

# Beyond mean field in cold gases: quantum droplets and three-body interactions

Dmitry Petrov

Laboratoire Physique Théorique et Modèles Statistiques (Orsay)

Alexandre Pricoupenko (LPTMS, Orsay)

Grecia Guijarro, Gregory Astrakharchik, Jordi Boronat (UPC, Barcelona)

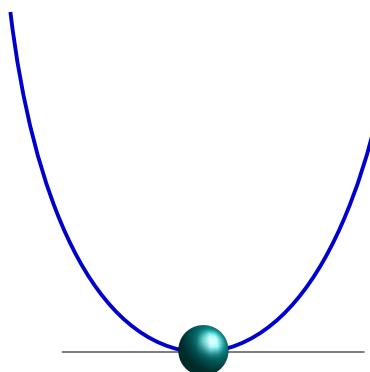
Alessio Recati (BEC Center, Trento)



European Research Council  
Established by the European Commission

# **Quantum stabilization**

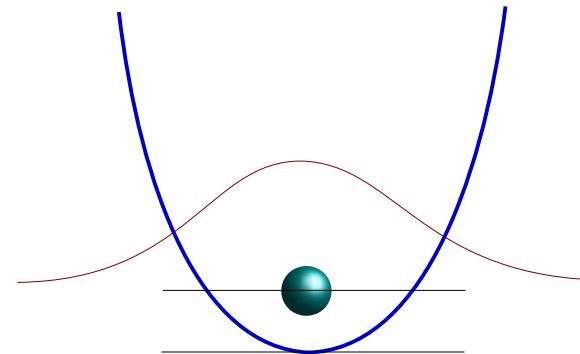
## Classical



BEC analog:  
Classical or mean-field limit =  
Gross-Pitaevskii equation

---

## Quantum



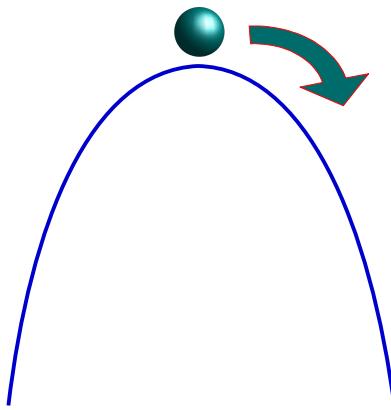
BEC analog:  
Mean field + Gaussian fluctuations  
= GP+LHY

Classical vacuum



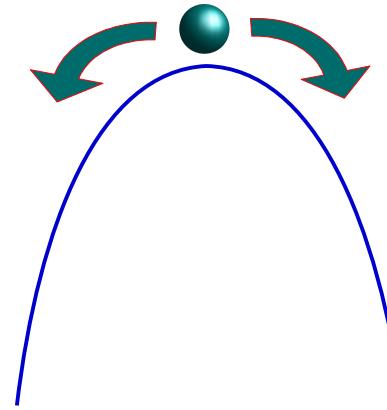
Bogoliubov vacuum

Classical



BEC analog:  
collapse

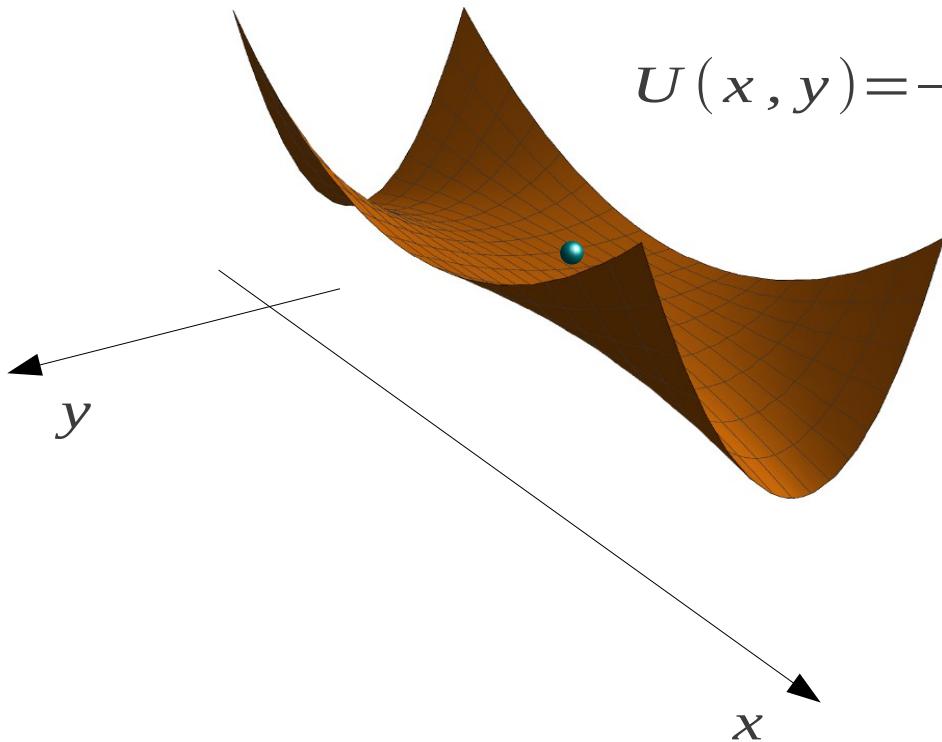
Quantum



BEC analog:  
collapse :(

Can there be a classically unstable system,  
yet stable when quantum mechanics is “switched on” ?

# Quantum stabilization idea

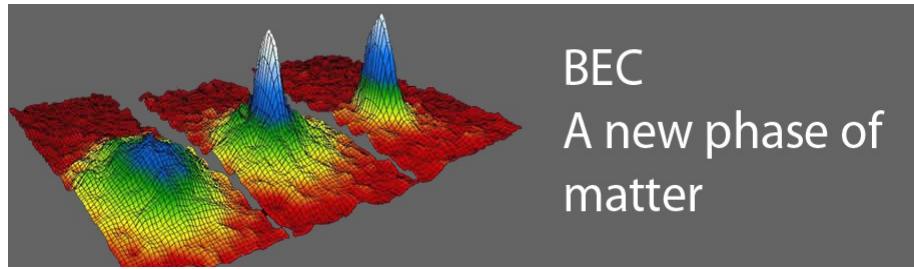


$$U(x, y) = -\alpha x^2 + \frac{\omega^2(x)}{2} y^2$$

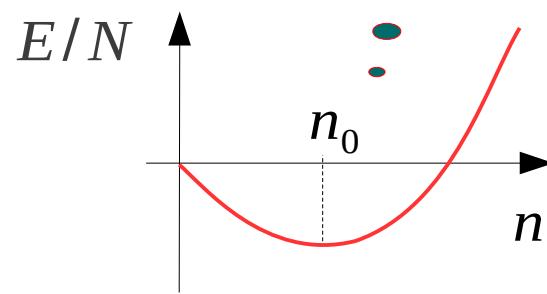
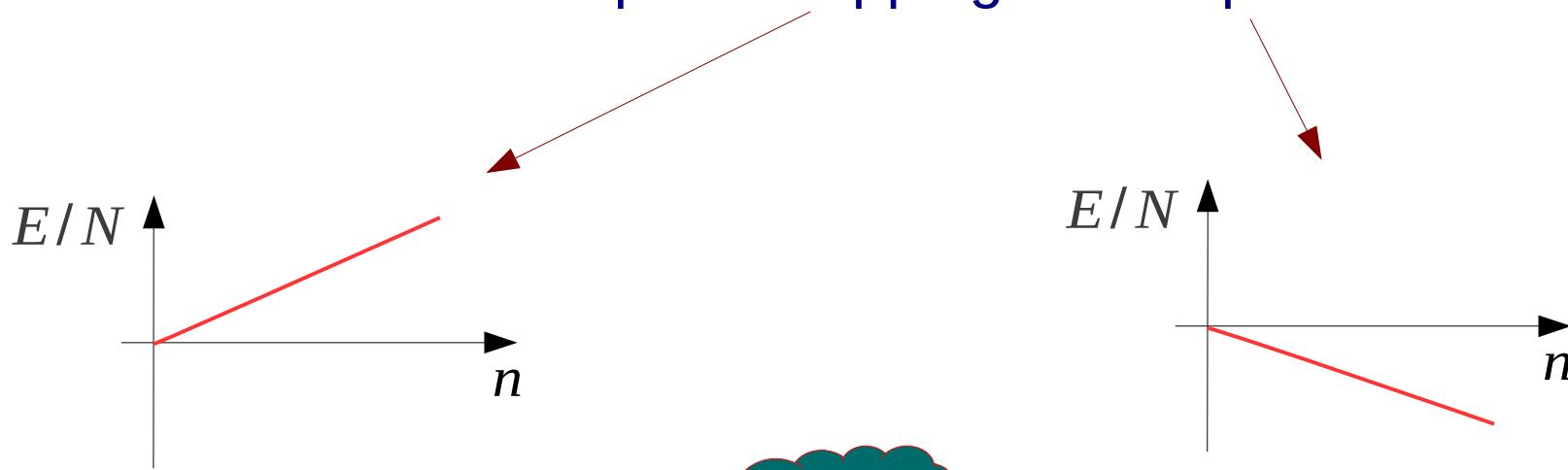
Stable for sufficiently fast growing  
 $\omega(x)$

Classically unstable degree of freedom stabilized by quantum fluctuations in another degree of freedom!

**BEC analog:  
quantum droplet!**

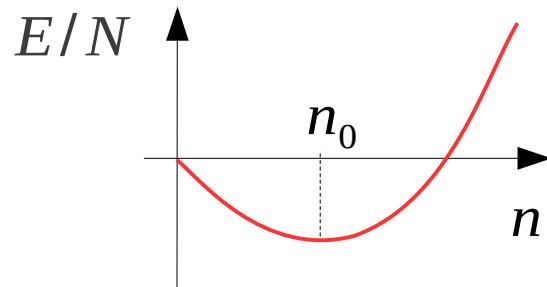


GAS requires trapping or collapses



$$E/N \propto g_2 n + g_{\alpha+1} n^{\alpha}, \quad \alpha > 1$$

The gas should remain dilute, otherwise short lifetime!

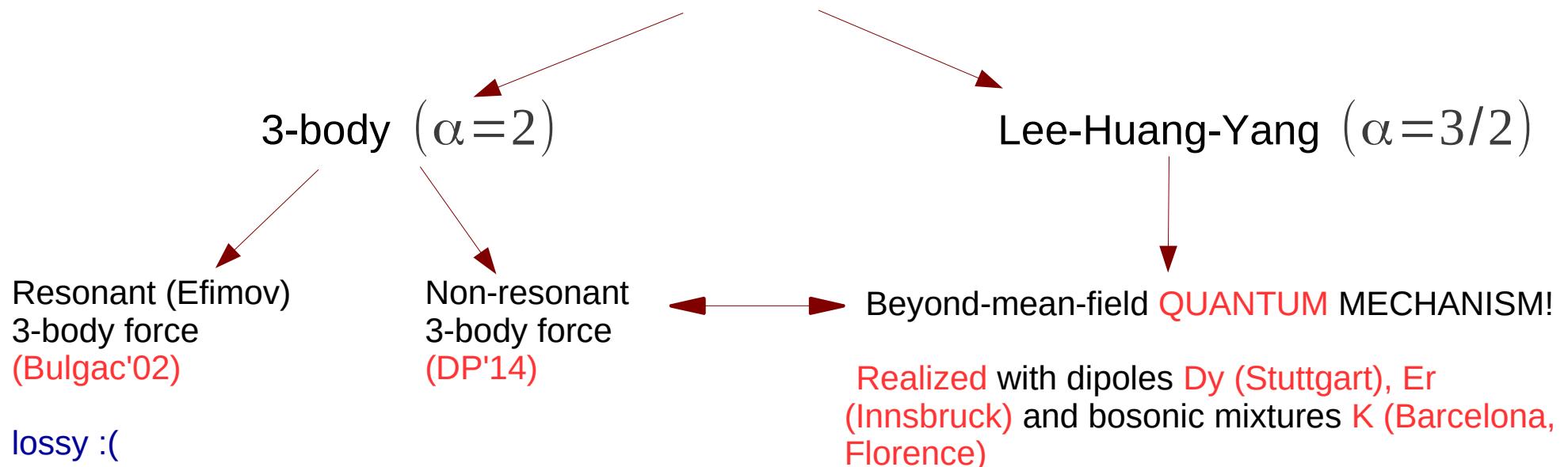


$$E/N \propto a n + L^{3\alpha-2} n^\alpha, \quad \alpha > 1$$



$$n_0 \sim \frac{1}{L^3} \left( \frac{a}{L} \right)^{\frac{1}{\alpha-1}}$$

Dilute = simultaneously small  $a$  and large  $L$   
and prefer small  $\alpha$



Lots of ``beyond-mean-field prospects''!

# **LHY mechanism**

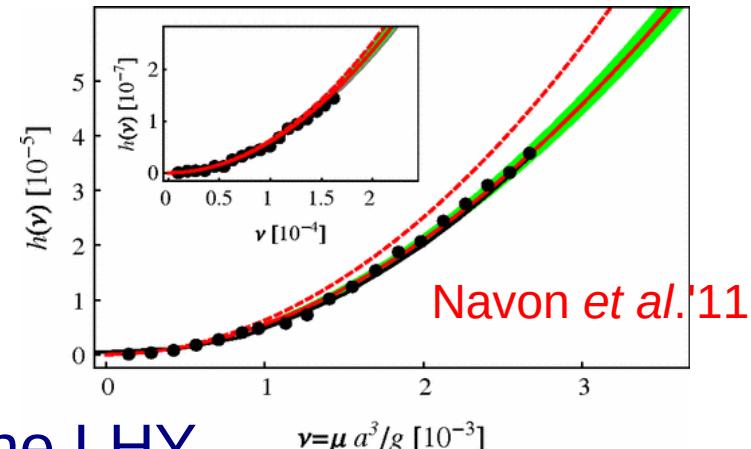
For spinless BEC:

$$\frac{E}{\text{Volume}} = \frac{g_2 n^2}{2} \left( 1 + \frac{128}{15} \sqrt{\frac{n a^3}{\pi}} + \dots \right)$$

Lee-Huang-Yang correction  $\propto g_2^{5/2} n^{5/2}$

LHY correction is **UNIVERSAL** (depends only on the scattering length) and **QUANTUM** (zero-point energy of Bogoliubov phonons)!

Observed in ultracold gases where the scattering length is tunable by using Feshbach resonances  
(Innsbruck, MIT, ENS, JILA, Rice)



Unfortunately, the effect is perturbative and the LHY term is smaller than the mean-field one!

# Bose-Bose mixture, mean field

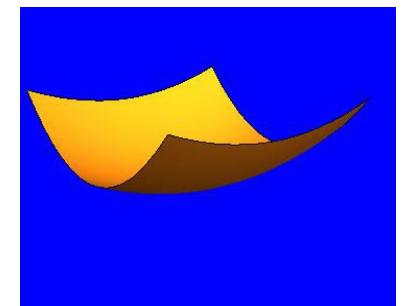
Mean-field energy density: 
$$\frac{E_{MF}}{\text{Volume}} = \frac{g_{11}n_1^2 + g_{22}n_2^2 + 2g_{12}n_1n_2}{2}$$

$g_{12}$  ↑  $g_{12} > \sqrt{g_{11}g_{22}}$  phase separation



mean-field stability

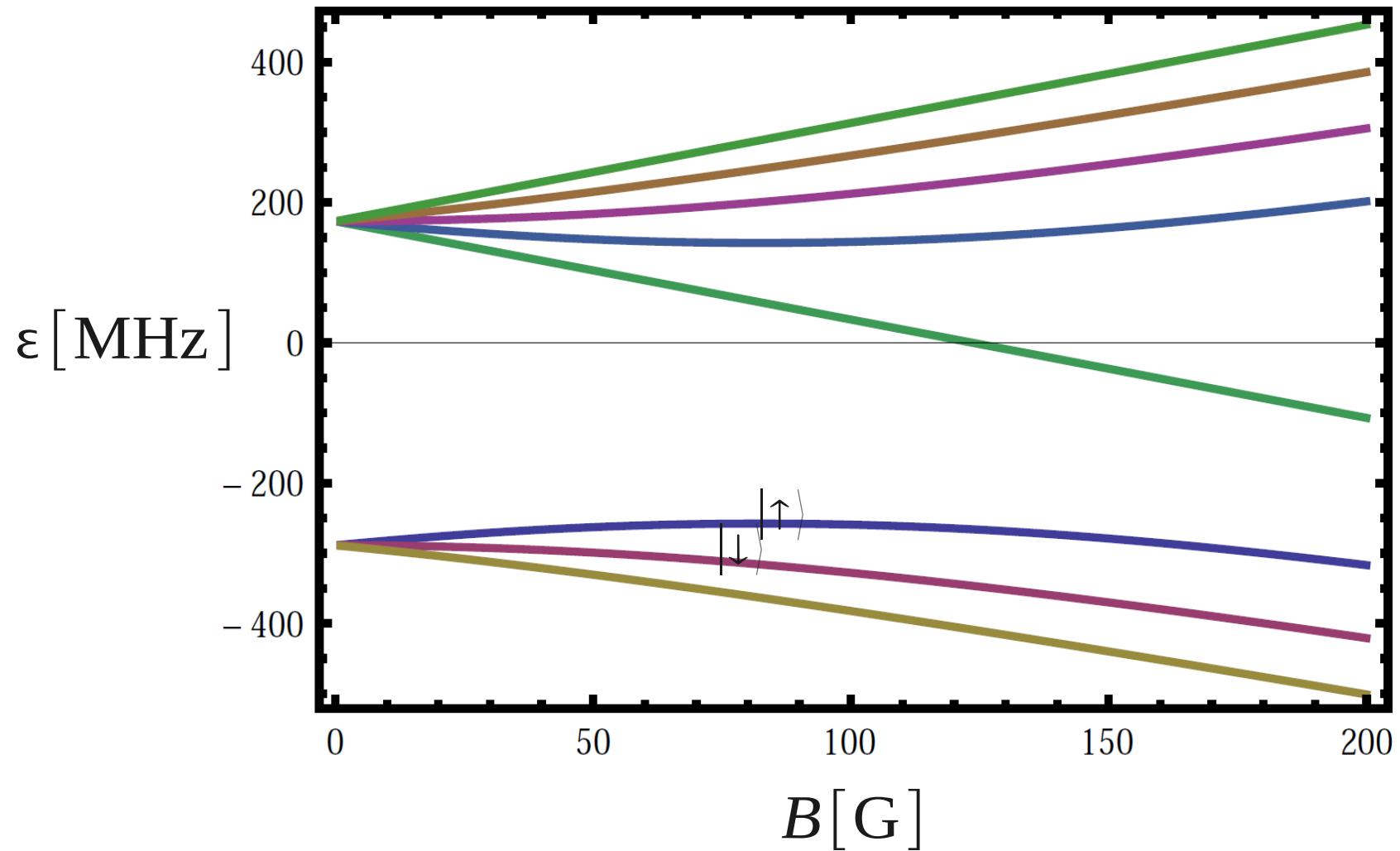
$g_{11} > 0, g_{22} > 0,$  and  $g_{12}^2 < g_{11}g_{22}$



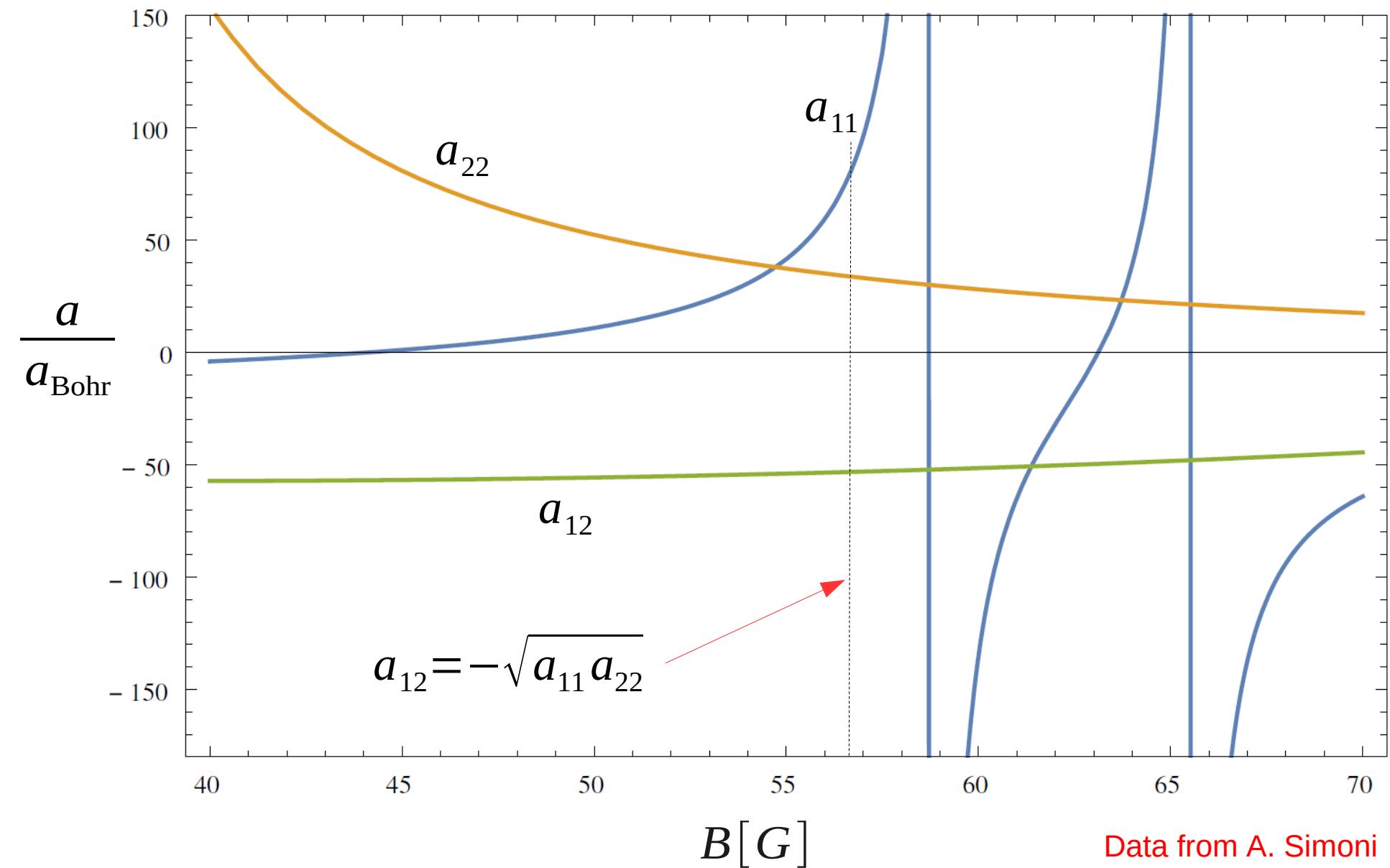
$g_{12} < -\sqrt{g_{11}g_{22}}$  collapse



$^{39}\text{K}$ :  $|F=1, m_F=0\rangle$  and  $|F=1, m_F=-1\rangle$



$^{39}\text{K}$ :  $|F=1, m_F=0\rangle$  and  $|F=1, m_F=-1\rangle$



# LHY correction

## Bogoliubov theory

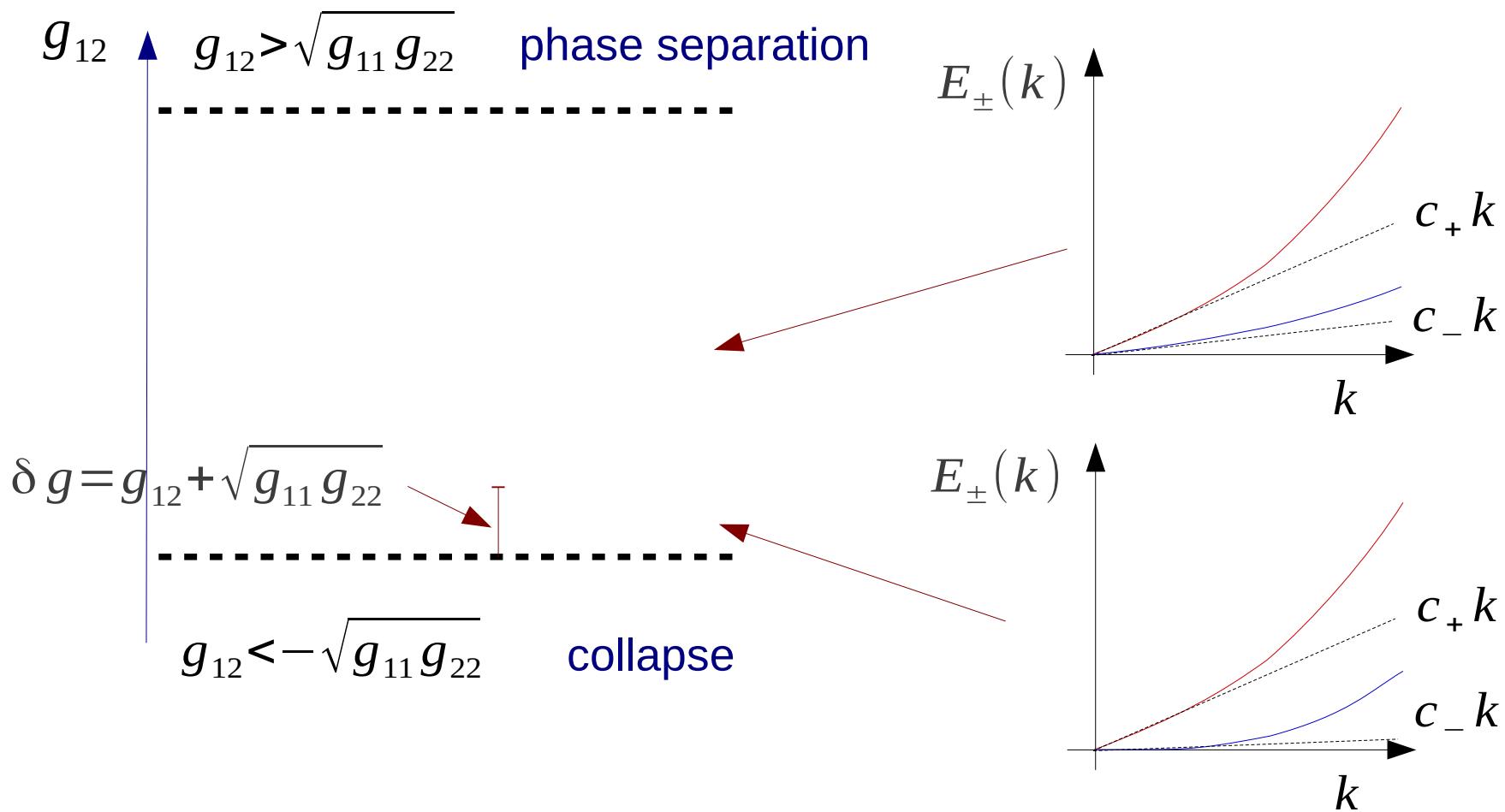
$$\begin{aligned}
 E_{\pm}(k) &= \sqrt{c_{\pm}^2 k^2 + k^4/4}; \quad c_{\pm}^2 = \frac{g_{11} n_1 + g_{22} n_2 \pm \sqrt{(g_{11} n_1 - g_{22} n_2)^2 + 4 g_{12}^2 n_1 n_2}}{2} \\
 \frac{E}{\text{Volume}} &= \frac{g_{11} n_1^2 + g_{22} n_2^2 + 2 g_{12} n_1 n_2}{2} + \frac{1}{2} \sum_{\pm} \sum_k [E_{\pm}(k) - k^2/2 - c_{\pm}^2] = \\
 &= \underbrace{\frac{g_{11} n_1^2 + g_{22} n_2^2 + 2 g_{12} n_1 n_2}{2}}_{\text{MF} \propto g n^2} + \underbrace{\frac{8}{15 \pi^2} (c_+^5 + c_-^5)}_{\text{LHY} \propto (g n)^{5/2}} \quad (\text{Larsen'63})
 \end{aligned}$$

In contrast to one-component case, MF and LHY depend on (different) combinations of  $g_{\sigma\sigma}, n_{\sigma}$

...and, thus, can be independently controlled!

# LHY correction

$$\frac{E}{\text{Volume}} = \frac{g_{11}n_1^2 + g_{22}n_2^2 + 2g_{12}n_1n_2}{2} + \frac{8}{15\pi^2} \frac{m^4}{\hbar^3} (c_+^5 + c_-^5) + \dots$$



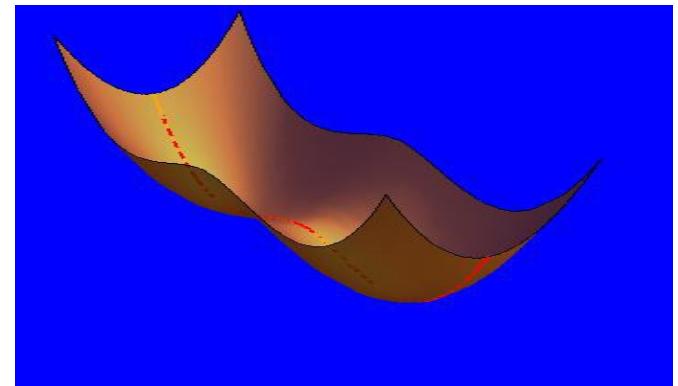
# Quantum stabilization

$$\delta g = g_{12} + \sqrt{g_{11} g_{22}} \ll \sqrt{g_{11} g_{22}} = g$$

The mean-field term "locks" the ratio  $\frac{n_2}{n_1} = \sqrt{\frac{g_{11}}{g_{22}}}$

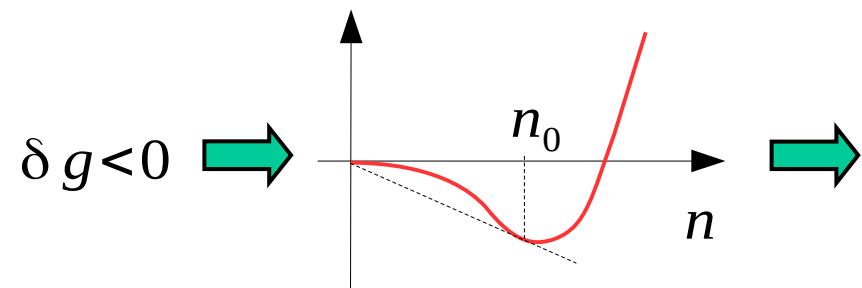
Softening of lower Bog. mode  $c_- \ll c_+ \propto \sqrt{gn/m}$

only the ``hard'' + branch contributes to the LHY term



The structure of the energy-density functional:

$$\frac{E}{\text{Volume}} = A_1 \times \delta g \times n^2 + A_2 \times (m/\hbar^2)^{3/2} (gn)^{5/2}$$



Gas exists in equilibrium with vacuum. Saturation density

$$n_0 \propto \frac{1}{a^3} \left( \frac{\delta g}{g} \right)^2$$

Density is tunable by modifying interaction parameters!

# Gross-Pitaevskii eq., droplet shape

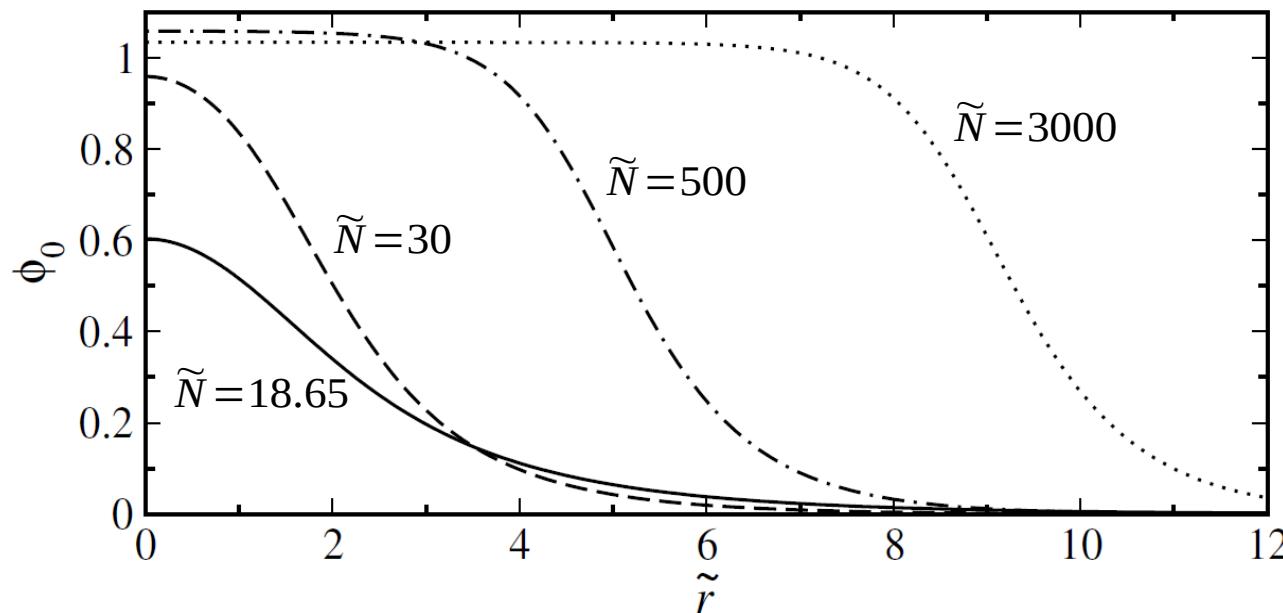
Rescaling  $\vec{r} = \xi \tilde{\vec{r}}$ ,  $t = \tau \tilde{t}$ ,  $N = n \xi^3 \tilde{N}$ , where  $\xi \propto 1/\sqrt{m|\delta g|n}$ ,  $\tau \propto 1/|\delta g|n$



$$i \partial_{\tilde{t}} \varphi = (-\nabla_{\tilde{\vec{r}}}^2/2 - 3|\varphi|^2 + 5|\varphi|^3/2 - \tilde{\mu}) \varphi$$

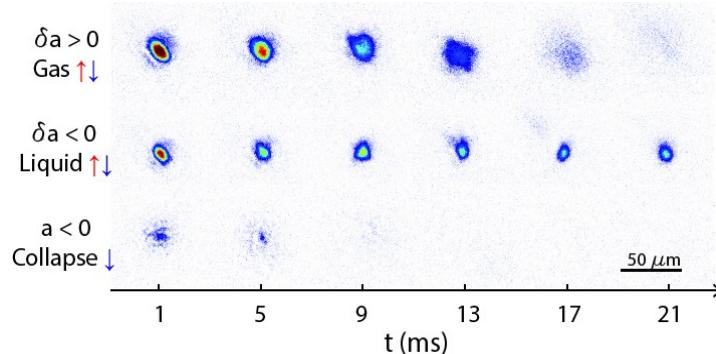
$$\tilde{N} = \int |\varphi|^2 d^3 \tilde{r}$$

Modified Gross-Pitaevskii equation  
cubic-quartic  
nonlinearities

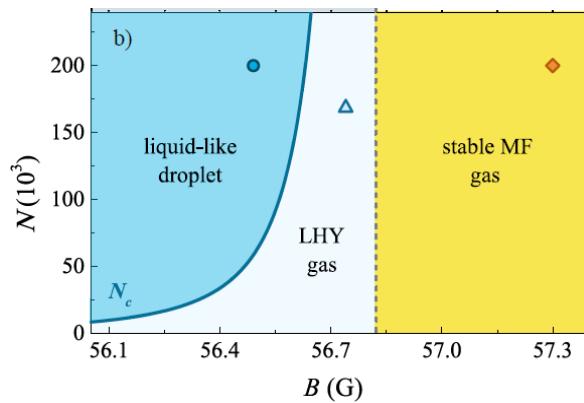


# Reality

## Observation of droplets

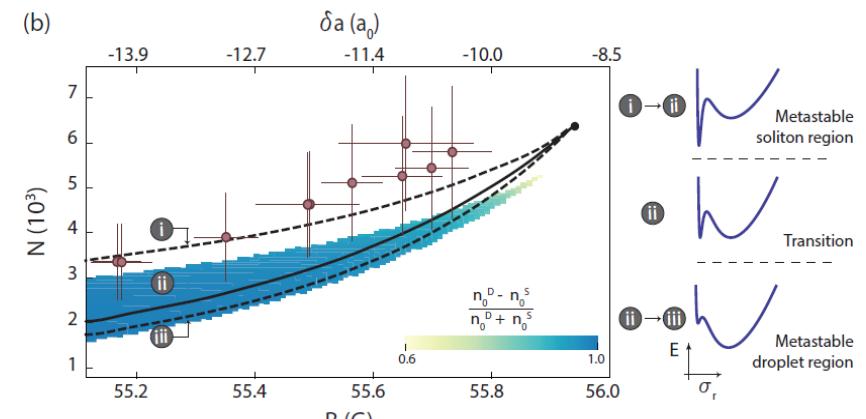


(Cabrera et al'18)



(Semeghini et al'18)

## Bright soliton to droplet transition



(Cheinay et al'18)

The main problem so far is the three-body recombination!

$$v_3 \propto \delta g^4$$

$$\frac{1}{\tau} \propto |\delta g|^3$$

$$N_c \propto |\delta g|^{-5/2}$$

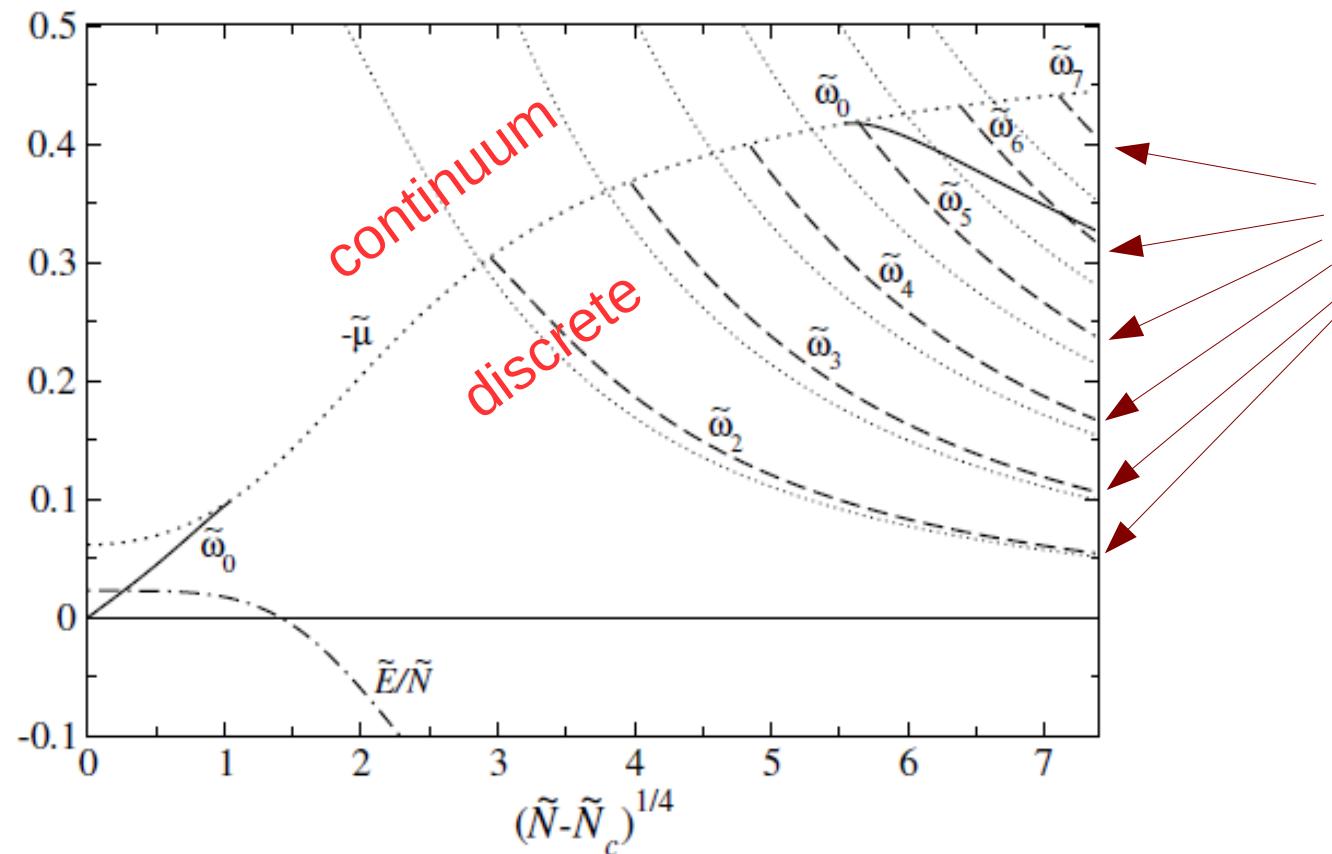
Increasing  $N$  by 10 decreases  $v_3 \tau$  by 2.5

# Bogoliubov-de Gennes eqs., excitations

$$\varphi(\tilde{t}, \tilde{r}) = \varphi_0(\tilde{r}) + \delta\varphi(\tilde{t}, \tilde{r})$$



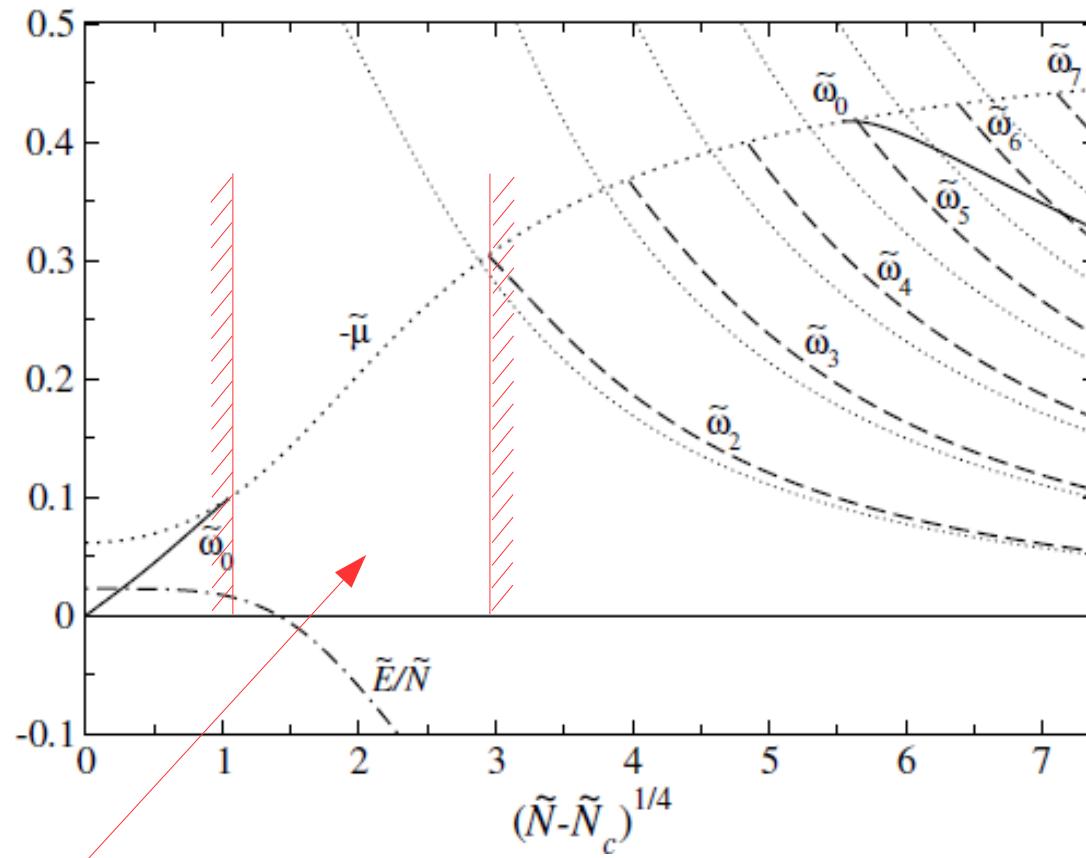
linearize  $i\partial_{\tilde{t}}\varphi = (-\nabla_{\tilde{r}}^2/2 - 3|\varphi|^2 + 5|\varphi|^3/2 - \tilde{\mu})\varphi$  with respect to small  $\delta\varphi(\tilde{t}, \tilde{r})$



Surface modes



# Zero-temperature object



No discrete modes  $\rightarrow$  the droplet evaporates itself to zero T!  
(by contrast,  ${}^4\text{He}$  droplets always have discrete modes)



Macroscopic zero-temperature object:

- is interesting by itself
- can be used for sympathetic cooling of other systems

# LHY depends on ...

$$\frac{E}{\text{Volume}} = \frac{g_{11}n_1^2 + g_{22}n_2^2 + 2g_{12}n_1n_2}{2} + \frac{1}{2} \sum_{\pm} \sum_k [E_{\pm}(k) - k^2/2 - c_{\pm}^2] =$$

number of components

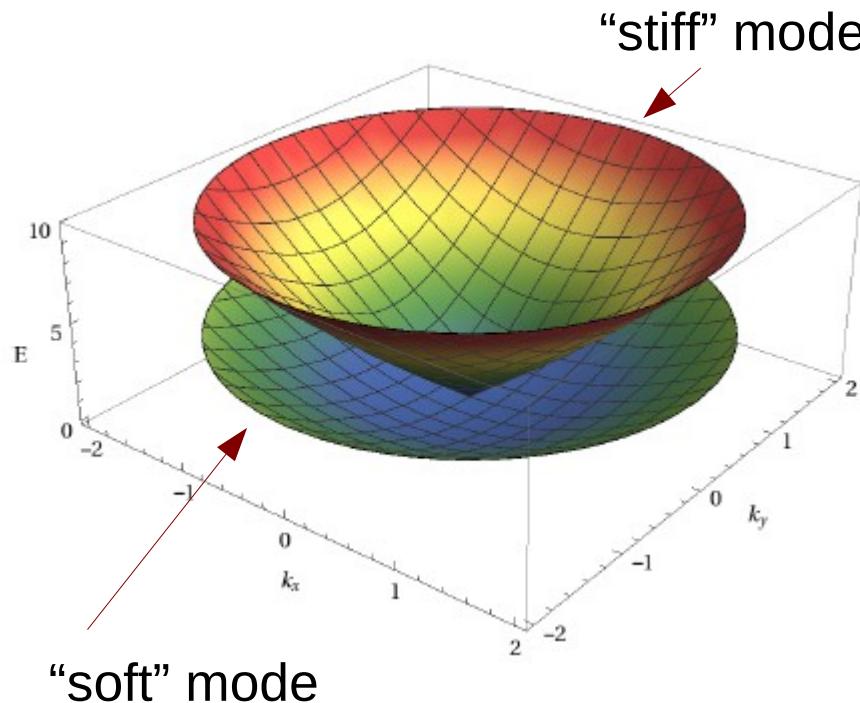
density of states (dimension)

shape of the Bogoliubov spectrum  
(anisotropy of the interaction,  
driving the mixture, etc.)

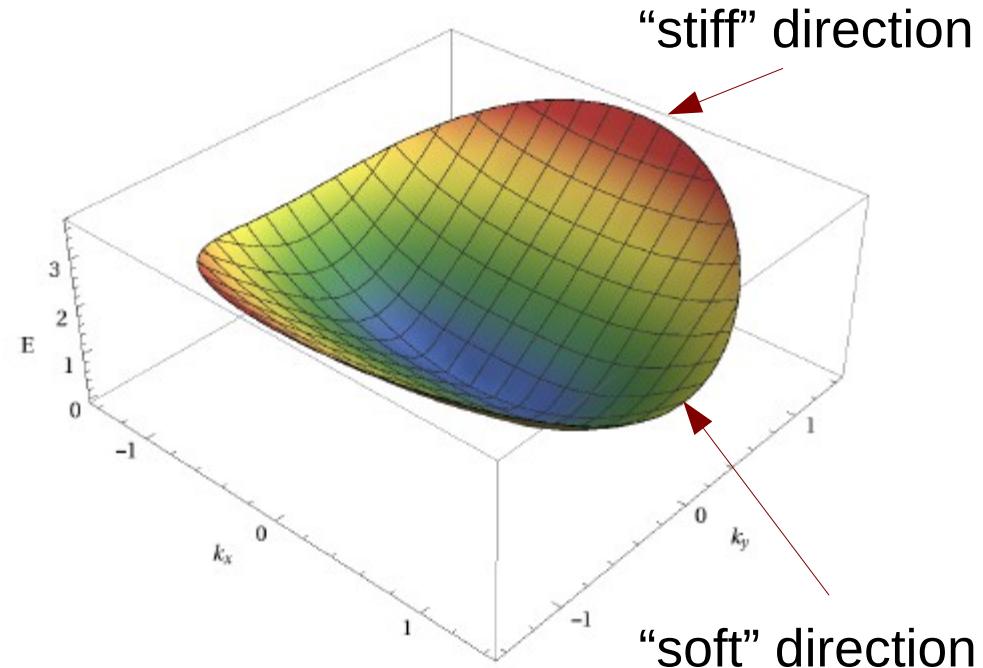
...and life becomes ~~harder~~ more interesting in the inhomogeneous case  
particularly if LDA is not valid

# Bose-Bose mixture vs Dy/Er

## Bose-Bose mixture



## dipolar Bose gas



$$\text{LHY} = \frac{8}{15\pi^2} \frac{m^4}{\hbar^3} (c_+^5 + c_-^5)$$

(Larsen'63)

$$\text{LHY} = \frac{8}{15\pi^2} \frac{m^4}{\hbar^3} \langle c^5(\hat{k}) \rangle_{\hat{k}}$$

(Lima&Pelster'11)

# **Low-dimensional case**

# Bogoliubov theory – ``Mean field + LHY'' (DP, Astrakharchik'16)

$$\frac{E}{\text{Volume}} = \frac{g_{11}n_1^2 + g_{22}n_2^2 + 2g_{12}n_1n_2}{2} + \frac{1}{2} \sum_{\pm} \sum_k [E_{\pm}(k) - k^2/2 - c_{\pm}^2]$$



Dimension enters here!

# Bogoliubov theory – ``Mean field + LHY'' (DP, Astrakharchik'16)

**3D:**  $\frac{E_{3D}}{\text{Volume}} = \frac{1}{2} \sum_{\sigma\sigma'} g_{\sigma\sigma'} n_\sigma n_{\sigma'} + \frac{8}{15\pi^2} \sum_{\pm} c_{\pm}^5 \sim \delta g n^2 + (gn)^{5/2}$

$$\sqrt{n g^3} \ll 1$$

**2D:**  $\frac{E_{2D}}{\text{Surface}} = \frac{1}{2} \sum_{\sigma\sigma'} g_{\sigma\sigma'} n_\sigma n_{\sigma'} + \frac{1}{8\pi} \sum_{\pm} c_{\pm}^4 \ln \frac{c_{\pm}^2 \sqrt{e}}{\kappa^2} \sim g^2 n^2 \ln \frac{n}{n_0}$

$$g_{\sigma\sigma'} = 2\pi / \ln(2e^{-\gamma}/a_{\sigma\sigma'}\kappa) \ll 1$$

**1D:**  $\frac{E_{1D}}{\text{Length}} = \frac{1}{2} \sum_{\sigma\sigma'} g_{\sigma\sigma'} n_\sigma n_{\sigma'} - \frac{2}{3\pi} \sum_{\pm} c_{\pm}^3 \sim \delta g n^2 - (gn)^{3/2}$

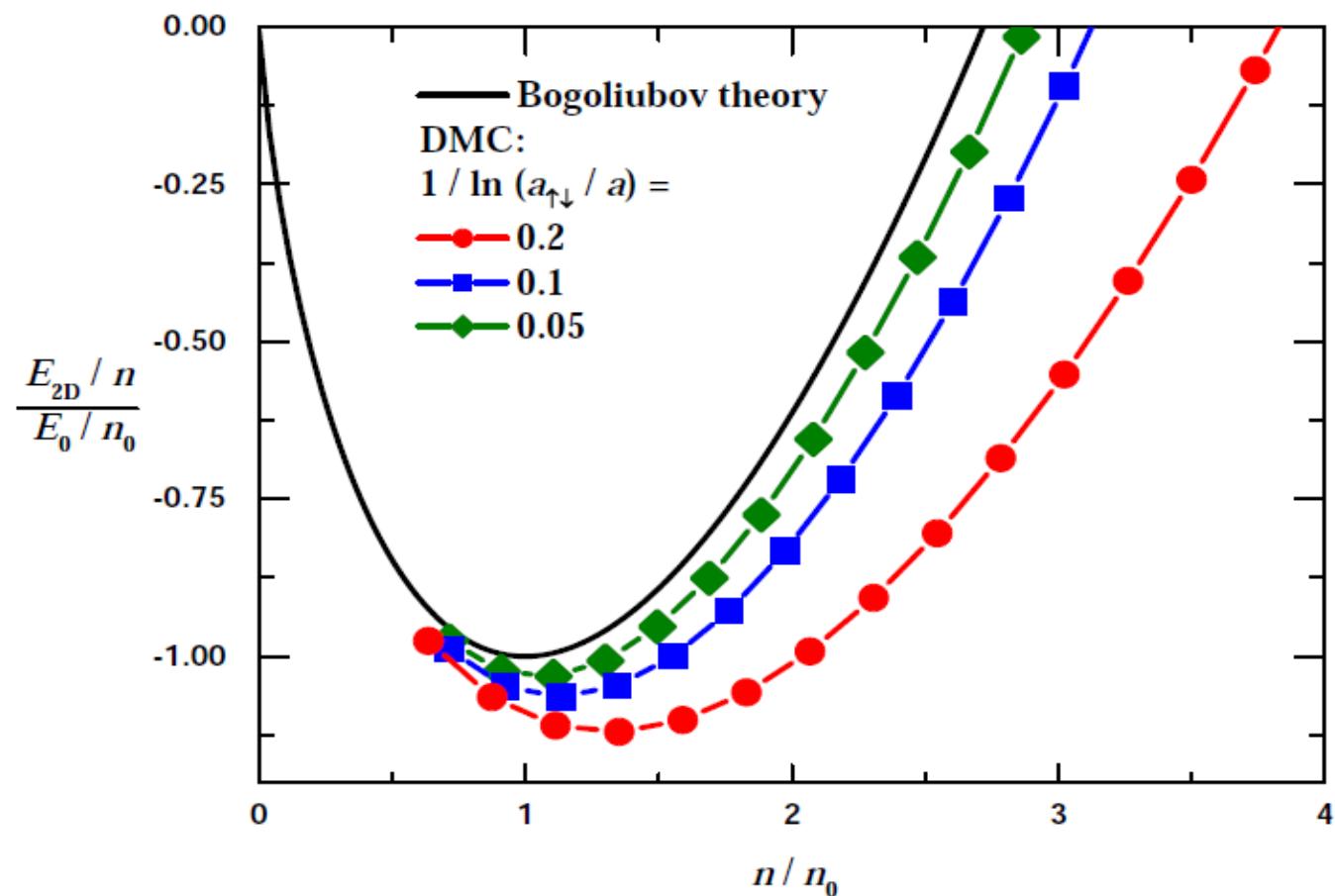
$$\sqrt{g/n} \ll 1$$

!

# 2D symmetric case

$$a_{\uparrow\uparrow} = a_{\downarrow\downarrow} = a \rightarrow n_{\uparrow} = n_{\downarrow} = n \rightarrow \frac{E_{2D}}{\text{Surface}} = \frac{8\pi n^2}{\ln^2(a_{\uparrow\downarrow}/a)} [\ln(n/n_0) - 1]$$

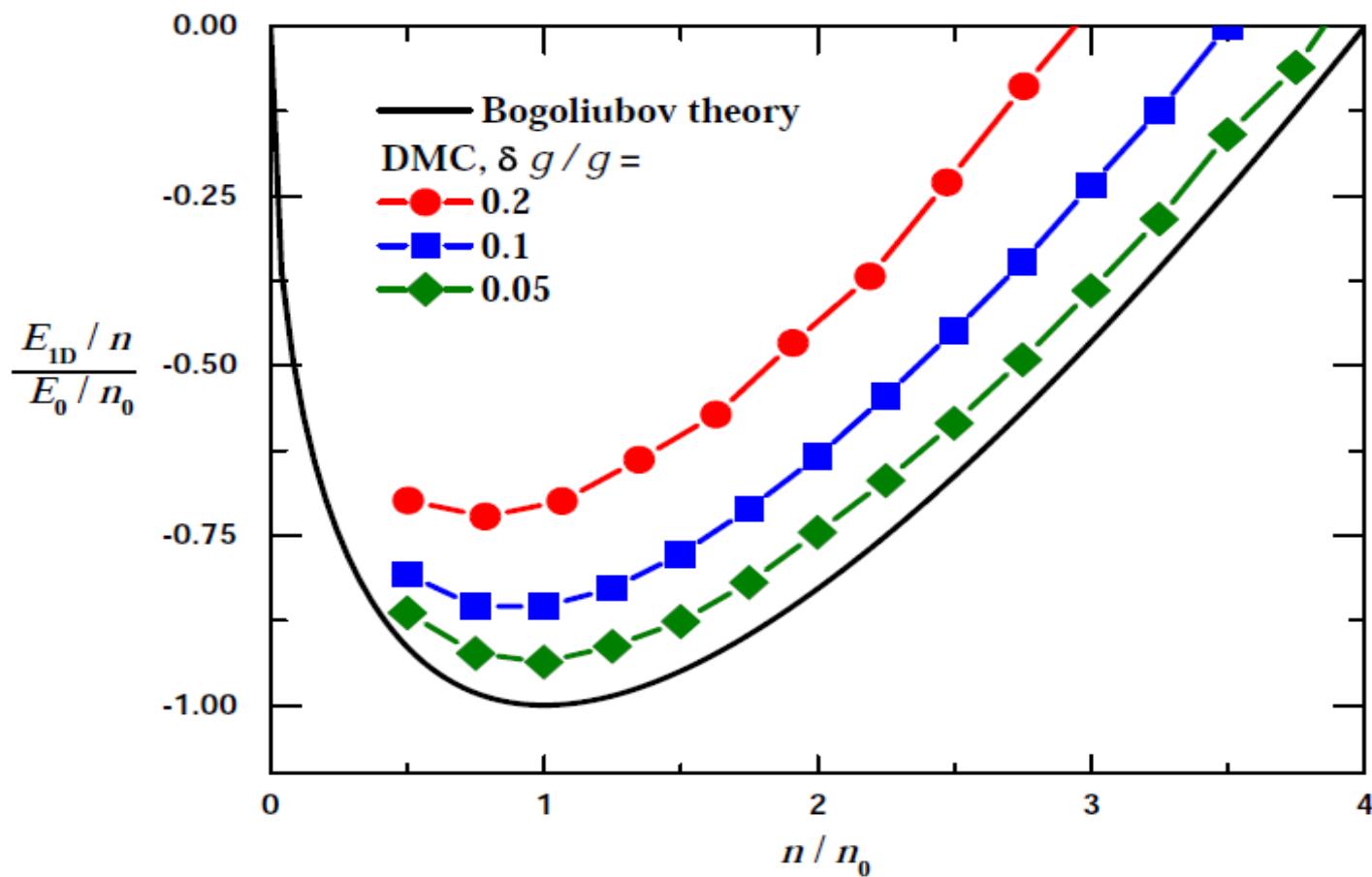
where  $n_0 = \frac{e^{-2\gamma-3/2}}{2\pi} \frac{\ln(a_{\uparrow\downarrow}/a)}{a a_{\uparrow\downarrow}}$



# 1D symmetric case

$$g_{\uparrow\uparrow}=g_{\downarrow\downarrow}=g \rightarrow n_{\uparrow}=n_{\downarrow}=n \rightarrow \frac{E_{1D}}{\text{Length}} = \delta g n^2 - \frac{4\sqrt{2}}{3\pi} (gn)^{3/2}$$

Minimum for  $\delta g > 0$  at  $n_0 = \frac{8g^3}{9\pi^2 \delta g^2}$



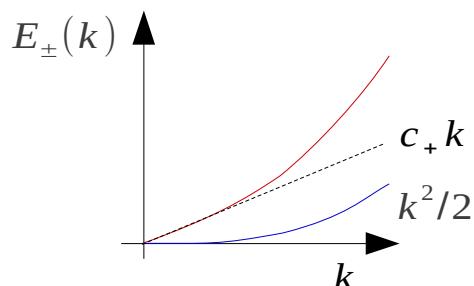
# **Connection between LHY and three-body force**

Symmetric case close to collapse  $g_{11}=g_{22}=-g_{12}=g$  and  $n_1=n_2=n/2$

without coupling

$$E_+(k) = \sqrt{(k^2/2)(k^2/2 + 2gn)}$$

$$E_-(k) = k^2/2$$

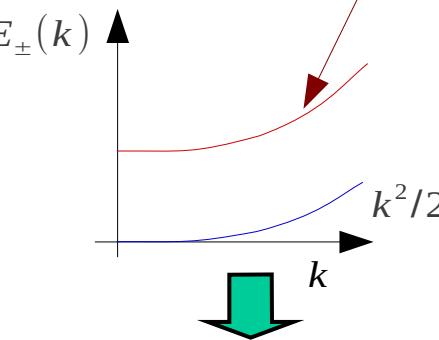


$$\text{LHY} = \frac{8}{15\pi^2} (gn)^{5/2}$$

with coupling (Goldstein&Meystre'97)

$$E_+(k) = \sqrt{(k^2/2 + \Omega)(k^2/2 + \Omega + 2gn)}$$

$$E_-(k) = k^2/2$$



$$\text{LHY} = \frac{2}{\pi^2} (gn)^{5/2} \int_0^1 \sqrt{x(1-x)\left(x + \frac{\Omega}{2gn}\right)} dx$$

$$\downarrow gn \ll \Omega$$

$$= \frac{\sqrt{\Omega} g^2}{2\sqrt{2}\pi} \frac{n^2}{2} + \frac{3g^3}{4\sqrt{2}\pi\sqrt{\Omega}} \frac{n^3}{3!} + \dots$$

Renormalization of  
two-body interaction

Effective three-body  
force

DSP and Recati, to be published... since a long time...

# Why interesting?

Bosons +  $g_2 < 0$   Collapse

Bosons +  $g_2 < 0 + g_3 > 0$   Free space → self-trapped droplet state Bulgac'02:

Neglecting surface tension, flat density profile  $n = 3|g_2|/2g_3$

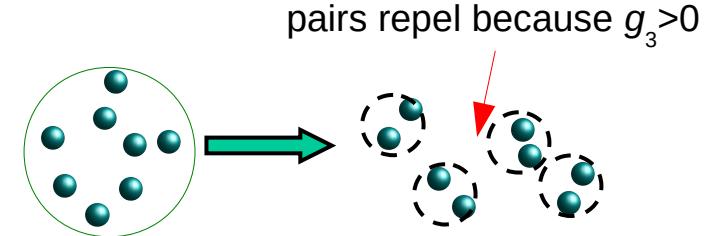
Including surface tension → surface modes



Increasing  $g_2 < 0$   bosonic pairing Nozieres&Saint James'82

Topological transition, not crossover!

Radzhovsky et al., Romans et al., Lee&Lee'04



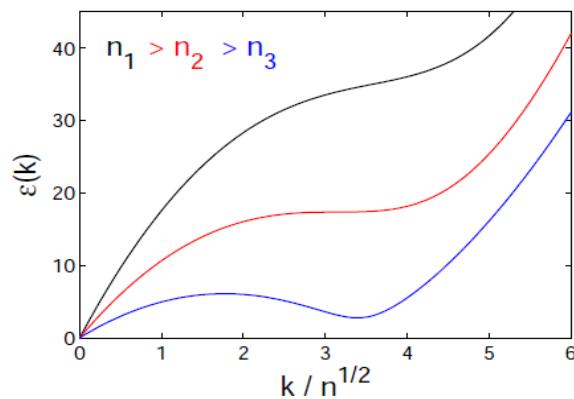
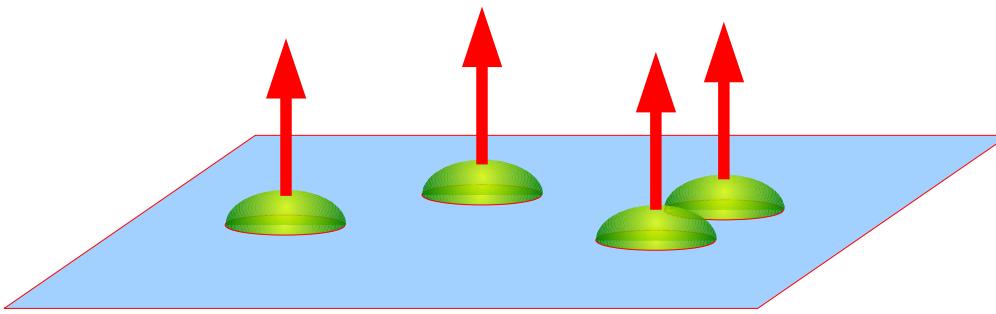
Pairing on a lattice with three-body constraint:

Daley et al.'09-, Ng&Yang'11, Bonnes&Wessel'12,...

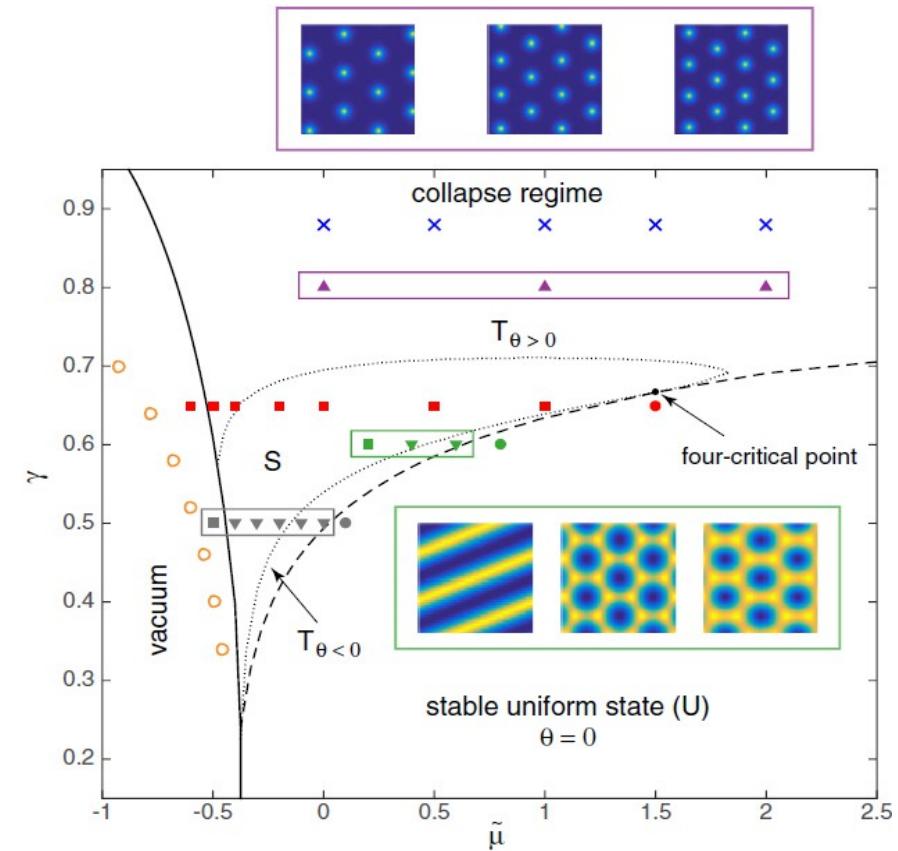
$g_3$  is necessary! = Pauli pressure in the BCS-BEC crossover!

# Why interesting? (contd.)

## 2D dipoles



## Phase diagram

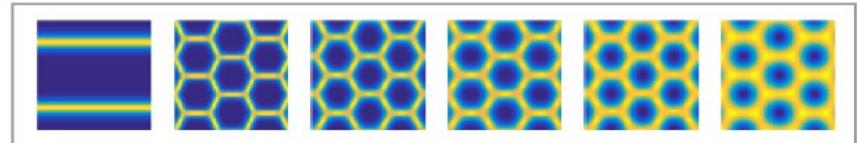


Rotonized superfluid & supersolid



Mechanical stability for  $g_3 > 0$

Lu et al'15



# Dimensions of X-dimensional coupling constants

X-dimensional 2-body scattering  $\longrightarrow g_2 \propto (\hbar^2/m) \times \text{length}^{X-2}$



X-dimensional 3-body scattering = (2X)-dimensional 2-body scattering



X-dimensional 3-body scattering  $\longrightarrow g_3 \propto (\hbar^2/m) \times \text{length}^{2X-2}$

In particular,

3D:  $g_3 \propto (\hbar^2/m) \times \text{length}^4$

2D:  $g_3 \propto (\hbar^2/m) \times \text{length}^2$

1D:  $g_3 \propto (\hbar^2/m) / \ln(k \times \text{length}) = \text{small parameter}$

# Dimensions of X-dimensional coupling constants

X-dimensional 2-body scattering  $\longrightarrow g_2 \propto (\hbar^2/m) \times \text{length}^{X-2}$



X-dimensional 3-body scattering = (2X)-dimensional 2-body scattering



X-dimensional 3-body scattering  $\longrightarrow g_3 \propto (\hbar^2/m) \times \text{length}^{2X-2}$

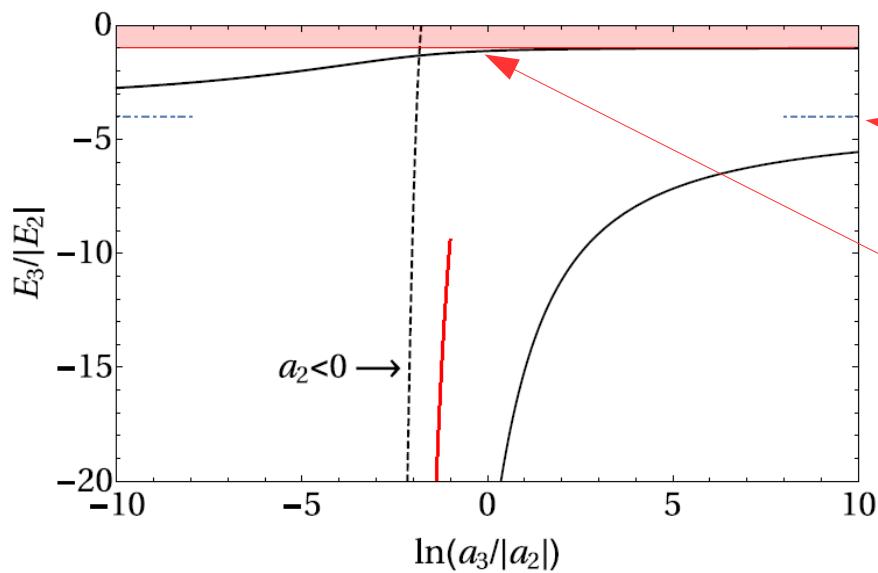
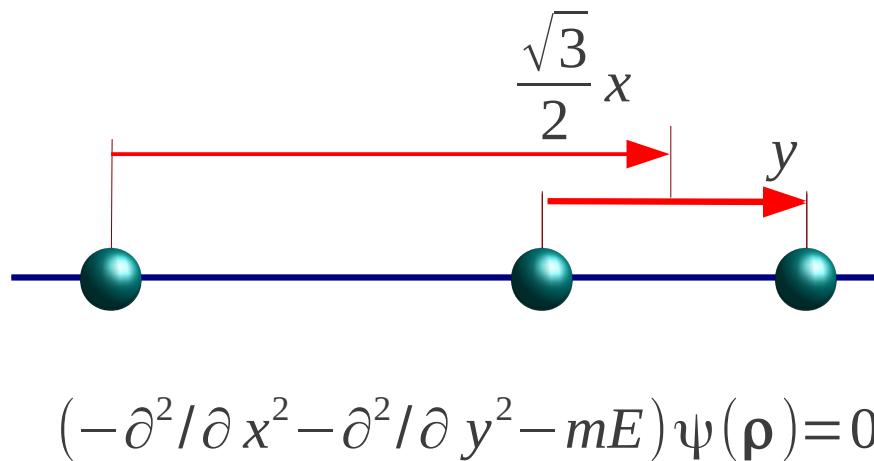
In particular,

3D:  $g_3 \propto (\hbar^2/m) \times \text{length}^4$

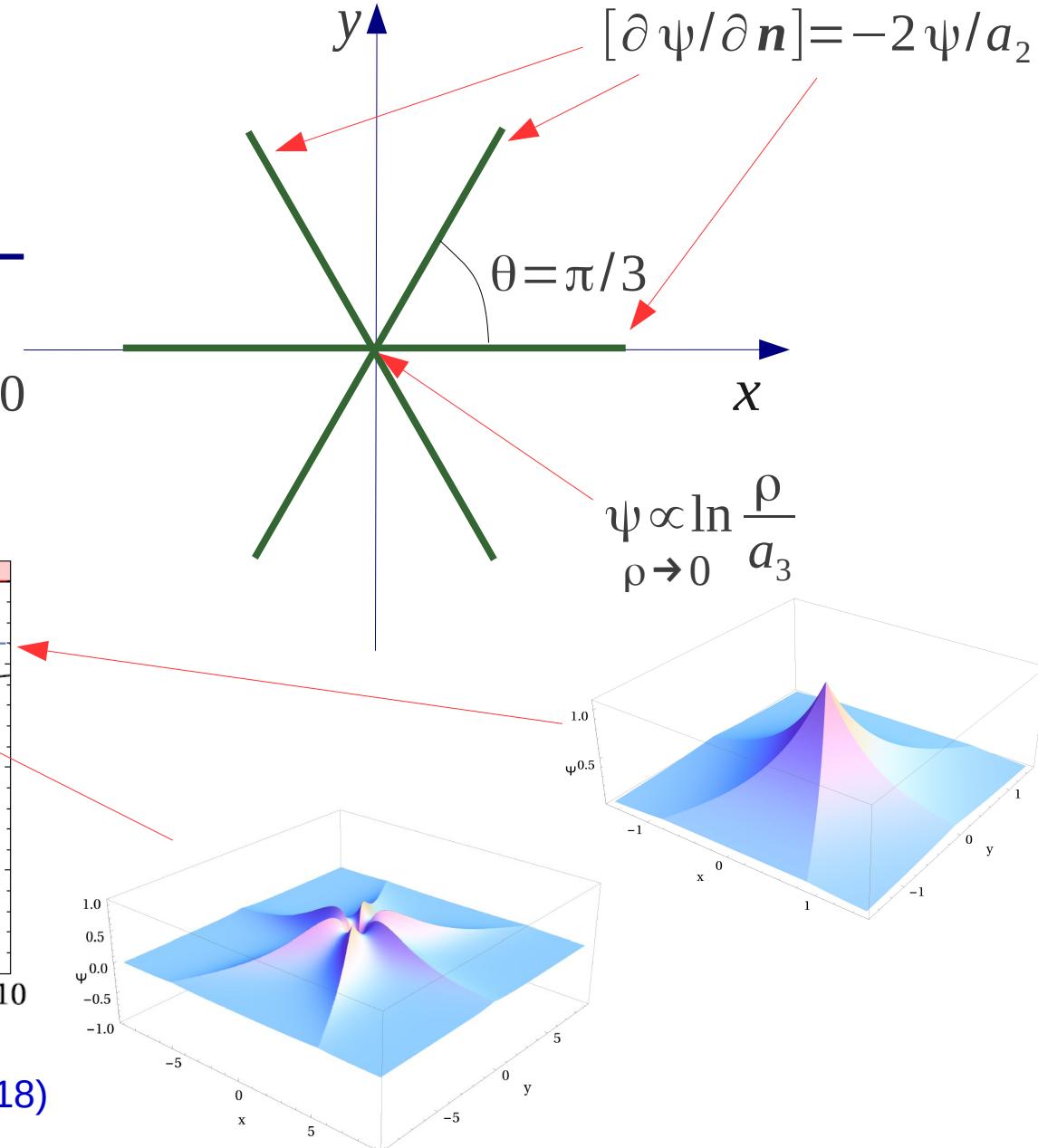
2D:  $g_3 \propto (\hbar^2/m) \times \text{length}^2$

1D:  $g_3 \propto (\hbar^2/m) / \ln(k \times \text{length}) = \text{small parameter}$

# Three-body problem with $a_2$ and $a_3$

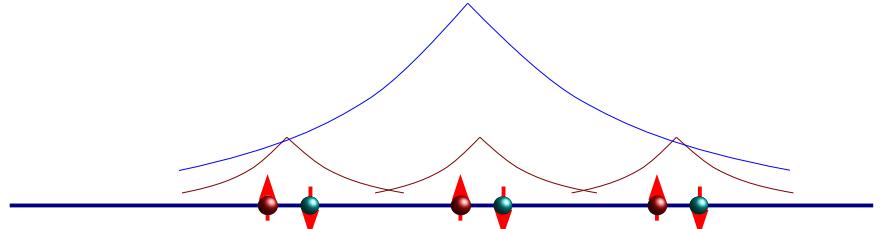
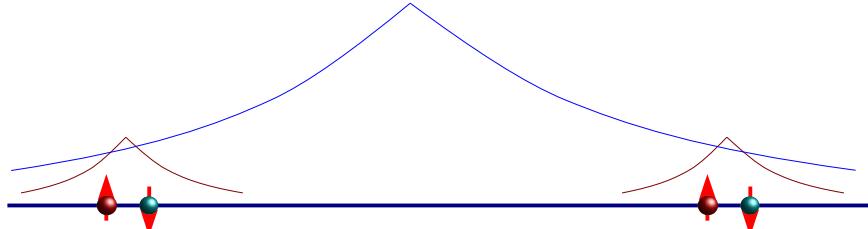


(Guijarro et al.'18, Nishida'18)



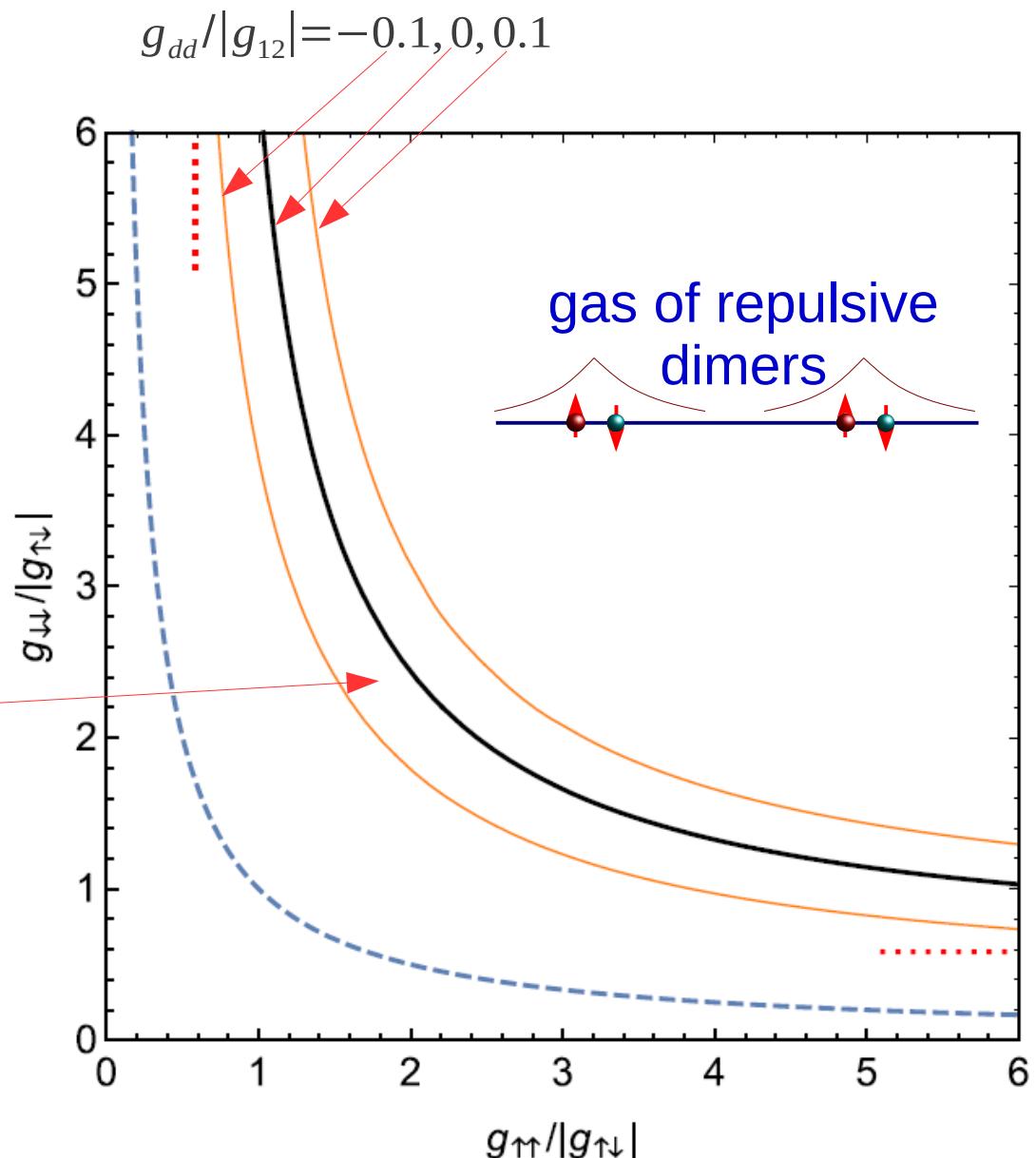
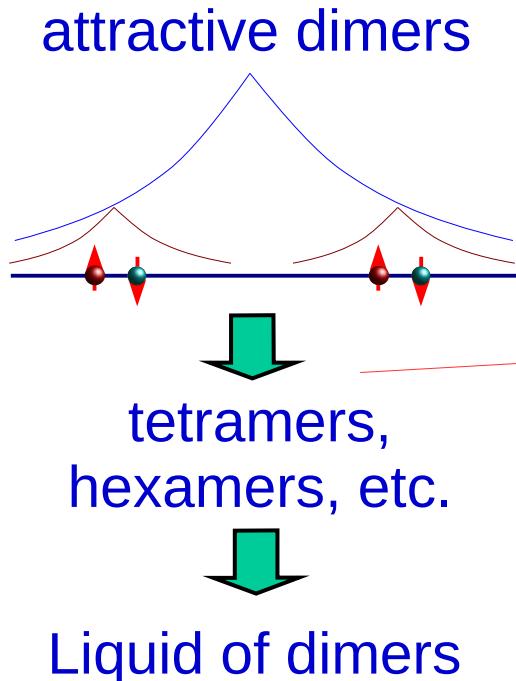
# Three-dimer interaction

Strategy: calculate tetramer and hexamer energies  
without three-dimer interaction they should differ by 4 (McGuire'64)



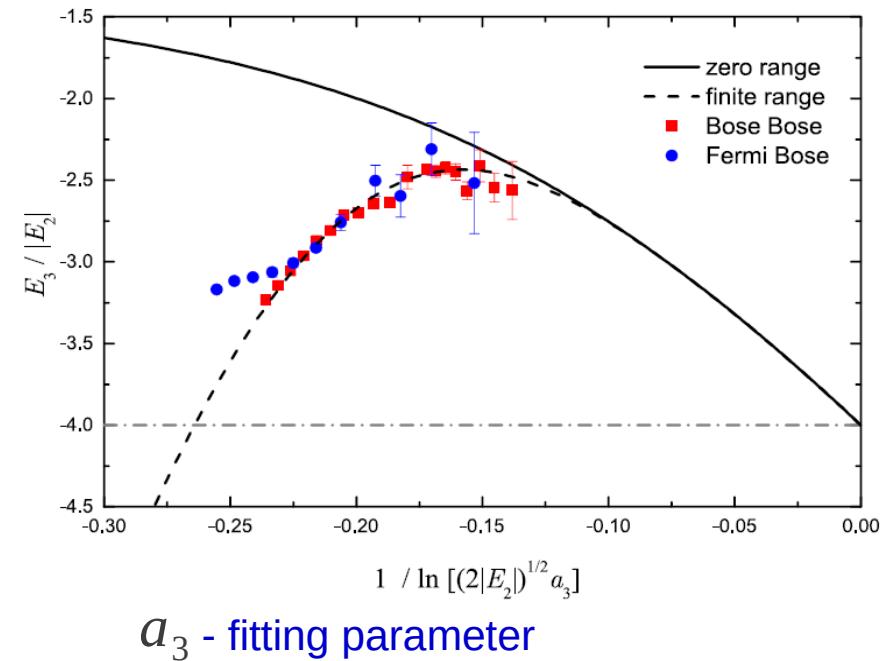
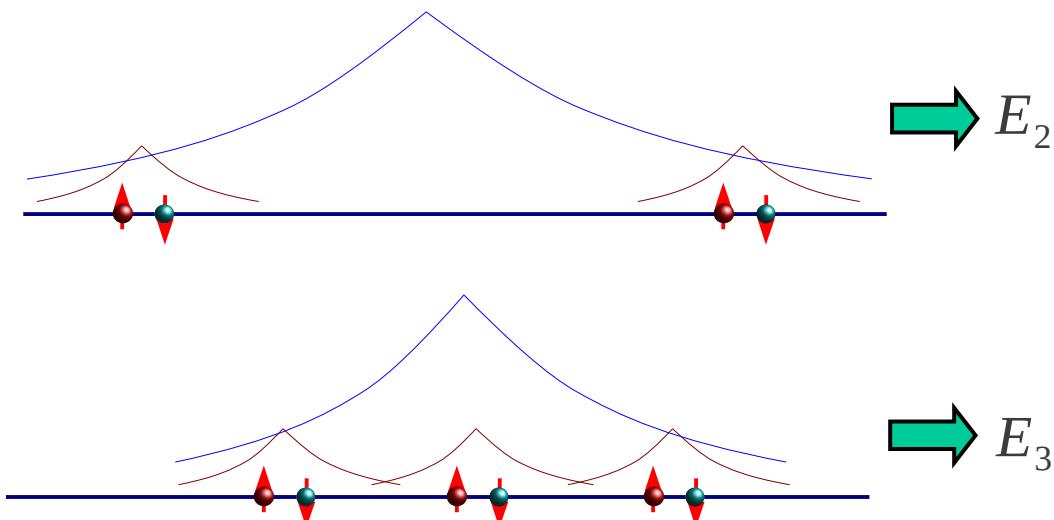
# 1D mixture phase diagram

(A. Pricoupenko&DP'18)



# Three-dimer interaction

Monte-Carlo calculation of the 2- and 3-dimer problem  
(Guijarro et al.'18)



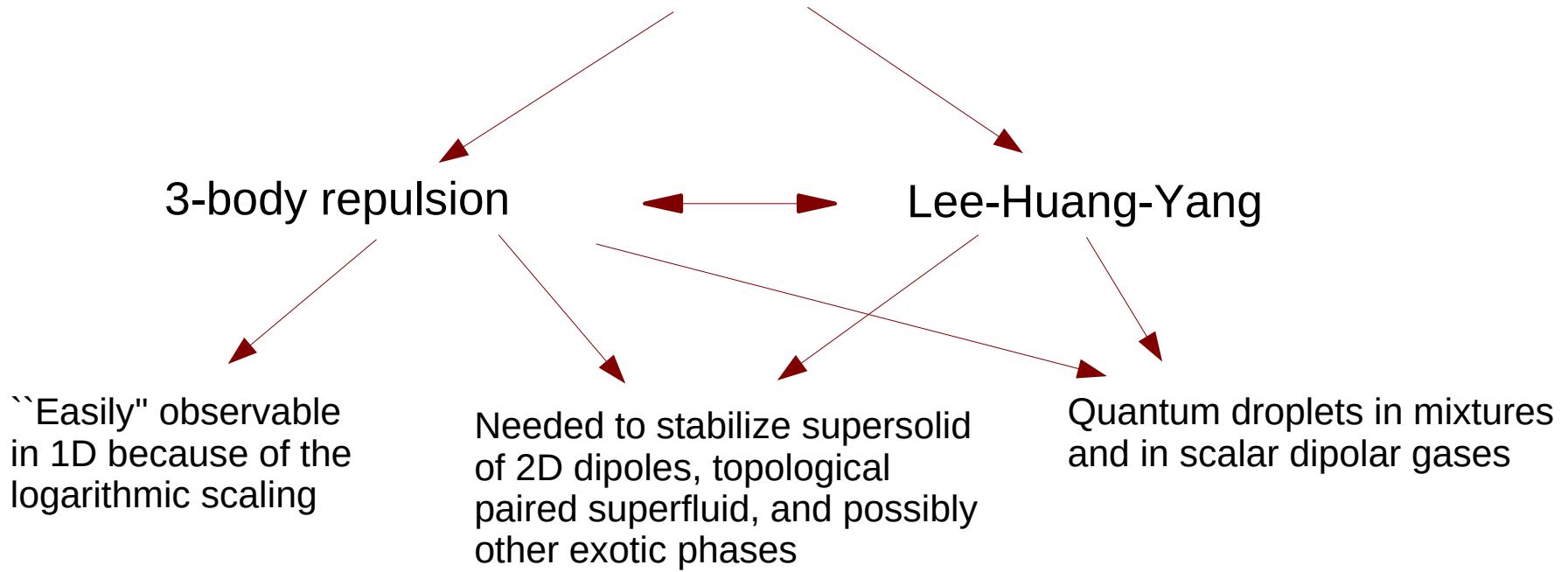
“Bose-Bose” case:  $g_{11}=g_{22} \approx -2.2 g_{12}$   $\rightarrow a_3 \approx 0.01 a_{12}$

“Fermi-Bose” case:  $g_{11}=\infty, g_{22} \approx -0.575 g_{12}$   $\rightarrow a_3 \approx 0.03 a_{12}$

True Fermi-Bose case can be realized with quasi-1D  
 $^{40}\text{K}$ - $^{41}\text{K}$  or  $^{40}\text{K}$ - $^{39}\text{K}$  mixtures!

# Summary

## Beyond-mean-field effects



### Main theoretical challenges:

- beyond mean field in the inhomogeneous case beyond LDA, dimensional crossover, etc.
- dynamics

Thank you for your attention!