$t\bar{t}$ Production at Hadron Colliders

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Plan of the Talk

- General Introduction
- Status of the $t\bar{t}$ Inclusive Cross Section
- NNLO Corrections:
  - Method (Analytic Calculation)
  - Results

R. B., A. Ferroglia, T. Gehrmann, D. Maitre, and C. Studerus, JHEP **0807** (2008) 129.
R. B., A. Ferroglia, T. Gehrmann, and C. Studerus, JHEP **0908** (2009) 067.

- Conclusions and Outlook

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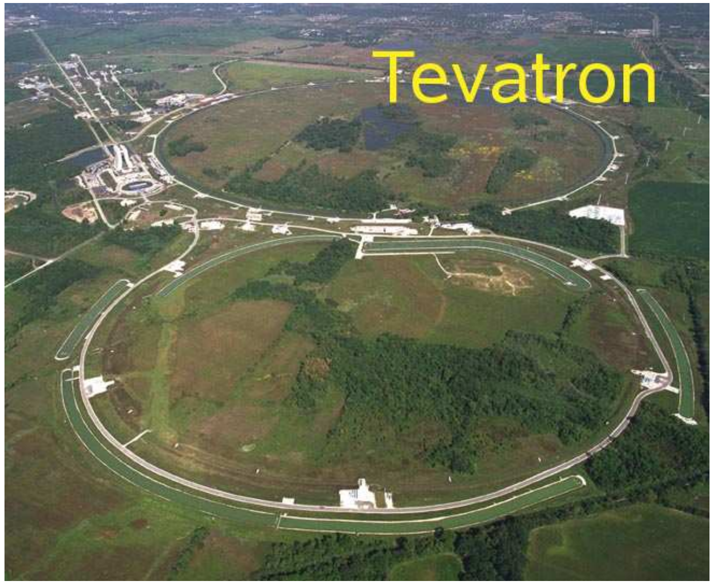
- To date the Top quark could be produced and studied only at the Tevatron, where it was discovered in 1995.
  - In total, $\mathcal{O}(10^3)$ $t\bar{t}$ pairs were produced at Tevatron since the discovery of the top.
  - the mass is measured at better than 1%
  - the total cross section $\sigma_{t\bar{t}}$ is measured at the 12% level (D0 arXiv:0903.5525: $\sigma_{t\bar{t}} = 8.18^{+0.98}_{-0.87}$ pb)



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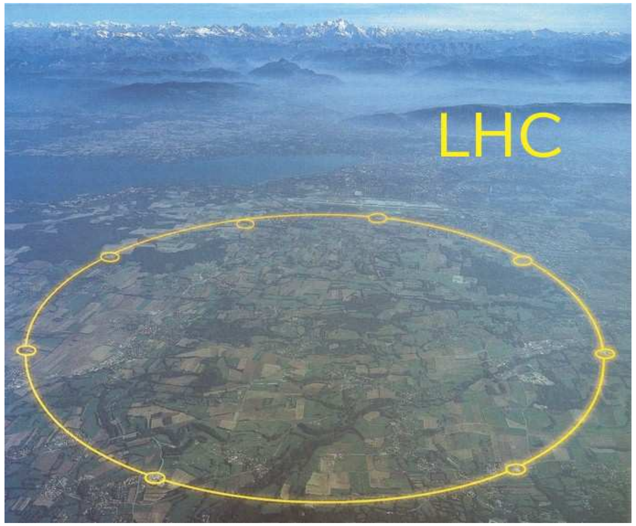
- At the LHC the situation is going to improve:
  - Even in the first low-luminosity phase we are expected to see millions of $t\bar{t}$ pairs per year!
  - With LHC at full speed, $\sigma_{t\bar{t}}$ is expected to be measured at better than 5%!!



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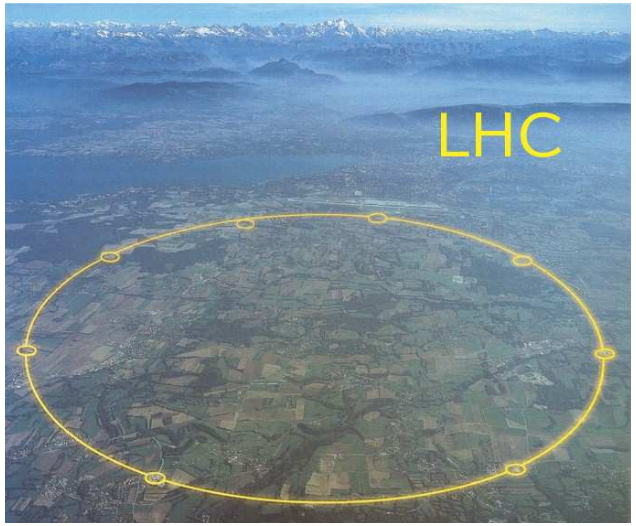

$\Longrightarrow$ At the LHC, top-quark physics will become “precision” physics.

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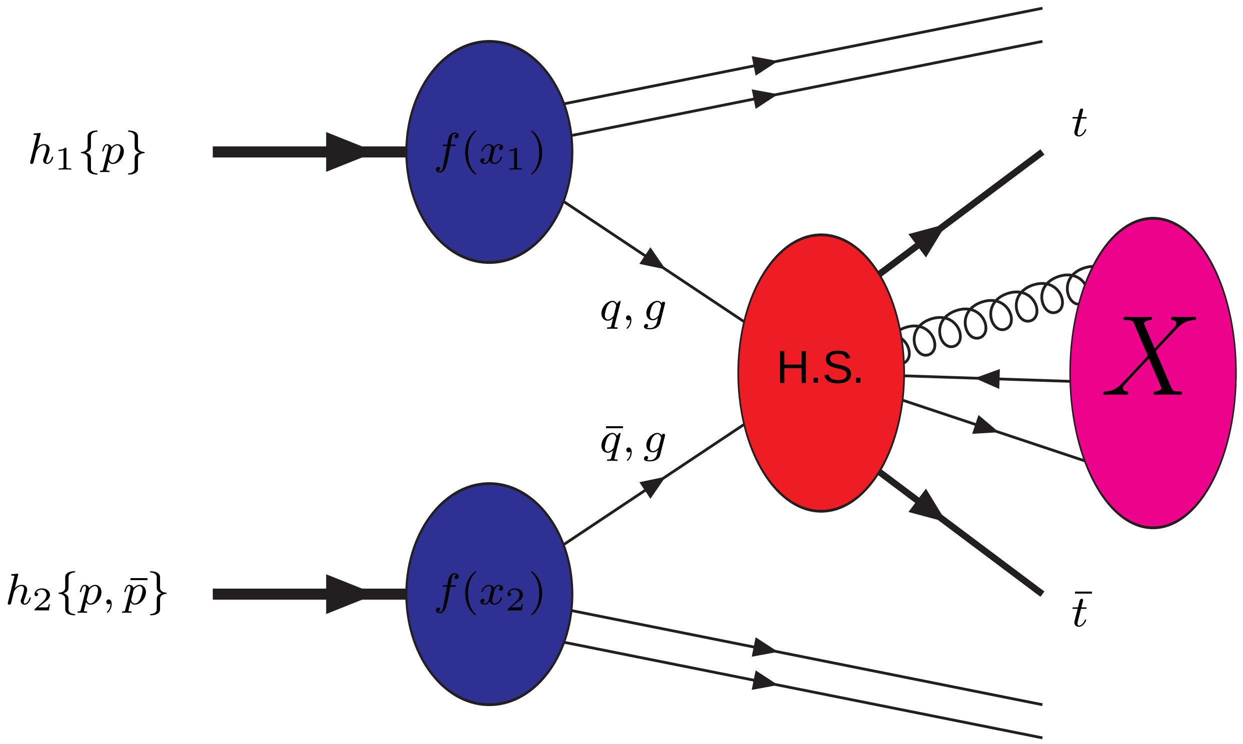
According to the factorization theorem, the process $h_1 + h_2 \to t\bar{t} + X$ can be sketched as in the figure:



$$\sigma^{t\bar{t}}_{h_1,h_2} = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 f_{h_1,i}(x_1, \mu_F) f_{h_2,j}(x_2, \mu_F)\ \hat{\sigma}_{ij}\left(\hat{s}, m_t, \alpha_s(\mu_R), \mu_F, \mu_R\right)$$

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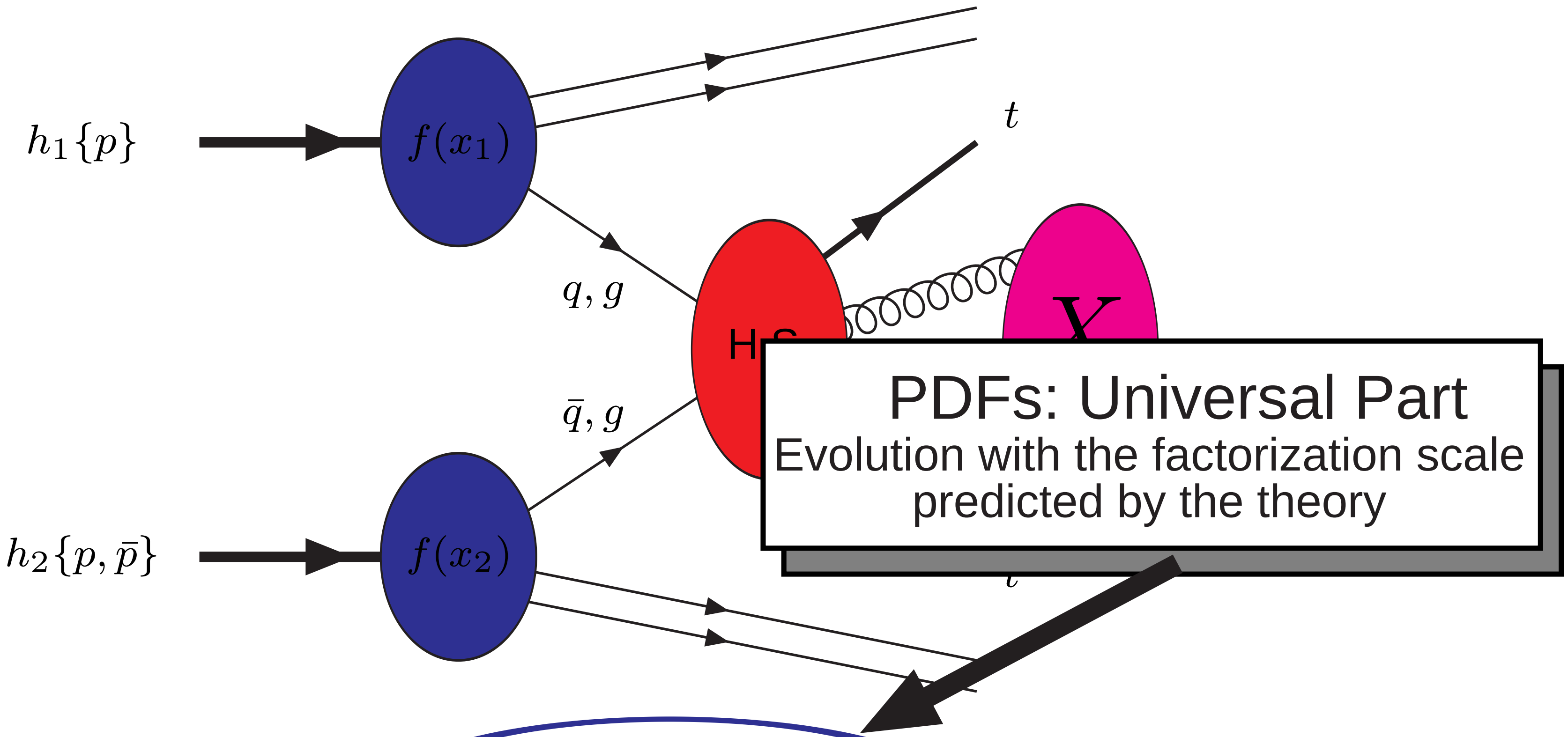

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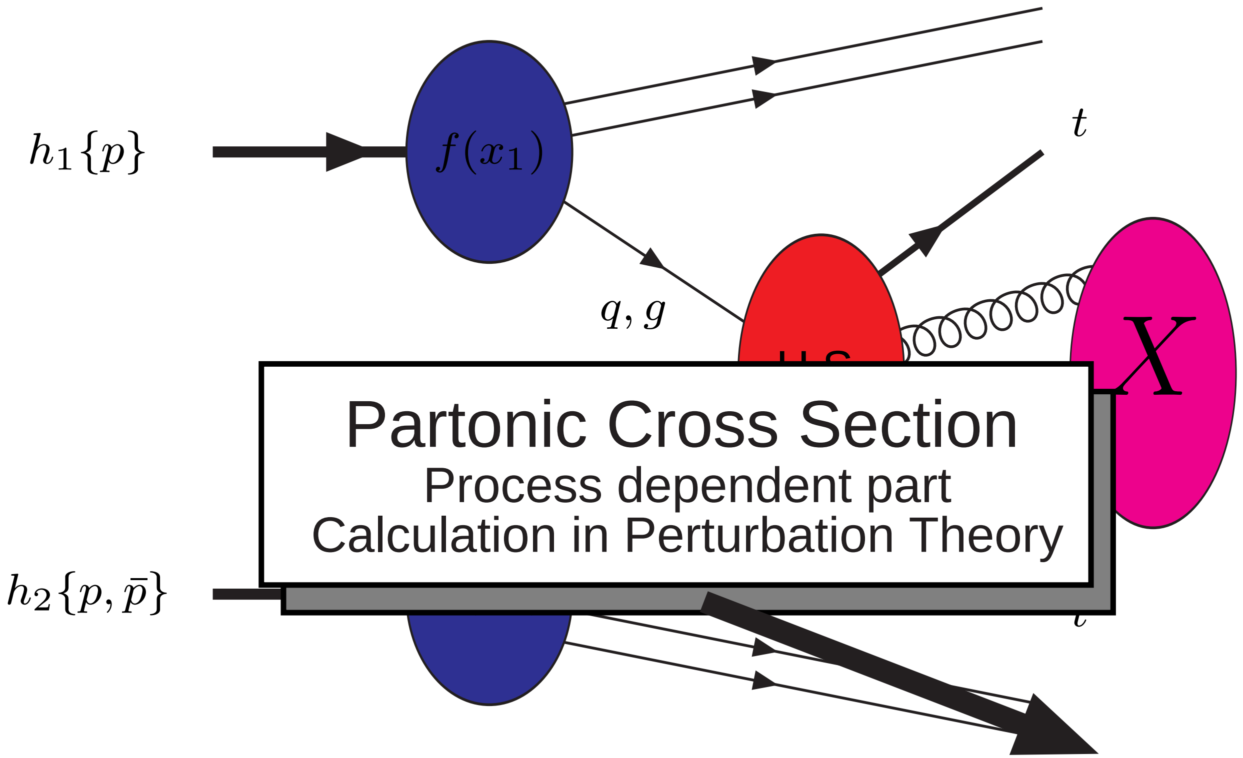

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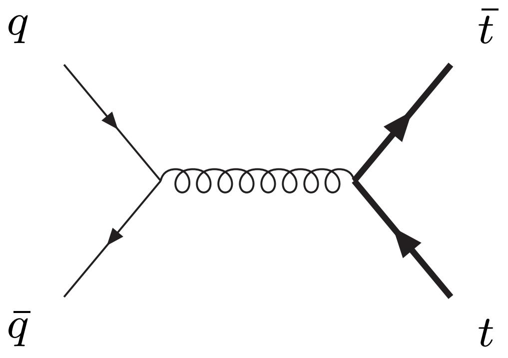

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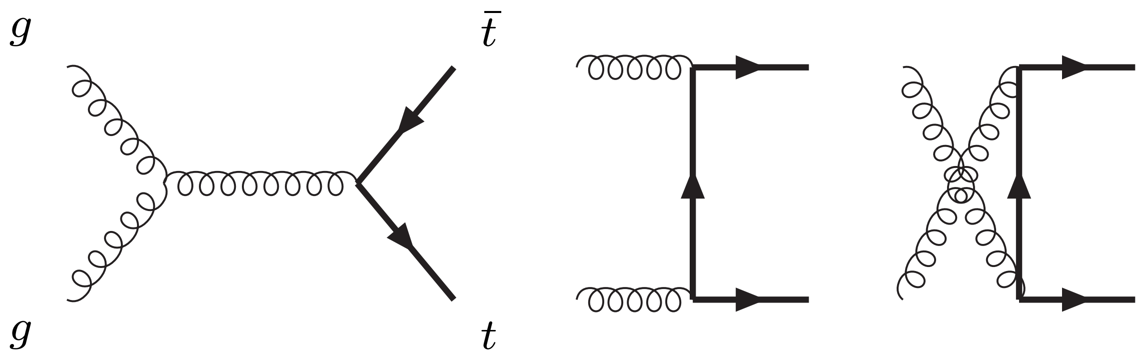

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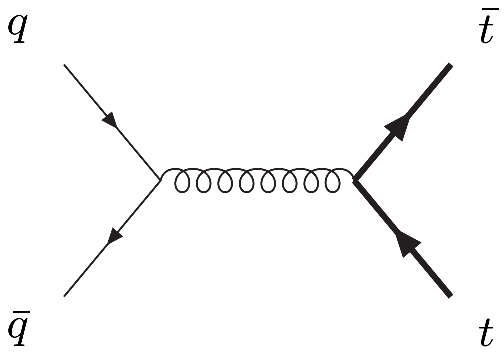

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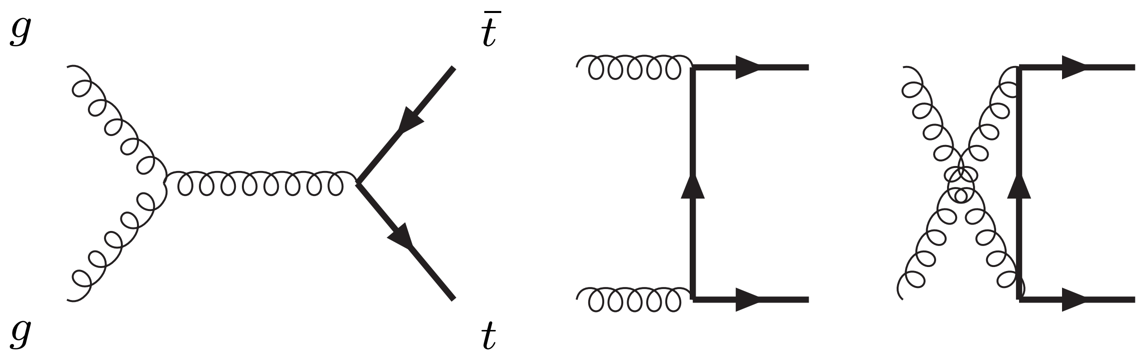

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$\sigma_{t\bar{t}}^{LO}(LHC, m_t = 171\,\text{GeV}) = 583\,\text{pb} \pm 30\%$

$\sigma_{t\bar{t}}^{LO}(Tev, m_t = 171\,\text{GeV}) = 5.92\,\text{pb} \pm 44\%$

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- The NLO QCD corrections are quite sizable: + 25% at Tevatron and +50% at LHC. Reduction of the th error to $\pm 15\%$.

  Nason, Dawson, Ellis ’88-’90; Beenakker, Kuijf, van Neerven, Smith ’89-’91; Mangano, Nason, Ridolfi ’92; Frixione et al. ’95; Czakon and Mitov ’08.

- Mixed NLO QCD-EW corrections are small: - 1% at Tevatron and -0.5% at LHC.

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$$\sigma \sim \sum_{n,m} C_{n,m} \alpha_S^n \ln^m (1-\rho) \qquad m \leq 2n$$

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- Even if $\alpha_S \ll 1$ (perturbative region) we can have at all orders $\alpha_S^n \ln^m (1-\rho) \sim \mathcal{O}(1)$

  Resummation $\Longrightarrow$ improved perturbation theory

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All-order Soft-Gluon Resummation

- Leading-Logs (LL)

  Laenen et al. ’92-’95; Berger and Contopanagos ’95-’96; Catani et al. ’96.

- Next-to-Leading-Logs (NLL)

  Kidonakis and Sterman ’97; R. B., Catani, Mangano, and Nason ’98-’03.

- Next-to-Next-to-Leading-Logs (NNLL) under study ...

  Moch and Uwer ’08; Beneke et al. ’09; Czakon et al. ’09; Kidonakis ’09.

The effect of the resummation up to NLL is to enhance the NLO cross section of +4% and to reduce the dependence on $\mu_{F/R}$ (to $\sim 2/3$ at the Tevatron).

NLO+NLL Theoretical Prediction

TEVATRON

$$\sigma_{t\bar{t}}^{\rm NLO+NLL}({\rm Tev}, m_t = 171~{\rm GeV}, {\rm CTEQ6.5}) = 7.61~^{+0.30(3.9\%)}_{-0.53(6.9\%)}~({\rm scales})~^{+0.53(7\%)}_{-0.36(4.8\%)}~({\rm PDFs})~~{\rm pb}$$

$$\sigma_{t\bar{t}}^{\rm NLO+NLL}({\rm Tev}, m_t = 171~{\rm GeV}, {\rm MRST2006nnlo}) = 7.93~^{+0.34(4.3\%)}_{-0.56(7.1\%)}~({\rm scales})~^{+0.24(3.1\%)}_{-0.20(2.5\%)}~({\rm PDFs})~~{\rm pb}\,.$$

$$\sigma_{t\bar{t}}^{\rm NLO}({\rm Tev}, m_t = 171~{\rm GeV}, {\rm CTEQ6.5}) = 7.35~^{+0.38(5.1\%)}_{-0.80(10.9\%)}~({\rm scales})~^{+0.49(6.6\%)}_{-0.34(4.6\%)}~({\rm PDFs})~~{\rm pb}$$

LHC

$$\sigma_{t\bar{t}}^{\rm NLO+NLL}({\rm LHC}, m_t = 171~{\rm GeV}, {\rm CTEQ6.5}) = 908~^{+82(9.0\%)}_{-85(9.3\%)}~({\rm scales})~^{+30(3.3\%)}_{-29(3.2\%)}~({\rm PDFs})~~{\rm pb}$$

$$\sigma_{t\bar{t}}^{\rm NLO+NLL}({\rm LHC}, m_t = 171~{\rm GeV}, {\rm MRST2006nnlo}) = 961~^{+89(9.2\%)}_{-91(9.4\%)}~({\rm scales})~^{+11(1.1\%)}_{-12(1.2\%)}~({\rm PDFs})~~{\rm pb}$$

$$\sigma_{t\bar{t}}^{\rm NLO}({\rm LHC}, m_t = 171~{\rm GeV}, {\rm CTEQ6.5}) = 875~^{+102(11.6\%)}_{-100(11.5\%)}~({\rm scales})~^{+30(3.4\%)}_{-29(3.3\%)}~({\rm PDFs})~~{\rm pb}$$

M. Cacciari, S. Frixione, M. Mangano, P. Nason, and G. Ridolfi, JHEP 0809:127,2008.

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.. this is to be compared with the experimental requirements for $\sigma_{t\bar{t}}$:

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Last year two groups, Kidonakis-Vogt and Moch-Uwer, presented “approximated” NNLO results for $\sigma_{t\bar{t}}$ including

- scale dependence at NNLO
- NNLL soft-gluon contributions
- Coulomb corrections

This drastically reduces the uncertainty (factorization/renormalization scale dependence) to the level predicted for LHC: $\boxed{\sim 4-6\%}$.

This results are “approximated” NNLO results.
Nevertheless, they indicate that a COMPLETE NNLO computation is indeed needed in order to match the experimental precision of LHC.

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Both matrix elements known for $t\bar{t} + j$ calculation, BUT subtraction up to 1 unresolved parton, while in a complete NNLO computation of $\sigma_{t\bar{t}}$ we need subtraction terms with up to 2 unresolved partons.

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$$|\mathcal{M}|^2(s,t,m,\varepsilon) = \frac{4\pi^2\alpha_s^2}{N_c}\left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi}\right)\mathcal{A}_1 + \left(\frac{\alpha_s}{\pi}\right)^2\mathcal{A}_2 + \mathcal{O}\left(\alpha_s^3\right)\right]$$

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- The coefficients D_i, E_i, F_i, and A are known analytically (agreement with num res)

  R. B., Ferroglia, Gehrmann, Maitre, and Studerus '08-'09

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$$|\mathcal{M}|^2(s,t,m,\varepsilon) = \frac{4\pi^2\alpha_s^2}{N_c}\left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi}\right)\mathcal{A}_1 + \left(\frac{\alpha_s}{\pi}\right)^2 \mathcal{A}_2 + \mathcal{O}\left(\alpha_s^3\right)\right]$$

$$\mathcal{A}_2 = \mathcal{A}_2^{(2\times0)} + \mathcal{A}_2^{(1\times1)}$$

$$\begin{aligned}\mathcal{A}_2^{(2\times0)} \quad = \quad & N_c C_F\Big[N_c^2 A + B + \frac{C}{N_c^2} + N_l\left(N_c D_l + \frac{E_l}{N_c}\right) \\ & + N_h\left(N_c D_h + \frac{E_h}{N_c}\right) + N_l^2 F_l + N_l N_h F_{lh} + N_h^2 F_h\Big]\end{aligned}$$

218 two-loop diagrams contribute to the 10 different color coefficients

- The whole $\mathcal{A}_2^{(2\times0)}$ is known numerically

  Czakon '08.

- The coefficients D_i, E_i, F_i, and A are known analytically (agreement with num res)

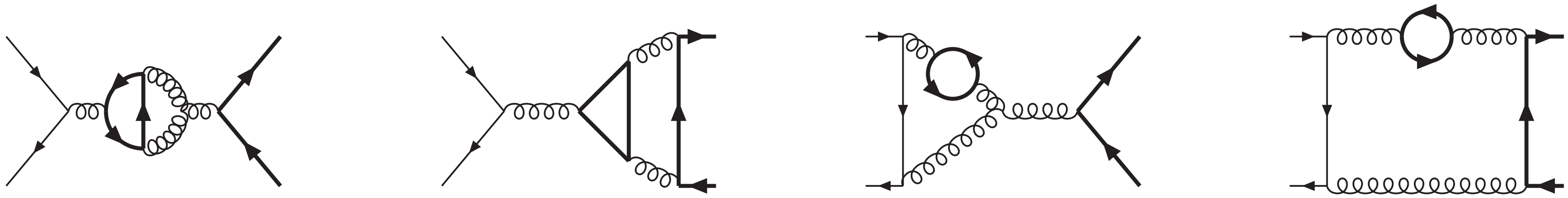
  R. B., Ferroglia, Gehrmann, Maitre, and Studerus '08-'09

- The poles of $\mathcal{A}_2^{(2\times0)}$ (and therefore of B and C) are known analytically

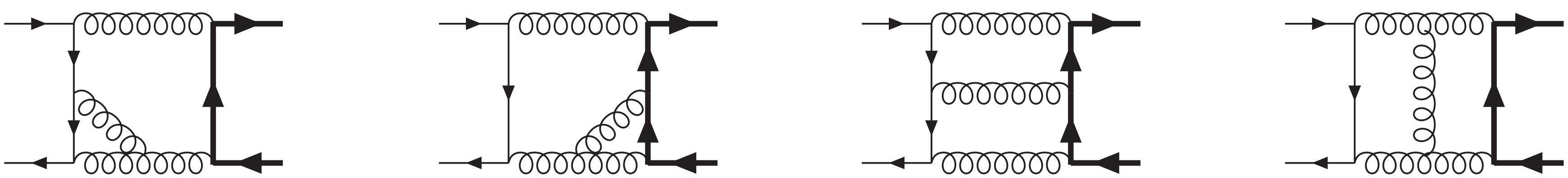
  Ferroglia, Neubert, Pecjak, and Li Yang '09

Two-Loop Corrections to $q\bar{q} \to t\bar{t}$

- $\boxed{D_i, E_i, F_i}$ come from the corrections involving a closed (light or heavy) fermionic loop:

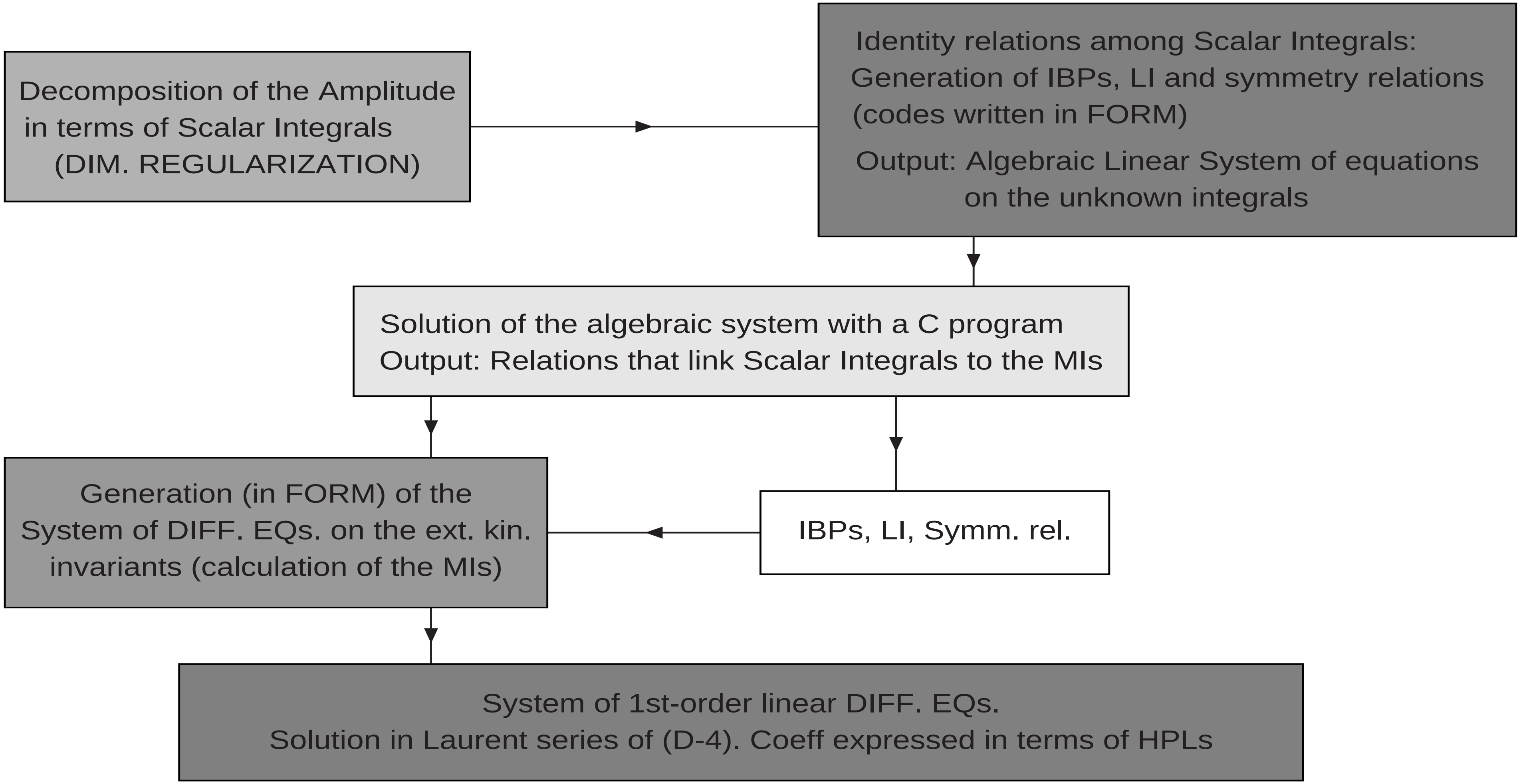


- $\boxed{A}$ the leading-color coefficient, comes from the planar diagrams:

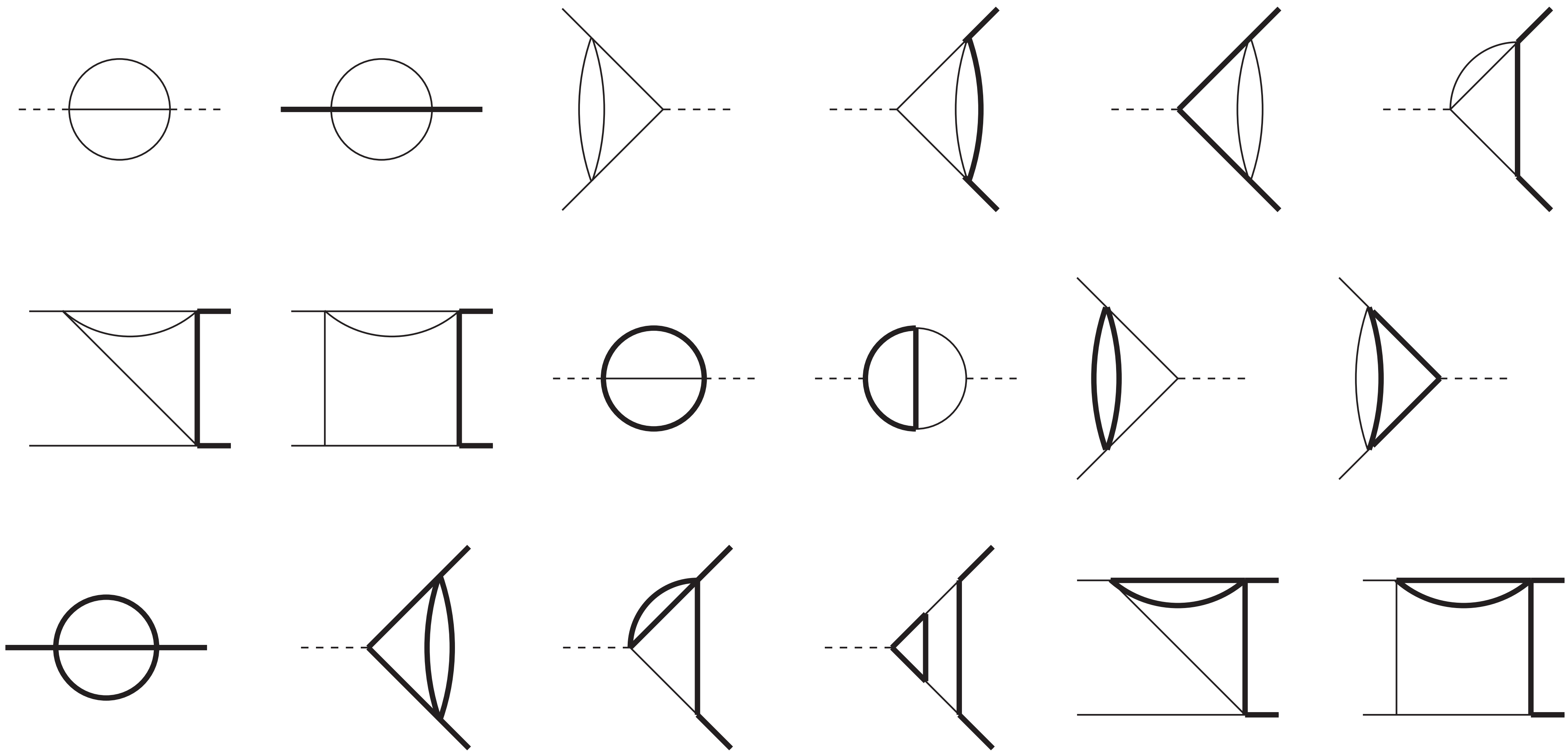


- The calculation is carried out analytically using:
  - Laporta Algorithm for the reduction of the dimensionally-regularized scalar integrals (in terms of which we express the $|\mathcal{M}|^2$) to the Master Integrals (MIs)
  - Differential Equations Method for the analytic solution of the MIs

Laporta Algorithm and Diff. Equations



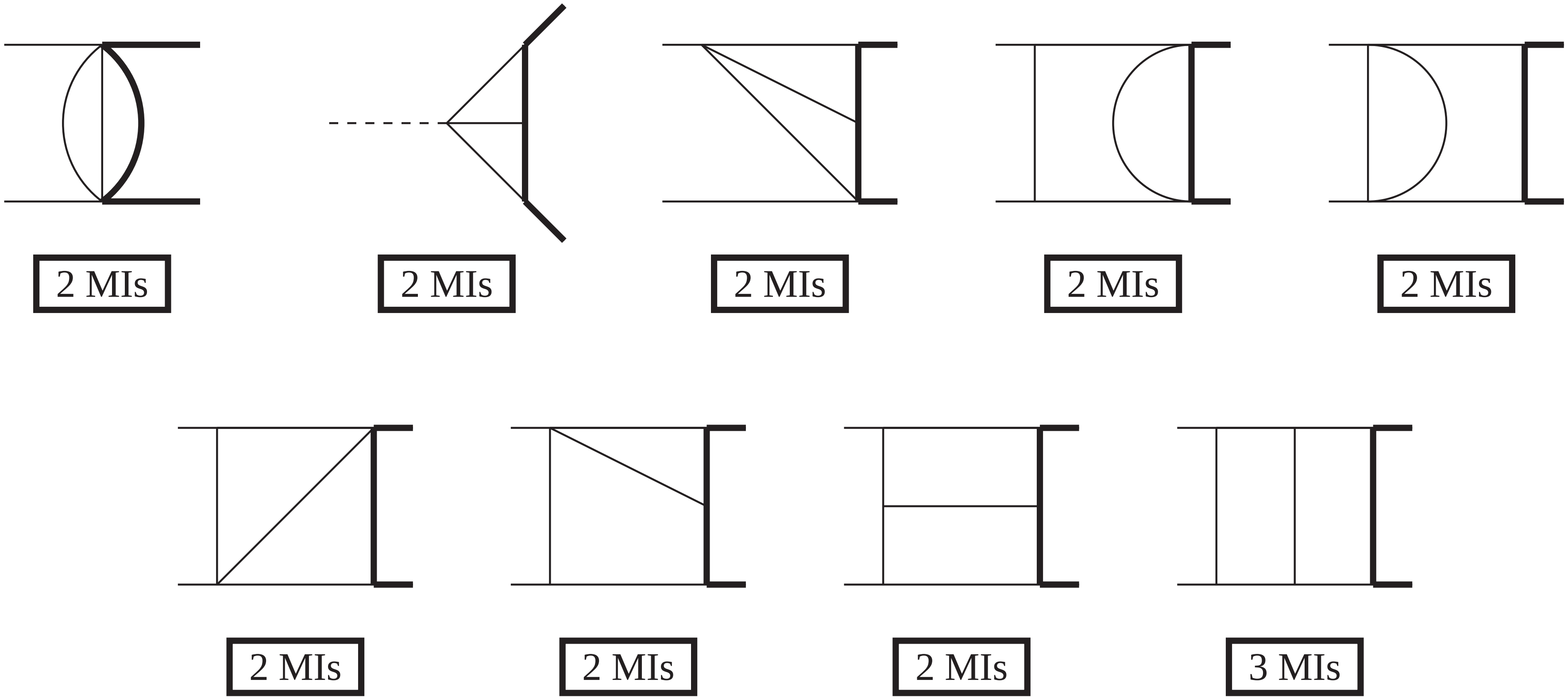
Master Integrals for N$_l$ and N$_h$



18 irreducible two-loop topologies (20 MIs)

R. B., A. Ferroglia, T. Gehrmann, D. Maitre, and C. Studerus, JHEP **0807** (2008) 129.

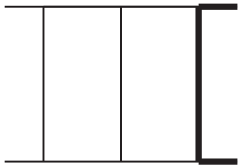
Master Integrals for the Leading Color Coeff



For the leading color coefficient there are 9 additional irreducible topologies (19 MIs)

R. B., A. Ferroglia, T. Gehrmann, and C. Studerus, JHEP **0908** (2009) 067.

Example



$$= \frac{1}{m^6} \sum_{i=-4}^{-1} A_i \epsilon^i + \mathcal{O}(\epsilon^0)$$

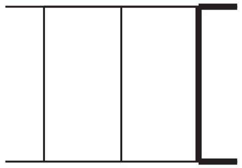
$$A_{-4} = \frac{x^2}{24(1-x)^4(1+y)} ,$$

$$A_{-3} = \frac{x^2}{96(1-x)^4(1+y)} \Big[-10G(-1;y) + 3G(0;x) - 6G(1;x) \Big] ,$$

$$A_{-2} = \frac{x^2}{48(1-x)^4(1+y)} \Big[-5\zeta(2) - 6G(-1;y)G(0;x) + 12G(-1;y)G(1;x) + 8G(-1,-1;y) \Big] ,$$

$$\begin{aligned} A_{-1} = \frac{x^2}{48(1-x)^4(1+y)} \Big[& -13\zeta(3) + 38\zeta(2)G(-1;y) + 9\zeta(2)G(0;x) + 6\zeta(2)G(1;x) - 24\zeta(2)G\left(-1/y;x\right) \\ & +24G(0;x)G(-1,-1;y) - 24G(1;x)G(-1,-1;y) - 12G\left(-1/y;x\right)G(-1,-1;y) \\ & -12G(-y;x)G(-1,-1;y) - 6G(0;x)G(0,-1;y) + 6G\left(-1/y;x\right)G(0,-1;y) + 6G(-y;x)G(0,-1;y) \\ & +12G(-1;y)G(1,0;x) - 24G(-1;y)G(1,1;x) - 6G(-1;y)G\left(-1/y,0;x\right) + 12G(-1;y)G\left(-1/y,1;x\right) \\ & -6G(-1;y)G(-y,0;x) + 12G(-1;y)G(-y,1;x) + 16G(-1,-1,-1;y) - 12G(-1,0,-1;y) \\ & -12G(0,-1,-1;y) + 6G(0,0,-1;y) + 6G(1,0,0;x) - 12G(1,0,1;x) - 12G(1,1,0;x) + 24G(1,1,1;x) \\ & -6G\left(-1/y,0,0;x\right) + 12G\left(-1/y,0,1;x\right) + 6G\left(-1/y,1,0;x\right) - 12G\left(-1/y,1,1;x\right) + 6G(-y,1,0;x) \\ & -12G(-y,1,1;x) \Big] \end{aligned}$$

Example



$$= \frac{1}{m^6} \sum_{i=-4}^{-1} A_i^i + \mathcal{O}(^0)$$

$$A_{-4} = \frac{x^2}{24(1-x)^4(1+y)},$$

$$A_{-3} = \frac{x^2}{96(1-x)^4(1+y)}\Big[-10G(-1;y) + 3G(0;x) - 6\ldots$$

$$A_{-2} = \frac{x^2}{48(1-x)^4(1+y)}\Big[-5\zeta(2) - 6G(-1;y)G(0;x) + \ldots$$

$$A_{-1} = \frac{x^2}{48(1-x)^4(1+y)}\Big[-13\zeta(3) + 38\zeta(2)G(-1;y) + 9\zeta(2)G(0;x) + 6\zeta(2)G(1;x) - 24\zeta(2)G(-1/y;x)$$

$$+24G(0;x)G(-1,-1;y) - 24G(1;x)G(-1,-1;y) - 12G(-1/y;x)G(-1,-1;y)$$

$$-12G(-y;x)G(-1,-1;y) - 6G(0;x)G(0,-1;y) + 6G(-1/y;x)G(0,-1;y) + 6G(-y;x)G(0,-1;y)$$

$$+12G(-1;y)G(1,0;x) - 24G(-1;y)G(1,1;x) - 6G(-1;y)G(-1/y,0;x) + 12G(-1;y)G(-1/y,1;x)$$

$$-6G(-1;y)G(-y,0;x) - 12G(-1;y)G(-y,1;x) + 16G(-1,-1,-1;y) - \ldots(-1,0,-1;y)$$

$$-12G(0,-1,-1;y) + 6G(0,0,-1;y) + 6G(1,0,0;x) - 12G(1,0,1;x) - 12G(1,1,0;x) + 24G(1,1,1;x)$$

$$-6G(-1/y,0,0;x) + 12G(-1/y,0,1;x) + 6G(-1/y,1,0;x) - 12G(-1/y,1,1;x) + 6G(-y,1,0;x)$$

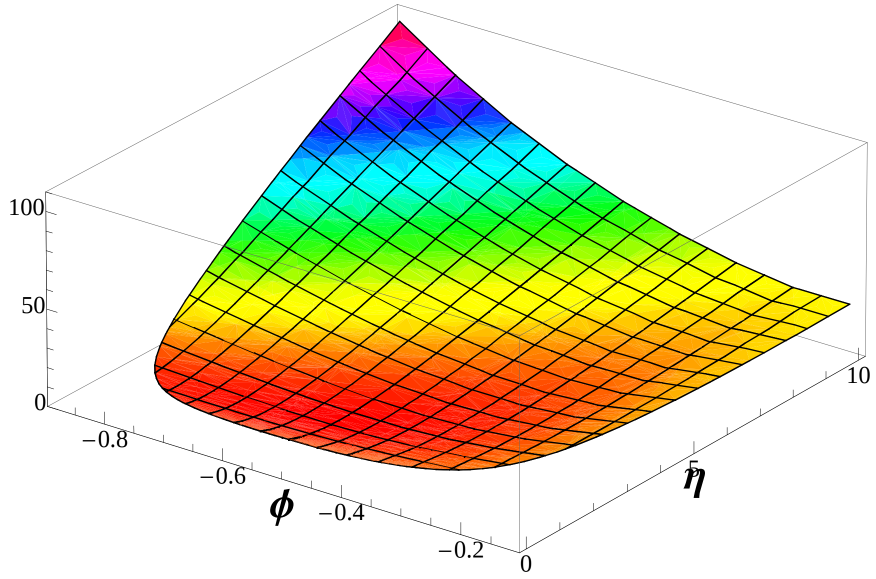
$$-12G(-y,1,1;x)\Big]$$

$$\rho = \frac{4m_t^2}{\hat{s}} \to 1$$

1- and 2-dim GHPLs

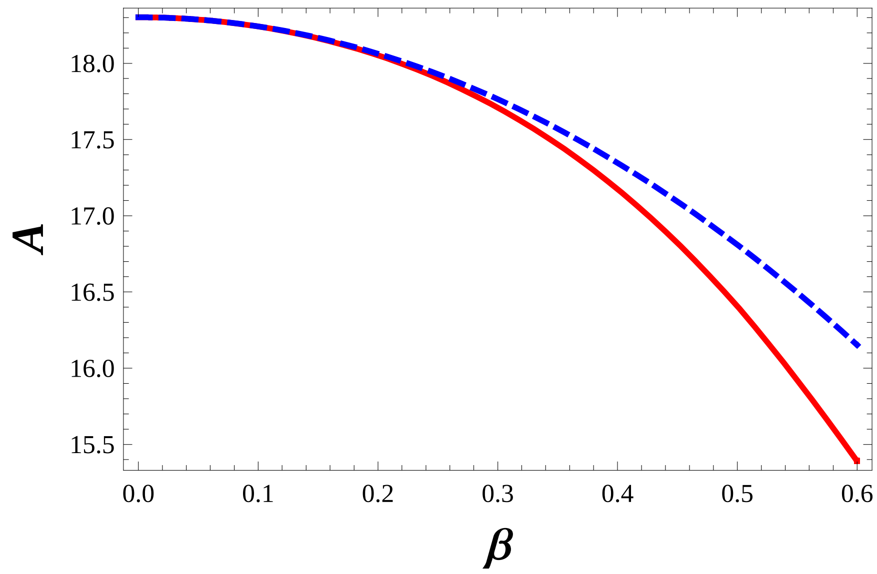
Coefficient A

Finite part of A



$$\eta = \frac{s}{4m^2} - 1\,, \quad \phi = -\frac{t - m^2}{s}$$

Threshold expansion versus exact result



$$\beta = \sqrt{1 - \frac{4m^2}{s}}$$

partonic c.m. scattering angle $= \frac{\pi}{2}$

Numerical evaluation of the GHPLs with GiNaC C++ routines.

Vollinga and Weinzierl ’04

Two-Loop Corrections to $gg \to t\bar{t}$

$$|\mathcal{M}|^2(s,t,m,\varepsilon) = \frac{4\pi^2\alpha_s^2}{N_c}\left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi}\right)\mathcal{A}_1 + \left(\frac{\alpha_s}{\pi}\right)^2\mathcal{A}_2 + \mathcal{O}\left(\alpha_s^3\right)\right]$$

$$\mathcal{A}_2 = \mathcal{A}_2^{(2\times 0)} + \mathcal{A}_2^{(1\times 1)}$$

$$\begin{aligned}\mathcal{A}_2^{(2\times 0)} \quad = \quad & (N_c^2-1)\Big(N_c^3 A + N_c B + \frac{1}{N_c} C + \frac{1}{N_c^3} D + N_c^2 N_l E_l + N_c^2 N_h E_h \\ & + N_l F_l + N_h F_h + \frac{N_l}{N_c^2} G_l + \frac{N_h}{N_c^2} G_h + N_c N_l^2 H_l + N_c N_h^2 H_h \\ & + N_c N_l N_h H_{lh} + \frac{N_l^2}{N_c} I_l + \frac{N_h^2}{N_c} I_h + \frac{N_l N_h}{N_c} I_{lh}\Big)\end{aligned}$$

789 two-loop diagrams contribute to 16 different color coefficients

- No numeric result for $\mathcal{A}_2^{(2\times 0)}$ yet
- The poles of $\mathcal{A}_2^{(2\times 0)}$ are known analytically

  Ferroglia, Neubert, Pecjak, and Li Yang '09
- The coefficients A, E_l–I_l can be evaluated analitically as for the $q\bar{q}$ channel

  R. B., Ferroglia, Gehrmann, and Studerus, in preparation.

Summary and Outlook

- Top physics will play a big role in the LHC program. LHC will be a heavy-quark factory and the properties of the top quark will become soon "precision" physics
- The total cross section for the production of $t\bar{t}$ pairs (which is a standard candle for SM checks in hadronic collisions) is suppose to be measured in the near future at the 5% accuracy level. The theoretical predictions are still far from this goal $\Longrightarrow$ need of a complete NNLO calculation that could match the accuracy needed at the LHC (see approximated NNLO studies)
- Among the ingredients of the NNLO calculation, it is now available a part of the two-loop corrections in analytic form, which is fast to be evaluated numerically (FORTRAN, C++, Mathematica numerical codes are available) and it is totally under control (threshold and asymptotic expansions, analytic continuation)
- Our goal is to complete, in the near future,the evaluation of the two-loop matrix elements and to address the problem of the subtraction terms at NNLO for the real part