

Few nucleons near unitarity

Sebastian König

GDR RESANET GT3 meeting

“Quelles sont les nouvelles frontières dans la description microscopique des noyaux?”

Saclay, France

November 13, 2018

SK, H.W. Griebhammer, H.-W. Hammer, U. van Kolck, PRL **118** 202501 (2017)

SK, J Phys. G **44** 064007 (2017)

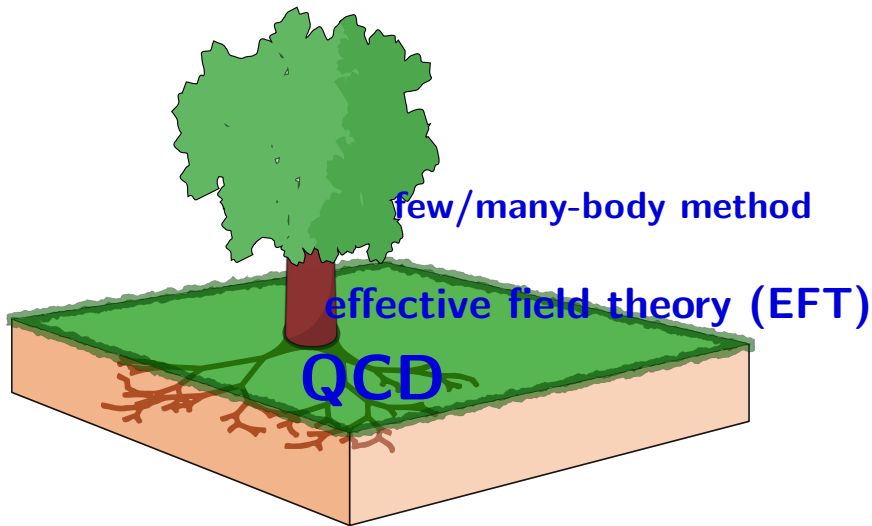
P. Klos, SK, J. Lynn, H.-W. Hammer, and A. Schwenk, PRC **98** 034004 (2018)



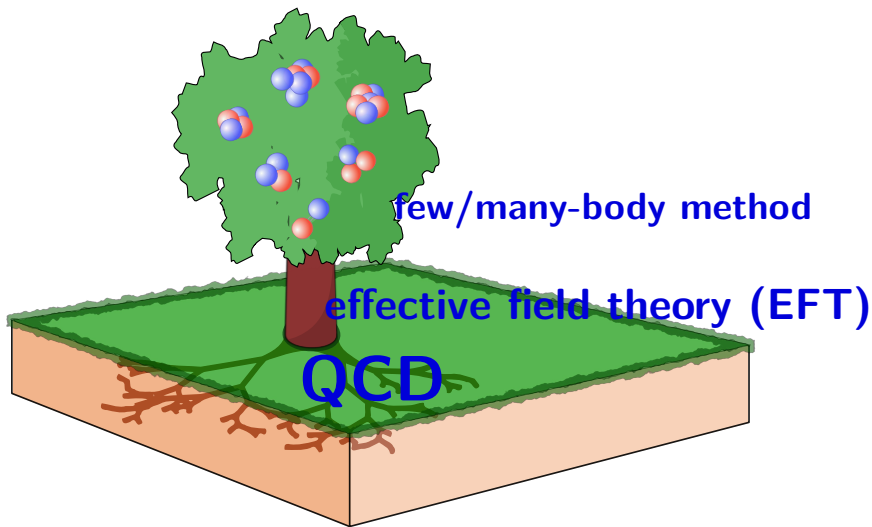
TECHNISCHE
UNIVERSITÄT
DARMSTADT



Nuclear paradise

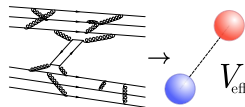


Nuclear paradise



Nuclear paradise?

- **QCD** = underlying theory of strong interaction
- **EFT** = effective description in terms of hadrons

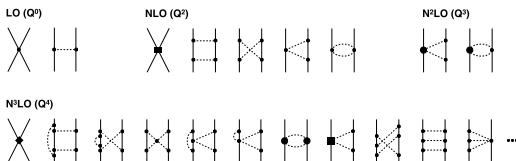


separation of scales + symmetries

↪ **systematic expansion of observables**

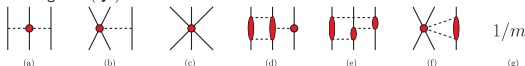
- **chiral EFT**: nucleons + pions, expand in $(Q \sim m_\pi)/M_{\text{QCD}}$

Weinberg (90); Rho (91); Ordoñez+van Kolck (92); van Kolck (93); Epelbaum *et al.* (98); Entem+Machleidt (03); ...



Epelbaum *et al.*, EPJA 51 53 (2015)

starting at $\mathcal{O}(Q^3)$:



Hebeler *et al.*, PRC 91 044001 (2015)

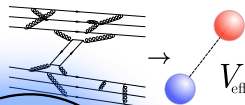
- **LO triton not very good!** e.g., ~ 4 MeV (LO) vs. 8.5 MeV (exp.)

Nogga *et al.*, PRC 72 054006 (2005)

Nuclear paradise?

- **QCD** = underlying theory of strong interaction
- **EFT** = effective description in terms of hadrons

separation of scales + symmetries

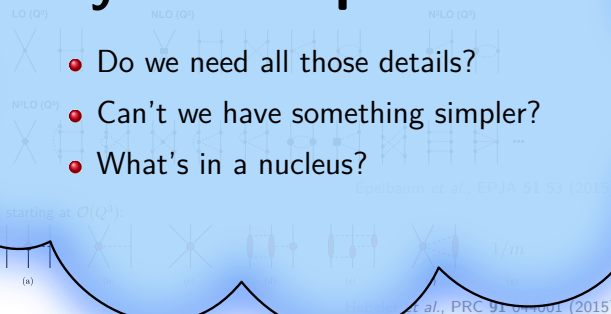


↪ systematic expansion of observables

- **chiral EFT**: nucleons + pions, expand in $(Q \sim m_\pi)/M_{\text{QCD}}$
Weinberg (1979); Doornik et al. (2000); Entem + Machleidt (03); ...

Why so complicated?

- Do we need all those details?
- Can't we have something simpler?
- What's in a nucleus?

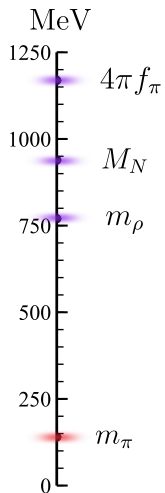


- **LO triton not very good!** e.g., ~ 4 MeV (LO) vs. 8.5 MeV (exp.)

Nogga et al., PRC 72 054006 (2005)

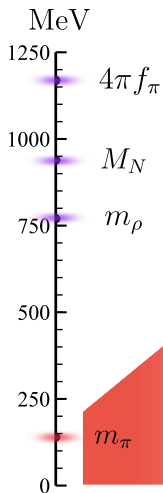
Nuclear scales

chiral expansion

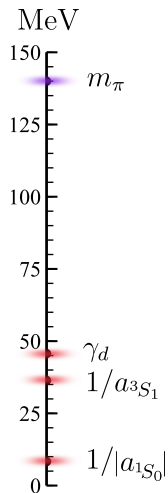


Nuclear scales

chiral expansion



pionless expansion



Universal trimers and tetramers

- **Efimov effect:** infinite tower of three-body states in unitarity limit

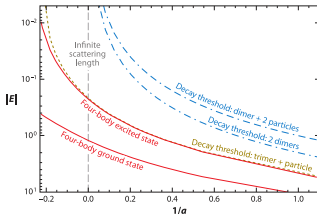
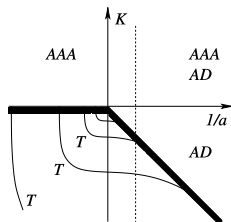
Efimov, PLB **33** 563 (1970)

- each state comes with two **associated tetramers**

Hammer+Platter, EPJA **32** 13 (2007)

- plus higher-body cluster states beyond that

von Stecher, JPB **43** 101002 (2010); Gattobigio *et al.*, PRA **84** 052503 (2011)



Hammer+Platter, Ann. Rev. Nucl. Part. Sci. **60** 207 (2010)

Braaten+Hammer, Phys. Rept. **428** 259 (2006)

- **at unitarity:** $B_4/B_3 \simeq 4.611$, $B_{4^*}/B_3 \simeq 1.002$

Deltuva, PRA **82** 040701 (2010)

Universal trimers and tetramers

- **Efimov effect:** infinite tower of three-body states in unitarity limit

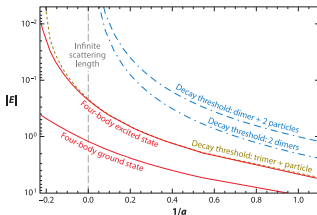
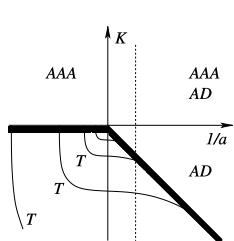
Efimov, PLB **33** 563 (1970)

- each state comes with two **associated tetramers**

Hammer+Platter, EPJA **32** 13 (2007)

- plus higher-body cluster states beyond that

von Stecher, JPB **43** 101002 (2010); Gattobigio et al., PRA **84** 052503 (2011)



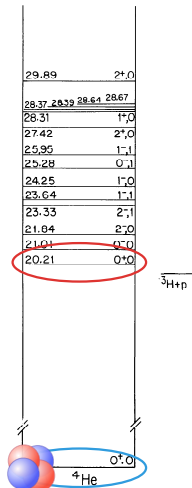
Hammer+Platter, Ann. Rev. Nucl. Part. Sci. **60** 207 (2010)

Braaten+Hammer, Phys. Rept. **428** 259 (2006)

- **at unitarity:** $B_4/B_3 \simeq 4.611$, $B_{4^*}/B_3 \simeq 1.002$

Deltuva, PRA **82** 040701 (2010)

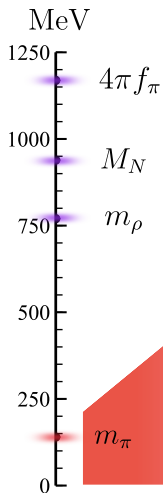
- **in ^4He :** ground state at $B_\alpha/B_H \simeq 3.66$,
resonance at $B_{\alpha^*}/B_H \simeq 1.05$ (where $B_H = 7.72$)



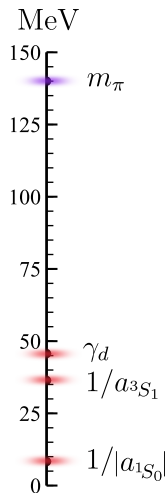
TUNL nuclear data

Nuclear scales

chiral expansion

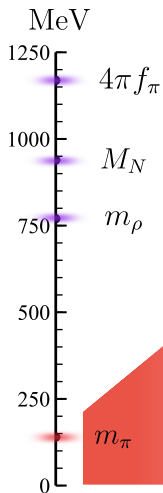


pionless expansion

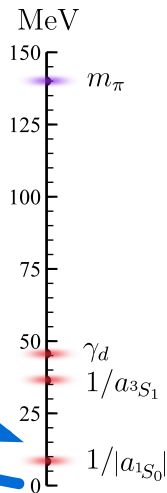


Nuclear scales

chiral expansion



pionless expansion



perturbation

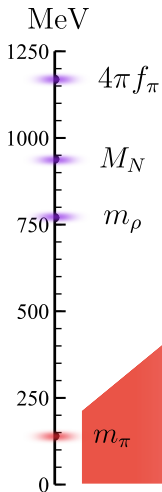


unitarity limit: $1/a = 0$

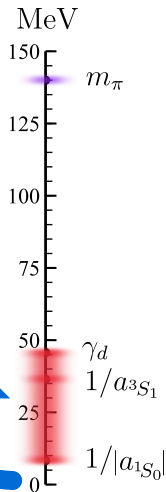
SK et al. JPG 43 055106 (2016)

Nuclear scales

chiral expansion



pionless expansion



perturbation

unitarity limit: $1/a = 0$

SK et al. PRL 118 202501 (2017)

Capture **gross features at leading order**, build up the rest as **perturbative “fine structure!”**

- shift focus away from two-body details
- zero-energy deuteron at LO and NLO
- **physics in universality regime**
 - **discrete scale invariance** as guiding principle (Efimov effect!)
 - near equivalence to **bosonic clusters**
 - exact $SU(4)_W$ **symmetry** at LO

Wigner, PR **51** 106 (1937); Mehen *et al.*, PRL **83** 931 (1999); Bedaque *et al.*, NPA **676** 357 (2000)
Vanasse+Phillips, FB Syst. **58** 26 (2017)
cf. Kievsky+Gattobigio, EPJ Web Conf. **113** 03001 (2016)

Capture **gross features at leading order**, build up the rest as **perturbative “fine structure!”**

- shift focus away from two-body details
- zero-energy deuteron at LO and NLO
- **physics in universality regime**
 - **discrete scale invariance** as guiding principle (Efimov effect!)
 - near equivalence to **bosonic clusters**
 - exact $SU(4)_W$ symmetry at LO

Wigner, PR **51** 106 (1937); Mehen *et al.*, PRL **83** 931 (1999); Bedaque *et al.*, NPA **676** 357 (2000)
Vanasse+Phillips, FB Syst. **58** 26 (2017)
cf. Kievsky+Gattobigio, EPJ Web Conf. **113** 03001 (2016)

Conjecture

Nuclear sweet spot

$$1/a_{s,t} < Q_A < 1/R \sim m_\pi$$

$$Q_A \sim \sqrt{2M_N B_A/A}$$

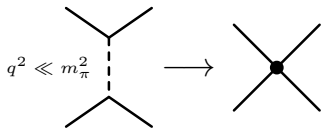
A	2	3	4	...	56
$Q_A R$	0.3	0.5	0.8	...	0.9

↪ iron not much different from ${}^4\text{He}$
(actual exp. parameter maybe smaller)

van Kolck (2018)

The unitarity expansion

- ① describe strong force with **contact interactions** (cutoff $\Lambda \rightsquigarrow$ smearing)

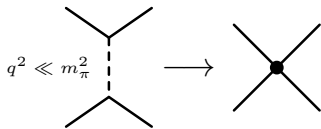


$$C_0 = \underbrace{C_0^{(0)}}_{\text{leading order (LO)}} + C_0^{(1)} + \dots$$

- ② fix $C_0^{(0)}$ to **get** $a = \infty$ for both NN S-wave channels ($s=^1S_0$, $t=^3S_1$)

The unitarity expansion

- ① describe strong force with **contact interactions** (cutoff $\Lambda \rightsquigarrow$ smearing)



$$C_0 = \underbrace{C_0^{(0)}}_{\text{leading order (LO)}} + C_0^{(1)} + \dots$$

- ② fix $C_0^{(0)}$ to **get** $a = \infty$ for both NN S-wave channels ($s=^1S_0$, $t=^3S_1$)
- ③ fix Efimov spectrum to physical $B(^3\text{H}) \leftrightarrow$ **LO three-body contact**

Bedaque et al., NPA 676 357 (2000)

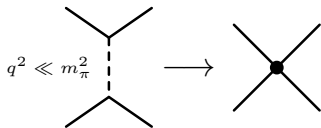


use triton as “anchor” at each order



The unitarity expansion

- 1 describe strong force with **contact interactions** (cutoff $\Lambda \rightsquigarrow$ smearing)



$$C_0 = \underbrace{C_0^{(0)}}_{\text{leading order (LO)}} + C_0^{(1)} + \dots$$

- 2 fix $C_0^{(0)}$ to **get** $a = \infty$ for both NN S-wave channels ($s=^1S_0$, $t=^3S_1$)

- 3 fix Efimov spectrum to physical $B(^3\text{H}) \leftrightarrow$ **LO three-body contact**

Bedaque *et al.*, NPA **676** 357 (2000)



use triton as “anchor” at each order



- 4 include in perturbation theory:

- finite a , Coulomb
- range effects
- higher-order corrections

- amplitudes $T = T^{(0)} + T^{(1)} + \dots$
- binding energies $B = B^{(0)} + B^{(1)} + \dots$

The unitarity expansion

- 1 describe strong force with **contact interactions** (cutoff $\Lambda \rightsquigarrow$ smearing)

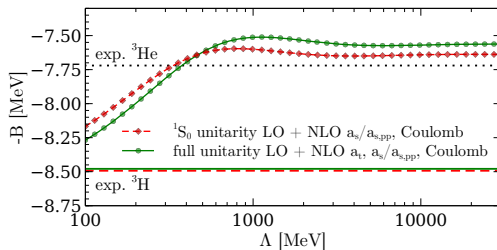
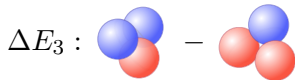
- 2 fix $C_0^{(0)}$ to match experimental data for 1S_0 (e.g. $s=1, t=^3S_1$)
 - 3 fix $C_0^{(1)}$ to match experimental data for 1S_0 (e.g. $s=1, t=^3S_1$)
- Leading order has a single parameter, all the rest is a perturbation!**

- 4 include in perturbation theory:

- finite a , Coulomb
- range effects
- higher-order corrections
- **binding energies** $B = B^{(0)} + B^{(1)} + \dots$

Trinucleon energy difference

- at LO ${}^3\text{H}$ and ${}^3\text{He}$ are degenerate (exact isospin symmetry)
- Coulomb correction enters together with $1/a_{s,pp}$ at NLO



	LO	NLO	exp.
${}^3\text{H}$	8.48	8.48	8.48
${}^3\text{He}$	8.48	7.6(2)	7.72

Range corrections

- **unitarity** and **standard pionless** expansions **paired**
- \rightsquigarrow range corrections enter at NLO
- however, treat $r_{s,np} = 2.73 \text{ fm} \neq r_{s,pp} = 2.79 \text{ fm}$ as higher order

SK et al. JPG 43 055106 (2016)

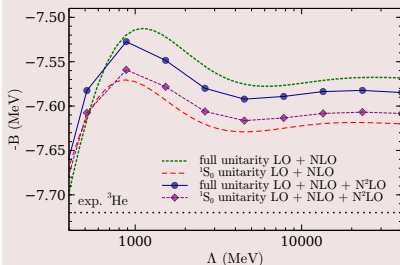
\hookrightarrow range corrections cancel at NLO in ΔE_3

Trinucleon energy difference

Various contributions at N^2LO ...

- 1 quadratic scattering-length corrections
- 2 two-photon exchange
- 3 quadratic range corrections, isospin-breaking: $r_{s,pp} \neq r_{s,np}$
- 4 mixed Coulomb and range corrections!

Zero-range calculation at N^2LO



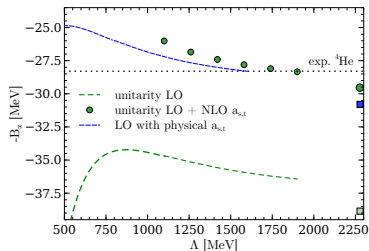
SK, J Phys. G **44** 064007 (2017)

- convergence of unitarity expansion ✓
- Coulomb perturbative ✓
- cutoff stability \rightarrow renormalization ✓
- **note:** don't expect convergence towards experiment here

Four nucleons

- unitarity expansion converges well in three-nucleon sector ✓
- **further test:** ${}^4\text{He}$, with $Q_4 \sim 115$ MeV
- good standard pionless LO description established previously

Platter *et al.*, PLB **607** 254 (2005); *cf.* also Platter, PhD thesis (2005)



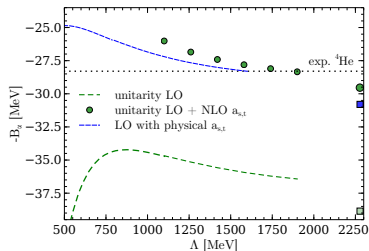
incomplete NLO
(finite- a corr. only) remarkably
close to LO with physical a

	LO	NLO	exp.
${}^3\text{H}$	8.48	8.48	8.48
${}^4\text{He}$	39(12)	30(9)	28.30

Four nucleons

- unitarity expansion converges well in three-nucleon sector ✓
- **further test:** ${}^4\text{He}$, with $Q_4 \sim 115$ MeV
- good standard pionless LO description established previously

Platter *et al.*, PLB **607** 254 (2005); *cf.* also Platter, PhD thesis (2005)

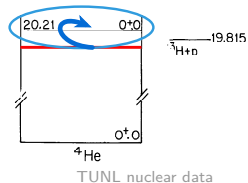


incomplete NLO
(finite- a corr. only) remarkably
close to LO with physical a

	LO	NLO	exp.
${}^3\text{H}$	8.48	8.48	8.48
${}^4\text{He}$	39(12)	30(9)	28.30

${}^4\text{He}$ monopole resonance

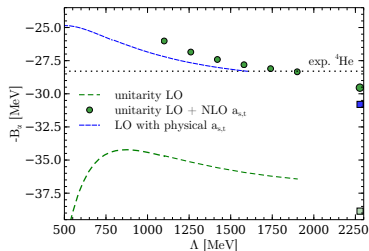
- ${}^4\text{He}$ resonance state 0.3 MeV above ${}^3\text{H} + p$ threshold
- just below threshold at unitarity LO
- boson calculations with nuclear scales
 \rightsquigarrow shift by about 0.2 – 0.5 MeV



Four nucleons

- unitarity expansion converges well in three-nucleon sector ✓
- **further test:** ^4He , with $Q_4 \sim 115 \text{ MeV}$
- good standard pionless LO description established previously

Platter *et al.*, PLB **607** 254 (2005); *cf.* also Platter, PhD thesis (2005)



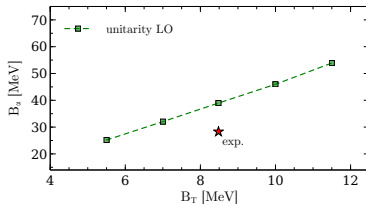
incomplete NLO
(finite- a corr. only) remarkably
close to LO with physical a

	LO	NLO	exp.
^3H	8.48	8.48	8.48
^4He	39(12)	30(9)	28.30

Tjon line

$$\text{4 nucleons} = f(\text{3 nucleons})$$

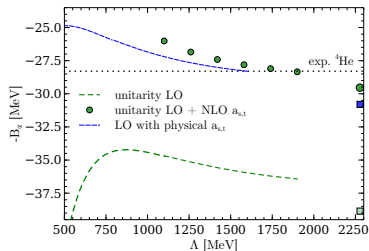
Tjon, PLB **56** 217 (1975)



Four nucleons

- unitarity expansion converges well in three-nucleon sector ✓
- **further test:** ${}^4\text{He}$, with $Q_4 \sim 115$ MeV
- good standard pionless LO description established previously

Platter *et al.*, PLB **607** 254 (2005); *cf.* also Platter, PhD thesis (2005)



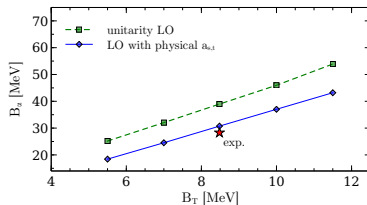
incomplete NLO
(finite- a corr. only) remarkably
close to LO with physical a

	LO	NLO	exp.
${}^3\text{H}$	8.48	8.48	8.48
${}^4\text{He}$	39(12)	30(9)	28.30

Tjon line

$$\text{Diagram of 4 nucleons} = f(\text{Diagram of 3 nucleons})$$

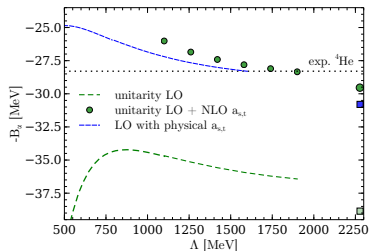
Tjon, PLB **56** 217 (1975)



Four nucleons

- unitarity expansion converges well in three-nucleon sector ✓
- **further test:** ^4He , with $Q_4 \sim 115 \text{ MeV}$
- good standard pionless LO description established previously

Platter *et al.*, PLB **607** 254 (2005); *cf.* also Platter, PhD thesis (2005)



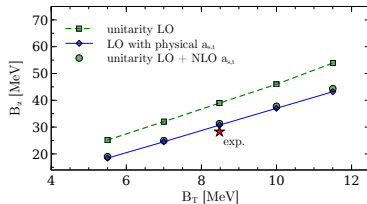
incomplete NLO
(finite- a corr. only) remarkably
close to LO with physical a

	LO	NLO	exp.
^3H	8.48	8.48	8.48
^4He	39(12)	30(9)	28.30

Tjon line

$$\text{Diagram of 4 nucleons} = f(\text{Diagram of 3 nucleons})$$

Tjon, PLB **56** 217 (1975)



Unitarity expansion summary

Novel approach to few-nucleon systems

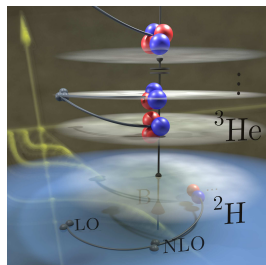
SK *et al.*, PRL **118** 202501 (2017)

SK, JPG **44** 064007 (2017)

	LO	NLO*	N ² LO	exp.
² H	0	0	1.41	2.22
³ H	8.48	8.48	8.48	8.48
³ He	8.48	7.56		7.72
⁴ He	38.86	29.50		28.30

*) four-body: no Coulomb, zero-range

NLO uncertainties: 0.2 MeV (³He), 9 MeV (⁴He)



Unitarity expansion summary

Novel approach to few-nucleon systems

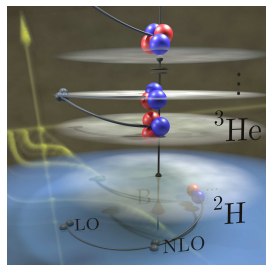
SK *et al.*, PRL **118** 202501 (2017)

SK, JPG **44** 064007 (2017)

	LO	NLO*	N ² LO	exp.
² H	0	0	1.41	2.22
³ H	8.48	8.48	8.48	8.48
³ He	8.48	7.56		7.72
⁴ He	38.86	29.50		28.30

*) four-body: no Coulomb, zero-range

NLO uncertainties: 0.2 MeV (³He), 9 MeV (⁴He)



- **emphasize three-body sector** over two-body precision
- **enhanced symmetry** and **only one parameter at leading order**
- **conjecture:** unitarity expansion useful beyond four nucleons
 - supported by bosonic cluster results
- Coester line from discrete scale invariance

Bazak + van Kolck, PRA **94** 052502 (2016), Carlson *et al.*, PRL **119** 223002 (2017)

van Kolck, Few-Body Syst. **58** 112 (2017)

Unitarity expansion summary

Novel approach to few-nucleon systems

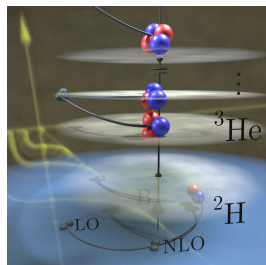
SK *et al.*, PRL **118** 202501 (2017)

SK, JPG **44** 064007 (2017)

	LO	NLO*	N ² LO	exp.
^2H	0	0	1.41	2.22
^3H	8.48	8.48	8.48	8.48
^3He	8.48	7.56		7.72
^4He	38.86	29.50		28.30

*) four-body: no Coulomb, zero-range

NLO uncertainties: 0.2 MeV (^3He), 9 MeV (^4He)



The great nuclear simplification

- EDFs constrained by unitarity
- saturation point from pionless-like model
- one-parameter description of He isotopes
- d - α universality (^6Li Phillips line)

Denis's talk earlier this morning

Kievsky *et al.*, PRL **121** 072701 (2018)

Fossez *et al.*, arXiv:1806.02936

Lei *et al.*, arXiv:1809.06351

**terra incognita
at the doorstep...**

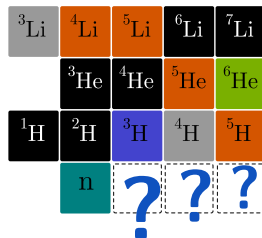
${}^3\text{Li}$	${}^4\text{Li}$	${}^5\text{Li}$	${}^6\text{Li}$	${}^7\text{Li}$
	${}^3\text{He}$	${}^4\text{He}$	${}^5\text{He}$	${}^6\text{He}$
${}^1\text{H}$	${}^2\text{H}$	${}^3\text{H}$	${}^4\text{H}$	${}^5\text{H}$
	n	?	?	?

terra incognita at the doorstep...

${}^3\text{Li}$	${}^4\text{Li}$	${}^5\text{Li}$	${}^6\text{Li}$	${}^7\text{Li}$
	${}^3\text{He}$	${}^4\text{He}$	${}^5\text{He}$	${}^6\text{He}$
${}^1\text{H}$	${}^2\text{H}$	${}^3\text{H}$	${}^4\text{H}$	${}^5\text{H}$
	n	?	?	?

- **neutron-neutron scattering length** is large but not known very well!
 $a_{nn,\text{exp}} = -16.1 \pm 0.4 \text{ fm}, -18.7 \pm 0.7 \text{ fm}, a_{nn,\text{th}} = -22.9 \pm 4.1 \text{ fm}$
Huhn *et al.*, PRL **85** 1190 (2000), González *et al.*, PRC **73** 034001 (2006); Phillips + Kirscher, PRC **84** 054004 (2011)
- **dineutron** bound at large pion masses
Orginos *et al.* (NPLQCD) PRD **92** (2015); Yamazaki *et al.* (PACS) PoS LATTICE2015 081 (2016)
- at physical pion mass: **not excluded** by pionless EFT
Hammer + SK, PLB **736** 208 (2014)
- however, **discouraged** by deuteron muon capture data
Marcucci + Machleidt, PRC **90** 054001 (2014)

terra incognita at the doorstep...



- recent indications for a **three-neutron resonance**...

Gandolfi *et al.*, PRL **118** 232501 (2017)

- ... although **excluded by previous work**

Offermann + Glöckle, NPA **318**, 138 (1979); Lazauskas + Carbonell, PRC **71** 044004 (2005)

- possible experimental evidence for **tetraneutron resonance**

Kisamori *et al.*, PRL **116** 052501 (2016)

Conflicting theoretical tetraneutron results!



Hiyama *et al.*, PRC **93** 044004 (2016); Deluva, PLB **782** 238 (2018)

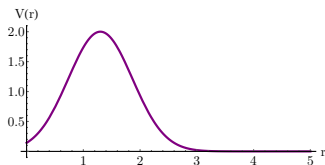


Shirokov *et al.* PRL **117** 182502 ('16); Gandolfi *et al.*, PRL **118** 232501 ('17); Fossez *et al.*, PRL **119** 032501 ('17)

How to tackle resonances?

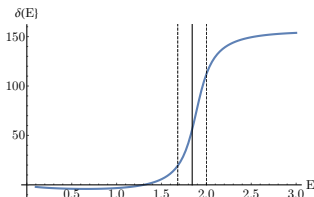
Resonances

- metastable states
- decay width \leftrightarrow lifetime



① Look for jump by π in scattering phase shift:

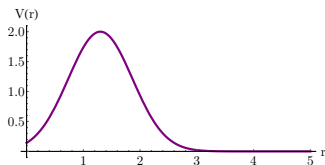
✓ simple ✗ possibly ambiguous (background), need 2-cluster system



How to tackle resonances?

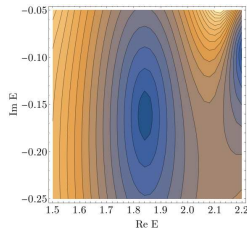
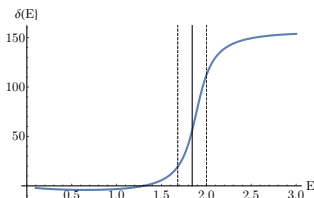
Resonances

- metastable states
- decay width \leftrightarrow lifetime



1 Look for jump by π in scattering phase shift:

✓ simple ✗ possibly ambiguous (background), need 2-cluster system



2 Find complex poles in S-matrix:

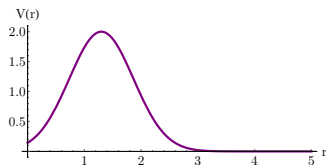
e.g., Glöckle, PRC **18** 564 (1978); Borasoy *et al.*, PRC **74** 055201 (2006); ...

✓ direct, clear signature ✗ technically challenging, needs analytic pot.

How to tackle resonances?

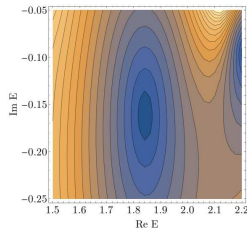
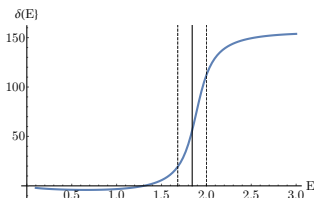
Resonances

- metastable states
- decay width \leftrightarrow lifetime



1 Look for jump by π in scattering phase shift:

✓ simple ✗ possibly ambiguous (background), need 2-cluster system



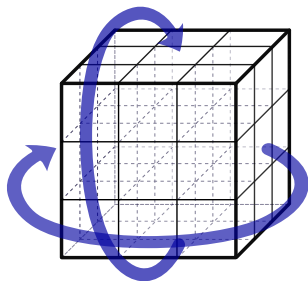
2 Find complex poles in S-matrix:

e.g., Glöckle, PRC **18** 564 (1978); Borasoy *et al.*, PRC **74** 055201 (2006); ...

✓ direct, clear signature ✗ technically challenging, needs analytic pot.

3 Put system into periodic box!

Finite periodic boxes



- physical system enclosed in **finite volume (box)**
- typically used:
periodic boundary conditions

⇒ **volume-dependent energies**

Lüscher formalism

Physical properties encoded in the L -dependent energy levels!

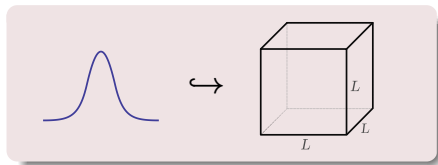
- infinite-volume S-matrix governs **discrete** finite-volume spectrum
- PBC natural for lattice calculations. . .
- . . . but can also be implemented with other methods

Bound-state volume dependence

$$\hat{H} |\psi_B\rangle = -\frac{\kappa^2}{2\mu} |\psi_B\rangle$$

binding momentum κ

\leftrightarrow intrinsic length scale



- for S-wave states, one finds $\Delta B(L) = -3\pi|\gamma|^2 \frac{e^{-\kappa L}}{\mu L} + \mathcal{O}(e^{-\sqrt{2}\kappa L})$

Lüscher, Commun. Math. Phys. **104** 177 (1986); ...

- in general, the prefactor is a polynomial in $1/\kappa L$

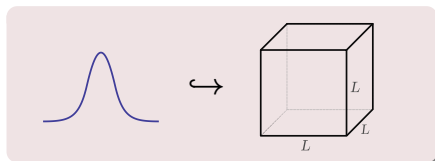
SK, Lee, Hammer, PRL **107** 112001 (2011); Annals Phys. **327**, 1450 (2012)

Bound-state volume dependence

$$\hat{H} |\psi_B\rangle = -\frac{\kappa^2}{2\mu} |\psi_B\rangle$$

binding momentum κ

\leftrightarrow intrinsic length scale



- for S-wave states, one finds $\Delta B(L) = -3\pi|\gamma|^2 \frac{e^{-\kappa L}}{\mu L} + \mathcal{O}(e^{-\sqrt{2}\kappa L})$

Lüscher, Commun. Math. Phys. **104** 177 (1986); ...

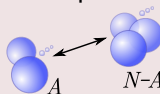
- in general, the prefactor is a polynomial in $1/\kappa L$

SK, Lee, Hammer, PRL **107** 112001 (2011); Annals Phys. **327**, 1450 (2012)

General N -body result

nearest breakup threshold determines volume dependence

$$\Delta B(L) \propto \exp\left(-\kappa_{A|N-A} L\right) / L^{(d-1)/2}$$



SK + Lee, PLB **779**, 9 (2018)

$$\kappa_{A|N-A} = \sqrt{2\mu_{A|N-A}(B_N - B_A - B_{N-A})}$$

Finite-volume resonance signatures

Lüscher formalism: phase shift \leftrightarrow box energy levels

$$p \cot \delta_0(p) = \frac{1}{\pi L} S(\eta) \quad , \quad \eta = \left(\frac{Lp}{2\pi} \right)^2 \quad , \quad p = p(E(L))$$

Lüscher, Nucl. Phys. B **354** 531 (1991); ...

resonance contribution \rightsquigarrow **avoided level crossing**

Wiese, Nucl. Phys. B (Proc. Suppl.) **9**, 609 (1989); ...

Finite-volume resonance signatures

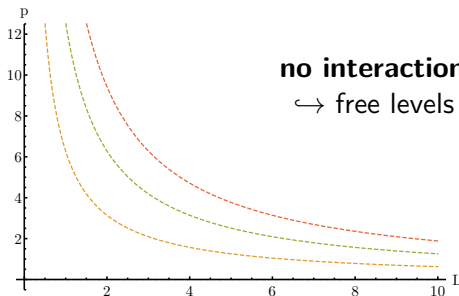
Lüscher formalism: phase shift \leftrightarrow box energy levels

$$p \cot \delta_0(p) = \frac{1}{\pi L} S(\eta) \quad , \quad \eta = \left(\frac{Lp}{2\pi} \right)^2 \quad , \quad p = p(E(L))$$

Lüscher, Nucl. Phys. B 354 531 (1991); ...

resonance contribution \rightsquigarrow **avoided level crossing**

Wiese, Nucl. Phys. B (Proc. Suppl.) 9, 609 (1989); ...



Finite-volume resonance signatures

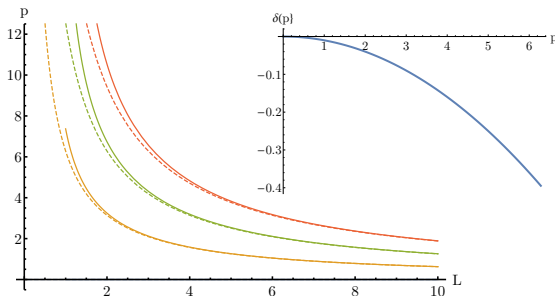
Lüscher formalism: phase shift \leftrightarrow box energy levels

$$p \cot \delta_0(p) = \frac{1}{\pi L} S(\eta) \quad , \quad \eta = \left(\frac{Lp}{2\pi} \right)^2 \quad , \quad p = p(E(L))$$

Lüscher, Nucl. Phys. B **354** 531 (1991); ...

resonance contribution \rightsquigarrow **avoided level crossing**

Wiese, Nucl. Phys. B (Proc. Suppl.) **9**, 609 (1989); ...



Finite-volume resonance signatures

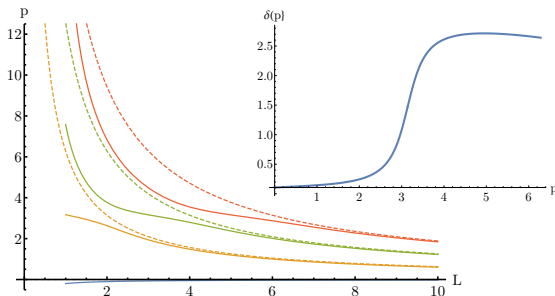
Lüscher formalism: phase shift \leftrightarrow box energy levels

$$p \cot \delta_0(p) = \frac{1}{\pi L} S(\eta) \quad , \quad \eta = \left(\frac{Lp}{2\pi} \right)^2 \quad , \quad p = p(E(L))$$

Lüscher, Nucl. Phys. B **354** 531 (1991); ...

resonance contribution \rightsquigarrow **avoided level crossing**

Wiese, Nucl. Phys. B (Proc. Suppl.) **9**, 609 (1989); ...



Discrete variable representation

Needed: calculation of several few-body energy levels

- difficult to achieve with QMC methods
- direct discretization possible, but not very efficient

Klos *et al.*, PRC **94** 054005 (2016)

↪ use a **Discrete Variable Representation (DVR)**

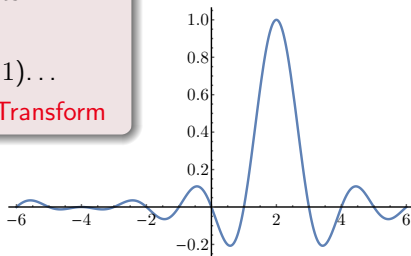
well established in quantum chemistry, suggested for nuclear physics by Bulgac+Forbes, PRC **87** 87, 051301 (2013)

Main features

- basis functions localized at grid points
- potential energy matrix diagonal
- kinetic energy matrix sparse (in $d > 1$)...
- ...or implemented via Fast Fourier Transform

periodic boundary conditions

↔ plane waves as starting point



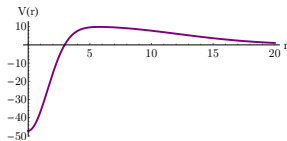
Three-body check

Study established three-body resonance from literature:

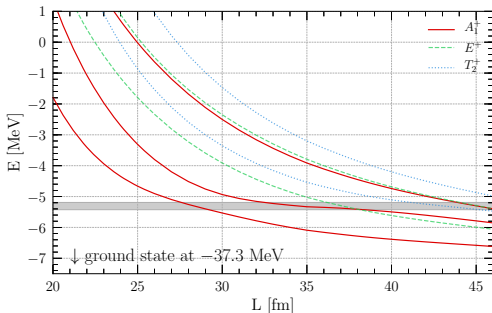
Fedorov *et al.*, Few-Body Syst. P **33** 153 (2003); Blandon *et al.*, PRA **75** 042508 (2007)

$$V(r) = V_0 \exp\left(-\left(\frac{r}{R_0}\right)^2\right) + V_1 \exp\left(-\left(\frac{r-a}{R_1}\right)^2\right)$$

$$V_0 = -55 \text{ MeV}, V_1 = 1.5 \text{ MeV}, R_0 = \sqrt{5} \text{ fm}, R_1 = 10 \text{ fm}, a = 5 \text{ fm}$$



- three spinless bosons with mass $m = 939.0 \text{ MeV}$
- **three-body resonance** at $-5.31 - i0.12 \text{ MeV}$ (Blandon *et al.*), $-5.96 - i0.40 \text{ MeV}$ (Fedorov *et al.*)

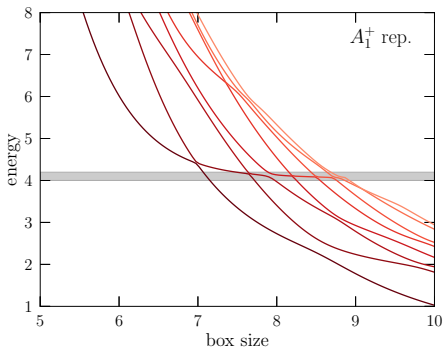
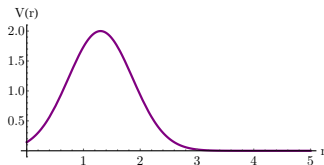


- **fit inflection point(s)** to extract resonance energy $\rightsquigarrow E_R = -5.32(1) \text{ MeV}$

Three bosons with shifted Gaussian interaction

three-boson system

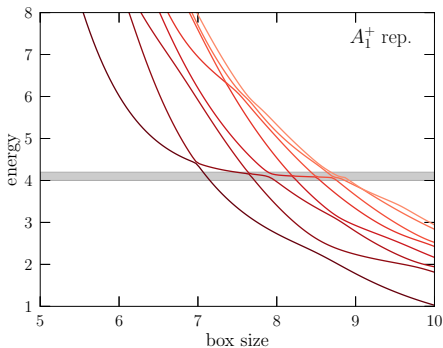
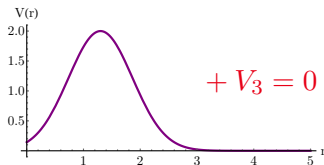
- shifted Gaussian 2-body potential
- **note:** no 2-body bound state!



Three bosons with shifted Gaussian interaction

three-boson system

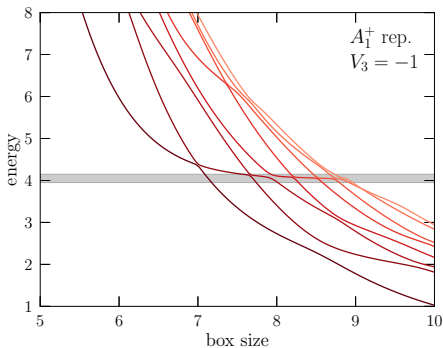
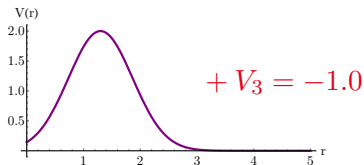
- shifted Gaussian 2-body potential
- **note:** no 2-body bound state!
- add short-range 3-body force



Three bosons with shifted Gaussian interaction

three-boson system

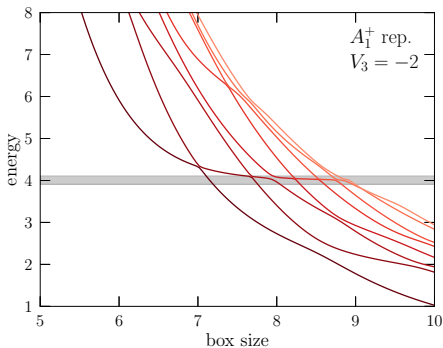
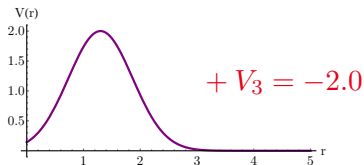
- shifted Gaussian 2-body potential
- **note:** no 2-body bound state!
- add short-range 3-body force



Three bosons with shifted Gaussian interaction

three-boson system

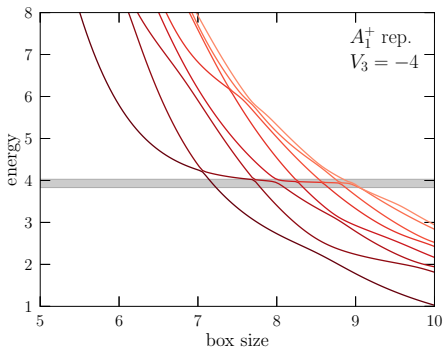
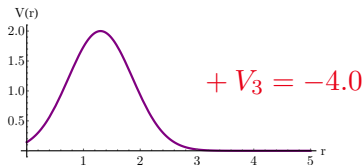
- shifted Gaussian 2-body potential
- **note:** no 2-body bound state!
- add short-range 3-body force



Three bosons with shifted Gaussian interaction

three-boson system

- shifted Gaussian 2-body potential
- **note:** no 2-body bound state!
- add short-range 3-body force



↪ possible to move three-body state ↔ spatially localized wf.

Current status

- ✓ **method established** for up to four particles
- ✓ handle **large N_{DVR} for three-body systems** (current record: 32)
- ✓ efficient **symmetrization and antisymmetrization**
- ✓ projection onto **cubic irreps.** ($H \rightarrow H + \lambda(1 - P_{\Gamma})$, λ large)

Current status

- ✓ **method established** for up to four particles
- ✓ handle **large N_{DVR} for three-body systems** (current record: 32)
- ✓ efficient **symmetrization and antisymmetrization**
- ✓ projection onto **cubic irreps.** ($H \rightarrow H + \lambda(1 - P_T)$, λ large)

Work in progress

- ✓ **chiral interactions** (non-diagonal due to spin dependence!)
 - application to **few-neutron systems**
 - **further optimization** (especially for spin-dep. potentials)
 - ↪ need to reach decent N_{DVR} for four-neutron calculation!
 - isospin degrees of freedom \rightsquigarrow **treat general nuclear systems**
 - **different boundary conditions** (e.g., antiperiodic)

Current status

- ✓ **method established** for up to four particles
- ✓ handle **large N_{DVR} for three-body systems** (current record: 32)
- ✓ efficient **symmetrization and antisymmetrization**
- ✓ projection onto **cubic irreps.** ($H \rightarrow H + \lambda(1 - P_T)$, λ large)

Work in progress

- ✓ **chiral interactions** (non-diagonal due to spin dependence!)
 - application to **few-neutron systems**
 - **further optimization** (especially for spin-dep. potentials)
 - ↪ need to reach decent N_{DVR} for four-neutron calculation!
 - isospin degrees of freedom \rightsquigarrow **treat general nuclear systems**
 - **different boundary conditions** (e.g., antiperiodic)

*** Thank you! ***