Observations gravitationnelles et incertitudes dans la mesure de distance

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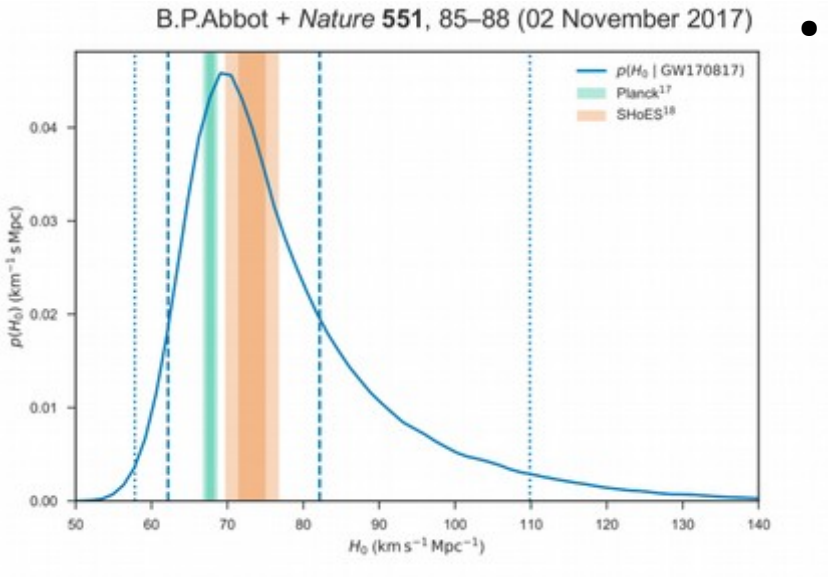
Context: measurement of the Hubble constant



So far, only 1 point in the *z*, D_L plane (GW170817)!

- Gravitational-wave "standard sirens"

Redshift from electromagnetic observation (e.g., host galaxy)

Luminosity distance from GW observations

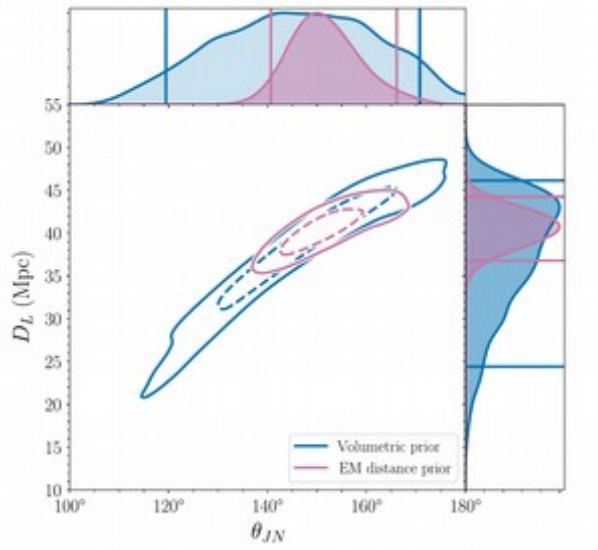
$h \propto 1/D_L$

$$cz = H_0 \, D_L$$

Infer Hubble constant

Error propagation : $$\left(\frac{\Delta H_0}{H_0}\right)^2 \sim \left(\frac{\Delta z}{z}\right)^2 + \left(\frac{\Delta D_L}{D_L}\right)^2$$

Initial intuitions



Abbott et al. Phys. Rev. X 9, 011001 2019

- Uncertainty largely due to distance/inclination degeneracy

$$h_+ \propto \frac{1+\cos^2\iota}{D_L} \qquad h_\times \propto \frac{\cos\iota}{D_L}$$

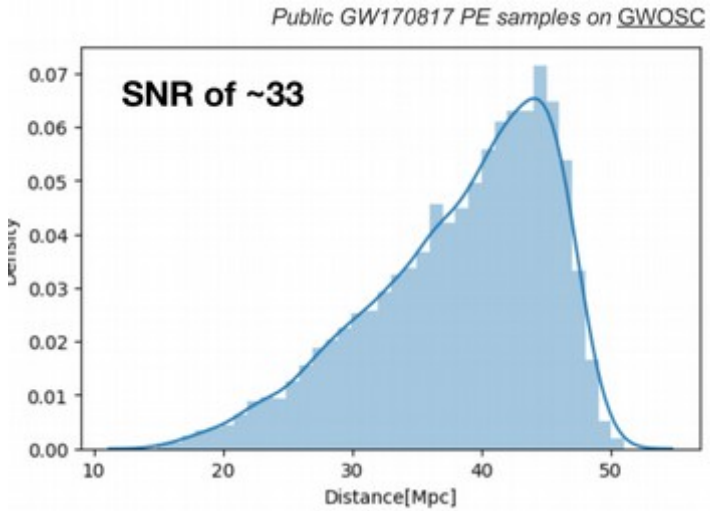
- Two polarizations may help to resolve this degeneracy

  Ex: GW170817 with significant SNR in both LIGO HL and Virgo

- Sky locations where distance uncertainty is smaller?

  “Golden spots” for H0 measurement?

Uncertainty estimate



Gaussian approx is an **oversimplification**

- **Fisher information matrix** → Gaussian approx
- **Require "beyond Gaussian" approx that**
  - Encodes the degeneracies
  - Is analytic (fast evaluation)

Seminal paper: Cutler & Flanagan – 1994

Log likelihood: $\log P(s|h(\theta)) \propto \langle s - h(\theta)|s - h(\theta)\rangle$

Newtonian waveform (**masses**, merger time, **sky position known**)

Free parameters: D_L, inclination, polarization angle, merger phase

Cutler & Flanagan – 1994

A, B = + or x

GW strain at detector:

distance
phase at merger
polar angle
beam pattern
inclination

$$\tilde{h}_a(f) = \frac{e^{-i(\phi_c + 2\pi f \tau_a)}}{D} R^A_B(2\psi) \hat{F}^B_a(\alpha, \delta) \chi_A(v) \tilde{k}(f; t_c, \mathcal{M})$$

Effective polarization amplitudes:

$$\mathcal{A}_B = \frac{e^{-i\phi_c}}{D} R^A_B(2\psi) \chi_A(v)$$

Rewrite scalar product:
using effective polarizations

$$\langle \vec{h} | \vec{h'} \rangle = \Re \left[\mathcal{A}^*_A \mathcal{A}'_B \Theta^{AB} \right] \langle \tilde{k} | \tilde{k} \rangle$$

Mixing matrix of the scalar product:

noise spectrum

$$\Theta^{AB}(\vec{n}) = \hat{F}^A_a(\vec{n}) \hat{F}^B_b(\vec{n}) \kappa^{ab}$$

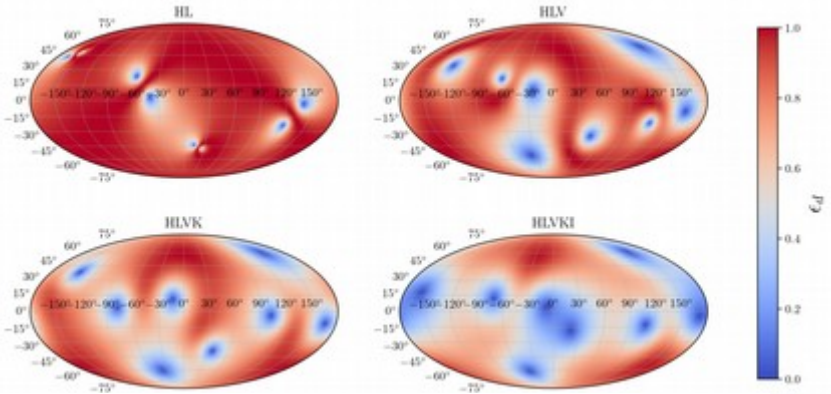
Diagonalize:

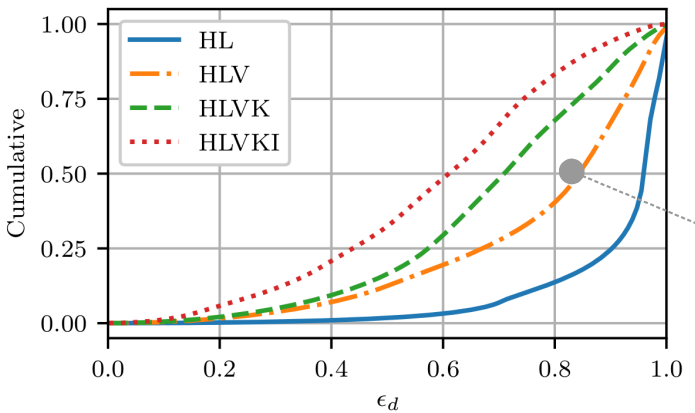
$$\bar{\Theta}^{AB} = \sigma_d \begin{pmatrix} 1+\epsilon_d & 0 \\ 0 & 1-\epsilon_d \end{pmatrix}$$

$$\log P(s|h(\theta)) \sim (1+\epsilon_d)|\mathcal{A}_+ - \mathcal{A}'_+| + (1-\epsilon_d)|\mathcal{A}_\times - \mathcal{A}'_\times|$$

Note: error in CF94 after marginalization

Degeneracy parameter ε_d

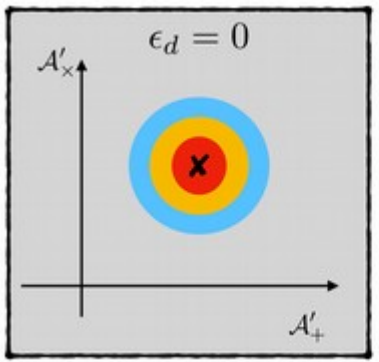


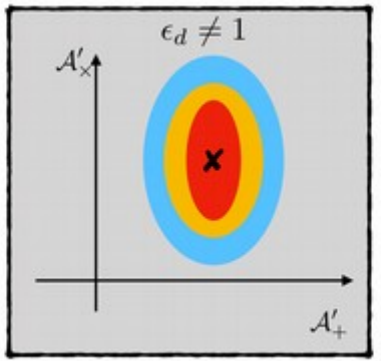


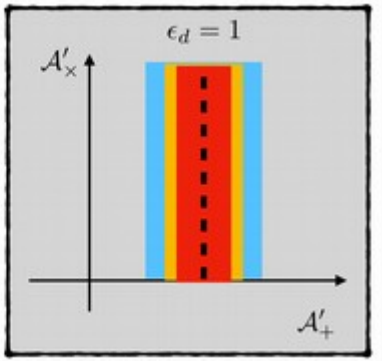
$$\log P(s|h(\theta)) \sim (1+\epsilon_d)|\mathcal{A}_+ - \mathcal{A}'_+| + (1-\epsilon_d)|\mathcal{A}_\times - \mathcal{A}'_\times|$$

Two unknowns – two equations

Two unknowns **one** equation





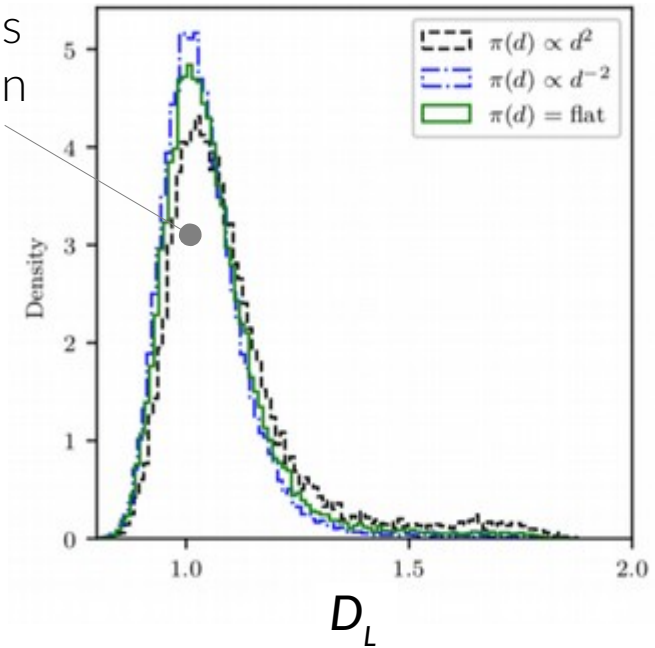


Increase the number of detectors,
decreases the number of degenerate cases

Effect of degeneracy

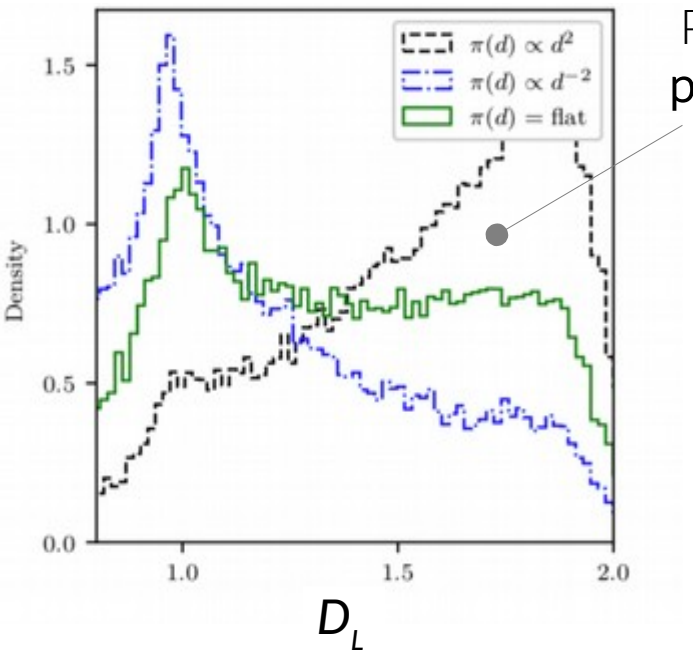
Non-degenerate $\varepsilon_d = 0$

Posterior is **data** driven

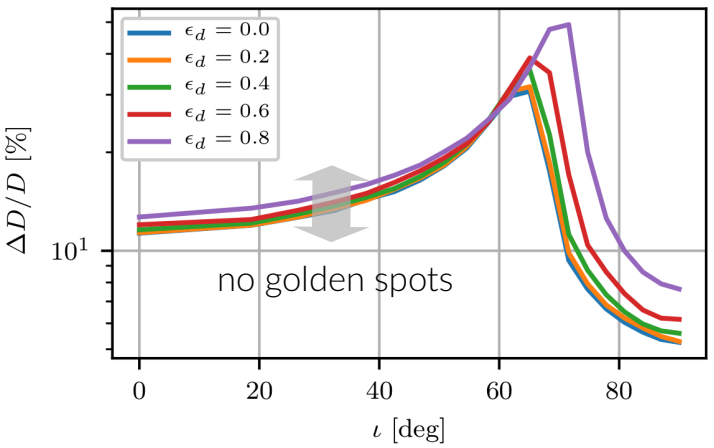


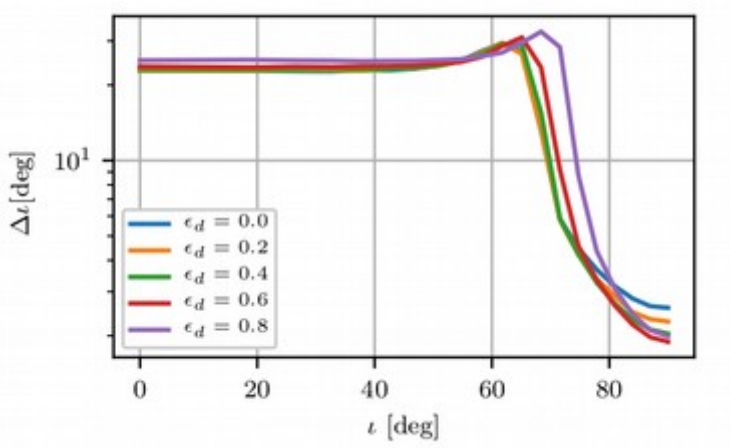
Degenerate $\varepsilon_d = 1$

Posterior is **prior** driven



Uncertainties in the non-degenerate case



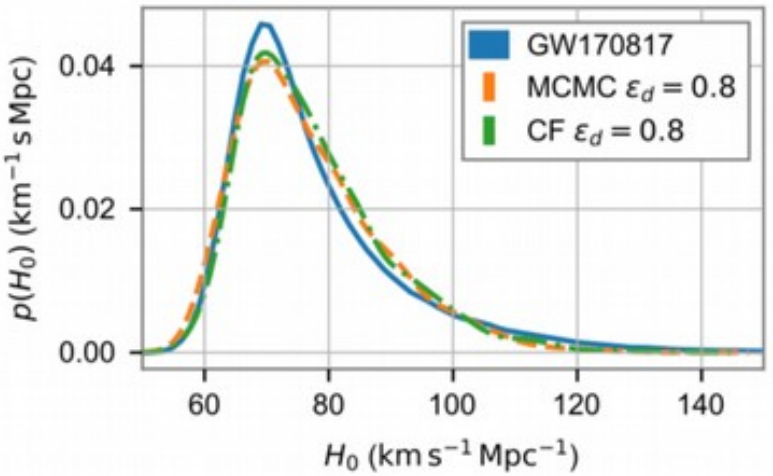


SNR = 20

Our predicted uncertainties for D_L or inclination ι are consistent with:

- Simulated binary neutron-star signals estimated using LAL Inf (nested sampling)
- Similar results in the literature obtained through simulations

Application to GW170817



$\varepsilon_d = 0.8$

SNR = 33

Computed using LAL Inf posteriors

$$H_0 = 70.0^{+12.0}_{-8.0}\ \mathrm{km\,s^{-1}Mpc^{-1}}$$

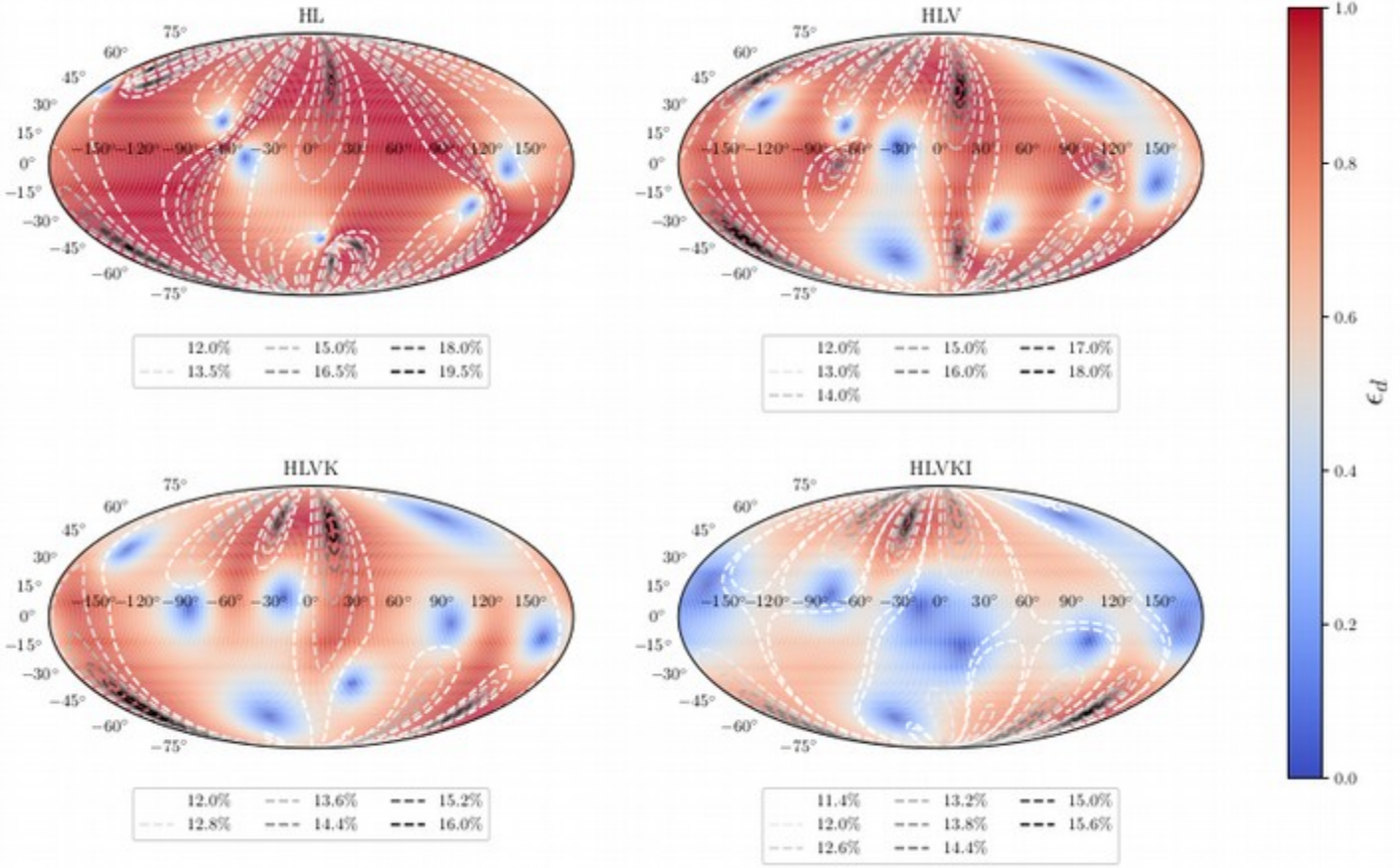
Based on analytical approx [CF'94]

$$H_0 = 69.8^{+11.2}_{-9.0}\ \mathrm{km\,s^{-1}Mpc^{-1}}$$

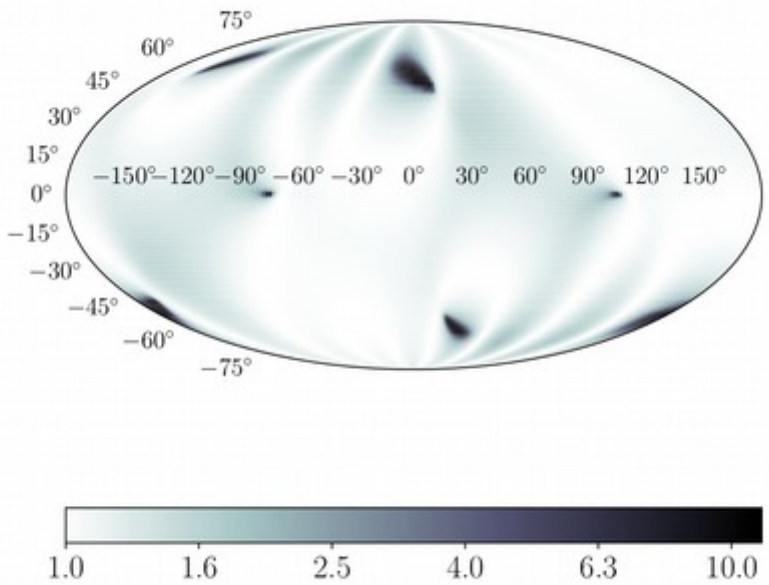
Concluding remarks

- **Analytical prediction for the distance uncertainties**
  - Able to capture the distance/inclination degeneracy
  - Consistent results with Bayesian estimates
  - Can be used for future projections of H0 measurements (200 BNS+)
- **Position-dependent predictions applied to the full sky**
  - No “golden” spots
  - Evidences the existence of degenerate sky locations – $\varepsilon_d = 1$

    Risk of bias due to prior-driven posteriors when $\epsilon_d \gtrsim 1 - \frac{1}{\rho_0^2}$


HL
12.0% 15.0% 18.0%
13.5% 16.5% 19.5%
HLV
12.0% 15.0% 17.0%
13.0% 16.0% 18.0%
14.0%
HLVK
12.0% 13.6% 15.2%
12.8% 14.4% 16.0%
HLVKI
11.4% 13.2% 15.0%
12.0% 13.8% 15.6%
12.6% 14.4%
ϵ_d

Cutler & Flanagan – 1994



Factor between our uncertainty prediction on distance and that of CF94 (difference due to a mistake in the calculation that we have corrected)