Gravitational wave polarimetry with quaternions

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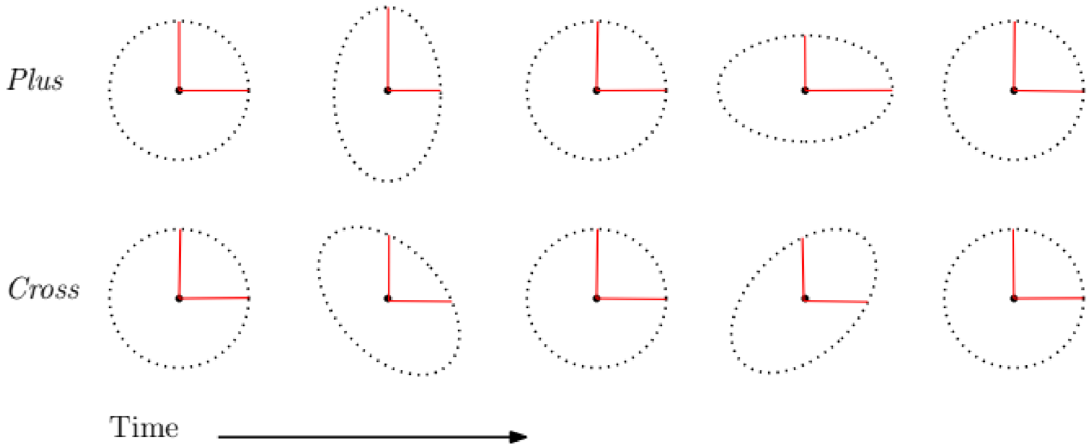
Gravitational wave polarizations

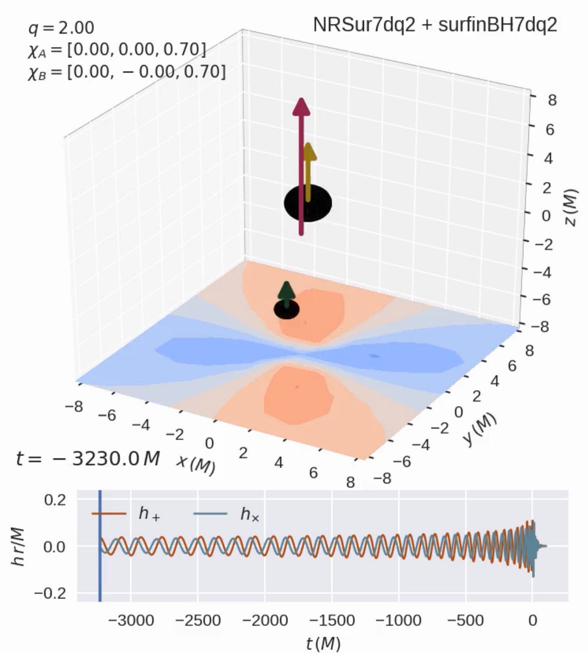


$$\frac{\Delta L(t)}{L} = h_+(t)F_+(\Theta) + h_\times(t)F_\times(\Theta)$$

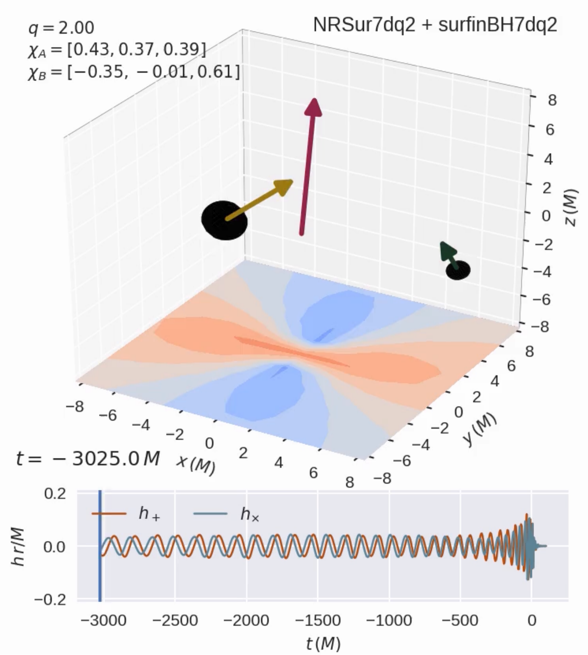
Credit : [arXiv:1811.06552], vijayvarma392.github.io/binaryBHexp

Example of polarization evolution

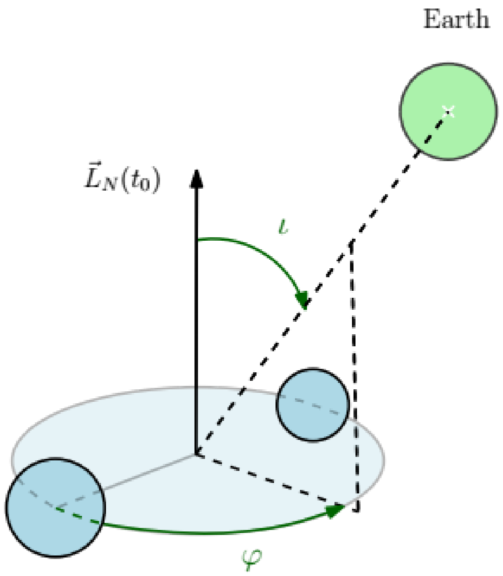
No precession



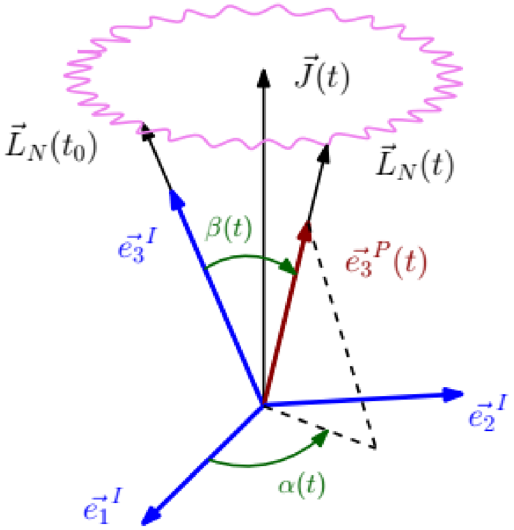
Precession

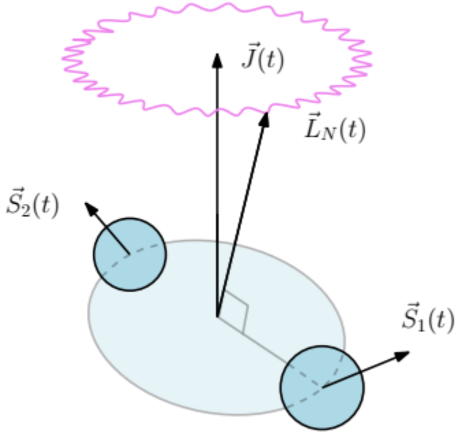


Precession parameters



(ι, φ) : line of sight





$\alpha(t)$, $\beta(t)$ tracking $L_N(t)$

Expected signal

Bivariate signal : $h(t) = h_+(t) - \mathbf{i}h_\times(t)$

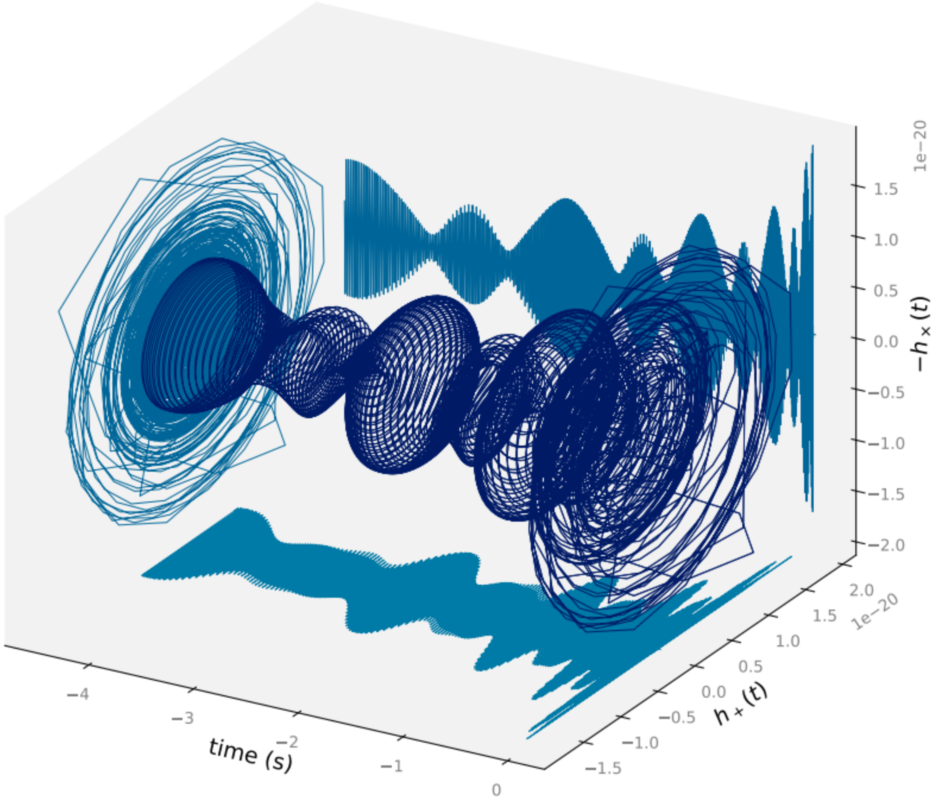
$$h(t) = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} h^P_{l,m}(t)_{-2}Y_{l,m}(\Omega(t))$$

dominated by : $h^P_{2,\pm 2}(t) = a_0(t)e^{\mp \mathbf{i}\Phi_0(t)}$

$$h(t) = h^P_{2,2}(t)_{-2}Y_{2,2}(\Omega(t)) + h^P_{2,-2}(t)_{-2}Y_{2,-2}(\Omega(t))$$

$\Omega(t)$ observer position in precessing frame,

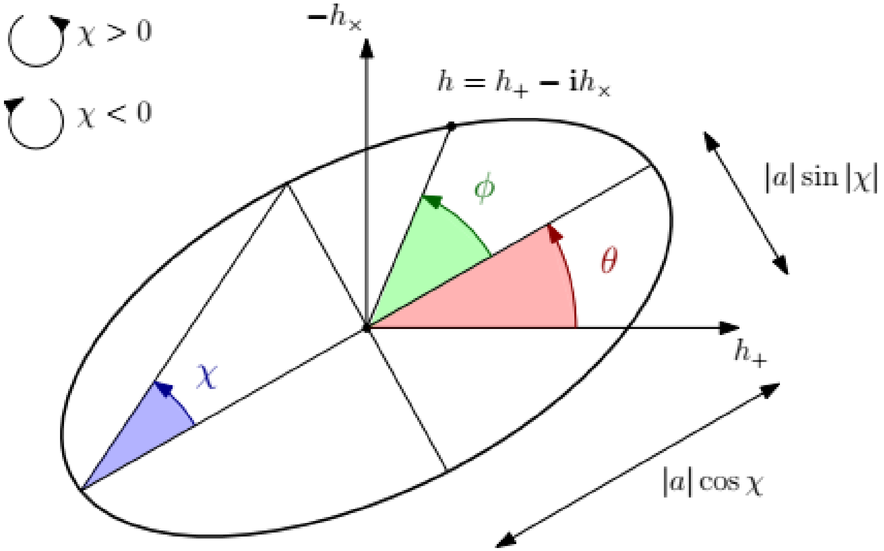
depends on $\alpha(t)$, $\beta(t)$, ι, φ.



Ref : Babak, Taracchini and Buonanno [arXiv:1607.05661]

Stokes parameters

$$h_{\mathbb{H}}(t) = h(t) + \mathcal{H}\{h\}(t)\mathbf{j}$$
$$= a(t)e^{\mathbf{i}\theta(t)}e^{-\mathbf{k}\chi(t)}e^{\mathbf{j}\phi(t)}$$



Polarized AM-FM if $|\phi'(t)| \gg |\chi'(t)|\,,\ |\theta'(t)|\,,\ \left|\frac{a'(t)}{a(t)}\right|$

Ref : Flamant, Le Bihan and Chainais [arXiv:1609.02463]

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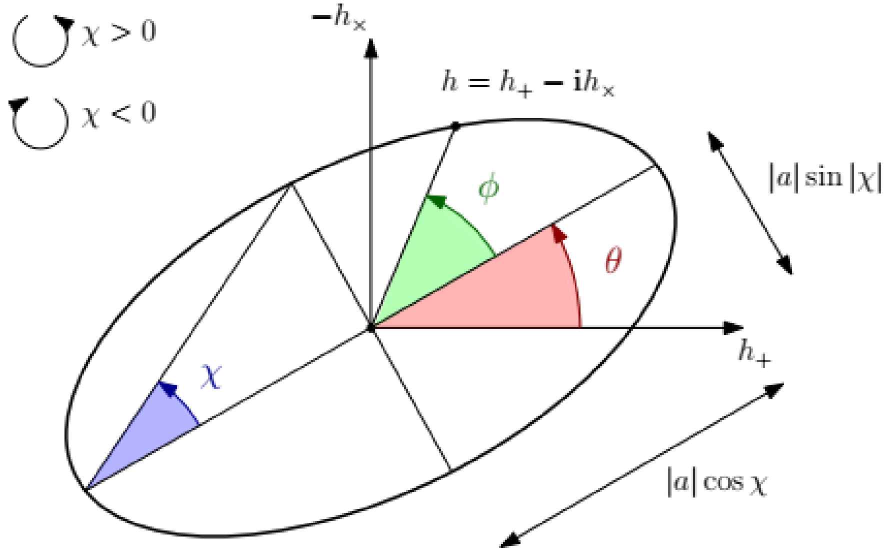
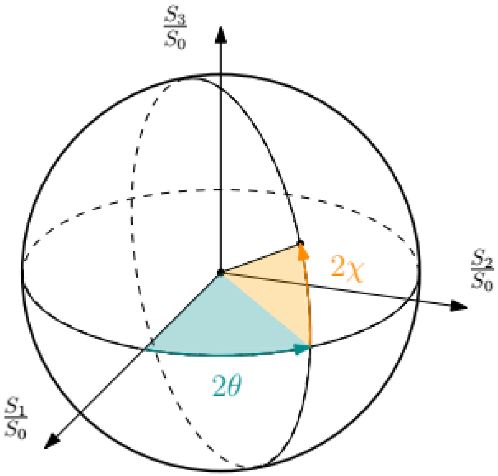


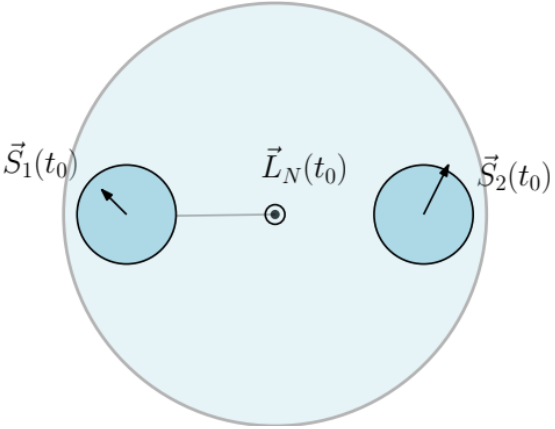
$$S_0(t) = a(t)^2$$
$$S_1(t) = a(t)^2 \cos 2\chi(t) \cos 2\theta(t)$$
$$S_2(t) = a(t)^2 \cos 2\chi(t) \sin 2\theta(t)$$
$$S_3(t) = a(t)^2 \sin 2\chi(t)$$



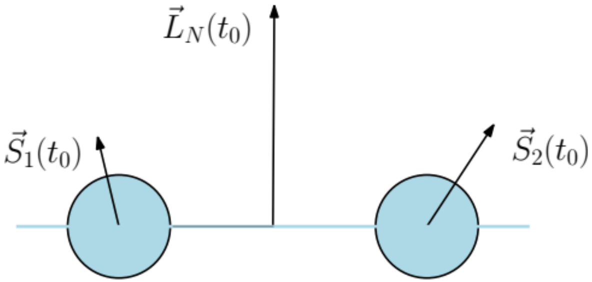
Ref : Flamant, Le Bihan and Chainais [arXiv:1609.02463]

Stokes parameters for small precession

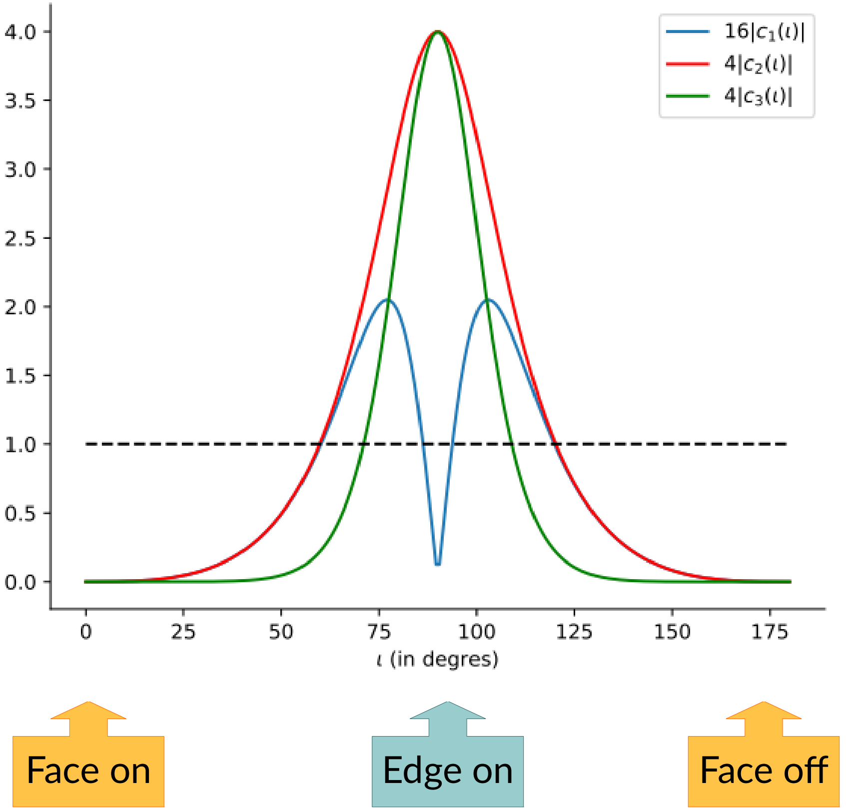
$$
\begin{aligned}
S_1/S_0 &= \quad b_1(\iota) + 16c_1(\iota)\cos(\alpha-\varphi)\beta + \mathcal{O}(\beta^2),\\
S_2/S_0 &= \quad 4c_2(\iota)\cos(\alpha-\varphi)\beta + \mathcal{O}(\beta^2),\\
S_3/S_0 &= -4b_3(\iota) - 4c_3(\iota)\cos(\alpha-\varphi)\beta + \mathcal{O}(\beta^2),
\end{aligned}
$$



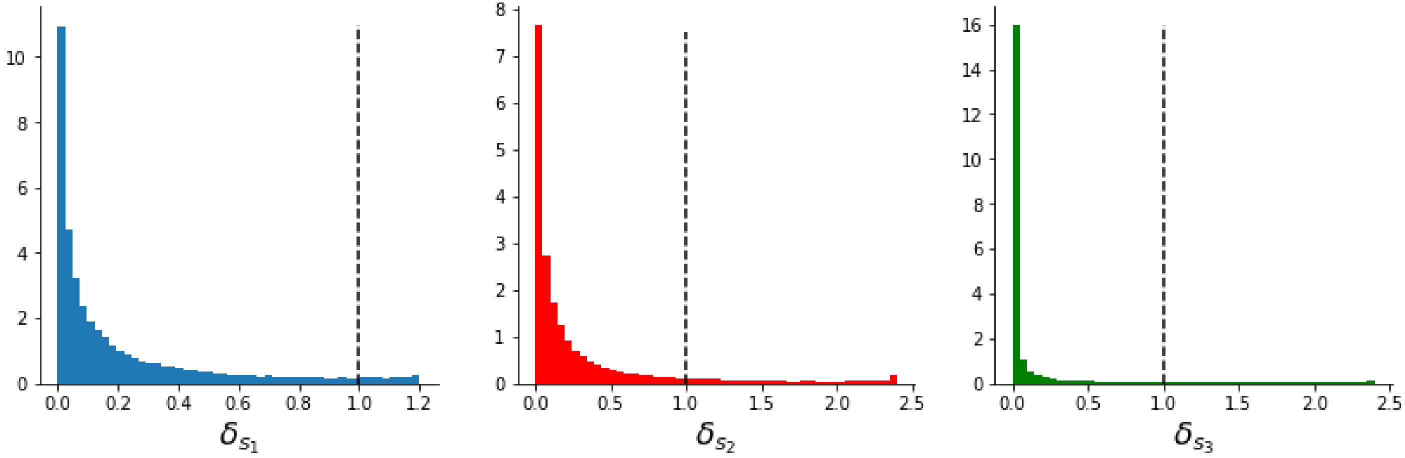
Face on



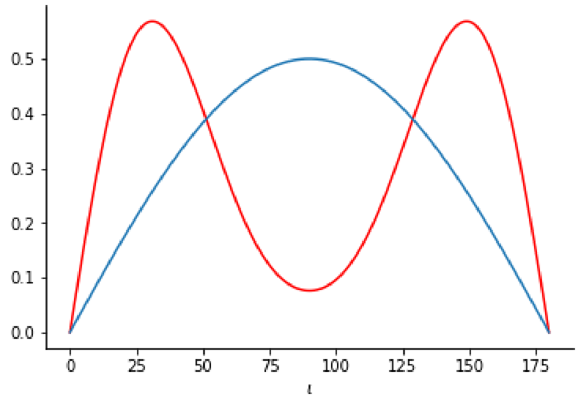
Edge on



Preliminar study on sensibility



Histograms of δ_{s_i} for $\beta\_\max = 17$ deg

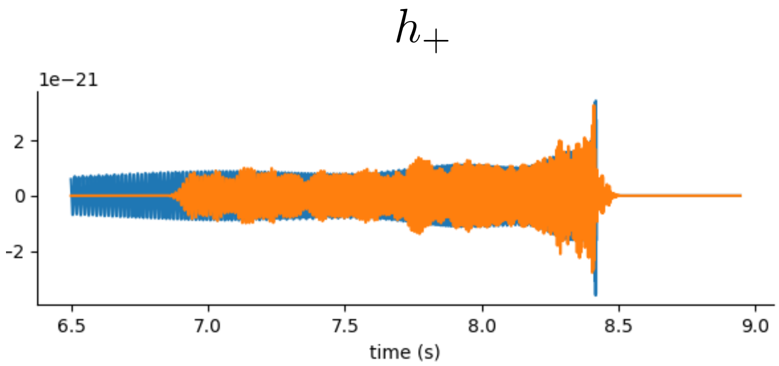


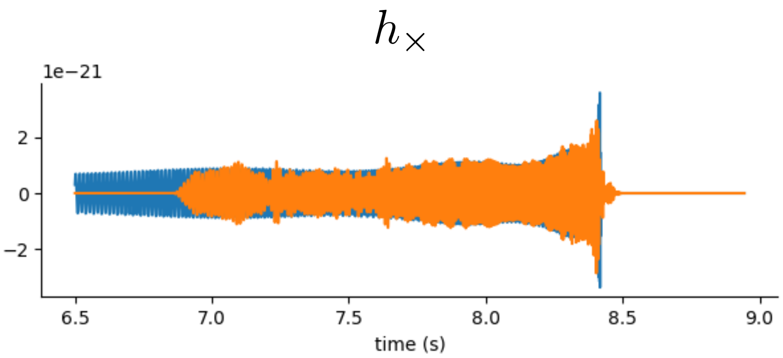
Distrib. of detect. ι

Schutz [arXiv:1102.5421]

$\beta\_\max$ deg	$P(\delta_{s_1} \geq 1)$	$P(\delta_{s_2} \geq 1)$	$P(\delta_{s_3} \geq 1)$
17	0.03	0.08	0.04
28	0.11	0.14	0.06
40	0.17	0.19	0.08

Results





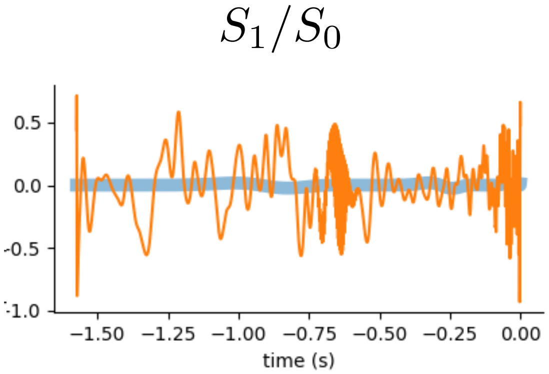
Line of sight : $\iota =$ **30** deg, $\varphi = 90$ deg

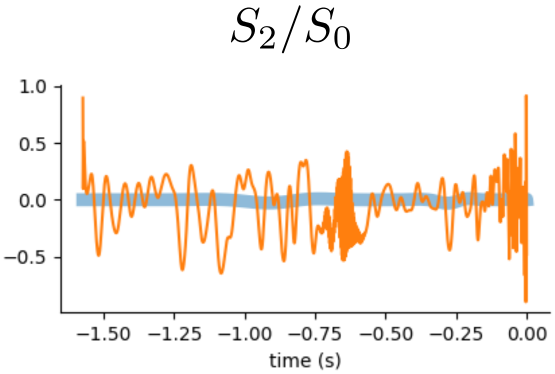
HLV network SNR : 237

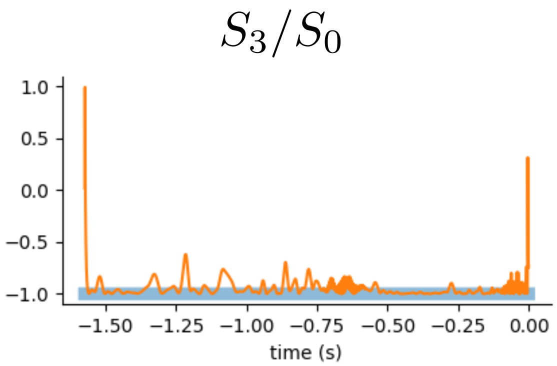
Configuration : $m_1 = 10\,M_\odot,\ m_2 = 1\,M_\odot,\ \chi_1 = (0.2, 0.2, 0.55),\ \chi_2 = (0, 0, 0)$

Precession angle : $\beta_{\rm max} \approx 32$ deg

Precession not detectable

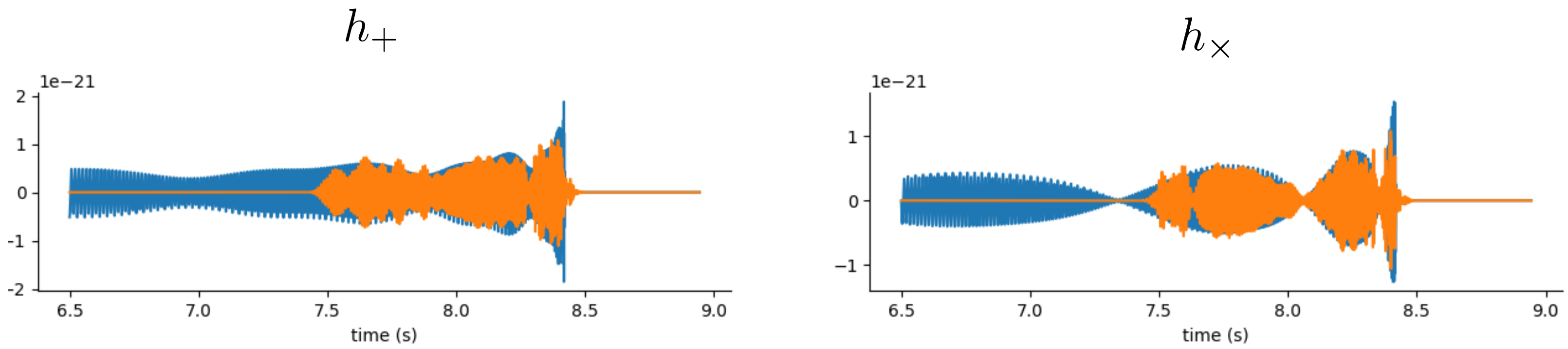






Results

Edge on



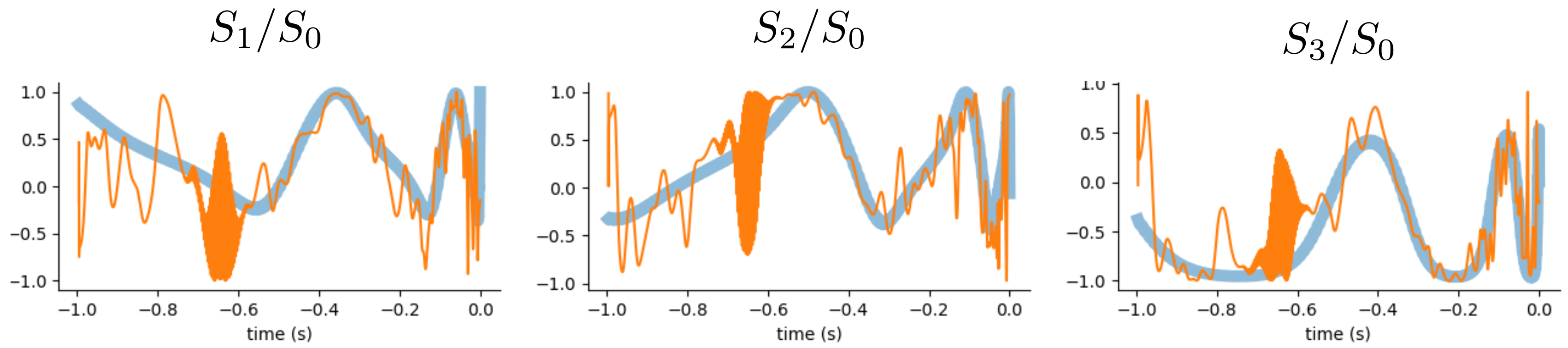
Line of sight : $\iota =$ **90** deg, $\varphi = 0$ deg

HLV network SNR : 97

Configuration : $m_1 = 10\, M_\odot$, $m_2 = 1\, M_\odot$, $\chi_1 = (0.2, 0.2, 0.55)$, $\chi_2 = (0, 0, 0)$

Precession angle : $\beta_{\max} \approx 32$ deg

Detectable precession !



Conclusion

- Polarimetric analysis of GW with Stokes parameters
- Application to precessing binaries
  - Linear dependency in precession angle
- General methodology

Perspectives

- Development of extraction method
- Detection and caracterisation of precession
- Application to other sources

Computation of Stokes parameters

Multiplication rules : $\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = -1$

$$\mathbf{ij} = -\mathbf{ji} = \mathbf{k}$$

$$\mathbf{ki} = -\mathbf{ki} = \mathbf{j}$$

$$\mathbf{jk} = -\mathbf{kj} = \mathbf{i}$$

For $q = a + \mathbf{i}b + \mathbf{j}c + \mathbf{k}d$

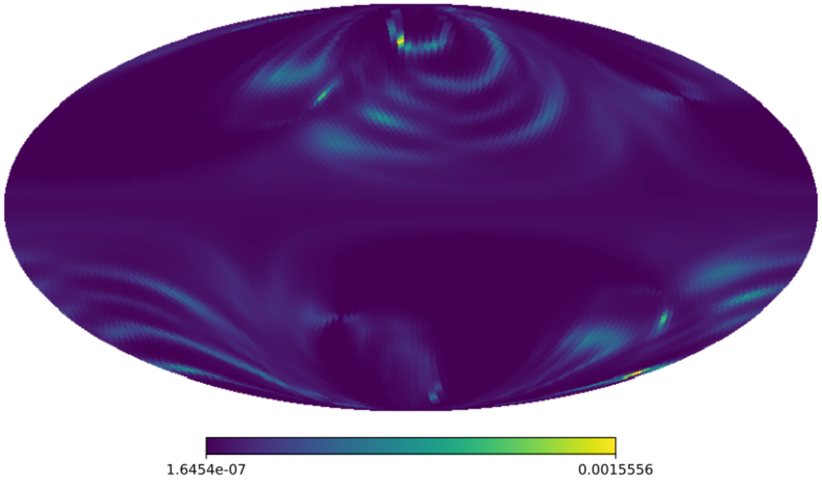
we define $q^{*\mathbf{j}} = \overline{q^{\mathbf{j}}} = \overline{q}^{\mathbf{j}} = a + \mathbf{i}b - \mathbf{j}c + \mathbf{k}d$

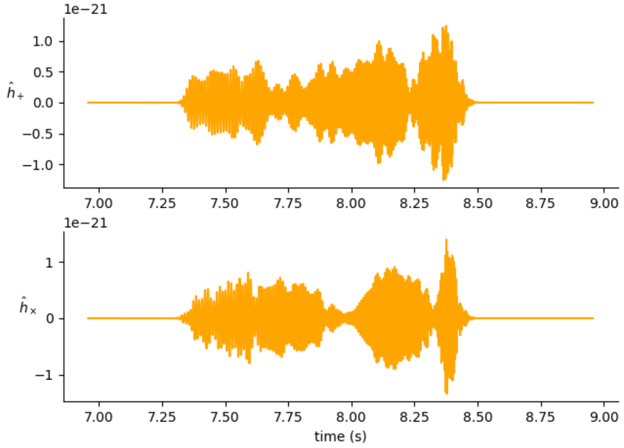
$$h(t) = h_+(t) - \mathbf{i}h_\times(t) \Longrightarrow h_{\mathbb{H}}(t) = h(t) + \mathcal{H}\{h\}(t)\mathbf{j} \Longrightarrow \begin{aligned} S_0(t) &= h_{\mathbb{H}}(t)^2 \\ h_{\mathbb{H}}(t)h_{\mathbb{H}}(t)^{*\mathbf{j}} &= S_1(t) + \mathbf{i}S_2(t) - \mathbf{k}S_3(t) \end{aligned}$$

Extraction algorithm

Inverse problem : $x(t-\tau_\Theta) = F_\Theta \mathbf{h}(t) + n(t)$

Two step :
- Maximizing likelyhood on skymap gives $\hat{\Theta}$
- Minimizing quadratic error with parsimony a priori gives $\hat{\mathbf{h}}(t) = \begin{bmatrix} \hat{h}_+(t) \\ \hat{h}_\times(t) \end{bmatrix}$





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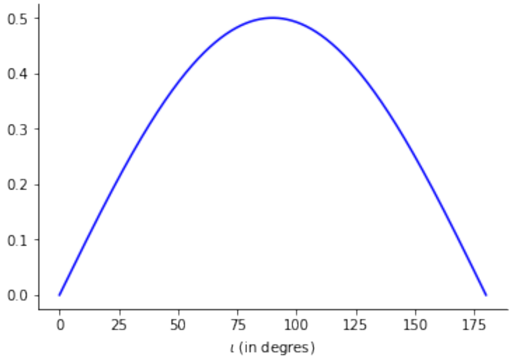
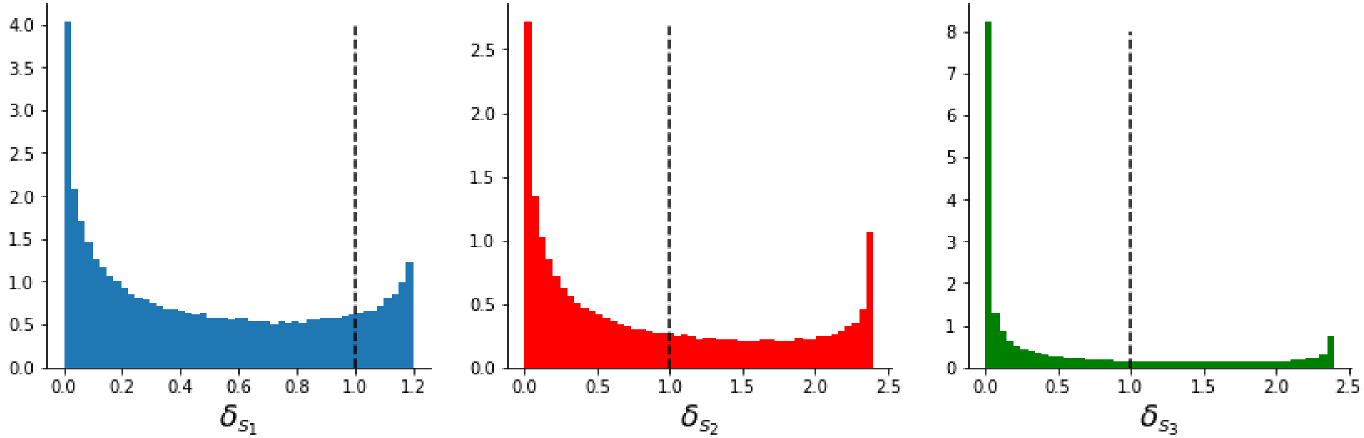


Distrib. of ι



Histograms of δ_{s_i} for $\beta\_\max = 17$ deg

$\beta\_\max$ deg	$P(\delta_{s_1} \geq 1)$	$P(\delta_{s_2} \geq 1)$	$P(\delta_{s_3} \geq 1)$
11	0	0.26	0.17
17	0.16	0.38	0.25
28	0.30	0.45	0.29