Black holes and (neutron) stars in the Minimal Theory of Massive Gravity

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based on: arXiv:1808.01403

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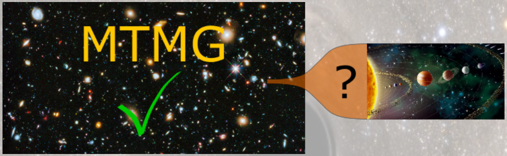
3ème Assemblée Générale du GdR Ondes Gravitationelles
10. October 2019

Motivation: Testing theories at astrophysical scales

- Most of alternative theories are build to solve large-scales problem

↪ What about astrophysical scales ?



⇒ Finding (or not) a black hole solution is a very good test as

- ↪ BHs seem to exist in real life,
- ↪ the post-Newtonian parameters β / γ are easily read from the metric,
- ↪ it opens the way to study the generation of GWs.

⇒ But stellar BHs do not "pop up" from nothing

- ↪ one has to find a safe mechanism for star collapses.

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Rapid reminder : "dRGT" ghost-free massive gravity

- General Relativity is a purely kinetic theory (regarding the graviton),

$\Rightarrow$ adding a potential provides $\Lambda_{\text{eff}} \sim m_g^2$.

- $\hookrightarrow$ no running of the cosmological constant,
- $\hookrightarrow$ OK with current bounds[1] : $m_g^2 \lesssim 1.2 \cdot 10^{-22}$ eV.

- The first viable (*ie.* ghost-free) theory of "massive gravity" in 4D was constructed by C. de Rham, G. Gabadazze and A. Tolley[2] in 2010

$$\mathcal{S}_{\text{dRGT}} = \mathcal{S}_{\text{EH}}\left[g_{\mu\nu}\right] + \mathcal{S}_{\text{m}}\left[g_{\mu\nu};\psi_m\right] + \frac{M_{\text{Pl}}^2 m_g^2}{2}\int \mathrm{d}^4x\sqrt{-g}\,\mathcal{U}\left(\sqrt{f^{-1}g}\right),$$

with $f_{\mu\nu}$ a fiducial metric.

- Then extended to "massive bi-gravity" (*ie.* with a dynamical fiducial sector) by S. Hassan and R. Rosen[3] in 2012.

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[1] B. P. Abbott *et al.* [LVC], PRL 116(2016)221101
[2] C. de Rham, G. Gabadadze & A. J. Tolley, PRL 106(2011)231101
[3] S. F. Hassan & R. A. Rosen, JHEP 1202(2012)126

Rapid reminder : "dRGT" ghost-free massive gravity

- dRGT massive gravity seems a nice theory :
  - ↪ "natural" explanation of the cosmological constant,
  - ↪ OK with current bounds on the graviton's mass,
  - ↪ no spurious degree of freedom,

BUT !

- ⇝ no stable Friedmann-Lemaître-Robertson-Walker cosmologie[4]
- ⇝ the static BH solutions are sick[5]
- ⇝ seems to break unitarity of the $\mathcal{S}$-matrix at some energy[6]
- ⇝ propagates 5 degrees of freedom,
  - ↪ notably a scalar one $\Rightarrow$ induces a fifth force that has to be screened.

[4] A. De Felice, A. E. Gumrukcuoglu & S. Mukohyama, PRL 109(2012)171101
[5] C. Deffayet & T. Jacobson, CQG 29(2012)065009
R. A. Rosen, JHEP 1710(2017)206
[6] C. de Rham, S. Melville & A. J. Tolley, JHEP 04(2018)083

Seeking for simplicity : the minimalistic approach

- Occam's rasor : *Pluralitas non est ponenda sine neccesitate*[7]
  in english : "Entities should not be multiplied without necessity"

$\Rightarrow$ Let's seek for a massive gravity with only 2 propagating degrees of freedom.

$\hookrightarrow$ Starting from a dRGT-inspired precursor theory, we kill the extra fields *via* Hamiltonian constraints.

$\hookrightarrow$ But it has a cost :
loosing temporal Lorentz invariance

$\hookrightarrow$ weakly, as the breaking is $\propto m_g^2$.

[7] its "ontological" version, *Quaestiones et decisiones* in "Quatuor libros Sententiarum cum centilogio theologico", (1319)

The Minimal Theory of Massive Gravity

- Occam's rasor : "Entities should not be multiplied without necessity"

Let's seek for a massive gravity with only 2 propagating *dof*.

$$\mathcal{S}_{\text{MTMG}} = \mathcal{S}_{\text{EH}}\left[g_{\mu\nu}\right] + \mathcal{S}_{\text{m}}\left[g_{\mu\nu};\psi_m\right] + \frac{M_{\text{Pl}}^2 m_g^2}{2}\int \mathrm{d}^4x\, N\sqrt{\gamma}\,\mathcal{W}\left[N, N_i, \gamma_{ij}; \tilde{N}, \tilde{\gamma}_{ij}; \lambda, \lambda^i\right] ,$$

$\{N, N_i, \gamma_{ij}\}$: ADM decomposition of the physical metric,

$$g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -N^2\mathrm{d}t^2 + \gamma_{ij}\left(N^i\mathrm{d}t + \mathrm{d}x^i\right)\left(N^j\mathrm{d}t + \mathrm{d}x^j\right) .$$

$\{\tilde{N}, \tilde{N}_i, \tilde{\gamma}_{ij}\}$: ADM decomposition of the fiducial metric
$\{\lambda, \lambda^i\}$: non-dynamical fields (Lagrange multipliers)

$\Rightarrow$ For technical details about the construction, see[8]

[8] A. De Felice & S. Mukohyama, PLB 752(2016)302

Cosmological phenomenology of MTMG

- By taking two FLRW metrics, two branches appear[9] :

⇒ The "self-accelerating" one, where the two scale factors are proportional,

- ↪ a natural cosmological constant ($w = -1$) appears,
- ↪ the scalar and vector sectors are the same as in GR (background and perturbations),
- ↪ the tensor sector acquires a mass.

⇒ The "normal" one, with more complicated conditions

- ↪ a dark fluid ($w \neq -1$) appears,
- ↪ the scalar sector obeys a modified Poisson equation,
- ⇒ interesting phenomenology, notably regarding ISW bounds[10],
- ↪ the tensor sector acquires a mass.

[9] A. De Felice & S. Mukohyama, JCAP04(2016)028
[10] N. Bolis, A. De Felice & S. Mukohyama, PRD 98(2018)024010

When MTMG is mimicking GR

- The metric equations of motion of MTMG can be written as

$$M_{\rm Pl}^2\,(G_{\mu\nu}+\Lambda_{\mu\nu}) = T^{\rm mat}_{\mu\nu}$$

↪ Let's take the ansatz $\lambda(x^\mu)$, $\lambda^i(x^\mu)$,

$$ds_g^2 = -N^2(x^\mu)\,dt^2 + a^2(t)\,\delta_{ij}\left[N^i(x^\mu)dt + dx^i\right]\left[N^j(x^\mu)dt + dx^j\right]\,,$$
$$ds_f^2 = -\tilde{N}^2(x^\mu)\,dt^2 + \tilde{a}^2(t)\,\delta_{ij}\left[\tilde{N}^i(x^\mu)dt + dx^i\right]\left[\tilde{N}^j(x^\mu)dt + dx^j\right]\,,$$

⇒ Then[11] in the first branch,

- the constraint equations are solved by $\tilde{a}(t) = \mathcal{X}\,a(t)$,
- the auxilliary functions can be safely set to their vanishing cosmological values,
- the mass term becomes

$$\Lambda_{\mu\nu} = \frac{m_g^2}{2}\left(c_4 + 2c_3\,\mathcal{X} + c_2\,\mathcal{X}^2\right)\,g_{\mu\nu} \equiv \Lambda_{\rm eff}\,g_{\mu\nu}\,.$$

[11] A De Felice, FL, S. Mukohyama & M. Oliosi, PRD 98(2018)104031

When MTMG is mimicking GR

Thus a nice lemma

Any metric with flat constant-time surfaces

$$\mathrm{d}s_g^2 = -N^2(x^\mu)\,\mathrm{d}t^2 + a^2(t)\,\delta_{ij}\left[N^i(x^\mu)\mathrm{d}t + \mathrm{d}x^i\right]\left[N^j(x^\mu)\mathrm{d}t + \mathrm{d}x^j\right],$$

that is solution of GR + Λ is solution of the self-accelerating branch of MTMG with $\Lambda_{\rm eff} \propto m_g^2$.

$\Rightarrow$ Can we use such a trick to construct BH and star solutions in MTMG ?

- *NB* : as diffeomorphisms are broken, two solutions that are similar under coordinate changes are not equivalent in MTMG.

Can BHs have flat constant-time surfaces ?

YES ![12,13&14]

- Let's take a spherically symmetric system.
  The metric can be written as

$$\mathrm{d}s^2 = -f(t,r)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1-\frac{2\mu(r,t)}{r}} + r^2\mathrm{d}\Omega^2\,,$$

↪ a coordinate change $t \to \tau + T(r,\tau)$ such that $(\partial_r T)^2 = \frac{2\mu}{f(r-2\mu)}$ yields

$$\mathrm{d}s^2 = -\frac{r\,f}{r-2\mu}\left(1+\partial_\tau T\right)^2\,\mathrm{d}\tau^2 + \left[\mathrm{d}r - f\,\partial_r T\left(1+\partial_\tau T\right)\mathrm{d}\tau\right]^2 + r^2\mathrm{d}\Omega^2\,,$$

[12] P. Painlevé, C. R. Acad. Sci. (Paris) 173(1921)677
[13] A. Gullstrand, Ark. Mat. Astron. Fys 16(1922)8
[14] G. Lemaître, Ann. Soc. Sci. Bruxelles A53(1933)51

Can BHs have flat constant-time surfaces ?

- Explicitly the Schwarzschild-de-Sitter solution in extended Painlevé-Gullstrand coordinates[15,16&17] together with $r \to a(\tau)r$ reads

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + a^2(\tau)\left[(\mathrm{d}r + \beta\,\mathrm{d}\tau)^2 + r^2\mathrm{d}\Omega^2\right].$$

with

$$\beta(\tau, r) = \frac{\partial_\tau a}{a}\,r \pm \sqrt{\frac{2M}{a^3 r} - \frac{\Lambda r^2}{3}}.$$

⇒ Cosmologically embedded static GR-like BHs are natural solution of the self-accelerating branch of MTMG !

⇒ In the $M \to 0$ limit, those solutions are strictly homogeneous, isotropic and free of strong coupling issues, which seems to be a unique feature among massive gravities.

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[15] P. Painlevé, C. R. Acad. Sci. (Paris) 173(1921)677
[16] A. Gullstrand, Ark. Mat. Astron. Fys 16(1922)8
[17] G. Lemaître, Ann. Soc. Sci. Bruxelles A53(1933)51

What about matter ?

- Usual fluid system can be described by

$$ds^2 = -N^2 d\tau^2 + (dr + \beta\, d\tau)^2 + r^2 d\Omega^2 \,.$$

- To avoid coordinate singularity, we impose that the extrinsic curvature remains regular and becomes isotropic at the center.

$\hookrightarrow$ The anisotropic part of the extrinsic curvature, $K_{rr} - K/3$ shall thus vanish at $r = 0$. This is realised for

$$\lim_{r\to 0}\left(\partial_r \beta - \frac{\beta}{r}\right) = 0\,,$$

$\Rightarrow$ Taking a usual fluid with density ρ and barotropic equation of state $P = P(\rho)$, it comes

$$\beta(\tau, r) = \pm \frac{N\, r}{M_{\rm Pl}} \sqrt{\frac{2\rho_{r=0}}{3}} + \mathcal{O}\left(r^3\right)\,.$$

Let's spin to win !

- Static BHs are nice theoretical objects, but real BHs are spinning...

⇒ Can we implement Kerr-de Sitter spacetimes in MTMG in the same convenient fashion ?

No ![18]

- Starting from KdS in usual Boyer-Lindquist coordinates, no change of coordinate can yield a spatially flat metric.

⇒ We have to go hunting the spinning BHs the hard way...

[18] A. De Felice, FL, S. Mukohyama & M. Oliosi, arXiv:1908.03456, *to be published*

Summary and perspectives

- Finding BHs and (neutron) stars is an important test for alternative theories of gravity.
  - ↪ Those objects seem to exist in real life,
  - ↪ They are needed to generate and study GWs.
  - ↪ They are small-scales objects, when the theories are developed to solve large-scale problems.

- The Minimal Theory of Massive Gravity is a newborn in the family of massive gravities.
  - ↪ It propagates only two GW polarisations,
  - ↪ It has a stable, self-accelerating cosmology,
  - ↪ It contains static BHs and (neutron) stars that are free from strong coupling issues.

- What's next ?
  - ↪ Seeking for rotating BHs,
  - ↪ Studying the perturbations of the BHs.


Thank you for your attention