$B \to \gamma\ell\nu$ as an avenue to B-meson LCDAs

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MB and J. Rohrwild, Eur.Phys.J. C71 (2011) 1818, arXiv:1110.3228 [hep-ph]
MB, V.M. Braun, Y. Ji, Y.B. Wei, JHEP 1807 (2018) 154, arXiv:1804.04962 [hep-ph]

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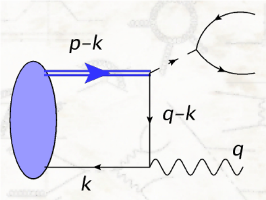
Introduction



Theory for $E_\gamma \gg \Lambda_{\rm QCD}$, formally $\mathcal{O}(m_b)$

$$\Gamma(\ell\nu) \propto f_B^2 \left(\frac{m_\ell}{m_B}\right)^2$$

$$\Gamma(\gamma\ell\nu) \propto f_B^2 \,\frac{\alpha_{\rm em}}{4\pi} \left(\frac{m_B}{\lambda_B}\right)^2$$

No helicity suppression.
Simplest, non-trivial, hard-exclusive B decay when $2E_\gamma = \mathcal{O}(m_b)$.

Involves B meson light-cone distribution amplitude:

$$iF_{\rm stat}(\mu)\Phi_{B+}(\omega,\mu) = \frac{1}{2\pi}\int dt\, e^{it\omega}\,\langle 0|(\bar q_s Y_s)(tn_-)\not{n}_-\gamma_5(Y_s^\dagger h_v)(0)|\bar B_v\rangle_\mu$$

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega}\,\Phi_{B+}(\omega,\mu), \qquad \sigma_n(\mu) = \lambda_B(\mu)\int_0^\infty \frac{d\omega}{\omega}\,\ln^n\frac{\mu_0}{\omega}\,\Phi_{B+}(\omega,\mu)$$

At LP, need only inverse, inverse-log moments, since $\omega \sim \Lambda_{\rm QCD}$.
Not related to local operators.

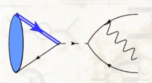
$B \to \gamma\ell\nu$ amplitude (to first order in QED, exact in QCD)

$$\langle \ell\bar\nu\gamma|\ell\gamma^\mu(1-\gamma_5)\nu\cdot\bar u\gamma_\mu(1-\gamma_5)b|B^-\rangle =$$

$$= -ie\epsilon_\nu^\star\left[\langle\ell\bar\nu|\bar\ell\gamma^\mu(1-\gamma_5)\nu)|0\rangle\cdot\int d^4x\, e^{iqx}\langle 0|\mathrm{T}\{j^\nu_{\rm em}(x)(\bar u\gamma_\mu(1-\gamma_5)b)(0)\}|B^-\rangle\right.$$

$$\left.+\int d^4x\, e^{iqx}\langle\ell\bar\nu|\mathrm{T}\{j^\nu_{\rm em}(x)(\bar\ell\gamma_\mu(1-\gamma_5)\nu)(0)\}|0\rangle\cdot\langle 0|\bar u\gamma^\mu(1-\gamma_5)b|B^-\rangle\right]$$

$$= e\epsilon_\nu^\star\,\bar u_\ell\gamma_\mu(1-\gamma_5)u_\nu\cdot T^{\nu\mu}(p,q) - ieQ_\ell f_B\cdot\bar u_\ell\not\epsilon^\star(1-\gamma_5)u_\nu$$



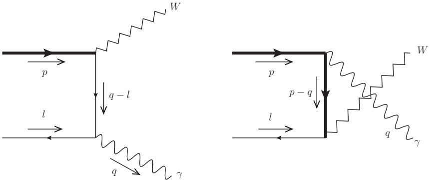
$$iT_{\nu\mu}(p,q) = i\epsilon_{\mu\nu\rho\sigma}v^\rho q^\sigma\, F_V(E_\gamma) + (g_{\mu\nu}v\cdot q - v_\nu q_\mu)\,\hat F_A(E_\gamma) + \frac{v_\nu v_\mu}{v\cdot q}f_B m_B + q_\nu\text{-terms}$$

Contact term fixed by the em Ward identity $q_\nu T^{\nu\mu} = -if_B p^\mu$ (Khodjamirian, Wyler, 2001). Replace

$$(g_{\mu\nu}v\cdot q - v_\nu q_\mu)\,\hat F_A(E_\gamma) + \frac{v_\nu v_\mu}{v\cdot q}f_B m_B \to (g_{\mu\nu}v\cdot q - v_\nu q_\mu)\,F_A(E_\gamma) + g_{\mu\nu}f_B$$

to cancel the lepton emission term. Then $F_A = \hat F_A + \dfrac{Q_\ell f_B}{E_\gamma}$ $\Rightarrow$ two form factors $F_{V,A}$

$B \to \gamma \ell \nu$ in QCD (Korchemsky, Pirjol, Yan, 1999 [$k_\perp$ factorization]; ...)



Intermediate light-quark propagator has hard-collinear virtuality $n_+q\, n_-l \sim m_b\Lambda$

$$\frac{i(\not{q}-\not{l})}{(q-l)^2} = -\frac{i\not{q}}{2q\cdot l} + \ldots \sim \frac{1}{\Lambda}$$

$$F_V = +F_A = \frac{Q_u m_B f_B}{2E_\gamma \lambda_B}$$

Photon inherits the helicity of the energetic up quark. Leading power.

Intermediate heavy-quark propagator has hard virtuality m_b^2

$$\frac{i(\not{p}-\not{q})+m_b}{(p-q)^2-m_b^2} = \frac{i\not{q}}{2p\cdot q} + \ldots \sim \frac{1}{m_b}$$

$$F_V = -F_A = \frac{Q_b m_B f_B}{2E_\gamma m_b}$$

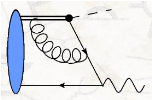
Photon has opposite helicity, because the weak current couples to the spectator quark. Power-suppressed.

Factorization at leading power in the heavy-quark mass expansion

(Lunghi, Pirjol, Wyler, 2002; Bosch, Hill, Lange, Neubert, 2003; Descotes-Genon, Sachrajda, 2002)

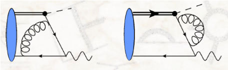
Integrate out hard virtualities:

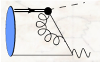
$$iT_{\nu\mu}(p,q) = C(E_\gamma,\mu)\int d^4x\, e^{iqx}\langle 0|\mathrm{T}\{[j_{\nu,\mathrm{em}}(x)]_{\mathrm{SCET}}, (\bar\xi W_c\gamma_\mu(1-\gamma_5)h_v)(0)_\mu\}|B^-\rangle$$



$$[j_{\nu,\mathrm{em}}(x)]_{\mathrm{SCET}} = (\bar q_s W_c^\dagger\gamma_\nu\xi)(x)$$
$$+\, i\int d^4y\mathrm{T}\{\bar\xi\Big[\gamma_\nu\frac{1}{in_+D_c}i\not{D}_\perp + i\not{D}_\perp\frac{1}{in_+D_c}\gamma_\nu\Big]\frac{\not{n}_+}{2}\xi(y), (\bar q_s W_c^\dagger i\not{D}_\perp\xi)(x)\}$$

Integrate out hard-collinear virtualities:





$$F_V(E_\gamma) = F_A(E_\gamma) = C(E_\gamma,\mu)\int dt\, J(E_\gamma,t,\mu)\underbrace{\langle 0|(\bar q_s Y_s)(tn_-)\not{n}_-\gamma_5(Y_s^\dagger h_v)(0)|\bar B_v\rangle_\mu}_{\tilde\Phi_{B+}(t,\mu)}$$

$$= \frac{Q_u m_B f_B}{2E_\gamma\lambda_B(\mu)}R(E_\gamma,\mu)$$

where R can be calculated in perturbation theory and depends on σ_i, $i\le 2n$ at $\mathcal{O}(\alpha_s^n)$.

Radiative corrections (NLL [NNLL in Sudakov coutning])

(MB, Rohrwild, 2011; Descotes-Genon, Sachrajda, 2002; Lunghi, Pirjol, Wyler, 2002; Bosch, Hill, Lange, Neubert, 2003)

$$R(E_\gamma,\mu) = C(E_\gamma,\mu_{h1}) K^{-1}(\mu_{h2}) \times U(E_\gamma,\mu_{h1},\mu_{h2},\mu) \times J(E_\gamma,\mu)$$

$$C(E_\gamma,\mu) = 1 + \frac{\alpha_s C_F}{4\pi}\left(-2\ln^2\frac{2E_\gamma}{\mu} + 5\ln\frac{2E_\gamma}{\mu} - \frac{3-2x}{1-x}\ln x - 2\mathrm{Li}_2(1-x) - 6 - \frac{\pi^2}{12}\right)$$

$$J(E_\gamma,\mu) = 1 + \frac{\alpha_s C_F}{4\pi}\left(\ln^2\frac{2E_\gamma\mu_0}{\mu^2} - 2\sigma_1(\mu)\ln\frac{2E_\gamma\mu_0}{\mu^2} - 1 - \frac{\pi^2}{6} + \sigma_2(\mu)\right).$$

$$K(\mu) = 1 + \frac{\alpha_s C_F}{4\pi}\left(\frac{3}{2}\ln\frac{m_b^2}{\mu^2} - 2\right).$$

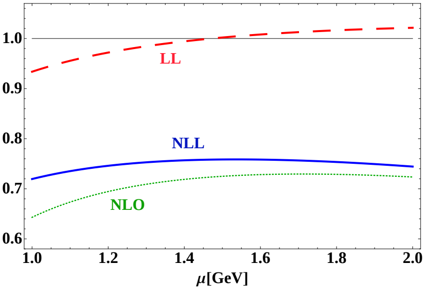
NLL renormalization-group evolution from the hard (m_b) to the hard-collinear scale ($(m_b\Lambda)^{1/2}$)

$$U(E_\gamma,\mu_{h1},\mu_{h2},\mu) = U_1(E_\gamma,\mu_{h1},\mu) U_2(\mu_{h2},\mu)^{-1}$$

$$\mu\frac{d}{d\mu}U_1(E_\gamma,\mu_h,\mu) = \left(\Gamma_{\rm cusp}(\alpha_s)\,\ln\frac{\mu}{2E_\gamma} + \gamma(\alpha_s)\right) U_1(E_\gamma,\mu_h,\mu)$$

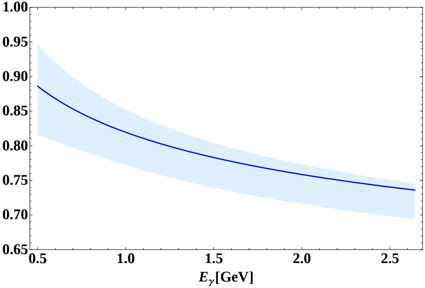
Two-loop anomalous dimension of the heavy-light SCET current from (Bonciani, Ferroglia; Asatrian, Greub, Pecjak; MB, Huber, Li; Bell 2008)

Size of radiative corrections (to the amplitude)



Hard-collinear scale dependence
($E_\gamma = 2\,\text{GeV}$)

Photon-energy dependence (and total scale uncertainty)



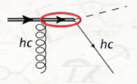
$1/m_b$ power corrections

Integrate out hard virtualities:

$$iT_{\nu\mu}(p,q) = \int d^4x\, e^{iqx} \langle 0|\mathrm{T}\{[j_{\nu,\mathrm{em}}(x)]_{\mathrm{SCET}}, (\bar{u}\gamma_\mu(1-\gamma_5)b)(0)_{\mathrm{SCET}}\}|B^-\rangle + \frac{1}{m_b}\langle 0|iS^{\mathrm{local}}_{\nu\mu}|B^-\rangle$$

Time-ordered products with sub-leading SCET interactions, subleading corrections to the currents, e.g.

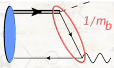
$$[\bar{u}\gamma_\mu(1-\gamma_5)Q](0) \to \int ds \sum_{i=1}^{3} \tilde{C}_i^{(A)}(s) J_\mu^{(A),i}(s) + \int ds\, ds' \sum_{i=1}^{4} \tilde{C}_i^{(B)}(s,s') J_\mu^{(B),i}(s,s')$$



and a local term.

At tree level only emission off the heavy quark and

$$\frac{i(\not{q}-\not{l})}{(q-l)^2} = -\frac{i\not{q}}{2q\cdot l} + \underbrace{\frac{i\not{l}}{2q\cdot l}}_{\text{power suppressed}} = -\frac{i\not{n}_-}{4l_-} + \left[\underbrace{\frac{il_+\not{n}_-}{4E_\gamma l_-} + \frac{i\not{l}_\perp}{2E_\gamma l_-}}_{\text{SCET time-ordered products}} + \underbrace{\frac{i\not{n}_+}{4E_\gamma}}_{\text{local}}\right]$$



Local operator is $S^{\mathrm{local}}_{\nu\mu} = \bar{q}_s \Gamma_{\mu\nu} \not{n}_\pm h_v \to f_B$.

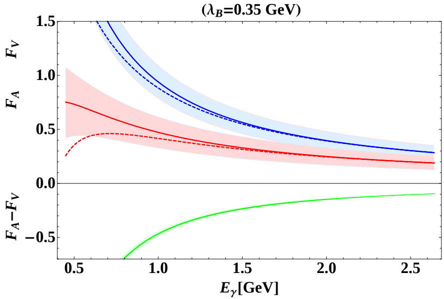
Beyond tree level: photon structure from collinear quark-photon coupling, three-particle B-meson distribution amplitudes, non-factorizable time-ordered products

$B \to \gamma$ form factors with radiative and power corrections

$$F_V(E_\gamma) = \frac{Q_u m_B f_B}{2E_\gamma \lambda_B(\mu)} R(E_\gamma,\mu) + \left[\xi(E_\gamma) + \frac{Q_b m_B f_B}{2E_\gamma m_b} + \frac{Q_u m_B f_B}{(2E_\gamma)^2}\right],$$

$$F_A(E_\gamma) = \frac{Q_u m_B f_B}{2E_\gamma \lambda_B(\mu)} R(E_\gamma,\mu) + \left[\xi(E_\gamma) - \frac{Q_b m_B f_B}{2E_\gamma m_b} - \frac{Q_u m_B f_B}{(2E_\gamma)^2} + \frac{Q_\ell f_B}{E_\gamma}\right].$$

Non-factorizable time-ordered product terms parameterized by $\xi(E_\gamma) = \pm f_B/(2E_\gamma)$. Irreducible uncertainty in determination of λ_B. Drops out in $F_V - F_A$.



$1/m_b$-suppressed "soft" form factor ξ

Factorization of ξ in SCET is not understood.

QCD sum rule calculation of the "symmetry-preserving", power-suppressed form factor (Braun, Khodjamirian [1210.4453])

$$\xi_{\rm BR}(E_\gamma) = c \times \frac{Q_u f_B}{2E_\gamma} \qquad c \in \left[-\frac{3}{2}, \frac{3}{2}\right]$$

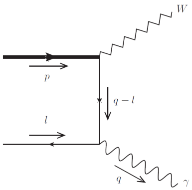
$$\xi_{\rm BK}(E_\gamma) = \frac{Q_u f_B}{2E_\gamma} \times \frac{m_B}{2E_\gamma} \times \hat{\xi}(E_\gamma)$$

where $\hat{\xi}$ is negative, nearly energy-independent, but depends strongly on λ_B.
No radiative and higher-twist corrections to the sum rule.

Improvements

- [1606.03080, Y. Wang] NLO radiative corrections to the twist-2 term
- [1804.04962, BBJW] NLO radiative corrections to the twist-2 term, LO twist-3 and 4 terms, study of B-meson LCDA-models
  $\Rightarrow$ revised strategy for the analysis of the anticipated data

Hard-collinear vs. soft power correction



Intermediate light-quark propagator has hard-collinear virtuality $m_b \Lambda$

- → Weak and electromagnetic currents separated by short distance
- → Light-cone expansion in soft background field [Balitsky, Braun, 1989]
- → Expressed in terms of moments of higher-twist and three-particle B-meson LCDAs, dependence on $\bar{\Lambda}$, λ_E^2, λ_B^2.

Intermediate light-quark propagator has soft virtuality $E_\gamma \omega \sim \Lambda^2$

- → Weak and electromagnetic currents separated by long distance, non-perturbative
- → Endpoint region of $\omega \sim \Lambda^2/E_\gamma$.
- → Evaluate through dispersion relation and light-cone QCD sum rule.

Endpoint divergence in hard-collinear region would indicate a log enhancement and relation of the two.

QCD sum rule for the power-suppressed form factor

Similar strategy as for the light-cone sum rule for the $\gamma^*\gamma \to \pi^0$ form factor [Khodjamirian, hep-ph/9712451; Agaev, Braun, Offen, Porkert, 1012.4671]

Calculate the $B \to \gamma^*$ form factors for Euclidean, hard-collinear virtuality, $-p^2$. Use dispersion relation and duality. Extract the power-suppressed contribution directly.

$$F_{B\to\gamma^*}(E_\gamma, p^2) = \frac{f_\rho F_{B\to\rho}(q^2)}{m_\rho^2 - p^2} + \frac{1}{\pi}\int_{s_0}^\infty ds\, \frac{\mathrm{Im} F_{B\to\gamma^*}(E_\gamma, s)}{s - p^2}$$

$$F^{\mathrm{QCDF}}_{B\to\gamma^*}(E_\gamma, p^2) = \frac{1}{\pi}\int_0^\infty ds\, \frac{\mathrm{Im}\, F^{\mathrm{QCDF}}_{B\to\gamma^*}(E_\gamma, s)}{s - p^2}$$

$$f_\rho F_{B\to\rho}(q^2) = \frac{1}{\pi}\int_0^{s_0} ds\, e^{-(s-m_\rho^2)/M^2}\, \mathrm{Im}\, F^{\mathrm{QCDF}}_{B\to\gamma^*}(E_\gamma, s)$$

$$F_{B\to\gamma}(E_\gamma) = \frac{1}{\pi}\int_0^{s_0} \frac{ds}{m_\rho^2}\, \mathrm{Im}\, F^{\mathrm{QCDF}}_{B\to\gamma^*}(E_\gamma, s)\, e^{-(s-m_\rho^2)/M^2} + \frac{1}{\pi}\int_{s_0}^\infty \frac{ds}{s}\, \mathrm{Im}\, F^{\mathrm{QCDF}}_{B\to\gamma^*}(E_\gamma, s)$$

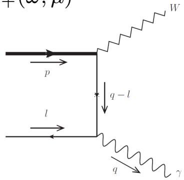
$$= F^{\mathrm{QCDF}}_{B\to\gamma}(E_\gamma) + \xi^{\mathrm{soft}}_{B\to\gamma}(E_\gamma)$$

$$\xi^{\mathrm{soft}}_{B\to\gamma}(E_\gamma) = \frac{1}{\pi}\int_0^{s_0} \frac{ds}{s}\left[\frac{s}{m_\rho^2} e^{-(s-m_\rho^2)/M^2} - 1\right] \mathrm{Im}\, F^{\mathrm{QCD}}_{B\to\gamma^*}(E_\gamma, s)$$

Twist-2, LO+NLO

$$F_V^{(\mathrm{LO})}(E_\gamma, p^2) = F_A^{(\mathrm{LO})}(E_\gamma, p^2) = e_u f_B m_B \int_0^\infty d\omega \, \frac{\phi_+(\omega,\mu)}{2E_\gamma\omega - p^2}$$

$$\xi^{\mathrm{soft}}_{(\mathrm{LO})}(E_\gamma) = \frac{e_u f_B m_B}{2E_\gamma} \int_0^{\frac{s_0}{2E_\gamma}} d\omega \left[\frac{2E_\gamma}{m_\rho^2} e^{-(2E_\gamma\omega - m_\rho^2)/M^2} - \frac{1}{\omega}\right] \phi_+(\omega,\mu)$$



Endpoint, anomalously small $\omega \ll \Lambda_{\mathrm{QCD}}$. Soft virtuality up to s_0 of the spectator quark propagator.

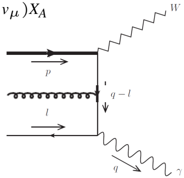
$$\xi^{\mathrm{soft}}_{(\mathrm{NLO})}(E_\gamma) = \frac{e_u f_B m_B}{2E_\gamma} C(E_\gamma, \mu_{h1}) K^{-1}(\mu_{h2}) U(E_\gamma, \mu_{h1}, \mu_{h2}, \mu) \times \int_0^{\frac{s_0}{2E_\gamma}} d\omega' \left[\frac{2E_\gamma}{m_\rho^2} \, \mathrm{e}^{-(2E_\gamma\omega' - m_\rho^2)/M^2} - \frac{1}{\omega'}\right] \phi_+^{\mathrm{eff}}(\omega',\mu)$$

$$\phi_+^{\mathrm{eff}}(\omega',\mu) = \phi_+(\omega',\mu) + \frac{\alpha_s(\mu) C_F}{4\pi} \left\{ \left(\ln^2 \frac{\mu^2}{2E_\gamma\omega'} + \frac{\pi^2}{6} - 1\right) \phi_+(\omega',\mu) + \left(2\ln\frac{\mu^2}{2E_\gamma\omega'} + 3\right) \omega' \int_{\omega'}^\infty d\omega \, \ln\frac{\omega-\omega'}{\omega'} \frac{d}{d\omega} \frac{\phi_+(\omega,\mu)}{\omega} - 2\ln\frac{\mu^2}{2E_\gamma\omega'} \int_0^{\omega'} d\omega \, \ln\frac{\omega'-\omega}{\omega'} \frac{d}{d\omega}\phi_+(\omega,\mu) + \int_0^{\omega'} d\omega \, \ln^2\frac{\omega'-\omega}{\omega'} \frac{d}{d\omega}\left[\frac{\omega'}{\omega}\phi_+(\omega,\mu) + \phi_+(\omega,\mu)\right] \right\}$$

Twist-3+4, LO (+ singular twist 5+6 quark condensate terms)

$$\langle 0|\bar{q}(nz_1) g_s G_{\mu\nu}(nz_2) \Gamma h_v(0)|\bar{B}(v)\rangle =$$

$$= \frac{1}{2} F_B(\mu)\,\mathrm{Tr}\Big\{\gamma_5 \Gamma P_+ \Big[(v_\mu\gamma_\nu - v_\nu\gamma_\mu)\big[\Psi_A - \Psi_V\big] - i\sigma_{\mu\nu}\Psi_V - (n_\mu v_\nu - n_\nu v_\mu) X_A$$
$$+ (n_\mu\gamma_\nu - n_\nu\gamma_\mu)\big[W + Y_A\big] - i\epsilon_{\mu\nu\alpha\beta} n^\alpha v^\beta \gamma_5 \widetilde{X}_A + i\epsilon_{\mu\nu\alpha\beta} n^\alpha \gamma^\beta \gamma_5 \widetilde{Y}_A$$
$$- (n_\mu v_\nu - n_\nu v_\mu) \not{n}\, W + (n_\mu\gamma_\nu - n_\nu\gamma_\mu)\not{n}\, Z\Big]\Big\}(z_1, z_2; \mu)\,.$$



Eight three-particle LCDAs. After eom relations one twist-3 and two twist-4 functions of (ω_1, ω_2) [Braun, Ji, Manashov, 1703.02446].

Hard-collinear contribution:

$$\xi^{\rm ht}_{1/E_\gamma} = \frac{e_u f_B m_B}{4E_\gamma^2}\left\{1 - 2\frac{\bar\Lambda}{\lambda_B} + 2\int_0^\infty d\omega\, \ln\omega\, \phi_-^{\rm t3}(\omega) + \int_0^\infty \frac{d\omega_1}{\omega_1}\int_0^\infty \frac{d\omega_2}{\omega_1+\omega_2}\left[\psi_4 - \widetilde{\psi}_4\right]^{\rm t4}(\omega_1,\omega_2)\right\}$$
$$= \frac{e_u f_B m_B}{4E_\gamma^2}\left\{-1 + 2\int_0^\infty d\omega\, \ln\omega\, \phi_-^{\rm t3}(\omega) - 2\int_0^\infty \frac{d\omega_2}{\omega_2}\,\phi_4(0,\omega_2)\right\},$$

Possible endpoint divergence at $\omega_i \to 0$ cancels. The corresponding soft contribution is therefore suppressed by two powers, $1/E_\gamma^2 \times s_0/(E_\gamma\lambda)$. Keep, since $s_0/(E_\gamma\lambda)$ is numerically $\mathcal{O}(1)$. There is also a $1/(m_b E_\gamma)$ terms, which is numerically less important.

B meson LCDA input

In total

$$\xi = \xi^{\rm ht}_{1/E_\gamma} + \xi^{\rm ht}_{1/m_b} + \xi^{\rm soft}_{\rm (NLO)} + \xi^{\rm soft}_{(tw-3,4)} + \xi^{\rm soft}_{(tw-5,6)}$$

LP depends only on λ_B and log-inverse-moments. "ht" also on positive moments $\bar\Lambda$, $\lambda^2_{E,H}$ of $\phi_+(\omega)$.

The soft form factor in the LCSR evaluation depends on the form of the LCDAs. We express higher-twist B-LCDA through $\phi_+(\omega)$ (ansatz, consistent with eom)

$$\eta_+^{(\mathrm{I})}(s,\mu_0) = {}_1F_1(1+2/b,2/b,-s\omega_0) = \left(1-\frac{1}{2}bs\omega_0\right)e^{-s\omega_0}$$

$$\eta_+^{(\mathrm{II})}(s,\mu_0) = {}_1F_1(2+a,2,-s\omega_0)\,, \qquad -0.5 < a < 1\,,$$

$$\eta_+^{(\mathrm{III})}(s,\mu_0) = {}_1F_1(3/2+a,3/2,-s\omega_0)\,, \qquad 0 < a < 0.5\,,$$

$$\phi_+^{(\mathrm{I})}(\omega,\mu_0) = \left[(1-b) + \frac{b\,\omega}{2\omega_0}\right]\frac{\omega}{\omega_0^2}\,\mathrm{e}^{-\omega/\omega_0}\,,$$

$$\phi_+^{(\mathrm{II})}(\omega,\mu_0) = \frac{1}{\Gamma(2+a)}\frac{\omega^{1+a}}{\omega_0^{2+a}}\mathrm{e}^{-\omega/\omega_0}\,,$$

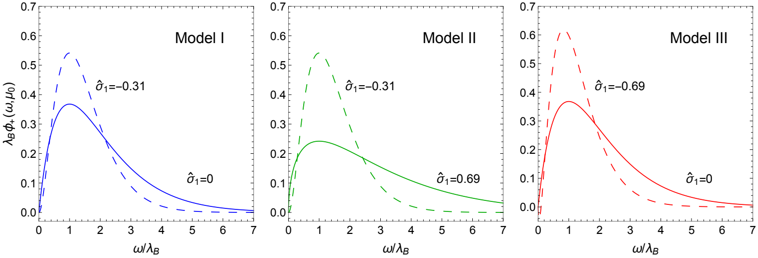
$$\phi_+^{(\mathrm{III})}(\omega,\mu_0) = \frac{\sqrt{\pi}}{2\Gamma(3/2+a)}\frac{\omega}{\omega_0^2}\,e^{-\omega/\omega_0}\,U(-a,3/2-a,\omega/\omega_0)$$

Assumed to hold at $\mu_0 = 1$ GeV. RG evolution can be done analytically. .

Parameters: λ_B, $\widehat\sigma_1$ ($\widehat\sigma_2$ is strongly correlated)

$$\phi_+(\omega) = \omega f(\omega) \qquad \widehat\sigma_n = \int_0^\infty d\omega\,\frac{\lambda_B}{\omega}\,\ln^n\frac{\lambda_B e^{-\gamma_E}}{\omega}\,\phi_+(\omega)$$

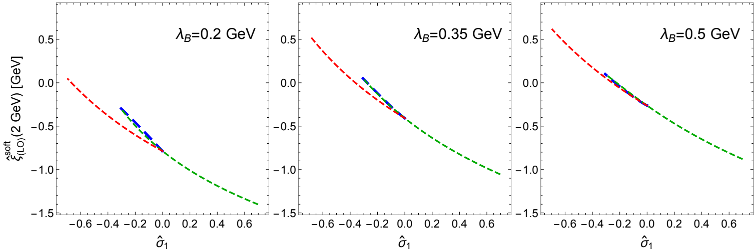
$$\phi_3(\omega_1,\omega_2) = -\frac{1}{2}\varkappa(\lambda_E^2-\lambda_H^2)\,\omega_1\omega_2^2 f'(\omega_1+\omega_2) \qquad \phi_4(\omega_1,\omega_2) = \frac{1}{2}\varkappa(\lambda_E^2+\lambda_H^2)\,\omega_2^2 f(\omega_1+\omega_2)$$



Note: We take the positive moments $\bar{\Lambda}$, $\lambda_{E,H}^2$ of $\phi_+(\omega)$ as independent parameters. The hard-collinear higher-twist contribution can be expressed in terms of these only (and λ_B). Better treatment requires consistent treatment of cut-off moments at higher-twist level.

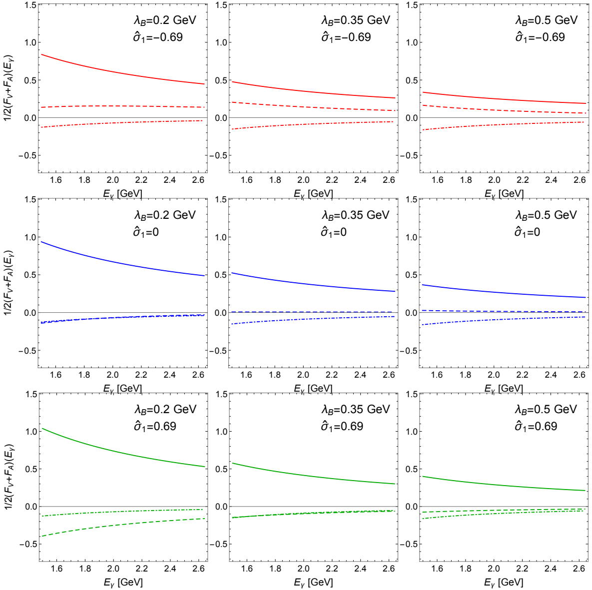
Discussion of results (1)

Leading-order soft correction normalized to the LO LP result



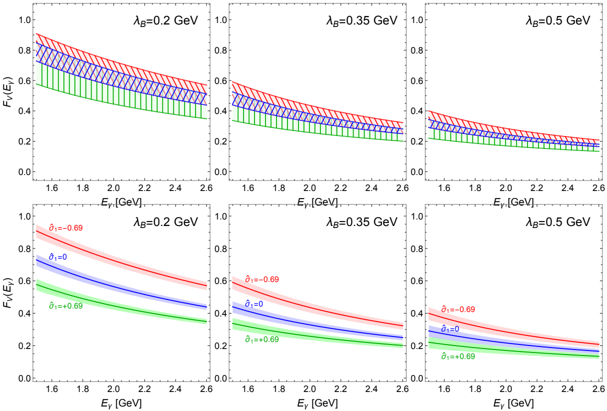
$$\xi^{\rm soft}_{\rm (LO)}(E_\gamma) = \frac{e_u f_B m_B}{2E_\gamma \lambda_B(\mu)}\, U_{\rm LL}\, \frac{\widehat{\xi}^{\rm soft}_{\rm (LO)}(E_\gamma)}{2E_\gamma}$$

- Strong dependence on λ_B <u>and</u> $\widehat{\sigma}_1$.
- Counteracting the dependence at leading power, such that the sensitivity of the form factor to the B-meson LCDA is reduced relative to LP alone.
  [$\rightarrow$ next plot]



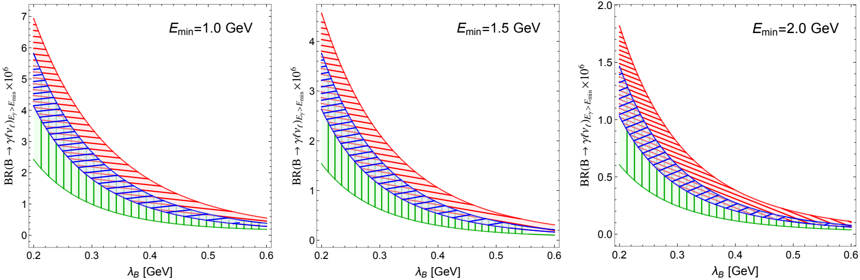
Perturbative (solid), soft (dashed) and higher-twist (dash-dotted) contributions to $(F_V + F_A)/2$.

Discussion of results (2) – Vector form factor $F_V(E_\gamma)$



- The uncertainty from all parameters except those of the leading-twist B-meson LCDA $\phi_+(\omega)$ is generally smaller than the dependence on $\phi_+(\omega)$ itself, which is large.
- The dependence of the form factors on the shape of the B-meson LCDA (which is mostly a dependence on $\widehat{\sigma}_1$) is as strong as on λ_B. Experiment should aim at the extraction of correlated values for λ_B and "shape parameters" $\widehat{\sigma}_1$, etc.

Discussion of results (3) – cut branching fraction



- $E_{\rm min} = 1.0\,\mathrm{GeV}$ probably too low for reliable prediction. Power correction becomes large.
- Must specify the scale of λ_B. Here $\lambda_B(1\,\mathrm{GeV})$.
- The width of the band is not the uncertainty, since it contains the dependence on $\widehat{\sigma}_1$, see above.

Measurements [BELLE, 1810.12976, slides from P. Goldenzweig talk at MIAPP, 13 May 2019]

Measure two quantities:

$$\Delta\mathcal{B}(B^+ \to \ell^+\nu_\ell\gamma)_{E_\gamma>1.0\,\mathrm{GeV}} \qquad \text{and} \qquad \mathcal{R}_\pi = \frac{\Delta\mathcal{B}(B^+ \to \ell^+\nu_\ell\gamma)_{E_\gamma>1.0\,\mathrm{GeV}}}{\mathcal{B}(B^+ \to \pi^0\ell^+\nu_\ell)}$$

This allows to extract λ_B independent of $|V_{ub}|$. In addition, some systematics cancel in the ratio $\mathcal{R}$.

ℓ	$\mathcal{B}(B^+ \to \pi^0\ell^+\nu_\ell)$ (10^{-5})	σ	$\Delta\mathcal{B}(B^+ \to \ell^+\nu_\ell\gamma)$ (10^{-6})	σ
e	$8.3^{+0.9}_{-0.8} \pm 0.9$	8.0	$1.7^{+1.6}_{-1.4} \pm 0.7$	1.1
μ	$7.5 \pm 0.8 \pm 0.6$	9.6	$1.0^{+1.4}_{-1.0} \pm 0.4$	0.8
e, μ	**$7.9 \pm 0.6 \pm 0.6$**	**12.6**	**$1.4 \pm 1.0 \pm 0.4$**	**1.4**

ℓ	$\Delta\mathcal{B}(B^+ \to \ell^+\nu_\ell\gamma)$ $(\times 10^{-6})$ limit @90% C.L.		
	BaBar (2009)[a]	Belle (2015)[b]	**This work**
e	-	< 6.1	**< 4.3**
μ	-	< 3.4	**< 3.4**
e, μ	< 14	< 3.5	**< 3.0**

Measurements (2) [BELLE, 1810.12976, slide from P. Goldenzweig talk at MIAPP, 13 May 2019]

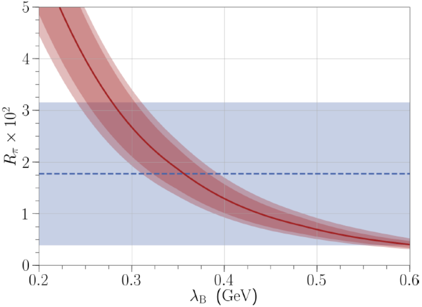
Extraction of λ_B

$$R_\pi = \frac{\Delta\mathcal{B}(B^+ \to \ell^+\nu_\ell\gamma)}{\mathcal{B}(B^+ \to \pi^0\ell^+\nu_\ell)} = \frac{\Delta\Gamma(\lambda_\mathrm{B})}{\Gamma(B^+ \to \pi^0\ell^+\nu_\ell)}$$

$$R_\pi^{\text{measured}} = (1.77 \pm 1.38) \times 10^{-2}$$

	λ_B (GeV) [exp., th.]
Model I	$0.36^{+0.25}_{-0.08}{}^{+0.03}_{-0.03}$
Model II	$0.38^{+0.25}_{-0.06}{}^{+0.05}_{-0.08}$
Model III	$0.32^{+0.24}_{-0.07}{}^{+0.05}_{-0.08}$

based on theoretical input from:
Beneke et al., JHEP 07:154 (2018)
HFLAV, Eur. Phys. J., C77:895, (2017)



$$\lambda_\mathrm{B} = 0.36^{+0.25}_{-0.08}\,[\text{exp.}] \pm 0.03\,[\text{th.}] \pm 0.03\,[\text{mod.}]$$

$\lambda_\mathrm{B} > 0.24$ GeV @ 90% C.L.

Result of Belle (2015) was $\lambda_\mathrm{B} > 0.238$ GeV

Correlated extraction of λ_B and $\hat{\sigma}_1$ from this data still to be done.
Increase $E_{\gamma,\text{min}}$.

Conclusion

- Factorization at leading power understood.
  Simple dependence on the B-meson LCDA.

- Recent efforts concentrate on estimating the power-suppressed for factor $\xi(E_\gamma)$ with LCSR.

- Main conclusion for future measurements of $B \to \gamma\ell\nu$

  The dependence of the form factors on the shape of the B-meson LCDA (which is mostly a dependence on $\widehat{\sigma}_1$) is as strong as on λ_B. Future experimental analyses should aim at the extraction of correlated values for λ_B and "shape parameters" $\widehat{\sigma}_1$, etc., rather than extracting λ_B alone and treating the "shape parameters" as theoretical uncertainty parameters.

- Still no SCET factorization of the NLP term. Need to understand the LCSR soft endpoint contribution in terms of SCET.

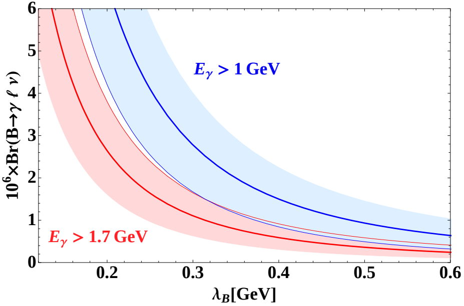
Extra Slides

Three reasons to study $B \to \gamma\ell\nu$ (at large E_γ)

- For its own.
- For a measurement of λ_B.
  λ_B appears in almost all exclusive B decays in LP in the heavy-quark expansion (spectator scattering)
- Factorization theory beyond LP.

Historical note: Radiative leptonic decays first mentioned in [Burdman, Goldman, Wyler, hep-ph/9405425 and Atwood, Eilam, Soni, hep-ph/9411367] but in the context of background for $B \to \ell\nu$ and for "monitoring the annihilation graph."

Branching fractions



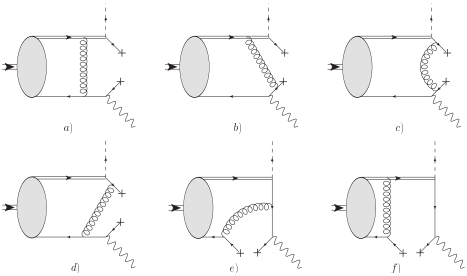
Example (based on the 2011 calculation):

$$\mathrm{Br}\,(B^- \to \gamma\ell\bar{\nu}, E_\gamma > 1.7\,\mathrm{GeV}) = (2.0 \pm 0.4) \times 10^{-6} \quad \to \quad \lambda_B = 228^{+76}_{-61}\,\mathrm{MeV}$$

Dominant theoretical uncertainty about equally from σ_1, σ_2 and ξ.

All terms

Some twist5+6 terms get promoted to $1/E_\gamma^2$ corrections due to $1/p^2 \rightarrow 1/m_\rho^2$ singularities (diagrams a,c,e)



In total

$$\xi = \xi^{\rm ht}_{1/E_\gamma} + \xi^{\rm ht}_{1/m_b} + \xi^{\rm soft}_{\rm (NLO)} + \xi^{\rm soft}_{(tw-3,4)} + \xi^{\rm soft}_{(tw-5,6)}$$

LP depends only on λ_B and log-inverse-moments. The soft form factor in the LCSR evaluation depends on the form of the LCDAs.