

# New microscopic *sd*-shell interactions from the No-Core Shell Model

B.R. Barrett, *University of Arizona*

I.J. Shin, Y. Kim, *Institute of Basic Science, Daejeon*

P. Maris, J.P. Vary, *Iowa State University*

A.M. Shirokov, *Moscow State University*

N.A. Smirnova, *CENBG/University of Bordeaux*

GDR Resanet: GT3 “Nuclear Theories”,  
Orme-des-Merisiers, November 12–13, 2018



# Plan

## Microscopic *sd*-shell interactions from the No-Core Shell Model (NCSM)

- ① Valence-space effective interactions from the NCSM: formalism
- ② Comparison of microscopic and phenomenological interactions
  - Monopole part versus multipole terms
  - Spectra and binding energies of O-isotopes
  - Spectra of odd-*A* F-isotopes
  - Collective properties:  $^{24}\text{Mg}$
- ③ Spin-tensor analysis
- ④ Charge-dependent *sd*-shell effective interaction from the NCSM
- ⑤ Conclusions and perspectives

# Introduction

Progress in  $NN$  potentials (CD-Bonn, AV18,  $\chi$  EFT potentials, JISP16, Daejeon16, ...) and *ab-initio* many-body methods for light nuclei (GFMC, NCSM, CC, ...) and heavy closed-shell (and open-shell) nuclei (CC, GGF, ...)

Effective approaches to medium-mass and heavy open-shell nuclei?  $\Rightarrow$  Shell Model, energy density functional theories, ...

Valence-space effective interactions for the shell model: renormalization

- $G$  matrix + MBPT

*Kuo, Brown, Nucl. Phys. 85 (1966); 114 (1968)*

*Hjorth-Jensen, Kuo, Osnes, Phys. Rep. 261, 125 (1995)*

- $V_{low-k}$  + MBPT

*Bogner, Kuo, Schwenk, Phys. Rep. 386 (2003)*

- IMSRG ( $NN + 3N$ )

*Stroberg, Hergert, Holt, Bogner, Schwenk, PRC93, 051301(R) (2016)*

*Stroberg, Calci, Hergert, Holt, Bogner, Roth, Schwenk, PRL118, 032502 (2017)*

- Coupled-cluster effective interaction method ( $NN + 3N$ )

*Jansen et al, PRC94, 011301(R) (2016)*

- NCSM

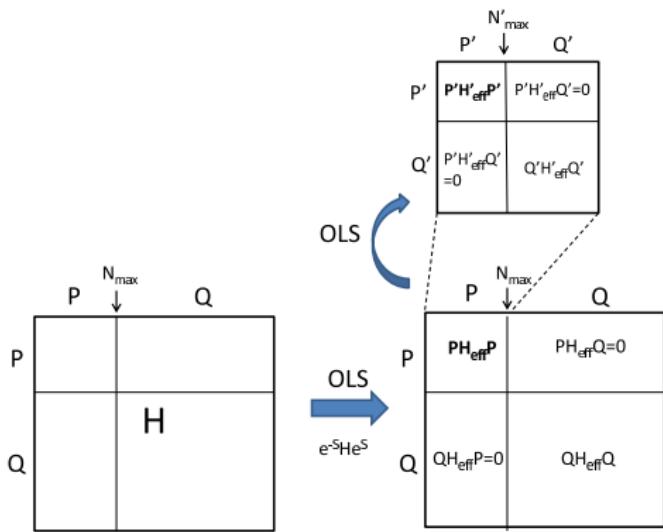
*Lisetskiy, Barrett, Kruse, Navratil, Stetcu, Vary, PRC78, 044302 (2008)*

*Dikmen, Lisetskiy, Barrett, Maris, Shirokov, Vary, PRC91, 064301 (2015)*



# Ab-initio effective interaction from the NCSM

$$H = \sum_{i < j=1}^A \frac{(\vec{p}_i - \vec{p}_j)^2}{2Am} + \sum_{i < j=1}^A V_{ij}^{NN} \left( + \sum_{i < j < k=1}^A V_{ijk}^{NNN} \right)$$



## Flow

- NCSM for  $^{18}\text{F}$  at  $N_{\max} = 4$  with  $V_{\text{eff}}$  (OLS)
- $\tilde{H}'_{\text{eff}}$  for  $^{18}\text{F}$  at  $N'_{\max} = 0$  (OLS)
- $^{16}\text{O}$  at  $N_{\max} = 4$  (core energy)
- $^{17}\text{O}, ^{17}\text{F}$  at  $N_{\max} = 4$  (one-body terms)
- S.p. energies and  $T = 0, 1$  TBMEs in  $sd$  shell

Okubo, Progr. Theor. Phys. 12 (1954); Suzuki, Lee, Prog. Theor. Phys. 68 (1980)

Dikmen, Lisetskiy, Barrett, Maris, Shirokov, Vary, PRC91, 064301 (2015)

# *Ab-initio* effective interaction from the NCSM

## Present results obtained from the NCSM

- $N_{max} = 4$  (first results for  $N_{max} = 6$ )
- $\hbar\Omega = 14$  MeV ( $\hbar\Omega = 12$  MeV – 20 MeV)
- $^{18}\text{F} \Rightarrow T = 0, 1$  TBMEs in  $sd$  shell (charge-independent)
- $^{18}\text{O}, ^{18}\text{F}, ^{18}\text{Ne} \Rightarrow \text{pp,nn, pn}$  ( $T = 0, 1$ ) TBMEs in  $sd$  shell (charge-dependent)
- MFDn code (*Vary, Maris et al, Iowa SU*) and Antoine code (*Caurier, Nowacki, Strasbourg*)

## NN potentials

- Chiral N3LO  
*Entem, Machleidt, PRC91, 041001 (2003)*
- $J$ -matrix Inverse Scattering Potential (JISP16)  
*Shirokov, Vary, Mazur, Weber, Phys. Lett. B644, 33 (2007)*
- Daejeon16 NN potential (SRG-evolved chiral N3LO, adjusted on light nuclei)  
*Shirokov, Shin, Kim, Maris, Sosonkina, Vary, Phys. Lett. B761, 81 (2016)*

# *Ab-initio* effective interaction from the NCSM

## *A*-dependence

$$H_a + H_{CM} = \sum_{i=1}^a \left[ \frac{\vec{p}_i^2}{2m} + \frac{1}{2} m\Omega^2 \vec{r}_i^2 \right] + \sum_{i < j=1}^a V_{ij}(\Omega, A),$$

$$V_{ij}(\Omega, A) = V_{ij}^{NN} - \frac{m\Omega^2}{2A} (\vec{r}_i - \vec{r}_j)^2.$$

$E_{core}$ , s.-p. energies and TBMEs are (weakly)  $A$ -dependent.

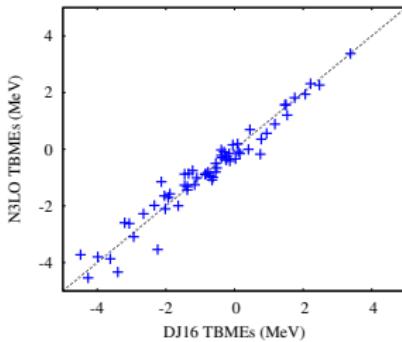
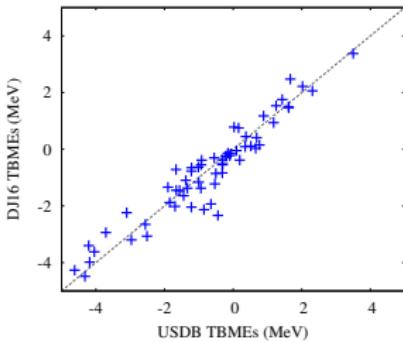
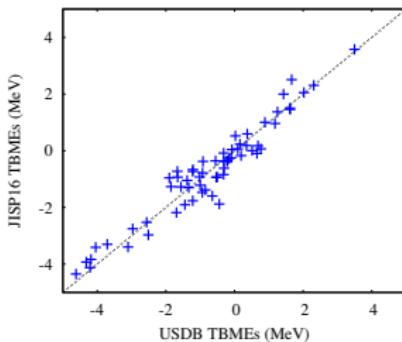
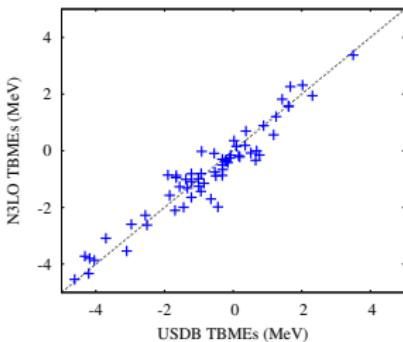
- $\langle V_\delta \rangle \sim (\hbar\Omega)^{3/2}$
- $\langle V_{Coul} \rangle \sim (\hbar\Omega)^{1/2}$
- $\langle V_{SR} \rangle \sim (\hbar\Omega) \sim A^{-1/3}$

Core energy and single-particle energies for the *sd* shell from Daejeon16 (in MeV)

	$A = 18$ $E_{core} = -121.817$			$A = 19$ $E_{core} = -121.783$		
	$1s_{1/2}$	$0d_{5/2}$	$0d_{3/2}$	$1s_{1/2}$	$0d_{5/2}$	$0d_{3/2}$
$\epsilon_\nu$	-3.576	-3.302	6.675	-3.572	-3.299	6.677
$\epsilon_\pi$	-0.077	0.291	9.974	-0.073	0.294	9.976
$\epsilon$ (USDB)	-3.208	-3.926	2.118			

# How to compare various sets of TBMEs?

1



# Multipole representation of the valence-space shell-model Hamiltonian

We work in a valence space of one (two) harmonic oscillator shells beyond a closed-shell core.

In a particular (harmonic-oscillator) basis:

$$\hat{H} = \sum_i \epsilon_i \mathbf{a}_i^+ \tilde{\mathbf{a}}_i + \sum_{ijkl, \Gamma} V_{ijkl}^\Gamma [\mathbf{a}_i^+ \mathbf{a}_j^+]^{(\Gamma)} [\tilde{\mathbf{a}}_k \tilde{\mathbf{a}}_l]^{(\Gamma)}, \quad \Gamma = (JT)$$

Re-write a two-body interaction in the multipole form:

$$\hat{V} = \sum_{ijkl, \gamma} w_{ijkl}^\gamma [\mathbf{a}_i^+ \tilde{\mathbf{a}}_k]^{(\gamma)} [\mathbf{a}_j^+ \tilde{\mathbf{a}}_l]^{(\gamma)} + \delta_{jk} \delta_{il} w_{ijjj}^{0\tau} [\mathbf{a}_i^+ \tilde{\mathbf{a}}_l]^{(0\tau)}, \quad \gamma = (\lambda\tau)$$

Monopole-multipole decomposition:

$$\hat{H} = \hat{H}_{mon} + \hat{H}_{mult}$$

$H_{mon}$  contains all  $\lambda = 0$  terms;  $\hat{H}_{mult}$  contains all the rest.

Dufour, Zuker (1995); Caurier et al, RMP77 (2005)

# Monopole part of the shell-model Hamiltonian

Monopole part (all terms with  $\lambda = 0$ ):

$$\hat{H}_{mon} = \sum_i \epsilon_{\nu_i} \hat{n}_{\nu_i} + \sum_i \epsilon_{\pi_i} \hat{n}_{\pi_i} + \sum_{ij} \overline{V}_{ij}^{\nu\pi} \hat{n}_{\nu_i} \hat{n}_{\pi_j} + \\ \sum_{i \leq j} \frac{\hat{n}_{\nu_i} (\hat{n}_{\nu_j} - \delta_{ij})}{1 + \delta_{ij}} \overline{V}_{ij}^{\nu\nu} + \sum_{i \leq j} \frac{\hat{n}_{\pi_i} (\hat{n}_{\pi_j} - \delta_{ij})}{1 + \delta_{ij}} \overline{V}_{ij}^{\pi\pi}$$

Centroids of the interaction:

$$\overline{V}_{ij}^{\rho\rho'} = \frac{\sum_J \langle ij | V | ij \rangle_J (2J+1)}{\sum_J (2J+1)}$$

## Interpretation

- $H_{mon}$ 
  - shell formation  $\Rightarrow$  *spherical mean field*;
  - dominant contribution to the *nuclear binding*.
- $H_{mult}$ 
  - contains nucleonic *correlations* (quadrupole-quadrupole, etc);

Dufour, Zuker (1995, 1996); Caurier et al, RMP77, 427 (2005)

# Schematic considerations

## Effective single-particle energies

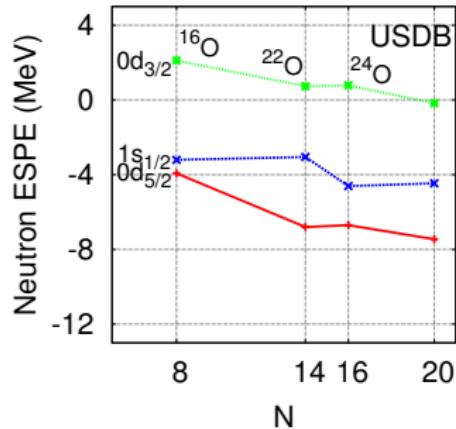
One-nucleon separation energies from a monopole Hamiltonian:

$$\tilde{\epsilon}_i(A) = \epsilon_i(A_0) + \sum_j \overline{V}_{ij} \langle \hat{n}_j \rangle$$

Centroids of the interaction:

$$\overline{V}_{ij}^{\rho\rho'} = \frac{\sum_J \langle ij | V | ij \rangle_J (2J+1)}{\sum_J (2J+1)}$$

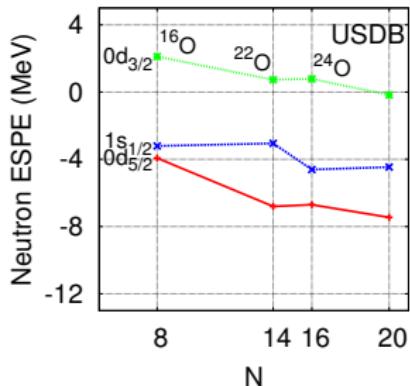
Bansal, French (1964); Poves, Zuker (1981);  
Otsuka et al (1999); Caurier et al, RMP (2005)



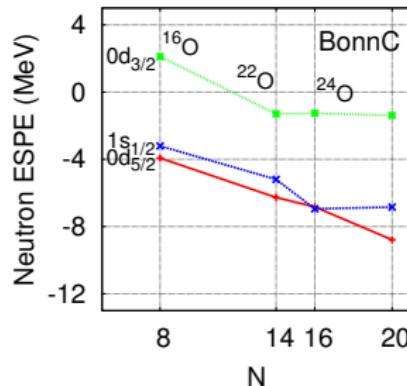
ESPEs from an *ab-initio* perspective: more caution is necessary

Duguet, Hagen, PRC85 (2012); Duguet et al, PRC92 (2015)

# What is wrong with microscopic effective interactions?



Fit to 608 data in the *sd* shell  
*Richter, Brown, PRC74 (2006)*



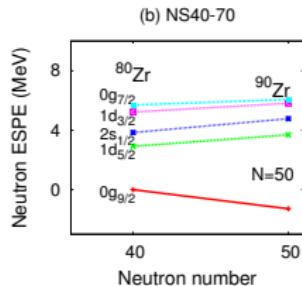
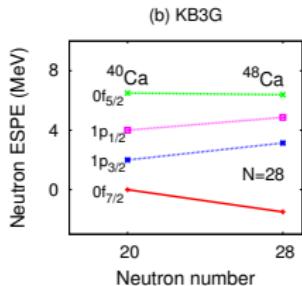
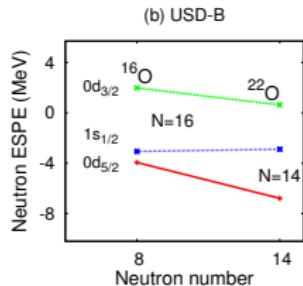
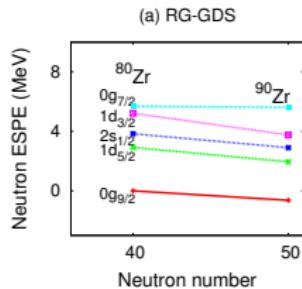
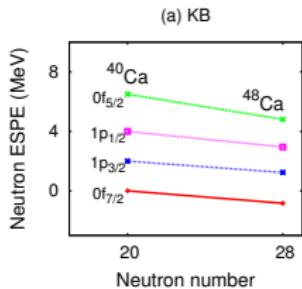
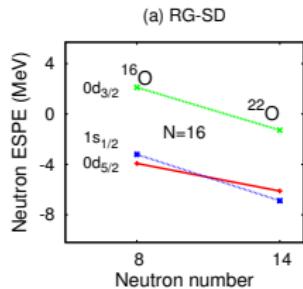
G-matrix + MBPT  
*Hjorth-Jensen et al, PR261 (1995)*

Microscopic effective interactions obtained from a *NN* potential

- They do not reproduce subshell closures  $\Rightarrow$  a poor-defined monopole term (centroids);
- Multipole terms are reasonable.
- Missing  $3N$  forces

*Poves, Zuker, PR70; PR71 (1981); Abzouzi, Caurier, Zuker, PRL66 (1991); Zuker, PRL90 (2003); Schwenk, Zuker, PRC74, 061302 (2006); Otsuka et al PRL (2010)*

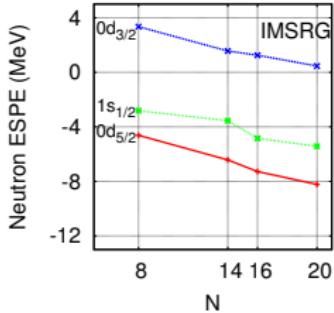
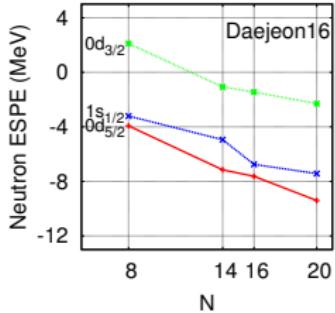
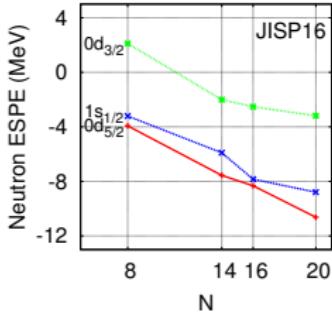
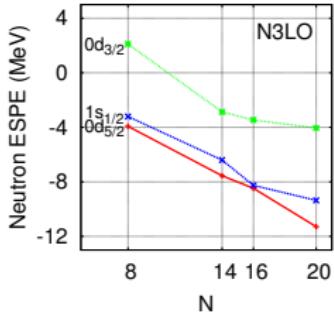
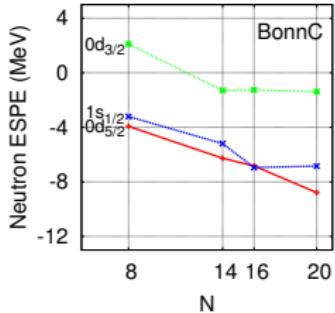
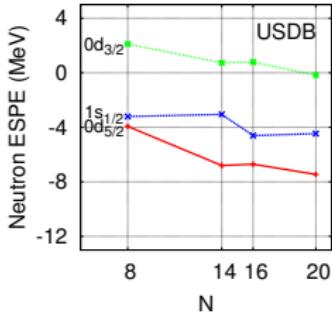
# Examples: microscopic versus phenomenological interactions



Microscopic interactions: missing  $N = 14$ ,  $N = 28$  and  $N = 50$  shell gaps  
from Smirnova, Heyde, Bally, Nowacki, Sieja, PRC86 (2012)

# Neutron ESPEs in O-isotopes

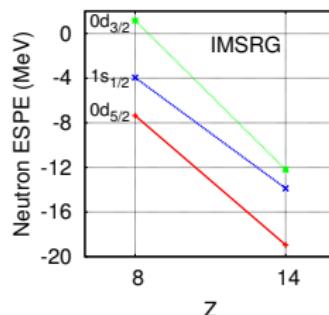
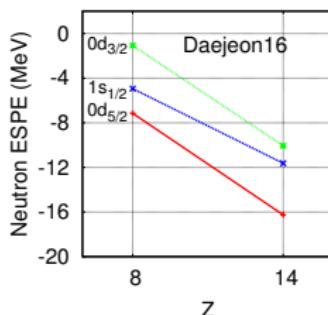
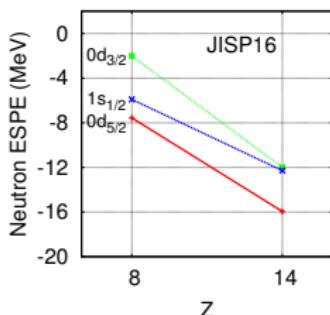
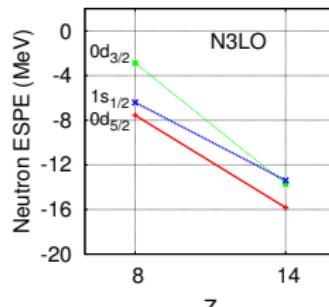
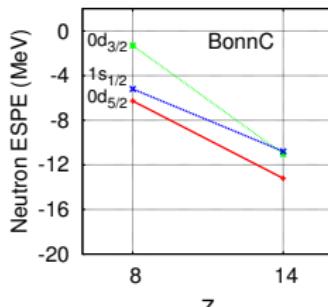
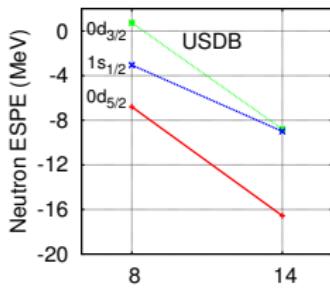
$$\tilde{\epsilon}_{\nu_i}(A) = \epsilon_{\nu_i}(A_0) + \sum_j \overline{V}_{ij}^{\nu\nu} n_{\nu_j}$$



BonnC: Hjorth-Jensen, Kuo, Osnes, PR261 (1995); IMSRG: Stroberg et al, PRL118 (2017)

# Neutron ESPEs in $N = 14$ isotones

$$\tilde{\epsilon}_{\nu_i}(A) = \epsilon_{\nu_i}(A_0) + \sum_j \overline{V}_{ij}^{\nu\pi} n_{\pi_j}$$



# Monopole-modified $V_{eff}$ from Daejeon16

## Modifications of the centroids

$$\Delta V_{d_{5/2}d_{5/2}}^{T=1} = +80 \text{ keV}$$

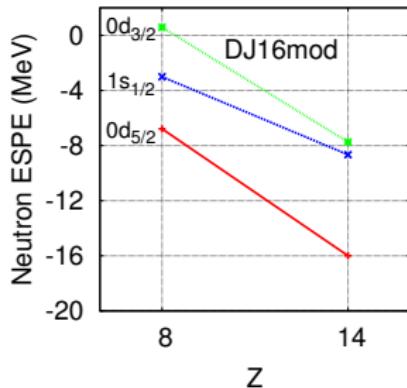
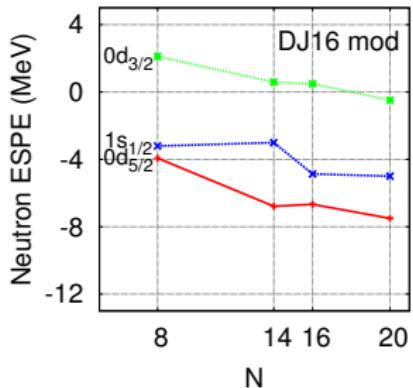
$$\Delta V_{d_{5/2}s_{1/2}}^{T=1} = +350 \text{ keV}$$

$$\Delta V_{d_{5/2}d_{3/2}}^{T=1} = +300 \text{ keV}$$

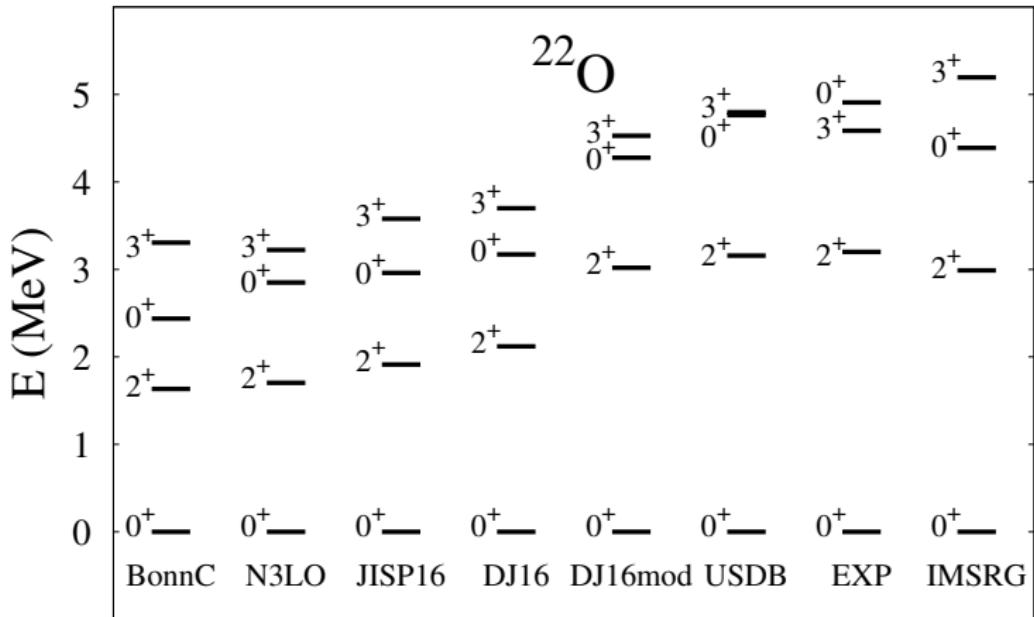
$$\Delta V_{d_{3/2}s_{1/2}}^{T=1} = +200 \text{ keV}$$

$$V_{d_{5/2}d_{5/2}}^{T=0} = -80 \text{ keV}$$

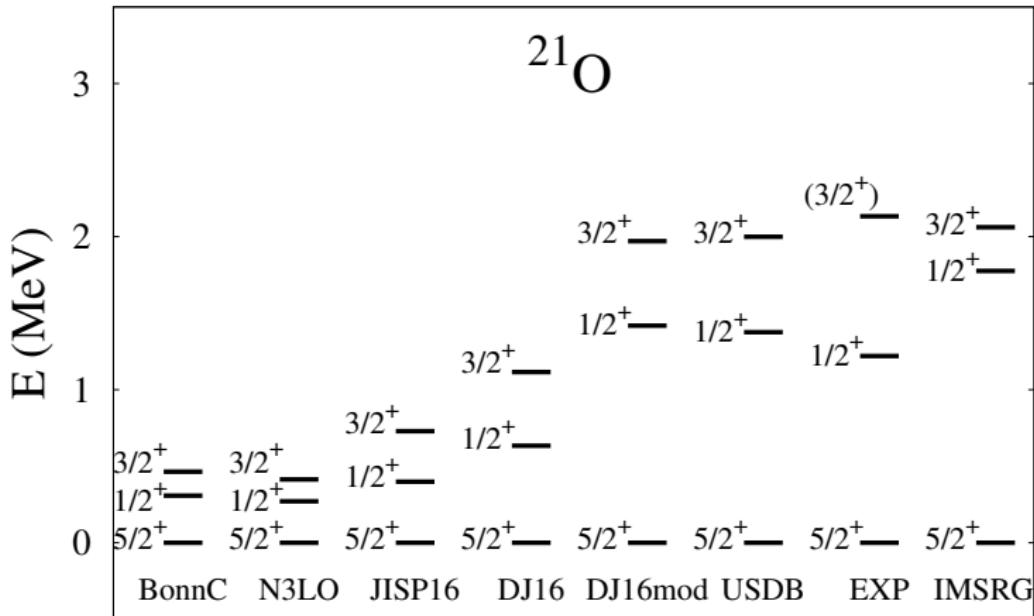
$$V_{d_{5/2}s_{1/2}}^{T=0} = +100 \text{ keV}$$



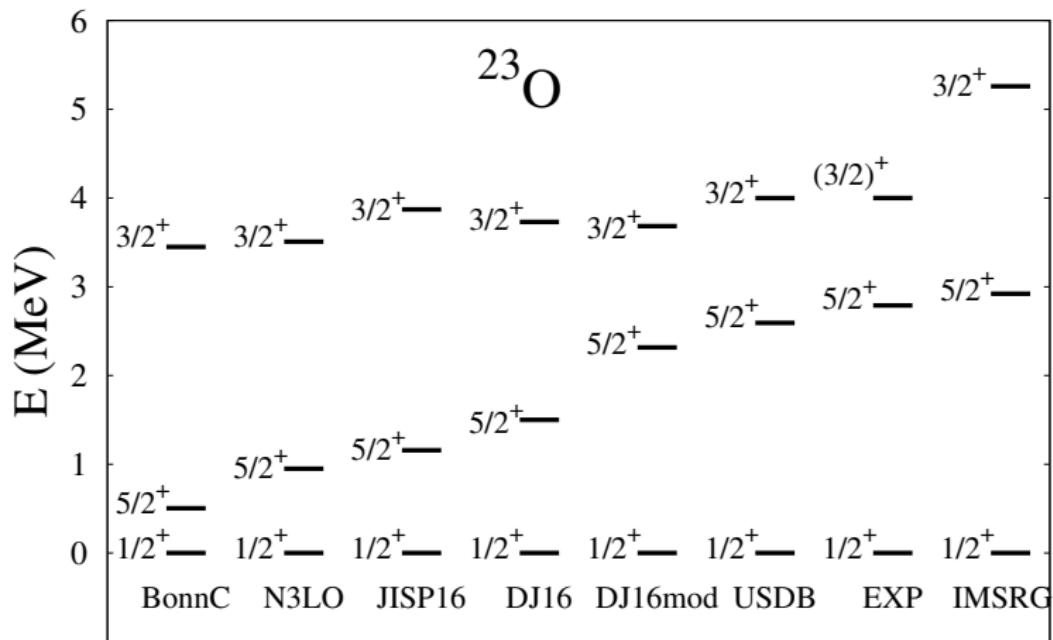
# Spectrum of $^{22}\text{O}$



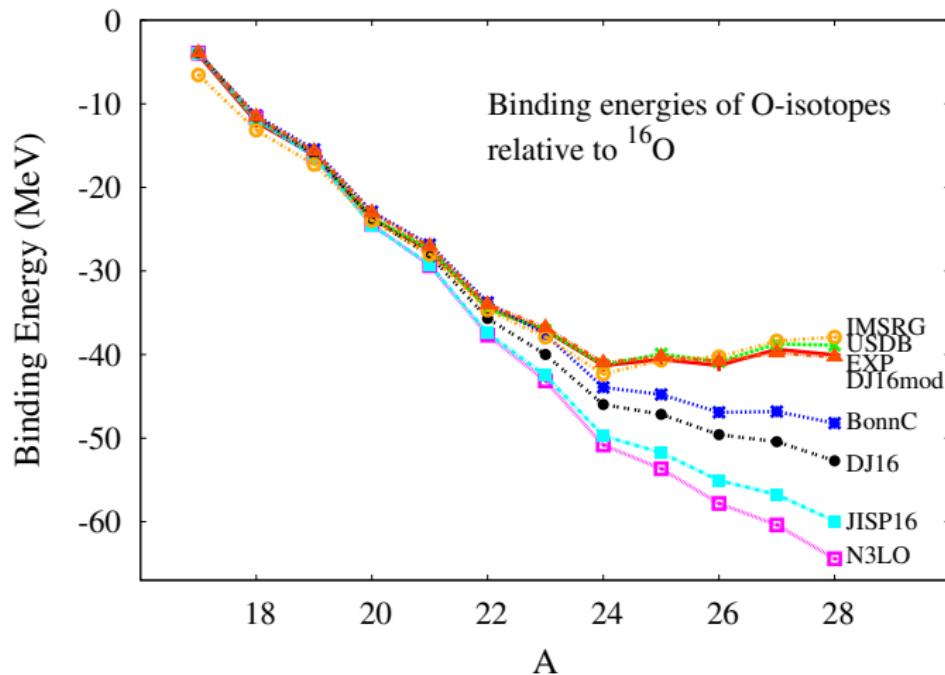
# Spectrum of $^{21}\text{O}$



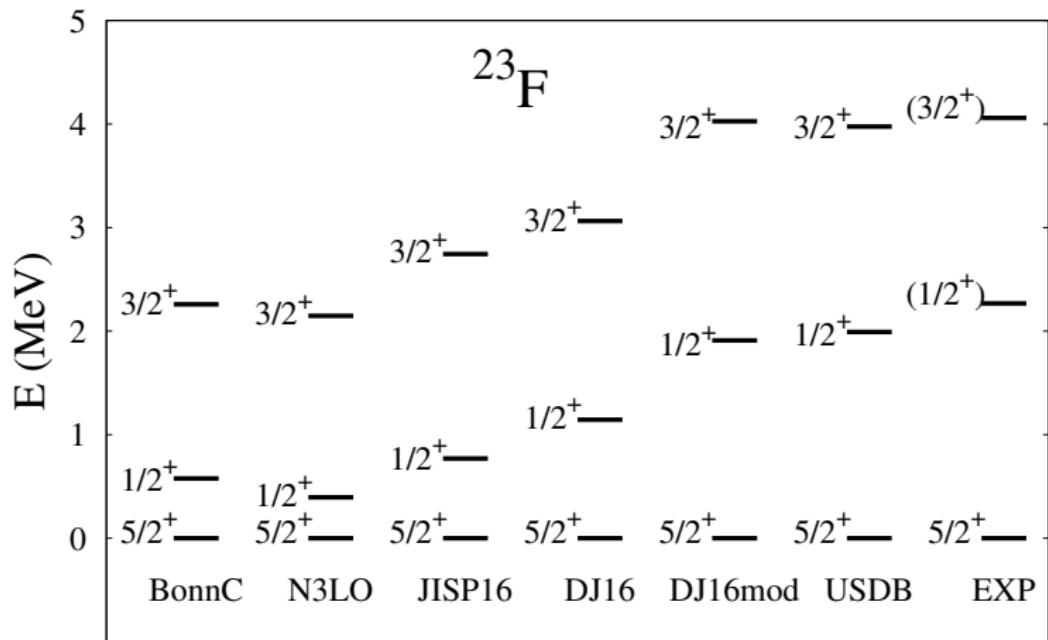
# Spectrum of $^{23}\text{O}$



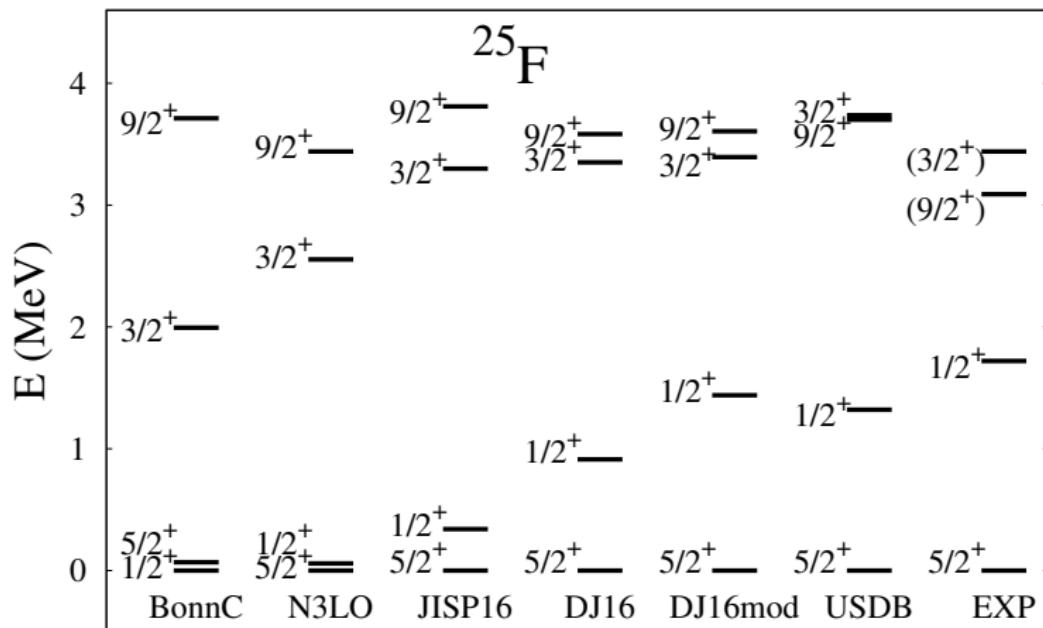
# Binding energies of O-isotopes



# Spectrum of $^{23}\text{F}$

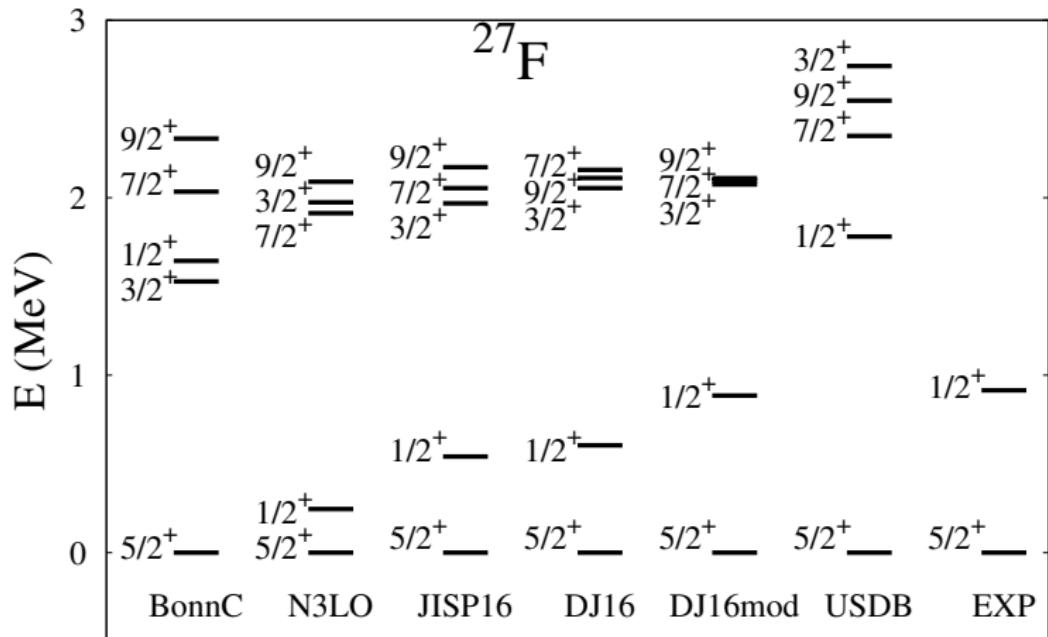


# Spectrum of $^{25}\text{F}$



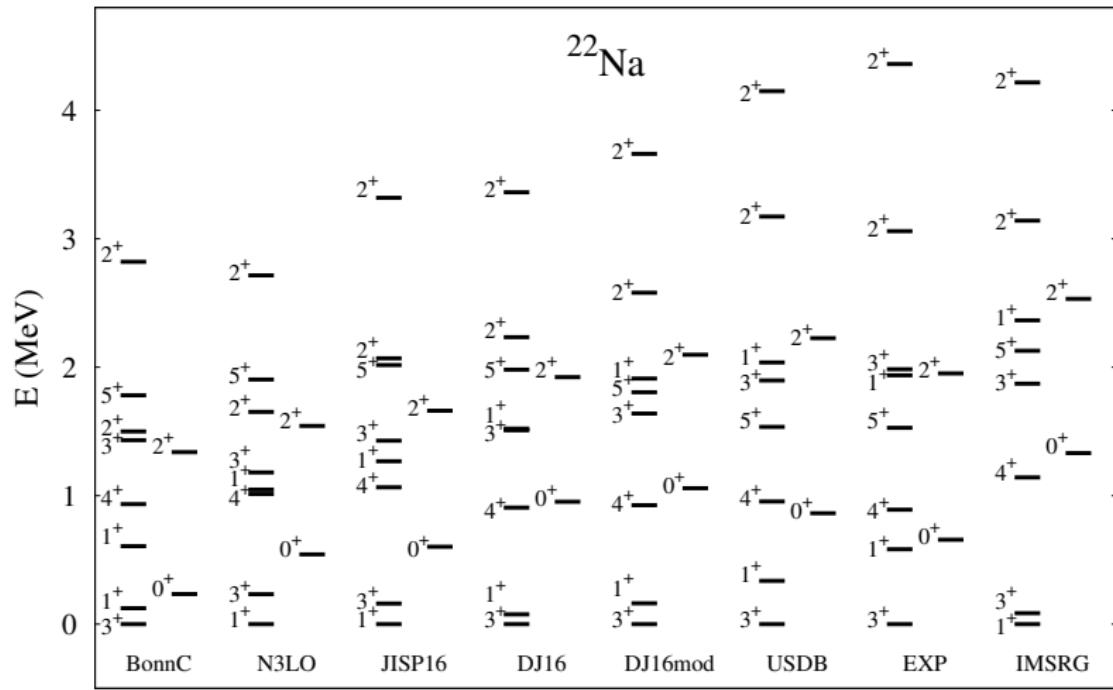
Experiment: P. Doornenbal et al, PRC95, 041301 (2017)

# Spectrum of $^{27}\text{F}$

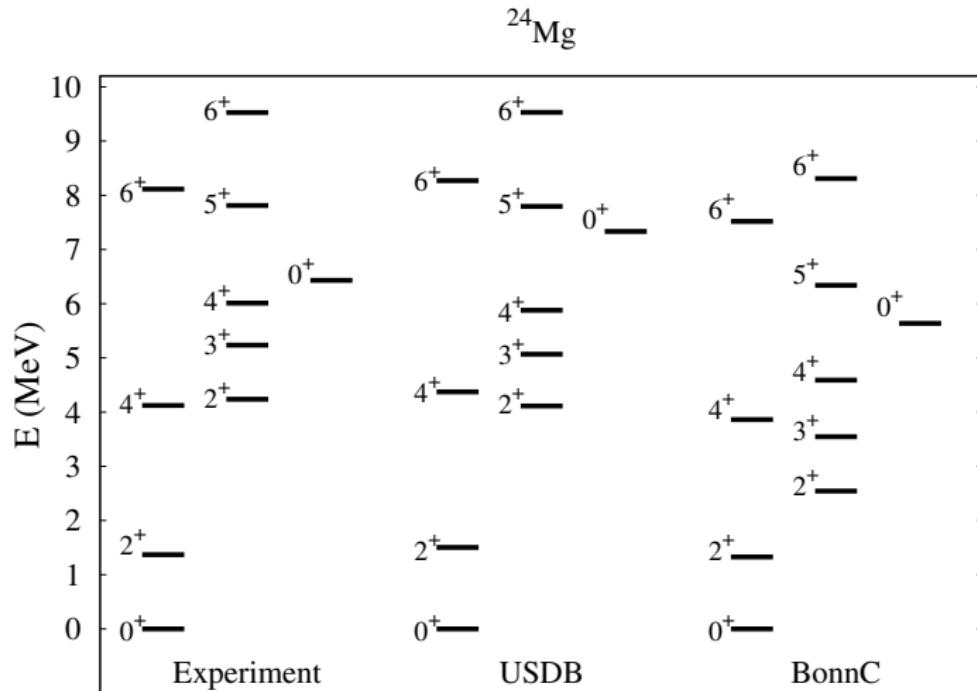


Experiment: P. Doornenbal et al, PRC95, 041301 (2017)

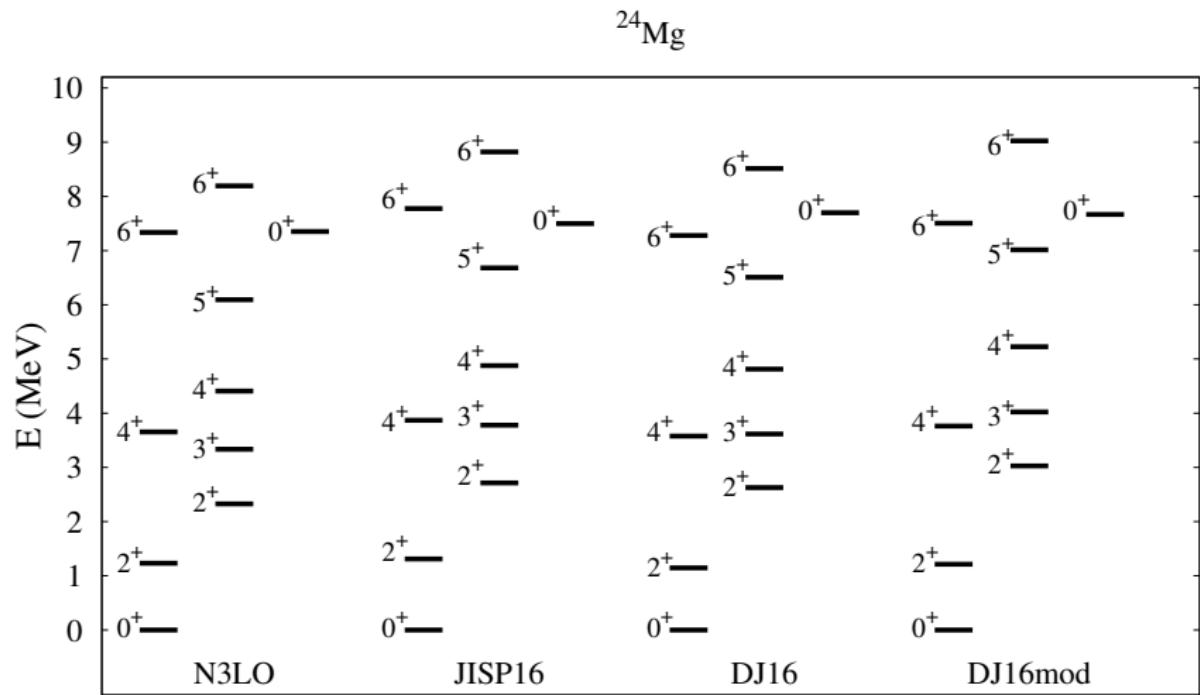
# Spectrum of $^{22}\text{Na}$



# Collective properties: $^{24}\text{Mg}$



# Collective properties: $^{24}\text{Mg}$



# $^{24}\text{Mg}$ : electromagnetic (E2) transition rates

	Exp	USDB	BonnC	N3LO	JISP16	DJ16
$^{24}\text{Mg}$						
$B(E2; 2_1^+ \rightarrow 0_1^+) (\text{e}^2\text{fm}^4)$	88(4)	95	108	107	106	104
$B(E2; 4_1^+ \rightarrow 2_1^+) (\text{e}^2\text{fm}^4)$	160(16)	124	143	140	138	138
$B(E2; 6_1^+ \rightarrow 2_1^+) (\text{e}^2\text{fm}^4)$		115	140	135	133	135
$Q(2_1^+) (\text{e fm}^2)$	-16.6(6)	-19.3	-18.3	-18.8	-19.1	-19.7

# RMS deviations (keV)

Interaction	BE(O)	$^{21-24}\text{O}$	$^{19-27}\text{F}-^{39}\text{K}$	$^{22}\text{Na}$	$^{28}\text{Si}, ^{32}\text{S}$	$^{24}\text{Mg}$
BonnC	3882	1460	795	878	1186	1116
N3LO	11621	1316	837	828	1331	1275
JISP16	9673	1151	725	595	993	939
DJ16	5960	931	506	512	1146	1096
DJ16mod	449	274	220	369	891	806
USDB	467	251	388	169	234	313
IMSRG	1177	738		497	1497	

# Spin-tensor decomposition of the two-body interaction

$S_1^{(0)} = 1, \quad S_2^{(0)} = [\vec{\sigma}_1 \times \vec{\sigma}_2]^{(0)}$	Scalar	TE, TO, SE, SO
$S_3^{(1)} = \vec{\sigma}_1 + \vec{\sigma}_2,$		LSE, LSO
$S_4^{(1)} = [\vec{\sigma}_1 \times \vec{\sigma}_2]^{(1)}$	Vector	ALS
$S_5^{(1)} = \vec{\sigma}_1 - \vec{\sigma}_2$		ALS
$S_6^{(2)} = [\vec{\sigma}_1 \times \vec{\sigma}_2]^{(2)}$	Tensor	TNE, TNO

Any two-nucleon interaction  $V$  can be decomposed as

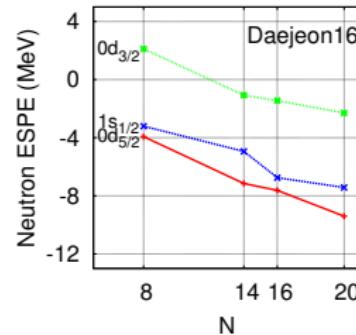
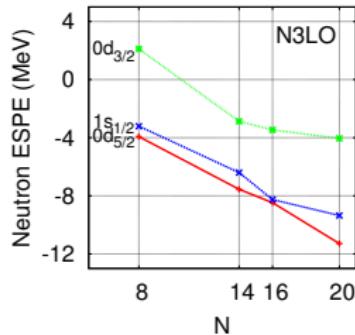
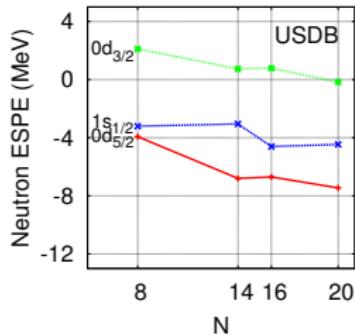
$$V = \sum_{k=0,1,2} \left( S^{(k)} \cdot Q^{(k)} \right) = \sum_{k=0,1,2} V^{(k)}$$

$$\langle (ab : LS, JMTM_T | V^{(k)} | cd : L'S', JMTM_T \rangle =$$

$$(2k+1)(-1)^J \left\{ \begin{array}{ccc} L & S & J \\ S' & L' & k \end{array} \right\} \sum_{J'} (-1)^{J'} (2J'+1) \left\{ \begin{array}{ccc} L & S & J' \\ S' & L' & k \end{array} \right\}$$

$$\times \langle ab : LS, J'MTM_T | V | cd : L'S', J'MTM_T \rangle$$

# Microscopic versus adjusted interaction: sd-shell



Evolution of  $N = 14$  shell gap from  $^{16}\text{O}$  to  $^{22}\text{O}$

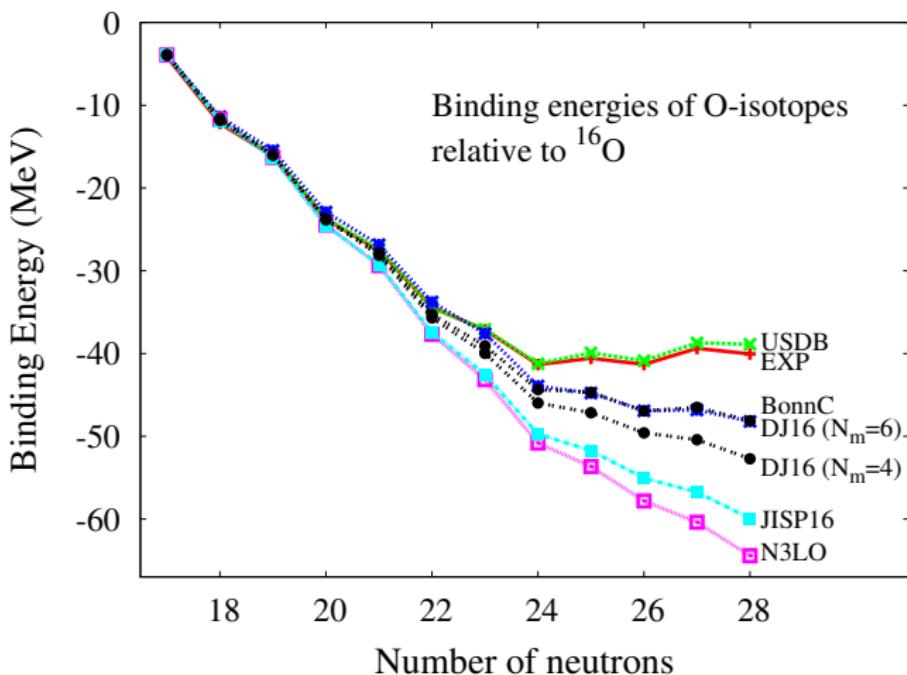
$\Delta E$	USDB (MeV)	N3LO (MeV)	DJ16 (MeV)
Total	<b>3.03</b>	<b>0.43</b>	<b>1.49</b>
Central	<b>1.89</b>	<b>0.65</b>	<b>0.90</b>
TO	1.31	0.24	0.76
SE	0.56	0.41	0.14
Vector	<b>1.26</b>	<b>0.33</b>	<b>0.78</b>
LS	0.27	0.07	0.24
ALS	0.99	0.26	0.53
Tensor	<b>-0.12</b>	<b>-0.55</b>	<b>-0.18</b>

# Evolution of $N = 14$ shell gap from $^{16}\text{O}$ to $^{22}\text{O}$

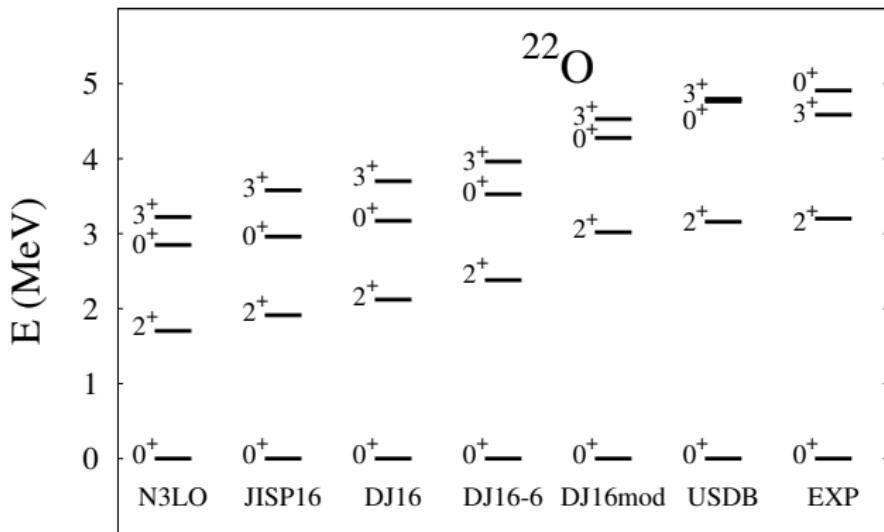
$T = 1$  centroids and their difference (in MeV)

	USDB			N3LO			Daejeon16		
	$d_{\frac{5}{2}} d_{\frac{5}{2}}$	$s_{\frac{1}{2}} d_{\frac{5}{2}}$	$\Delta V$	$d_{\frac{5}{2}} d_{\frac{5}{2}}$	$s_{\frac{1}{2}} d_{\frac{5}{2}}$	$\Delta V$	$d_{\frac{5}{2}} d_{\frac{5}{2}}$	$s_{\frac{1}{2}} d_{\frac{5}{2}}$	$\Delta V$
Tot	<b>-0.63</b>	<b>0.01</b>	<b>0.64</b>	<b>-0.79</b>	<b>-0.60</b>	<b>0.19</b>	<b>-0.71</b>	<b>-0.34</b>	<b>0.37</b>
Cen	<b>-0.39</b>	<b>0.01</b>	<b>0.40</b>	<b>-0.77</b>	<b>-0.54</b>	<b>0.23</b>	<b>-0.56</b>	<b>-0.31</b>	<b>0.24</b>
TO	0.39	0.57	0.18	0.10	0.13	0.03	0.21	0.31	0.11
SE	-0.78	-0.57	0.21	-0.76	-0.63	0.14	-0.87	-0.67	0.20
Vec	<b>-0.26</b>	<b>0.00</b>	<b>0.26</b>	<b>-0.14</b>	<b>-0.06</b>	<b>0.08</b>	<b>-0.19</b>	<b>-0.02</b>	<b>0.17</b>
LS	-0.06	-0.00	0.06	-0.06	-0.04	0.02	-0.02	0.03	0.05
ALS	-0.20	0.00	0.20	-0.09	-0.03	0.06	-0.17	-0.05	0.12
Ten	<b>0.03</b>	<b>0.00</b>	<b>-0.03</b>	<b>0.12</b>	<b>0.00</b>	<b>-0.12</b>	<b>0.04</b>	<b>0.00</b>	<b>-0.04</b>

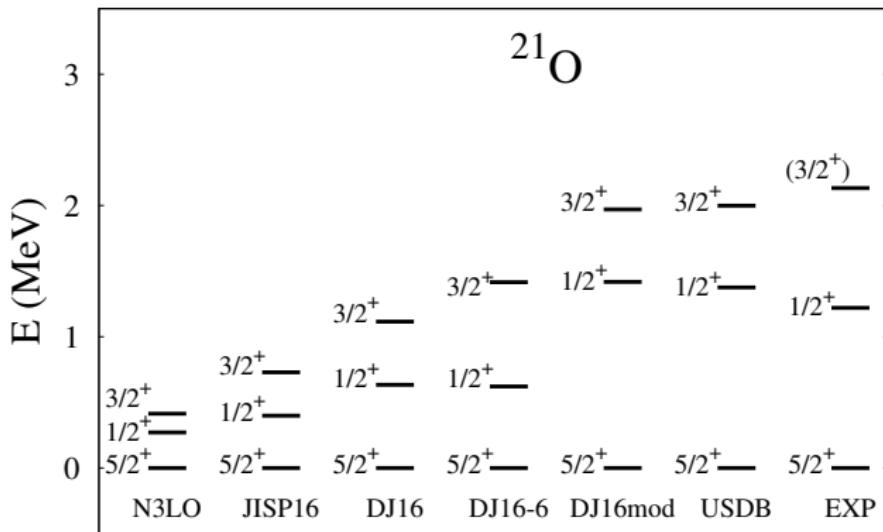
# $N_{\max} = 6$ : preliminary results



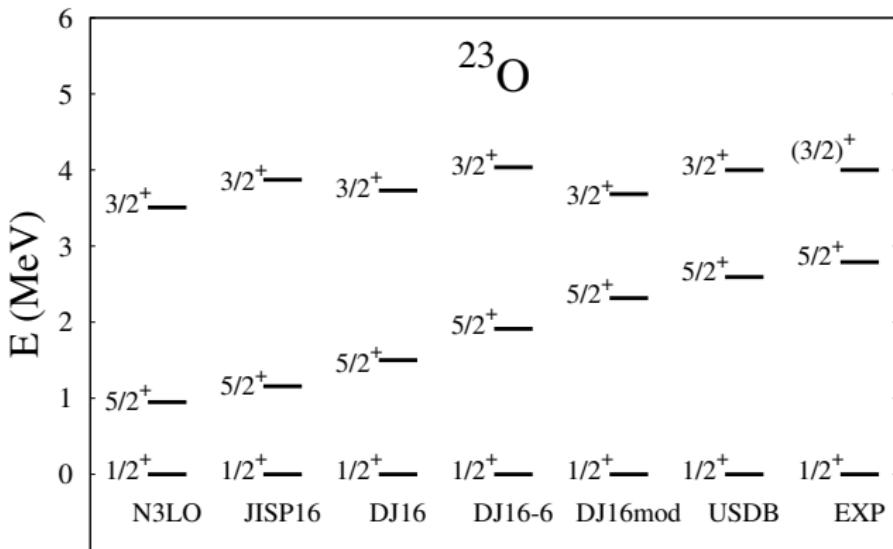
# $N_{\max} = 6$ : preliminary results



# $N_{\max} = 6$ : preliminary results



# $N_{\max} = 6$ : preliminary results



# Charge-dependent $NN$ forces

Classification of two-nucleon forces (*Henley, Miller, 1979*):

- Class I:  $V_I = \alpha + \beta \hat{\mathbf{t}}(1) \cdot \hat{\mathbf{t}}(2)$
- Class II:  $V_{II} = \alpha \hat{\mathbf{t}}_3(1) \hat{\mathbf{t}}_3(2)$
- Class III:  $V_{III} = \alpha (\hat{\mathbf{t}}_3(1) + \hat{\mathbf{t}}_3(2))$
- Class IV:  $V_{IV} = \alpha (\hat{\mathbf{t}}_3(1) - \hat{\mathbf{t}}_3(2)) + \beta [\hat{\mathbf{t}}(1) \times \hat{\mathbf{t}}(2)]_3$

$$V_I > V_{II} > V_{III} > V_{IV}$$

## Theory of ISB $NN$ forces

- Meson-exchange models

*Miller et al, Phys. Rep. 194, 1 (1990); Machleidt, PRC64, 024001 (2001), ...*

- $\chi$  EFT (ISB  $NN$  and  $3N$  forces)

*Van Kolck, Friar, Epelbaum, Meissner, ...*

*see for review Epelbaum et al, RMP81, 1773 (2009)*

## Light nuclei

GFMC with CD interactions (AV18 + IL7) for  $^3\text{H}$  -  $^3\text{He}$ ,  $A = 7, 8$ , isospin-mixing in  $^8\text{Be}$   
*e.g., Wiringa, Pastore, Pieper, Miller, PRC88, 044333 (2013)*

# Isospin-nonconserving shell model

- We start with an isospin-symmetry invariant shell-model Hamiltonian

$$\hat{H}\Psi_{TT_z} \equiv (\hat{H}_0 + \hat{V})\Psi_{TT_z} = E_T\Psi_{TT_z}, \quad \Psi_{TT_z} = \sum_k a_{Tk}\Phi_{TT_z k}$$

- We consider an isospin-symmetry non-conserving term

$$\hat{V}_{INC} = \underbrace{\lambda_C \hat{V}_C}_{Coulomb} + \underbrace{\lambda_1 \hat{V}^{(1)}}_{CSB} + \underbrace{\lambda_2 \hat{V}^{(2)}}_{CIB} + \underbrace{\hat{H}_0^{IV}}_{\sum_\alpha (\varepsilon_\alpha^p - \varepsilon_\alpha^n)}$$

- Within perturbation theory:

$$\langle \Psi_{TT_z} | \hat{V}_{INC} | \Psi_{TT_z} \rangle = E^{(0)}(\alpha, T) + E^{(1)}(\alpha, T)T_z + E^{(2)}(\alpha, T) [3T_z^2 - T(T+1)]$$

- Wigner's Isobaric Mass Multiplet Equation (IMME):

$$M(\alpha, T, T_z) = a(\alpha, T) + b(\alpha, T)T_z + c(\alpha, T)T_z^2,$$

- Diagonalization of the INC Hamiltonian  $\hat{H}_{INC} = \hat{H} + \hat{V}_{INC}$

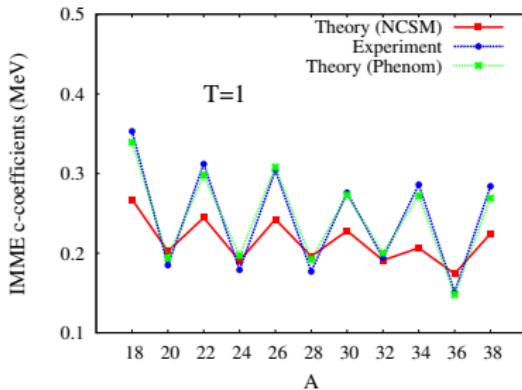
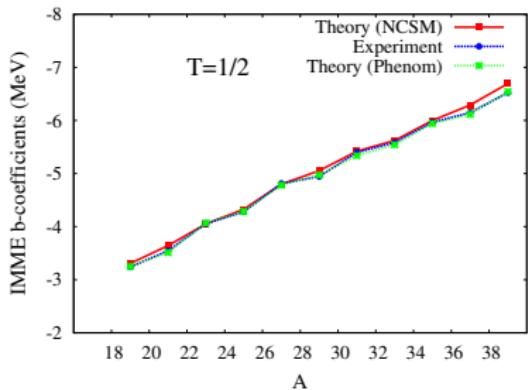
$$\hat{H}_{INC}\Psi = E\Psi$$

Lam, Smirnova, Caurier, PRC87 (2013).

# *Ab-initio* effective interaction from the NCSM

Present results obtained from the NCSM

- Daejeon16 *NN* potential (charge-independent)
- $^{18}\text{O}$ ,  $^{18}\text{F}$ ,  $^{18}\text{Ne} \Rightarrow \text{pp}, \text{nn}, \text{pn}$  ( $T = 0, 1$ ) TBMEs in *sd* shell (charge-dependent)



- *b*-coefficients: rms (NCSM)  $\approx 86$  keV, rms (phen)  $\approx 30$  keV
- *c*-coefficients: rms (NCSM)  $\approx 51$  keV, rms (phen)  $\approx 11$  keV

*Smirnova, Barrett, Kim et al, in preparation*

*sdpf*: Holt, Menendez, Schwenk, PRL110 (2013);

*pf-shell*: Ormand, Brown, Hjorth-Jensen, PRC96 (2017)

# Summary and Perspectives

- *Ab-initio* effective *sd*-shell interactions from the **NCSM**: promising results.
- $NN$  potential versus  $NN + 3N$
- Monopole part (Daejeon16): some deficiencies in  $T = 1$  part (improving at  $N_{max} = 6$ ), although robust proton-neutron centroids. Small monopole modifications allow to visibly improve the spectroscopy of O and F-isotopes
- Multipole part (Daejeon16): still to explore
- Charge-dependence: robust agreement with the data regarding Coulomb. Need for CSB and CIB  $NN$  terms.
- Heavier nuclei (*pf*-shell): challenging
- Possibility to explore  $1\hbar\omega$ , ....