



Nuclear Interactions: Status and Perspectives

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Outline

- Effective Field Theories
- EFT's for nuclear physics
- Chiral EFT
- EFT for weakly bound states
 - Pionless EFT
 - Halo/cluster EFT
- Summary and Outlook

Collaborators: S. König, U. van Kolck (Review article in preparation)

Effective Field Theory

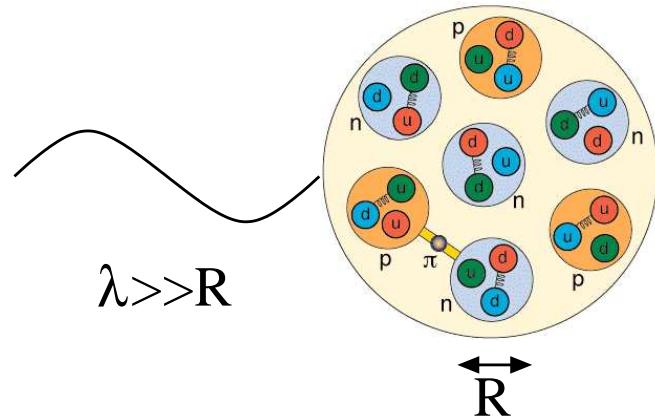


- Separation of scales:

$$1/Q = \lambda \gg R$$

- Limited resolution at low energy:

→ expand in powers of QR



Effective Field Theory



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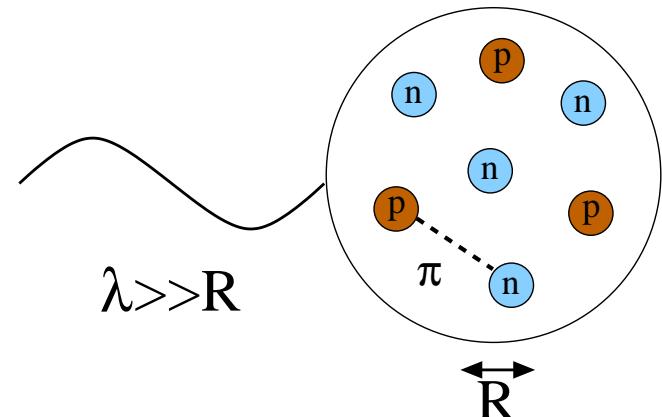
→ expand in powers of QR

- Short-distance physics not resolved

→ capture in low-energy constants using renormalization

→ include long-range physics explicitly

- Systematic, model independent → error estimates



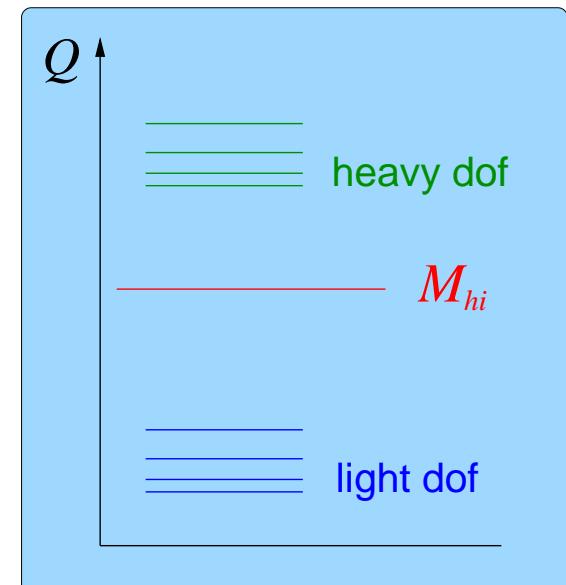


Construction of an EFT

- Construct most general \mathcal{L}_{eff} respecting underlying symmetries
- Exploit separation of scales
- Non-renormalizable in classical sense, but finite number of operators at each order

$$\mathcal{L}_{eff} = \mathcal{L}_{light} + \mathcal{L}_{heavy}$$

- \mathcal{L}_{light} describes dynamics of light fields
- \mathcal{L}_{heavy} contains low-energy constants
- Work at low energies: $Q \ll M_{hi} \sim 1/R$
 - Fix low-energy constants (LECs) from matching
 - Calculate observables in expansion in Q/M_{hi}
- Two scenarios: top-down & bottom-up



Light-By-Light Scattering

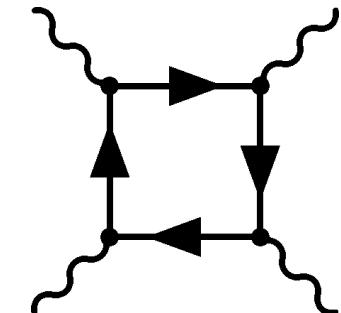


- Classic Example (Euler, Heisenberg, 1936)

- Photon energy E , electron mass m_e

- Separation of scales: $E \ll m_e$

⇒ theory simplifies: $\mathcal{L}_{QED}[\psi, \bar{\psi}, A_\mu] \rightarrow \mathcal{L}_{\text{eff}}[A_\mu]$



- Invariants: $F_{\mu\nu}F^{\mu\nu}$, $(F_{\mu\nu}F^{\mu\nu})^2$, $(F_{\mu\nu}\tilde{F}^{\mu\nu})^2$, ...

$$\mathcal{L}_{\text{eff}} = \frac{1}{8\pi}(\vec{E}^2 - \vec{B}^2) + \frac{e^4}{360\pi^2 m_e^4} \left[(\vec{E}^2 - \vec{B}^2)^2 + 7(\vec{E} \cdot \vec{B})^2 \right] + \dots$$

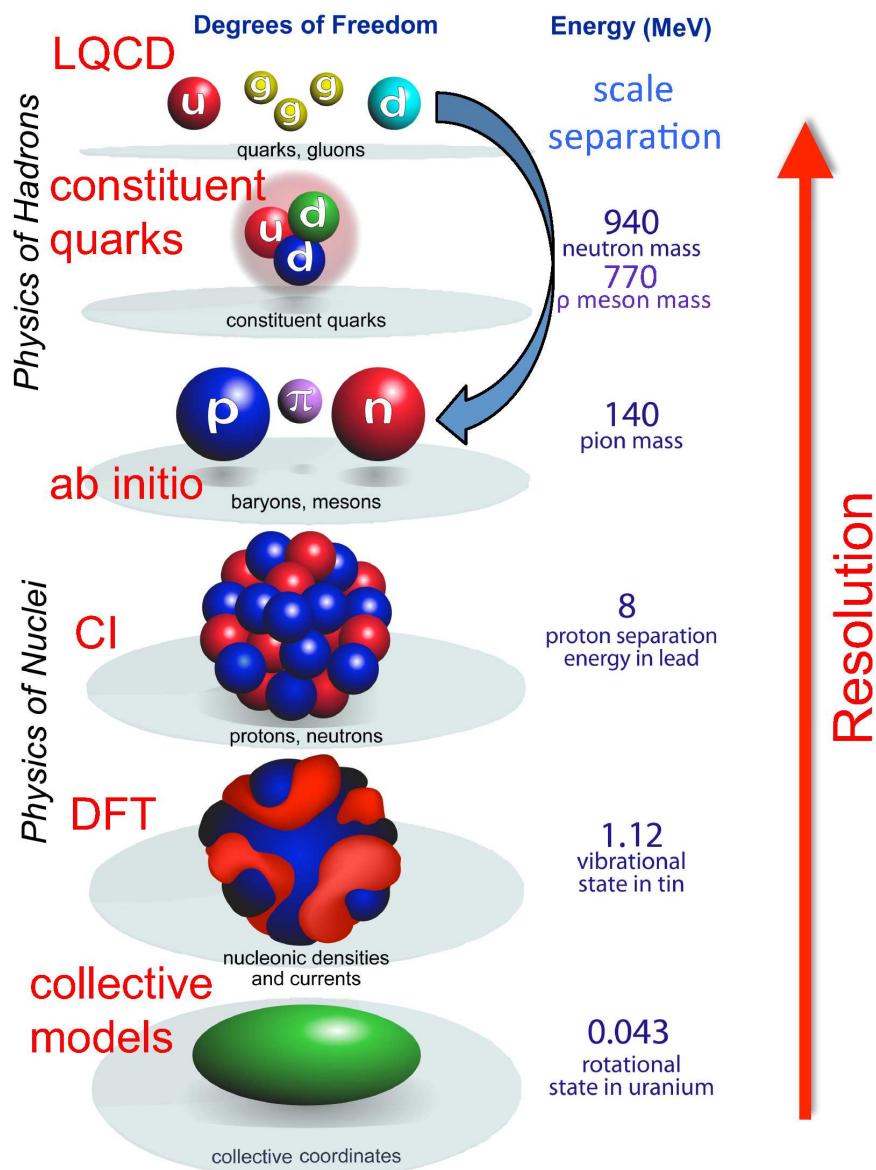
- Energy expansion: $(E/m_e)^{2n}$

- Cross section: $\sigma(E) = \frac{1}{16\pi^2} \frac{973}{10125\pi} \frac{e^8}{m_e^2} \frac{E^6}{m_e^6} + \mathcal{O}(E^8)$

- Observation in Pb-Pb collisions at LHC

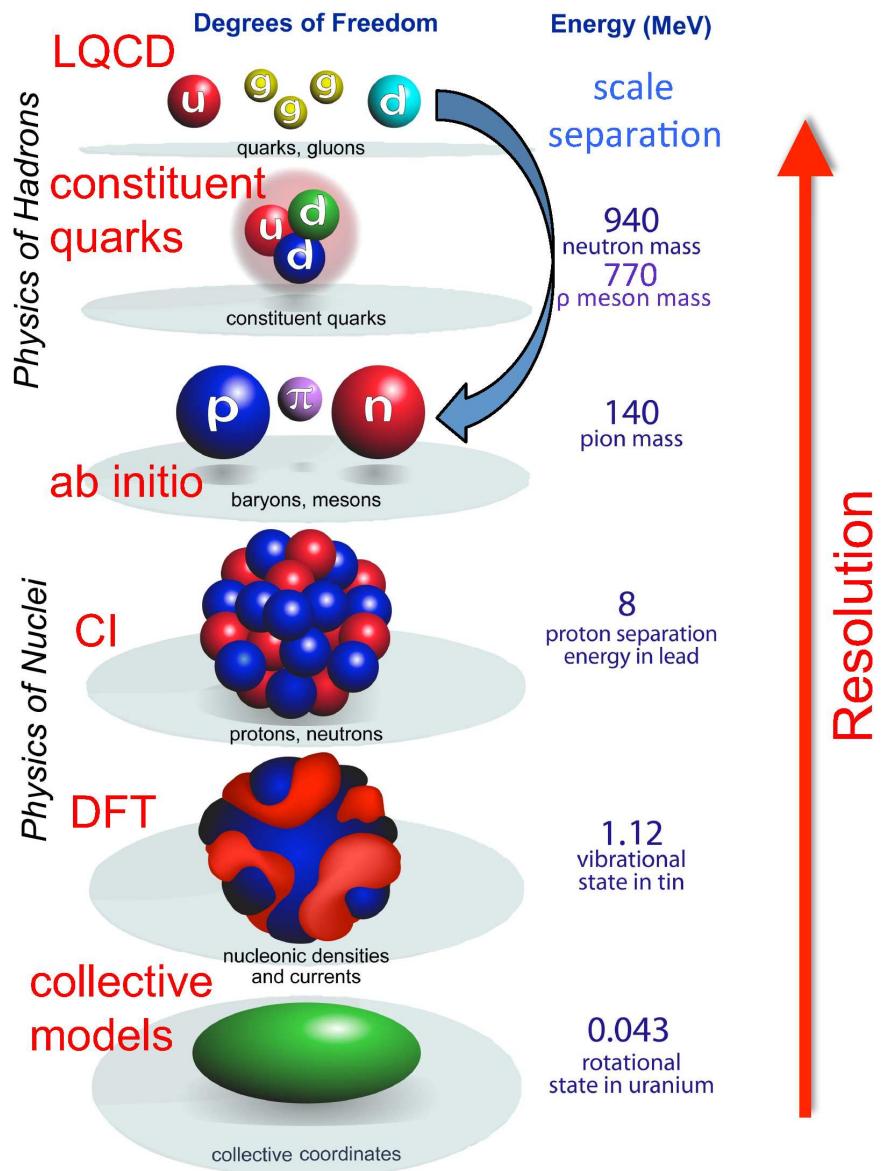
(ATLAS collaboration, Nature Physics **13** (2017) 852)

Nuclear EFTs



from Bertsch, Dean, Nazarewicz (2007)

Nuclear EFTs



EFT exploits separation of scales



EFT depends on resolution scale

- Chiral EFT: $N, \pi, (\Delta)$
- Pionless EFT: N
- Halo EFT: N , clusters
- EFT for deformed nuclei: collective dof
- EFT at Fermi surface: quasi- N
- ...

from Bertsch, Dean, Nazarewicz (2007)



Nuclear forces from chiral EFT

$$e^{iZ[J]} = \int \mathcal{D}q \mathcal{D}\bar{q} \mathcal{D}G e^{i \int d^4x \mathcal{L}_{QCD}[q, \bar{q}, G; J]} \Leftrightarrow \int \mathcal{D}\pi \mathcal{D}N e^{i \int d^4x \mathcal{L}_{eff}[\pi, N; J]}$$

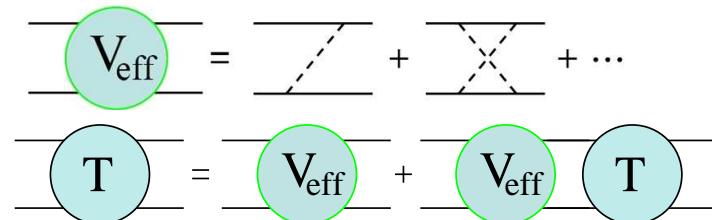
- Write down most general \mathcal{L}_{eff} consistent with chiral symmetry

$$\mathcal{L}_{QCD} \rightarrow \mathcal{L}_{eff} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \dots$$

- Compute S-Matrix elements perturbatively in chiral expansion
- Fix low-energy constants (LECs) & make predictions...

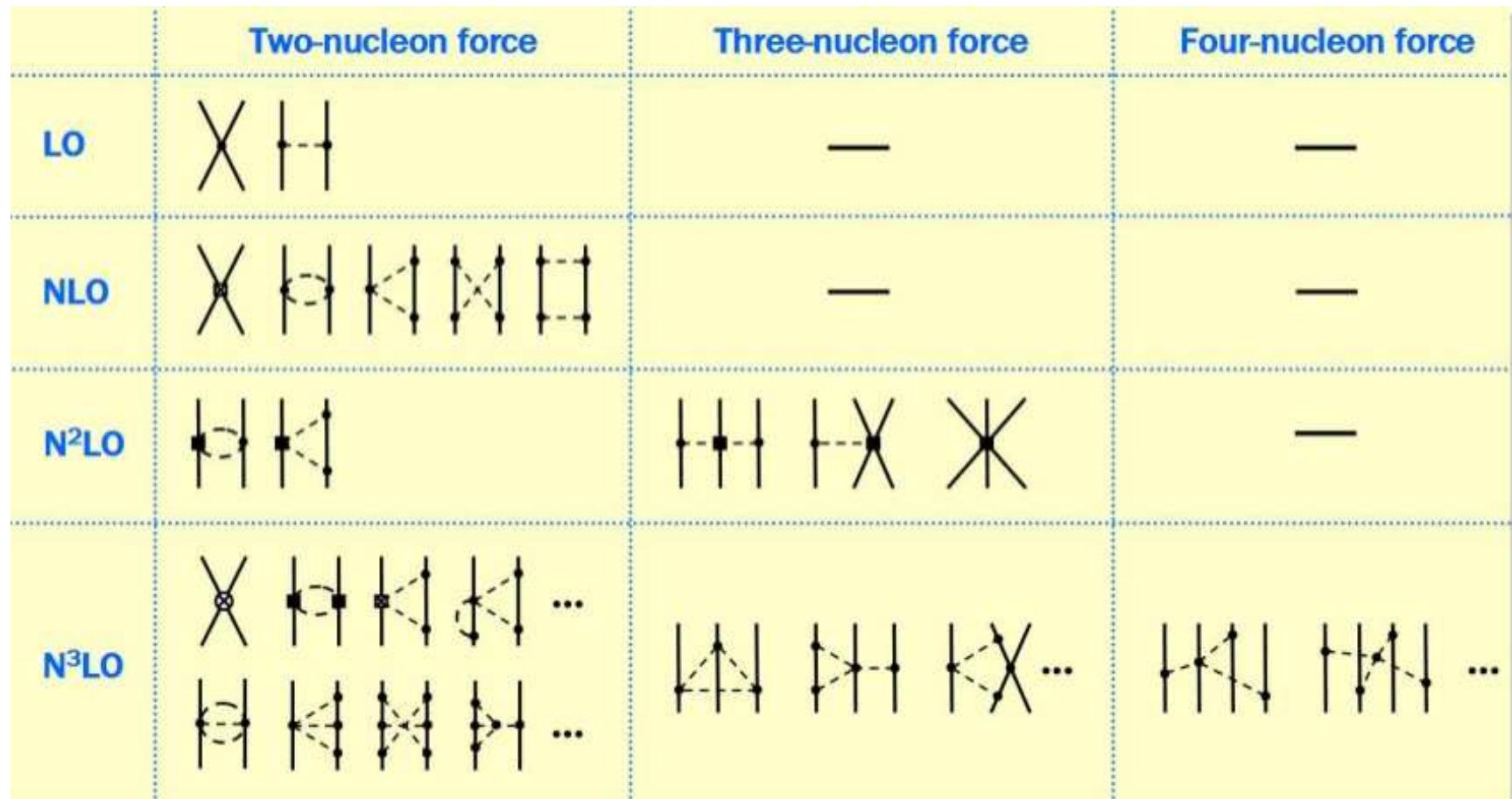
Complication for $\geq 2N$: large scattering length ($\Rightarrow {}^2H$, nuclei)

- Weinberg approach: (Weinberg, 1990, 1991; Ordóñez and van Kolck, 1992; ...)
 - Use \mathcal{L}_{eff} to compute V_{eff}
(syst. expansion in Q/Λ_χ and m_π/Λ_χ)
 - Solve LS equation with V_{eff}
 - UV regulator: cutoff Λ





Nuclear forces from chiral EFT



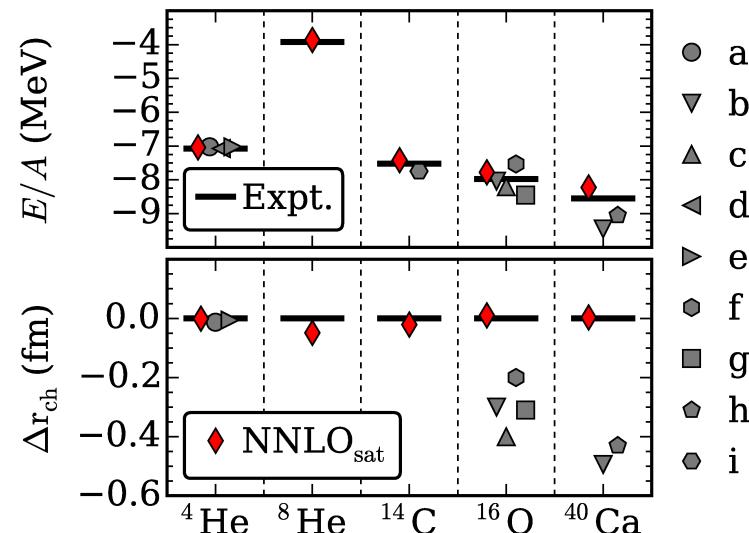
- Hierarchy of nuclear forces: $V_{2N} \gg V_{3N} \gg V_{4N} \gg \dots$
- Successful phenomenology using FY, NCSM, CC, IMSRG, Lattice EFT, ...



Nuclear forces from chiral EFT

- Successful phenomenology

(e.g., Ekström et al., 2015)

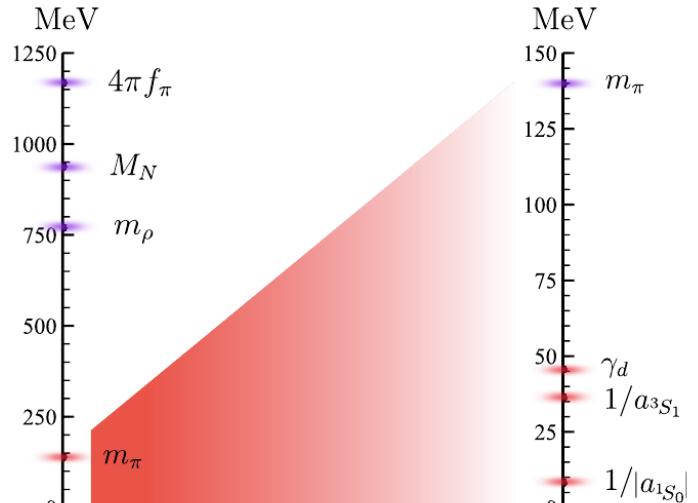


- But some issues remain to be settled:
 - Narrow range of cutoffs ($\Lambda \approx 450\ldots 600$ MeV)
⇒ Is exact RG invariance even possible?
 - Some observables show dependence on type of regulator
(local vs. non-local, r -space vs. p -space etc.)
- Alternative approaches available but untested for $A > 3$
(Nogga, Timmermans, van Kolck, 2005;)

Nuclear Scales and the Unitary Limit



- Chiral EFT: $Q \sim m_\pi$
⇒ contact interactions and pion exchange between nucleons
- Pionless EFT: $Q \sim \gamma_d \sim 1/a$
⇒ contact interactions only
⇒ expansion around unitary limit
- Unitary limit: $a \rightarrow \infty, R \sim r_e, \dots \rightarrow 0$



(cf. Bertsch problem, 2000)

$$\mathcal{T}_2(k, k) \propto \left[-1/a + r_e k^2/2 + \dots - ik \right]^{-1} \implies i/k$$

- Scattering amplitude scale invariant, saturates unitarity bound
- Standard pionless EFT: include scattering lengths exactly at LO
- Alternative: strictly expand around $1/a = 0$ and include finite a in perturbation theory (talk by S. König)

EFT for the Unitary Limit



- Effective Lagrangian

(Kaplan, 1997; Bedaque, HWH, van Kolck, 1999)

$$\mathcal{L}_{eff} = \text{---} + \text{---} + \text{---} + \text{---} + \text{---} + \dots$$

- 2-body amplitude:

$$\text{---} = \text{---} + \text{---} + \dots$$

- 2-body coupling g_2 near fixed point ($1/a = 0$)

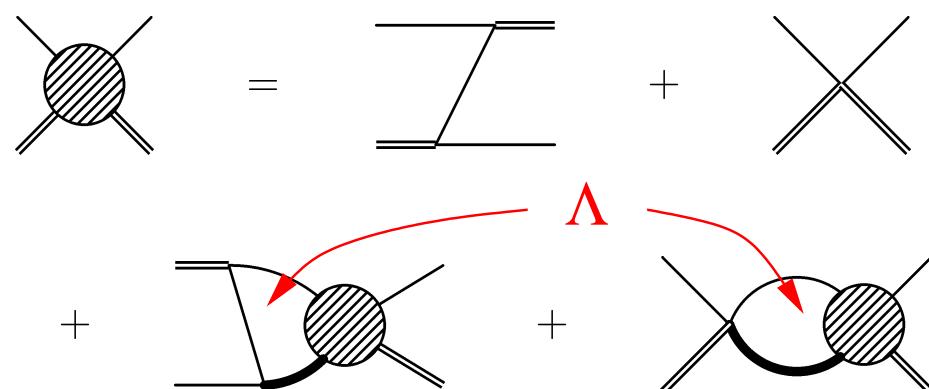
⇒ scale and conformal invariance \iff unitary limit

(Mehen, Stewart, Wise, 2000; Nishida, Son, 2007; ...)

- 3-body amplitude:

$g_3(\Lambda) \Rightarrow$ Enhanced/NDA

⇒ limit cycle & DSI





Three-Body Force: Limit Cycle

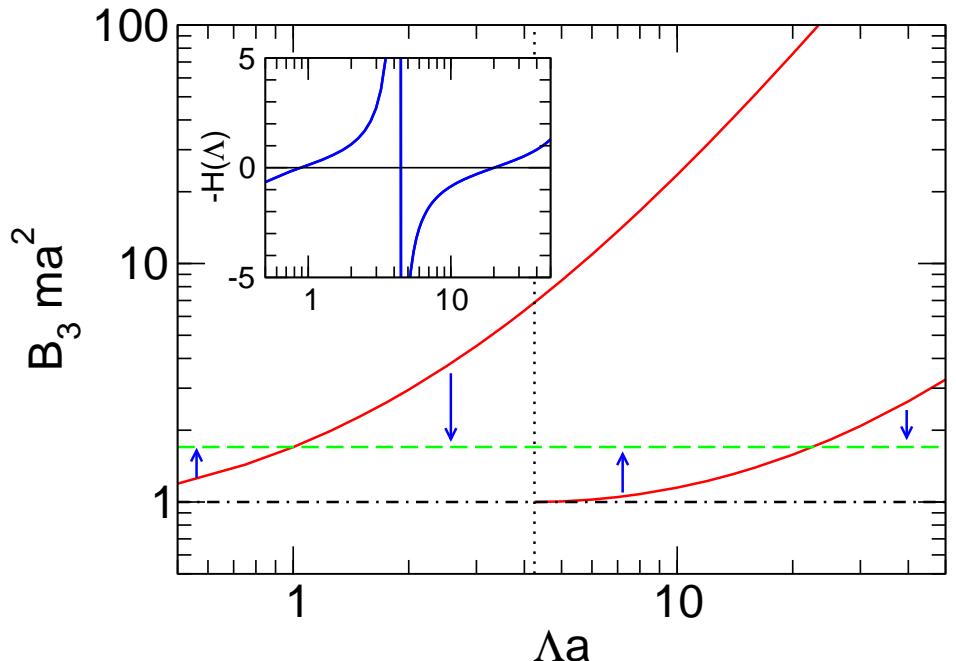
- RG invariance \implies running coupling $H(\Lambda) = g_3 \Lambda^2 / (9g_2^2)$

- $H(\Lambda)$ periodic: limit cycle

$$\Lambda \rightarrow \Lambda e^{n\pi/s_0} \approx \Lambda (22.7)^n$$

(cf. Wilson, 1971)

- Anomaly: scale invariance broken to discrete subgroup



$$H(\Lambda) \approx \frac{\cos(s_0 \ln(\Lambda/\Lambda_*) + \arctan(s_0))}{\cos(s_0 \ln(\Lambda/\Lambda_*) - \arctan(s_0))}, \quad s_0 \approx 1.00624$$

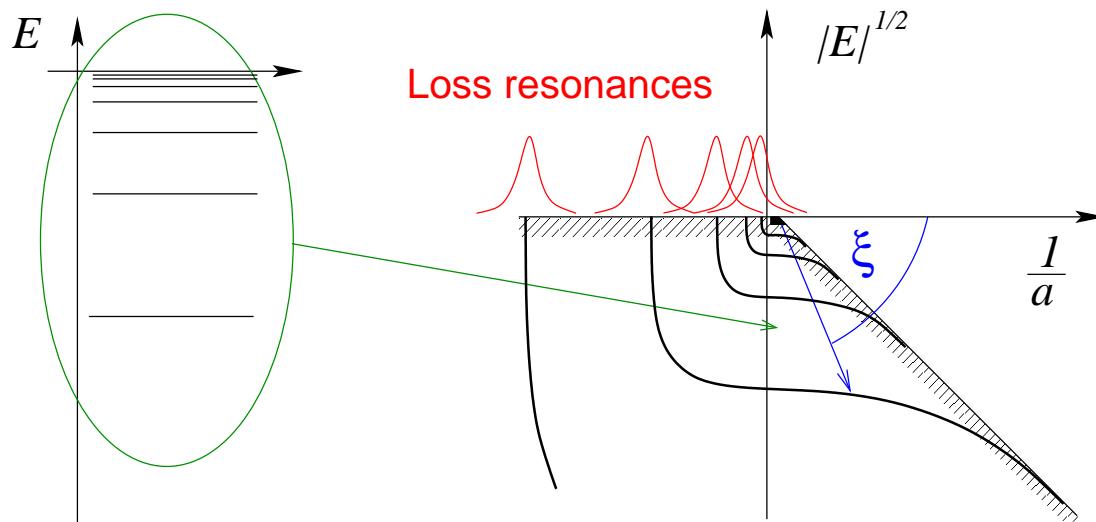
(Bedaque, HWH, van Kolck, 1999)

- Limit cycle \iff Discrete scale invariance \iff Efimov physics

Limit Cycle: Efimov Physics



- Universal spectrum of three-body states (Efimov, 1970)



- Discrete scale invariance for fixed angle ξ
- Geometrical spectrum for $1/a \rightarrow 0$

$$B_3^{(n)} / B_3^{(n+1)} \xrightarrow{1/a \rightarrow 0} e^{2\pi/s_0} = 515.035\dots$$

- Ultracold atoms \Rightarrow variable scattering length \Rightarrow loss resonances
- Nuclei \Rightarrow universal correlations and scaling relations

Universal Physics at Different Scales

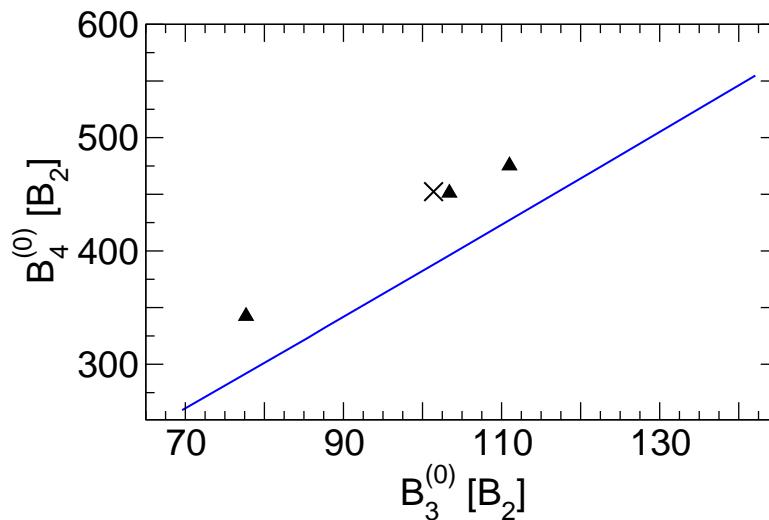
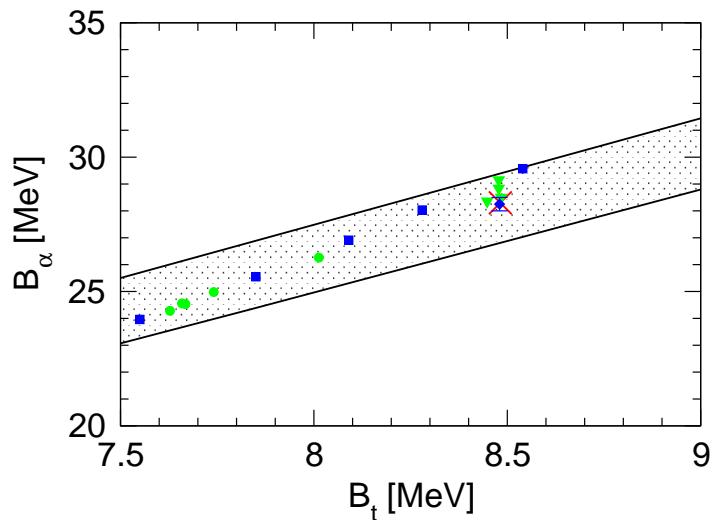


- Atomic physics:
 - ${}^4\text{He}$: $a \approx 104 \text{ \AA} \gg r_e \approx 7 \text{ \AA} \sim l_{vdW} \longrightarrow B_d \approx 100 \text{ neV}$
 - ${}^4\text{He}$ ground state trimer observed (Bruehl et al., 2005)
 - ${}^4\text{He}$ excited trimer observed (Kunitski et al., 2015)
 - Cold atoms near Feshbach resonance: Cs, Li, ...
- Nuclear physics: dripline nuclei, e.g.
 - $2N$ -system: $|a| \gg r_e \sim 1/m_\pi \longrightarrow B_d \approx 2.2 \text{ MeV}$
 - ${}^{11}\text{Be} \implies {}^{10}\text{Be} + n \longrightarrow B_d \approx 0.5 \text{ MeV}$
- Particle physics: hadronic molecules
 - $X(3872)$ as a $D^0 \bar{D}^{0*}$ molecule? $\longrightarrow B_d = (-0.01 \pm 0.2) \text{ MeV}$
- Condensed matter physics:
 - Spin systems/Magnons (Nishida et al., 2013)
 - Excitons (Omachi et al., 2013)



Universal Correlations

- 2 Parameters at LO \Rightarrow 3-body observables are correlated
 \Rightarrow Phillips line (Efimov, Tkachenko, 1985; Bedaque, HWH, van Kolck, 2000)
- No four-body parameter at LO (Platter, HWH, Mei β nner, 2004)
 \Rightarrow 4-body observables are correlated \Rightarrow Tjon line



- Variation of 3-body parameter generates correlations
- RG-evolved interactions: Λ dependence traces correlations
(cf. Nogga, Bogner, Schwenk, 2004)

Halo Effective Field Theory



- Separation of scales:

$$1/k = \lambda \gg R_{core}$$

- Limited resolution at low energy:

→ expand in powers of kR_{core}

- Short-distance physics not resolved

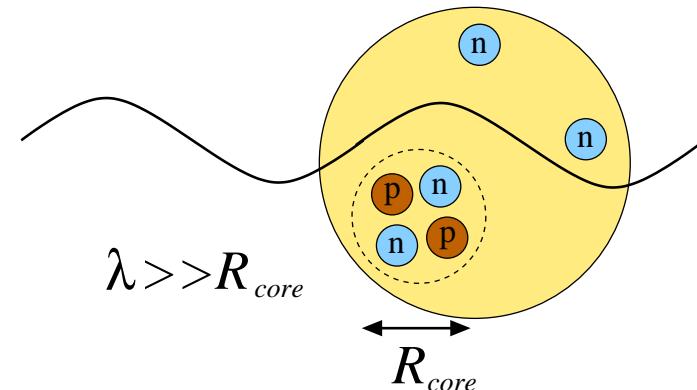
→ capture in low-energy constants using renormalization

→ include long-range physics explicitly

(Bertulani, HWH, van Kolck, 2002; Bedaque, HWH, van Kolck, 2003; ...)

- Halo nuclei: P - and higher partial waves

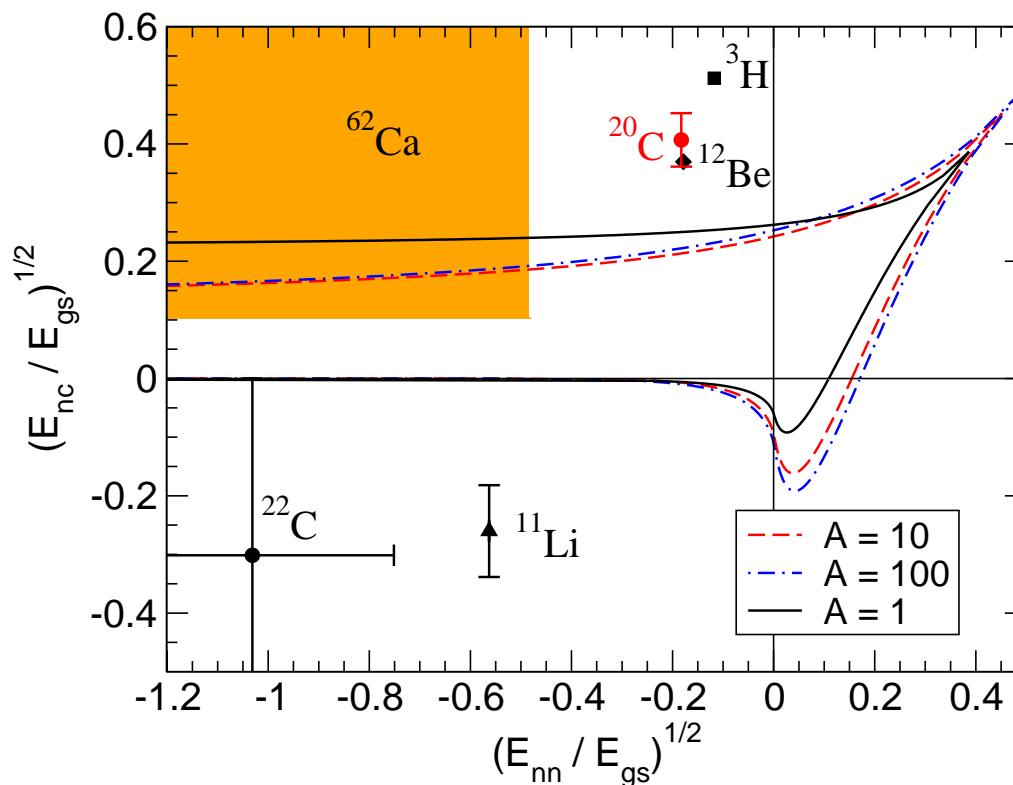
more sensitivity to short distances ⇒ more parameters ⇒ less universal



Scaling Relations in Halo Nuclei



- Efimov effect in halo nuclei? (Fedorov, Jensen, Riisager, 1994)
⇒ excited states obeying scaling relations
- Correlation plot: $E_{nn} \leftrightarrow E_{nc}$ (Amorim, Frederico, Tomio, 1997; Hammer, Canham, 2008)





Summary and Outlook

- Effective Field Theories in Nuclear Physics
 - Controlled, systematic approach to exploit scale separation
 \implies error estimates
 - Different EFT's with different range of applicability
- Chiral limit \iff Unitary limit
- Chiral EFT
 - Successful phenomenology, rapid progress in applications
 - Some issues to be resolved
- EFT for unitary limit (Pionless/Halo EFT)
 - more limited scope
 - structure and reactions of weakly-bound nuclei
- Bayesian statistics
- Connection to density functionals