

# Jet angular correlation in vector-boson fusion processes at the LHC

Kentarou Mawatari (ITP, Heidelberg U.)



with Kaoru Hagiwara (KEK), Qiang Li (Karlsruhe U.)

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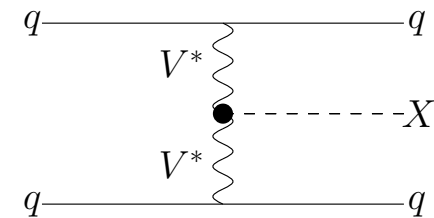
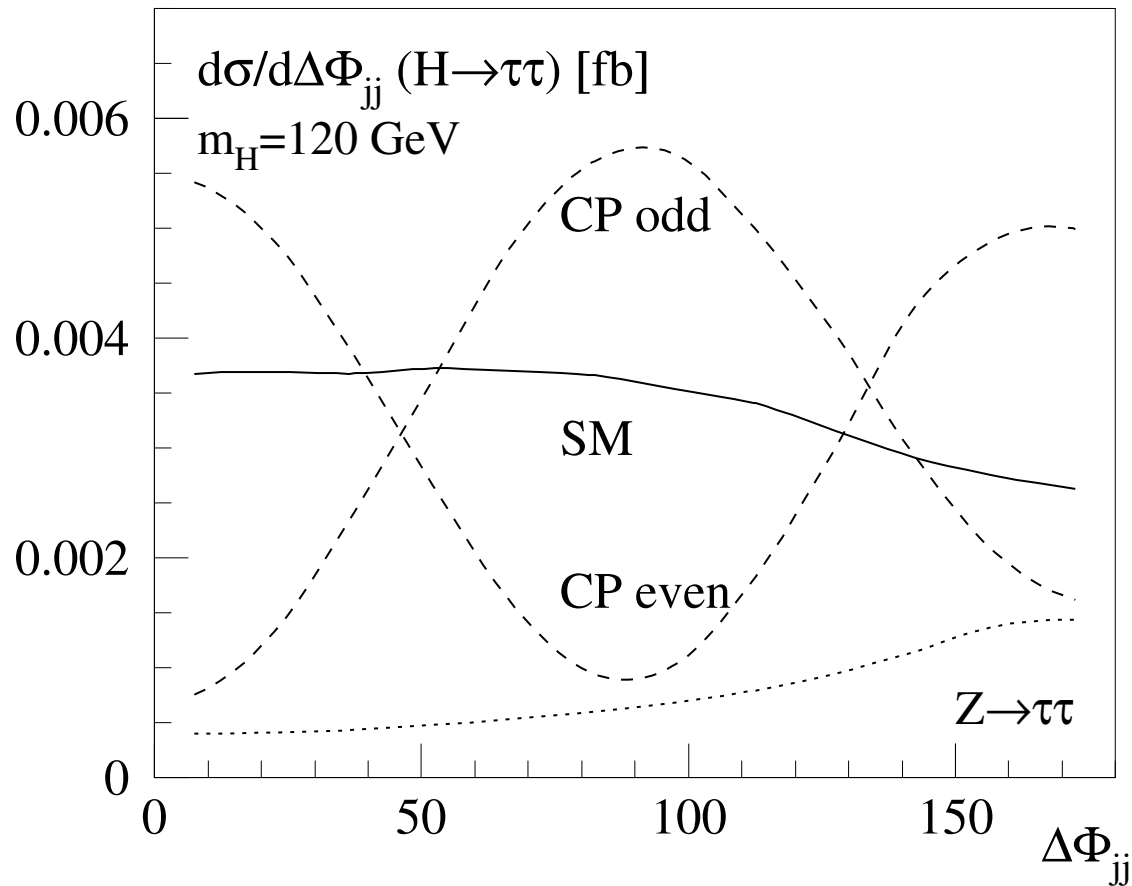
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## Introduction

- If new particles are discovered at the LHC, the main challenge will be to find which of the many competing models (SUSY, extra dimensions, compositeness, ...) correctly predicts their properties: most fundamentally, their **masses** and **spins**.
- **Angular correlation of the two accompanying jets in Higgs boson productions at the LHC** has been known as a potential tool to study its **spin** and ***CP*** nature.

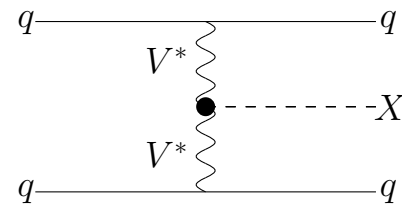
# Azimuthal correlation between jets in $Hjj$ events

T.Plehn, D.Rainwater, D.Zeppenfeld (2002)



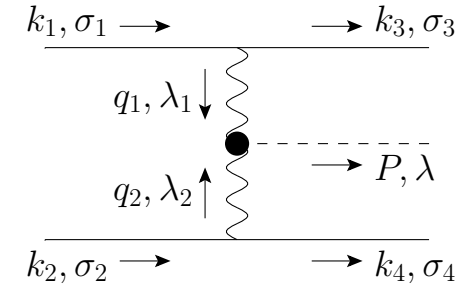
- Azimuthal correlations reflect the tensor structures of the  $HVV$  coupling. Why does each tensor structure give such a distribution?
- How about the correlation for spin-2 massive gravitons?

# Contents



1. Introduction
2. Helicity formalism and kinematics
3. Helicity amplitudes for VBF processes
4. Azimuthal correlations between the two jets
  - Higgs boson ( $J = 0$ ) productions
  - Massive graviton ( $J = 2$ ) productions
5. Summary

# The VBF helicity amplitudes



The helicity amplitudes for VBF processes

$$\mathcal{M}_{\sigma_1\sigma_3,\sigma_2\sigma_4}^\lambda = J^{\mu'_1}(k_1, k_3; \sigma_1, \sigma_3) \frac{-g_{\mu'_1\mu_1} + \frac{q_1^{\mu'_1} q_{1\mu_1}}{m_V^2}}{q_1^2 - m_V^2} J^{\mu'_2}(k_2, k_4; \sigma_2, \sigma_4) \frac{-g_{\mu'_2\mu_2} + \frac{q_2^{\mu'_2} q_{2\mu_2}}{m_V^2}}{q_2^2 - m_V^2} \Gamma_{XVV}^{\mu_1\mu_2}(q_1, q_2; \lambda)^*$$

can be expressed by using

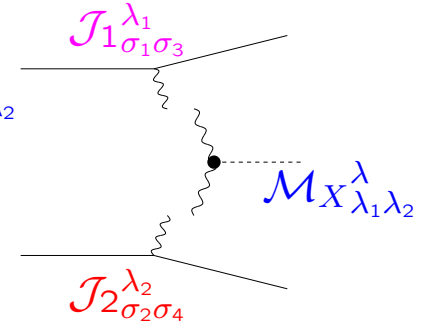
completeness relation  $-g_{\mu'\mu} + \frac{q_i^{\mu'} q_{i\mu}}{q_i^2} = \sum_{\lambda_i=\pm,0} (-1)^{\lambda_i+1} \epsilon_{\mu'}(q_i, \lambda_i)^* \epsilon_\mu(q_i, \lambda_i)$

current conservation  $q_{i\mu} J^\mu(k_i, k_{i+2}; \sigma_i, \sigma_{i+2}) = 0$

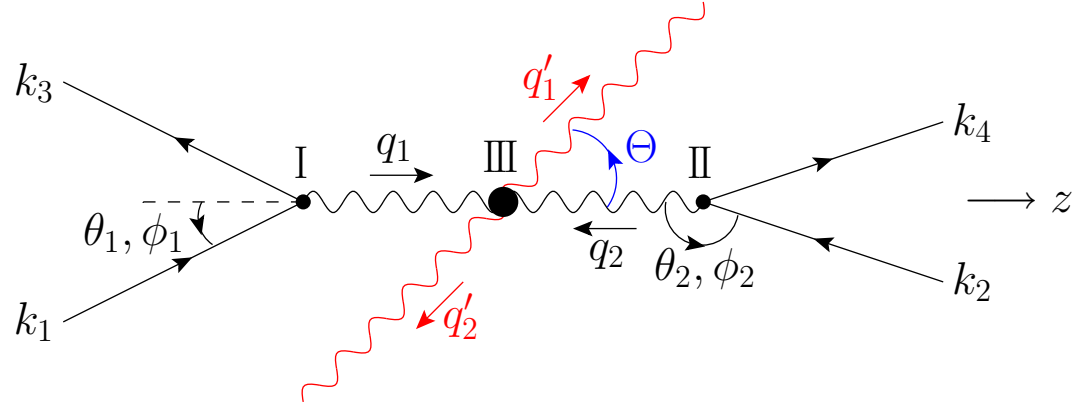
as the product of the three helicity amplitudes summed over the polarization of the intermediate vector-bosons:

$$\begin{aligned} \mathcal{M}_{\sigma_1\sigma_3,\sigma_2\sigma_4}^\lambda &= \frac{1}{q_1^2 - m_V^2} J^{\mu'_1}(k_1, k_3; \sigma_1, \sigma_3) \sum_{\lambda_1=\pm,0} (-1)^{\lambda_1+1} \epsilon_{\mu'_1}(q_1, \lambda_1)^* \epsilon_{\mu_1}(q_1, \lambda_1) \\ &\times \frac{1}{q_2^2 - m_V^2} J^{\mu'_2}(k_2, k_4; \sigma_2, \sigma_4) \sum_{\lambda_2=\pm,0} (-1)^{\lambda_2+1} \epsilon_{\mu'_2}(q_2, \lambda_2)^* \epsilon_{\mu_2}(q_2, \lambda_2) \\ &\times \Gamma_{XVV}^{\mu_1\mu_2}(q_1, q_2; \lambda)^* \end{aligned}$$

$$= \frac{1}{(q_1^2 - m_V^2)(q_2^2 - m_V^2)} \sum_{\lambda_1=\pm,0} \sum_{\lambda_2=\pm,0} \mathcal{J}_{1\sigma_1\sigma_3}^{\lambda_1} \mathcal{J}_{2\sigma_2\sigma_4}^{\lambda_2} \mathcal{M}_{X\lambda_1\lambda_2}^\lambda$$



## Kinematics



I)  $q_1$  Breit frame ( $Q_1 = \sqrt{-q_1^2}$ ,  $0 < \theta_1 < \pi/2$ ):

$$q_1^\mu = k_1^\mu - k_3^\mu = (0, 0, 0, Q_1)$$

$$k_1^\mu = \frac{Q_1}{2 \cos \theta_1} (1, \sin \theta_1 \cos \phi_1, \sin \theta_1 \sin \phi_1, \cos \theta_1)$$

$$k_3^\mu = \frac{Q_1}{2 \cos \theta_1} (1, \sin \theta_1 \cos \phi_1, \sin \theta_1 \sin \phi_1, -\cos \theta_1)$$

II)  $q_2$  Breit frame ( $Q_2 = \sqrt{-q_2^2}$ ,  $\pi/2 < \theta_2 < \pi$ ):

$$q_2^\mu = k_2^\mu - k_4^\mu = (0, 0, 0, -Q_2)$$

$$k_2^\mu = -\frac{Q_2}{2 \cos \theta_2} (1, \sin \theta_2 \cos \phi_2, \sin \theta_2 \sin \phi_2, \cos \theta_2)$$

$$k_4^\mu = -\frac{Q_2}{2 \cos \theta_2} (1, \sin \theta_2 \cos \phi_2, \sin \theta_2 \sin \phi_2, -\cos \theta_2)$$

III) VBF frame ( $X$  rest frame):

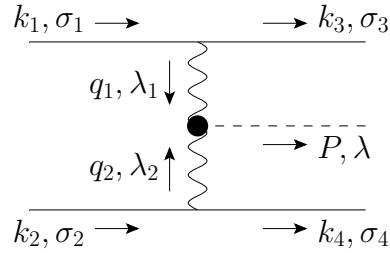
$$q_1^\mu + q_2^\mu = P^\mu = q_1'^\mu + q_2'^\mu = (M, 0, 0, 0)$$

$$q_1^\mu = \frac{M}{2} \left( 1 - \frac{Q_1^2 - Q_2^2}{M^2}, 0, 0, \beta \right);$$

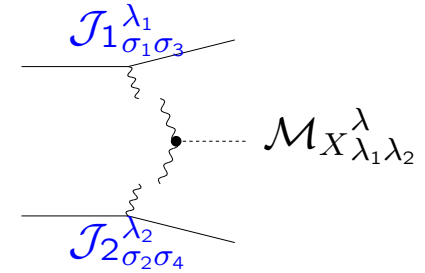
$$q_1'^\mu = \frac{M}{2} \left( 1 + \frac{Q_1'^2 - Q_2'^2}{M^2}, \beta' \sin \Theta, 0, \beta' \cos \Theta \right)$$

$$q_2^\mu = \frac{M}{2} \left( 1 - \frac{Q_2^2 - Q_1^2}{M^2}, 0, 0, -\beta \right);$$

$$q_2'^\mu = \frac{M}{2} \left( 1 + \frac{Q_2'^2 - Q_1'^2}{M^2}, -\beta' \sin \Theta, 0, -\beta' \cos \Theta \right)$$



## Current amplitudes



$$\mathcal{J}_{i\sigma_i\sigma_{i+2}}^{\lambda_i} = (-1)^{\lambda_i+1} J^\mu(k_i, k_{i+2}; \sigma_i, \sigma_{i+2}) \epsilon_\mu(q_i, \lambda_i)^*$$

- Quark current vectors

$$J_{Vff'}^\mu(k_i, k_{i+2}; \sigma_i, \sigma_{i+2}) = g_{\sigma_i}^{Vff'} \bar{u}_{f'}(k_{i+2}, \sigma_{i+2}) \gamma^\mu u_f(k_i, \sigma_i)$$

- Wavefunctions for the quarks

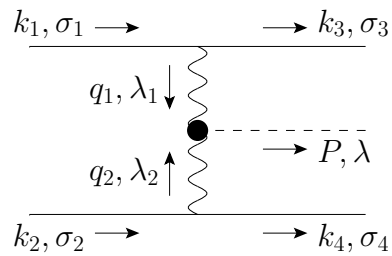
$$u(k_1, +) = \sqrt{2E_1} \begin{pmatrix} 0 \\ 0 \\ \cos(\theta_1/2) \\ \sin(\theta_1/2) e^{i\phi_1} \end{pmatrix}; \quad u(k_1, -) = \sqrt{2E_1} \begin{pmatrix} -\sin(\theta_1/2) e^{-i\phi_1} \\ \cos(\theta_1/2) \\ 0 \\ 0 \end{pmatrix}$$

$$u(k_3, +) = \sqrt{2E_1} \begin{pmatrix} 0 \\ 0 \\ \sin(\theta_1/2) \\ \cos(\theta_1/2) e^{i\phi_1} \end{pmatrix}; \quad u(k_3, -) = \sqrt{2E_1} \begin{pmatrix} -\cos(\theta_1/2) e^{-i\phi_1} \\ \sin(\theta_1/2) \\ 0 \\ 0 \end{pmatrix}$$

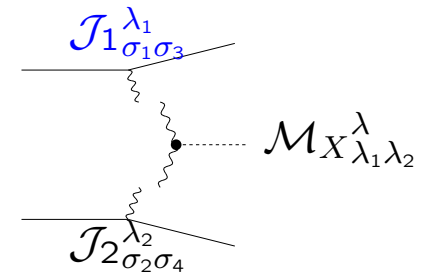
- Wavefunctions for the  $t$ -channel vector-bosons

$$\epsilon^\mu(q_1, \pm) = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0)$$

$$\epsilon^\mu(q_1, 0) = (1, 0, 0, 0)$$



Current amplitudes




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$\mathcal{J}_{1\sigma_1\sigma_3}^{\lambda_1}$  (quark)

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$$\begin{aligned} \mathcal{J}_{1++}^+ &= -(\mathcal{J}_{1--}^-)^* & \frac{1}{2 \cos \theta_1} (1 + \cos \theta_1) e^{-i\phi_1} \\ \mathcal{J}_{1++}^0 &= \mathcal{J}_{1--}^0 & -\frac{1}{\sqrt{2} \cos \theta_1} \sin \theta_1 \\ \mathcal{J}_{1++}^- &= -(\mathcal{J}_{1--}^+)^* & -\frac{1}{2 \cos \theta_1} (1 - \cos \theta_1) e^{i\phi_1} \\ \mathcal{J}_{1+-}^{\lambda_1} &= \mathcal{J}_{1-+}^{\lambda_1} & 0 \end{aligned}$$


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- $VV \rightarrow X$  fusion amplitudes:

$$\mathcal{M}_{X\lambda_1\lambda_2} = \epsilon_\mu(q_1, \lambda_1) \epsilon_\nu(q_2, \lambda_2) \Gamma_{XVV}^{\mu\nu}(q_1, q_2; \lambda)^*$$

- Effective Lagrangian:

$$\mathcal{L}_{H,A} = -\frac{1}{4}g_{Hgg}HF_{\mu\nu}^aF^{a,\mu\nu} - \frac{1}{4}g_{Agg}AF_{\mu\nu}^a\tilde{F}^{a,\mu\nu}$$

$$\mathcal{L}_G = -\frac{1}{\Lambda}T^{\mu\nu}G_{\mu\nu}$$

- $XVV$  vertex:

$X$	$(\lambda)$	$V$	$\Gamma_{XVV}^{\mu\nu}/g_{XVV}$
$H$	$(0)$	$W, Z$	$g^{\mu\nu}$
$H$	$(0)$	$\gamma, Z/\gamma, g$	$q_1 \cdot q_2 g^{\mu\nu} - q_2^\mu q_1^\nu$
$A$	$(0)$	$\gamma, Z/\gamma, g$	$\epsilon^{\mu\nu\alpha\beta} q_{1\alpha} q_{2\beta}$
$G$	$(\pm 2, \pm 1, 0)$	$W, Z, \gamma, g$	$\epsilon_{\alpha\beta} \hat{\Gamma}_{GVV}^{\alpha\beta\mu\nu}$

\*  $\epsilon^{\alpha\beta}(P, \lambda)$ : the polarization tensor;  $\hat{\Gamma}_{GVV}^{\alpha\beta\mu\nu}(q_1, q_2)$ : the  $GVV$  vertex

## Wavefunction and vertex for gravitons

The polarization tensor for a spin-2 particle:

$$\epsilon^{\mu\nu}(p, \pm 2) = \epsilon^\mu(p, \pm) \epsilon^\nu(p, \pm)$$

$$\epsilon^{\mu\nu}(p, \pm 1) = \frac{1}{\sqrt{2}} [\epsilon^\mu(p, \pm) \epsilon^\nu(p, 0) + \epsilon^\mu(p, 0) \epsilon^\nu(p, \pm)]$$

$$\epsilon^{\mu\nu}(p, 0) = \frac{1}{\sqrt{6}} [\epsilon^\mu(p, +) \epsilon^\nu(p, -) + \epsilon^\mu(p, -) \epsilon^\nu(p, +) + 2 \epsilon^\mu(p, 0) \epsilon^\nu(p, 0)]$$

The  $GVV$  vertex:

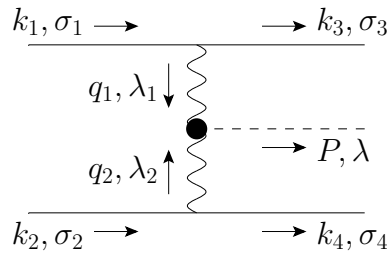
$$\hat{\Gamma}_{GVV}^{\mu\nu, \rho\sigma}(q_1, q_2) = (m_V^2 + q_1 \cdot q_2) C^{\mu\nu, \rho\sigma} + D^{\mu\nu, \rho\sigma}(q_1, q_2) + \xi^{-1} E^{\mu\nu, \rho\sigma}(q_1, q_2)$$

where  $\xi$  is the gauge-fixing parameter, and

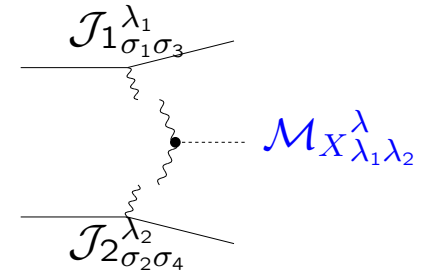
$$C^{\mu\nu, \rho\sigma} = g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho} - g^{\mu\nu} g^{\rho\sigma}$$

$$D^{\mu\nu, \rho\sigma}(q_1, q_2) = g^{\mu\nu} q_1^\sigma q_2^\rho - [g^{\mu\sigma} q_1^\nu q_2^\rho + g^{\mu\rho} q_1^\sigma q_2^\nu - g^{\rho\sigma} q_1^\mu q_2^\nu + (\mu \leftrightarrow \nu)]$$

$$E^{\mu\nu, \rho\sigma}(q_1, q_2) = g^{\mu\nu} (q_1^\rho q_1^\sigma + q_2^\rho q_2^\sigma + q_1^\rho q_2^\sigma) - [g^{\nu\sigma} q_1^\mu q_1^\rho + g^{\nu\rho} q_2^\mu q_2^\sigma + (\mu \leftrightarrow \nu)]$$



$V^*V^* \rightarrow H/A$  amplitudes



$\lambda$	$(\lambda_1 \lambda_2)$	$CP$ -even		$CP$ -odd
		$H$ (WBF)	$H$ (loop-induced)	$A$
0	$(\pm\pm)$	-1	$-\frac{1}{2}(M^2 + Q_1^2 + Q_2^2)$	$\mp \frac{i}{2} \sqrt{(M^2 + Q_1^2 + Q_2^2)^2 - 4Q_1^2 Q_2^2}$
0	$(00)$	$\frac{M^2 + Q_1^2 + Q_2^2}{2Q_1 Q_2}$	$Q_1 Q_2$	0

For  $Q_1, Q_2 \ll M$ , where the VBF contributions dominant,

- WBF  $H$ : produced by the **longitudinally** polarized vector-bosons.
- GF  $H/A$ : produced by the **transversely** polarized vector-bosons.

## Azimuthal correlations for Higgs bosons

The  $J = 0$  VBF amplitudes are the sum of the three amplitudes:

$$\begin{aligned} \mathcal{M}_{\sigma_1\sigma_3,\sigma_2\sigma_4}^{\lambda=0} &= \frac{1}{(Q_1^2 + m_V^2)(Q_2^2 + m_V^2)} \sum_{\lambda_1=\pm,0} \sum_{\lambda_2=\pm,0} \mathcal{J}_{1\sigma_1\sigma_3}^{\lambda_1} \mathcal{J}_{2\sigma_2\sigma_4}^{\lambda_2} \mathcal{M}_{X\lambda_1\lambda_2}^{\lambda=0} \\ &\sim \mathcal{J}_{1\sigma_1\sigma_3}^+ \mathcal{J}_{2\sigma_2\sigma_4}^+ \mathcal{M}_{X^{00}}^0 e^{-i(\phi_1-\phi_2)} + \mathcal{J}_{1\sigma_1\sigma_3}^0 \mathcal{J}_{2\sigma_2\sigma_4}^0 \mathcal{M}_{X^{00}}^0 \\ &\quad + \mathcal{J}_{1\sigma_1\sigma_3}^- \mathcal{J}_{2\sigma_2\sigma_4}^- \mathcal{M}_{X^{00}}^0 e^{i(\phi_1-\phi_2)} \end{aligned}$$

The squared amplitudes are

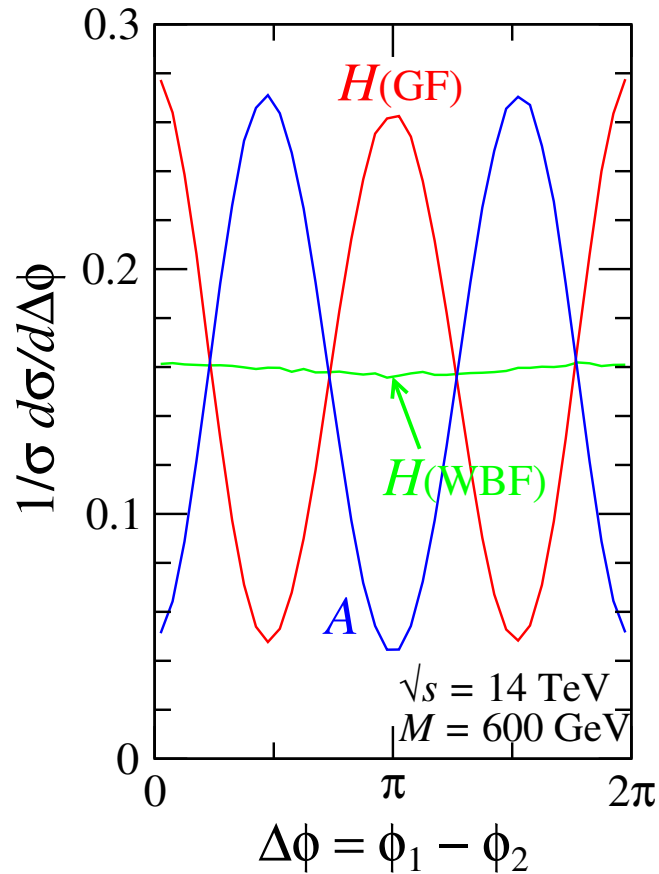
$$\sum_{\sigma_{1,\dots,4}} |\mathcal{M}_{\sigma_1\sigma_3,\sigma_2\sigma_4}^{\lambda=0}|^2 = \Sigma_0 + \Sigma_1 \cos \Delta\phi + \Sigma_2 \cos 2\Delta\phi \quad (\Delta\phi \equiv \phi_1 - \phi_2)$$

The azimuthal correlation is manifestly expressed by the interference among different helicity states of the intermediate vector-bosons.

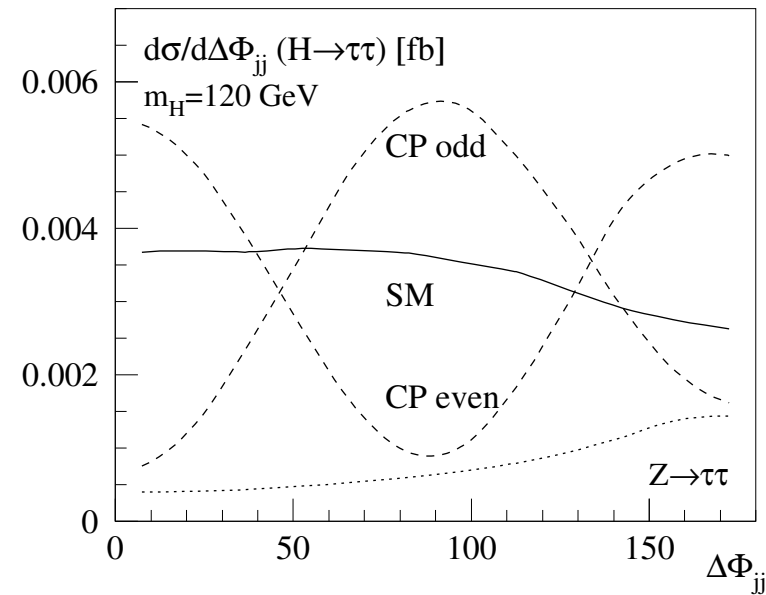
The different tensor structures of the  $XVV$  couplings give rise to the different azimuthal angle dependences:

$$\begin{aligned} H(\text{WBF}) : \quad \mathcal{M}_{00} \gg \mathcal{M}_{++} = \mathcal{M}_{--} &\quad \Rightarrow \quad d\hat{\sigma}/d\Delta\phi \sim \text{constant} \\ H(\text{GF}) : \quad \mathcal{M}_{00} \ll \mathcal{M}_{++} = \mathcal{M}_{--} &\quad \Rightarrow \quad d\hat{\sigma}/d\Delta\phi \sim \Sigma_0 + |\Sigma_2| \cos 2\Delta\phi \\ A : \quad \mathcal{M}_{00} = 0, \mathcal{M}_{++} = -\mathcal{M}_{--} &\quad \Rightarrow \quad d\hat{\sigma}/d\Delta\phi \sim \Sigma_0 - |\Sigma_2| \cos 2\Delta\phi \end{aligned}$$

## $\Delta\phi$ distributions for Higgs bosons



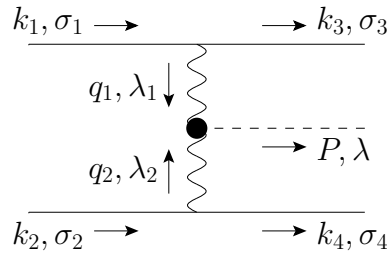
Plehn, Rainwater, Zeppenfeld (2002)



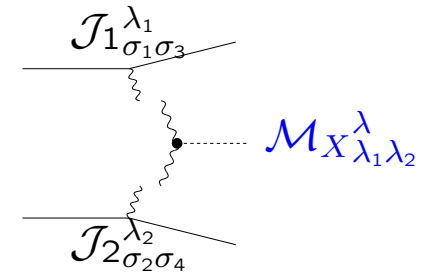
$$H(\text{WBF}) \Rightarrow d\sigma/d\Delta\phi \sim \text{constant}$$

$$H(\text{GF}) \Rightarrow d\sigma/d\Delta\phi \sim \Sigma_0 + |\Sigma_2| \cos 2\Delta\phi$$

$$A \Rightarrow d\sigma/d\Delta\phi \sim \Sigma_0 - |\Sigma_2| \cos 2\Delta\phi$$



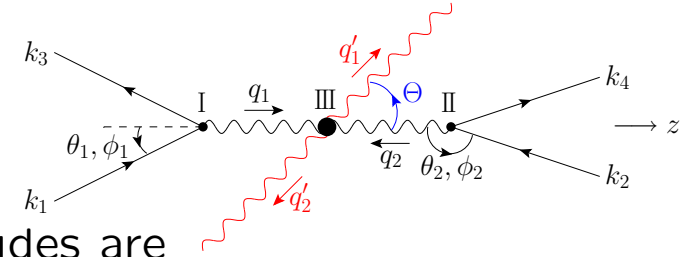
$V^*V^* \rightarrow G$  amplitudes



$\lambda$	$(\lambda_1 \lambda_2)$	$G$
$\pm 2$	$(\pm \mp)$	$-(M^2 + Q_1^2 + Q_2^2)$
$\pm 1$	$(\pm 0)$	$\frac{1}{\sqrt{2}M} Q_2 (M^2 - Q_1^2 + Q_2^2)$
$\pm 1$	$(0 \mp)$	$\frac{1}{\sqrt{2}M} Q_1 (M^2 + Q_1^2 - Q_2^2)$
$0$	$(\pm \pm)$	$\frac{1}{\sqrt{6}M^2} [(Q_1^2 - Q_2^2)^2 + M^2(Q_1^2 + Q_2^2)]$
$0$	$(00)$	$-\frac{4}{\sqrt{6}} Q_1 Q_2$

For  $Q_1, Q_2 \ll M$ , the  $\lambda = \pm 2$  states are dominantly produced through the collisions of the vector-bosons which have the **opposite-sign transverse** polarization.

# Azimuthal correlations for gravitons



The VBF  $G$  production plus its 2-body decay amplitudes are

$$\begin{aligned} \mathcal{M}_{\sigma_1, \dots, 4; \sigma_5, 6} &= \frac{1}{Q_1^2 Q_2^2} \sum_{\lambda_1} \sum_{\lambda_2} \mathcal{J}_{1\sigma_1\sigma_3}^{\lambda_1} \mathcal{J}_{2\sigma_2\sigma_4}^{\lambda_2} \mathcal{M}_{G\lambda_1\lambda_2}^{\lambda=\lambda_1-\lambda_2} \frac{d_{\lambda, \lambda'}^2(\Theta)}{P^2 - M^2 + iM\Gamma} \mathcal{M}'_{G\sigma_5\sigma_6}{}^{\lambda'=\sigma_5-\sigma_6} \\ &\sim \mathcal{J}_{1\sigma_1\sigma_3}^+ \mathcal{J}_{2\sigma_2\sigma_4}^- \mathcal{M}_{G_{+-}^{+2}} e^{-i(\phi_1+\phi_2)} d_{+2, \lambda'}^2(\Theta) \\ &\quad + \mathcal{J}_{1\sigma_1\sigma_3}^- \mathcal{J}_{2\sigma_2\sigma_4}^+ \mathcal{M}_{G_{-+}^{-2}} e^{i(\phi_1+\phi_2)} d_{-2, \lambda'}^2(\Theta) \end{aligned}$$

The squared amplitudes are

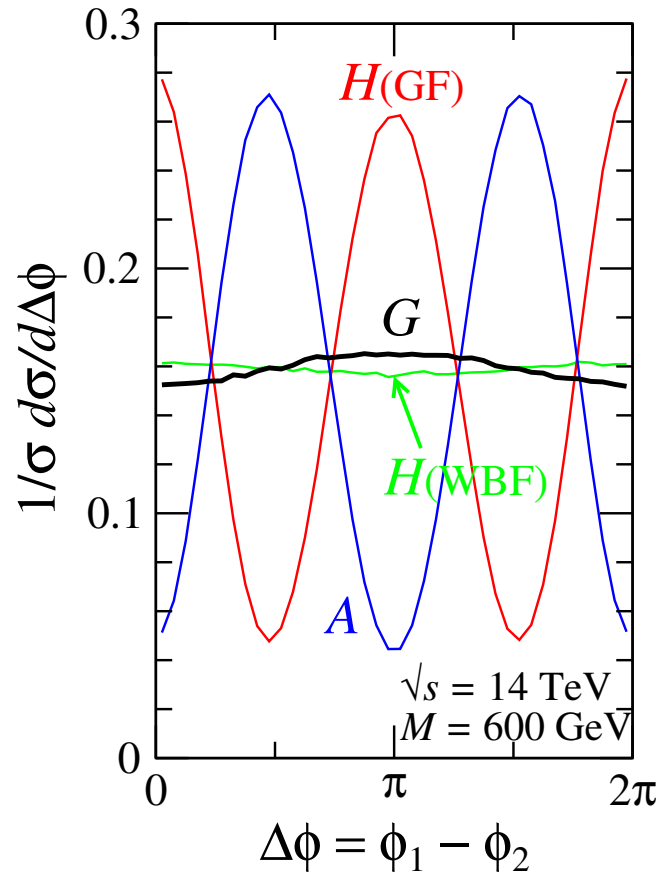
$$\sum_{\sigma_1, \dots, 4} |\mathcal{M}_{\sigma_1, \dots, 4; \sigma_5, 6}|^2 = \Sigma_0 + \Sigma_1 \cos 2\Phi \quad (\Phi \equiv \phi_1 + \phi_2)$$

where

$$\begin{aligned} \Sigma_0 &\propto (d_{+2, \lambda'}^2(\Theta))^2 + (d_{-2, \lambda'}^2(\Theta))^2 = \begin{cases} \frac{1}{8}(1 + 6 \cos^2 \Theta + \cos^4 \Theta) & \text{for } \lambda' = \pm 2 \\ \frac{1}{2}(1 - \cos^4 \Theta) & \text{for } \lambda' = \pm 1 \\ \frac{3}{4} \sin^4 \Theta & \text{for } \lambda' = 0 \end{cases} \\ \Sigma_1 &\propto 2 d_{+2, \lambda'}^2(\Theta) d_{-2, \lambda'}^2(\Theta) = \begin{cases} +\frac{1}{8} \sin^4 \Theta & \text{for } \lambda' = \pm 2 \\ -\frac{1}{2} \sin^4 \Theta & \text{for } \lambda' = \pm 1 \\ +\frac{3}{4} \sin^4 \Theta & \text{for } \lambda' = 0 \end{cases} \end{aligned}$$

$\Rightarrow$  The azimuthal  $\Phi$  correlations depend on  $\Theta$  and  $\lambda'$ .

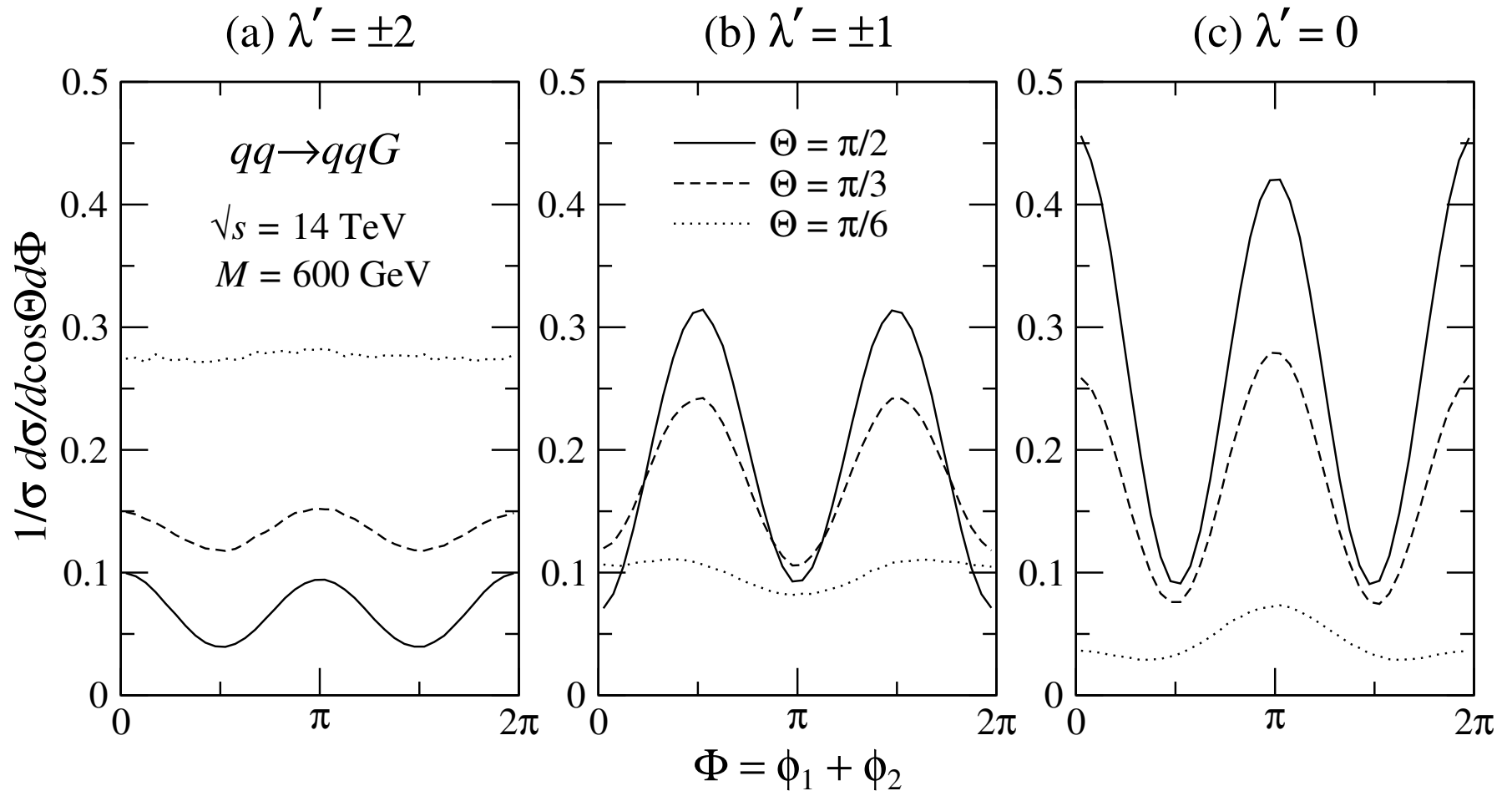
## $(\phi_1 - \phi_2)$ distributions for gravitons



“HELAS and MadGraph/MadEvent with spin-2 particles”  
K.Hagiwara, J.Kanzaki, Q.Li, KM, EPJC56(2008)435  
(The code is available at <http://madgraph.kek.jp/KEK/>.)



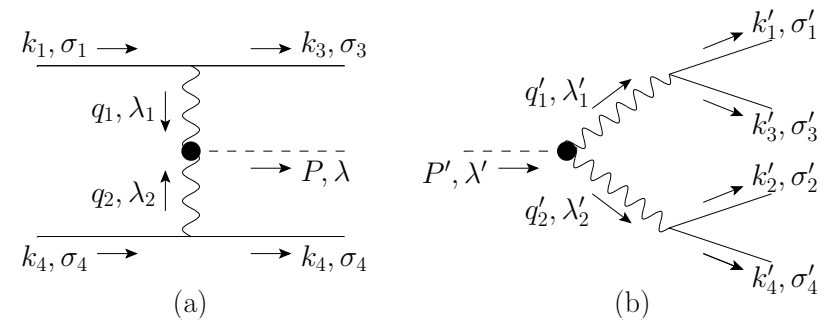
## ( $\phi_1 + \phi_2$ ) distributions for gravitons



$$d\sigma/d\cos\Theta d\Phi \sim \Sigma_0 + \Sigma_1 \cos 2\Phi; \quad \Sigma_1 \propto \begin{cases} +\frac{1}{100} \sin^4 \Theta & \text{for } \lambda' = \pm 2 \\ -\frac{1}{2} \sin^4 \Theta & \text{for } \lambda' = \pm 1 \\ +\frac{3}{4} \sin^4 \Theta & \text{for } \lambda' = 0 \end{cases}$$

The  $\Theta$  and  $\lambda'$  dependent azimuthal  $\Phi$  correlations !

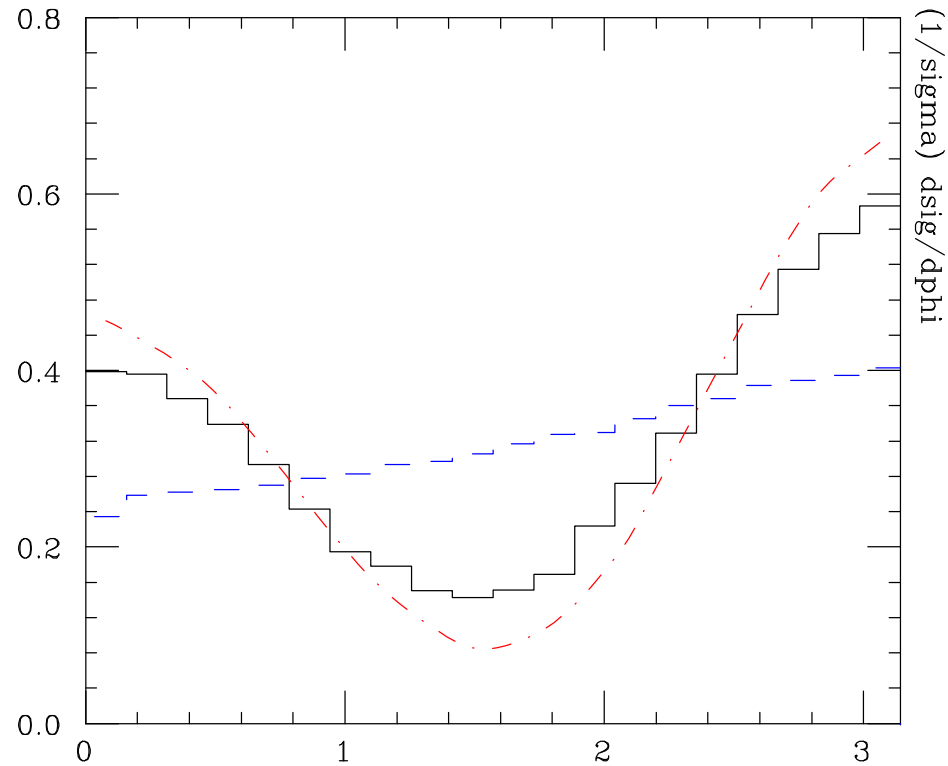
## Summary



- We have studied
  - heavy particle ( $H/A$  and  $G$ ) productions in association with two jets via VBF processes at the LHC.
  - (their decays into 4 leptons/jets via a vector-boson pair.)
- We showed
  - the helicity amplitudes explicitly for the VBF subprocesses.
  - non-trivial azimuthal correlations of the jets are manifestly expressed as the quantum interference among different helicity states of the intermediate vector-bosons.
- These correlations reflect the spin and  $CP$  nature of the produced heavy particles.

Back up

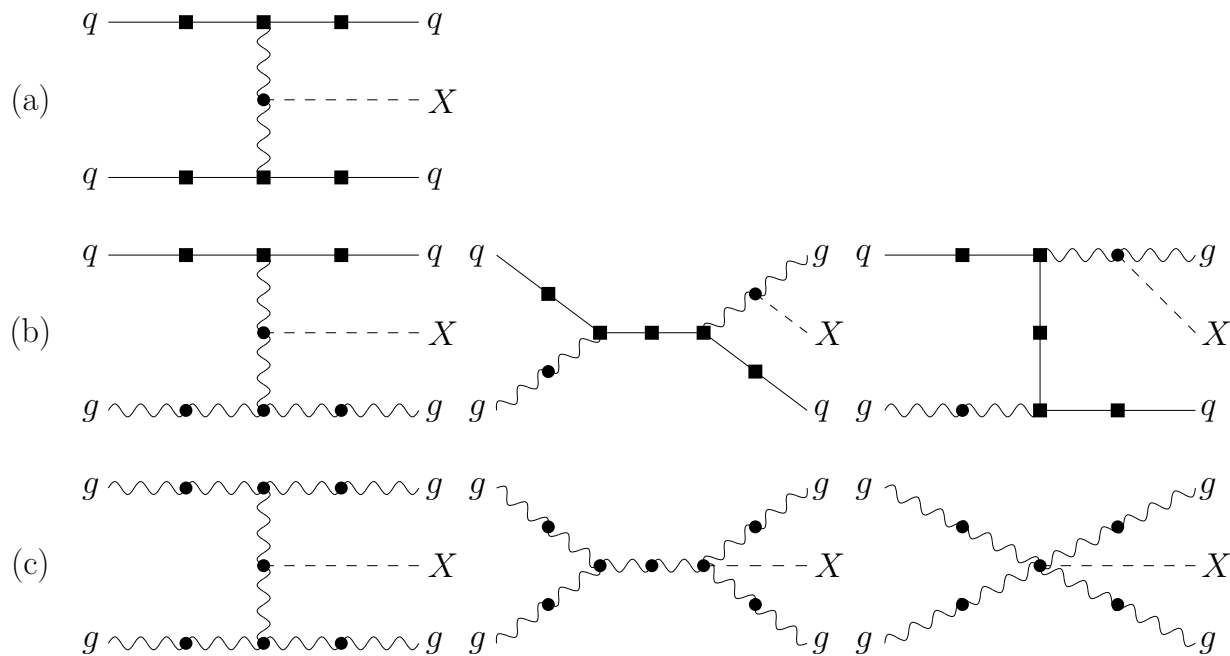
## Parton-shower effect



V. Del Duca et al. (2006)

The present consensus seems to be that **the azimuthal angle correlations predicted in the leading-order corrections may survive.**

## Subprocesses for $X + 2$ jet events



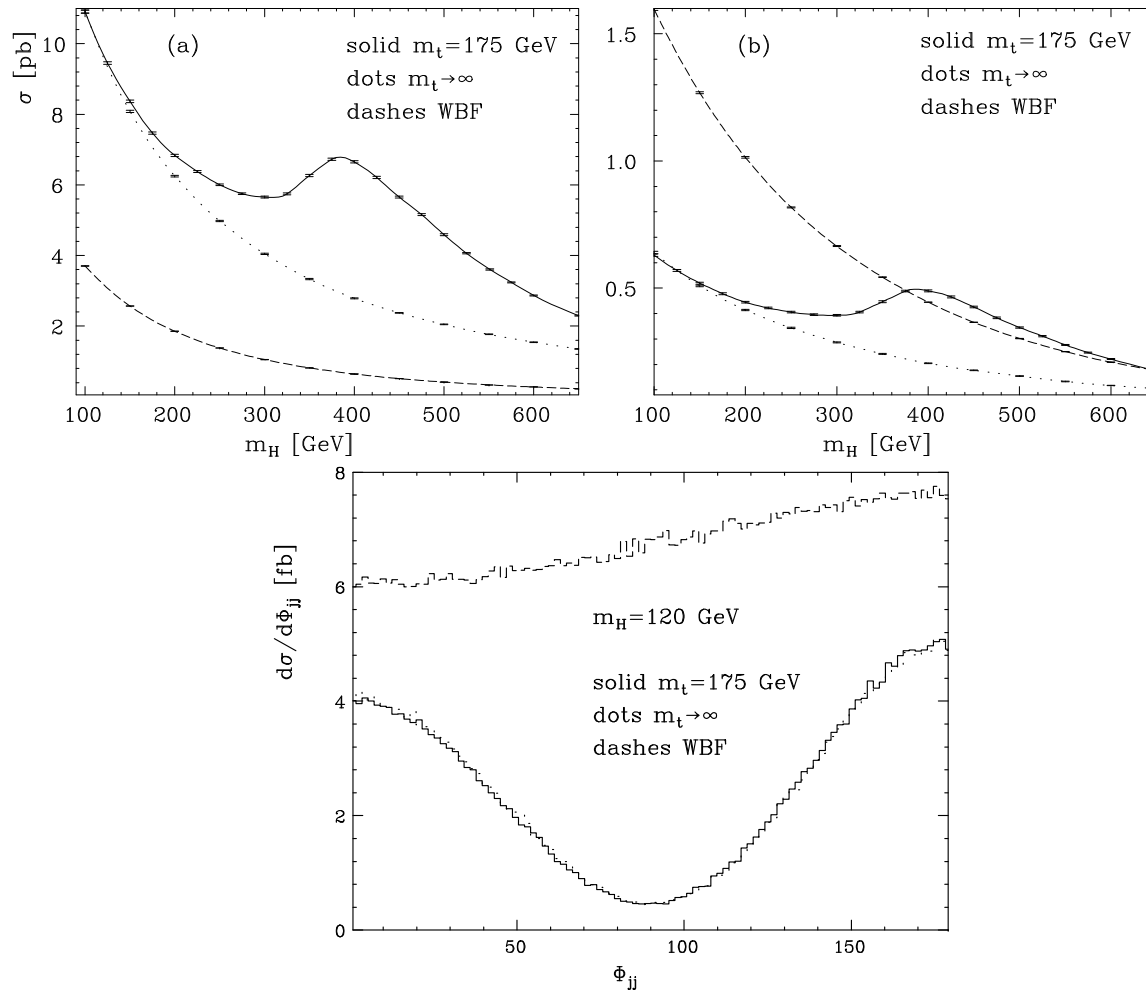
Higgs bosons: emitted from each of ●

KK gravitons: emitted from each of ● and □

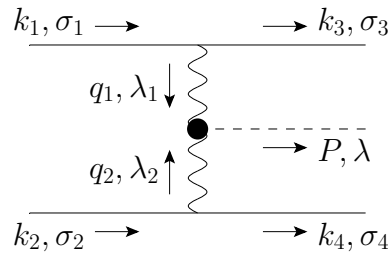
Due to the  $t$ -channel propagators, the  $Xjj$  events via the VBF processes are dominantly produced when  $Q_i^2$  are small, and hence the initial partons scatter to far forward and backward.

⇒ The large rapidity separation cut, or the VBF cut, can select the VBF diagrams among the full diagrams.

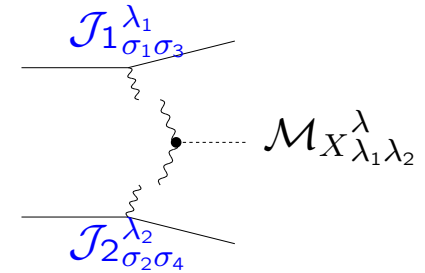
# WBF vs. GF



Del Duca, Kilgore, Oleari, Schmidt, Zeppenfeld (2001)



## Current amplitudes



$$\mathcal{J}_{i\sigma_i\sigma_{i+2}}^{\lambda_i} = (-1)^{\lambda_i+1} J^\mu(k_i, k_{i+2}; \sigma_i, \sigma_{i+2}) \epsilon_\mu(q_i, \lambda_i)^*$$

- Quark current vectors

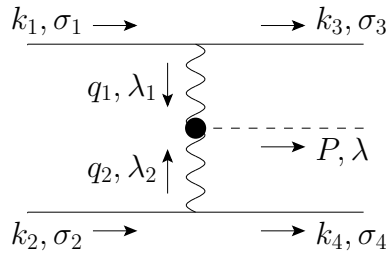
$$J_{Vff'}^\mu(k_i, k_{i+2}; \sigma_i, \sigma_{i+2}) = g_{\sigma_i}^{Vff'} \bar{u}_{f'}(k_{i+2}, \sigma_{i+2}) \gamma^\mu u_f(k_i, \sigma_i)$$

- Gluon current vectors

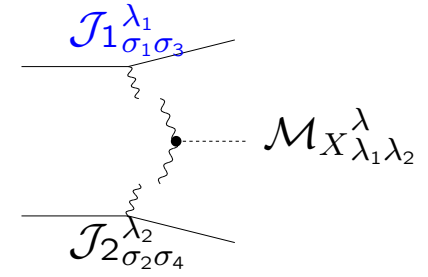
$$J_{ggg}^\mu(k_i, k_{i+2}; \sigma_i, \sigma_{i+2}) = g_s f^{abc} \epsilon_\alpha^b(k_i, \sigma_i) \epsilon_\beta^c(k_{i+2}, \sigma_{i+2})^* \\ \times \left[ -g^{\alpha\beta} (k_i + k_{i+2})^\mu - g^{\beta\mu} (-k_{i+2} + q_i)^\alpha - g^{\mu\alpha} (-q_i - k_i)^\beta \right]$$

- Wavefunctions for the  $t$ -channel vector-bosons

$$\begin{aligned} \epsilon^\mu(q_1, \pm) &= \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0); & \epsilon^\mu(q_2, \pm) &= \frac{1}{\sqrt{2}}(0, \mp 1, i, 0) \\ \epsilon^\mu(q_1, 0) &= (1, 0, 0, 0); & \epsilon^\mu(q_2, 0) &= (-1, 0, 0, 0) \end{aligned}$$



## Current amplitudes




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### $\hat{\mathcal{J}}_{1\sigma_1\sigma_3}^{\lambda_1}$ (quark)

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$$\begin{aligned} \hat{\mathcal{J}}_{1++}^+ &= -(\hat{\mathcal{J}}_{1--}^-)^* & \frac{1}{2 \cos \theta_1} (1 + \cos \theta_1) e^{-i\phi_1} \\ \hat{\mathcal{J}}_{1++}^0 &= \hat{\mathcal{J}}_{1--}^0 & -\frac{1}{\sqrt{2} \cos \theta_1} \sin \theta_1 \\ \hat{\mathcal{J}}_{1++}^- &= -(\hat{\mathcal{J}}_{1--}^+)^* & -\frac{1}{2 \cos \theta_1} (1 - \cos \theta_1) e^{i\phi_1} \\ \hat{\mathcal{J}}_{1+-}^{\lambda_1} &= \hat{\mathcal{J}}_{1-+}^{\lambda_1} & 0 \end{aligned}$$


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### $\hat{\mathcal{J}}_{1\sigma_1\sigma_3}^{\lambda_1}$ (gluon)

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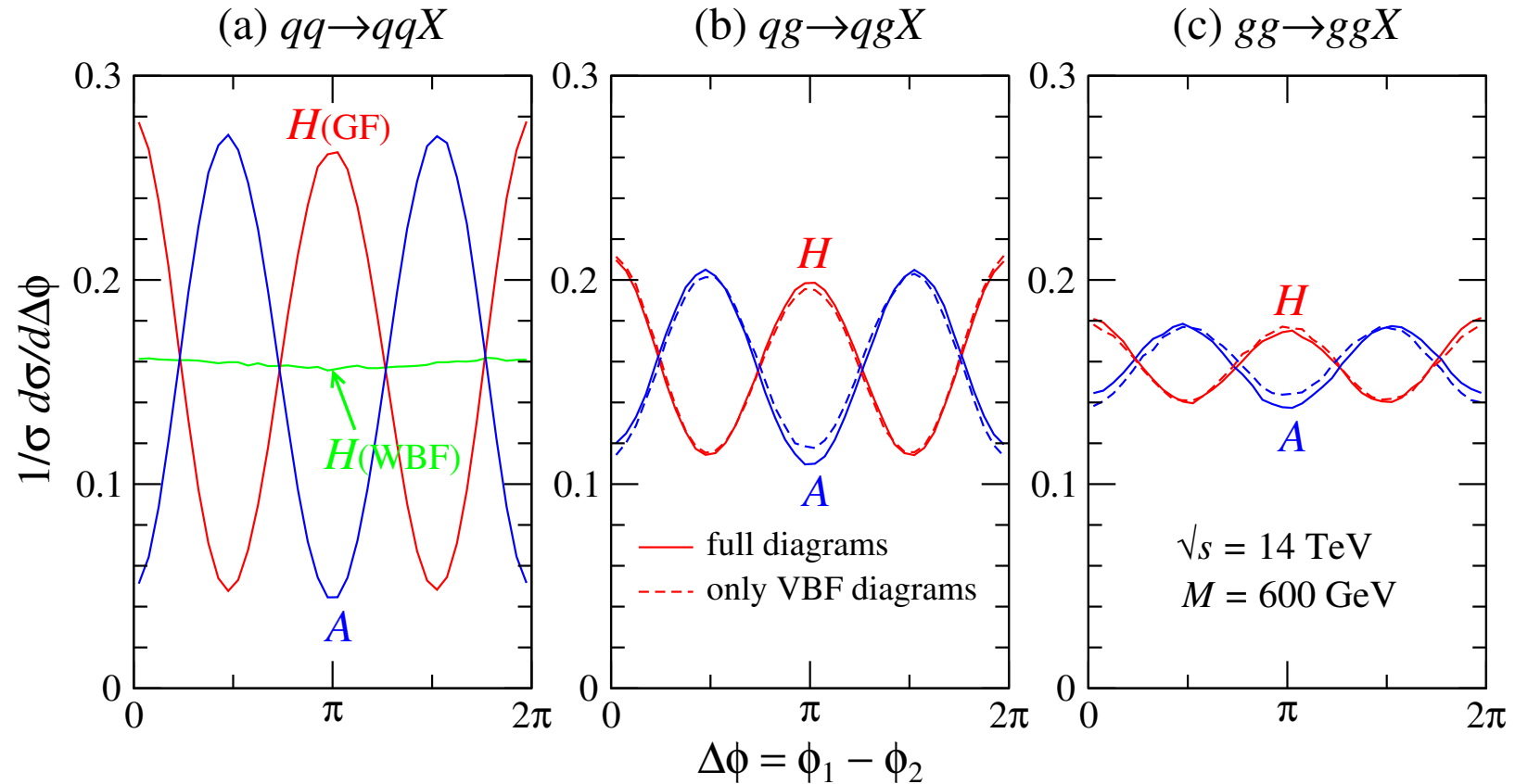
$$\begin{aligned} \hat{\mathcal{J}}_{1++}^+ &= -(\hat{\mathcal{J}}_{1--}^-)^* & \frac{1}{2 \sin \theta_1 \cos \theta_1} (1 + \cos \theta_1)^2 e^{-i\phi_1} \\ \hat{\mathcal{J}}_{1++}^0 &= \hat{\mathcal{J}}_{1--}^0 & -\frac{1}{\sqrt{2} \cos \theta_1} \\ \hat{\mathcal{J}}_{1++}^- &= -(\hat{\mathcal{J}}_{1--}^+)^* & -\frac{1}{2 \sin \theta_1 \cos \theta_1} (1 - \cos \theta_1)^2 e^{i\phi_1} \\ \hat{\mathcal{J}}_{1+-}^+ &= -(\hat{\mathcal{J}}_{1-+}^-)^* & -\frac{2}{\tan \theta_1} e^{i\phi_1} \\ \hat{\mathcal{J}}_{1+-}^{0/-} &= \hat{\mathcal{J}}_{1-+}^{0/+} & 0 \end{aligned}$$


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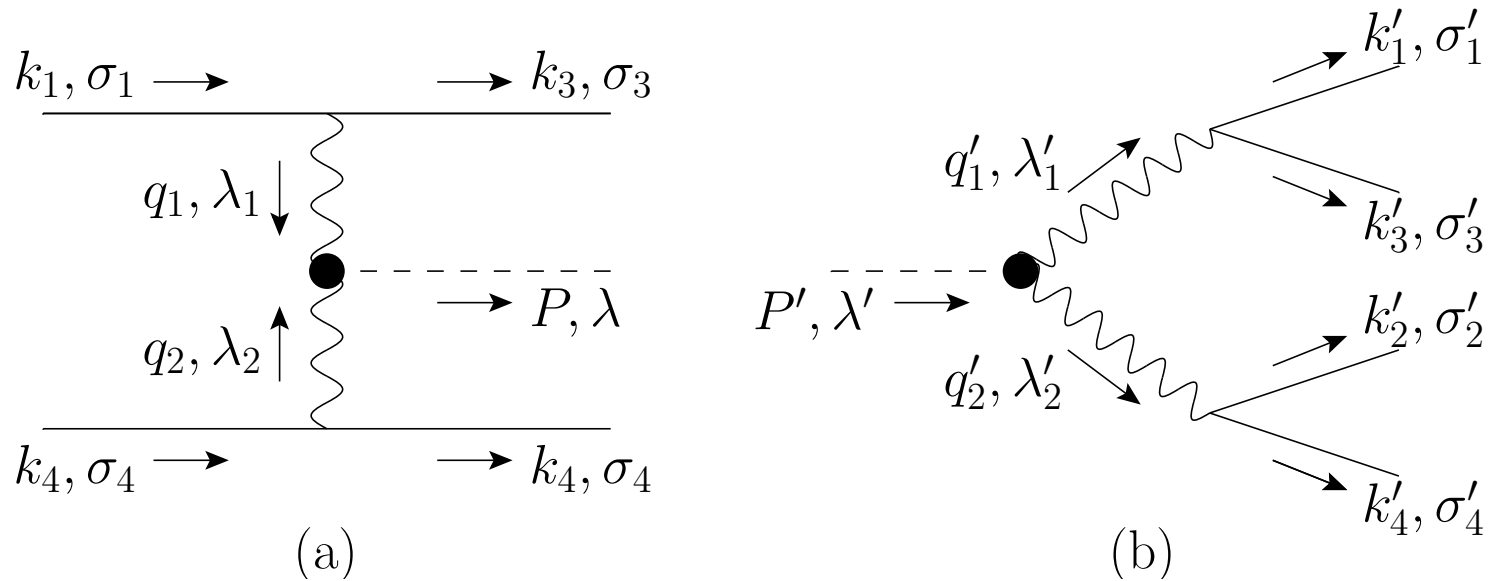
# $\Delta\phi$ distributions for Higgs bosons

The VBF cuts:  $\eta_{j_1} > 0 > \eta_{j_2}$ ,  $\Delta\eta_{jj} = \eta_{j_1} - \eta_{j_2} > 4$



$\implies$  By imposing the VBF (large rapidity separation) cuts, the VBF contributions can reproduce the distributions with the exact matrix elements very well even for the GF processes.

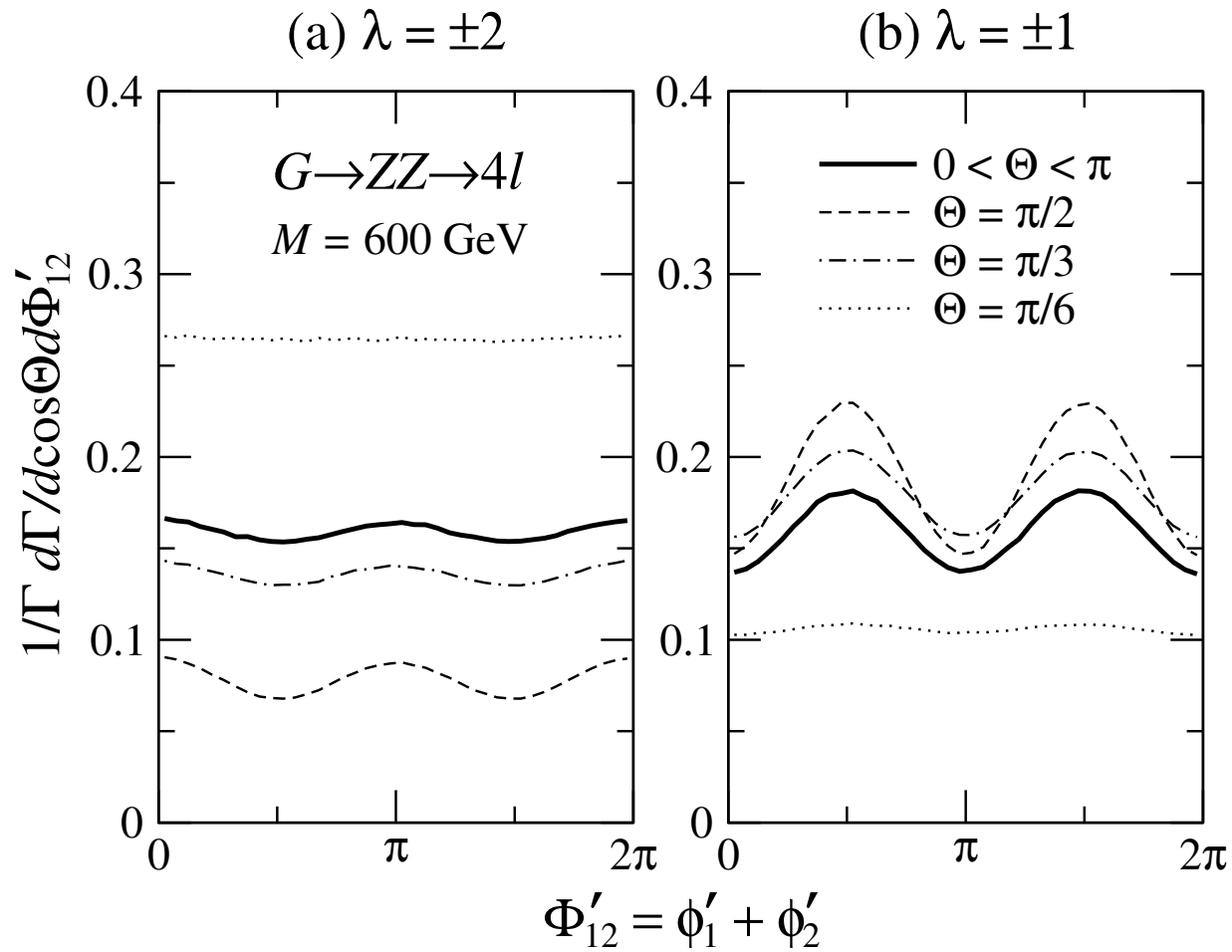
## $X$ decay to a vector-boson pair



Angular correlations in heavy-particle decays are also promising tools to determine their properties.

The process of  $X$  decays into 4 jets/leptons via a vector-boson pair is related by crossing symmetry to the VBF process.

$(\phi'_1 + \phi'_2)$  distributions for  $pp \rightarrow G \rightarrow ZZ \rightarrow 4\ell$



The  $\Theta$  and  $\lambda$  dependent azimuthal  $\cos 2\Phi'$  correlations !