On the QED effects on R_K and R_{K^*}

based on 1605.07633
in collaboration with G.Isidori and A.Pattori

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GDR-InF workshop:
QED corrections to (semi-) leptonic B decays
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R_K and R_{K^*} within the SM

Which are the sources of LFUV contributions to $R_{K^{(*)}}$ in the SM?

- kinematics and form factor effects $\sim \frac{m_\ell^2}{q^2}$
- naive estimation of QED corrections $\sim \frac{\alpha}{\pi} \log^2\left(\frac{m_\ell^2}{q^2}\right)$ [T.Huber,T.Hurth,E.Lunghi T.Huber,E.Lunghi,M.Misiak,D.Wyler]
- interplay between kinematic effects and QED corrections

Can we trust the $\mathcal{O}(10^{-3}/10^{-4})$ uncertainties that are quoted in the literature?

semi-analytic calculation of radiative corrections

Calculation setup for the region $q^2 \in [1, 6]$ GeV^2

- $B \to K^* \ell\ell(\gamma)$ and $B \to K\ell\ell(\gamma)$ decays can be treated in complete analogy (NB: real+virtual QED effects $\Rightarrow$ IR safe observables)
- limit $m_\ell^2 \ll q^2$
- interested to extract log-enhanced terms $\sim \frac{\alpha}{\pi} \log\left(\frac{m_\ell^2}{q^2}\right)$ and $\sim \frac{\alpha}{\pi} \log^2\left(\frac{m_\ell^2}{q^2}\right)$
  - since they depend on m_ℓ they can be the only terms responsible of LFU violation
  - can be extracted from term associated with collinear and soft divergences due to the photon emission
- neglect $\mathcal{O}(\alpha/\pi)$ finite corrections ($\sim 0.2\%$)
- radiation from meson leg is negligible (not log-enhanced)

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Main purpose: understand if QED effects are correctly taken in account in the experimental analysis + estimate residual th. error due to unknown finite corrections

Radiator function

$\omega(x, x_\ell)$: probability density function that a dilepton system retains a fraction $\sqrt{x}$ of its original invariant mass q_0^2 after bremstrahlung

$$\omega(x, x_\ell, \theta_K) = \omega_1(x, x_\ell, \theta_K)\theta(1 - x - x_*) + \omega_2(x, x_\ell, x_*)\delta(1 - x)$$

- ω_1: real emission

- $x = q^2/q_0^2$
- $x_\ell = m_\ell^2/q_0^2$
- x_*: IR regulator
- θ_K: angle between $K^{(*)}$ and the photon

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- ω_1: real emission
- ω_2: soft emission and virtual corrections, obtained from

$$\int_{-1}^{1} \mathrm{d}\cos\theta_K \int_{2x_\ell}^{1} \mathrm{d}x \; \omega(x, x_\ell, \theta_K) = 1 + \mathcal{O}\left(\frac{\alpha}{\pi}\right)$$

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- $x_\ell = m_\ell^2/q_0^2$
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Implementation of the radiator into the non radiative spectrum

Double-differential decay width

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}q_0^2\mathrm{d}x}\,(B \to K\ell\ell(\gamma)) = \mathcal{F}_K^{(0)}(q_0^2)\omega(x, x_\ell, \theta_K)$$

$\mathcal{F}_K^{(0)}(q^2)$: non radiative spectrum of the decay $B \to K\ell\ell$
To obtain the radiative-spectrum we need to perform the following **convolution**

$$\mathcal{F}_K^{\ell}(q^2) = \int_{q^2}^{q^2_{0,\max}} \frac{\mathrm{d}q_0^2}{q_0^2}\;\mathcal{F}_K^{(0)}(q_0^2)\;\omega\left(\frac{q^2}{q_0^2}, \frac{2m_\ell^2}{q_0^2}, \theta_K\right)$$

where the kinematical region of integration depends on experimental cuts, namely m_B^{rec}

Modelling the J/Ψ

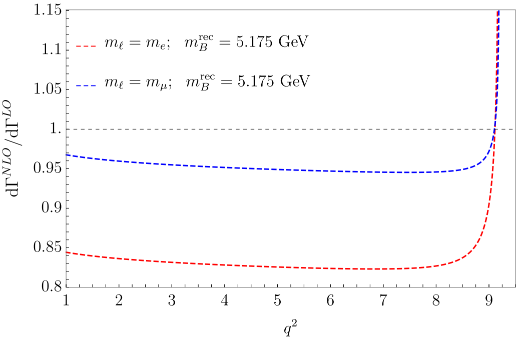
Non-perturbative spectrum

$$C_9(q^2) = C_9^{\text{pert}} + \kappa_\Psi \frac{q^2}{q^2 - m_\Psi^2 + i m_\Psi \Gamma_\Psi}$$

- C_9^{pert} ensures the behaviour at low q^2 region
- BW reproduces the presence of J/Ψ, κ_Ψ normalised to $\mathcal{B}(B \to K^{(*)} J/\Psi)$
- relative phase between C_9^{pert} and BW doesn't affect the result
- we do not claim this is the "true" shape of the resonance, but still it is suitable toy to study the behaviour around the J/Ψ
- interference not included in PHOTOS-based models

J/Ψ tail

m_B^{rec}: reconstructed mass of the B meson from charged tracks



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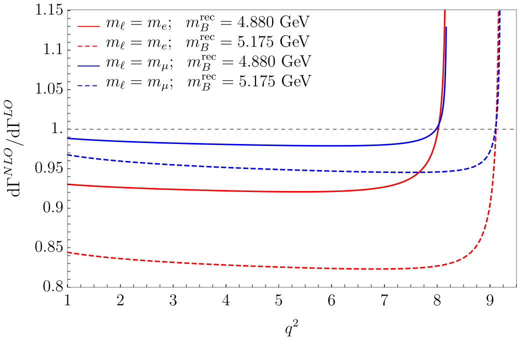
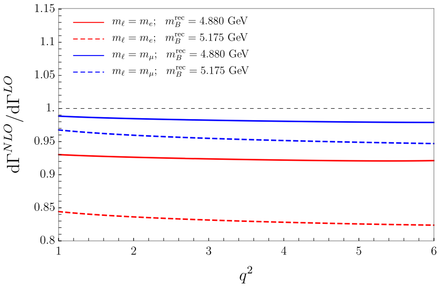


- **Key-variable**: m_B^{rec}, that determines the size of the effect of radiation we need to take in account
- even with the looser cut $m_B^{\mathrm{rec}} = 4.880$ GeV the tail is safely above the interesting region $[1, 6]$ GeV^2

$B \to K\ell\ell(\gamma)$ for $1\ \text{GeV}^2 \leq q^2 \leq 6\ \text{GeV}^2$



m_B^{rec}	$\ell = e$	$\ell = \mu$
4.880 GeV	-7.60%	-1.8%
5.175 GeV	-16.9%	-4.6%

- radiative correction can be sizable

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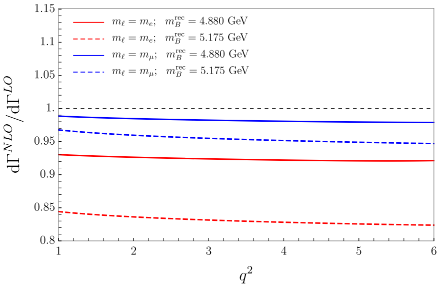

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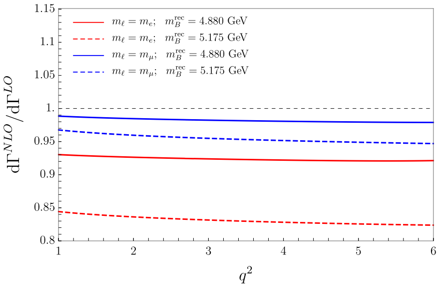

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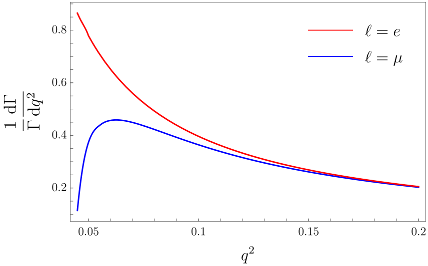
- The estimate effect on R_K is: $\Delta R_K = 3\%$
- The effect is the same for R_{K^*}
- Most important, we are in agreement at the ‰level with PHOTOS-based signal model
- The shift has already been taken in account in the analysis
- We associate a conservative error of ± 0.01

$q^2 \leq 1$ GeV^2

1. Kinematic effect are non universal for electron and muons and they may cause distortion
   - the radiator for QED corrections must include the complete mass dependence
   - the error due to the form factors is not negligible
2. Light-quark resonances (η, f_0, ...) provide non-bremsstrahlung terms not included in PHOTOS-based signal model



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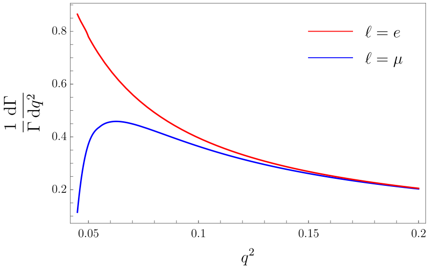
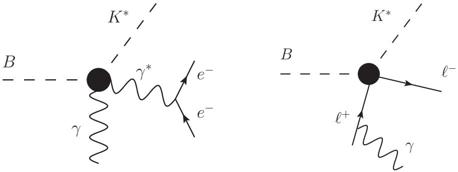


Main results:

- The theory uncertainty on R_{K^*} increases to a few %
- $\mathcal{O}(1\%)$ negative shift in R_{K^*}

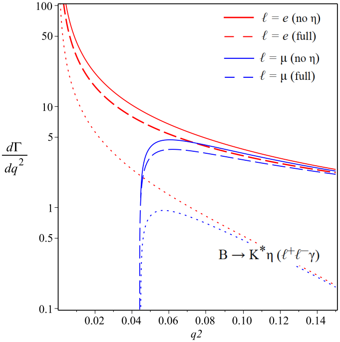
Non-bremsstrahlung effects

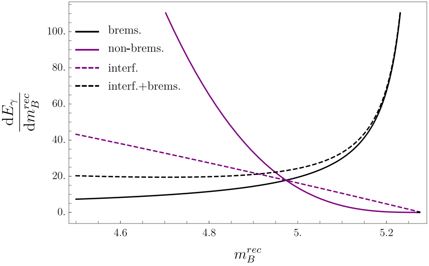


- The radiative tails is modified by the non-bremsstrahlung (direct emission) terms
- The effect is enhanced for the electron, due to kinematic effects and the cuts on m_B^{rec}
- Potentially sizeable effects due to

$$\mathcal{B}(B \to K^* \eta(\to e^+ e^- \gamma)) \sim 30\% \mathcal{B}(B \to K^* e^+ e^-, q^2 < 0.1 \text{ GeV}^2)$$

The η case





Given the cuts on q^2 we estimate a shift of $\Delta R_{K^*} \sim -0.017$ to which we assign an error of $\mathcal{O}(100\%)$

$$R_{K^*}[0.045, 1.1]^{\mathsf{SM}} = 0.906 \pm 0.020_{\mathsf{QED}} \pm 0.020_{\mathsf{FF}}$$

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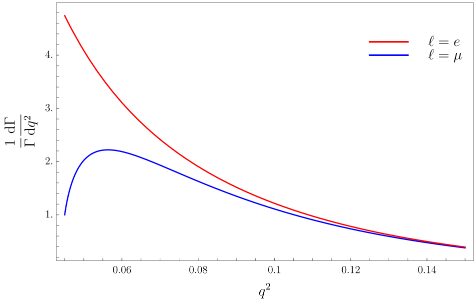

- The non-bremsstrahlung effects lead to a non-vanishing contribution to R_{K^*} only near the threshold
- If we look at the region $q^2 > 0.1$ GeV^2 all these effects are negligible, and

$$R_{K^*}[0.1, 1.1]^{\mathsf{SM}} = 0.983 \pm 0.010_{\mathsf{QED}} \pm 0.010_{\mathsf{FF}}$$

We recommend for future analysis to take in account the $q^2 \in [0.1, 1.1]$ GeV^2

Summary

- In the central region $q^2 \in [1.1, 6]$ GeV^2

$$R_{K^*} = 1.00 \pm 0.01$$

where the uncertainty is driven by QED corrections

- In the $q^2 \in [0.045, 1.1]$ GeV^2 region

$$R_{K^*} = 0.91 \pm 0.03$$

where the central value is shifted by the presence of non-bremsstrahlung contribution due to light resonances and the uncertainty is due to QED and non vanishing form factor effects

Prospects

[Bordone, Isidori, Lancerini, Zwicky, WIP]

- Implementation of a “theory” Montecarlo

- Prediction for angular observables in $B \to K^* \ell^+ \ell^-$