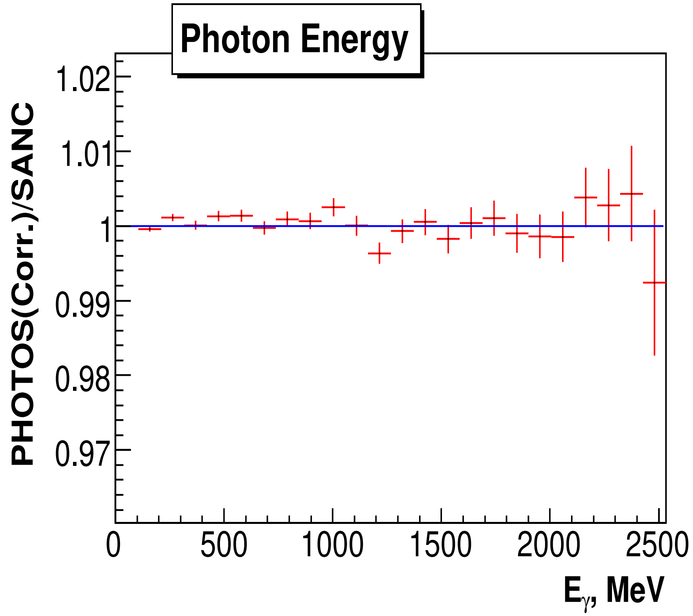


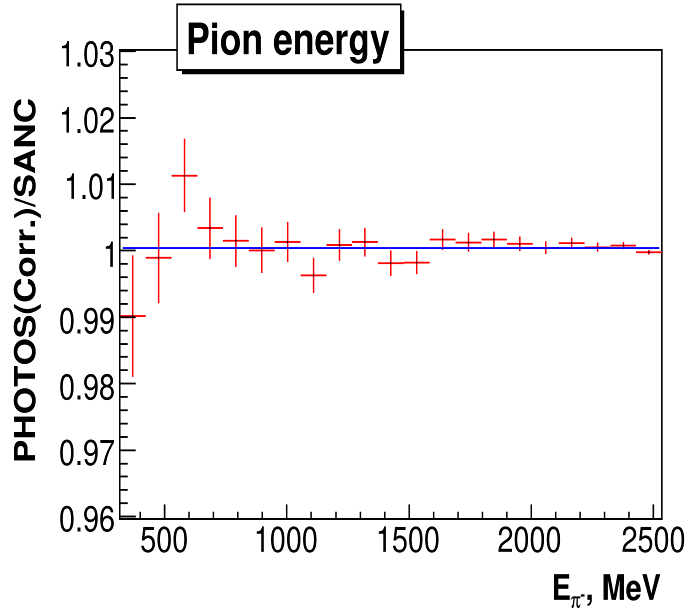
$B^0 \to \pi^- K^+$: NLO improved PHOTOS Looks good ...

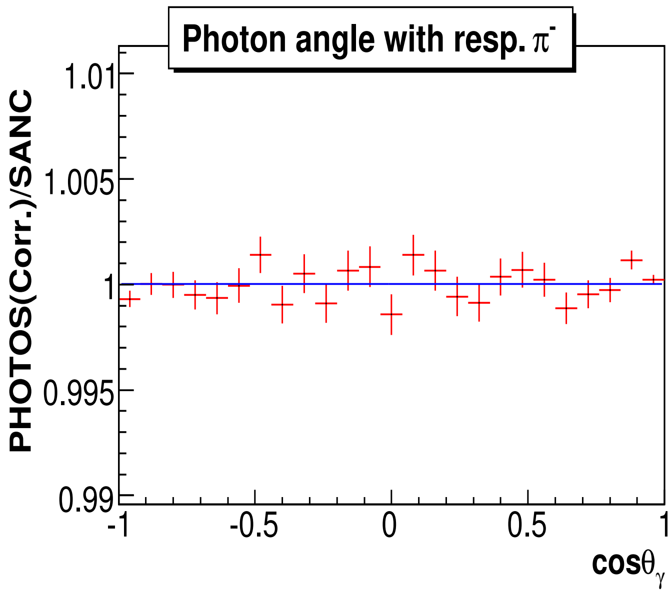


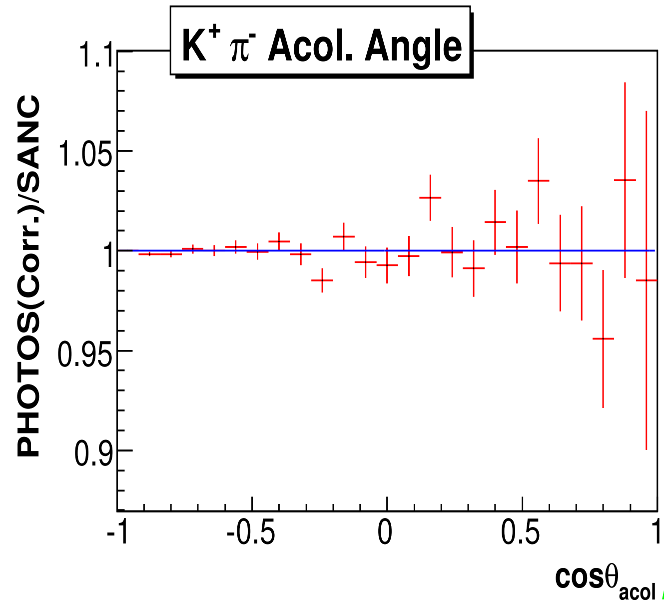
4) Other processes single photon ME , from questionable models but precisely.

$B^0 \to \pi^- K^+$; NLO improved PHOTOS ... also perfect !









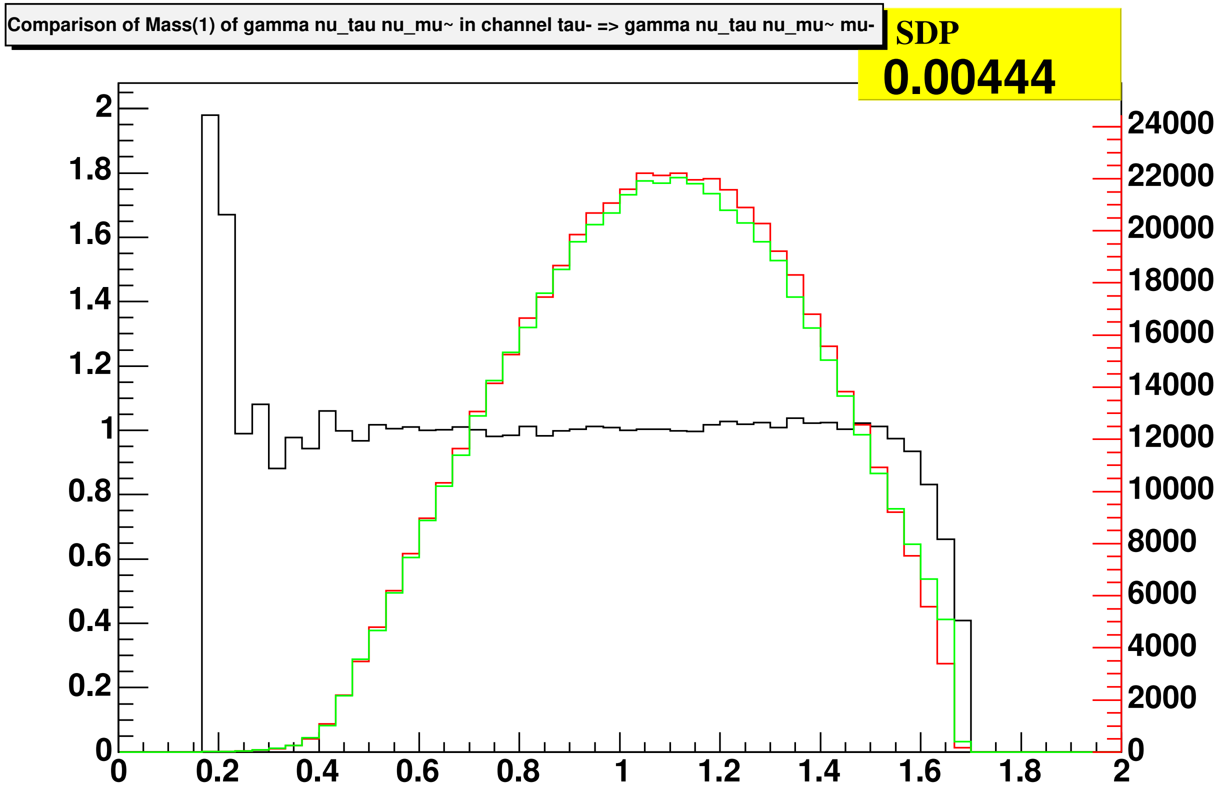
Other processes

Results for W, H, and $\gamma^* \to \pi^+\pi^-$ are of similar quality.

Tests for $K^\pm \to l^\pm \nu \pi^+\pi^-$ are going on. In general for more than two body decays effort of tests was never matching the one presented earlier.

$\tau \to l\nu\bar{\nu}(\gamma)$ PHOTOS vs TAUOLA

Plot of worst agreement for the channel. Distribution of $\gamma\nu_\tau\nu_\mu$ system mass is shown .

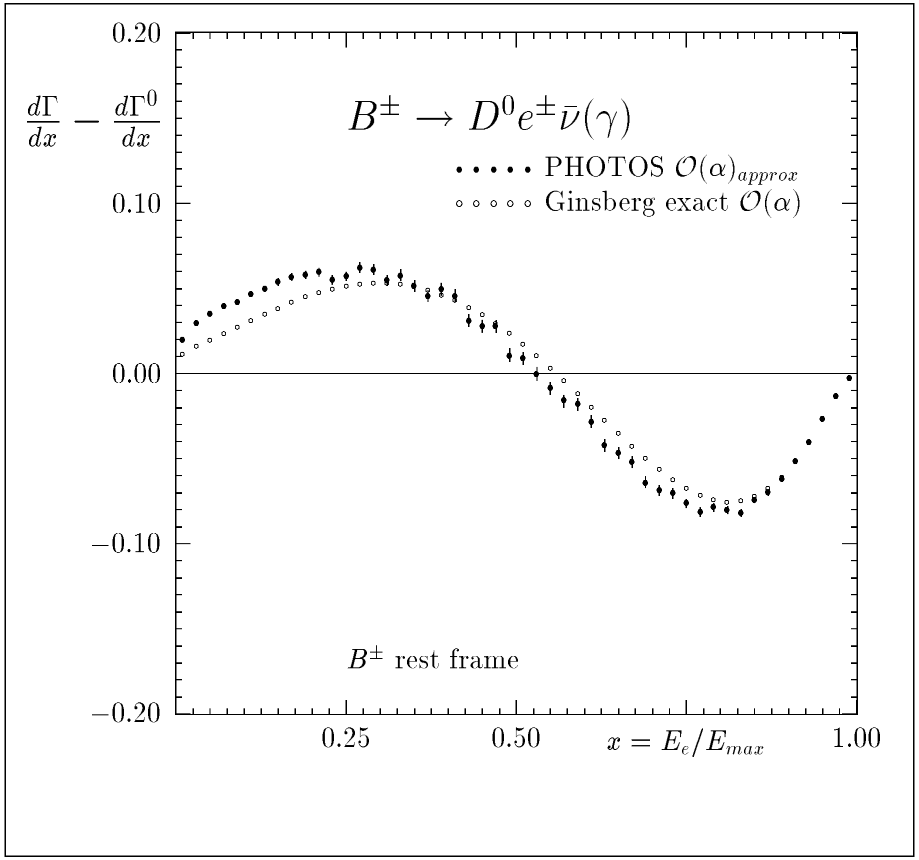


Also the fraction of events with photon above threshold agrees better than permille level.

In TAUOLA complete matrix element, comparison test PHOTOS approximations and design.

another scattered test from 1993

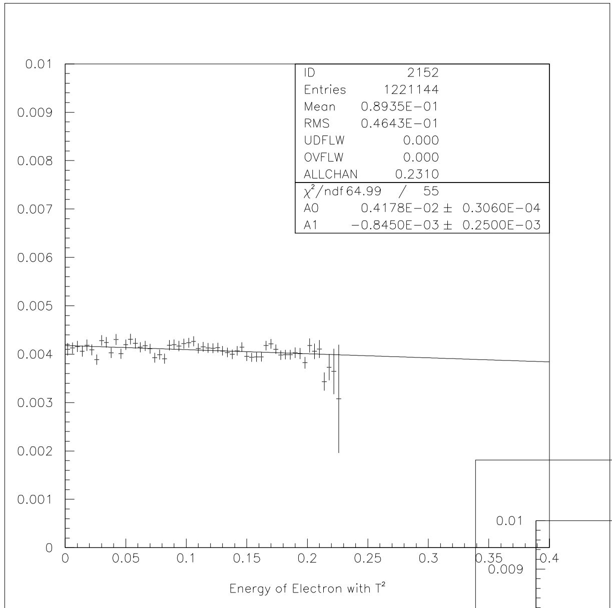
Phys. Lett, B 303 (1993) 163-169

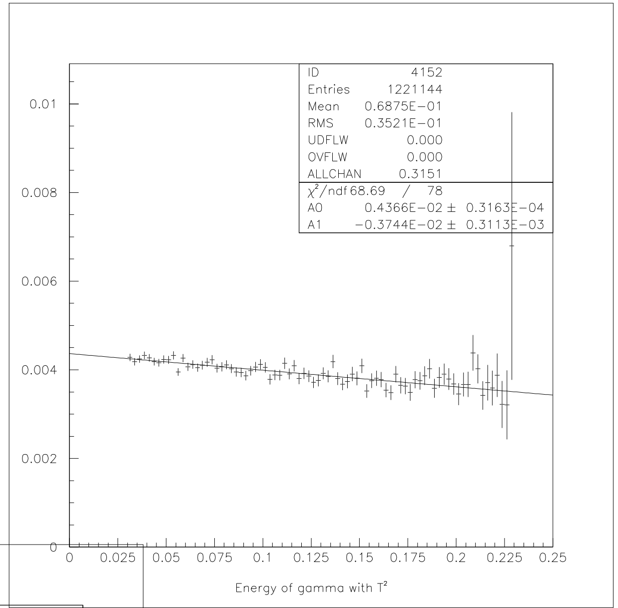


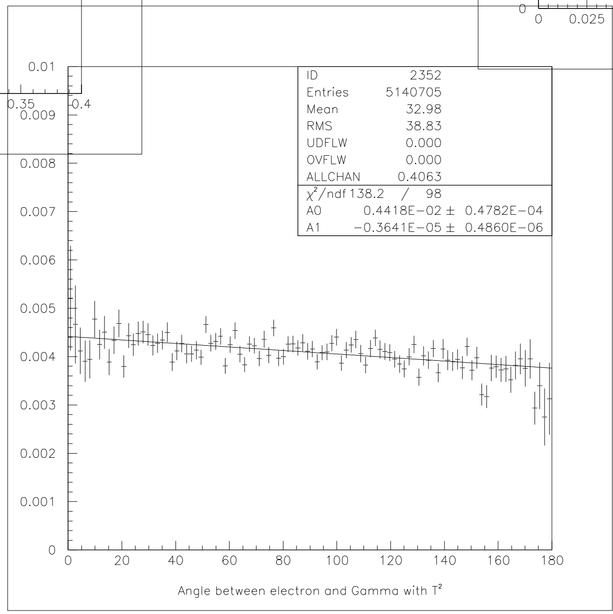
Radiative correction to the decay rate $(d\Gamma/dx - d\Gamma^0/dx)$ for $B^\pm \rightarrow D^0 e^\pm \bar{\nu}(\gamma)$ in the $B^\pm$ rest frame. Open circles are from the exact analytical formula [2], points with the marked statistical errors from PHOTOS applied to JETSET 7.3. A total of 10^7 events have been generated. The results are given in units of $(G_\mu^2 m_B^5/32\pi^3) N_\eta |V_{cb}|^2 |f_+^D|^2$, where $N_\eta = \eta^5 \int_0^1 x^2(1-x)^2/(1-\eta x)dx$ and $\eta = 1 - m_D^2/m_B^2$.

- *"QED bremsstrahlung in semileptonic B and leptonic τ decays"* by E. Richter-Was.
- agreement up to 1%
- disagreement in the low-x region due to missing sub-leading terms
- study performed in 1993.

$K \to \pi e \nu (\gamma)$ PHOTOS w/Interf vs Gasser







This was OK in 2005 but it is not systematic work.

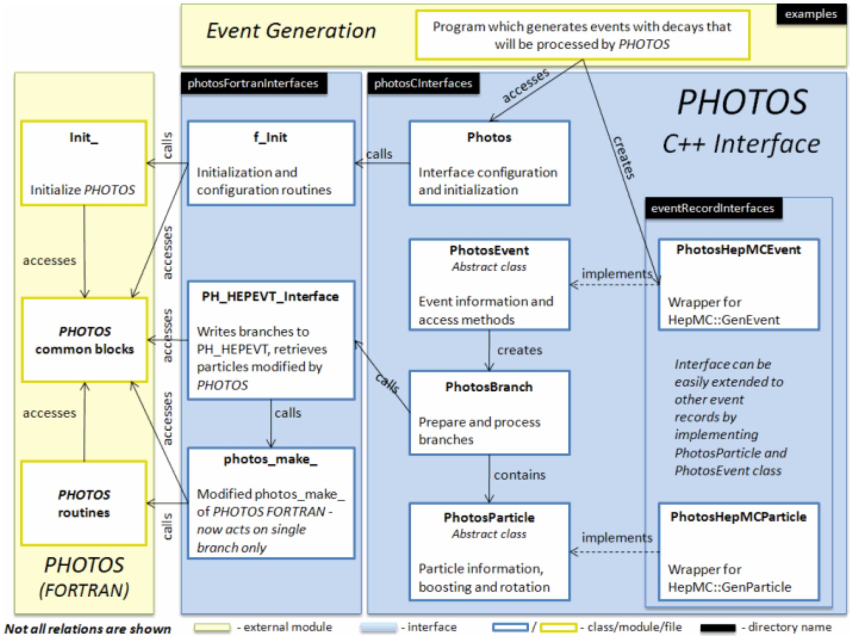
Events with and without photon:

$R = \frac{\Gamma_{K_{e3\gamma}}}{\Gamma_{K_{e3}}}$	PHOTOS %	GASSER %
$5 < E_\gamma < 15\,MeV$	2.38	2.42
$15 < E_\gamma < 45\,MeV$	2.03	2.07
$\Theta_{e,\gamma} > 20$	0.876	0.96

courtesy of NA48 and Prof. L.Litov

This results can be obtained starting from PHOTOS version 2.13.

PHOTOS in C++



- Version in C++ is available and complete.
- Physicswise already more sophisticated than F77, eg. with emission of light fermion pairs is.
- Better access to decaying particle frame. Necessary for installation of process dependent matrix elements.
- `HepMC` event format is a standard, `HEPEVT` interface exists still.
- documentation completed.
- Regurarly updated web page: http://photospp.web.cern.ch/photospp/ with tar balls of program versions.

• It was a pleasure to give a talk on the project which was started when I was regularily visiting Paris (from Marseille). At that time nobody was dreaming that it could become so popular and for long! Potential was underesimated too. **TODAY:**

• Program is based on exact multiphoton phase space and matrix element

• Only for some channels, kernels of exact first order matrix elements installed.

• Spin amplitudes were essential in program development.

• In construction we rely on properties of factorization and those properties of QED matrix elements which lead to YFS exponentiation. Studies of double emission amplitudes were necessary. Eg. $e^+e^- \to \nu_e\bar{\nu}_e\gamma\gamma$, EPJC C44 (2005) 489 paper, emissions from fermions and vectors studied.

• Program can be used as a precision tool. Benchmarks better than 0.1 % .

• Program works as after-burner. This is advantageous, events generated by other programs can be modified. It is a challenge too. Numerical stability and correctness of PHOTOS operation depends on quality of events read in.

• Implementation for FORTRAN HEPEVT and C++ HepMC.

- For some meson decay channels matrix elements are implemented exactly.
- Often in itself this does not improve precision

-*-*-*-

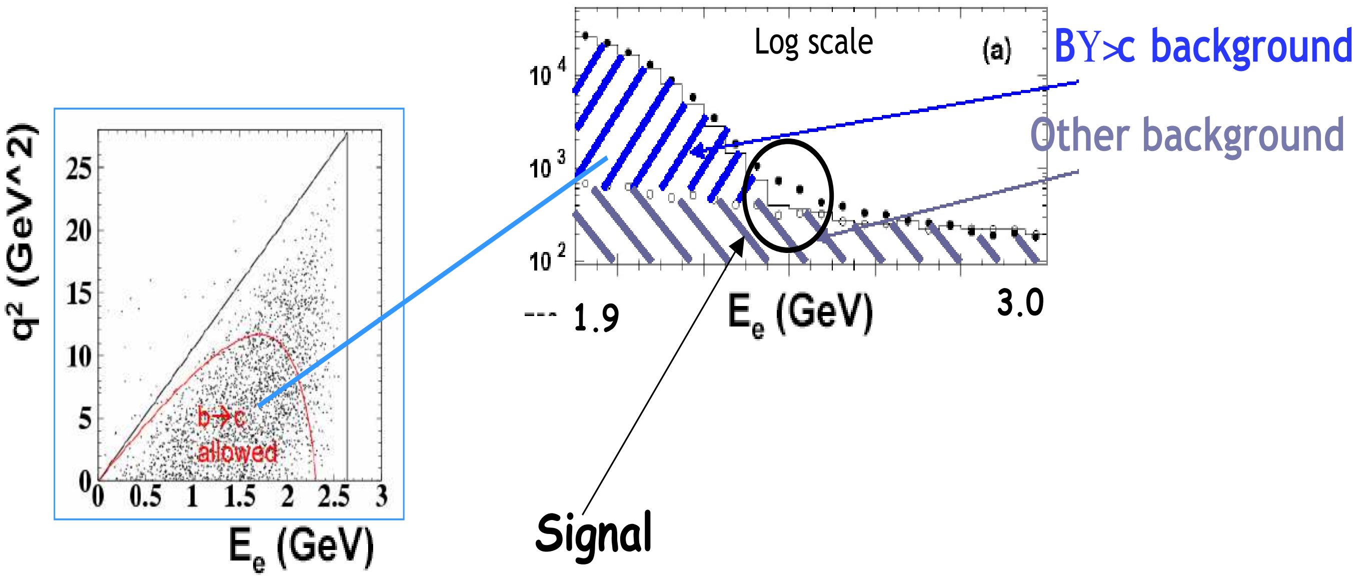
- but **opens the gate** for the data adjusted form-factors •

- **That is your activity of the forthcoming years** •

- **that is the message of my talk** •

Extra transparencies

electron endpoint



Systematic uncertainty:
Model dependent signal efficiencies
$B\_X_c\ell Y$ background estimation

$$\Delta Br(X_u l\nu) = \frac{N(X_u l\nu)}{2N_{BB}\,\varepsilon_{MC}}$$

$$|V_{ub}| = \sqrt{\frac{(1+\delta_{rad})\times\Delta Br(X_u l\nu)}{\tau_B}\frac{1}{R}}$$

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1. we are using **STANDARD and FORMAL** parametrizations of Lorentz group. One can express it with the help of consecutive boosts and rotations.
2. Convenient for Monte Calro event construction!
3. For the definition of coordinate system in the P-rest frame the $\hat{x}$ and $\hat{y}$ axes of the laboratory frame boosted to the rest frame of P can be used. The orthogonal right-handed system can be constructed with their help in a standard way.
4. We choose polar angles θ_1 and ϕ_1 defining the orientation of the four momentum $\bar{k}_2$ in the rest frame of P. In that frame $\bar{k}_1$ and $\bar{k}_2$ are back to back[a], see fig. (1).
5. The previous two points would complete the definition of the two-body phase space, if both $\bar{k}_1$ and $\bar{k}_2$ had no measurable spin degrees of freedom visualizing themselves e.g. through correlations of the secondary decay products' momenta. Otherwise we need to know an additional angle ϕ_X to complete the set of Euler angles defining the relative orientation of the axes of the P rest-frame system with the coordinate system used in the rest-frame of $\bar{k}_2$ (and possibly also of $\bar{k}_1$), see fig. (2).

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[a] In the case of phase space construction for multi-body decays $\bar{k}_2$ should read as a state representing the sum of all decay products of P but $\bar{k}_1$.

6. If both rest-frames of $\bar{k}_1$ and $\bar{k}_2$ are of interest, their coordinate systems are oriented with respect to P with the help of θ_1, ϕ_1, ϕ_X. We assume that the coordinate systems of $\bar{k}_1$ and $\bar{k}_2$ are connected by a boost along the $\bar{k}_2$ direction, and in fact share axes: $z' \uparrow\downarrow z''$, $x' \uparrow\uparrow x''$, $y' \uparrow\downarrow y''$.

7. For the three-body phase space: We take the photon energy k_γ in P rest frame. We calculate: photon, k_1 and k_2 energies, all in $k_1 + k_2$ frame.

8. We use the angles θ, ϕ, in the rest-frame of the $k_1 + k_2$ pair: angle θ is an angle between the photon and k_1 direction (i.e. $-z''$). Angle ϕ defines the photon azimuthal angle around z'', with respect to x'' axis (of the k_2 rest-frame), see fig. (3).

9. If all k_1, k_2 and $k_1 + k_2$ rest-frames exist, then the x-axes for the three frames are chosen to coincide. It is OK, all frmes connected by boosts along z'' see fig. (3).

10. To define orientation of k_2 in P rest-frame coordinate system, and to complete construction of the whole event, we will re-use Euler angles of $\bar{k}_2$: ϕ_X, θ_1 and ϕ_1 (see figs. 4 and 5), defined again of course in the rest frame of P.

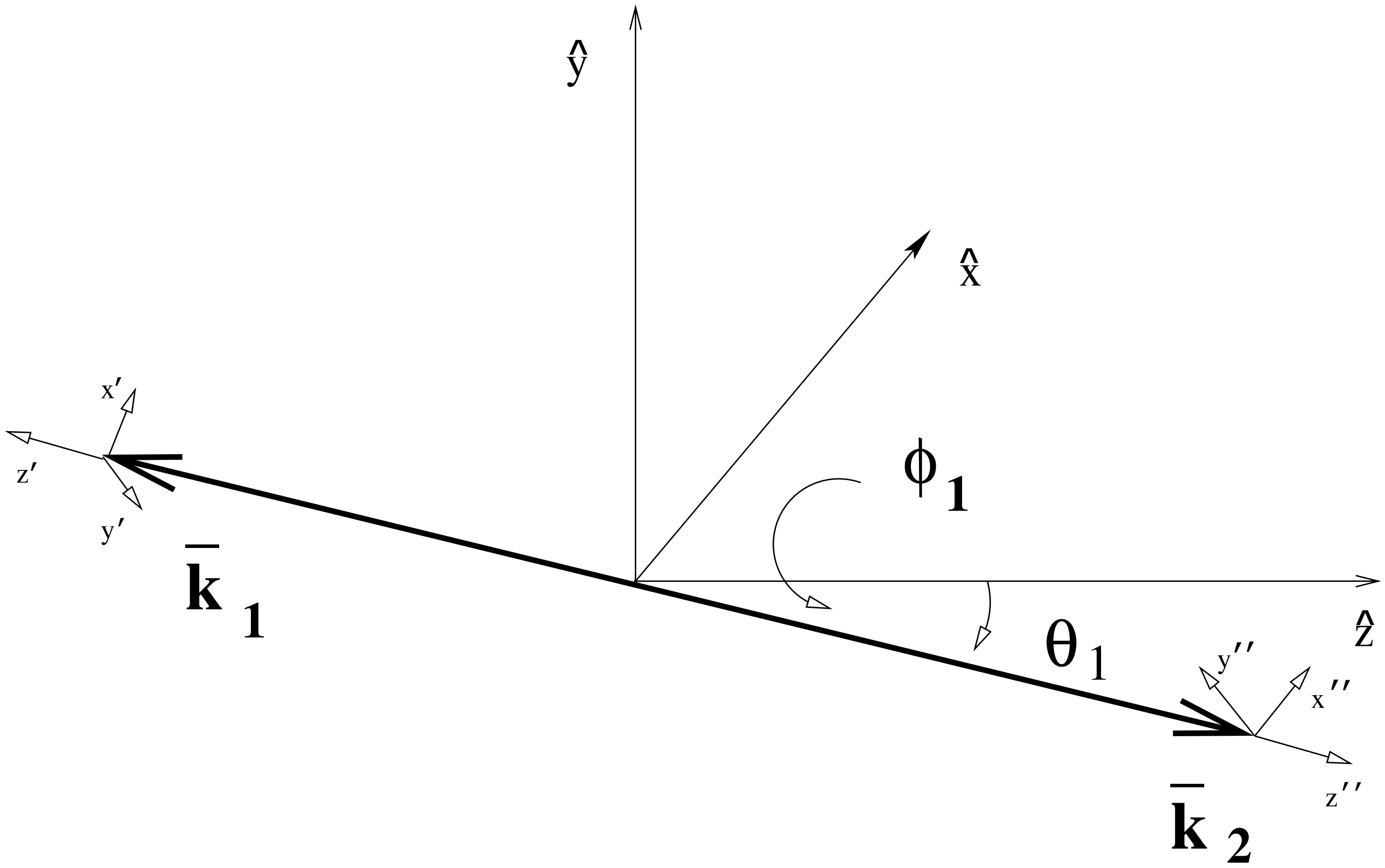

Figure 1: The angles θ_1, ϕ_1 defined in the rest-frame of P and used in parametrization of two-body phase-space.

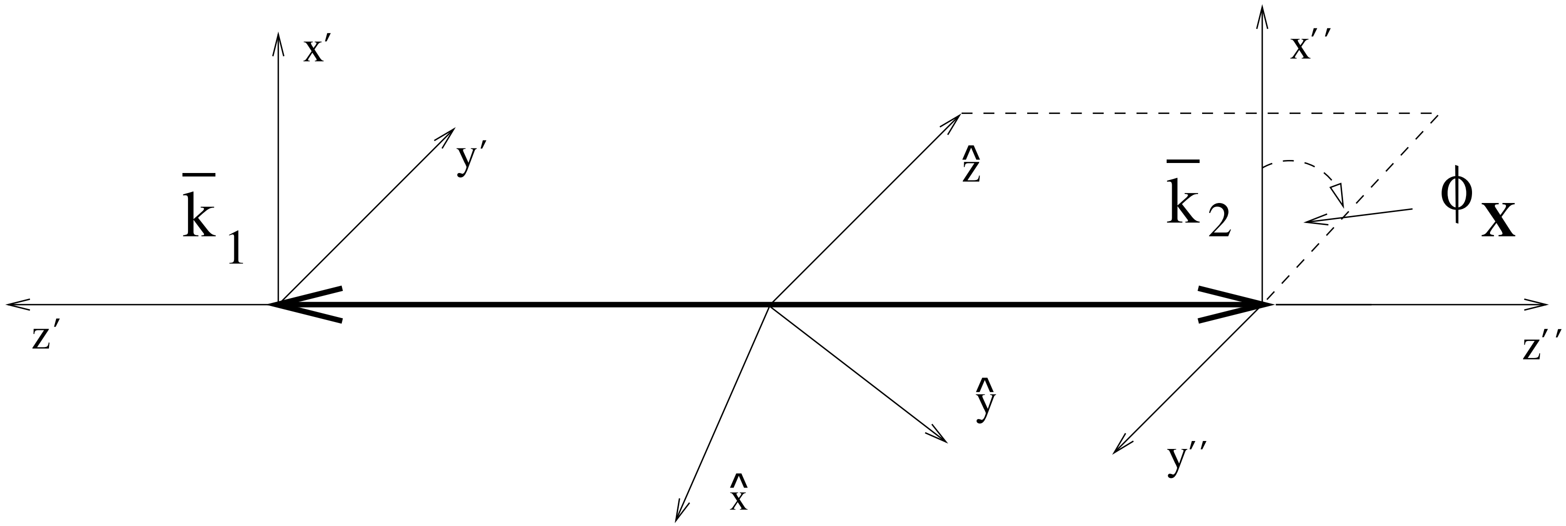

Figure 2: Angle ϕ_X is also defined in the rest-frame of P as an angle between (oriented) planes spanned on: (i) $\bar{k}_1$ and $\hat{z}$-axis of the P rest-frame system, and (ii) $\bar{k}_1$ and x''-axis of the $\bar{k}_2$ rest frame. It completes definition of the phase-space variables if internal orientation of $\bar{k}_1$ system is of interest. In fact, Euler angle ϕ_X is inherited from unspecified in details, parametrization of phase space used to describe possible future decay of $\bar{k}_2$ (or $\bar{k}_1$).

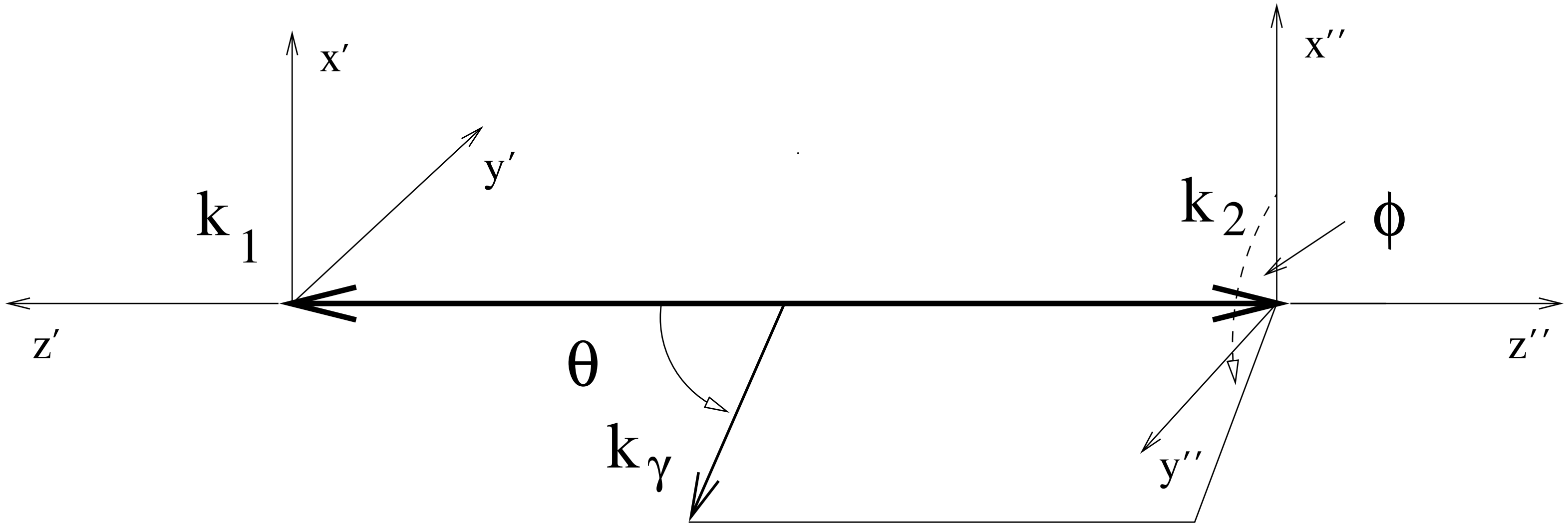

Figure 3: The angles θ, ϕ are used to construct the four-momentum of k_γ in the rest-frame of $k_1 + k_2$ pair (itself not yet oriented with respect to P rest-frame). To calculate energies of k_1, k_2 and photon, it is enough to know m_1, m_2, M and photon energy k_γ of the P rest-frame.

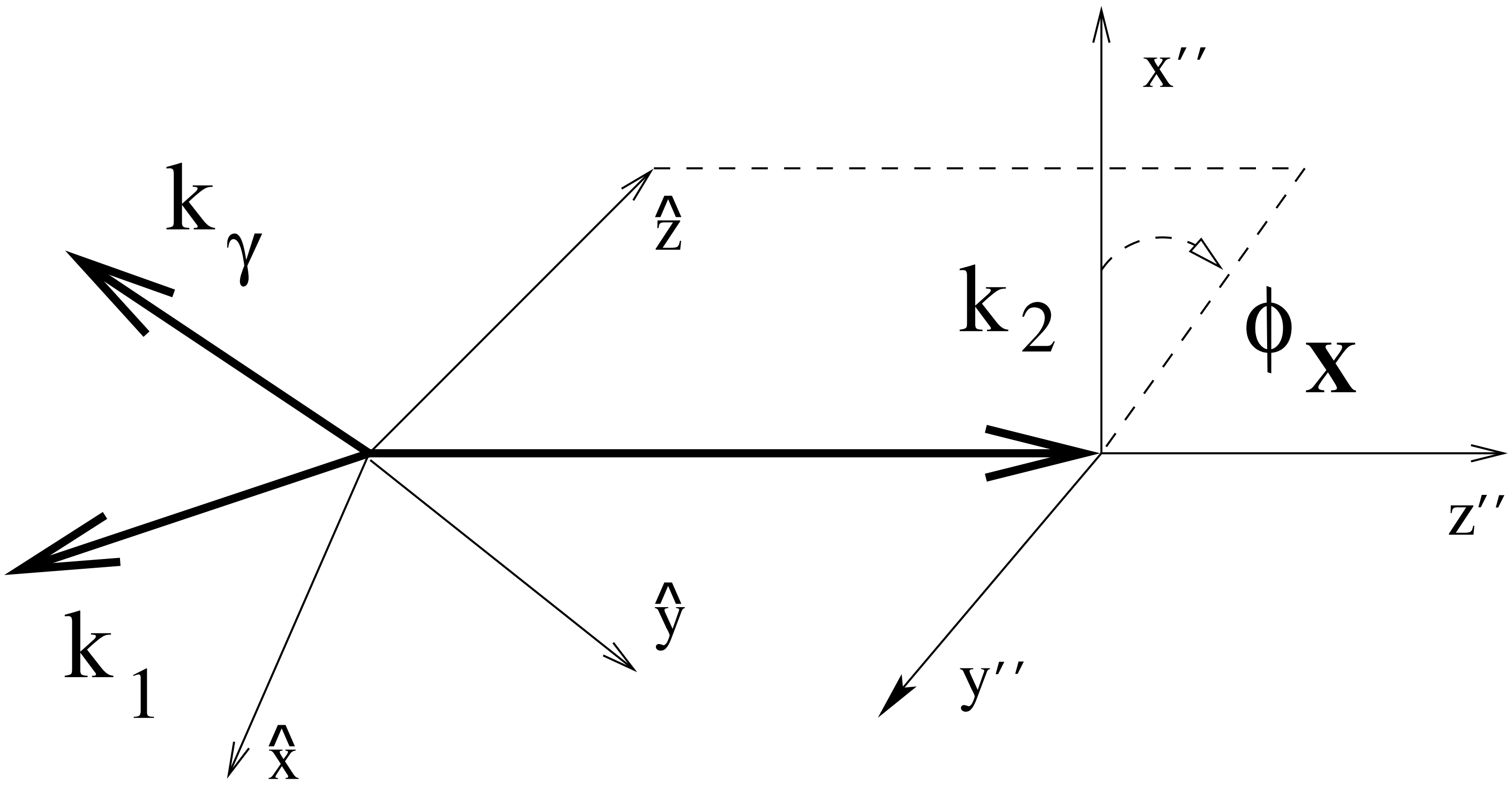

Figure 4: Use of angle ϕ_x in defining orientation of k_1, k_2 and photon in the rest-frame of P. At this step only the plane spanned on P frame axis $\hat{z}$ and k_2 is oriented with respect to $k_2 \times x''$ plane.

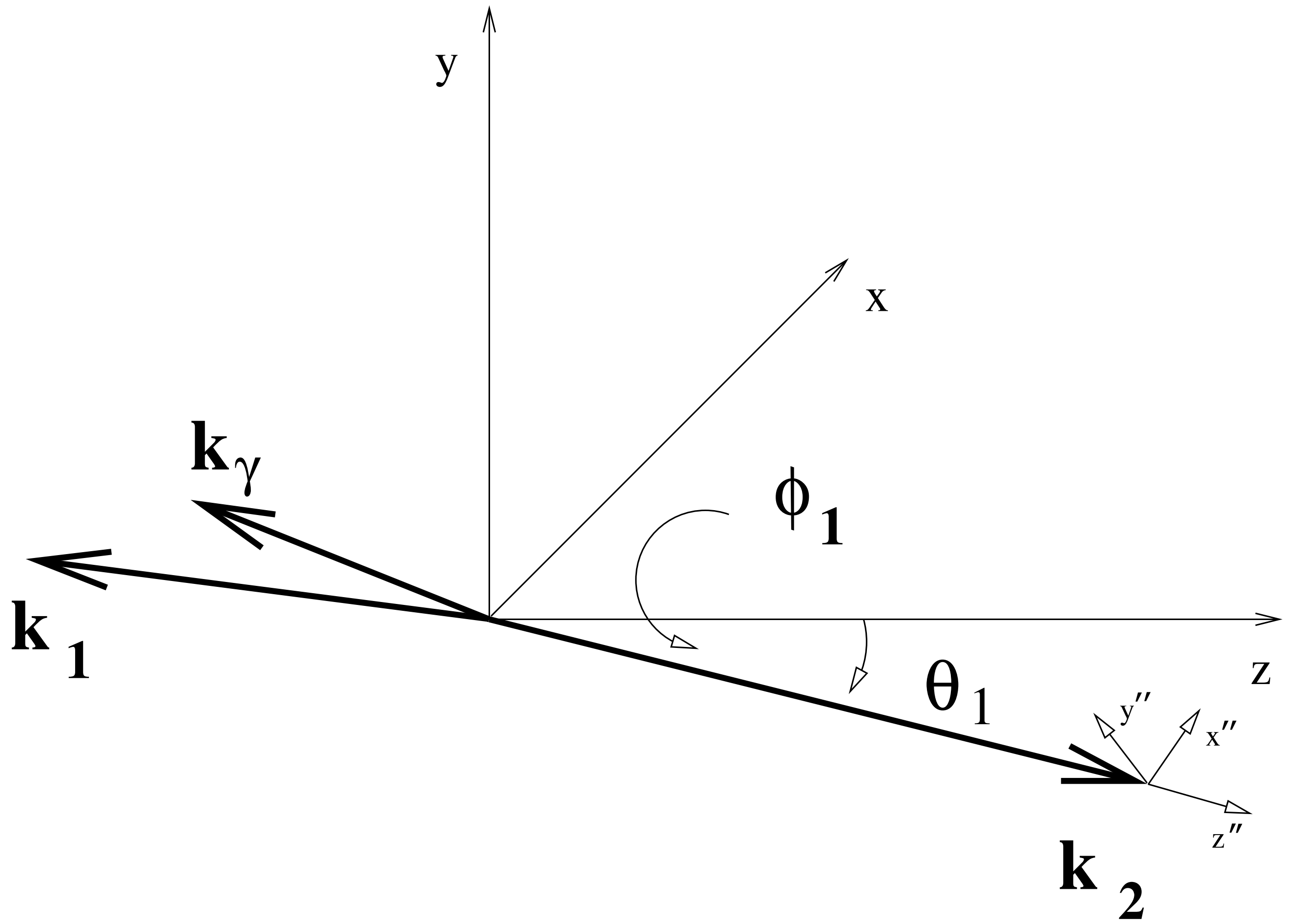

Figure 5: Final step in event construction. Angles θ_1, ϕ_1 are used. The final orientation of k_2 coincide with this of $\bar{k}_2$.

$\mathcal{M}$atrix $\mathcal{E}$lement: $q\bar{q} \to Jgg$ - part proportional to $T^A T^B$ fermion spinors dropped

$$I_{lr}^{(1,2)} = \left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{k_2\cdot e_1}{k_2\cdot k_1} - \frac{\not{e}_1\not{k}_1}{2p\cdot k_1}\right)\not{J}\left(\frac{\not{k}_2\not{e}_2}{2q\cdot k_2} + \frac{k_1\cdot e_2}{k_1\cdot k_2} - \frac{q\cdot e_2}{q\cdot k_2}\right)$$

$$I_{ll}^{(1,2)} = \frac{p\cdot k_2}{p\cdot k_1 + p\cdot k_2 - k_1\cdot k_2}\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{k_2\cdot e_1}{k_2\cdot k_1} - \frac{\not{e}_1\not{k}_1}{2p\cdot k_1}\right)\left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2} - \frac{\not{e}_2\not{k}_2}{2p\cdot k_2}\right)\not{J}$$

$$I_{rr}^{(1,2)} = \not{J}\,\frac{q\cdot k_1}{q\cdot k_1 + q\cdot k_2 - k_1\cdot k_2}\left(\frac{q\cdot e_1}{q\cdot k_1} - \frac{k_2\cdot e_1}{k_2\cdot k_1} - \frac{\not{k}_1\not{e}_1}{2q\cdot k_1}\right)\left(\frac{q\cdot e_2}{q\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2} - \frac{\not{k}_2\not{e}_2}{2q\cdot k_2}\right)$$

$$I_{e}^{(1,2)} = \not{J}\left(1 - \frac{p\cdot k_2}{p\cdot k_1 + p\cdot k_2 - k_1\cdot k_2} - \frac{q\cdot k_1}{q\cdot k_1 + q\cdot k_2 - k_1\cdot k_2}\right)\left(\frac{k_1\cdot e_2}{k_1\cdot k_2}\,\frac{k_2\cdot e_1}{k_1\cdot k_2} - \frac{e_1\cdot e_2}{k_1\cdot k_2}\right)$$

Remainder:

$$I_{p}^{(1,2)} = -\frac{1}{4}\,\frac{1}{p\cdot k_1 + p\cdot k_2 - k_1\cdot k_2}\left(\frac{\not{e}_1\not{k}_1\not{e}_2\not{k}_2 - \not{e}_2\not{k}_2\not{e}_1\not{k}_1}{k_1\cdot k_2}\right)\not{J}$$

$$I_{q}^{(1,2)} = -\frac{1}{4}\,\not{J}\,\frac{1}{q\cdot k_1 + q\cdot k_2 - k_1\cdot k_2}\left(\frac{\not{k}_1\not{e}_1\not{k}_2\not{e}_2 - \not{k}_2\not{e}_2\not{k}_1\not{e}_1}{k_1\cdot k_2}\right)$$

Matrix Element: $q\bar{q} \to Jgg$ - part proportional to $T^B T^A$ fermion spinors dropped

$$I_{lr}^{(2,1)} = \left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2} - \frac{\not{e}_2\not{k}_2}{2p\cdot k_2}\right)\not{J}\left(\frac{\not{k}_1\not{e}_1}{2q\cdot k_1} + \frac{k_2\cdot e_1}{k_2\cdot k_1} - \frac{q\cdot e_1}{q\cdot k_1}\right)$$

$$I_{ll}^{(2,1)} = \frac{p\cdot k_1}{p\cdot k_2 + p\cdot k_1 - k_2\cdot k_1}\left(\frac{p\cdot e_2}{p\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2} - \frac{\not{e}_2\not{k}_2}{2p\cdot k_2}\right)\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{k_2\cdot e_1}{k_2\cdot k_1} - \frac{\not{e}_1\not{k}_1}{2p\cdot k_1}\right)\not{J}$$

$$I_{rr}^{(2,1)} = \not{J}\,\frac{q\cdot k_2}{q\cdot k_2 + q\cdot k_1 - k_2\cdot k_1}\left(\frac{q\cdot e_2}{q\cdot k_2} - \frac{k_1\cdot e_2}{k_1\cdot k_2} - \frac{\not{k}_2\not{e}_2}{2q\cdot k_2}\right)\left(\frac{q\cdot e_1}{q\cdot k_1} - \frac{k_2\cdot e_1}{k_2\cdot k_1} - \frac{\not{k}_1\not{e}_1}{2q\cdot k_1}\right)$$

$$I_{e}^{(2,1)} = \not{J}\left(1 - \frac{p\cdot k_1}{p\cdot k_2 + p\cdot k_1 - k_2\cdot k_1} - \frac{q\cdot k_2}{q\cdot k_2 + q\cdot k_1 - k_2\cdot k_1}\right)\left(\frac{k_2\cdot e_1}{k_2\cdot k_1}\;\frac{k_1\cdot e_2}{k_2\cdot k_1} - \frac{e_2\cdot e_1}{k_2\cdot k_1}\right)$$

$$I_{p}^{(2,1)} = -\frac{1}{4}\,\frac{1}{p\cdot k_2 + p\cdot k_1 - k_2\cdot k_1}\left(\frac{\not{e}_2\not{k}_2\not{e}_1\not{k}_1 - \not{e}_1\not{k}_1\not{e}_2\not{k}_2}{k_2\cdot k_1}\right)\not{J}$$

$$I_{q}^{(2,1)} = -\frac{1}{4}\,\not{J}\,\frac{1}{q\cdot k_2 + q\cdot k_1 - k_2\cdot k_1}\left(\frac{\not{k}_2\not{e}_2\not{k}_1\not{e}_1 - \not{k}_1\not{e}_1\not{k}_2\not{e}_2}{k_2\cdot k_1}\right)$$

For QCD we have separation too; 12 gauge invariant parts

- Terms like

$$\left(\frac{p\cdot e_1}{p\cdot k_1} - \frac{k_2\cdot e_1}{k_2\cdot k_1} - \frac{\not e_1 \not k_1}{2p\cdot k_1}\right) \qquad A$$

once integrated over part of phase space give Atarelli-Parisi kernel

- Terms

$$\frac{q\cdot k_1}{q\cdot k_1 + q\cdot k_2 - k_2\cdot k_1} \qquad B$$

if combined with phase space Jacobians can be used to redefine fermionic fields from $v(q)$ to $v(q-k_2)$ for example. Term of such type appeared already in scalar QED (normalization of hadronic current).

- No QCD prototype, for amplitudes $\longrightarrow$ my paper with A. van Hameren.

- For studies of amplitudes used in PHOTOS see my papers with: P. Golonka, G. Nanava (Z W H decays), G.Nanava Qinjun Xy (B γ^* decays), soon $K^\pm \to l^\pm \nu \pi^+ \pi^-$.