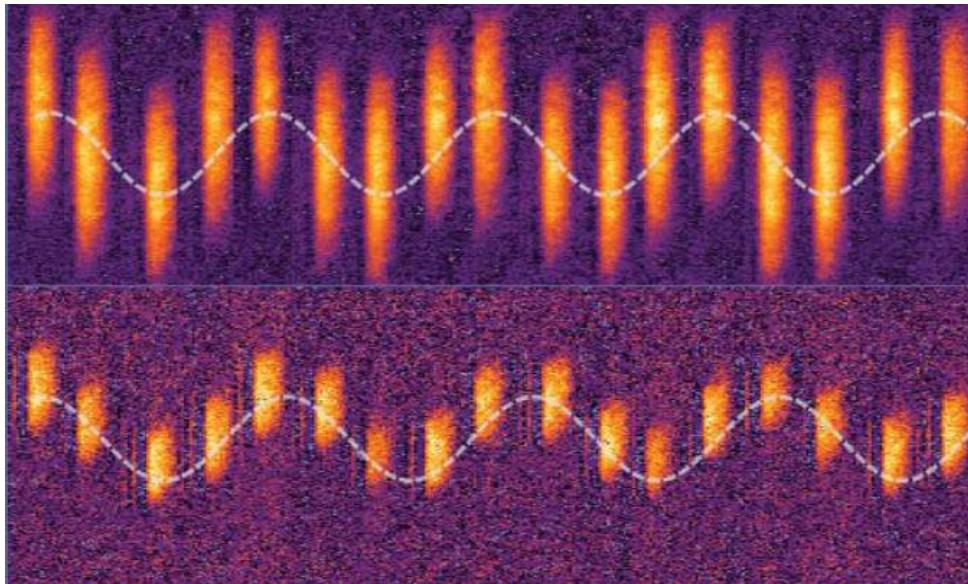


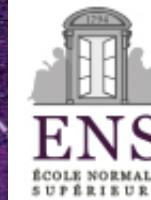
Ultracold Fermi gases: status and perspectives



C. Salomon



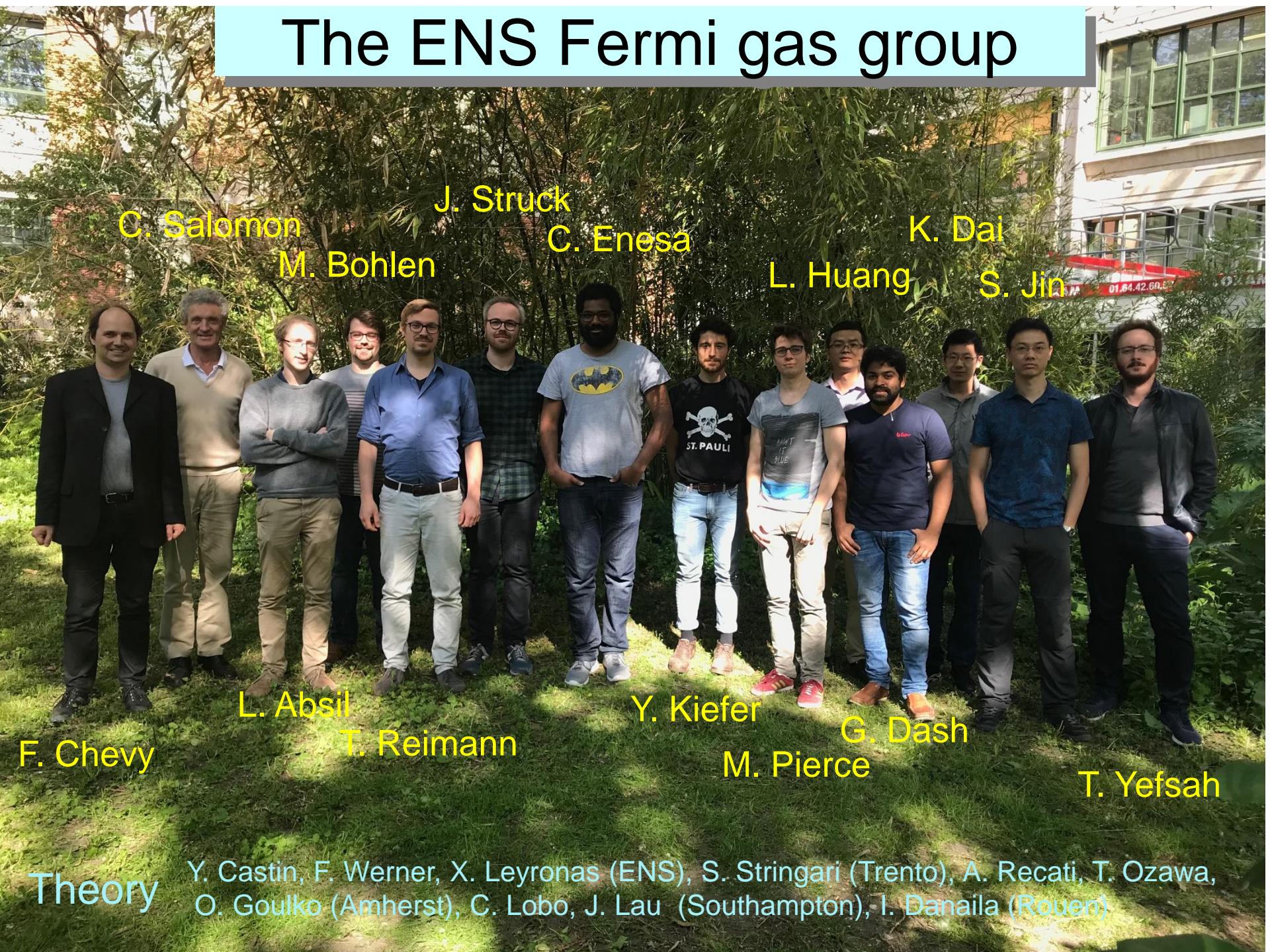
Orsay, November 12, 2018



Alexander von Humboldt
Stiftung / Foundation



The ENS Fermi gas group



Theory

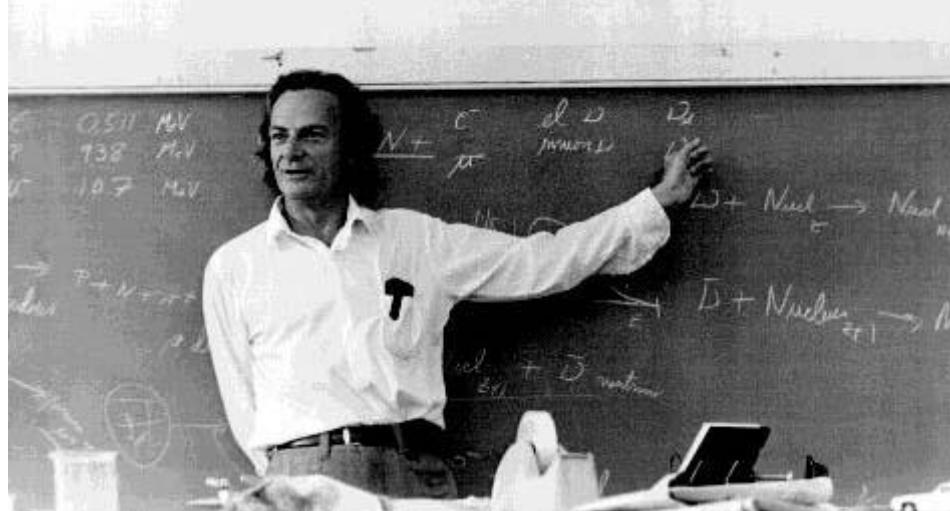
Y. Castin, F. Werner, X. Leyronas (ENS), S. Stringari (Trento), A. Recati, T. Ozawa,
O. Goulko (Amherst), C. Lobo, J. Lau (Southampton), I. Danaila (Rouen)

Outline

- Quantum simulation with ultracold atoms
- Overview of recent advances
- Exploring the link between Bose condensation and fermionic superfluidity: equation of state
- Dual Bose-Fermi superfluid mixture

The vision

Simulating Physics with Computers
Richard P. Feynman
Received May 7, 1981



Can we simulate quantum Physics with computers ?

Exponential growth of the Hilbert space when increasing the number of interacting particles:  untractable, in particular for fermions

1) Universal Quantum Computer

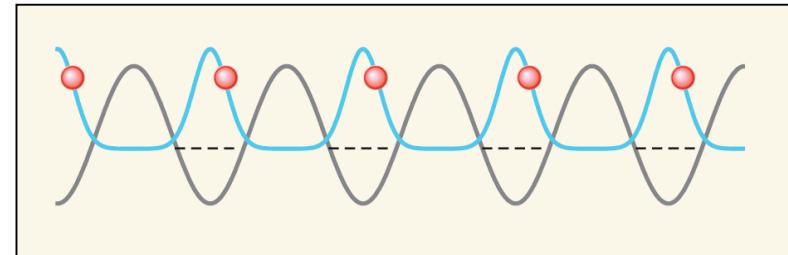
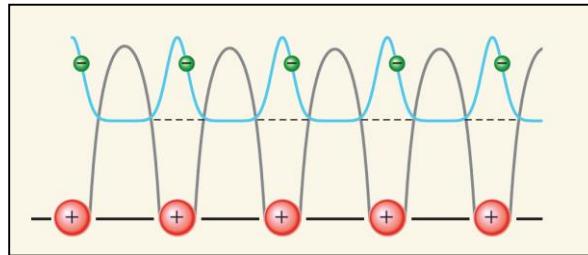
Ongoing research but extremely challenging

2) Analog simulation

- Write an Hamiltonian to describe a physical system
- Find a well controlled quantum system to simulate this Hamiltonian
- Measure the system's properties like ground state energy, excitation spectrum, collective modes,...
- Explore new parameter regimes

Dilute ultracold gases are good quantum simulators

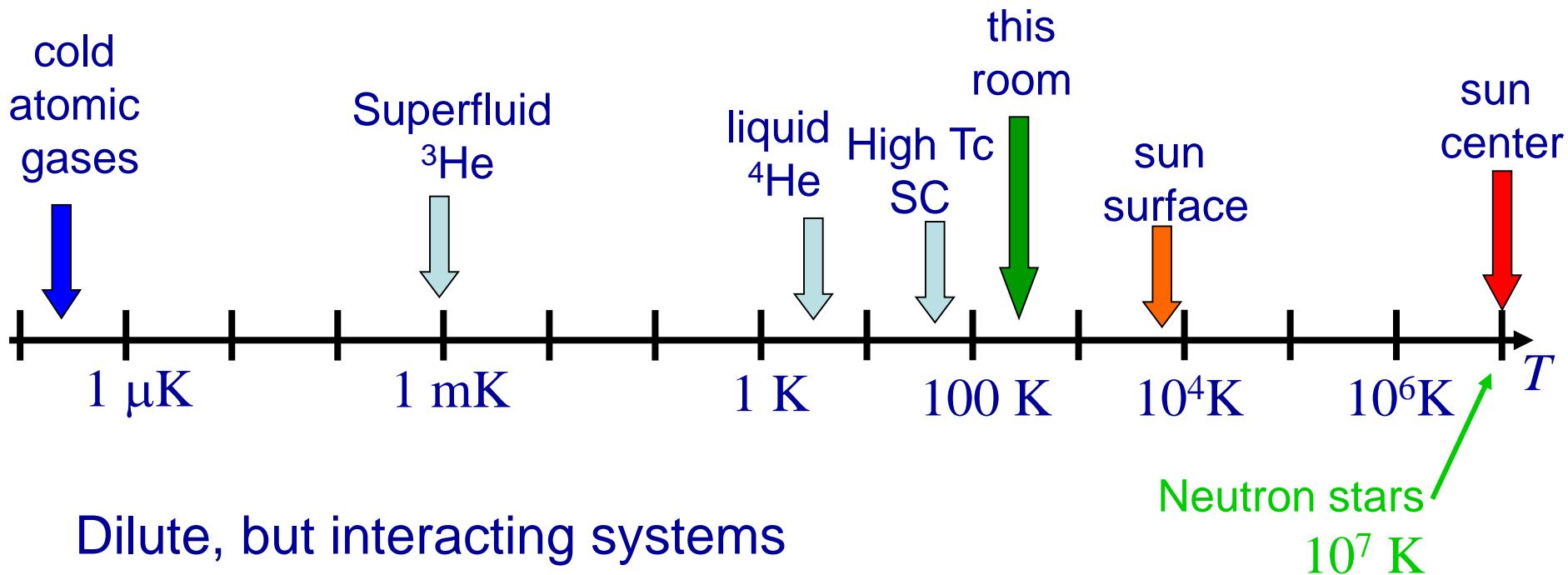
Interacting ultracold atoms enlarged simulators for condensed matter physics



Why ultracold atoms?

- Complete control of microscopic parameters
- Clean systems, no impurities
- Dynamics on observable timescales
- Understood from first principles
- Large inter-particle spacing makes optical imaging/manipulation possible: access to new observables such as density and spin correlation functions

Temperature scale of cold gases



Typical density: $\rho = 10^{13}$ to 10^{15} atoms/cm³

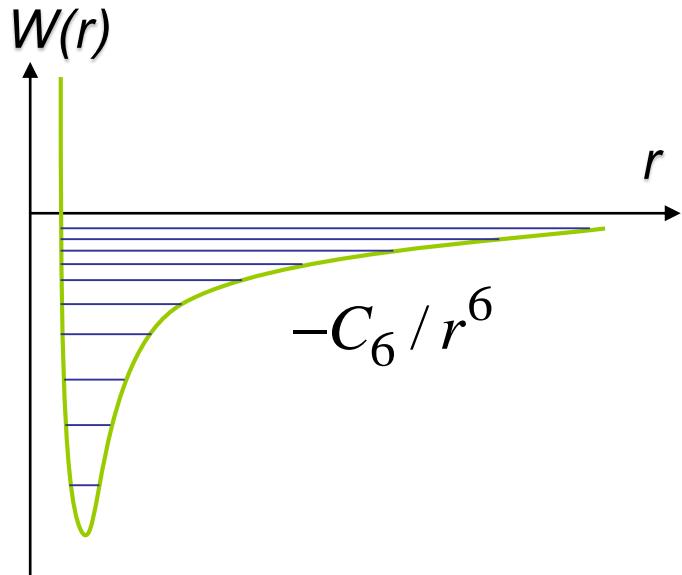
Interatomic distance 0.1 to 0.5 μm \gg range of interatomic potentials

$E_{\text{int}} \gg \hbar\omega$ quantum of motion in the trap or box

$E_{\text{int}} \gg k_B T$ thermal energy

Equilibrium properties and dynamics are governed by interactions

Atom-atom interactions



The magnitude and sign of a depend sensitively on the detailed shape of long range potential
Importance of position of last bound state

At low temperature,
only s wave collisions, $l = 0$

$$\psi(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} - \frac{a}{r} e^{ikr}$$

$$a = -\lim_{k \rightarrow 0} \frac{\tan \delta_0(k)}{k}$$

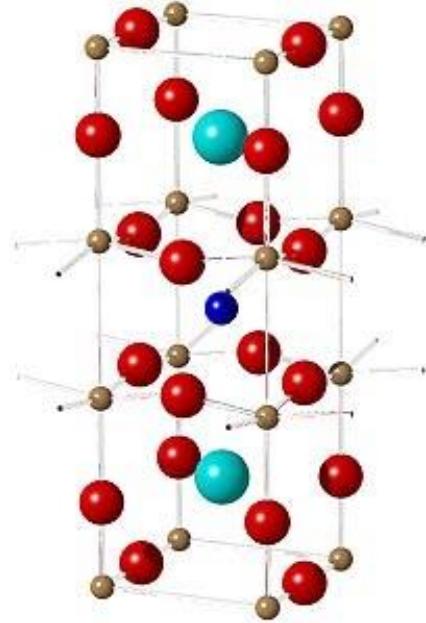
a : scattering length
 $|a| \sim 1$ to 10 nm

$$V(\vec{r}_1 - \vec{r}_2) = \frac{4\pi\hbar^2 a}{m} \delta(\vec{r}_1 - \vec{r}_2)$$

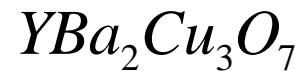
Tuning interactions via
Fano-Feshbach resonance

$a > 0$: effective repulsive interaction
 $a < 0$: effective attractive interaction

Quantum simulation with ultracold fermions

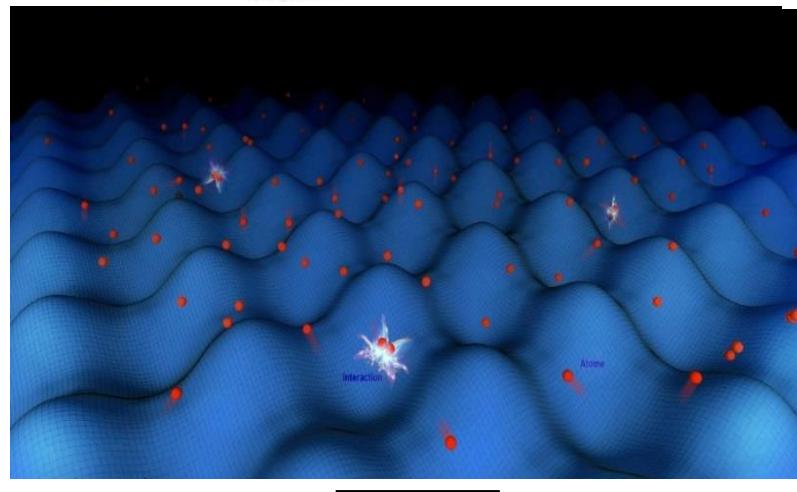


Ba
O
Y
Cu
c
b
a



$$d \sim 0.4 \text{ nm}$$

$$T_c = 92K$$



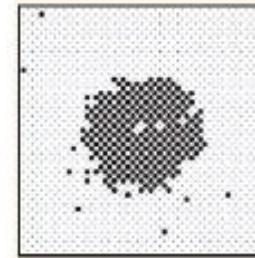
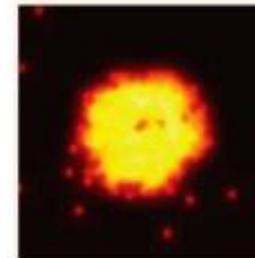
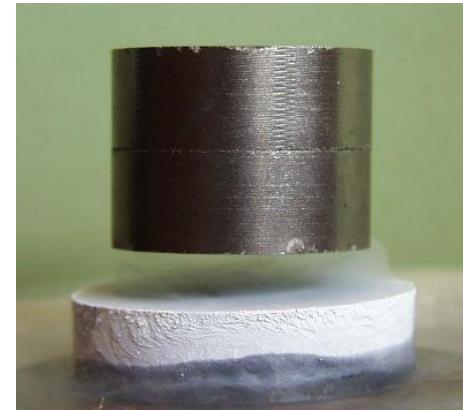
0.5 micron

$$d \sim 0.5 \mu\text{m}$$

$$T \sim 1 \mu\text{K}$$

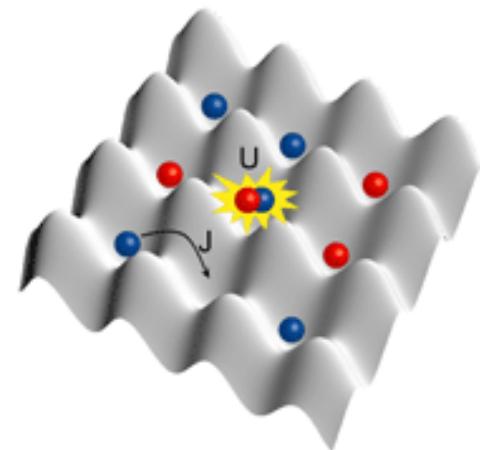
Seeing atoms
one by one !

Bloch et al., MPQ 2010
Greiner et al., Harvard



$n=1$

High T_c superconductivity and Fermi Hubbard model

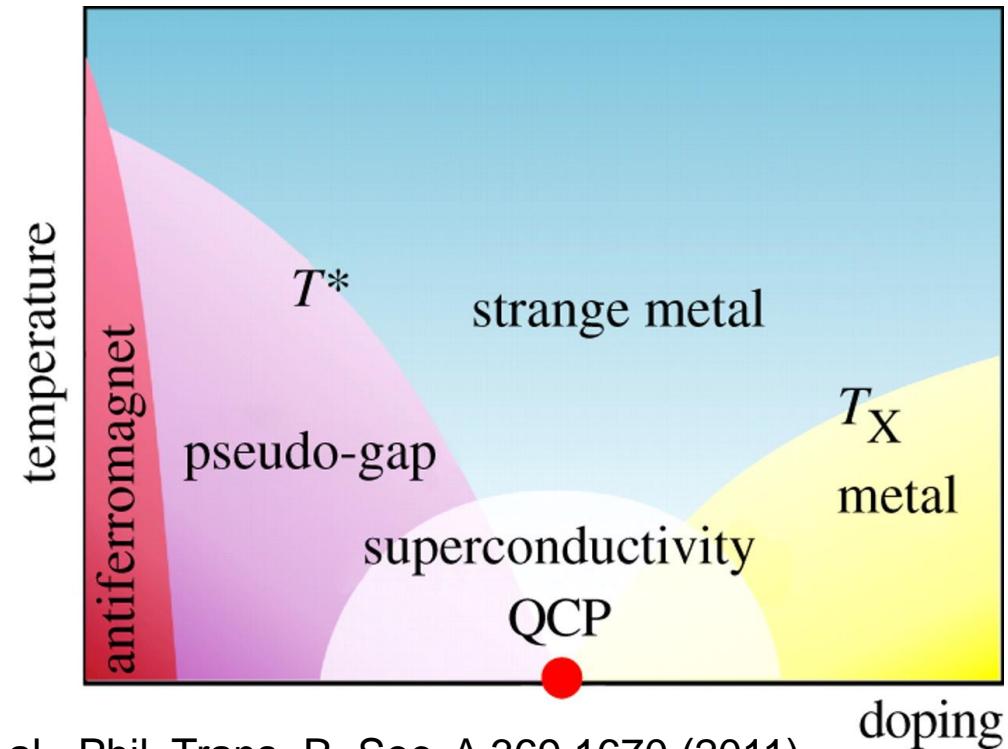


$$\mathcal{H} = -t \sum_{\langle ij \rangle, \sigma} (c_{i,\sigma}^\dagger c_{j,\sigma} + c_{j,\sigma}^\dagger c_{i,\sigma}) + U \sum_i n_{i,\uparrow} n_{i,\downarrow}$$

Realized naturally with cold atoms in optical lattices with fully tunable parameters.

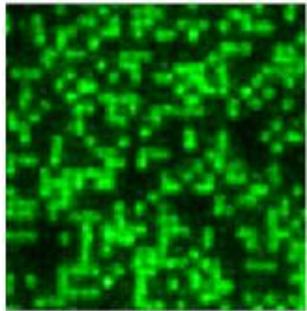
Rich phase diagram:

commensurate/
incommensurate AFM,
pseudogap, strange metal,
d-wave superconductivity...

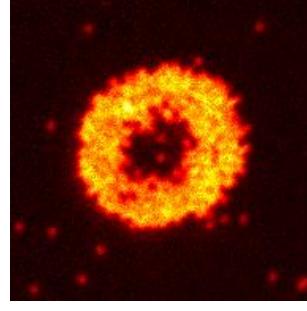


Quantum gas microscopy

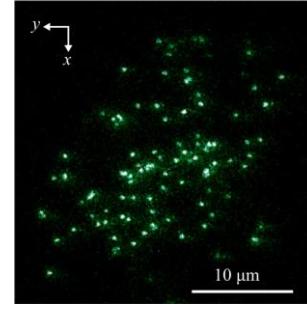
- Boson microscopes: 2010



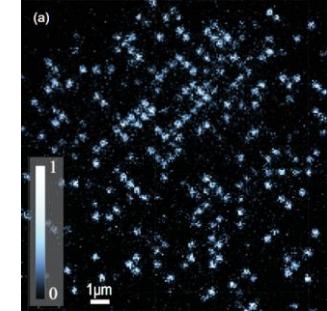
Harvard



MPQ

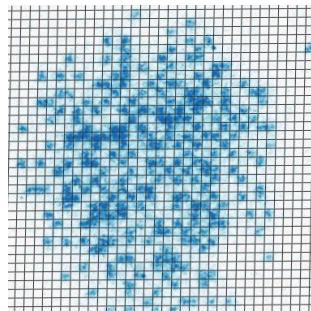


Kyoto

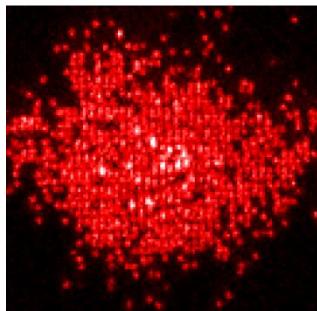


Tokyo

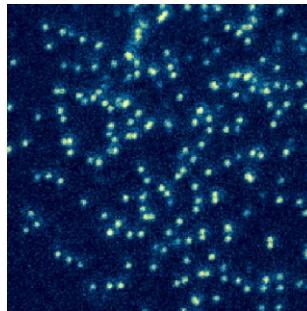
- Fermion microscopes: 2015



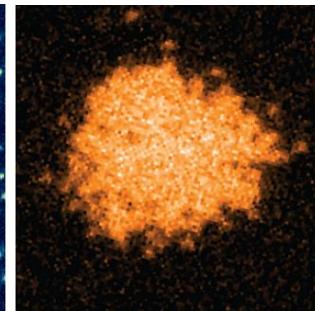
Harvard



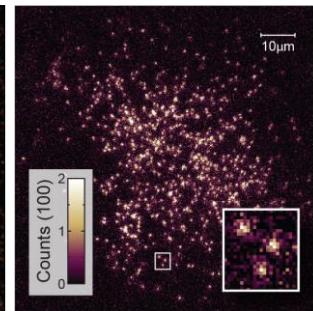
MPQ



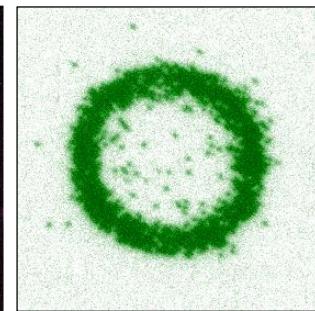
Strathclyde



MIT

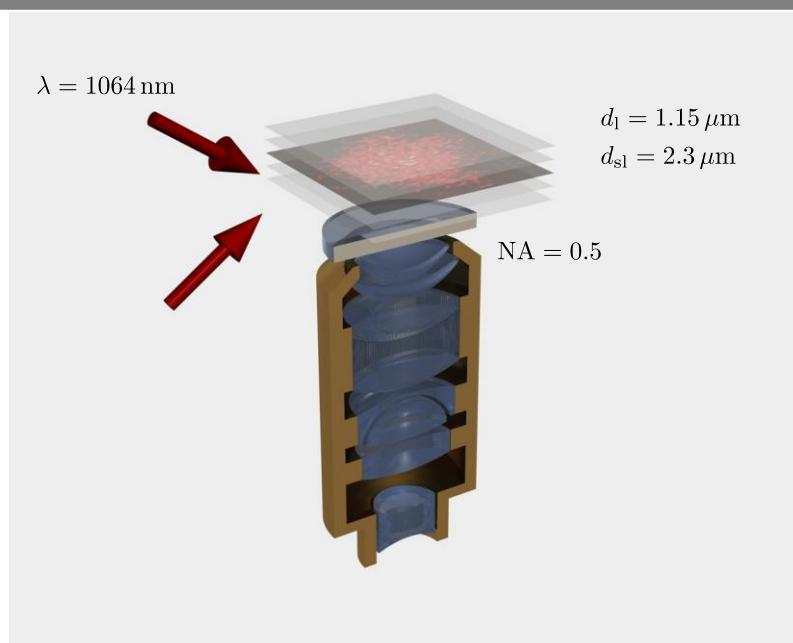


Toronto

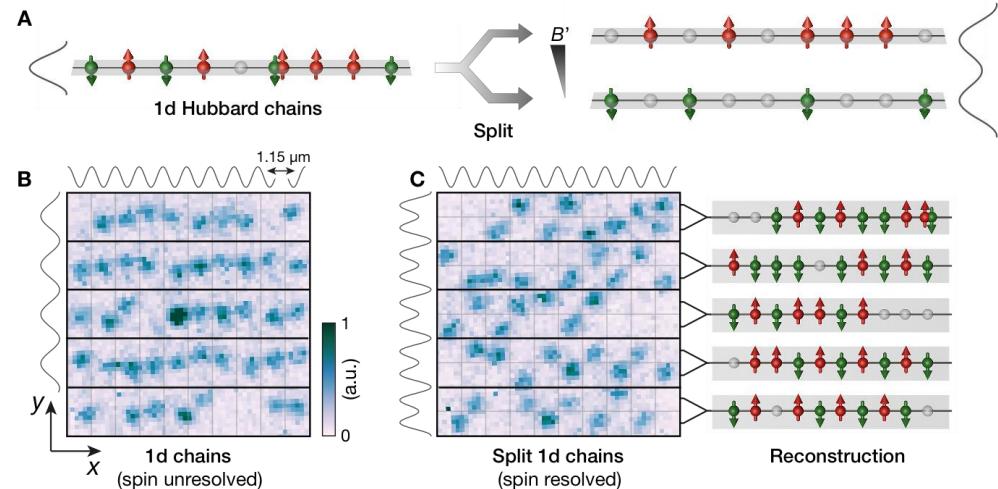


Princeton

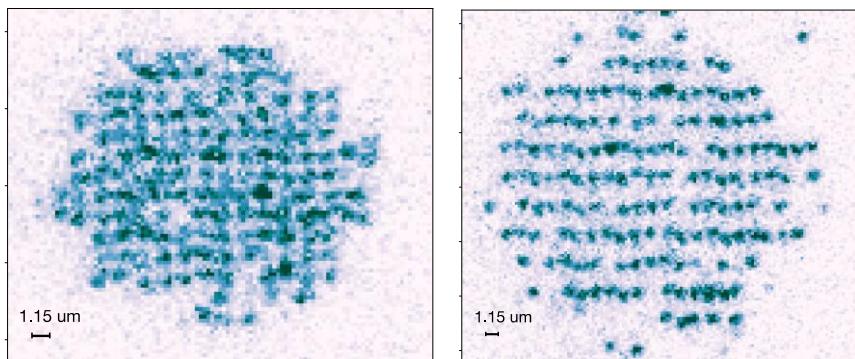
^6Li Quantum gas microscope in Munich



Spin resolved imaging



M. Boll, T. Hilker, G. Salomon *et al.* *Science* **353**, 6305 (2016)



2D
1D
Mott insulators

**Direct access to spin, charge,
holes and doublons**

Measurement of arbitrary spin/density
correlations

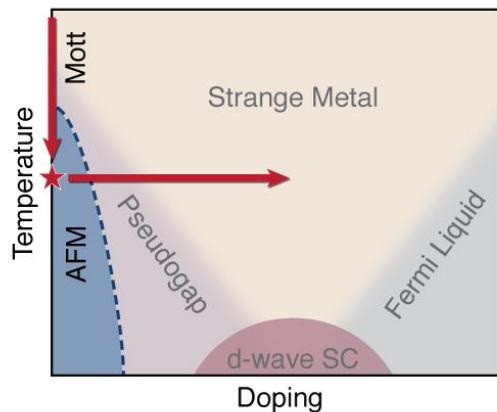


The Fermi Hubbard model in 2D

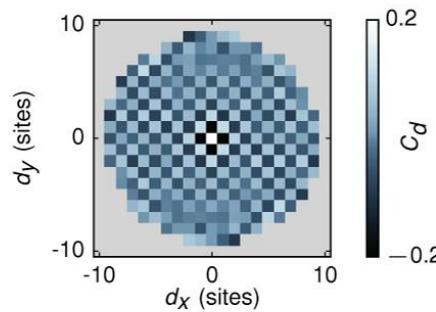
$$\hat{H} = -t \sum_{\sigma, \langle i,j \rangle \in \Omega} (\hat{c}_{i,\sigma}^\dagger \hat{c}_{j,\sigma} + h.c.) + U \sum_{i \in \Omega} \hat{n}_{i,\downarrow} \hat{n}_{i,\uparrow},$$

M. Greiner, Harvard
 I. Bloch, MPQ
 M. Koehl, Bonn
 S. Kuhr, Glasgow
 M. Zwierlein MIT
 J. Thywissen, Toronto

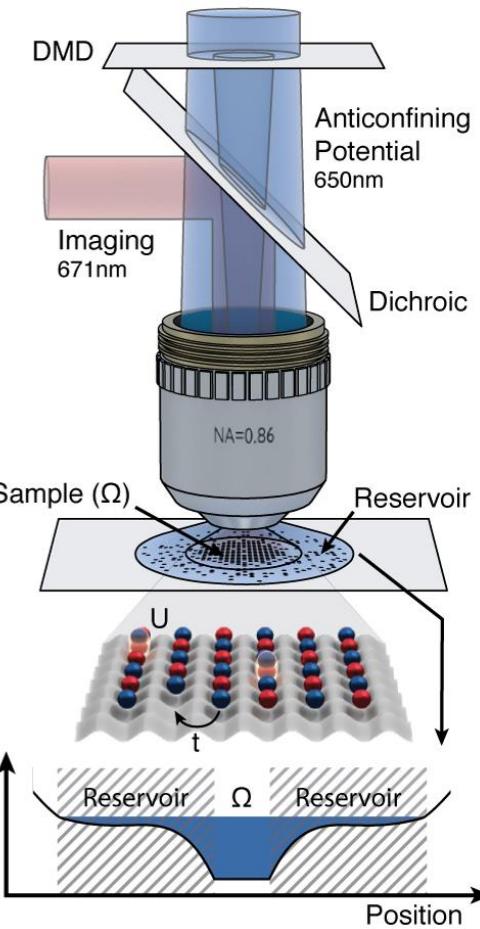
The Hubbard Model



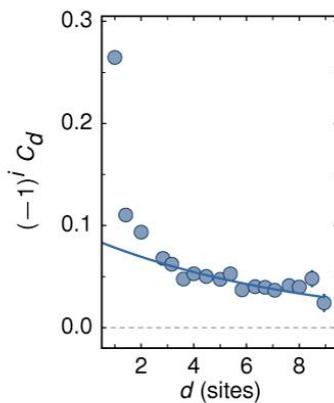
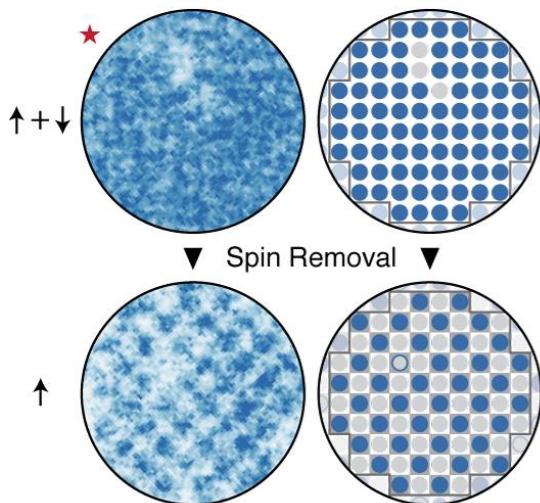
Spin Correlation Function



Entropy Redistribution



Long-range Antiferromagnet



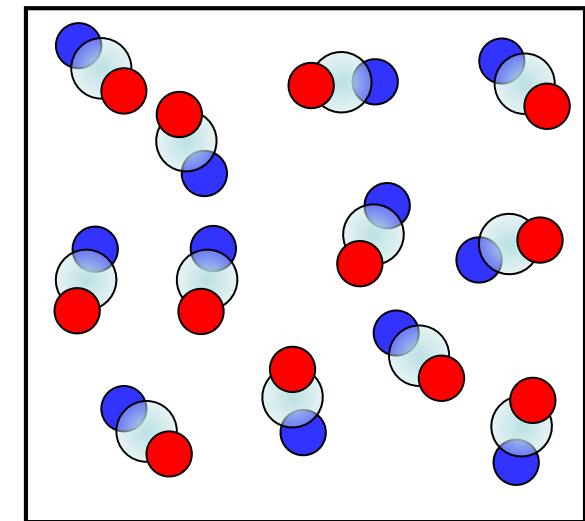
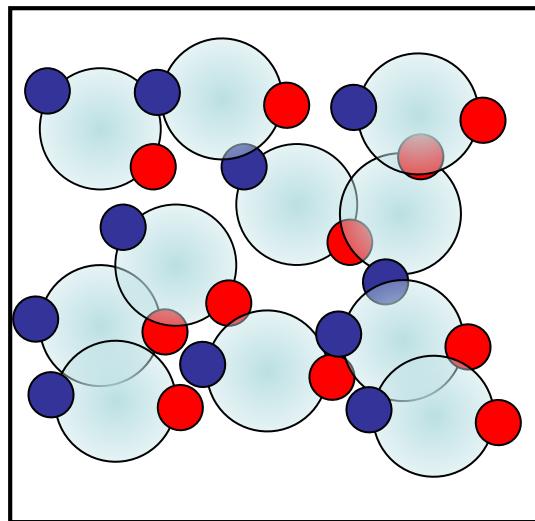
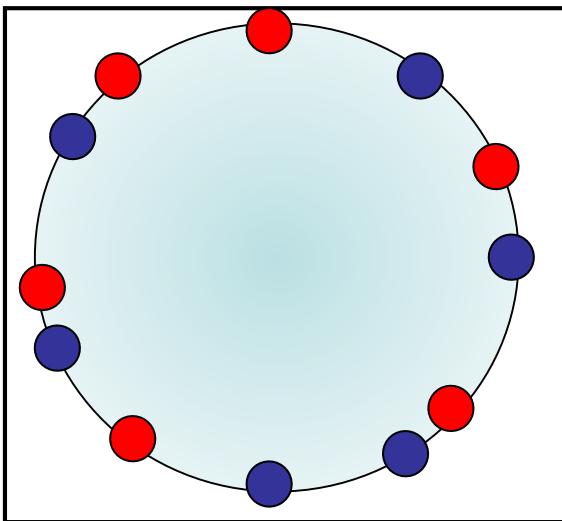
Exploring the link between BCS superfluidity and Bose-Einstein condensation

Leggett, Nozières, Schmidt-Rink,.. '80

Fermions with two spin states and attractive interaction the BCS-BEC Crossover

Increasing attraction strength

$$T_c \approx T_F e^{-\pi/2k_F|a|}$$



BCS regime:

$$k_F|a| \ll 1$$

Cooper pairs $k, -k$

Well localized in

Momentum: $k \sim k_F$

Delocalized in position

On resonance

$$na^3 \gg 1$$

$$k_F a \geq 1$$

Pairs stabilized by

Fermi sea

Size of pairs

$$\hbar v_F / \Delta \sim k_F^{-1}$$

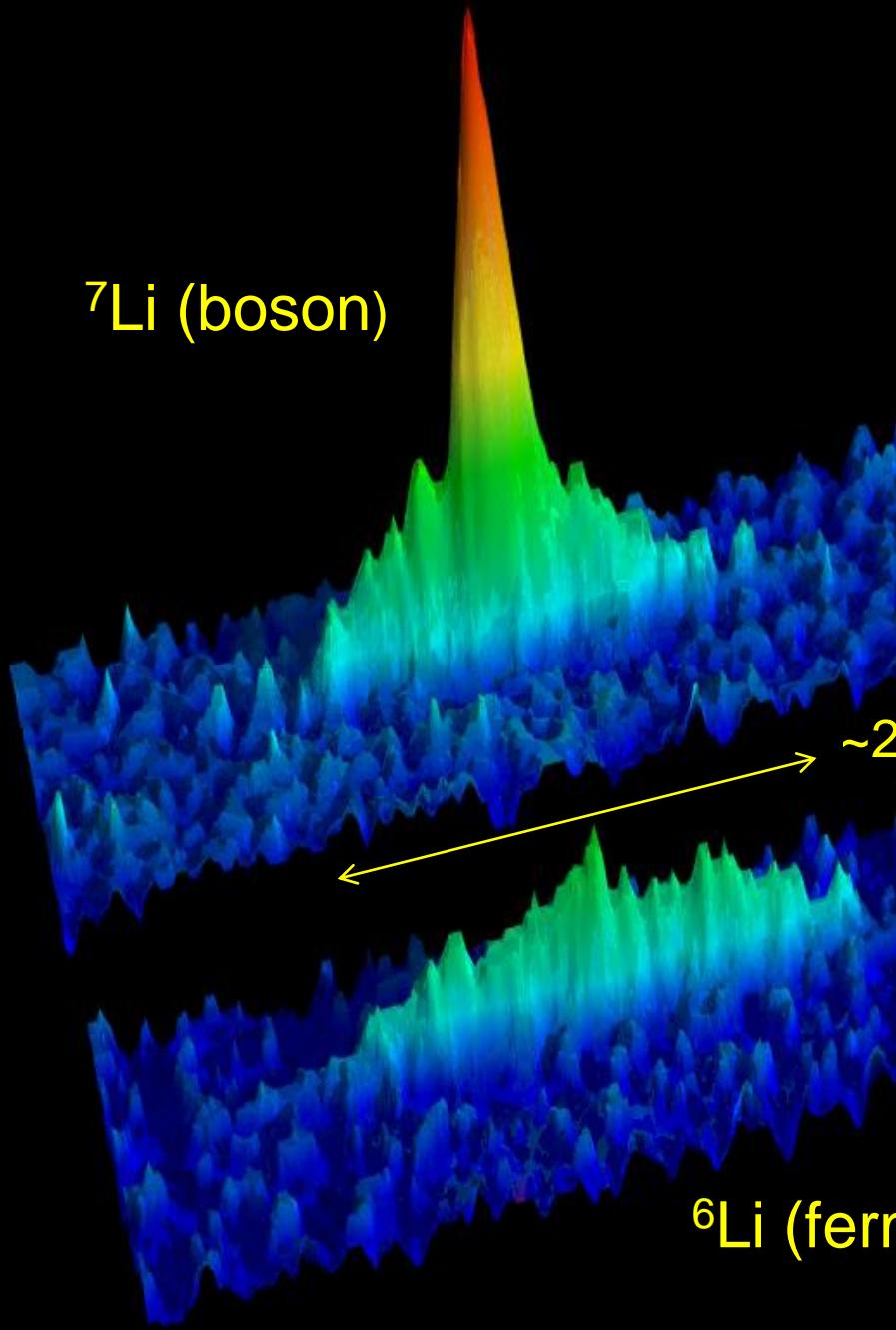
Molecular Condensate

A two-body bound state
strongly bound molecules

Size: $a \ll n^{-1/3}$

$n^{-1/3}$: average distance
between particles

ENS 2001



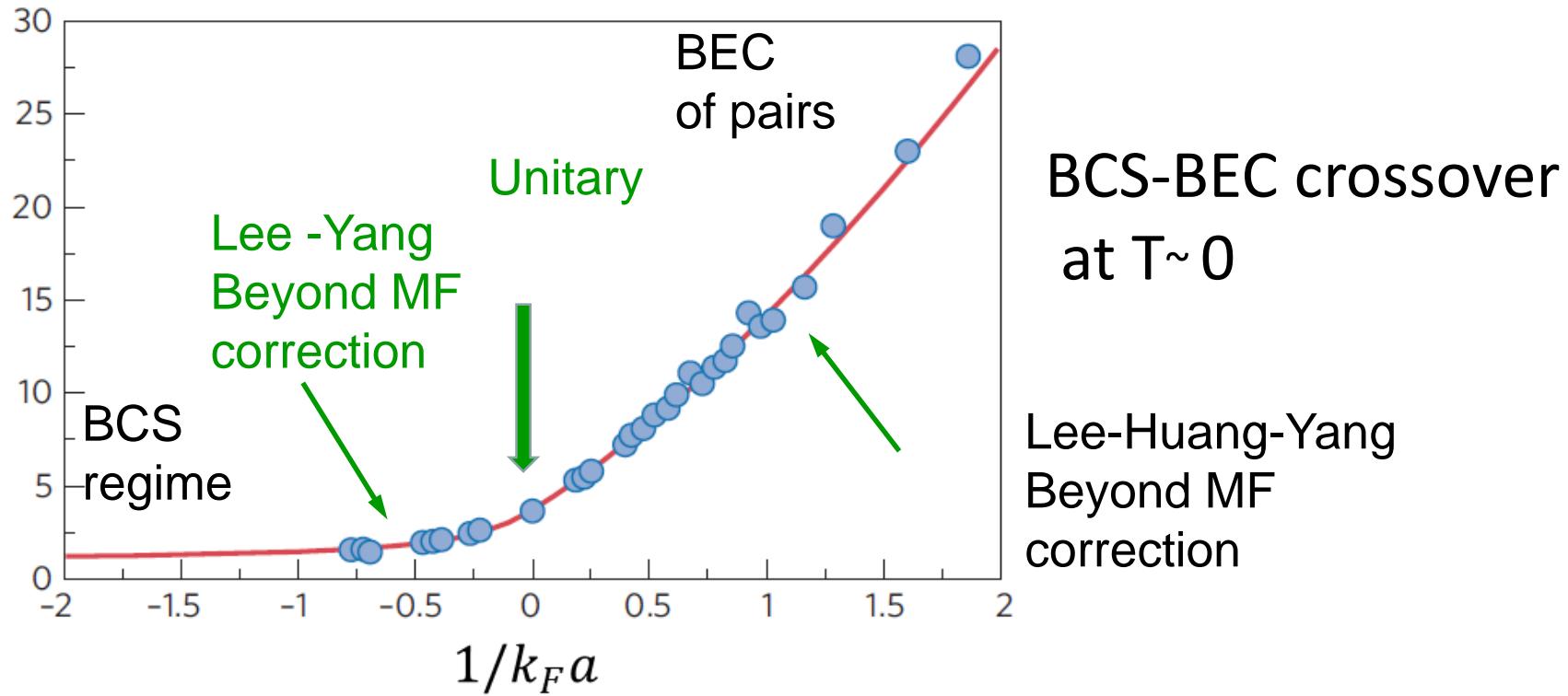
Bose-Einstein
condensate

Fermi sea

${}^6\text{Li}$ (fermion)

Equation of State of Fermi gas in the BEC-BCS crossover

Pressure equation of state $P/P_0 = f(1/k_F a)$



An example of quantum simulation in the strongly correlated regime

Universal Equation of State at unitarity

$$1/k_F a = 0$$

Thermodynamics is universal

T. L. Ho, E. Mueller, '04

The system has continuous scale invariance

Pressure depends only on $\mu/k_B T$

$$T_c = 0.16 T_F$$

At $T=0$

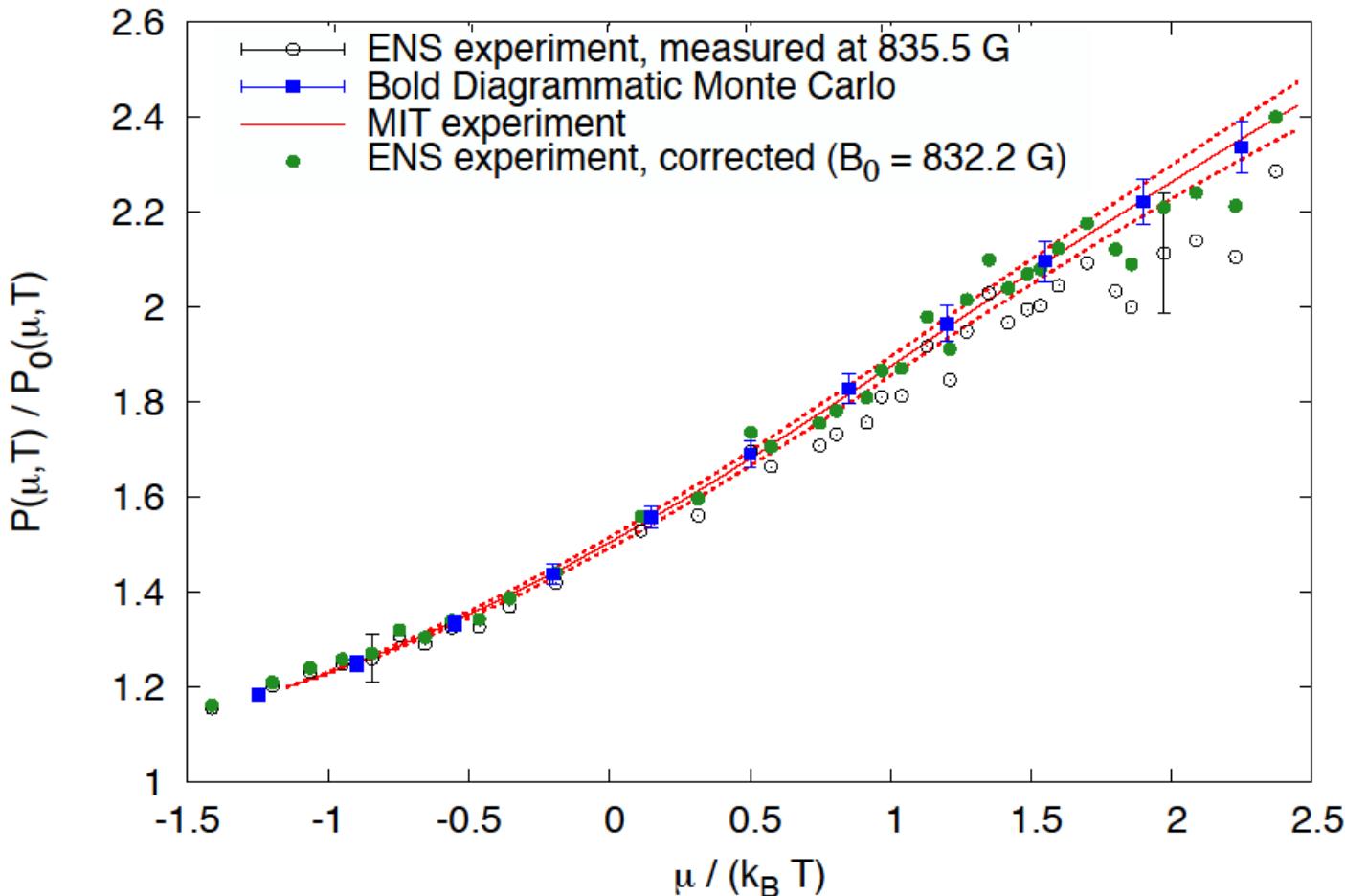
$$\mu = 0.37 E_F$$

Bertsch parameter

MIT 2012
Ku et al.

Universal Equation of State at Unitarity

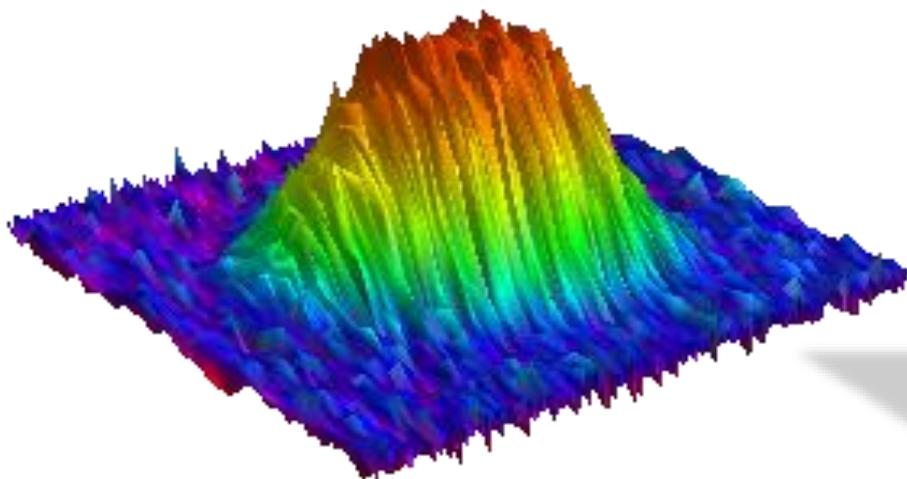
Comparison with MIT 2012 and Bold diag MC simulation



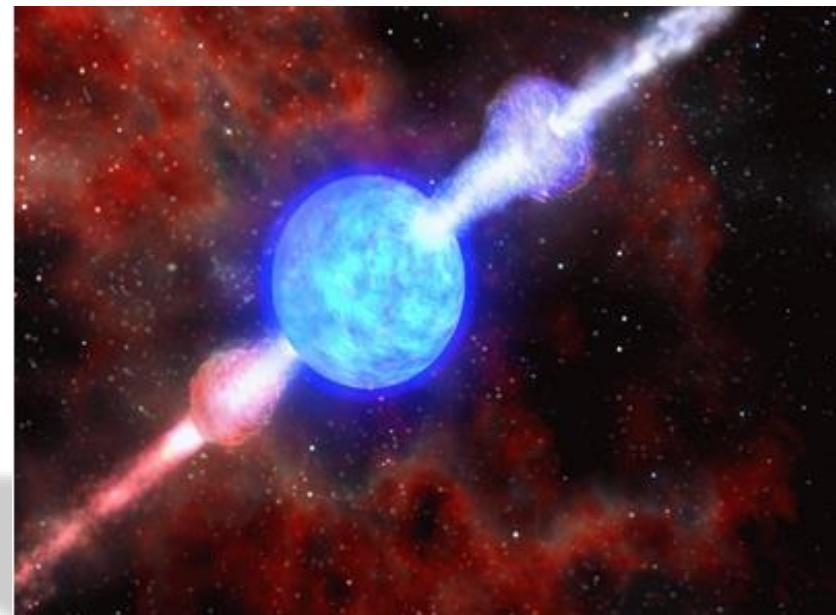
Theory:
Van Houcke,
Werner,
Kosik, Prokof'ev,
Svistunov,
Ku, Sommer
Cheuk, Schirotzek
Zwierlein
Nature Phys.,
2012

5% agreement with a Many-Body theory in strongly interacting regime

Comparison with neutron stars



lithium 6 atoms, spin $\frac{1}{2}$,
 $n \sim 10^{13} \text{ cm}^{-3}$, $T = 10^{-8} \text{ Kelvin}$
A superfluid 1 million times
thinner than air !



Neutron star, Spin $\frac{1}{2}$
 $a = -18.6 \text{ fm}$, $n \sim 2 \cdot 10^{36} \text{ cm}^{-3}$
• $T_c = 10^{10} \text{ K}$, $T = T_F/100$
• $k_F a \sim -4, -10, \dots$
1000 billion times denser than Earth !
Baym, Carlson, Bertsch, Schwenk...

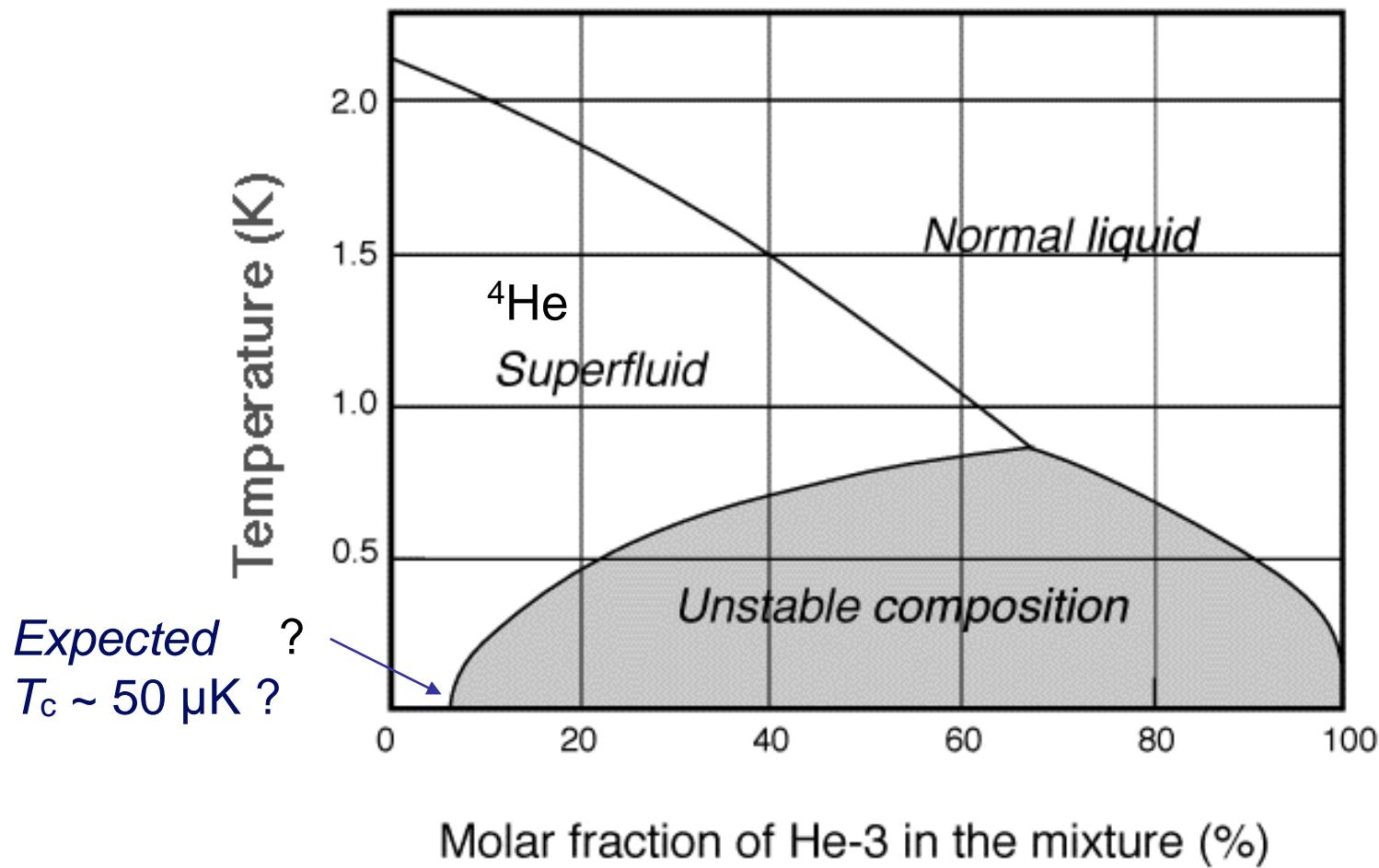
Final example

A novel system

Bose-Fermi superfluid mixture

I. Ferrier-Barbut, M. Delehaye, S. Laurent, A. T. Grier, M. Pierce,
B. S. Rem, F. Chevy, and C. Salomon, **Science**, **345**, 1035, 2014

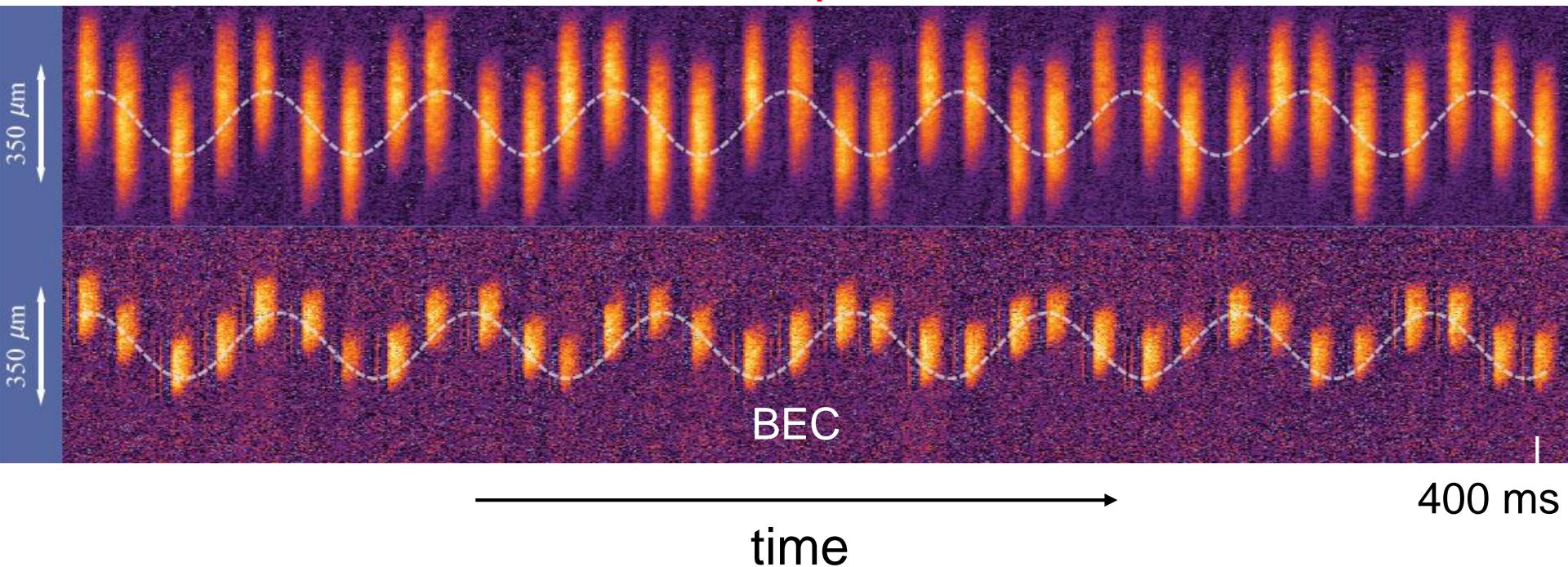
Searching for superfluid Bose-Fermi systems: ^4He - ^3He mixture



We use lithium 6 unitary Fermi gas mixed with lithium 7 bosons

Long-lived Oscillations of Superfluid Counterflow

Fermi Superfluid



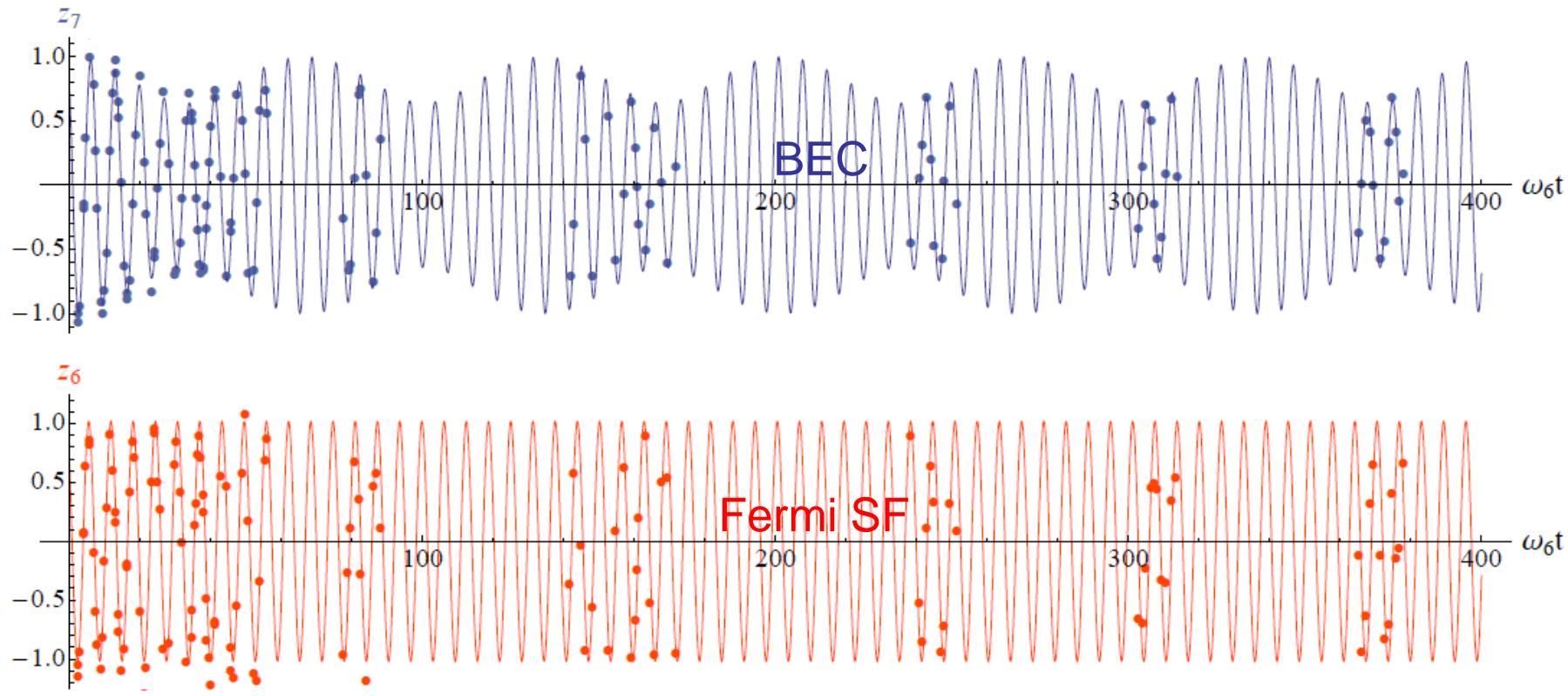
$$\tilde{\omega}_6 = 2\pi \times 17.06(1) \text{Hz}$$

$$\tilde{\omega}_7 = 2\pi \times 15.40(1) \text{Hz}$$

I. Ferrier-Barbut et al., Science, **345**, 1035, (2014)

Also, C. Hammer et al Phys. Rev. Lett. **106**, 065302 (2011) for boson-boson superfluid counterflow

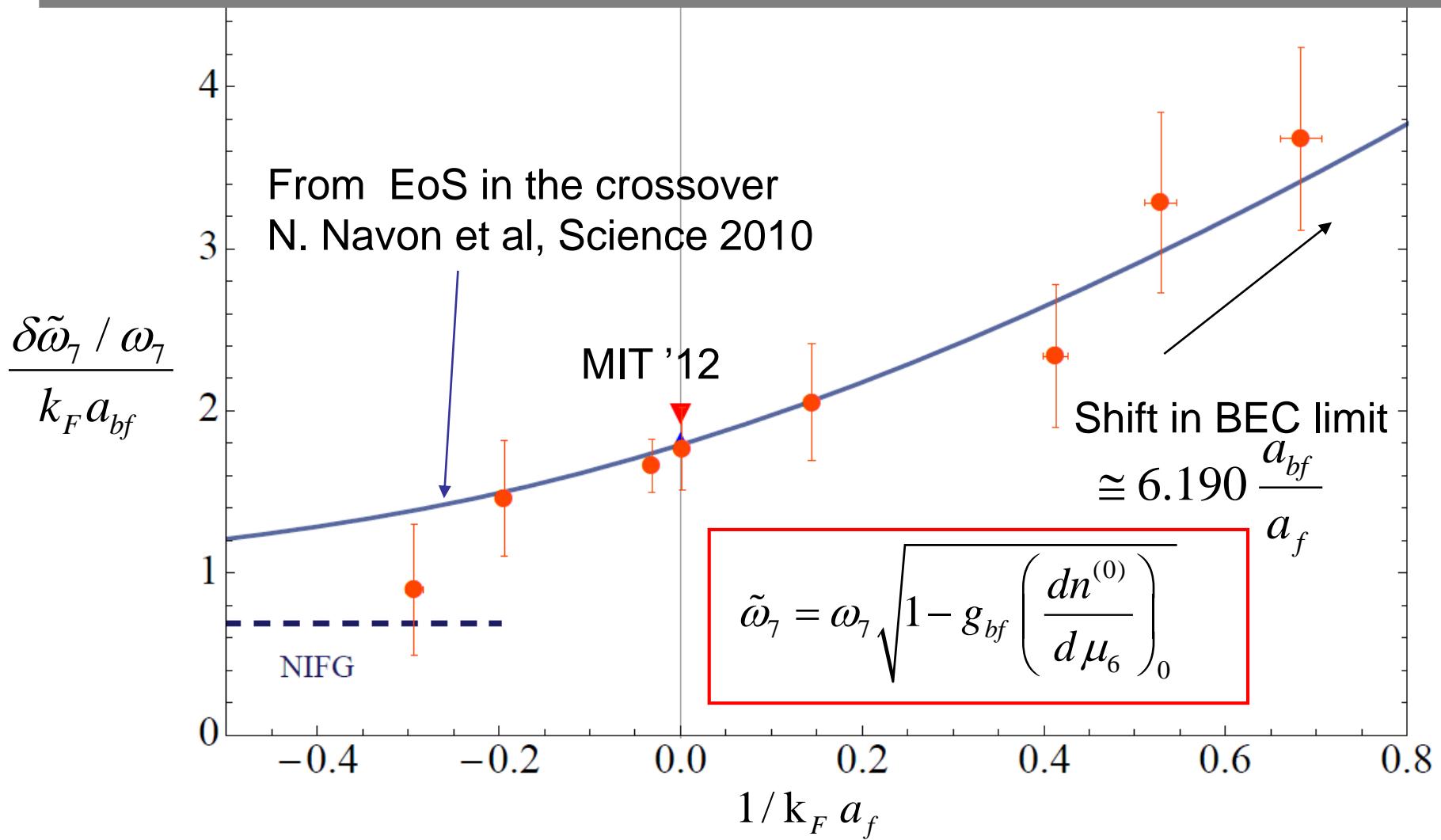
Oscillations of both superfluids



0 Very small damping: superfluid counterflow 4 s
Modulation of the ${}^7\text{Li}$ BEC amplitude by $\sim 30\%$ at $(\tilde{\omega}_6 - \tilde{\omega}_7)/2\pi$
Coherent energy exchange between the two oscillators

Frequencies can be measured very precisely !

Equation of State and Bose-Fermi Coupling in BEC-BCS crossover



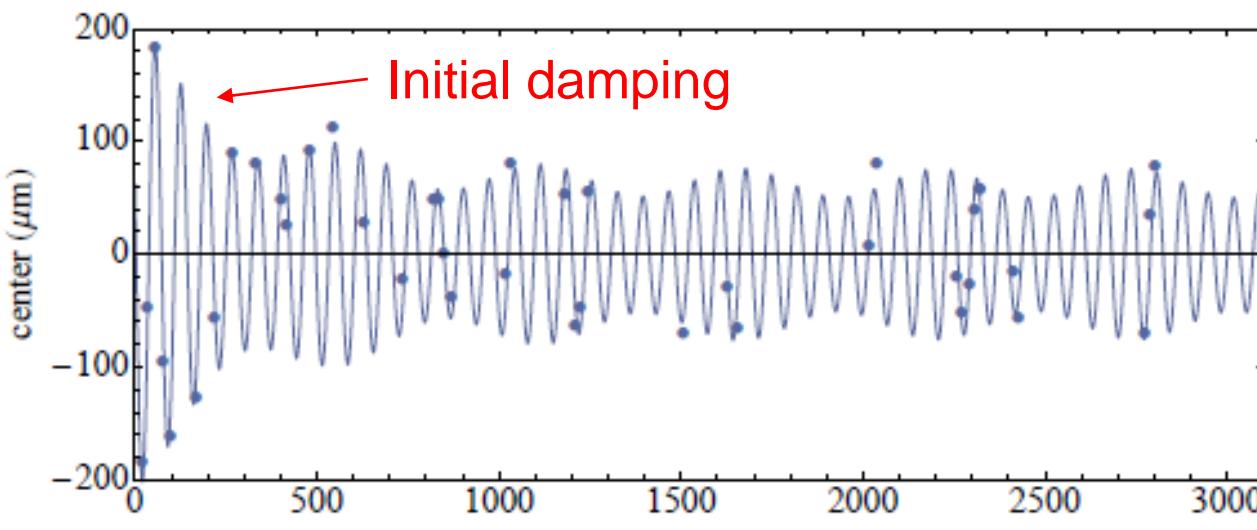
What is the critical velocity for superfluid counterflow ?

Increase initial displacement



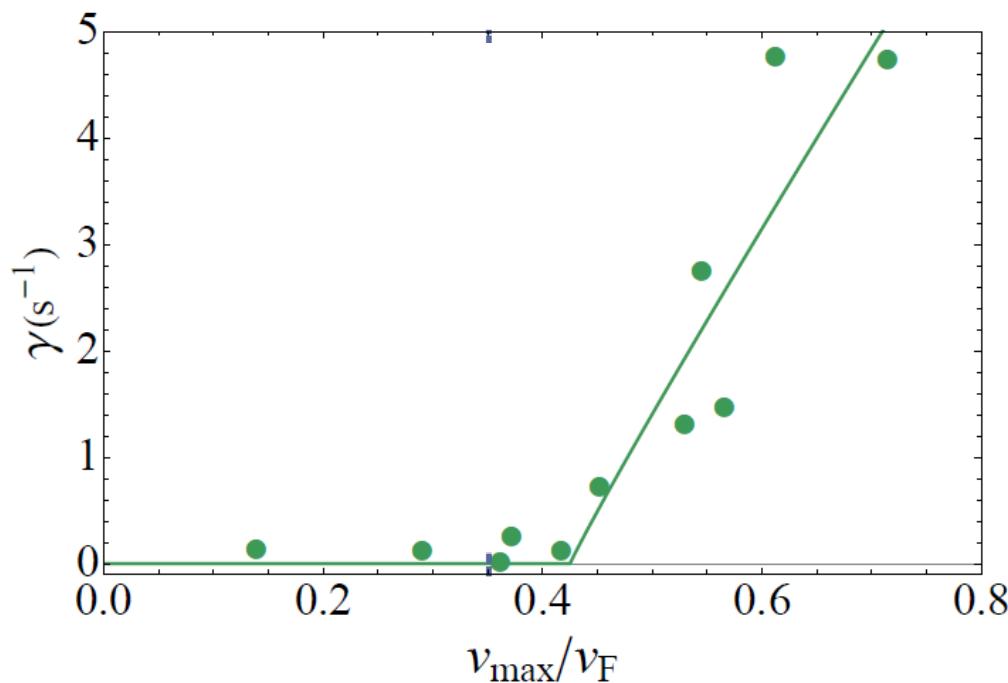
Increase relative velocity

Critical velocity for superfluid counterflow



$$d = d_0 \exp(-\gamma t) + d'$$
$$\gamma = 3.1 \text{ s}^{-1}$$

Time(ms)



$V_c = 2 \text{ cm/s}$
is quite high !

Landau criterion



Momentum Conservation : $M\mathbf{V} = M\mathbf{V}' + \hbar \mathbf{k}$

Energy Conservation : $MV^2 / 2 = MV'^2 / 2 + \varepsilon_{\mathbf{k}}$

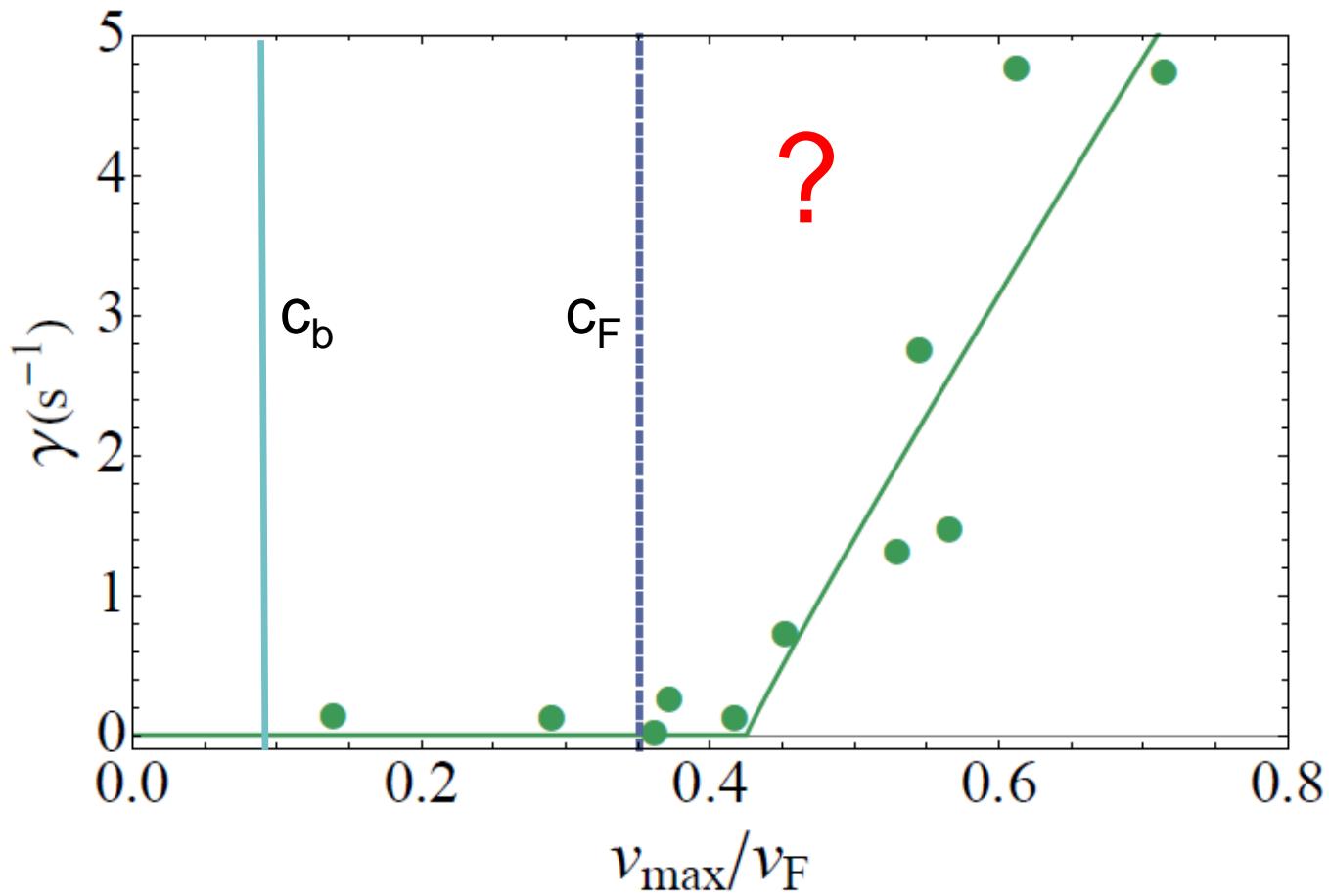
$$\hbar k V \geq \hbar \mathbf{k} \cdot \mathbf{V} = \varepsilon_{\mathbf{k}} + \hbar^2 k^2 / 2M \geq \varepsilon_{\mathbf{k}}$$

Motion of impurity is damped by the creation of elementary excitations if:

$$V \geq V_c = \min_k \left(\frac{\varepsilon_{\mathbf{k}}}{\hbar k} \right)$$

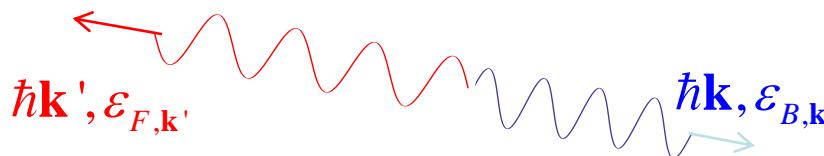
For a linear excitation spectrum $\varepsilon_{\mathbf{k}} = \hbar k c$, $V_c = c$, the sound velocity

Critical velocities



Revisiting Landau criterion for a Bose-Fermi mixture @ T=0

Y. Castin, I. Ferrier-Barbut and C. Salomon
Comptes-Rendus Acad. Sciences, Paris, **16**, 241 (2015)



1 Excitation in the bosonic superfluid $E_{B,\mathbf{k}} = \varepsilon_{B,\mathbf{k}} + \hbar\mathbf{k} \cdot \mathbf{V}_B$

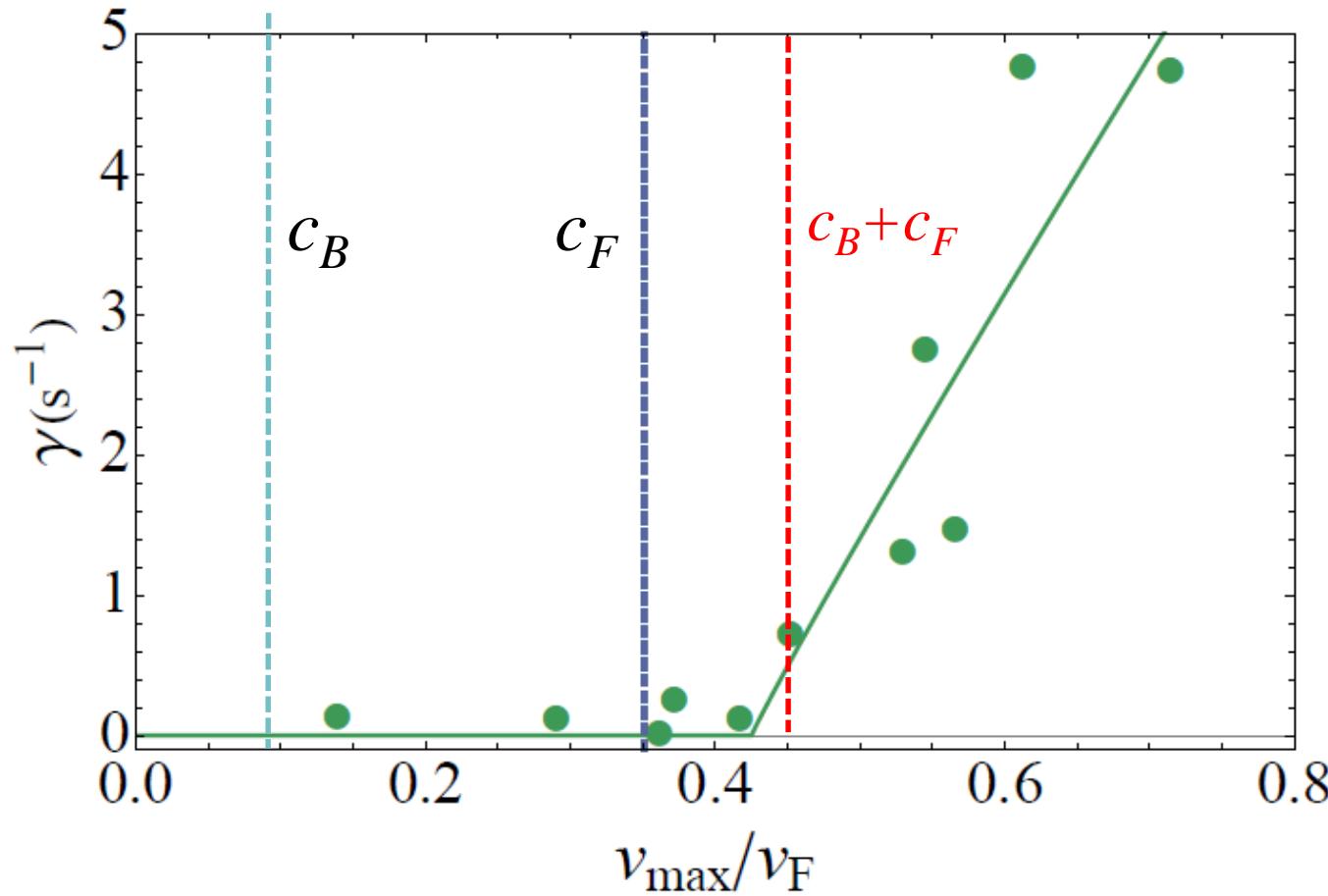
1 Excitation in the fermionic superfluid $E_{F,\mathbf{k}'} = \varepsilon_{F,\mathbf{k}'} + \hbar\mathbf{k}' \cdot \mathbf{V}_F$

Energy-momentum conservation: $E_{B,\mathbf{k}} + E_{F,\mathbf{k}'} = 0 \quad \mathbf{k} + \mathbf{k}' = 0$

$$|\mathbf{V}_B - \mathbf{V}_F| \geq \min_k \left(\frac{\varepsilon_{B,k} + \varepsilon_{F,-k}}{\hbar k} \right) \quad \text{Sound Modes: } V_c = c_B + c_F$$

See also Abbad et al. EPJD 69, 126 (2015), F. Chevy, PRA **91**, 063606 (2015),
W. Zheng and H. Zhai, Phys. Rev. Lett. 113, 265304 (2014)

Counter-flow critical velocity



M. Delehaye, S. Laurent, I. Ferrier-Barbut, S. Jin, F. Chevy, C. Salomon, PRL 2015

Related studies on Fermi gas at MIT, Miller PRL 2008, Hamburg, Weimer PRL 2015

Summary

- 3 examples of quantum simulation
- Explore further the cold atom-condensed matter interface: ex: spin polarization, FFLO phase
- Doped Fermi-Hubbard model in 2D lattices
- Dynamics of quantum systems: time dependent Hamiltonian, quantum quenches,.....
- Long range interactions: dipole-dipole interaction
- Topological bands and lattice physics
- Spin-orbit coupling, integer quantum Hall states, and fractional QH.
- Mixed dimensions: 3D-2D, 3D-1D, 3D-0D.