Determination of the CKM matrix element $|V_{cb}|$, the $B \to X_s\gamma$ decay rate, and the b-quark mass

Bestimmung des CKM Matrixelementes $|V_{cb}|$, der $B \to X_s\gamma$ Zerfallsrate, und der b-Quarkmasse

DISSERTATION

zur Erlangung des akademischen Grades

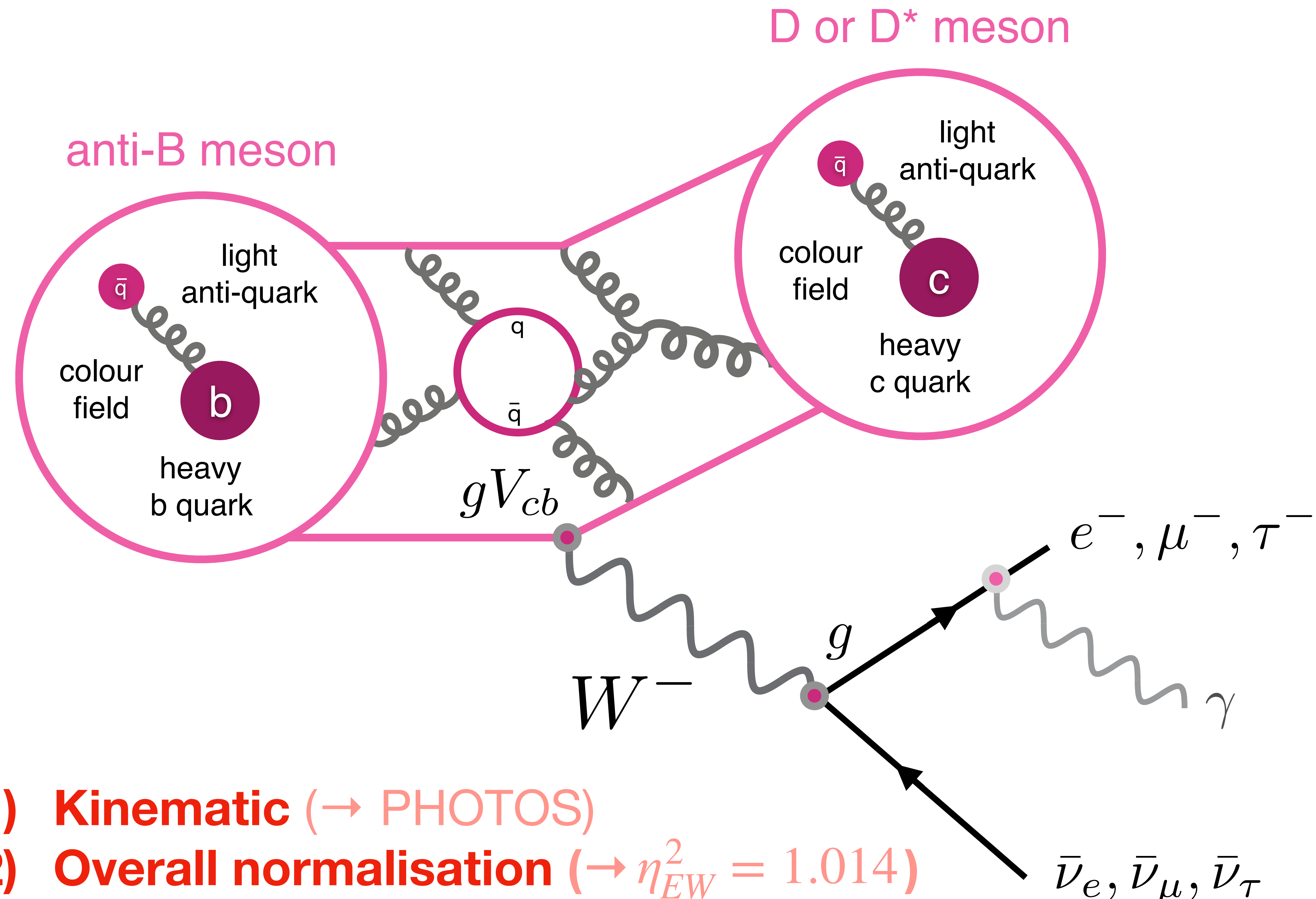
Dr. rer. nat.
im Fach Physik

[arXiv:1010.5997] with M. Schonherr

[hep-ph/0406006] Troy Andre

Impact of QED Corrections on the determination of $|V_{ub}|$ and $|V_{cb}|$

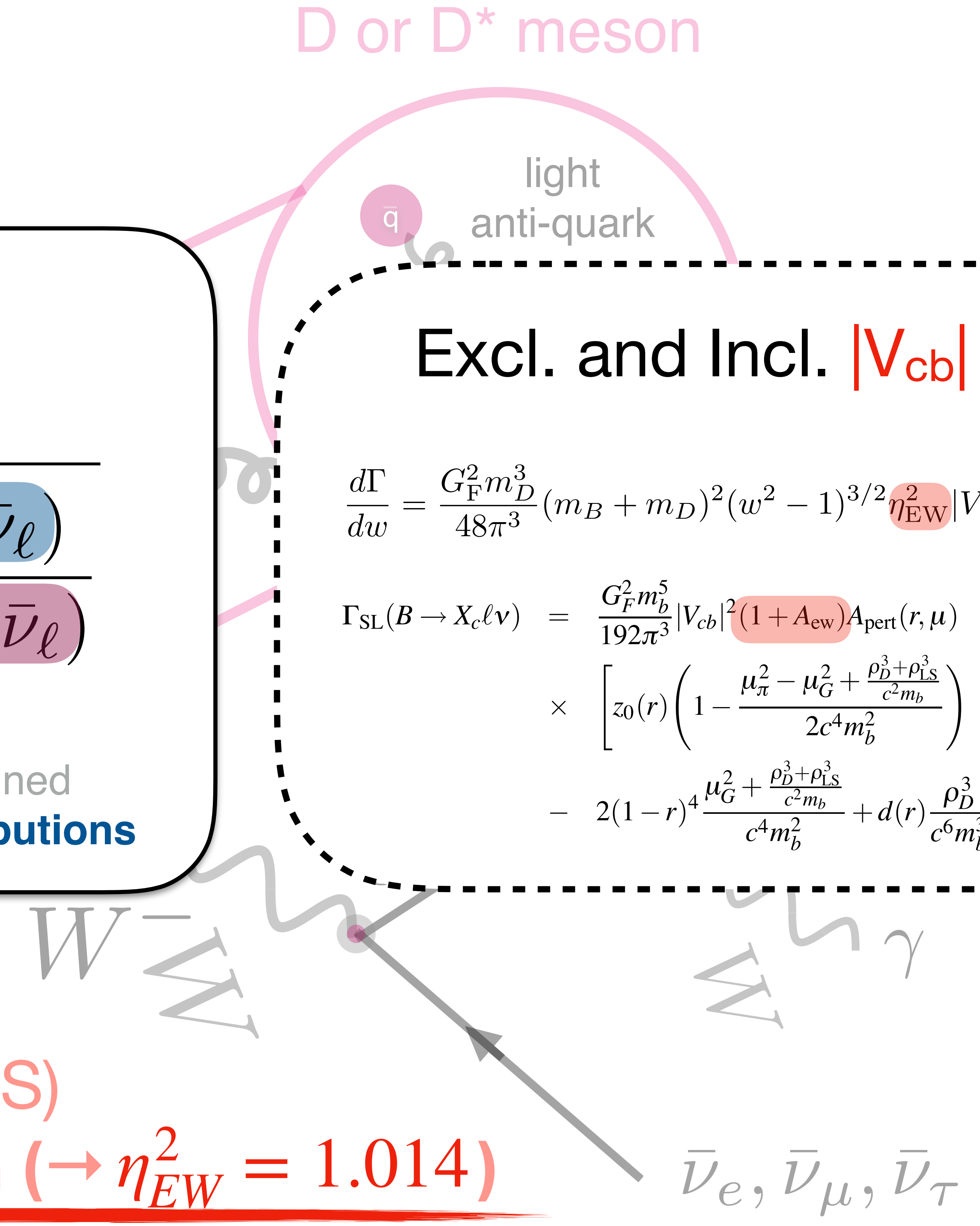
Impact on $|V_{ub}|$ and $|V_{cb}|$ measurements



1) **Kinematic** (→ PHOTOS)
2) **Overall normalisation** (→ $\eta_{EW}^2 = 1.014$)

Sirlin '82 quark-level calculation

Impact on $|V_{ub}|$ and $|V_{cb}|$ measurements



Measured
Branching Fraction

$$|V_{qb}| = \sqrt{\frac{\mathcal{B}(\bar{B} \to X_q\, \ell\, \bar{\nu}_\ell)}{\tau\, \Gamma(\bar{B} \to X_q\, \ell\, \bar{\nu}_\ell)}}$$

Prediction from
Theory but often also constrained
from **measured differential distributions**

Excl. and Incl. $|V_{cb}|$

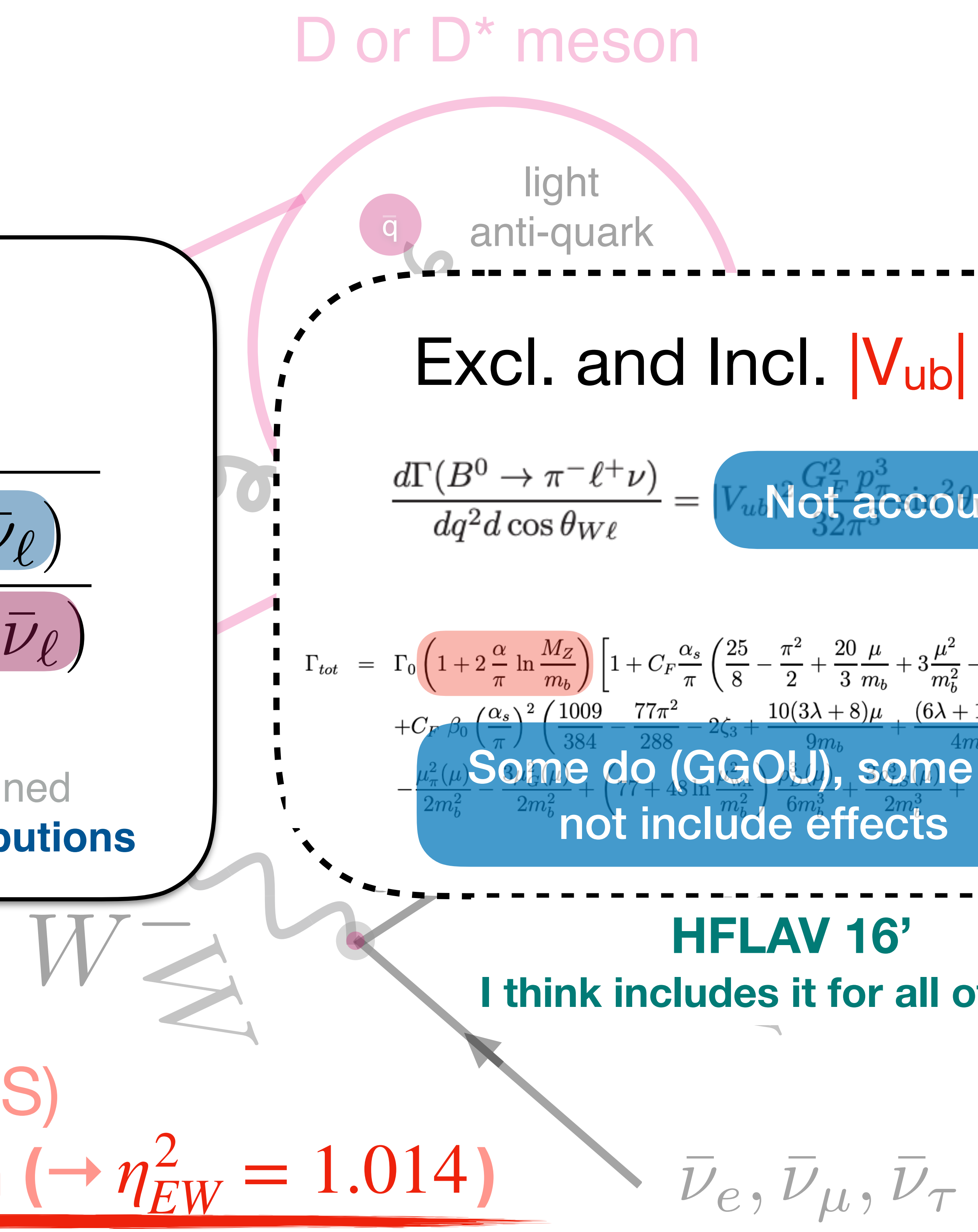
$$\frac{d\Gamma}{dw} = \frac{G_F^2 m_D^3}{48\pi^3}(m_B + m_D)^2(w^2-1)^{3/2}\eta_{EW}^2|V_{cb}|^2|\mathcal{G}(w)|^2 \, ,$$

$$\begin{aligned}\Gamma_{SL}(B \to X_c \ell \nu) &= \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2(1+A_{ew})A_{pert}(r,\mu) \\ &\times \left[z_0(r)\left(1 - \frac{\mu_\pi^2 - \mu_G^2 + \frac{\rho_D^3+\rho_{LS}^3}{c^2 m_b}}{2c^4 m_b^2}\right) \right. \\ &- \left. 2(1-r)^4 \frac{\mu_G^2 + \frac{\rho_D^3+\rho_{LS}^3}{c^2 m_b}}{c^4 m_b^2} + d(r)\frac{\rho_D^3}{c^6 m_b^3} + \mathcal{O}(1/m_b^4)\right],\end{aligned}$$

$$\eta_{EW}^2 = (1 + A_{ew})$$

1) **Kinematic** (→ PHOTOS)
2) **Overall normalisation** ($\to \eta_{EW}^2 = 1.014$)

Impact on $|V_{ub}|$ and $|V_{cb}|$ measurements



Measured
Branching Fraction

$$|V_{qb}| = \sqrt{\frac{\mathcal{B}(\bar{B} \to X_q\, \ell\, \bar{\nu}_\ell)}{\tau\, \Gamma(\bar{B} \to X_q\, \ell\, \bar{\nu}_\ell)}}$$

Prediction from
Theory but often also constrained
from **measured differential distributions**

Excl. and Incl. $|V_{ub}|$

$$\frac{d\Gamma(B^0 \to \pi^- \ell^+ \nu)}{dq^2 d\cos\theta_{W\ell}} = |V_{ub}|^2 \frac{G_F^2\, p_\pi^3}{32\pi^3} \ldots$$

Not accounted for

$$\Gamma_{tot} = \Gamma_0 \left(1 + 2\frac{\alpha}{\pi}\ln\frac{M_Z}{m_b}\right)\left[1 + C_F\frac{\alpha_s}{\pi}\left(\frac{25}{8} - \frac{\pi^2}{2} + \frac{20}{3}\frac{\mu}{m_b} + 3\frac{\mu^2}{m_b^2} - \frac{77}{9}\frac{\mu^3}{m_b^3}\right) + C_F\beta_0\left(\frac{\alpha_s}{\pi}\right)^2\left(\frac{1009}{384} - \frac{77\pi^2}{288} - 2\zeta_3 + \frac{10(3\lambda+8)\mu}{9m_b} + \frac{(6\lambda+13)\mu^2}{4m_b^2} - \frac{77(\lambda+2)\mu^3}{18m_b^3}\right) - \frac{\mu_\pi^2(\mu)}{2m_b^2} - \ldots + C_{WA} B_{WA}(\mu_{WA})\right], \quad (45)$$

Some do (GGOU), some do not include effects

HFLAV 16'
I think includes it for all of them

1) **Kinematic** (→ PHOTOS)
2) **Overall normalisation** (→ $\eta_{EW}^2 = 1.014$)

QED corrections as a systematic error

- Typically measurements either assumed that the current experimental uncertainties are larger than the **uncertainties on any QED** correction or 'cook' up an uncertainty

One of the **Recipes**:

- produce MC w/o PHOTOS
- Assign **20-30%** of the difference to the nominal result as the uncertainty on the QED modelling
- No strong argument why this size; in many analyses it's not a huge effect

on dedicated control samples. The uncertainty arising from radiative corrections is studied by comparing the results using PHOTOS [30] to simulate final state radiation (default case) with those obtained with PHOTOS turned off. We take 25 % of the difference as an error. The un-

- **As discussed: Sirlin's** correction applied, when extracting $|V_{cb}|$ and sometimes for $|V_{ub}|$, **sometimes** without any additional uncertainty

HFLAV 16'

where $\eta_{EW} = 1.0066 \pm 0.0050$ has been used. The central value of this number corresponds to the electroweak correction only. The uncertainty has been increased to accomodate the Coulomb effect. Based on Eq. (172), this results in

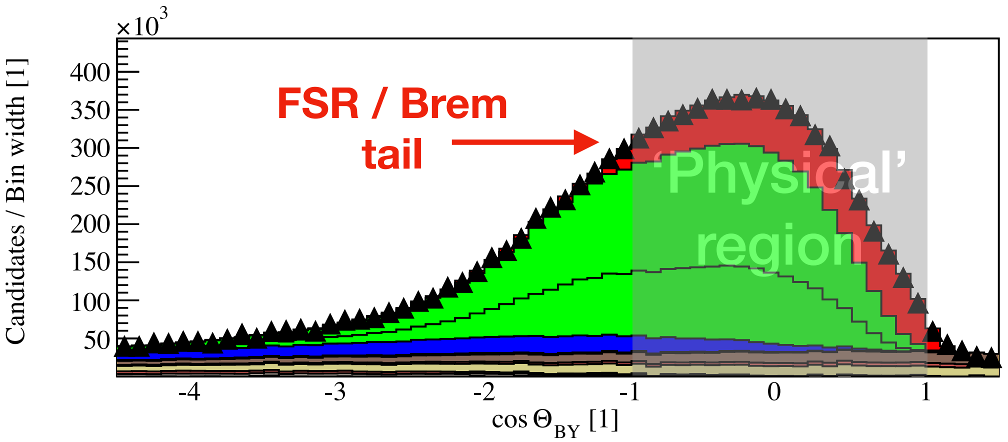
Example: Impact on QED on Global $|V_{cb}|$ Fit

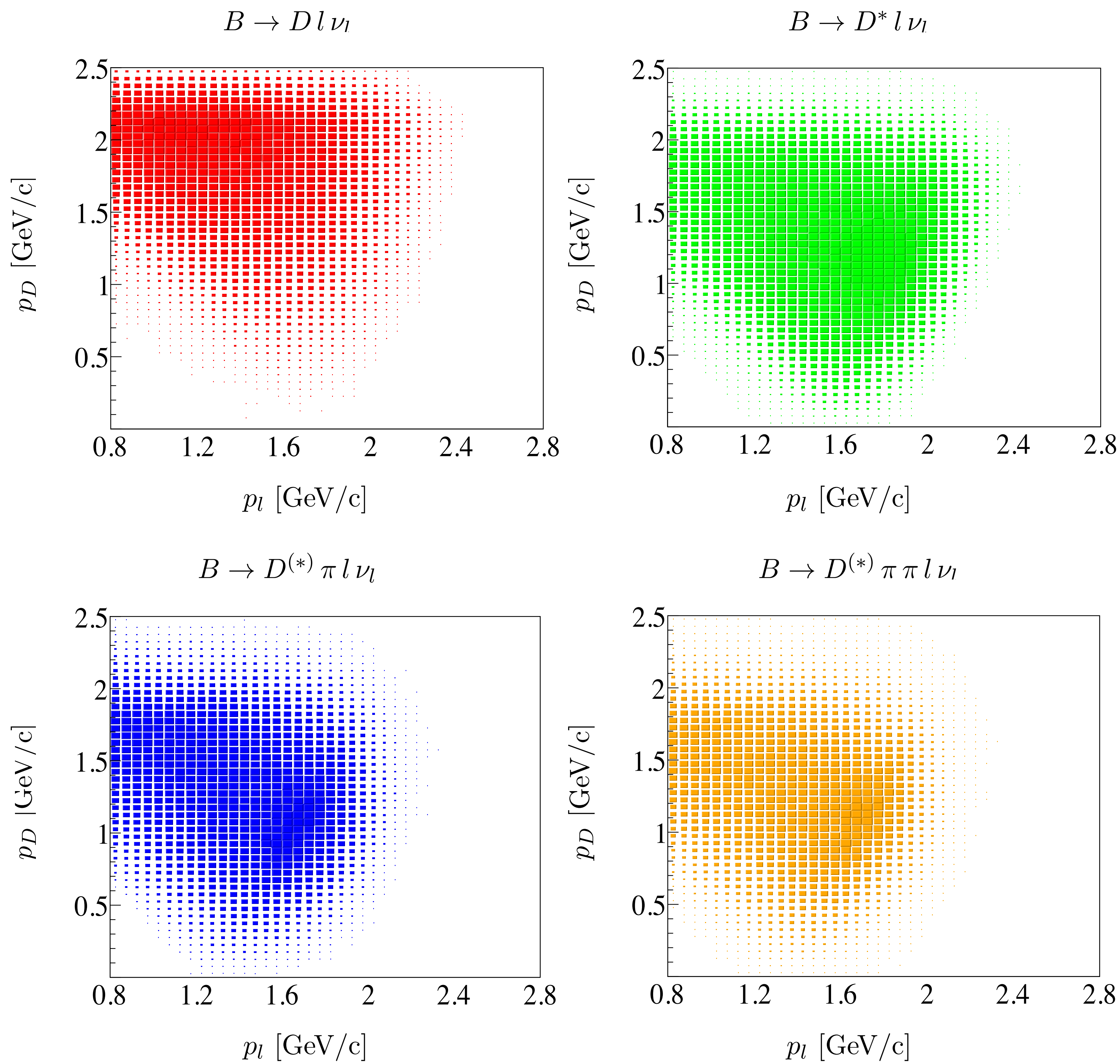
BaBar Global Fit
[arXiv:0809.0828]
Phys.Rev.D79:012002,2009

3D Fit of $B \to D X \ell \nu_\ell$
in lepton momentum,
D momentum,
cosBY

$$\cos\theta_{BY} = \frac{2 E_B\, E_{Dl} - m_B^2 - m_{Dl}^2}{2\, |\vec{p}_B|\, |\vec{p}_{Dl}|}$$

QED effects change shape of all of these variables





Systematics: Table for Global $|V_{cb}|$ Fit

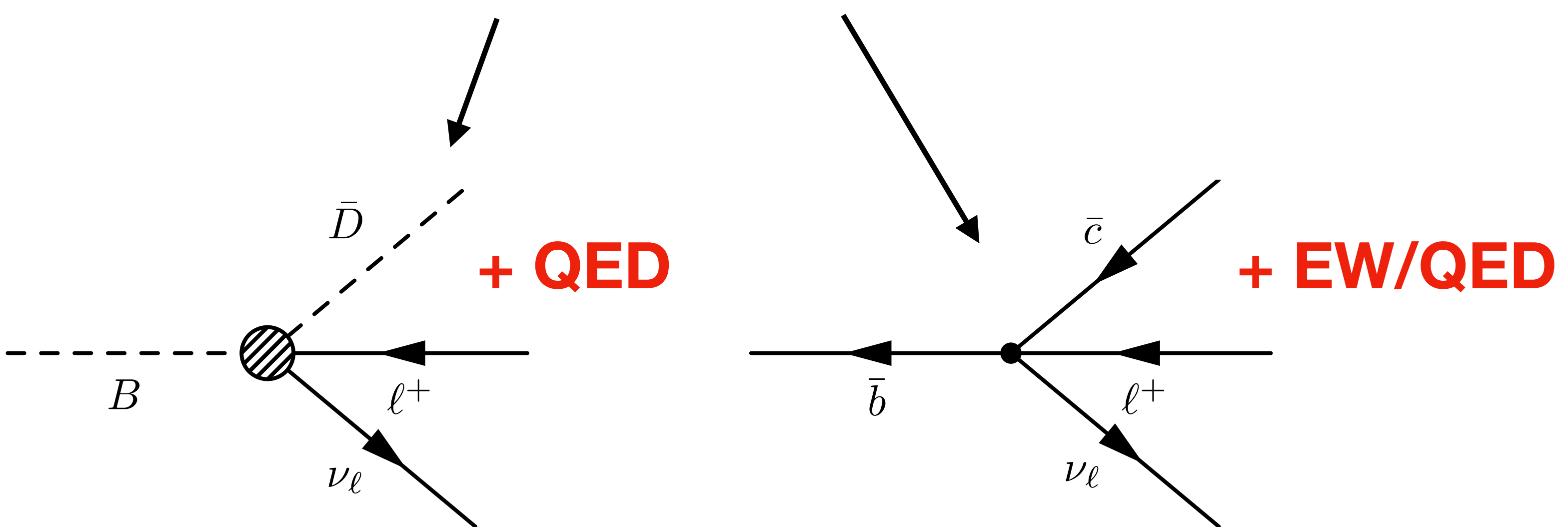
BaBar Global Fit
[arXiv:0809.0828]
Phys.Rev.D79:012002,2009

	Electron sample						Muon sample					
item	ρ^2_D	$\rho^2_{D^*}$	$\mathcal{B}(D\ell\overline{\nu})$	$\mathcal{B}(D^*\ell\overline{\nu})$	$\mathcal{G}(1)\|V_{cb}\|$	$\mathcal{F}(1)\|V_{cb}\|$	ρ^2_D	$\rho^2_{D^*}$	$\mathcal{B}(D\ell\overline{\nu})$	$\mathcal{B}(D^*\ell\overline{\nu})$	$\mathcal{G}(1)\|V_{cb}\|$	$\mathcal{F}(1)\|V_{cb}\|$
R_1'	0.44	2.74	0.71	−0.38	0.60	0.71	0.50	2.67	0.74	−0.40	0.63	0.70
R_2'	−0.40	1.02	−0.18	0.30	−0.32	0.49	−0.45	0.96	−0.19	0.30	−0.33	0.48
D^{**} slope	−1.42	−2.52	−0.07	−0.09	−0.82	−0.87	−1.42	−2.58	−0.10	−0.10	−0.77	−0.92
D^{**} FF approximation	−0.87	0.33	−0.12	0.19	−0.54	0.20	−0.99	0.59	−0.12	0.21	−0.59	0.30
$\mathcal{B}(B^- \to D^{(*)}\pi\ell\overline{\nu})$	0.28	−0.27	−0.22	−0.80	0.04	−0.49	0.59	−0.32	−0.13	−0.86	0.24	−0.54
$f_{D_2^*/D_1}$	−0.39	0.16	−0.38	0.16	−0.41	0.13	−0.50	0.17	−0.41	0.18	−0.47	0.15
$f_{D_0^* D\pi/D_1 D_2^*}$	−2.30	1.12	−1.53	0.97	−2.07	0.85	−3.13	1.23	−1.53	1.02	−2.41	0.93
$f_{D_1' D^*\pi/D_1 D_2^*}$	1.82	−1.14	1.30	−0.65	1.65	−0.70	2.44	−1.15	1.35	−0.72	1.91	−0.75
$f_{D\pi/D_0^*}$	−0.88	−1.28	0.36	0.17	−0.31	−0.34	−0.83	−1.23	0.31	0.18	−0.27	−0.33
$f_{D^*\pi/D_1'}$	−0.21	−0.05	−0.13	0.21	−0.18	0.09	−0.30	−0.04	−0.15	0.23	−0.23	0.10
NR D^*/D ratio	0.58	−0.16	0.11	−0.09	0.38	−0.04	0.66	−0.16	0.11	−0.09	0.40	−0.03
$\mathcal{B}(B^- \to D^{(*)}\pi\pi\ell\overline{\nu})$	1.19	−1.97	0.25	−1.28	0.78	−1.28	1.98	−1.71	0.40	−1.20	1.20	−1.18
X^*/X and Y^*/Y ratio	0.61	−1.15	0.09	−0.27	0.39	−0.52	0.74	−1.02	0.08	−0.24	0.42	−0.47
X/Y and X^*/Y^* ratio	0.76	−0.83	0.21	−0.65	0.52	−0.60	1.09	−0.76	0.25	−0.63	0.68	−0.57
$D_1 \to D\pi\pi$	2.22	−1.54	0.74	−1.08	1.63	−1.05	2.74	−1.48	0.76	−1.06	1.81	−1.03
$f_{D_2^*}$	−0.14	−0.01	−0.10	0.07	−0.12	0.03	−0.16	−0.01	−0.10	0.07	−0.13	0.03
$\mathcal{B}(D^{*+} \to D^0\pi^+)$	0.73	−0.01	0.43	−0.34	0.62	−0.17	0.80	−0.00	0.41	−0.33	0.61	−0.17
$\mathcal{B}(D^0 \to K^-\pi^+)$	0.69	0.02	−0.21	−1.63	0.29	−0.80	0.92	0.12	−0.27	−1.68	0.35	−0.80
$\mathcal{B}(D^+ \to K^-\pi^+\pi^+)$	−1.46	−0.42	−2.17	0.30	−1.89	0.01	−1.43	−0.42	−2.10	0.28	−1.77	−0.01
τ_{B^-}/τ_{B^0}	0.26	0.16	0.63	0.27	0.46	0.19	0.22	0.16	0.58	0.28	0.41	0.19
f_{+-}/f_{00}	0.88	0.43	0.66	−0.53	0.82	−0.12	0.91	0.48	0.57	−0.52	0.75	−0.10
Number of $B\overline{B}$ events	0.00	−0.00	−1.11	−1.11	−0.55	−0.55	0.00	−0.00	−1.11	−1.11	−0.55	−0.55
Off-peak Luminosity	0.05	0.01	−0.02	−0.00	0.02	0.00	0.07	0.00	−0.02	−0.00	0.02	−0.00
B momentum distrib.	−0.96	0.63	1.29	−0.54	−1.15	0.48	1.30	−0.10	1.27	−0.64	1.31	−0.35
Lepton PID eff	0.52	0.16	1.21	0.82	0.90	0.46	3.30	0.06	5.11	5.83	1.99	2.90
Lepton mis-ID	0.03	0.01	−0.01	−0.01	0.01	−0.00	2.65	0.70	−0.59	−0.50	1.06	−0.01
Kaon PID	0.07	0.80	0.28	0.23	0.18	0.38	1.02	0.71	0.35	0.29	0.70	0.39
Tracking eff	−1.02	−0.43	−3.35	−2.00	−2.25	−1.15	−0.63	−0.28	−3.37	−2.09	−2.02	−1.14
Radiative corrections	−3.13	−1.04	−2.87	−0.74	−3.02	−0.71	−0.76	−0.61	−0.82	−0.25	−0.79	−0.33
Bremsstrahlung	0.07	0.00	−0.13	−0.28	−0.04	−0.14	0.00	0.00	0.00	0.00	0.00	0.00
Vertexing	0.83	−0.64	0.63	0.60	0.78	0.09	1.79	−0.76	0.97	0.54	1.41	0.01
Background total	1.39	1.12	0.64	0.34	1.07	0.51	1.58	1.09	0.67	0.38	1.16	0.49
Total	**6.25**	**5.66**	**6.01**	**4.03**	**5.99**	**3.20**	**8.12**	**5.47**	**7.35**	**7.07**	**6.06**	**4.23**

Back to the global fit

- I was a bit annoyed, that a QED effect should be one of the largest systematics.
- Thus I implemented a NLO model and benchmarked it against PHOTOS
- It builds on some assumptions: (some of them likely not entirely great!)
  - First, I assumed I can split long-distance and short-distance physics

$$\mathcal{M}_0^1 = \mathcal{M}_{0,\text{ld}}^1(\Lambda) + \mathcal{M}_{0,\text{sd}}^1(\Lambda) \,.$$



Matching on scale , $\Lambda \sim m_D$; used to regularize any UV divergencies in LD part

More assumptions

- Short-distance parts: Sirlin

LARGE m_W, m_Z BEHAVIOUR OF THE $O(\alpha)$ CORRECTIONS TO SEMILEPTONIC PROCESSES MEDIATED BY W

A. SIRLIN[1]

Department of Physics, New York University, 4 Washington Place, New York, NY 10003, USA

Received 17 August 1981

Using the current algebra formulation of radiative corrections and working in the framework of the $SU(2)_L \times U(1) \times SU(3)_c$ theory, we derive a theorem that governs the large m_W, m_Z behaviour of the $O(\alpha)$ corrections to general semileptonic processes mediated by W. The leading asymptotic dependence is logarithmic with a universal coefficient not affected by the strong interactions. As a byproduct, we obtain the leading asymptotic effect induced perturbatively by the strong interactions, which is of $O(\ln \ln (m_W/\Lambda))$.

The aim of this paper is to analyze the large m_W, m_Z behaviour of the $O(\alpha)$ corrections to semileptonic processes mediated by the W meson, in the framework of the $SU(2)_L \times U(1) \times SU(3)_c$ theory.

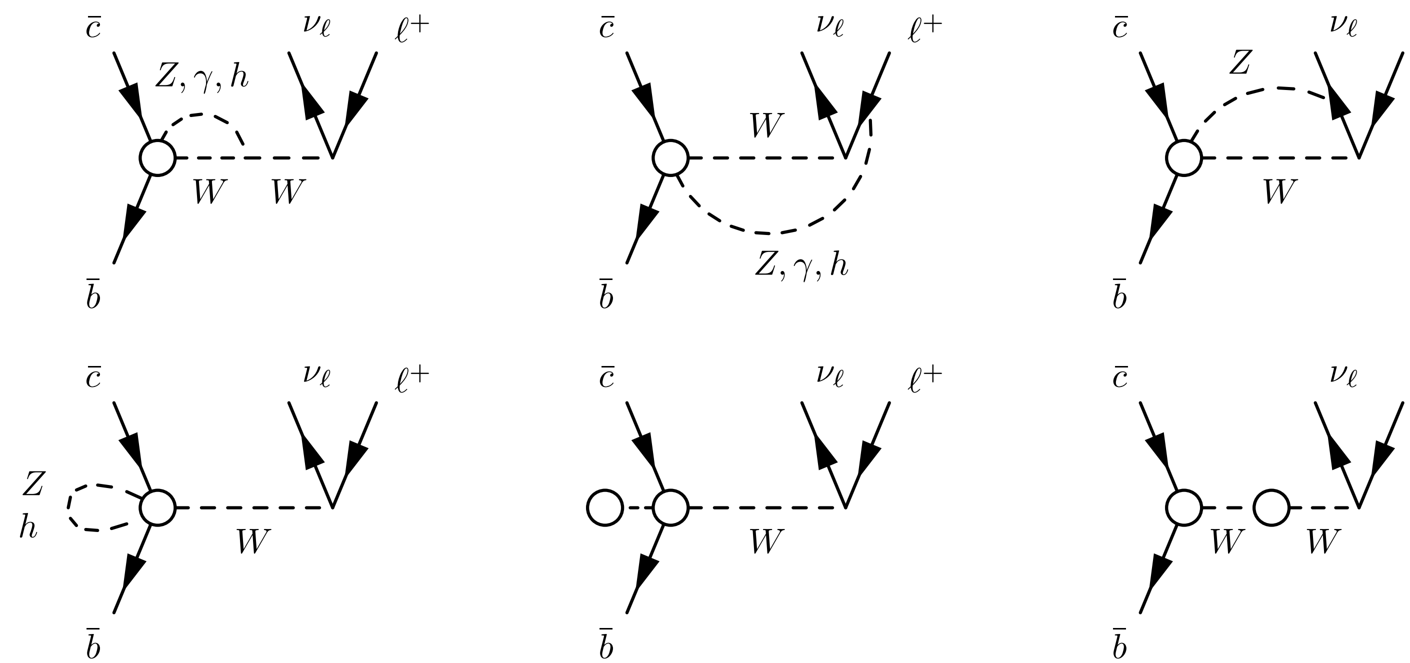
Our main results are summarized in the following theorem:

(a) In the simplest version of the theory in which $\cos \theta_W = m_W/m_Z$ at the tree level, the leading asymptotic behaviour in m_Z of the $O(\alpha)$ corrections to an arbitrary semileptonic process mediated by W is given by

$$\frac{M}{M^0} = 1 + \frac{3\alpha}{4\pi}(1 + 2\bar{Q}) \ln \frac{m_Z}{\mu} + \cdots , \qquad (1)$$

where M is the amplitude up to terms of $O(\alpha)$, M^0 is the zeroth-order amplitude but expressed in terms of the conventionally defined* muon decay coupling constant G_μ, μ is an unspecified mass scale characteristic of the process, and $\bar{Q}$ is the average charge of the quarks in a $SU(2)_L$ isodoublet. Henceforth . . . indicates non-leading contributions as m_W^2 or $m_Z^2 \to \infty$. For the usual charge assignments, $\bar{Q} = \frac{1}{6}$. It is also

$$\eta_{EW}^2 = 1.014$$



$$\boxed{\mathcal{M}^1_{0,\mathrm{sd}} = \frac{\alpha_{\mathrm{em}}}{\pi} \ln \frac{m_Z}{\Lambda} \mathcal{M}^0_0 + \ldots .}$$

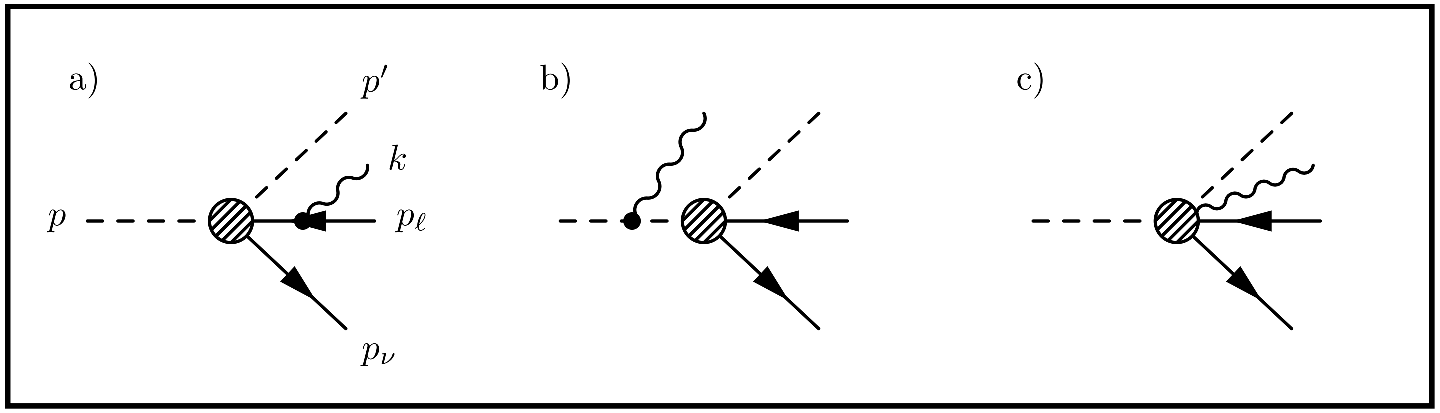
[115] Sirlin, A. : Current Algebra Formulation of Radiative Corrections in Gauge Theories and the Universality of the Weak Interactions. In: *Rev. Mod. Phys.* 50 (1978), S. 573. http://dx.doi.org/10.1103/RevModPhys.50.573. – DOI 10.1103/RevModPhys.50.573

[116] Sirlin, A. : Large m(W), m(Z) Behavior of the O(alpha) Corrections to Semileptonic Processes Mediated by W. In: *Nucl. Phys.* B196 (1982), S. 83. http://dx.doi.org/10.1016/0550-3213(82)90303-0. – DOI 10.1016/0550–3213(82)90303–0

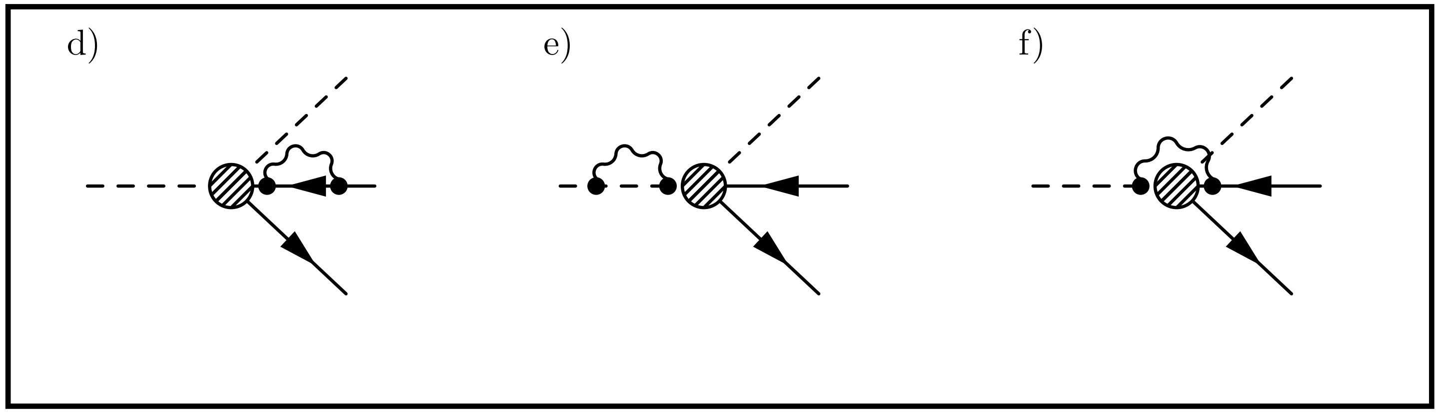
More assumptions

- Long-distance part: Scalar QED with QCD evolution

$$\mathcal{L}_{\mathrm{W}} = \frac{G_{\mathrm{F}}}{\sqrt{2}} V_{\mathrm{cb}} \left[(f_+ + f_-)\, \phi' \, \partial^\mu \phi + (f_+ - f_-)\, \phi \, \partial^\mu \phi' \right] \bar{\psi}_\nu \, P_{\mathrm{R}} \, \gamma_\mu \, \psi_\ell + \mathrm{h.c.}\,,$$



→ **Will create off-shell hadronic matrix elements**



More assumptions

- Long-distance part: Scalar QED with QCD evolution

$$\mathcal{L}_{\mathrm{W}} \quad = \quad \frac{G_{\mathrm{F}}}{\sqrt{2}} V_{\mathrm{cb}} \left[(f_+ + f_-)\, \phi' \, \partial^\mu \phi + (f_+ - f_-)\, \phi \, \partial^\mu \phi' \right] \, \bar{\psi}_\nu \, P_{\mathrm{R}} \, \gamma_\mu \, \psi_\ell + \mathrm{h.c.}\,,$$

- Assumed that the off-shell hadronic current can be modelled using the on-shell current; in particular that the form factors depend on **t = q²** only

$$\langle D(p') | \widehat{V}_\mu - \widehat{A}_\mu | B(p-k) \rangle \quad = \quad \widehat{f}_+(t', r', s') \big(p - k + p'\big)_\mu + \widehat{f}_-(t', r', s') \big(p - k - p'\big)_\mu \,,$$

t' = (p - p' - k)²

r', s': other lorentz scalars

↓

$$H'_\mu(t') \quad = \quad \langle D(p') | \widehat{V}_\mu - \widehat{A}_\mu | B(p-k) \rangle \quad = \quad f_+(t') \big(p - k + p'\big)_\mu + f_-(t') \big(p - k - p'\big)_\mu \,,$$

More formal

- More formal: coupling an electromagnetic current to LO decay results in

lepton leg coupling **hadronic coupling**

$$i\, e\, \frac{G_\mathrm{F}}{\sqrt{2}} V_\mathrm{cb}\, \bar{u}_\nu\, \gamma^\mu\, P_\mathrm{L} \left(-\frac{H_\mu}{2 p_\ell \cdot k} \left(\gamma_\nu \not{k} + 2 (p_\ell)_\nu \right) + V_{\mu\nu} - A_{\mu\nu} \right) v_l \,,$$

hadronic current

$$H_\mu(t) \;=\; \langle D(p') | \widehat{V}_\mu - \widehat{A}_\mu | B(p) \rangle$$

with a non-local operator describing the B-ɣ and D-ɣ coupling

$$V_{\mu\nu} - A_{\mu\nu} \;=\; \int \mathrm{d}^4 x\, e^{ik\cdot x} \langle D | \mathcal{T}\{ h_\mu(0)\, J_\nu^\mathrm{em}(x) \} | B \rangle \,,$$

Ward identities

$$\begin{aligned} k^\nu\, V_{\mu\nu} &= H_\mu \,, \\ k^\nu\, A_{\mu\nu} &= 0 \,, \end{aligned}$$

which can be expanded around first few resonant states

$$\begin{aligned} V_{\mu\nu} - A_{\mu\nu} \;=\; & \frac{\langle D(p') | \widehat{V}_\mu - \widehat{A}_\mu | B(p-k) \rangle \langle B(p-k) | J_\nu^\mathrm{em} | B(p) \rangle}{m_B^2 - (p-k)^2} \\ & + \frac{\langle D(p') | \widehat{V}_\mu - \widehat{A}_\mu | B^*(p-k) \rangle \langle B^*(p-k) | J_\nu^\mathrm{em} | B(p) \rangle}{m_{B^*}^2 - (p-k)^2} \\ & + \frac{\langle D(p'-k) | J_\nu^\mathrm{em} | D^*(p') \rangle \langle D^*(p') | \widehat{V}_\mu - \widehat{A}_\mu | B(p) \rangle}{m_{D^*}^2 - (p'-k)^2} + \ldots \,, \end{aligned}$$

More formal

- More formal: coupling an electromagnetic current to LO decay results in

lepton leg coupling hadronic coupling

$$i\,e\,\frac{G_F}{\sqrt{2}}\,V_{cb}\,\bar{u}_\nu\,\gamma^\mu\,P_L\left(-\frac{H_\mu}{2p_\ell\cdot k}\left(\gamma_\nu \not{k} + 2(p_\ell)_\nu\right) + V_{\mu\nu} - A_{\mu\nu}\right) v_l\,,$$

hadronic current

$$H_\mu(t) = \langle D(p')|\widehat{V}_\mu - \widehat{A}_\mu|B(p)\rangle$$

with a non-local operator describing the B-ɣ and D-ɣ coupling

$$V_{\mu\nu} - A_{\mu\nu} = \int d^4x\, e^{ik\cdot x}\langle D|\mathcal{T}\{h_\mu(0)\,J_\nu^{\text{em}}(x)\}|B\rangle\,,$$

Ward identities

$$k^\nu\,V_{\mu\nu} = H_\mu\,,\qquad k^\nu\,A_{\mu\nu} = 0\,,$$

which can be expanded around first few resonant states

$$V_{\mu\nu} - A_{\mu\nu} = \frac{\langle D(p')|\widehat{V}_\mu - \widehat{A}_\mu|B(p-k)\rangle\langle B(p-k)|J_\nu^{\text{em}}|B(p)\rangle}{m_B^2 - (p-k)^2} + \frac{\langle D(p')|\widehat{V}_\mu - \widehat{A}_\mu|B^*(p-k)\rangle\langle B^*(p-k)|J_\nu^{\text{em}}|B(p)\rangle}{m_{B^*}^2 - (p-k)^2} + \frac{\langle D(p'-k)|J_\nu^{\text{em}}|D^*(p')\rangle\langle D^*(p')|\widehat{V}_\mu - \widehat{A}_\mu|B(p)\rangle}{m_{D^*}^2 - (p'-k)^2} + \ldots\,,$$

→

$$k^\nu\,V_{\mu\nu} = H'_\mu(t')\frac{k\cdot(2p-k)}{2p\cdot k} + \ldots$$

with

$$\langle B(p-k)|J_\nu^{\text{em}}|B(p)\rangle = (2p-k)_\nu\,F_{\text{em}}\,,$$

$$F_{\text{em}} \approx 1$$

in soft photon limit

Continued

$$k^\nu V_{\mu\nu} = H'_\mu(t') \frac{k \cdot (2p - k)}{2p \cdot k} + \dots$$

- For the hadronic current: Taylor expand

$$k^\nu V_{\mu\nu} = H_\mu(t) + k' \frac{\mathrm{d}H'_\mu}{\mathrm{d}t'}\Big|_{k'=0} + k'^2 \frac{\mathrm{d}^2 H'_\mu}{\mathrm{d}t'^2}\Big|_{k'=0} + \dots ,$$

and neglect all higher order terms, plus introduce seagull terms to make sure matrix element fulfils the Ward identity / is gauge invariant:

$V_{\mu\nu} - A_{\mu\nu}$

$$i\, e \frac{G_\mathrm{F}}{\sqrt{2}} V_\mathrm{cb}\, \bar{u}_\nu\, \gamma^\mu\, P_\mathrm{L} \Bigg(- \frac{H_\mu}{2p_\ell \cdot k} \left(\gamma_\nu \not{k} + 2(p_\ell)_\nu\right) + \frac{H_\mu\, p_\nu}{p \cdot k} + f_3(t) \left(\frac{k_\mu\, p_\nu}{p \cdot k} - g_{\mu\nu}\right) + \left(-2(p - p')^\alpha \frac{\mathrm{d}H_\mu(t')}{\mathrm{d}t'}\Big|_{k'=0}\right) \left(\frac{k_\alpha\, p_\nu}{p \cdot k} - g_{\alpha\nu}\right)\Bigg) v_l ,$$

Assumptions assumptions

1. The non-local operator Eq. (3.4) was expanded in a number of matrix elements which correspond to intermediate resonances allowed in the soft photon part of phase-space.
2. The off-shell hadronic current was approximated by the on-shell hadronic current.
3. The higher order terms of Eq. (3.13) which are ambiguous and depend on the parametrization of the on-shell matrix element were neglected.
4. No intermediate excited resonances were considered.

- Certainly not a bad set of approximations in the soft-photon limit, **unclear how well this** describes **reality** if one goes beyond
  - Hope during my thesis: this only can happen for a handful of events, so maybe not so relevant
  - Plus: I wanted to graduate ;-) And this is already better than an arbitrary variation of 25%

More conceptual problems

- Form factors enter into all of the calculations
  - All of them are measured by "factoring out QED"
    - E.g. tagged measurements simply use q^2 as calculated from hadronic systems

$$q^2 = (p_B - p_D)^2 = \left(p_\ell + p_\nu + \sum_i k_i \right)^2$$

    - Even untagged measurements factorize QED effects out, i.e. change the shape of the templates are calculated using "true" q^2 values as defined w/o QED corrections

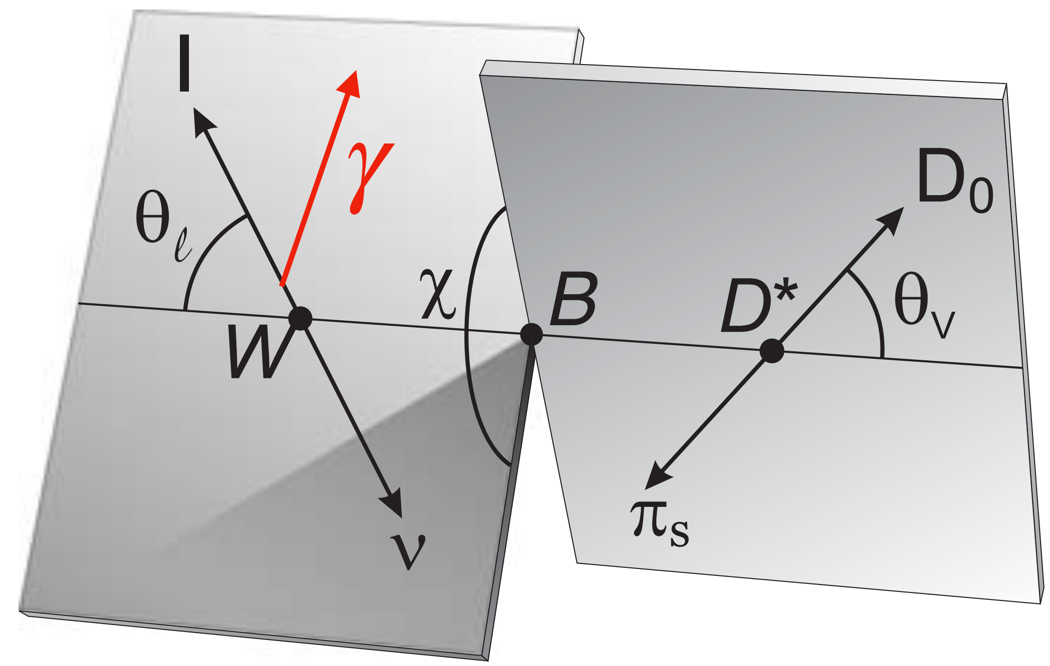
Shape differences due to inadequate QED modelling are just absorbed into form factor parameters.

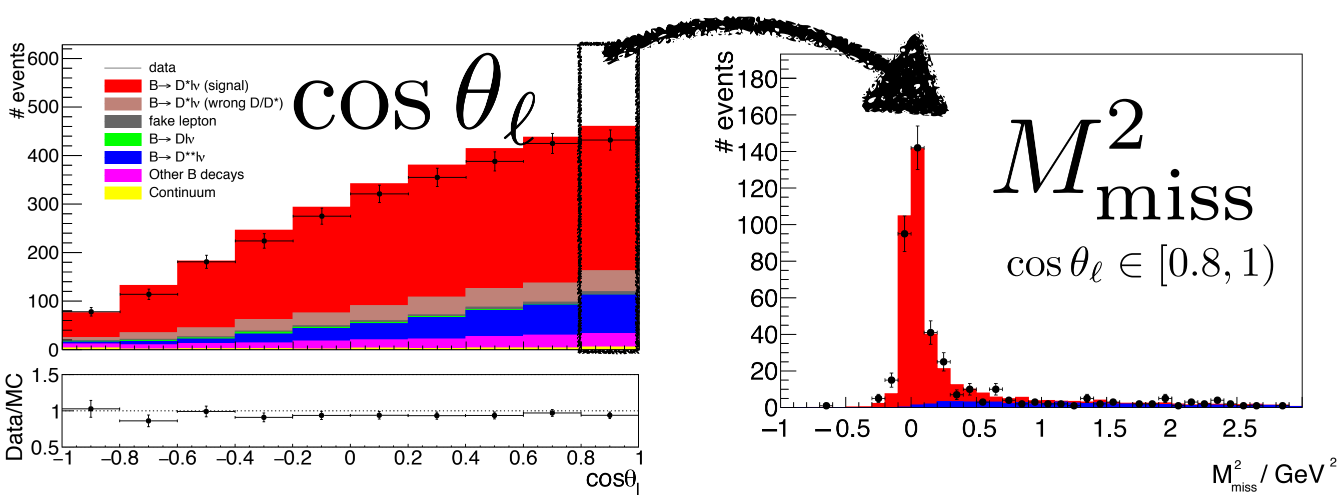
Thus any theory based prediction you make on how fundamental parameters change based on kinematic changes, will likely not be valid.

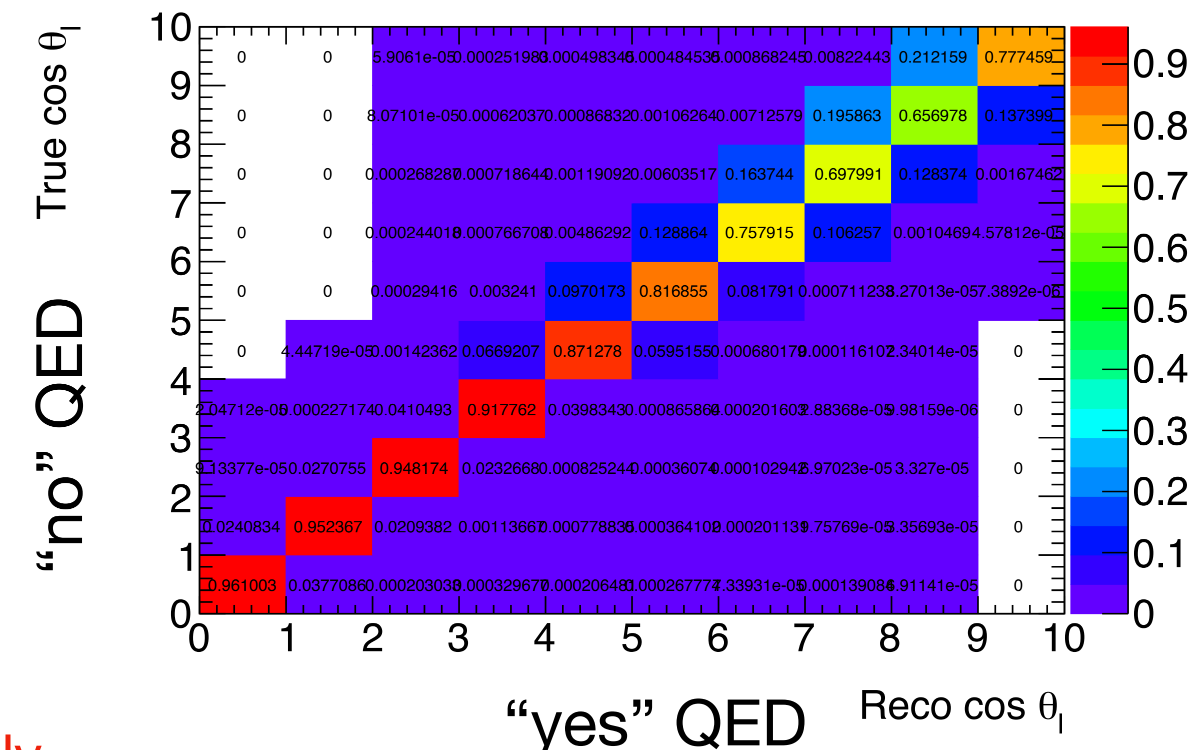
Example: Impact on Tagged Analyses

Belle Tagged and Untagged
[arXiv:1702.01521]
[arXiv:1809.03290]

- Reconstruct $q^2 = (p_B - p_{D^{(*)}})^2$
- For helicity angles: treat QED as a resolution effect when **unfolding** or **folding**
- Yield extraction via nearly model-independent fits:



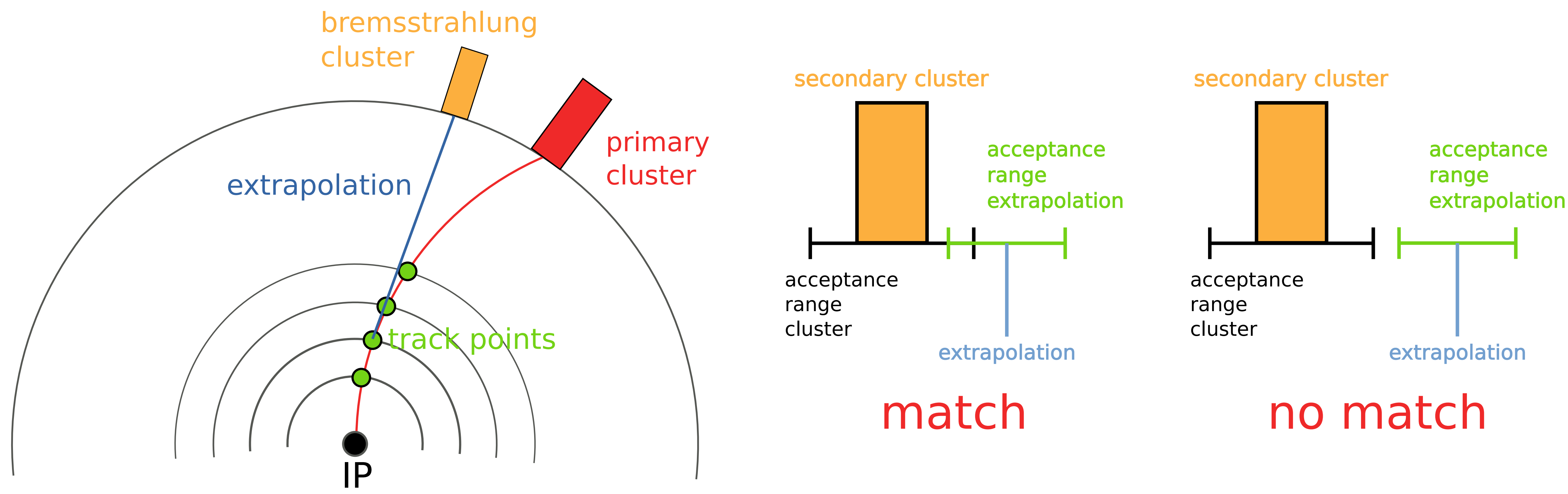




$M^2_{\rm miss} = (p_B - p_{D^*} - p_\ell)^2$

↖ neglects FSR photons partially

Why partially? Because of Brem-recovery in electron reconstruction



- BaBar & Belle: look for photons within a cone of the original trajectory as determined from **IP → this will find some collinear FSR photons,** correct 4-momentum as $p_\ell = p'_\ell + k$
- Belle II: do this, but extrapolate from each material layer, are looking into redoing track fit with photon information

Regularisation / Integration

- I regularised the UV poles using **Pauli-Villars,** as this made it easy to Sirlin's quark-level calculation

$$\mathcal{L}_{\mathrm{PV}} = \frac{1}{4}\tilde{F}^2 + \Lambda^2 \tilde{A}^2 \,, \quad \xrightarrow{\text{e.g.}} \quad \begin{array}{lcl} B_0(m_\ell^2, m_\ell^2, \lambda^2) & \to & B_0(m_\ell^2; m_\ell^2, \lambda^2) - B_0(m_l^2; m_\ell^2, \Lambda^2)\,, \\ \dot{B}_i(m_\ell^2, m_\ell^2, \lambda^2) & \to & \dot{B}_i(m_\ell^2, m_\ell^2, \lambda^2) - \dot{B}_i(m_\ell^2, m_\ell^2, \Lambda^2)\,, \end{array}$$

plus sign instead of -

- After that I integrated the total rate via

Born + Virtual

$$\mathrm{d}\Gamma_0^0 + \mathrm{d}\Gamma_0^1 = \frac{1}{64\pi^3 m}\left(\left|\mathcal{M}_0^0\right|^2 + 2\Re \sum_{d)-h)} \mathcal{M}_0^0 \mathcal{M}_0^1 + 2\left|\mathcal{M}_0^0\right|^2 \left(\frac{\alpha_{\mathrm{em}}}{\pi} \ln \frac{m_Z}{\Lambda}\right)\right) \mathrm{d}E' \, \mathrm{d}E_\ell \,,$$

Real emission

$$\mathrm{d}\Gamma_1^1 = \frac{1}{(2\pi)^{12}} \delta^{(4)}(m - p' - p_\ell - p_{\nu_l} - k) \left| \sum_{a)-c)} \mathcal{M}_1^{\frac{1}{2}} \right|^2 \frac{\mathrm{d}^3 p'}{E'} \frac{\mathrm{d}^3 p_\ell}{E_\ell} \frac{\mathrm{d}^3 p_{\nu_l}}{E_{\nu_l}} \frac{\mathrm{d}^3 k}{E_\gamma} \,, \quad \longrightarrow \quad \text{same IR cut-off } \lambda$$

- And produced **NLO events** using the corresponding matrix elements and mixed them according to their integrals

Correction to Sirlin's correction

- I also calculated corrections to Sirlin's correction using

Sirlin's correction — revised EW correction

$$\Gamma_0^0 + \Gamma_0^1 + \Gamma_1^1 = (1 + \delta_{\rm sd} + \delta_{\rm ld})\, \Gamma_0^0 = \eta_{\rm EW}^2\, \Gamma_0^0\,,$$

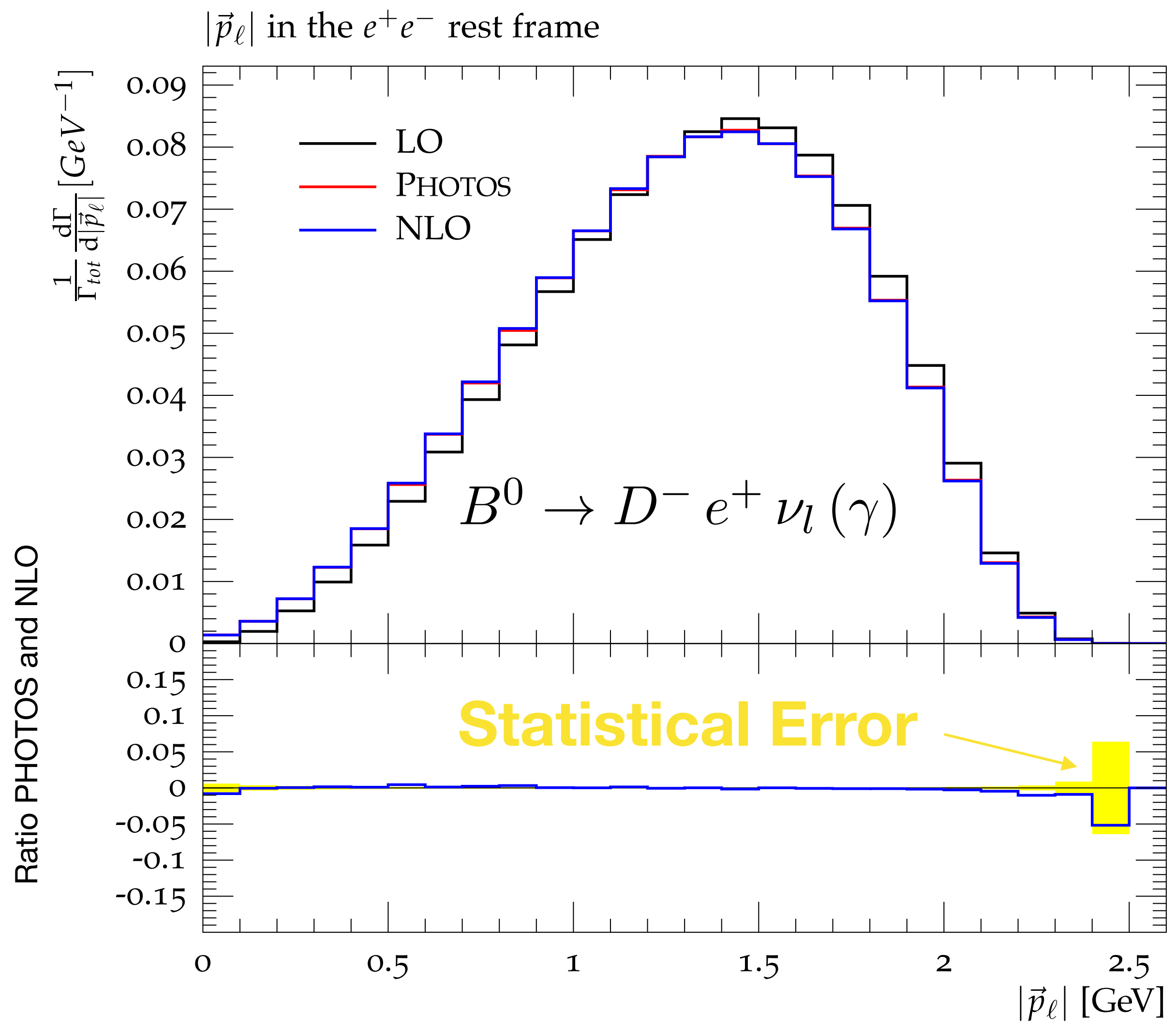
My new long-distance correction

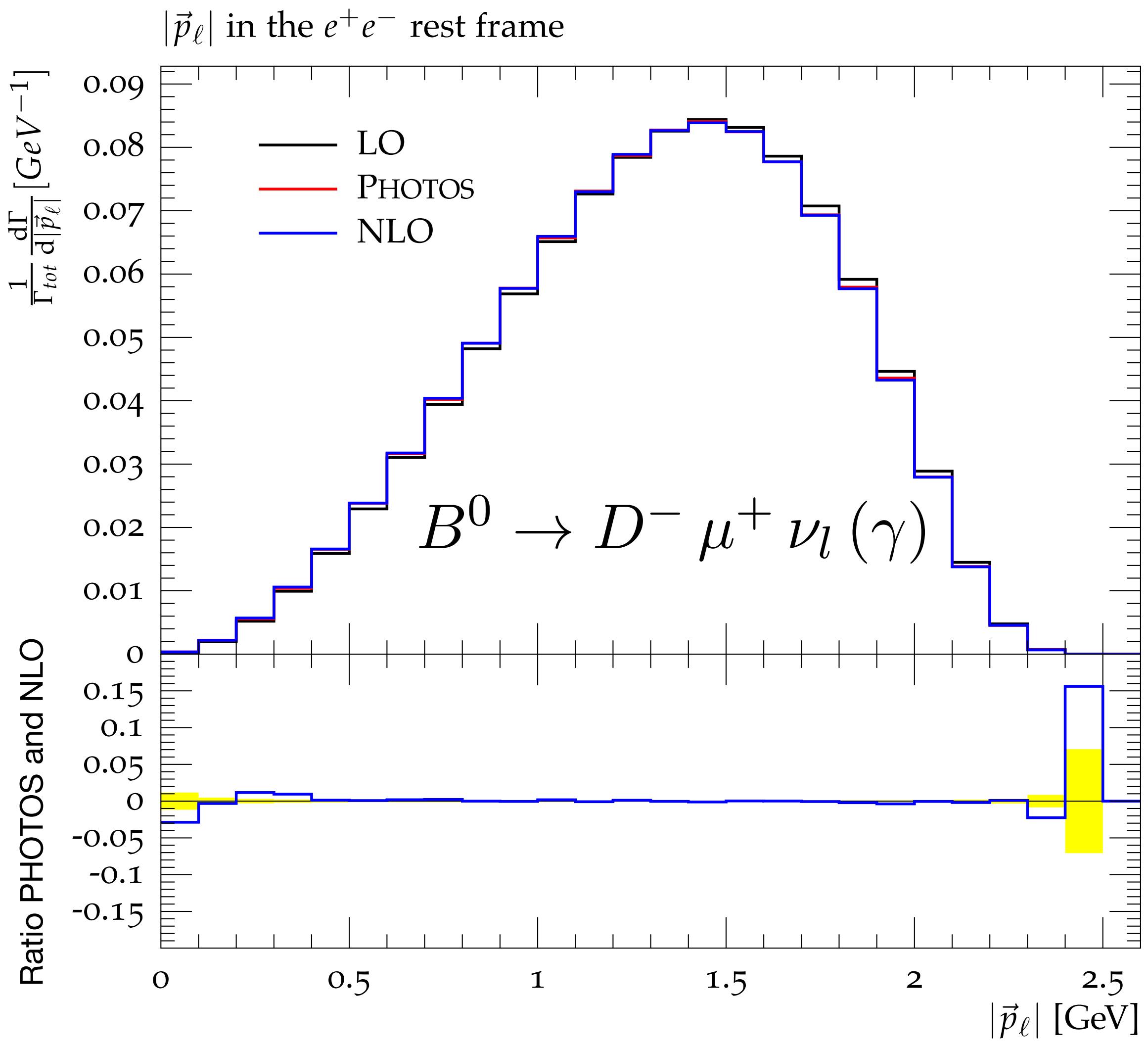
- Numerical input

$$\begin{aligned}
m_{\Upsilon(4S)} &= 10.579 \quad \text{GeV}/c^2\,,\\
\Gamma_{\Upsilon(4S)} &= 20.5 \quad \text{MeV}/c^2\,,\\
m_{B^+} &= 5.279 \quad \text{GeV}/c^2\,,\\
m_{B^0} &= 5.280 \quad \text{GeV}/c^2\,,\\
m_{D^+} &= 1.870 \quad \text{GeV}/c^2\,,\\
m_{D^0} &= 1.865 \quad \text{GeV}/c^2\,,\\
\lambda &= 10^{-7} \quad \text{GeV}/c^2\\
m_Z &= 91.188 \quad \text{GeV}/c^2\\
\alpha_{\rm em} &= 0.00729735\,,\\
G_{\rm F} &= 1.16637 \times 10^{-5} \quad \text{GeV}^{-2}\ (\hbar c)^3\,.
\end{aligned}$$

Results and Comparisons with PHOTOS

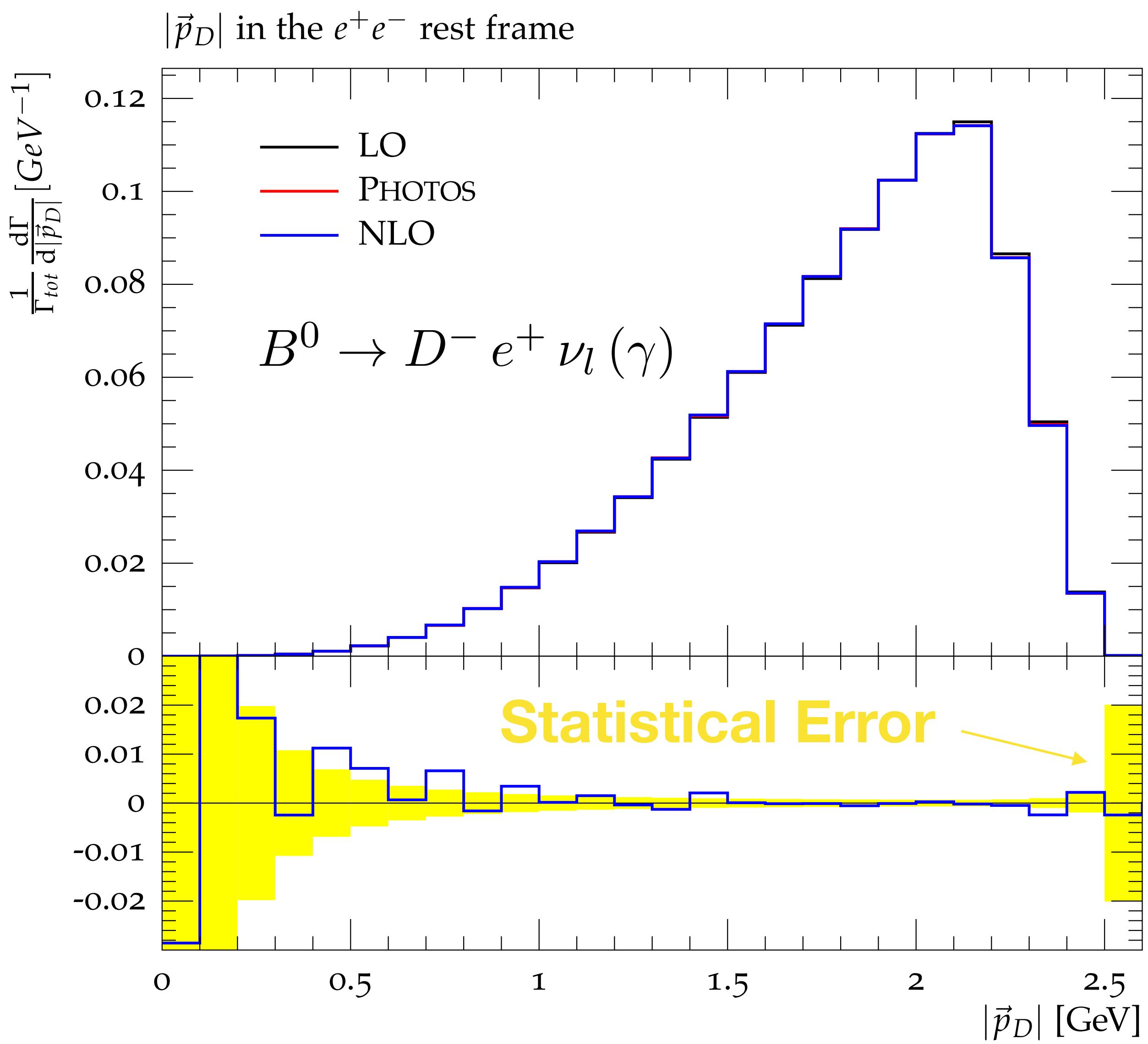
Lepton momentum spectrum in CM frame

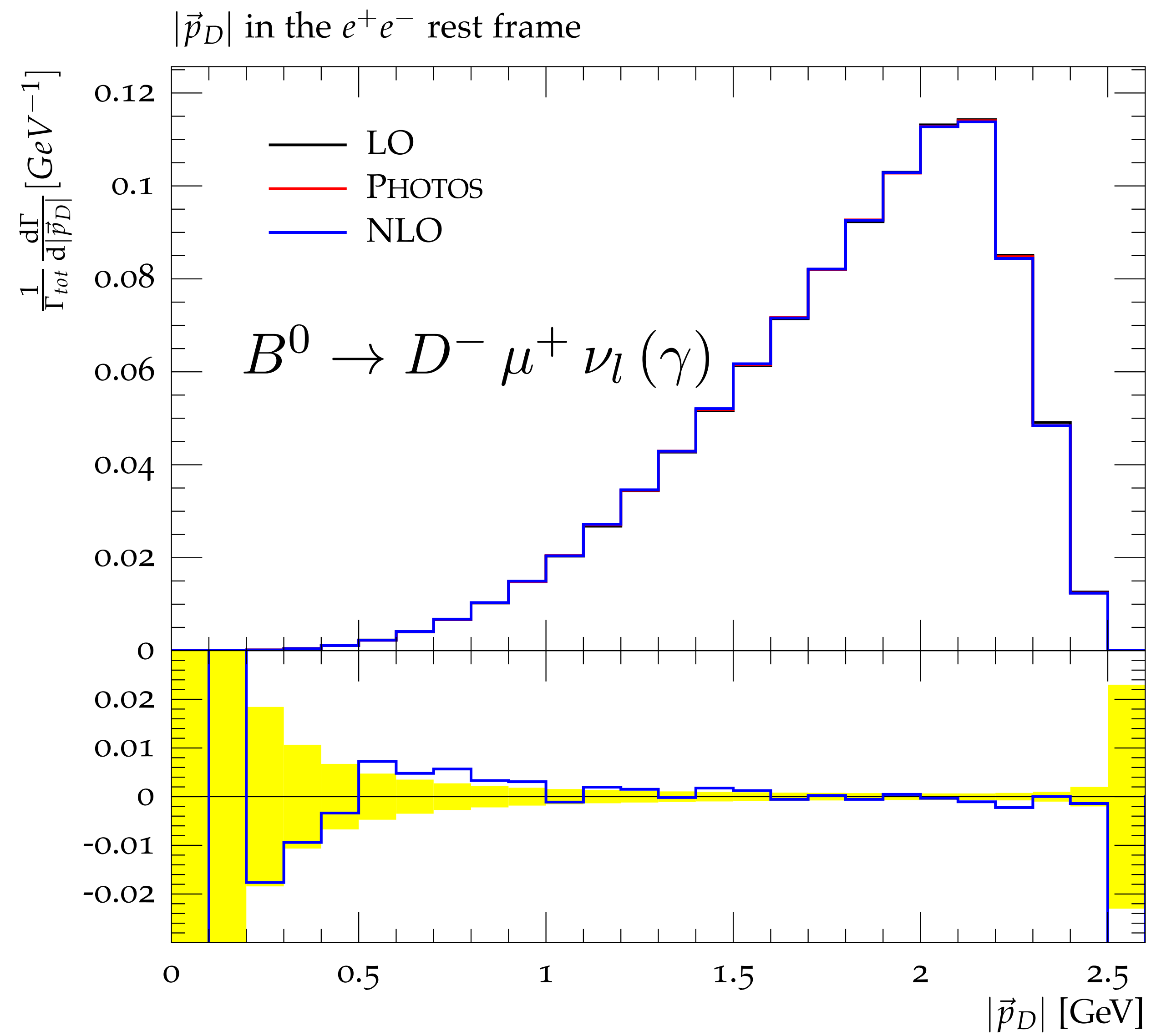




Results and Comparisons with PHOTOS

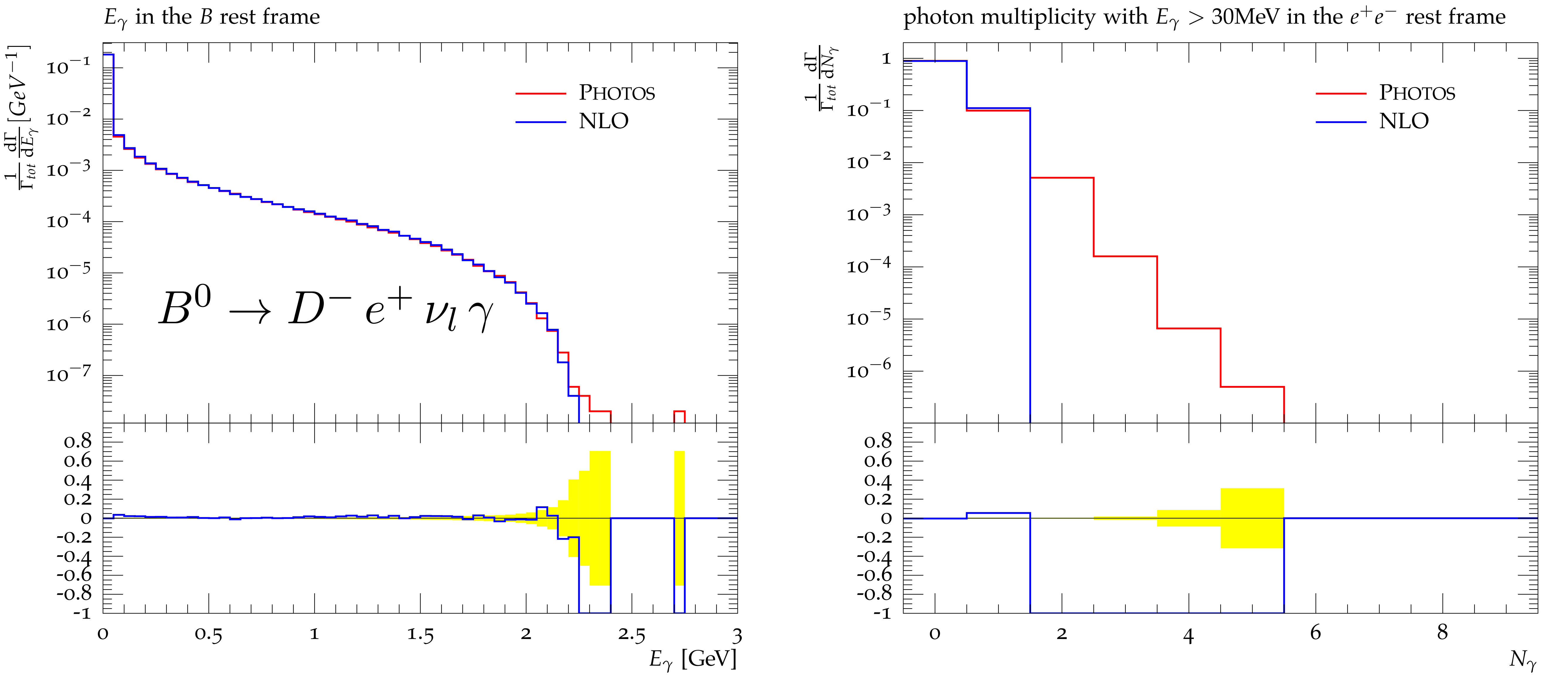
D momentum spectrum in CM frame





Photon Energy spectrum

Photon energy spectrum



Updated η_{EW}^2 Values

$B^0 \to D^- e^+ \bar{\nu}_e(\gamma)$

$$\eta_{\rm EW}^2 = 1.0235 \pm 0.0002_{\rm stat} \pm 0.0023_{\rm theo}\,,$$

$B^+ \to \bar{D}^0 e^+ \bar{\nu}_e(\gamma)$

$$\eta_{\rm EW}^2 = 1.0147 \pm 0.0001_{\rm stat} \pm 0.0045_{\rm theo}\,,$$

$B^0 \to D^- \mu^+ \bar{\nu}_\mu(\gamma)$

$$\eta_{\rm EW}^2 = 1.0237 \pm 0.0001_{\rm stat} \pm 0.0020_{\rm theo}\,,$$

$B^+ \to \bar{D}^0 \mu^+ \bar{\nu}_\mu(\gamma)$

$$\eta_{\rm EW}^2 = 1.0150 \pm 0.0001_{\rm stat} \pm 0.0045_{\rm theo}\,,$$

Sirlin's correction: $\eta_{EW}^2 = 1.014$

- **Theory errors:** Variation of the matching scale Λ

Decay	Λ	$\eta_{\rm EW}^2$
$B^0 \to D^- e^+ \nu_l$	$2m_D^+$	1.0220
	m_D^+	1.0235
	$\frac{1}{2}m_D^+$	1.0256

Decay	Λ	$\eta_{\rm EW}^2$
$B^0 \to D^- \mu^+ \nu_l$	$2m_D^+$	1.0220
	m_D^+	1.0237
	$\frac{1}{2}m_D^+$	1.0254

Decay	Λ	$\eta_{\rm EW}^2$
$B^+ \to \bar{D}^0 e^+ \nu_l$	$2m_D^+$	1.0110
	m_D^+	1.0147
	$\frac{1}{2}m_D^+$	1.0189

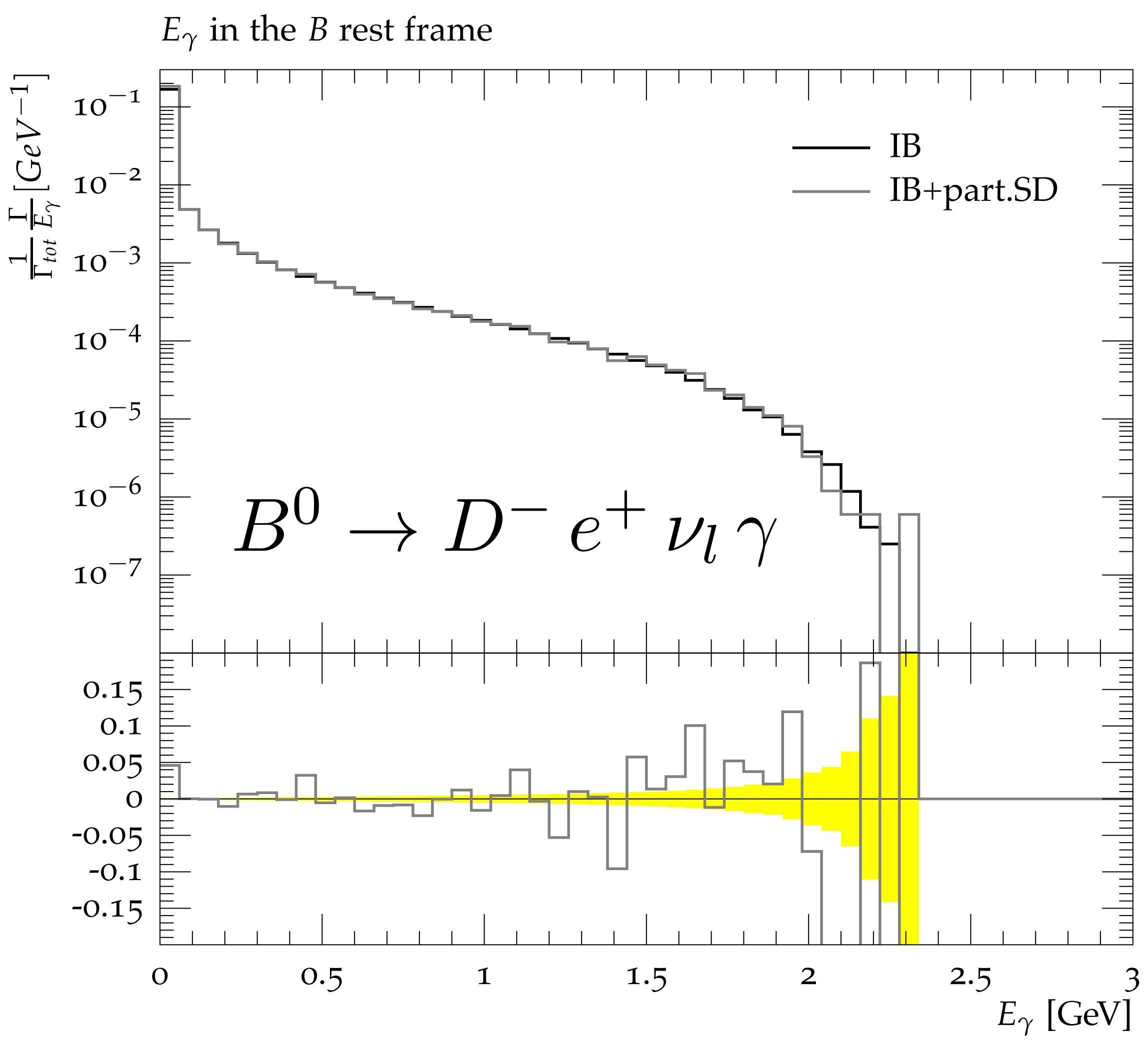
Decay	Λ	$\eta_{\rm EW}^2$
$B^+ \to \bar{D}^0 \mu^+ \nu_l$	$2m_D^+$	1.0113
	m_D^+	1.0150
	$\frac{1}{2}m_D^+$	1.0192

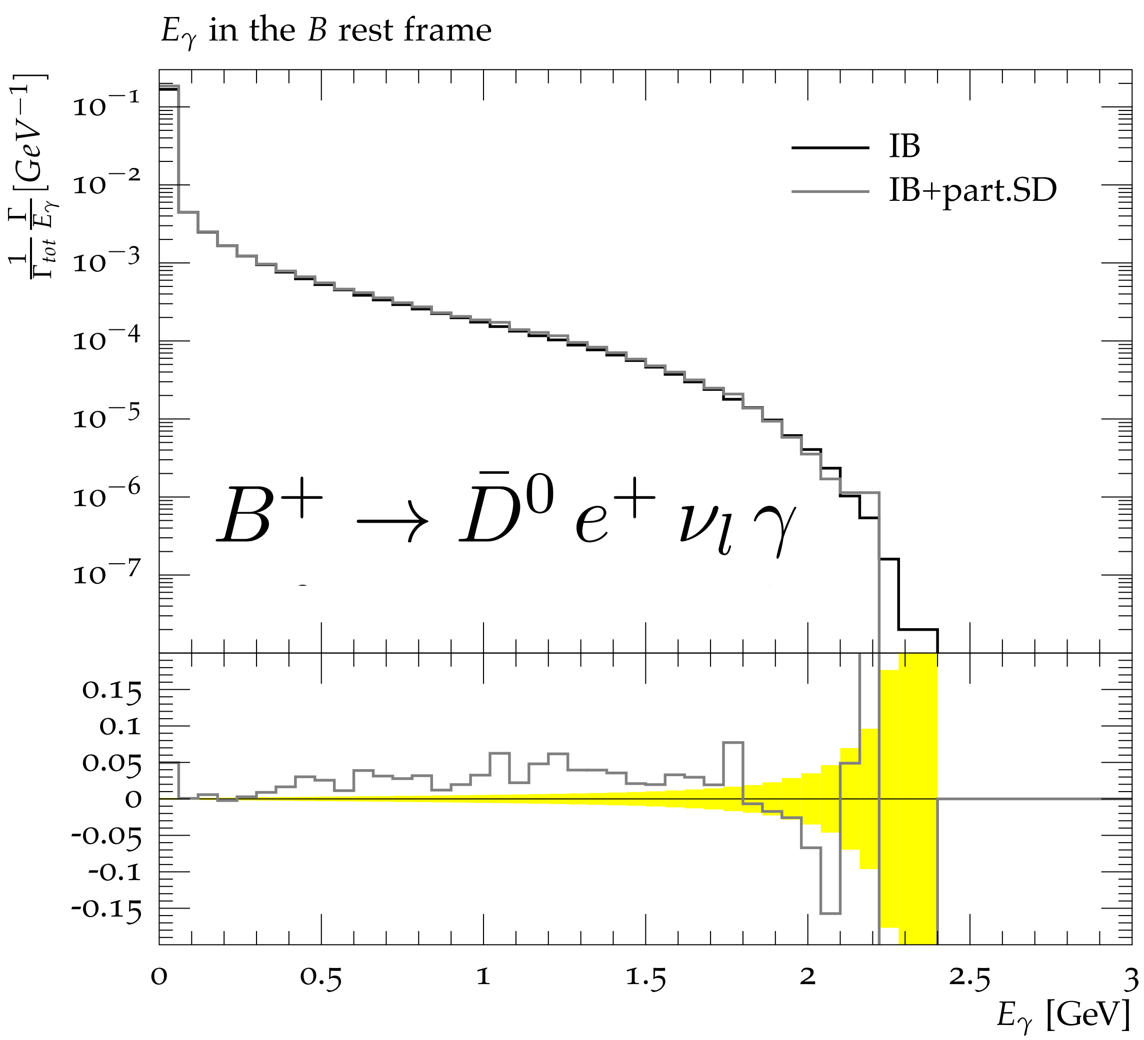
Theory Errors

Damir Becirevic,
Nejc Kosnik
[arXiv:0910.5031]

- Structure dependent contributions in the real emission; used different gauge invariant terms to assess impact and results from Damir's paper

$$\frac{\langle D(p'-k)|J_\nu^{\rm em}|D^*(p')\rangle\langle D^*(p')|\widehat{V}_\mu - \widehat{A}_\mu|B(p)\rangle}{m_{D^*}^2-(p'-k)^2}$$





Updated systematic table

	ρ_D^2	$\rho_{D^*}^2$	$\mathcal{B}(D^0\, l\, \nu_l)$	$\mathcal{B}(D^{*0}\, l\, \nu_l)$
$R_1'(1)$	1.248	3.046	0.841	−0.253
$R_2'(1)$	1.351	−1.343	0.550	−0.481
f_{D_2/D_1}	−0.206	0.051	−0.153	0.057
$f_{\mathcal{A}_1/D_0}$	−0.637	−0.641	0.165	0.071
$f_{\mathcal{A}_2/D_1'}$	−0.224	−0.163	−0.134	0.240
$f_{D_0\mathcal{A}_1/D_1D_2}$	−1.199	0.430	−0.576	0.327
$f_{D_1'\mathcal{A}_2/D_1D_2}$	0.572	−0.284	0.335	−0.109
f_{+0}	1.334	0.444	0.786	−0.529
τ_{+0}	0.253	0.108	0.438	0.176
f_{D_2}	−0.089	−0.004	−0.048	0.027
$\mathcal{B}(B^+ \to D^{(*)}\,\pi\, l\, \nu_l)$	0.490	−0.350	−0.130	−0.736
$\mathcal{B}(D^0 \to K^+\, \pi^-)$	1.032	0.026	−0.138	−1.612
$\mathcal{B}(D^+ \to K^+\, \pi^-\, \pi^+)$	−1.932	−0.361	−1.966	0.253
$\mathcal{B}(D^{*+} \to \bar{D}^0\, \pi^+)'$	1.116	−0.019	0.464	−0.314
$\mathcal{B}(D^{*+} \to D^+\, \pi^0)'$	0.508	−0.008	0.212	−0.143
Tracking	−0.371	−0.157	−1.000	−0.732
Vertexing	−0[illegible]	[illegible]	[illegible]	−0.698
Lepton mis-ID	0[illegible]	[illegible]	[illegible]	−0.010
Lepton PID	0[illegible]	[illegible]	[illegible]	1.469
Kaon PID	−0[illegible]	[illegible]	[illegible]	0.065
Bremsstrahlung	−0[illegible]	[illegible]	[illegible]	0.290
D^{**} Slope	−1[illegible]	[illegible]	[illegible]	−0.189
D^{**} FF approximation	0.920	−0.511	0.145	−0.195
Number of $B\bar{B}$ events	−0.123	−0.100	−0.670	−0.669
Off-resonance luminosity	0.059	0.003	−0.019	−0.003
Radiative corrections for $B \to D\, l\, \nu_l$	−0.126	−0.056	−0.289	0.045
Radiative corrections for $B \to D^*\, l\, \nu_l$	1.657	0.056	0.574	1.187
Radiative corrections for $B \to D^{**}\, l\, \nu_l$	−0.023	0.072	0.111	0.298
Correction to off-resonance	−1.057	0.155	−0.236	0.064
$D^{**}(2S) \to D^{(*)}\pi$ contributions	−0.463	−0.998	−0.184	−0.374
$B \to D^{(*)}\,\pi\,\pi\, l\, \nu_l$ contributions	0.876	0.364	0.245	0.445
Further background	0.595	0.699	0.354	0.099
Total	4.856	4.515	3.318	3.124

From difference of PHOTOS &'NLO'

item	Electron sample					
	ρ_D^2	$\rho_{D^*}^2$	$\mathcal{B}(D\ell\overline{\nu})$	$\mathcal{B}(D^*\ell\overline{\nu})$	$\mathcal{G}(1)\lvert V_{cb}\rvert$	$\mathcal{F}(1)\lvert V_{cb}\rvert$
R_1'	0.44	2.74	0.71	−0.38	0.60	0.71
R_2'	−0.40	1.02	−0.18	0.30	−0.32	0.49
D^{**} slope	−1.42	−2.52	−0.07	−0.09	−0.82	−0.87
D^{**} FF approximation	−0.87	0.33	−0.12	0.19	−0.54	0.20
$\mathcal{B}(B^- \to D^{(*)}\pi\ell\overline{\nu})$	0.28	−0.27	−0.22	−0.80	0.04	−0.49
$f_{D_2^*/D_1}$	−0.39	0.16	−0.38	0.16	−0.41	0.13
$f_{D_0^*D\pi/D_1D_2^*}$	−2.30	1.12	−1.53	0.97	−2.07	0.85
$f_{D_1'D^*\pi/D_1D_2^*}$	1.82	−1.14	1.30	−0.65	1.65	−0.70
$f_{D\pi/D_0^*}$	−0.88	−1.28	0.36	0.17	−0.31	−0.34
$f_{D^*\pi/D_1'}$	−0.21	−0.05	−0.13	0.21	−0.18	0.09
NR D^*/D ratio	0.58	−0.16	0.11	−0.09	0.38	−0.04
$\mathcal{B}(B^- \to D^{(*)}\pi\pi\ell\overline{\nu})$	1.19	−1.97	0.25	−1.28	0.78	−1.28
X^*/X and Y^*/Y ratio	0.61	−1.15	0.09	−0.27	0.39	−0.52
X/Y and X^*/Y^* ratio	0.76	−0.83	0.21	−0.65	0.52	−0.60
$D_1 \to D\pi\pi$	2.22	−1.54	0.74	−1.08	1.63	−1.05
$f_{D_2^*}$	−0.14	−0.01	−0.10	0.07	−0.12	0.03
$\mathcal{B}(D^{*+} \to D^0\pi^+)$	0.73	−0.01	0.43	−0.34	0.62	−0.17
$\mathcal{B}(D^0 \to K^-\pi^+)$	0.69	0.02	−0.21	−1.63	0.29	−0.80
$\mathcal{B}(D^+ \to K^-\pi^+\pi^+)$	−1.46	−0.42	−2.17	0.30	−1.89	0.01
τ_{B^-}/τ_{B^0}	0.26	0.16	0.63	0.27	0.46	0.19
f_{+-}/f_{00}	0.88	0.43	0.66	−0.53	0.82	−0.12
Number of $B\overline{B}$ events	0.00	−0.00	−1.11	−1.11	−0.55	−0.55
Off-peak Luminosity	0.05	0.01	−0.02	−0.00	0.02	0.00
B momentum distrib.	−0.96	0.63	1.29	−0.54	−1.15	0.48
Lepton PID eff	0.52	0.16	1.21	0.82	0.90	0.46
Lepton mis-ID	0.03	0.01	−0.01	−0.01	0.01	−0.00
Kaon PID	0.07	0.80	0.28	0.23	0.18	0.38
Tracking eff	−1.02	−0.43	−3.35	−2.00	−2.25	−1.15
Radiative corrections	−3.13	−1.04	−2.87	−0.74	−3.02	−0.71
Bremsstrahlung	0.07	0.00	−0.13	−0.28	−0.04	−0.14
Vertexing	0.83	−0.64	0.63	0.60	0.78	0.09
Background total	1.39	1.12	0.64	0.34	1.07	0.51
Total	**6.25**	**5.66**	**6.01**	**4.03**	**5.99**	**3.20**

Summary and some more pointers

- It would be nice to establish a consistent treatment of QED effects when extraction e.g. form factors
  - Not so important right right now, but Belle II and LHCb will reach O(1.2 - 1.4%) experimental precision on exclusive $|V_{ub}|$ and $|V_{cb}|$

Observables	Belle (2017)	Belle II 5 ab^{-1}	Belle II 50 ab^{-1}
$\lvert V_{cb}\rvert$ incl.	$42.2 \cdot 10^{-3} \cdot (1 \pm 1.8\%)$	1.2%	–
$\lvert V_{cb}\rvert$ excl.	$39.0 \cdot 10^{-3} \cdot (1 \pm 3.0\%_{\rm ex.} \pm 1.4\%_{\rm th.})$	1.8%	1.4%
$\lvert V_{ub}\rvert$ incl.	$4.47 \cdot 10^{-3} \cdot (1 \pm 6.0\%_{\rm ex.} \pm 2.5\%_{\rm th.})$	3.4%	3.0%
$\lvert V_{ub}\rvert$ excl. (WA)	$3.65 \cdot 10^{-3} \cdot (1 \pm 2.5\%_{\rm ex.} \pm 3.0\%_{\rm th.})$	2.4%	1.2%

- Can we fit the SD contributions with appropriate lattice input in a full NLO parametrisation (?)
  - Maybe start with pseudo-scalar final states; less complicated

Summary and some more pointers

- If you do studies, please benchmark them against PHOTOS
  - This is the most useful comparison you can do
  - Also: if you can provide MC generated events, we can do truth-level studies and find out what the impact on our measurements are
    - Interplay between reconstruction, Brem-recovery, more or less affected signal extraction variables, etc. is **very very** complicated
    - QED effects on kinematics typically are folded out of description → very hard to assess what is going on with our measured distributions
      - If you use measured FFs: any parametric difference will have been absorbed into the form factor parameters themselves
      - Could study tension between Lattice and Data in the future

- Thanks for organizing this workshop, it is great!
  — I wish I could have attended 10 years ago! ;-)