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PoPe verification of VOICE 1D-1V multispecies kinetics

Thomas Cartier-Michaud, Philippe Ghendrih,
Virginie Grandgirard, Frédéric Schwander,
Eric Serre, Eric Sonnendrücker et al.

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AMIDEX/TOP


FR FCM

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$$\mathcal{O}_c(X) = \sum_m \mathcal{O}_m(X) \quad \text{Maths}$$

on the computer (s)

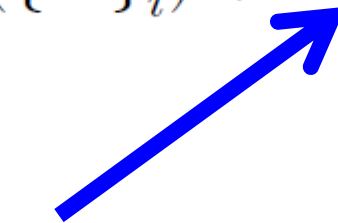
$$\mathcal{O}_c^{(s)}(\{X\}_i) = \sum_m \mathcal{O}_m^{(s)}(\{X\}_i) \quad \text{Simulation}$$



Pope process

$$\mathcal{O}_c^{(r)}(\{X\}_i) = \sum_m \left(1 + \delta a_m(\{X\}_i) \right) \mathcal{O}_m^{(r)}(\{X\}_i) + E(\{X\}_i)$$

if $\mathcal{O}_c^{(r)} = \mathcal{O}_c$
& $\mathcal{O}_m^{(r)} = \mathcal{O}_m$



Pope output

$\delta a_m(\{X\}_i)$ simulation **weight** error on $\mathcal{O}_m^{(s)}$

Error of code run

$$\begin{aligned} \mathcal{E}(\{X\}_i) &= \mathcal{O}_c^{(r)}(\{X\}_i) - \sum_m \mathcal{O}_m^{(r)}(\{X\}_i) \\ &= \sum_m \delta a_m(\{X\}_i) \mathcal{O}_m^{(r)}(\{X\}_i) + E(\{X\}_i) \end{aligned}$$

Weight error
= vicinity of solution
 \Rightarrow statistics

Residue = numerical
generation of new
operator

$$\partial_t f_e + v \partial_x f_e + \partial_x (\phi + \phi_{ext}) \partial_v f_e = 0$$

VOICE 1 species

$$\partial_x^2 \phi(x, t) = \int dv f_e(x, v, t) - 1$$

$$\mathcal{O}_1(x, v, t) = -v \partial_x f_e(x, v, t)$$

$$\mathcal{O}_2(x, v, t) = -\partial_x (\phi + \phi_{ext}) \partial_v f_e(x, v, t)$$

$$X = (x, v, t)$$

$$\mathcal{O}_c \longleftrightarrow \partial_t f_e(x, v, t)$$

Simulation: $\phi(x, t)$

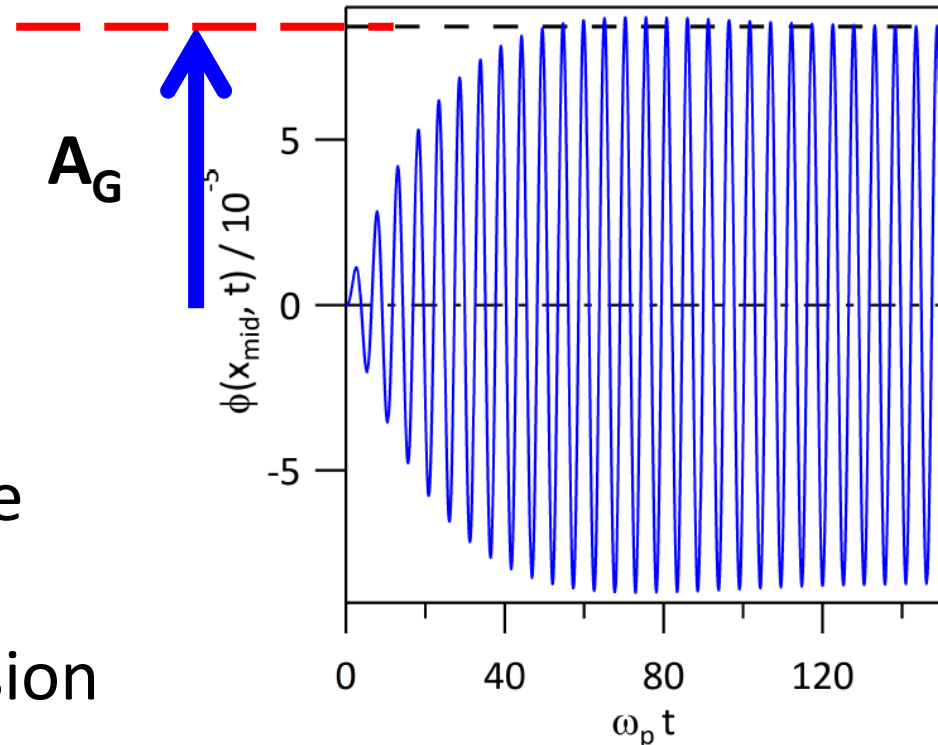
oscillations at ω_{ext}

transient to plateau

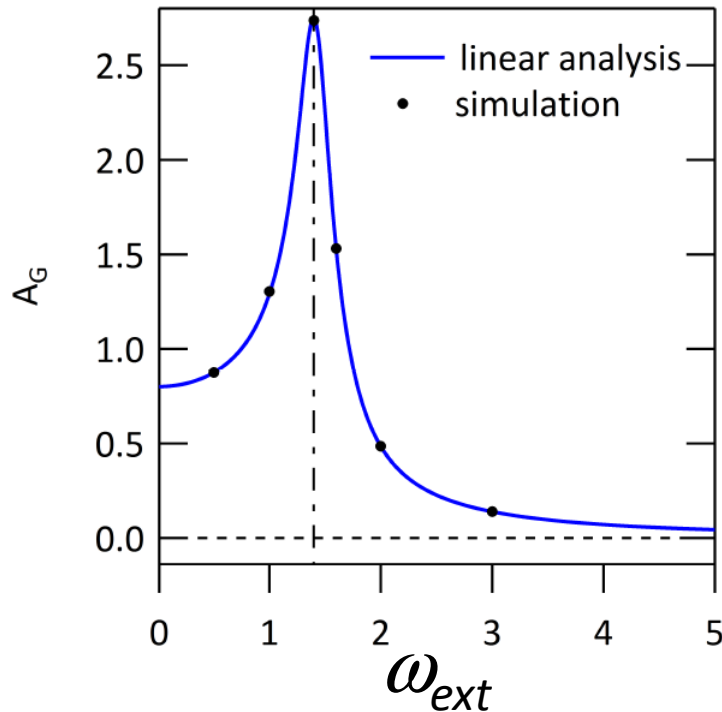
stroboscopy \Rightarrow amplitude

amplitude plateau

\exists analytical expression



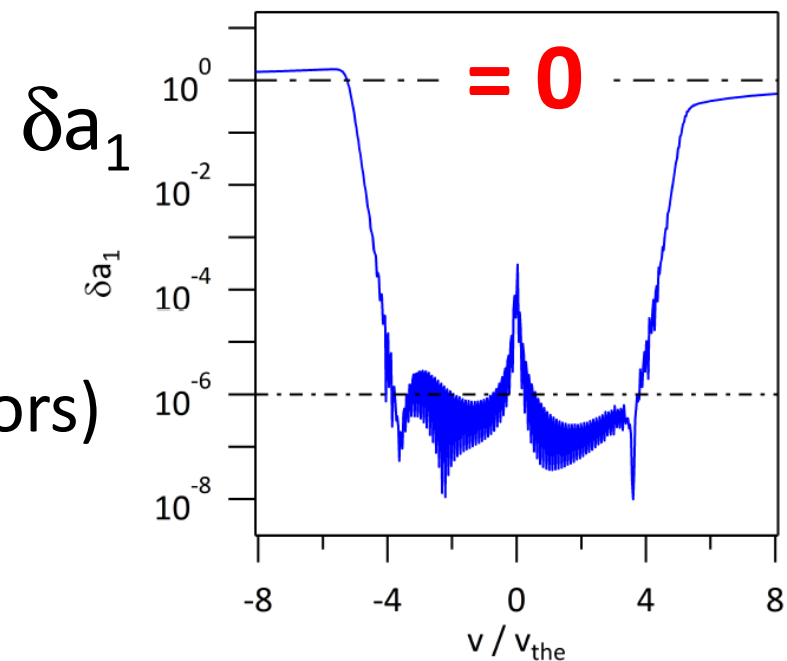
cea Recovering linear = not enough



Standard verification
= **excellent agreement**

BUT PoPe error **POPE = 0**

PoPe index = $-\log_{10}(\delta a_1)$



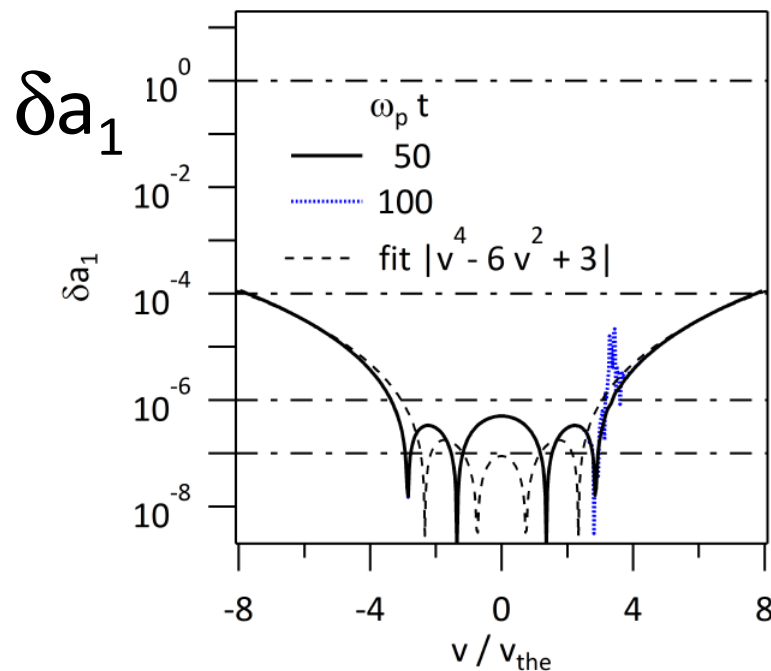
High precision at resonance
Issue at $v = 0$ (vanishing operators)
Error at large v ,
propagation to small $|v|$

VOICE modification

order 4 finite difference in v (previous Fourier)

PoPe index = $-\log_{10}(\delta a_1)$

PoPe index = 4

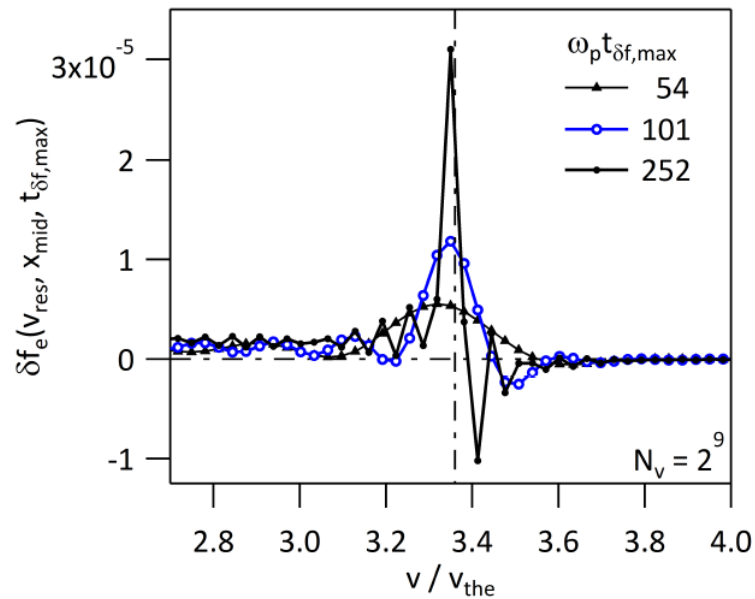


error on $\delta a_1 = \text{fit}$

governed by

order 4 finite difference

new issue @ resonance



$N_v = 512, N = 9$

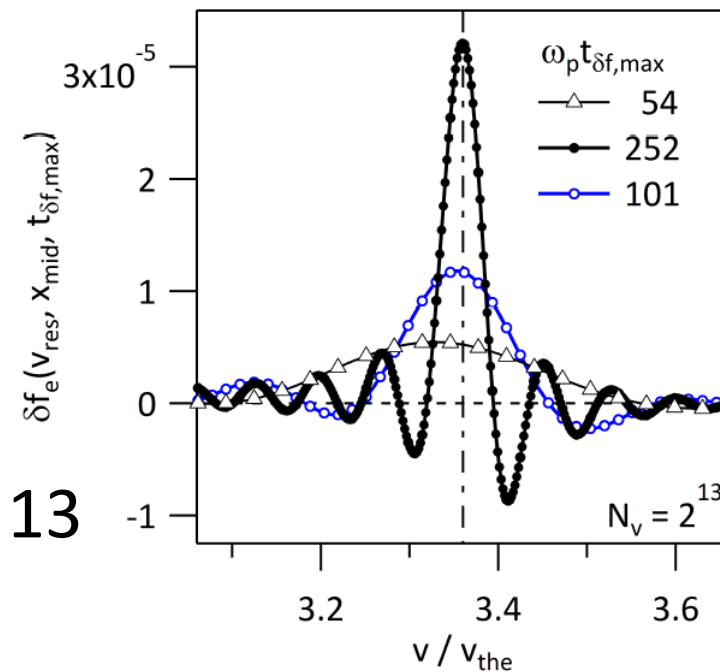
$N_v = 8192, N = 13$

$$N_v = 2^N$$

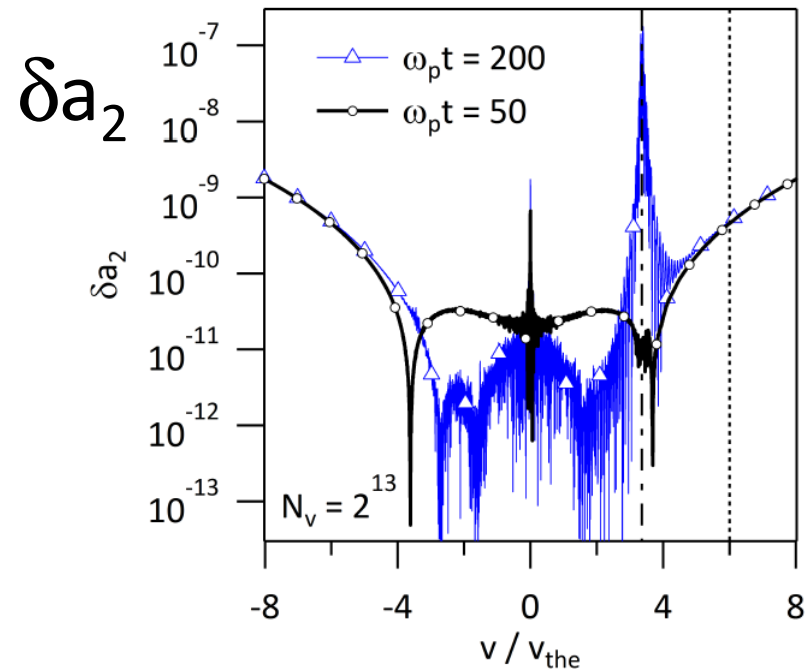
Impact of N_v

Most simulations

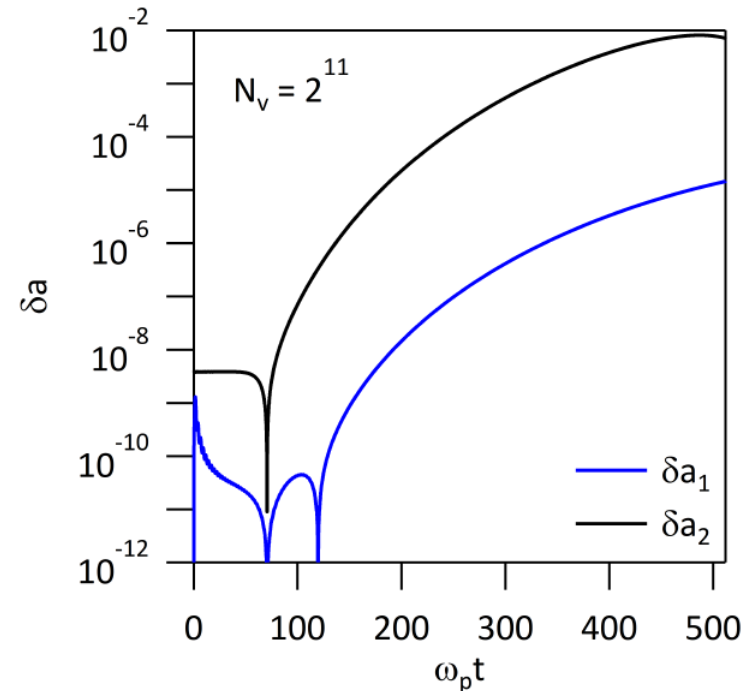
@ constrained resolution

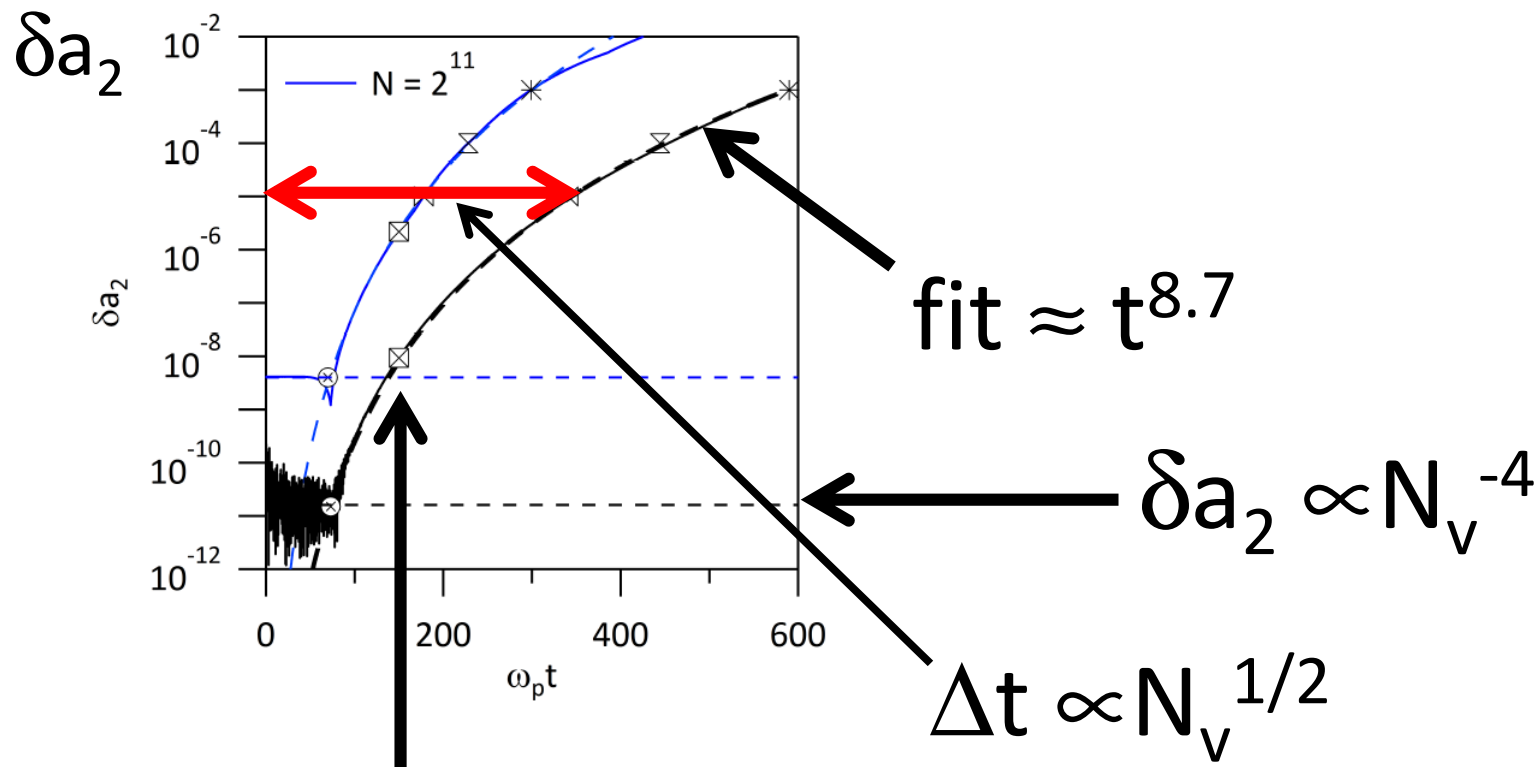


Resonance: error generation



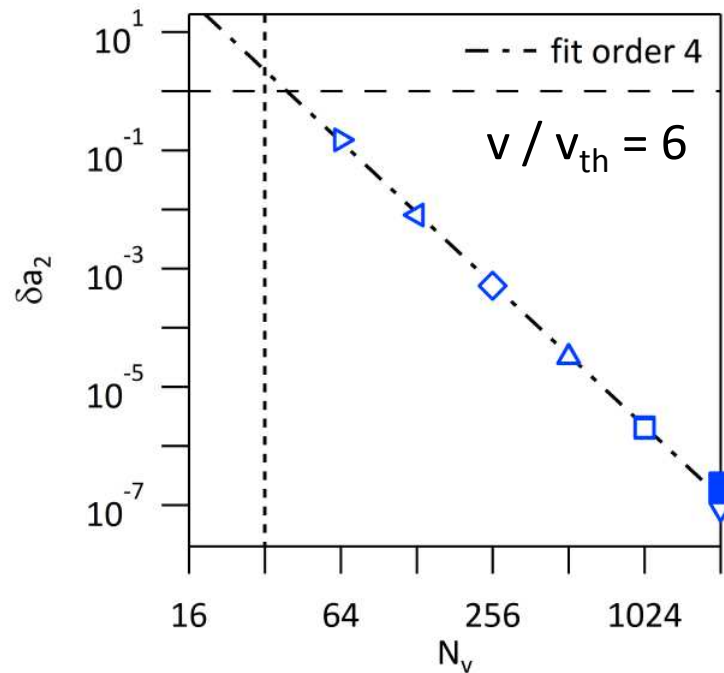
$\delta a_2 \gg \delta a_1$
Strong evolution
at v_{res}





$\omega_p t = 150$
 $\delta a_2 \propto N_v^{-4}$

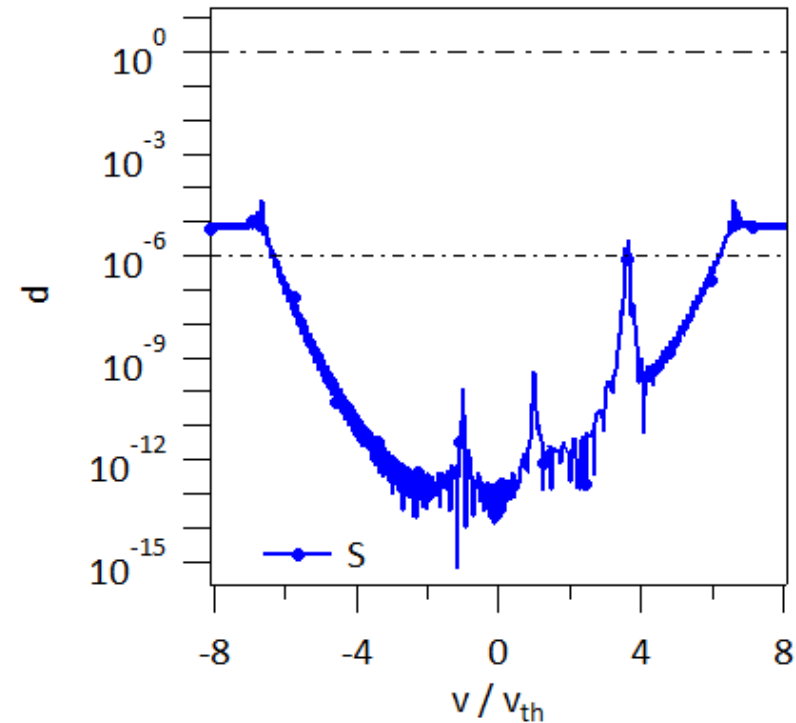
**"filamentation"
 known finite time
 breakdown**



PoPe index = 0

$N_v = 32$

order 4 fit of PoPe index



$d < 10^{-6}$, verified $d \approx 0$

Projection on diffusion operator

Fourier scheme :

peak at resonance

high velocity

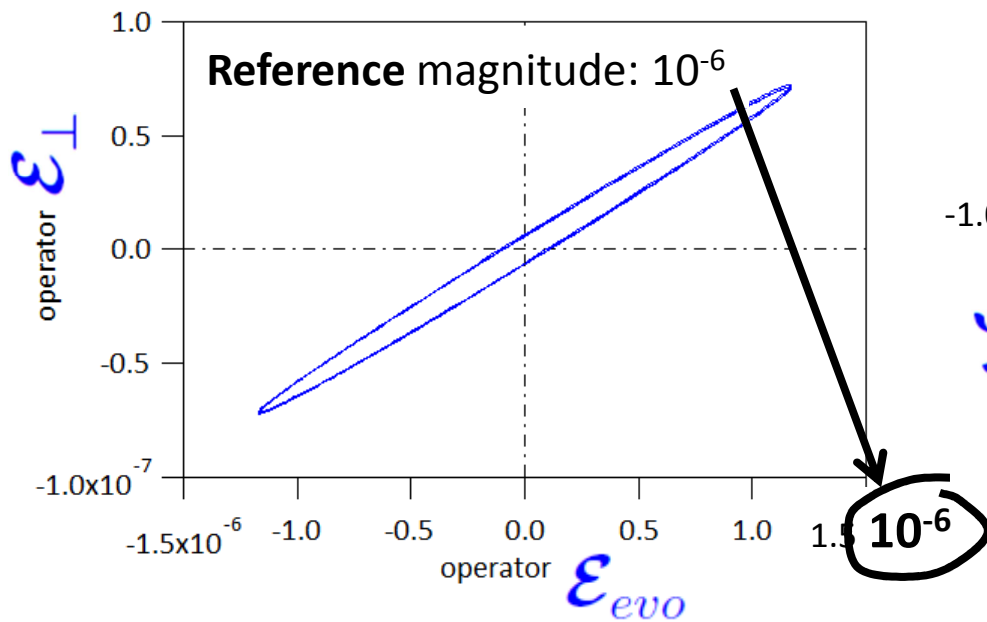
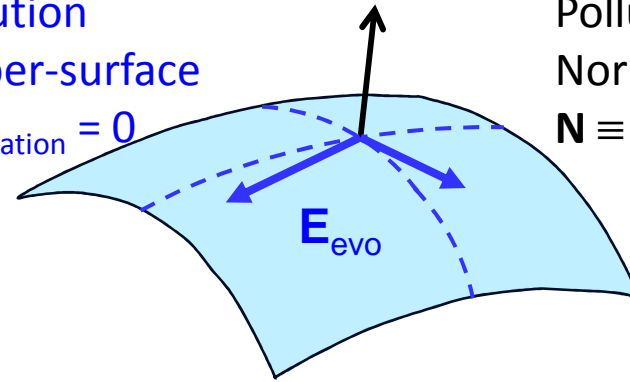
PoPe

Projection on Proper elements

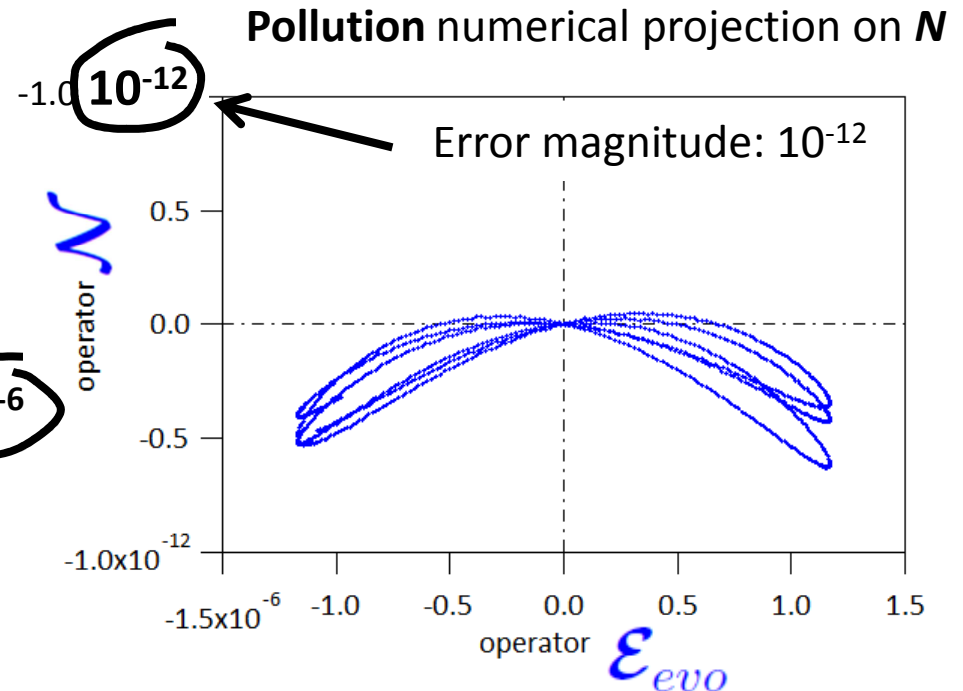
Application: VOICE code

Solution
Hyper-surface
 $S_{\text{equation}} = 0$

Pollution
Normal vector
 $N \equiv \nabla S_{\text{equation}}$



close to colinear operators
(oscillation in x & t)



PoPe

Projection **o**n **P**roper **e**lements

Application: **VOICE** code

Code Verification with data from production run

On the fly verification

In depth analysis of numerical scheme and equation