Gravitational-wave form working group
Progress of post-Newtonian calculations

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Nonidi 19 Vendémiaire CCXXVIII

Gravitational waveforms (GWF)

first term of the $1/R$ expansion of the form

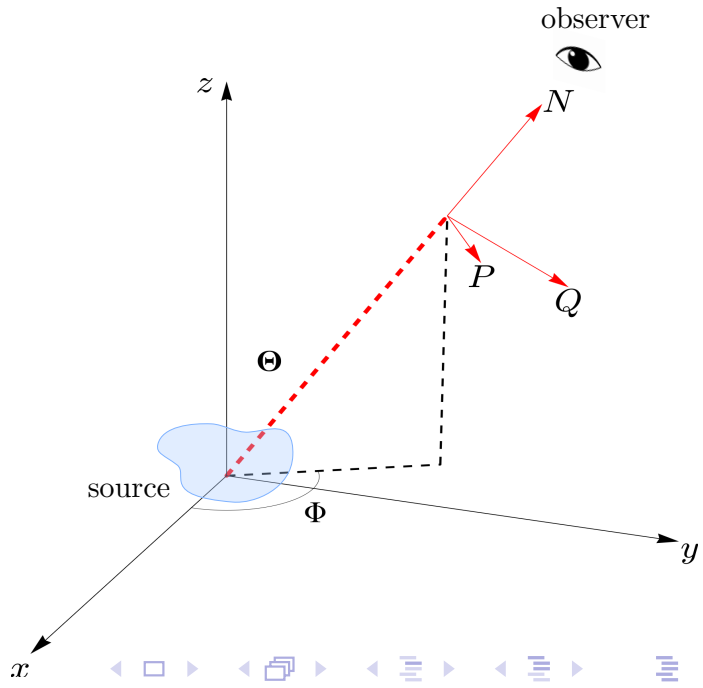
$$h_{ij}^{\rm rad}(\boldsymbol{X},T) = \frac{4G}{c^4 R}\sum_{\ell=2}^{+\infty}\frac{1}{c^{\ell-2}\ell!}\left\{ \underbrace{N_{L-2}}_{N_{i_1}\dots N_{i_{\ell-2}}} \underbrace{U_{ij}L-2}_{i_1\dots i_\ell} - \frac{2\ell}{c(2\ell+1)} N_{aL-2}\epsilon_{ab(i}V_{j)bL-2}\right\}^{\rm TT}(T-R/c)$$

Generalizes the *quadrupole formula* [Einstein (1918)]

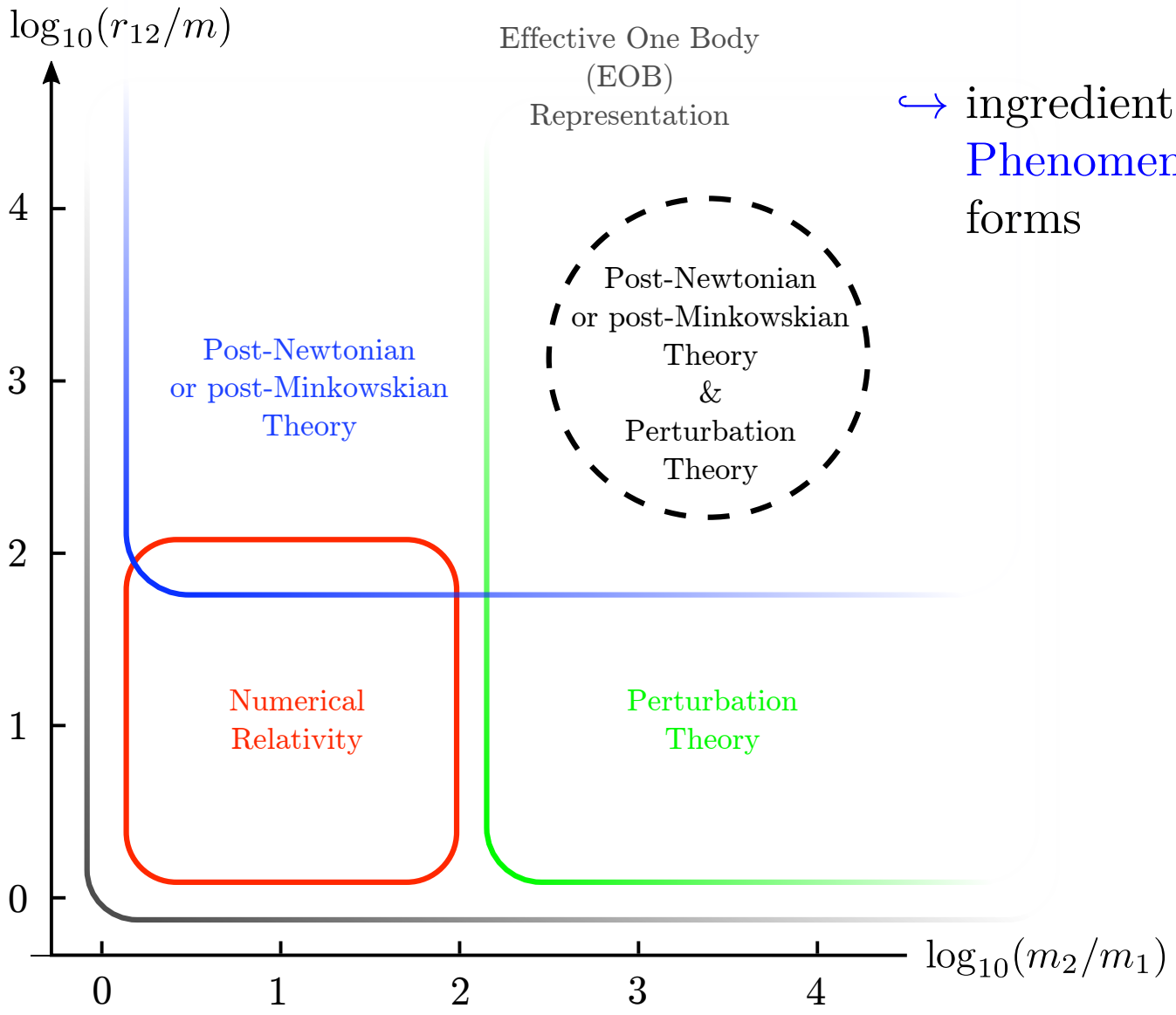
$$h_{ij}^{\rm rad}(\boldsymbol{X},T) = \frac{2G}{c^4 R}U_{ij}^{\rm TT}(T-R/c)$$

with $U_{ij} = I_{ij}^{(2)} + \mathcal{O}(G) = Q_{ij}^{(2)} + \mathcal{O}\left(\frac{1}{c^2}\right)$

$h_+ = h_{PP} = -h_{QQ} \quad h_\times = h_{PQ}$



Approximation methods for GW generation



$\hookrightarrow$ ingredients for EOB and Phenomenological wave-forms

1. First workshop of the GWF working group
2. Towards the 4PN GW phase of inspiraling binaries
3. Tidal effects in inspiraling binary GWF
4. Conclusion

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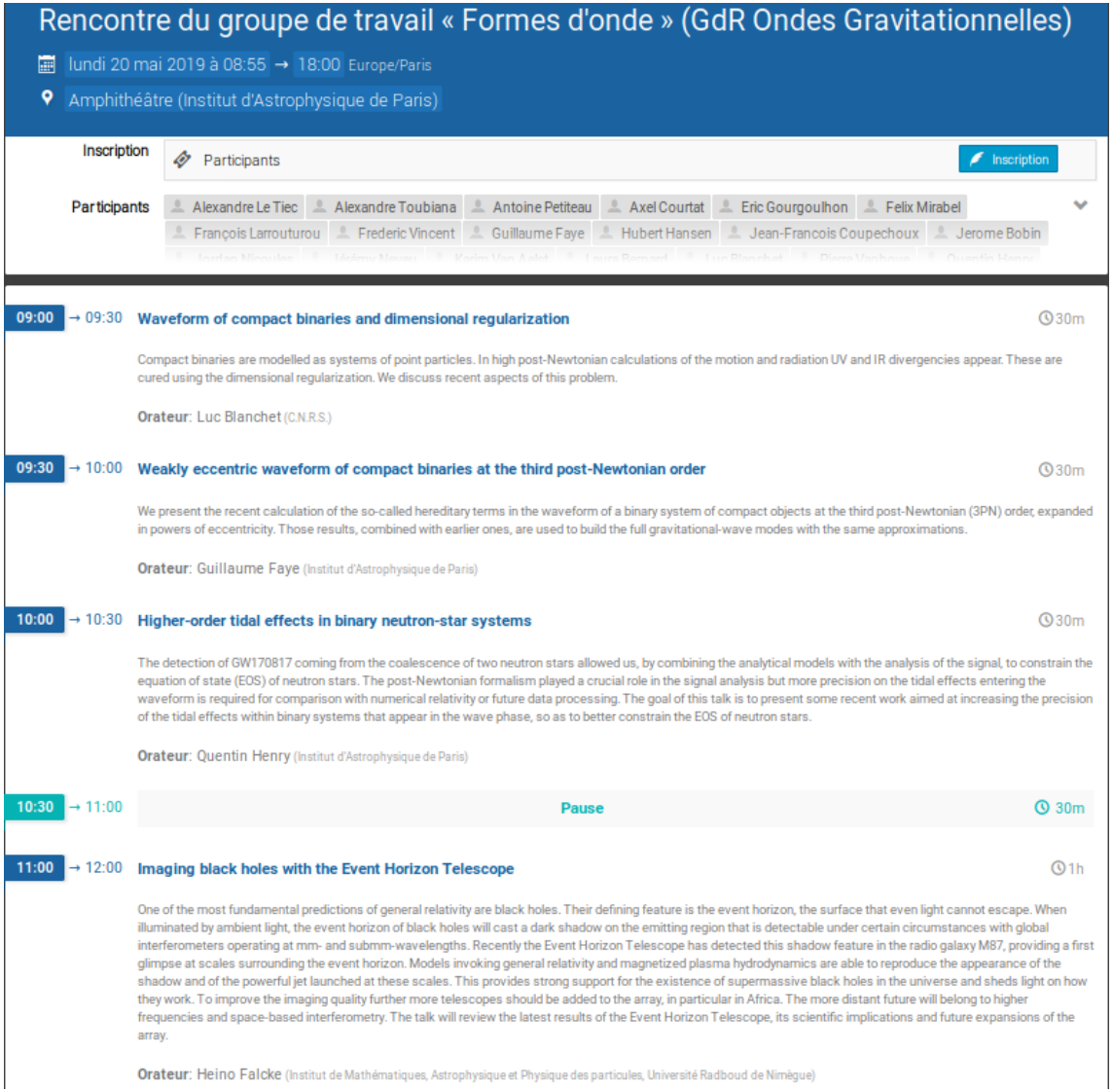
Workshop program I



Rencontre du groupe de travail « Formes d'onde » (GdR Ondes Gravitationnelles)

lundi 20 mai 2019 à 08:55 → 18:00 Europe/Paris

Amphithéâtre (Institut d'Astrophysique de Paris)

Inscription — Participants — Inscription

Participants: Alexandre Le Tiec, Alexandre Toubiana, Antoine Petiteau, Axel Courtat, Eric Gourgoulhon, Felix Mirabel, François Larrouturou, Frederic Vincent, Guillaume Faye, Hubert Hansen, Jean-Francois Coupechoux, Jerome Bobin

09:00 → 09:30 **Waveform of compact binaries and dimensional regularization** 30m

Compact binaries are modelled as systems of point particles. In high post-Newtonian calculations of the motion and radiation UV and IR divergencies appear. These are cured using the dimensional regularization. We discuss recent aspects of this problem.

Orateur: Luc Blanchet (C.N.R.S.)

09:30 → 10:00 **Weakly eccentric waveform of compact binaries at the third post-Newtonian order** 30m

We present the recent calculation of the so-called hereditary terms in the waveform of a binary system of compact objects at the third post-Newtonian (3PN) order, expanded in powers of eccentricity. Those results, combined with earlier ones, are used to build the full gravitational-wave modes with the same approximations.

Orateur: Guillaume Faye (Institut d'Astrophysique de Paris)

10:00 → 10:30 **Higher-order tidal effects in binary neutron-star systems** 30m

The detection of GW170817 coming from the coalescence of two neutron stars allowed us, by combining the analytical models with the analysis of the signal, to constrain the equation of state (EOS) of neutron stars. The post-Newtonian formalism played a crucial role in the signal analysis but more precision on the tidal effects entering the waveform is required for comparison with numerical relativity or future data processing. The goal of this talk is to present some recent work aimed at increasing the precision of the tidal effects within binary systems that appear in the wave phase, so as to better constrain the EOS of neutron stars.

Orateur: Quentin Henry (Institut d'Astrophysique de Paris)

10:30 → 11:00 Pause 30m

11:00 → 12:00 **Imaging black holes with the Event Horizon Telescope** 1h

One of the most fundamental predictions of general relativity are black holes. Their defining feature is the event horizon, the surface that even light cannot escape. When illuminated by ambient light, the event horizon of black holes will cast a dark shadow on the emitting region that is detectable under certain circumstances with global interferometers operating at mm- and submm-wavelengths. Recently the Event Horizon Telescope has detected this shadow feature in the radio galaxy M87, providing a first glimpse at scales surrounding the event horizon. Models invoking general relativity and magnetized plasma hydrodynamics are able to reproduce the appearance of the shadow and of the powerful jet launched at these scales. This provides strong support for the existence of supermassive black holes in the universe and sheds light on how they work. To improve the imaging quality further more telescopes should be added to the array, in particular in Africa. The more distant future will belong to higher frequencies and space-based interferometry. The talk will review the latest results of the Event Horizon Telescope, its scientific implications and future expansions of the array.

Orateur: Heino Falcke (Institut de Mathématiques, Astrophysique et Physique des particules, Université Radboud de Nimègue)

- Duration: one day
  ↪ May 20th 2018
- Organizers: F., Gourgoulhon, Le Tiec
- Venue: IAP, Paris
- Format:
  - 30 minute talks
  - 1 review talk
  - discussion session

  ↪ ILP seminar inserted
- Number of participants: 22
  ↪ includes people from IP2I
  (H. Hansen, J.-F. Coupechoux)

Workshop program II

13:30 → 14:00 **Gravitational waves in LISA band from bodies orbiting the Galactic Center black hole** ¶ 30m

A fully relativistic study of gravitational radiation from bodies in circular equatorial orbits around the massive black hole at the Galactic Center, Sgr A*, has been performed, taking into account the Roche limit induced by tidal forces in the Kerr metric. The signal-to-noise ratio for LISA, as well as the time spent in LISA band, has been computed. It is found that brown dwarfs, main sequence stars and compact objects are all detectable in one year of LISA data with a signal-to-noise ratio above 10 during at least 10^5 yr in the slow inspiral towards either the innermost stable circular orbit (compact objects) or the Roche limit (main-sequence stars and brown dwarfs). The longest times in-band, of the order of 10^6 yr, are achieved for primordial black holes (mass < 10^{-3} solar mass), as well as for brown dwarfs, just followed by white dwarfs and low mass main-sequence stars.

Orateur: Éric Gourgoulhon (LUTH, Observatoire de Paris)

14:00 → 15:00 **Modeling the strong-field dynamics of binary neutron star merger** 1h

The observation of gravitational and electromagnetic waves from a binary neutron star merger in August 2017 conveyed key information on the nature of matter at supranuclear densities, on the origin of short-gamma ray burst, on the production site of of heavy elements via r-process nucleosynthesis, and on cosmography. Thus, multimessenger observations of compact binary mergers hold the promise to unprecedent insights on some of the most fundamental physics questions. A crucial and necessary ingredient to intepret such observations is the precise knowledge of the dynamics of the sources. I will talk about recent developments on the modeling of neutron star mergers using numerical simulations in general relativity. I will focus on the numerical exploration of the the merger remnant and mass ejecta and their dependence on the binary parameters. I will discuss detailed models of the gravitational waves and kilonova light curves, highlighting the prospect of using them in joint analysis of multimessenger data.

Orateur: Sebastiano Bernuzzi (Université d'Iéna)

15:00 → 15:30 **The formation of BH-HMXBs and BBHs: Impact in Gravitationl Waves Astrophysics and Cosmology** 30m

Theoretical models and observations suggest that primordial Stellar Black Holes (Pop-III-BHs) were prolifically formed in HMXBs, which are powerful relativistic jet sources of synchrotron radiation called Microquasars (MQs). Large populations of BH-HMXB-MQs at cosmic dawn produce a smooth synchrotron cosmic radio background (CRB) that could account for the possible excess amplitude of atomic hydrogen absorption at z~17, recently reported by EDGES. BH-HMXB-MQs at cosmic dawn precede supernovae, neutron stars and dust. BH-HMXB-MQs promptly inject hard X-rays and relativistic jets into the IGM, which overtake the slower expanding HII regions ionized by progenitor Pop-III stars, heating and partially ionizing the IGM over larger distance scales. BH-HMXBs are channels for the formation of Binary-Black-Holes (BBHs). The large masses of BBHs detected by gravitational waves, relative to the masses of BHs detected by X-rays, and the high rates of BBH-mergers, are consistent with high formation rates of BH-HMXBs and BBHs in the early universe.

Orateur: Felix Mirabel (CEA)

15:30 → 16:00 **Pause** 30m

16:00 → 16:45 **Discussion** 45m

Highlights

- Progress in computing analytic waveforms at 4PN, i.e., v^8/c^8
- Review talk by S. Bernuzzi about neutron-star merger dynamics
  $\hookrightarrow$ useful for IP2I members starting num. sim. of neutron-star mergers
- GW from bodies orbiting Sgr A* (brown dwarf, MS stars) for LISA
- **Discussions:**
  - acknowledgment that the GWF community is small in France
  - consequences:
    - no more than one working group (WG) meeting /year
    - proposal: joint meetings with the WG "test of GR" or "astrophysics"
    - next meeting after the current general assembly with the WG "test of GR"

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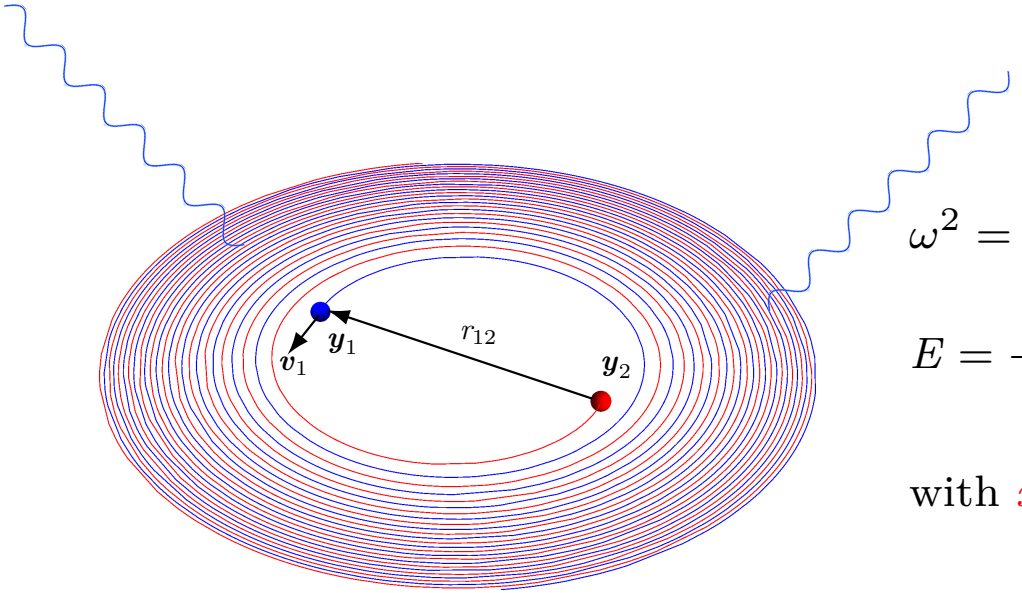
Compact binaries

- Leading order flux $= \dfrac{G}{5c^5}\left[\overset{\cdots}{Q}_{ij}(T_{\rm ret})\overset{\cdots}{Q}_{ij}(T_{\rm ret}) + \mathcal{O}\left(\dfrac{1}{c^2}\right)\right] \rightarrow$ Newtonian order

balance equations for E, $\boldsymbol{J} \Rightarrow$

- $e \searrow 0$ for isolated binaries
- E and $r_{12} \searrow$ at a rate $\sim \epsilon^{5/2}$

$= 2.5$PN order



$$\omega^2 = \frac{Gm}{r_{12}^3}\left[1 + \left(\frac{Gm}{r_{12}\,c^2}\right)(...) + ... + \left(\frac{Gm}{r_{12}\,c^2}\right)^4(...)\right]$$

$$E = -\frac{\mu c^2 x}{2}\left[1 + x(...) + ... + x^4(...)\right]$$

with $x = \left(\dfrac{Gm\omega}{c^3}\right)^{2/3}$

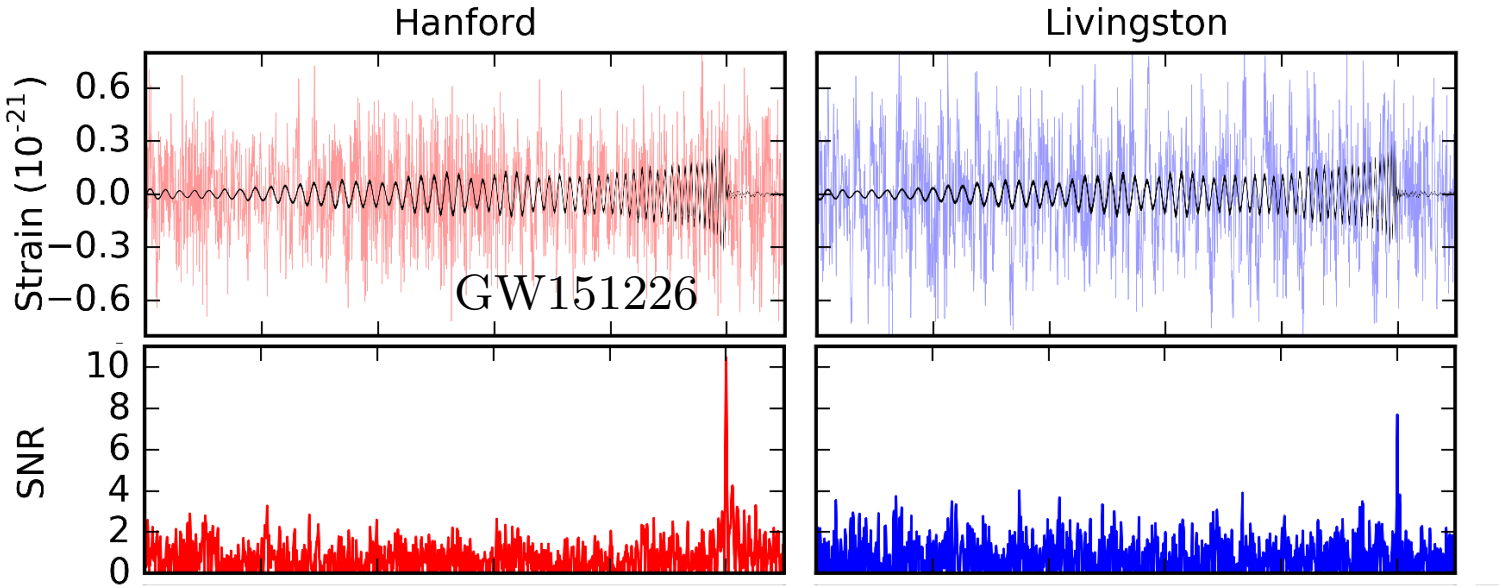
state of the art

- For circular orbits: $E = E(x)$, $\mathcal{F} = \mathcal{F}(x)$, $h_{+,\times} = h_{+,\times}(x)$ gauge invariant

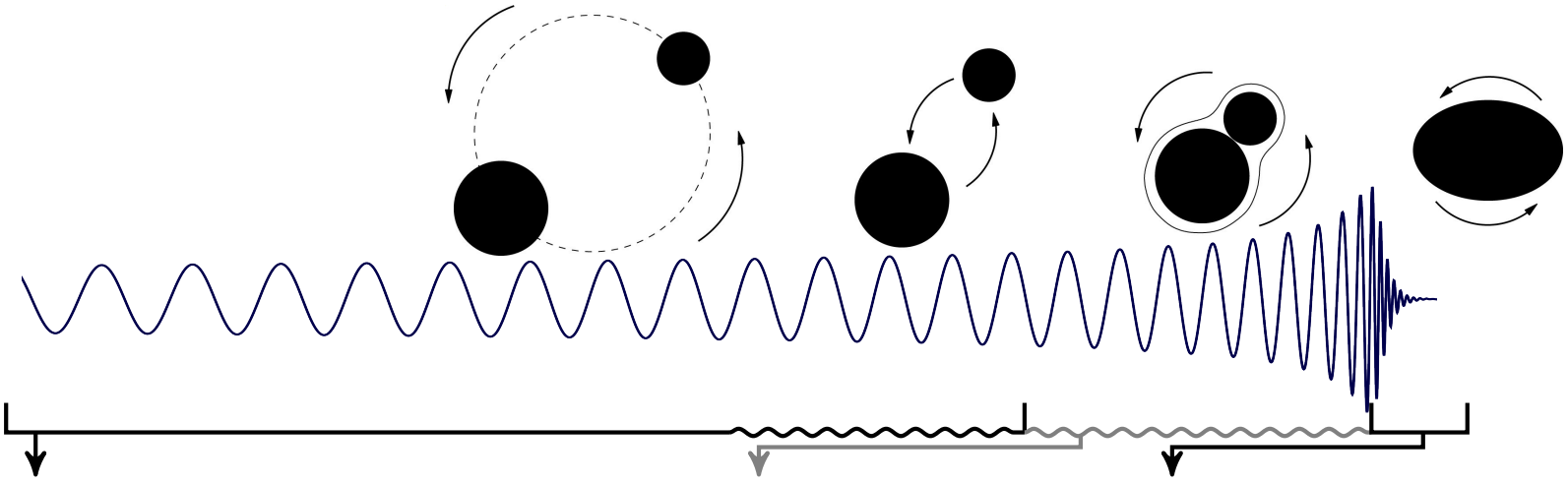
$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\mathcal{F} \quad \rightarrow \quad \text{ODE for } x$$

$\Rightarrow$ orbital phase ϕ

A key ingredient to model compact binaries



Credit: LVC



Inspiral
post-Newtonian (PN) theory
Effective-one-body (EOB)

Merger
no analyt. model

Ringdown
perturbation theory

Numerical Relativity (NR)

Credit: F. Ohme

Post-Newtonian vs Post-Minkowskian

- Post-Minkowskian (PM) expansion: expansion in powers of $\dfrac{GM}{c^2 R_{\rm sys}} \ll 1$

  $\hookrightarrow$ weak field

- Post-Newtonian (PN) expansion: expansion in powers of $\left(\dfrac{v_{\rm max\ sys}}{c}\right)^2 \ll 1$

  $\hookrightarrow$ small velocity

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN	
1PM	$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ v^8$	$+\ v^{10}$	$+\ v^{12}$	$+\ v^{14}$	$+\ \ldots)\ G^1$
2PM		$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ v^8$	$+\ v^{10}$	$+\ v^{12}$	$+\ \ldots)\ G^2$
3PM			$+\ (1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ v^8$	$+\ v^{10}$	$+\ \ldots)\ G^3$
4PM				$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ v^8$	$+\ \ldots)\ G^4$
5PM					$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ \ldots)\ G^5$
6PM						$(1$	$+\ v^2$	$+\ v^4$	$+\ \ldots)\ G^6$

[see Bern, Cheung, Roiban, Shen, Solon, Zeng (2019)]

[Damour, Jaranowski, Schäfer (2014); Jaranowski, Schäfer (2015); Bernard, Blanchet, Bohé, F., Marsat (216, 2017a,b); Marchand, Bernard, Blanchet, F. (2018); Foffa, Sturani (2019); Foffa, Porto, Rothstein, Sturani (2019)]

PN iteration in harmonic coordinates

Harmonic gauge equations

$$\partial_\nu h^{\mu\nu} = 0 \qquad \text{(Gauge Cond on shell)}$$

$$\Box h^{\mu\nu} = \frac{16\pi G}{c^4}\tau^{\mu\nu} \equiv \frac{16\pi G}{c^4}|g|\,T^{\mu\nu} + \Lambda^{\mu\nu}(\partial h, \partial h) \qquad \text{(Relaxed EE)}$$

PN expansion: $h^{\mu\nu}$ searched in the form $\sum_{m \geq m_0(\mu,\nu)} c^{-m} h^{\mu\nu}_{[m]}$

Solution for $h^{\mu\nu}_{[m]}$

$$h^{\mu\nu}_{[m]} = 16\pi G \left\{ \overline{\Box}^{-1}_{\mathrm{R}} \left[\tau^{\mu\nu}(\overline{h}^{\alpha\beta}) \right] + (\text{homogeneous outgoing sol.})^{\mu\nu} \right\}$$

finite part of $\overline{\Box}^{-1}_{\mathrm{R}}(r/r_0)^B[\,]$

Outside the source:

$$\Box h^{\mu\nu} = \Lambda^{\mu\nu} \quad \text{(REE)} \qquad \partial_\nu h^{\mu\nu} = 0 \quad \text{(Gauge)}$$

PM expansion: $h^{\mu\nu}$ searched in the form $h^{\mu\nu} = \sum_{n=1}^{+\infty} G^n h^{\mu\nu}_{(n)}$

Blanchet-Damour-Iyer formalism

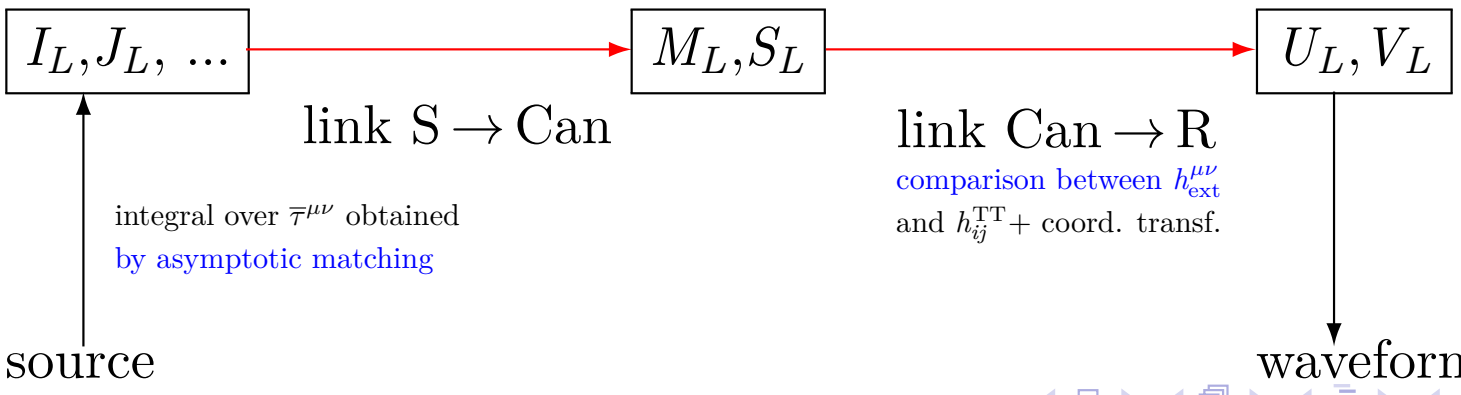
- Most general retarded solution at linear order, with $\Lambda^{\mu\nu} \longrightarrow 0$

$$h^{\mu\nu}_{(1)} = \sum_{\ell \geq 0} \overbrace{\partial_{i_1 \ldots i_\ell}}^{\frac{\partial}{\partial x^{i_1}} \cdots \frac{\partial}{\partial x^{i_\ell}}} \left(\frac{\mathcal{H}^{\mu\nu}_{i_1 \ldots i_\ell}\left(t - \frac{r}{c}\right)}{r} \right) = h^{\mu\nu}_{(1)}[\underbrace{I_L, J_L,}_{\text{source moments}} \underbrace{W_L,\ X_L, Y_L, Z_L}_{\text{gauge moments}}]$$

- Solution for $h^{\mu\nu}_{(n)}$

$$h^{\mu\nu}_{(n)} = \underbrace{\mathrm{FP}\Box^{-1}_{\mathrm{R}}}_{\text{finite-part regularized retarded integral}} \Lambda^{\mu\nu}_{(n)} + (\text{homogeneous outgoing sol.})^{\mu\nu}$$

Link between the various multipole moment sets



Computation of I_{ij} for the 4PN flux

[Blanchet, F., Henry, Larrouturou, Marchand, Marsat (in preparation)]

General expression of the mass 2^ℓ-pole moment

$$I_L(t) = \underset{B=0}{\mathrm{FP}} \int \mathrm{d}^3\mathbf{x}\,|\mathbf{x}|^B \int_{-1}^{1} \mathrm{d}z \Big\{ \delta_\ell(z)\,\hat{x}_L \underbrace{\Sigma}_{\text{effective mass density}} - \frac{4(2\ell+1)\delta_{\ell+1}(z)}{c^2(\ell+1)(2\ell+3)}\,\hat{x}_{iL} \underbrace{\dot{\Sigma}_i}_{\text{current density}}$$
$$+ \frac{2(2\ell+1)}{c^4(\ell+1)(\ell+2)(2\ell+5)}\,\delta_{\ell+2}(z)\,\hat{x}_{ijL} \underbrace{\ddot{\Sigma}_{ij}}_{\text{stress density}} \Big\}(\mathbf{x}, t+z|\mathbf{x}|/c)$$

Derivation in 3 steps (integration over z trivial)

- Integrations by part aimed at writing the integrand in an explicit form
  $\hookrightarrow$ surface integral at infinity to compute [in progress]
- Divergences near source singularities (UV) dealt with dim. reg.
  - initial computation performed with 3D finite-part type reg. ✓
  - dim. reg. corrections obtained separately [in progress]
- Divergences when $r \to \infty$ dealt with dim. reg. + B-type regularization
  - initial computation performed with 3D $\mathrm{FP}_{B=0}$ reg. ✓
  - dim. reg. corrections obtained separately

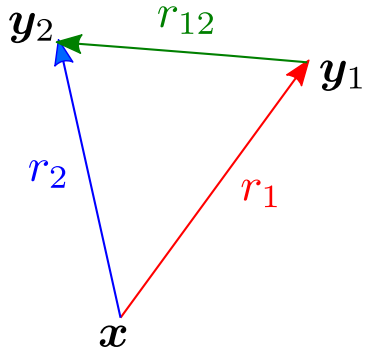
Example of metric potential

- 4PN metric parametrized by brick potentials: $V \approx -\Phi_{\rm N},\ V_i,\ ...,\ \hat{M}_{ij}$

$$\Box\,\hat{W}_{ij} = -4\pi\, G\,(T_{ij} - T_{kk}\,\delta_{ij}) - \partial_i V \partial_j V$$

$$\text{with } V^{(\rm sym)} = \frac{Gm_1}{r_1} + \frac{1}{2!}\partial_t^2\left(\frac{Gm_1 r_1}{c^2}\right) + \frac{1}{4!}\partial_t^4\left(\frac{Gm_1 r_1^3}{c^4}\right) + (1 \leftrightarrow 2) + \mathcal{O}\left(\frac{1}{c^6}\right)$$

$$\begin{aligned}\hat{W}_{ij}^{(\rm sym)} &= \Box^{-1}[\text{source}] \\ &= (\Delta^{-1} + \frac{1}{c^2}\partial_t^2\Delta^{-2} + \frac{1}{c^4}\partial_t^4\Delta^{-3} + ...)[\text{source}]\end{aligned}$$



$\Rightarrow$ kernels $\Delta^{-n} r_1^{2p-1} r_2^{2q-1}$ needed

Example

$$\Delta^{-1}\frac{1}{r_1\, r_2} = \ln(r_1 + r_2 + r_{12})$$

$$\begin{aligned}\Delta^{-1} r_1\, r_2 = {} & \frac{1}{120}[r_{12}^4 - 3(r_1^4 + r_2^4) + 2r_{12}^2(r_1^2 + r_2^2) + 6r_1^2 r_2^2]\ln(r_1 + r_2 + r_{12}) \\ & + \frac{1}{3600}\Big[63(r_1^4 + r_2^4) + 90 r_1 r_2(r_1^2 + r_2^2) - 126 r_1^2 r_2^2 + 90 r_{12}(r_1 + r_2)(r_1 - r_2)^2 \\ & \quad - 62 r_{12}^2\Big(r_1^2 + r_2^2 + \frac{15}{31} r_1 r_2\Big) - 30 r_{12}^3(r_1 + r_2) - r_{12}^4\Big]\end{aligned}$$

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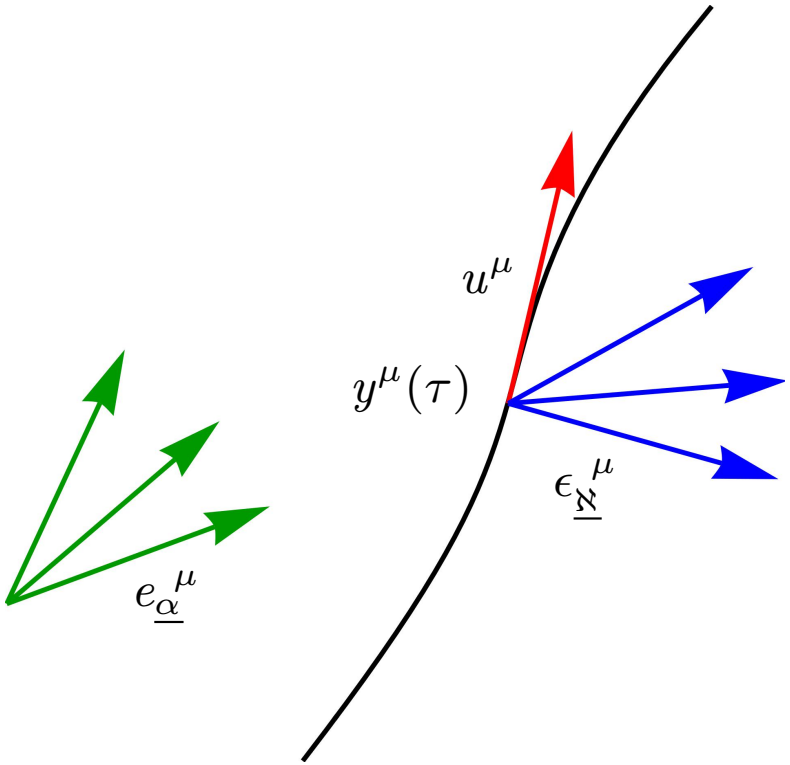
Effective action for extended bodies



$\epsilon_{\underline{\aleph}} = \Lambda_{\underline{\aleph}}{}^{\underline{\alpha}} e_{\underline{\alpha}}$

- $e_{\underline{\alpha}}{}^{\mu}$: field tetrad
- $e_{\underline{\aleph}}{}^{\mu}$: tetrad attached to the body
- $\Lambda_{\underline{\aleph}}{}^{\underline{\alpha}}$: Lorentz matrices (6 d.o.f.)

$$S = \int \mathrm{d}\tau\, L[u^\mu, \Omega^{\mu\nu}, g_{\mu\nu}, R_{\mu\nu\rho\sigma}, \nabla_\lambda R_{\mu\nu\rho\sigma}, ...]$$
$$= S_{\text{grav}} + S_{\text{point part.}} + S_{\text{non-minimal}}$$

Description of the system dynamics

GR action for point particles

$$S_{\rm grav}+S_{\rm point\ part.}=\frac{c^3}{16\pi G}\int {\rm d}^4x\sqrt{-g}\left[g^{\mu\nu}(\Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\rho}-\Gamma^\rho_{\mu\nu}\Gamma^\lambda_{\rho\lambda})-\underbrace{\frac{1}{2}g_{\mu\nu}\Gamma^\mu\Gamma^\nu}_{\text{gauge fixing term}}\right]$$

$$-\sum_A c^2\int {\rm d}t\left[m_A\sqrt{-(g_{\mu\nu})_A v_A^\mu v_A^\nu/c^2}+\frac{1}{2}S^A_{\mu\nu}\Omega^{\mu\nu}_A\right]$$

Non-minimal pieces

$$L_{\rm tides}=\sum_{\ell\geq 2}\frac{1}{\ell!}\left\{\frac{1}{2}\,\mu^{(\ell)}\,(G_L(\tau))^2+\frac{1}{2c^2}\,\frac{\ell}{\ell+1}\,\sigma^{(\ell)}\,(H_L(\tau))^2+\frac{1}{2c^2}\,\mu'^{(\ell)}\,(\dot G_L(\tau))^2+\ldots\right\}$$

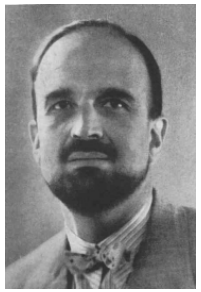
mass-like tidal moments; current-like tidal moments

[for a relativistic def. of Love numbers, see Damour, Nagar (2009); Binnington, Poisson (2009)]

Corresponding piece of the stress energy tensor: $T^{\mu\nu}_{\rm tides}=\dfrac{2}{\sqrt{-g}}\dfrac{\delta S_{\rm tides}}{\delta g_{\mu\nu}}$

Fokker action

- Start from the action $S[h^{\alpha\beta}, \boldsymbol{y_A}]$ (metric perturbation)
- Compute $h^{\alpha\beta}_{\text{(FE)}}$ given by $\left(\dfrac{\delta S}{\delta h^{\alpha\beta}}\right) = 0$ (REE)
- Build a Fokker action: $S_{\text{Fokker}} = S[h^{\alpha\beta}_{\text{(FE)}}[\boldsymbol{y_A}], \boldsymbol{y_B}]$



Credit: Huygens ING

Adriaan Fokker

$$\left(\frac{\delta S}{\delta y^i_A}\right)_{h=h_{\text{(FE)}}[\boldsymbol{y}_A]} = 0 \quad \text{and} \quad h_{\text{(FE)}}[\boldsymbol{y}_A] \text{ built with } \Box^{-1}_{\text{sym.}}$$

$$\Leftrightarrow \quad \frac{\delta S_{\text{Fokker}}}{\delta y^i_A} = 0$$

- If $h^{\mu\nu}_{\text{(FE)}} = \overline{h}^{\mu\nu}_0 + \delta h^{\mu\nu}$ then

$$S^{\text{red}} = S[\overline{h}^{\alpha\beta}_0[\boldsymbol{y}_A], \boldsymbol{y}_B] + \int \mathrm{d}^4x \underbrace{\frac{\delta S}{\delta h^{\mu\nu}}}_{=\mathcal{O}(\delta h)}[\overline{h}^{\alpha\beta}_0[\boldsymbol{y}_A], \boldsymbol{y}_B]\,\delta h^{\mu\nu} + \ldots$$

$\Rightarrow$ tidal effects can be ignored in $S_{\text{point particle}}$

2PN COM energy for circular orbits

[Henry, F., Blanchet (in preparation)]

Noether theorem $\Rightarrow$ **10 conserved quantities :** $\{E, P^i, J^i, G^i\}$

Choice of the center-of-mass frame

$$\boldsymbol{G = 0}$$

$\hookrightarrow$ convenient to introduce (anti-)sym. quantities $\mu^{(2)} \to \tilde{\mu}^{(2)}_{\pm}$, $\sigma^{(2)} \to \tilde{\sigma}^{(2)}_{\pm}$

$$\begin{aligned} E \;=\; & -\frac{\mu c^2 x}{2}\left\{1+\left(-\frac{3}{4}-\frac{\nu}{12}\right)x+\left(-\frac{27}{8}+\frac{19}{8}\nu-\frac{\nu^2}{24}\right)x^2\right. \\ & -18\,\tilde{\mu}^{(2)}_{+}x^5+\left[\left(-\frac{121}{2}+33\nu\right)\tilde{\mu}^{(2)}_{+}-\frac{55}{2}\delta\,\tilde{\mu}^{(2)}_{-}-176\,\tilde{\sigma}^{(2)}_{+}\right]x^6 \\ & +\left[\left(-\frac{20865}{56}+\frac{5434}{21}\nu-\frac{91}{4}\nu^2\right)\tilde{\mu}^{(2)}_{+}+\delta\left(-\frac{11583}{56}+\frac{715}{12}\nu\right)\tilde{\mu}^{(2)}_{-}\right. \\ & \left.\left.\left(-\frac{2444}{3}+\frac{1768}{3}\nu\right)\tilde{\sigma}^{(2)}_{+}-\frac{884}{3}\delta\,\tilde{\sigma}^{(2)}_{-}-130\,\tilde{\mu}^{(3)}_{+}\right]x^7\right\} \end{aligned}$$

Balance equation and phase

[Henry, F., Blanchet (in preparation)]

$$\mathcal{F} = -\frac{\mathrm{d}E}{\mathrm{d}t} \text{ with } \mathcal{F} = \sum_{\ell=2}^{+\infty} \frac{G}{c^{2\ell+1}} \left[c_\ell \left(I_L^{(\ell+1)} \right)^2 + \frac{d_\ell}{c^2} \left(J_L^{(\ell+1)} \right)^2 + O\left(\frac{G}{c^3} \right) \right] \quad \Rightarrow \quad \Psi(\omega)$$

EOB tidal phase of the GW without spin (in the SPA, $x \equiv \left(\frac{GM\omega_f}{c^3} \right)^{2/3}$)

$$\Psi^{\text{tides,EOB}}_{\text{2PN}}(x) = -\kappa^{(2)}_{\text{T}} \frac{39}{4} x^{5/2} \left[1 + \frac{3115}{1248} x - \pi x^{3/2} + \left(\frac{23073805}{3302208} + \frac{20}{351} \beta \right) x^2 \right]$$

with $\kappa^{(2)}_{\text{T}} = \kappa^{(2)}_1 + \kappa^{(2)}_2$, $\kappa^{(2)}_A = 2k^{(2)}_A \frac{m_B}{m_A} \left(\frac{R_A c^2}{G(m_A + m_B)} \right)^5$ [Damour, Nagar, Villain (2012)]

Preliminary result: $\beta = \frac{137056687}{5080320}$

1 First workshop of the GWF working group

2 Towards the 4PN GW phase of inspiraling binaries

3 Tidal effects in inspiraling binary GWF

4 Conclusion

Other works

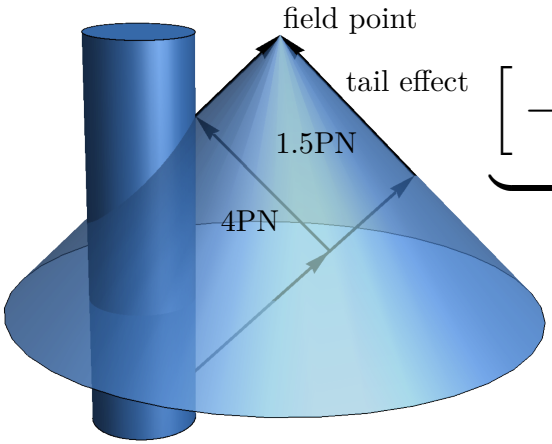
- Computation of the full 3PN waveform for eccentric inspirals with small e

  [Boetzel, Mishra, F., Gopakumar, Iyer; Ebersold, Boetzel, F., Mishra, Iyer, Jetzer (2019)]

- Computations of terms involving logs due to higher-order tails in E

  [Blanchet, Foffa, Larrouturou, Sturani (in preparation)]

$$E(x) = -\frac{1}{2} m\nu x c^2 \left\{ 1 + \ldots + \underbrace{\frac{448}{15}\nu x^4 \ln x}_{\text{leading tail effect at 4PN}} + \underbrace{\left[-\frac{4988}{35} - \frac{656}{5}\nu \right] \nu x^5 \ln x}_{\text{next-to-leading tail 5PN}} \right.$$



$$\left. \underbrace{\left[-\frac{1967284}{8505} + \frac{914782}{945}\nu + \frac{32384}{135}\nu^2 \right] \nu x^6 \ln x}_{\text{next-to-next-to leading 6PN}} \right\}$$

- Computation of the 4.5PN radiation reaction force in the binary motion

  [Bernard, Blanchet, F. (in preparation)]