Higgs boson masses in the NMSSM

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Based on: *G.Degrassi and P.S., arXiv:0907.4682*

Why the NMSSM?

The μ problem of the MSSM

In SUSY extensions of the SM we must introduce two Higgs doublets with opposite hypercharge:

- To give mass to both up- and down-type quarks
- To cancel gauge anomalies
- To allow for a higgsino mass term

Higgs/higgsino mass term in the superpotential

$$\mathcal{L} \supset \mu \int d^2\theta \; H_1 \, H_2$$

There are also soft SUSY-breaking mass terms for the Higgses in the scalar potential

$$V_{\rm soft} \supset m^2_{H_1} \, |H_1|^2 + m^2_{H_2} \, |H_2|^2 - B_\mu \, (H_1 \, H_2 + {\rm h.c.})$$

EWSB imposes a tree-level relation between μ, $\tan\beta \equiv \langle H_2 \rangle / \langle H_1 \rangle$ and the soft masses

$$m_Z^2 = \left(m^2_{H_2} \, \tan\beta - m^2_{H_1} \, \cot\beta \right) \tan 2\beta - 2\,\mu^2$$

In the MSSM, μ is the only superpotential term with the dimension of a mass

The μ problem: if μ is allowed in the SUSY limit, why is it not of $\mathcal{O}(M_P)$?

The *Giudice-Masiero* solution: (1988)

μ is forbidden in the SUSY limit, and is generated at the SUSY-breaking scale together with the soft parameters

NMSSM alternative: generate μ at the weak scale through the vev of a light singlet

$$\mathcal{L} \supset \lambda \int d^2\theta \; S \, H_1 \, H_2 \quad \longrightarrow \quad \mu_{\text{eff}} = \lambda \, \langle S \rangle$$

This brings along an extended Higgs sector (scalar & pseudoscalar singlet, singlino) and a whole new set of soft SUSY-breaking parameters

Half-empty glass: more complicated, less predictive than the MSSM

Half-full glass:

- extra particles, richer phenomenology at colliders
- relax the upper bound on the lightest Higgs mass

The Higgs sector of the NMSSM

Superpotential and soft SUSY-breaking terms: $W \supset -\lambda\, S H_1 H_2 + \frac{\kappa}{3} S^3$

$$V_{\rm soft} \supset m_{H_1}^2 H_1^\dagger H_1 + m_{H_2}^2 H_2^\dagger H_2 + m_S^2\, S^* S + \left(-\lambda\, A_\lambda\, S H_1 H_2 + \frac{\kappa}{3} A_\kappa\, S^3 + \text{ h.c.}\right)$$

The scalar, pseudoscalar and fermionic components of S mix with their MSSM counterparts

$$H_i^0 = v_i + \frac{1}{\sqrt{2}}(S_i + iP_i) \quad (i = 1, 2)\ , \qquad S = v_s + \frac{1}{\sqrt{2}}(S_3 + iP_3)$$

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R^S \begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix}, \qquad \begin{pmatrix} G^0 \\ A_1 \\ A_2 \end{pmatrix} = R^P \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix}, \qquad \begin{pmatrix} \chi_1^0 \\ \chi_2^0 \\ \chi_3^0 \\ \chi_4^0 \\ \chi_5^0 \end{pmatrix} = N \begin{pmatrix} -i\tilde{b} \\ -i\tilde{w}^0 \\ \tilde{h}_1^0 \\ \tilde{h}_2^0 \\ \tilde{s} \end{pmatrix}$$

The charged-Higgs and chargino sectors are the same as in the MSSM, once we identify

$$\mu \equiv \lambda\, v_s\ , \qquad B_\mu \equiv \lambda\, v_s\, (A_\lambda + \kappa v_s) - \lambda^2\, v_1 v_2\ , \qquad \tan\beta \equiv \frac{v_2}{v_1}$$

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In the limit $v_s^2 \gg v^2 \equiv v_1^2 + v_2^2$ the singlet decouples from the MSSM doublets

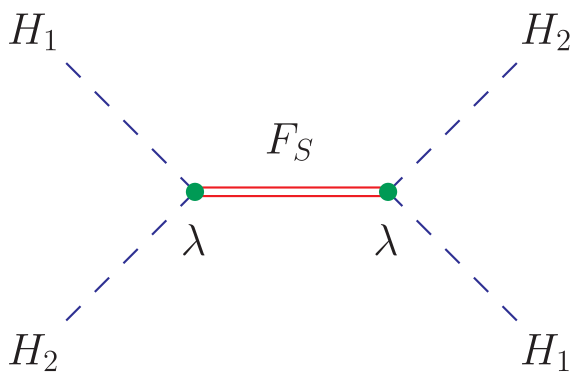
$$m_{A_1}^2 = \frac{2\,B_\mu}{\sin 2\beta} + \mathcal{O}(v^2)\,, \qquad m_{A_2}^2 = \frac{3\,\kappa^2}{w}\,v_s^2 + \mathcal{O}(v^2)$$

$$m_{h_2}^2 = m_{A_1}^2 + \mathcal{O}(v^2)\,, \qquad m_{h_3}^2 = \frac{4w-1}{3}\,m_{A_2}^2 + \mathcal{O}(v^2)$$

$$m_{h_1}^2 = M_Z^2\cos^2 2\beta + \lambda^2\,v^2\left\{\sin^2 2\beta - \frac{\left[\frac{\lambda}{k} + \left(\frac{A_\lambda}{2wA_\kappa} - 1\right)\sin 2\beta\right]^2}{1-\frac{1}{4w}}\right\} + \mathcal{O}(v^4)$$

where $w \equiv \frac{1}{4}\left(1 + \sqrt{1 - 8\,\frac{m_S^2}{A_\kappa^2}}\right) > \frac{1}{3}$

Additional, F-term induced contribution to the MSSM Higgs quartic coupling:



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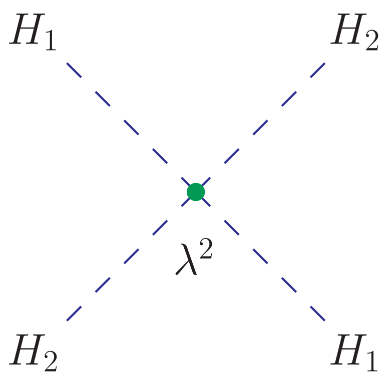

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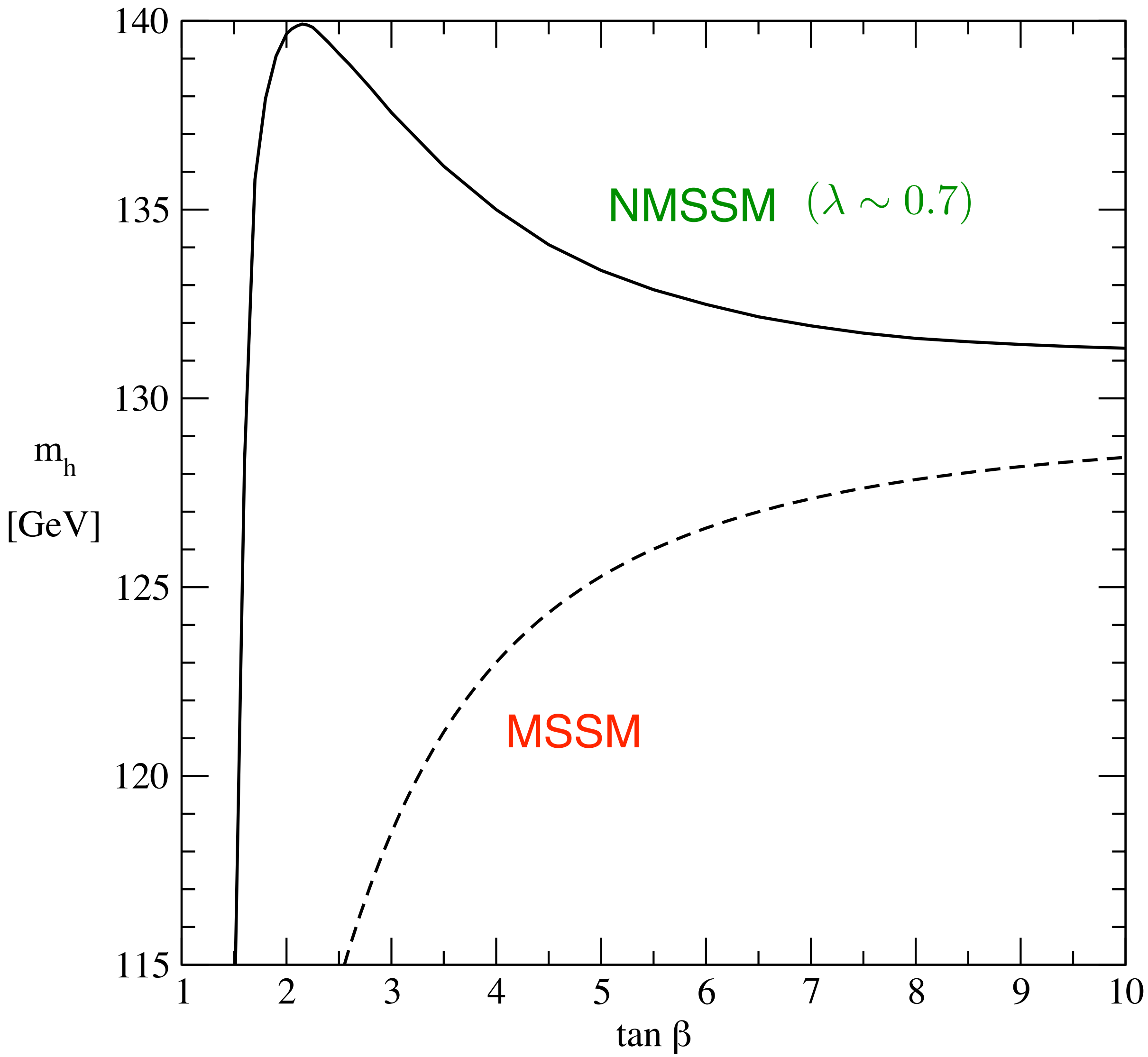
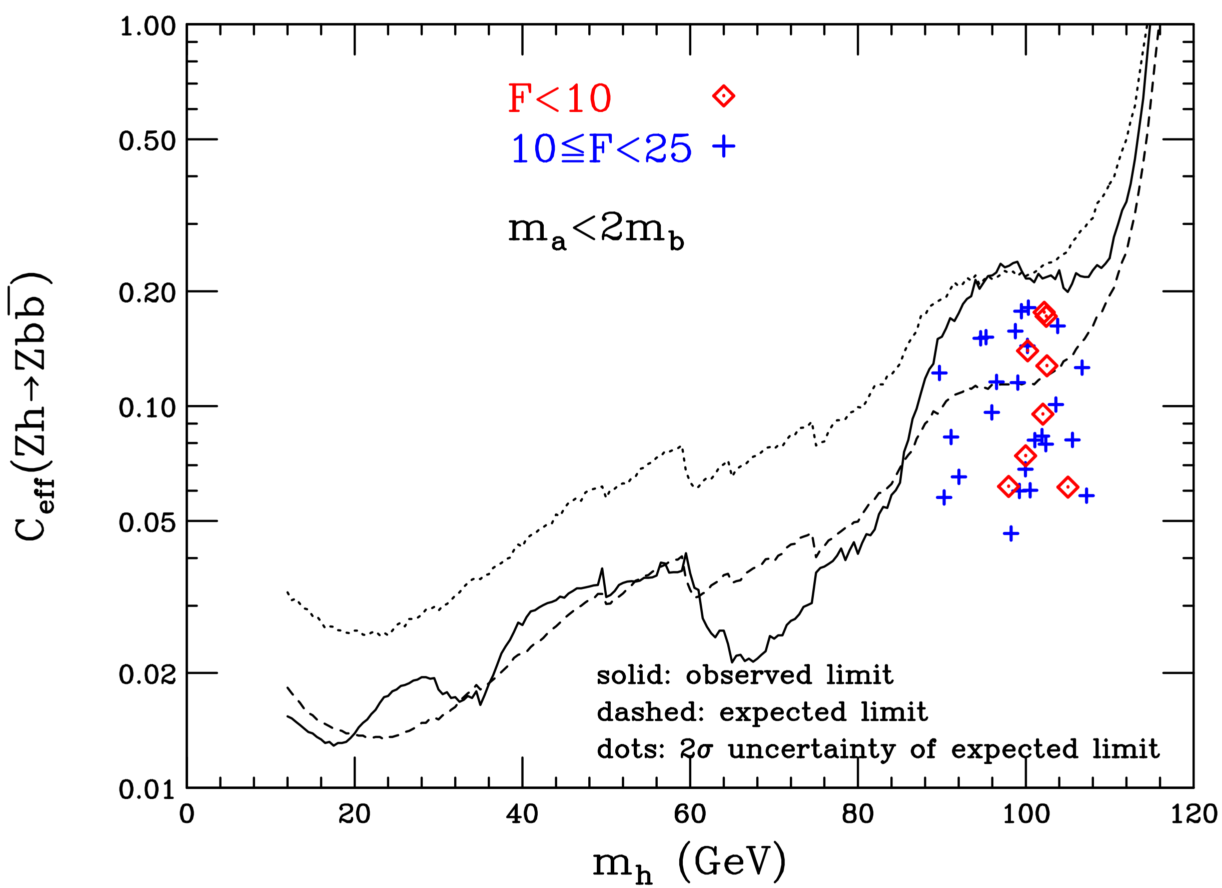
In the NMSSM the lightest Higgs boson mass is maximized at low $\tan\beta$



(plot from Ellwanger & Hugonie, hep-ph/0612133v2)

An interesting possibility: a 100-GeV Higgs boson might not be ruled out by the LEP



(plot from Dermisek & Gunion, hep-ph/0510322)

$$F \;=\; \mathrm{Max}_p \left| \frac{d \log m_Z}{d \log p} \right|$$

$$C_{\rm eff} \;=\; [g^2_{ZZh}/g^2_{ZZh_{\rm SM}}]\,\mathrm{BR}(h_1 \to b\bar{b})$$

The coupling to the Z can be reduced if h_1 has a sizeable singlet component

The BR into bottom can be reduced if $h_1 \;\to\; 2\,A_1 \;\to\; 4\,\tau$ (with $m_{A_1} < 2\,m_b$)

Precise calculation of the NMSSM Higgs masses

Nearly two decades of radiative corrections in the (real) MSSM

- First computation of the one-loop $\mathcal{O}(\alpha_t)$ effects: Okada *et al.* (1991), Ellis, Ridolfi & Zwirner (1991), Haber & Hempfling (1991);
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The situation for the NMSSM *(not quite as advanced!)*

- One-loop (s)top/(s)bottom contributions in the effective potential approach: Ellwanger (1993), Elliott, King & White (1993), Pandita (1993);

- One-loop logarithmic gauge/ino and Higgs/ino contributions in the RG approach: Ellwanger & Hugonie (2005);

- Leading-logarithmic two-loop contributions borrowed from the MSSM:

$$\left(\Delta\mathcal{M}_S^2\right)_{22}^{\mathrm{LL}} = \frac{6\,m_t^2}{\pi^2}\left(\alpha_t\alpha_s - \frac{3}{8}\alpha_t^2\right)\log^2\frac{M_S^2}{m_t^2}$$

All of these corrections are implemented in the public computer code NMHDECAY of Ellwanger, Gunion & Hugonie (2004-2006)

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G.Degrassi & P.S., *On the radiative corrections to the neutral Higgs boson masses in the NMSSM,* arXiv:0907.4682

- A complete one-loop calculation of the neutral Higgs masses in the NMSSM

- Effective potential calculation of the two-loop $\mathcal{O}(\alpha_t\alpha_s + \alpha_b\alpha_s)$ corrections

One-loop corrections to the neutral Higgs mass matrices

$$\left(\mathcal{M}_S^2\right)_{ij}^{\rm 1loop} \;=\; \left(\mathcal{M}_S^2\right)_{ij}^{\rm tree} \;+\; \frac{1}{\sqrt{2}}\,\frac{\delta_{ij}}{v_i}\,T_i \;-\; \Pi_{s_i s_j}(p^2)$$

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The tree-level mass matrices are expressed in terms of $\overline{\rm DR}$ running parameters

$$g, \quad g', \quad \lambda, \quad \kappa, \quad v, \quad v_s, \quad \tan\beta$$

Three parameters can be extracted from SM observables (the others are direct inputs)

$$v^{-2} \;=\; 2\sqrt{2}\,G_\mu\left(1 - \frac{\Pi^T_{WW}(0)}{M_W^2} - \delta_{\rm VB}\right)$$

$$g^2 + g'^{\,2} \;=\; 2\,v^{-2}\,M_Z^2\left(1 + \frac{\Pi^T_{ZZ}(M_Z^2)}{M_Z^2}\right)\,, \qquad g^2 \;=\; 2\,v^{-2}\,M_W^2\left(1 + \frac{\Pi^T_{WW}(M_W^2)}{M_W^2}\right)$$

Hand-made, PBMZ-like computation of $\Pi_{s_i s_j}, \;\; \Pi_{p_i p_j}, \;\; T_i, \;\; \Pi^T_{WW}, \;\; \Pi^T_{ZZ}$

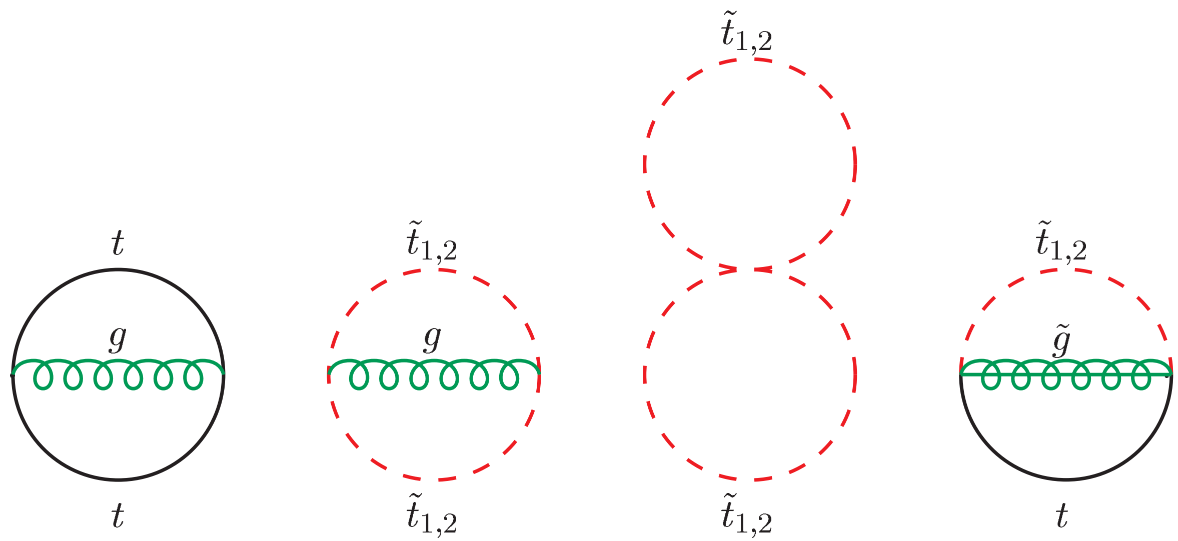
We checked that for $\lambda \to 0$ with $\mu = \lambda v_s$ constant we recover the MSSM results

Two-loop corrections in the effective potential approach

$$\left(\mathcal{M}_S^2\right)_{ij}^{\rm eff} = \left(\mathcal{M}_S^2\right)_{ij}^{\rm tree} - \frac{1}{\sqrt{2}}\,\frac{\delta_{ij}}{v_i}\,\frac{\partial \Delta V}{\partial S_i}\bigg|_{\rm min} + \frac{\partial^2 \Delta V}{\partial S_i \partial S_j}\bigg|_{\rm min}$$

$$\left(\mathcal{M}_P^2\right)_{ij}^{\rm eff} = \left(\mathcal{M}_P^2\right)_{ij}^{\rm tree} - \frac{1}{\sqrt{2}}\,\frac{\delta_{ij}}{v_i}\,\frac{\partial \Delta V}{\partial S_i}\bigg|_{\rm min} + \frac{\partial^2 \Delta V}{\partial P_i \partial P_j}\bigg|_{\rm min}$$

The top/stop, gluon/gluino contributions to ΔV induce the $\mathcal{O}(\alpha_t \alpha_s)$ corrections



Compute ΔV in terms of field-dependent parameters, take the derivatives w.r.t. S_i and P_i

The parameters in the top-stop sector depend on the Higgs fields through the left-right mixing

$$X \;\equiv\; |X|\, e^{i\varphi} \;=\; h_t\, H_2^0 \;, \qquad \widetilde{X} \;\equiv\; |\widetilde{X}|\, e^{i\tilde{\varphi}} \;=\; h_t\, \left(A_t\, H_2^0 - \lambda\, S^*\, H_1^{0\,*}\right)$$

A set of five field-dependent top/stop parameters can be chosen as

$$m_t^2 = |X|^2 \;, \qquad m^2_{\tilde{t}_{1,2}} = \frac{1}{2}\left[(m_Q^2 + m_U^2 + 2\,|X|^2\,) \pm \sqrt{(m_Q^2 - m_U^2)^2 + 4\,|\widetilde{X}|^2}\,\right]$$

$$\sin 2\,\bar{\theta}_{\tilde{t}} = \frac{2\,|\widetilde{X}|}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}} \;, \qquad \cos\left(\varphi - \tilde{\varphi}\right) = \frac{\mathrm{Re}(\widetilde{X})\,\mathrm{Re}(X) + \mathrm{Im}(\widetilde{X})\,\mathrm{Im}(X)}{|\widetilde{X}|\,|X|}$$

We use the chain rule to express the corrections in terms of the derivatives w.r.t.

$$m_t^2, \quad m^2_{\tilde{t}_1}, \quad m^2_{\tilde{t}_2}, \quad \cos^2 2\bar{\theta}_{\tilde{t}}, \quad \cos\left(\varphi - \tilde{\varphi}\right)$$

The mass matrix for the CP-even Higgses:

$$\left(\Delta\mathcal{M}_S^2\right)_{11} = \frac{1}{2}\,h_t^2\,\mu^2\,s_{2\theta_t}^2\,F_3 \;+\; h_t^2\,\tan\beta\,\frac{\mu\,A_t}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\,F\,,$$

$$\left(\Delta\mathcal{M}_S^2\right)_{12} = -h_t^2\,\mu\,m_t\,s_{2\theta_t}\,F_2 \;-\; \frac{1}{2}\,h_t^2\,A_t\,\mu\,s_{2\theta_t}^2\,F_3 \;-\; h_t^2\,\frac{\mu\,A_t}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\,F\,,$$

$$\left(\Delta\mathcal{M}_S^2\right)_{22} = 2\,h_t^2\,m_t^2\,F_1 \;+\; 2\,h_t^2\,A_t\,m_t\,s_{2\theta_t}\,F_2 \;+\; \frac{1}{2}\,h_t^2\,A_t^2\,s_{2\theta_t}^2\,F_3 \;+\; h_t^2\,\cot\beta\,\frac{\mu\,A_t}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\,F\,,$$

$$\left(\Delta\mathcal{M}_S^2\right)_{13} = \frac{1}{2}\,h_t\,\lambda\,m_t\,\mu\,\cot\beta\,s_{2\theta_t}^2\,F_3 \;-\; h_t\,\lambda\,m_t\,\frac{A_t - 2\,\mu\,\cot\beta}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\,F\,,$$

$$\left(\Delta\mathcal{M}_S^2\right)_{23} = -h_t\,\lambda\,m_t^2\,\cot\beta\,s_{2\theta_t}\,F_2 \;-\; \frac{1}{2}\,h_t\,\lambda\,A_t\,m_t\,\cot\beta\,s_{2\theta_t}^2\,F_3 \;-\; h_t\,\lambda\,\cot\beta\,\frac{m_t\,A_t}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\,F\,,$$

$$\left(\Delta\mathcal{M}_S^2\right)_{33} = \frac{1}{2}\,\lambda^2\,m_t^2\,\cot^2\beta\,s_{2\theta_t}^2\,F_3 \;+\; \lambda^2\,\cot\beta\,\frac{m_t^2\,A_t}{\mu\,(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)}\,F$$

$$F_1 = \frac{\partial^2 \Delta V}{(\partial m_t^2)^2} + \frac{\partial^2 \Delta V}{(\partial m_{\tilde{t}_1}^2)^2} + \frac{\partial^2 \Delta V}{(\partial m_{\tilde{t}_2}^2)^2} + 2\,\frac{\partial^2 \Delta V}{\partial m_t^2 \partial m_{\tilde{t}_1}^2} + 2\,\frac{\partial^2 \Delta V}{\partial m_t^2 \partial m_{\tilde{t}_2}^2} + 2\,\frac{\partial^2 \Delta V}{\partial m_{\tilde{t}_1}^2\, \partial m_{\tilde{t}_2}^2}\ ,$$

$$F_2 = \frac{\partial^2 \Delta V}{(\partial m_{\tilde{t}_1}^2)^2} - \frac{\partial^2 \Delta V}{(\partial m_{\tilde{t}_2}^2)^2} + \frac{\partial^2 \Delta V}{\partial m_t^2 \partial m_{\tilde{t}_1}^2} - \frac{\partial^2 \Delta V}{\partial m_t^2 \partial m_{\tilde{t}_2}^2}$$
$$- \frac{4\,c_{2\theta_t}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \left(\frac{\partial^2 \Delta V}{\partial c_{2\bar{\theta}_t}^2\, \partial m_t^2} + \frac{\partial^2 \Delta V}{\partial c_{2\bar{\theta}_t}^2\, \partial m_{\tilde{t}_1}^2} + \frac{\partial^2 \Delta V}{\partial c_{2\bar{\theta}_t}^2\, \partial m_{\tilde{t}_2}^2} \right)\ ,$$

$$F_3 = \frac{\partial^2 \Delta V}{(\partial m_{\tilde{t}_1}^2)^2} + \frac{\partial^2 \Delta V}{(\partial m_{\tilde{t}_2}^2)^2} - 2\,\frac{\partial^2 \Delta V}{\partial m_{\tilde{t}_1}^2\, \partial m_{\tilde{t}_2}^2} - \frac{2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \left(\frac{\partial \Delta V}{\partial m_{\tilde{t}_1}^2} - \frac{\partial \Delta V}{\partial m_{\tilde{t}_2}^2} \right)$$
$$+ \frac{16\,c_{2\theta_t}^2}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} \left(c_{2\theta_t}^2\, \frac{\partial^2 \Delta V}{(\partial c_{2\bar{\theta}_t}^2)^2} + 2\,\frac{\partial \Delta V}{\partial c_{2\bar{\theta}_t}^2} \right) - \frac{8\,c_{2\theta_t}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \left(\frac{\partial^2 \Delta V}{\partial c_{2\bar{\theta}_t}^2\, \partial m_{\tilde{t}_1}^2} - \frac{\partial^2 \Delta V}{\partial c_{2\bar{\theta}_t}^2\, \partial m_{\tilde{t}_2}^2} \right)\ ,$$

$$F = \frac{\partial \Delta V}{\partial m_{\tilde{t}_1}^2} - \frac{\partial \Delta V}{\partial m_{\tilde{t}_2}^2} - \frac{4\,c_{2\theta_t}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\, \frac{\partial \Delta V}{\partial c_{2\bar{\theta}_t}^2}$$

$$
\begin{aligned}
&- \;6\,g^2 \sum_{\tilde u/\tilde d}\sum_{i,j=1}^{2} \left(R^{\tilde u}_{i1}\,R^{\tilde d}_{j1}\right)^2\,\widetilde{B}_{22}(m_{\tilde u_i},m_{\tilde d_j}) \;-\; 2\,g^2 \sum_{\tilde e/\tilde\nu}\sum_{i=1}^{2} \left(R^{\tilde e}_{i1}\right)^2\,\widetilde{B}_{22}(m_{\tilde e_i},m_{\tilde\nu})\\
&+ \;\sum_{i=1}^{5}\sum_{j=1}^{2}\Bigg[\left(|\lambda_{W\chi^0_i\chi^+_j}|^2+|\lambda_{W\chi^0_i\chi^-_j}|^2\right)\,H(m_{\chi^0_i},m_{\chi^\pm_j})\\
&\qquad\qquad + 4\,m_{\chi^0_i}\,m_{\chi^\pm_j}\,\mathrm{Re}\left(\lambda^*_{W\chi^0_i\chi^+_j}\lambda_{W\chi^0_i\chi^-_j}\right)\,B_0(m_{\chi^0_i},m_{\chi^\pm_j})\Bigg]\;, \qquad \text{(B18)}
\end{aligned}
$$

where the sums in the fermion contributions and the first sums in the sfermion contributions run over the three families of (s)quarks and (s)leptons, and the couplings $\lambda_{W\chi^0_i\chi^+_j}$ and $\lambda_{W\chi^0_i\chi^-_j}$ are obtained by combining eqs. (A29) and (A30).

Appendix C: derivatives of the two-loop effective potential

We provide in this appendix the explicit formulae for the derivatives of the two-loop contribution to the effective potential involving top, stop, gluon and gluino. In units of $\alpha_s\,C_F N_c\,/(4\pi)$, where $C_F = 4/3$ and $N_c = 3$ are colour factors, and in terms of the field-dependent quantities defined in eqs. (20)–(22), the $\mathcal{O}(\alpha_s)$ contribution to $V_{\rm eff}$ reads

$$
\begin{aligned}
\Delta V \;&=\; 2\,J(m_t^2,m_t^2) - 4\,m_t^2\,I(m_t^2,m_t^2,0) + \\
&+\;\Big\{2\,m_{\tilde t_1}^2\;I(m_{\tilde t_1}^2,m_{\tilde t_1}^2,0) + 2\,L(m_{\tilde t_1}^2,m_{\tilde g}^2,m_t^2) - 4\,m_t\,m_{\tilde g}\,s_{2\bar\theta}\,c_{\varphi-\tilde\varphi}\,I(m_{\tilde t_1}^2,m_{\tilde g}^2,m_t^2)\\
&\quad+\frac{1}{2}\,(1+c^2_{2\bar\theta})\,J(m_{\tilde t_1}^2,m_{\tilde t_1}^2) + \frac{s^2_{2\bar\theta}}{2}J(m_{\tilde t_1}^2,m_{\tilde t_2}^2) \;\;+\;\; \left[m_{\tilde t_1}\leftrightarrow m_{\tilde t_2}\,,\; s_{2\bar\theta}\to -s_{2\bar\theta}\right]\Big\}, \qquad \text{(C1)}
\end{aligned}
$$

where the functions $I\,,J$ and L are defined in appendix D. The derivatives of ΔV that involve only the field-dependent stop mixing angle $\bar\theta$ and phase difference $\varphi-\tilde\varphi$ can be straightforwardly computed from eq. (C1). In term of the usual angle θ_t which diagonalizes the stop mass matrices at the minimum of the effective potential, they read

$$
\begin{aligned}
\frac{\partial\Delta V}{\partial c^2_{2\bar\theta_t}} \;&=\; \frac{1}{2}\left[J(m_{\tilde t_1}^2,m_{\tilde t_1}^2)+J(m_{\tilde t_2}^2,m_{\tilde t_2}^2)\right] - J(m_{\tilde t_1}^2,m_{\tilde t_2}^2)\\
&\quad+2\,\frac{m_{\tilde g}\,m_t}{s_{2\theta_t}}\left[I(m_{\tilde t_1}^2,m_{\tilde g}^2,m_t^2) - I(m_{\tilde t_2}^2,m_{\tilde g}^2,m_t^2)\right]\;, \qquad \text{(C2)}\\
\frac{\partial^2\Delta V}{(\partial c^2_{2\bar\theta_t})^2} \;&=\; -\frac{z_t}{4\,s^4_{2\theta_t}}\,\frac{\partial\Delta V}{\partial c_{\varphi_t-\tilde\varphi_t}} \;=\; \frac{m_{\tilde g}\,m_t}{s^3_{2\theta_t}}\left[I(m_{\tilde t_1}^2,m_{\tilde g}^2,m_t^2) - I(m_{\tilde t_2}^2,m_{\tilde g}^2,m_t^2)\right]\;, \qquad \text{(C3)}
\end{aligned}
$$

The explicit expressions for derivatives of ΔV that involve the quark or squark masses are somewhat lengthier. They read

$$
\begin{aligned}
\frac{\partial\Delta V}{\partial m^2_{\tilde t_1}} \;&=\; -6\,m^2_{\tilde t_1} + 2\,m_{\tilde g}\,m_t\,s_{2\theta_t} + 4\,m_t^2\left(1-\log\frac{m_t^2}{m^2_{\tilde t_1}}\right) + 4\,m_{\tilde g}^2\left(1-\log\frac{m_{\tilde g}^2}{m^2_{\tilde t_1}}\right)\\
&\quad+\left[\left(5-c^2_{2\theta_t}\right)\,m^2_{\tilde t_1} - s^2_{2\theta_t}\,m^2_{\tilde t_2} - 4\,m_{\tilde g}\,m_t\,s_{2\theta_t}\right]\,\log\frac{m^2_{\tilde t_1}}{Q^2}
\end{aligned}
$$

$$
\begin{aligned}
&+\left(-3+c^2_{2\theta_t}\right)\,m^2_{\tilde t_1}\;\log^2\frac{m^2_{\tilde t_1}}{Q^2} + s^2_{2\theta_t}\,m^2_{\tilde t_2}\;\log\frac{m^2_{\tilde t_1}}{Q^2}\;\log\frac{m^2_{\tilde t_2}}{Q^2}\\
&-\left[2\,\left(m_{\tilde g}^2+m_t^2-m^2_{\tilde t_1}\right) - 2\,m_{\tilde g}\,m_t\,s_{2\theta_t}\right]\left(\log\frac{m_t^2}{Q^2}\;\log\frac{m^2_{\tilde t_1}}{m^2_{\tilde g}} + \log\frac{m^2_{\tilde t_1}}{Q^2}\;\log\frac{m^2_{\tilde g}}{Q^2}\right)\\
&+\left[\frac{2}{m_t^2}\,\left(\Delta+2\,m_{\tilde g}^2\,m_t^2\right) - \frac{2\,m_{\tilde g}\,s_{2\theta_t}}{m_t}\,\left(m_{\tilde g}^2+m_t^2-m^2_{\tilde t_1}\right)\right]\,\Phi(m^2_{\tilde t_1},m^2_{\tilde g},m_t^2)\;, \qquad \text{(C4)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2\Delta V}{(\partial m^2_{\tilde t_1})^2} \;&=\; -\left(1+c^2_{2\theta_t}\right) + \frac{4}{m^2_{\tilde t_1}}\,\left(m_{\tilde g}^2+m_t^2-m_{\tilde g}\,m_t\,s_{2\theta_t}\right) - s^2_{2\theta_t}\,\frac{m^2_{\tilde t_2}}{m^2_{\tilde t_1}}\,\left(1-\log\frac{m^2_{\tilde t_2}}{Q^2}\right)\\
&\quad+\left[3+c^2_{2\theta_t}+\frac{8\,m_{\tilde g}^2\,m_t^2}{\Delta} - \frac{4\,m_{\tilde g}\,m_t\,s_{2\theta_t}}{\Delta}\,\left(m_{\tilde g}^2+m_t^2-m^2_{\tilde t_1}\right)\right]\,\log\frac{m^2_{\tilde t_1}}{Q^2}\\
&\quad-\frac{4\,m_t^2}{\Delta\,m^2_{\tilde t_1}}\,\left[\Delta - m_{\tilde g}^2\,\left(m_{\tilde g}^2-m_t^2-m^2_{\tilde t_1}\right) + \,m_{\tilde g}\,m_t\,s_{2\theta_t}\,\left(m_{\tilde g}^2-m_t^2+m^2_{\tilde t_1}\right)\right]\,\log\frac{m_t^2}{Q^2}\\
&\quad-\frac{4\,m_{\tilde g}^2}{\Delta\,m^2_{\tilde t_1}}\,\left[\Delta + m_t^2\,\left(m_{\tilde g}^2-m_t^2+m^2_{\tilde t_1}\right) - \,m_{\tilde g}\,m_t\,s_{2\theta_t}\,\left(m_{\tilde g}^2-m_t^2-m^2_{\tilde t_1}\right)\right]\,\log\frac{m_{\tilde g}^2}{Q^2}\\
&\quad+\left(-3+c^2_{2\theta_t}\right)\,\log^2\frac{m^2_{\tilde t_1}}{Q^2} + 2\,\left(\log\frac{m_t^2}{Q^2}\;\log\frac{m^2_{\tilde t_1}}{m^2_{\tilde g}} + \log\frac{m^2_{\tilde t_1}}{Q^2}\;\log\frac{m^2_{\tilde g}}{Q^2}\right)\\
&\quad-\frac{2}{\Delta\,m_t^2}\,\left[\left(m_{\tilde g}^2+m_t^2-m^2_{\tilde t_1}\right)\,\left(\Delta-2\,m_{\tilde g}^2\,m_t^2\right)+4\,m_{\tilde g}^3\,m_t^3\,s_{2\theta_t}\right]\,\Phi(m^2_{\tilde t_1},m^2_{\tilde g},m_t^2)\;, \qquad \text{(C5)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2\Delta V}{\partial c^2_{2\bar\theta_t}\,\partial m^2_{\tilde t_1}} \;&=\; \left[m^2_{\tilde t_2}\,\left(1-\log\frac{m^2_{\tilde t_2}}{Q^2}\right) - m^2_{\tilde t_1}\,\left(1-\log\frac{m^2_{\tilde t_1}}{Q^2}\right)\right]\,\log\frac{m^2_{\tilde t_1}}{Q^2}\\
&\quad-\frac{m_{\tilde g}\,m_t}{s_{2\theta_t}}\Bigg[1-2\,\log\frac{m^2_{\tilde t_1}}{Q^2} + \log\frac{m^2_{\tilde t_1}}{m^2_{\tilde g}}\;\log\frac{m_t^2}{Q^2} + \log\frac{m^2_{\tilde t_1}}{Q^2}\;\log\frac{m^2_{\tilde g}}{Q^2}\\
&\qquad\qquad - \frac{1}{m_t^2}\,\left(m_{\tilde g}^2+m_t^2-m^2_{\tilde t_1}\right)\;\Phi(m^2_{\tilde t_1},m^2_{\tilde g},m_t^2)\Bigg]\;, \qquad \text{(C6)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2\Delta V}{\partial m_t^2\partial m^2_{\tilde t_1}} \;&=\; \frac{m_{\tilde g}\,s_{2\theta_t}}{m_t} + \frac{4\,m_{\tilde g}^2}{\Delta}\,\left[m^2_{\tilde t_1}-m_{\tilde g}^2-m_t^2+2\,m_t\,m_{\tilde g}\,s_{2\theta_t}\right]\,\log\frac{m_{\tilde g}^2}{Q^2}\\
&\quad+\frac{4}{\Delta}\,\left[2\,m_{\tilde g}^2\,m_t^2 - m_t\,m_{\tilde g}\,s_{2\theta_t}\,\left(m_{\tilde g}^2+m_t^2-m^2_{\tilde t_1}\right)\right]\,\log\frac{m_t^2}{Q^2}\\
&\quad+\frac{2}{\Delta}\,\left[2\,m_{\tilde g}^2\,\left(m_{\tilde g}^2-m_t^2-m^2_{\tilde t_1}\right) - \frac{m_{\tilde g}\,s_{2\theta_t}}{m_t}\,\left(\Delta-2\,m_t^2\,(m_t^2-m_{\tilde g}^2-m^2_{\tilde t_1})\right)\right]\,\log\frac{m^2_{\tilde t_1}}{Q^2}\\
&\quad+\left(-2+\frac{m_{\tilde g}\,s_{2\theta_t}}{m_t}\right)\left(\log\frac{m^2_{\tilde t_1}}{m^2_{\tilde g}}\;\log\frac{m_t^2}{Q^2} + \log\frac{m^2_{\tilde t_1}}{Q^2}\;\log\frac{m^2_{\tilde g}}{Q^2}\right)\\
&\quad+\frac{1}{\Delta\,m_t^2}\Bigg\{\frac{m_{\tilde g}\,s_{2\theta_t}}{m_t}\,\left[\Delta\,\left(m^2_{\tilde t_1}-m_{\tilde g}^2-3\,m_t^2\right)+2\,m_t^2\,\left((m_t^2-m^2_{\tilde t_1})^2-m_{\tilde g}^4\right)\right]\\
&\quad+2\,\left(m_{\tilde g}^2-m^2_{\tilde t_1}\right)^3 + 2\,m_t^2\,\left[\Delta+\left(2\,m^2_{\tilde t_1}-m_t^2\right)\,\left(m_{\tilde g}^2+m^2_{\tilde t_1}\right)\right]\Bigg\}\,\Phi(m^2_{\tilde t_1},m^2_{\tilde g},m_t^2)\;, \qquad \text{(C7)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 \Delta V}{\partial m^2_{\tilde t_1}\,\partial m^2_{\tilde t_2}} &= s^2_{2\theta_t}\,\log\frac{m^2_{\tilde t_1}}{Q^2}\,\log\frac{m^2_{\tilde t_2}}{Q^2}\,, \qquad \text{(C8)}\\
\frac{\partial^2 \Delta V}{(\partial m_t^2)^2} &= -2 - \frac{5\,m_{\tilde g}\,m^2_{\tilde t_1}\,s_{2\theta_t}}{2\,m_t^3} + 6\,\log^2\frac{m_t^2}{Q^2}\\
&+\frac{4\,m_{\tilde g}^2}{\Delta}\left[m_{\tilde g}^2 - m_t^2 - m^2_{\tilde t_1} + \frac{m_t\,s_{2\theta_t}}{m_{\tilde g}}\left(m_t^2 - m_{\tilde g}^2 - m^2_{\tilde t_1}\right)\right]\log\frac{m_t^2}{Q^2}\\
&-\frac{4\,m_{\tilde g}^2}{\Delta}\left[m_{\tilde g}^2 - m_t^2 + m^2_{\tilde t_1} + \frac{m_{\tilde g}\,s_{2\theta_t}}{m_t}\left(m_t^2 - m_{\tilde g}^2 + m^2_{\tilde t_1}\right)\right]\log\frac{m_{\tilde g}^2}{Q^2}\\
&+\left[\frac{8\,m_{\tilde g}^2\,m^2_{\tilde t_1}}{\Delta} + \frac{2\,m_{\tilde g}\,m^2_{\tilde t_1}\,s_{2\theta_t}}{m_t^3}\left(1 - \frac{2\,m_t^2}{\Delta}\,(m_t^2 + m_{\tilde g}^2 - m^2_{\tilde t_1})\right)\right]\log\frac{m^2_{\tilde t_1}}{Q^2}\\
&-\left(2 - \frac{m_{\tilde g}\,m^2_{\tilde t_1}\,s_{2\theta_t}}{2\,m_t^3}\right)\log\frac{m_t^2}{m^2_{\tilde t_1}}\,\log\frac{m_{\tilde g}^2}{Q^2} - \left(2 + \frac{m_{\tilde g}\,m^2_{\tilde t_1}\,s_{2\theta_t}}{2\,m_t^3}\right)\log\frac{m_t^2}{Q^2}\,\log\frac{m^2_{\tilde t_1}}{Q^2}\\
&+\frac{m_{\tilde g}\,s_{2\theta_t}}{2\,m_t^3}\left(m_{\tilde g}^2 + 3\,m_t^2\right)\log\frac{m_t^2}{m_{\tilde g}^2}\,\log\frac{m^2_{\tilde t_1}}{Q^2}\\
&-\frac{2}{\Delta\,m_t^2}\Bigg\{\frac{m_{\tilde g}\,s_{2\theta_t}}{m_t^3}\left[\frac{\Delta^2}{4} + m_t^2\left(m_{\tilde g}^2 - 2\,m_t^2 + m^2_{\tilde t_1}\right)\Delta + m_t^4\left(m_{\tilde g}^2 - m_t^2 + m^2_{\tilde t_1}\right)^2\right]\\
&\qquad - m^2_{\tilde t_1}\left(\Delta + (m_{\tilde g}^2 + m_t^2)\,(2\,m_t^2 - m^2_{\tilde t_1})\right) - \left(m_{\tilde g}^2 - m_t^2\right)^3\Bigg\}\,\Phi(m^2_{\tilde t_1}, m_{\tilde g}^2, m_t^2)\\
&+\left\{m_{\tilde t_1} \to m_{\tilde t_2}\,,\quad s_{2\theta_t} \to -s_{2\theta_t}\right\}, \qquad \text{(C9)}\\
\frac{\partial^2 \Delta V}{\partial c^2_{2\bar\theta_t}\,\partial m_t^2} &= -\frac{m_{\tilde g}}{2\,m_t\,s_{2\theta_t}}\Bigg\{5\,m^2_{\tilde t_1} - 4\,m^2_{\tilde t_1}\,\log\frac{m^2_{\tilde t_1}}{Q^2} + \left(m_{\tilde g}^2 - 3\,m_t^2\right)\log\frac{m_{\tilde g}^2}{m_t^2}\,\log\frac{m^2_{\tilde t_1}}{Q^2}\\
&\qquad + m^2_{\tilde t_1}\left(\log\frac{m^2_{\tilde t_1}}{m_{\tilde g}^2}\,\log\frac{m_t^2}{Q^2} + \log\frac{m_{\tilde g}^2}{Q^2}\,\log\frac{m^2_{\tilde t_1}}{Q^2}\right)\\
&\qquad + \left[\frac{\Delta}{m_t^2} - 2\left(m_{\tilde g}^2 - m_t^2 + m^2_{\tilde t_1}\right)\right]\Phi(m^2_{\tilde t_1}, m_{\tilde g}^2, m_t^2)\Bigg\}\\
&+\left\{m_{\tilde t_1} \to m_{\tilde t_2}\,,\quad s_{2\theta_t} \to -s_{2\theta_t}\right\}, \qquad \text{(C10)}
\end{aligned}
$$

where Q is the renormalization scale at which the $\overline{\rm DR}$ parameters entering the one-loop part of the corrections are expressed, the function $\Phi(x,y,z)$ is defined in appendix D, and we have used the shortcut $\Delta \equiv \Delta(m^2_{\tilde t_1}, m^2_{\tilde g}, m^2_t)$, where the function $\Delta(x,y,z)$ is also defined in appendix D. Finally, the derivatives of ΔV that involve $m^2_{\tilde t_2}$ can be obtained from eqs. (C4)–(C7) with the replacements $m_{\tilde t_1} \leftrightarrow m_{\tilde t_2}$ and $s_{2\theta_t} \to -s_{2\theta_t}$.

Appendix D: two-loop functions

We provide in this appendix the explicit formulae for the two-loop functions appearing in the $\mathcal{O}(\alpha_t\alpha_s)$ corrections to the Higgs mass matrices. Differently from the approach of ref. [18], we choose to renormalize the two-loop effective potential *before* taking its derivatives. As first shown in ref. [36], this is equivalent to using the "minimally subtracted" two-loop functions:

$$
J(x,y) = x\,y\left(1 - \overline{\log}\,x\right)\left(1 - \overline{\log}\,y\right), \qquad \text{(D1)}
$$

$$
\begin{aligned}
I(x,y,z) &= \frac{1}{2}\left[(x-y-z)\,\overline{\log}\,y\,\overline{\log}\,z + (y-x-z)\,\overline{\log}\,x\,\overline{\log}\,z + (z-x-y)\,\overline{\log}\,x\,\overline{\log}\,y\right]\\
&- \frac{5}{2}\,(x+y+z) + 2\left(x\,\overline{\log}\,x + y\,\overline{\log}\,y + z\,\overline{\log}\,z\right) - \frac{\Delta(x,y,z)}{2\,z}\,\Phi(x,y,z)\,, \qquad \text{(D2)}
\end{aligned}
$$

$$
L(x,y,z) = J(y,z) - J(x,y) - J(x,z) - (x-y-z)\,I(x,y,z)\,. \qquad \text{(D3)}
$$

In the above formulae, $\overline{\log}\,x$ stands for $\log(x/Q^2)$, where Q is the renormalization scale. The functions Δ and Φ read, respectively,

$$
\Delta(x,y,z) = x^2 + y^2 + z^2 - 2\,(xy + xz + yz)\,, \qquad \text{(D4)}
$$

$$
\Phi\,(x,y,z) = \frac{1}{\lambda}\left[2\,\log x_+\,\log x_- - \log u\,\log v - 2\left({\rm Li}_2(x_+) + {\rm Li}_2(x_-)\right) + \frac{\pi^2}{3}\right], \qquad \text{(D5)}
$$

where ${\rm Li}_2(z) = -\int_0^z {\rm d}t\,[\log(1-t)/t]$ is the dilogarithm function and the auxiliary (complex) variables are:

$$
u = \frac{x}{z}\,, \qquad v = \frac{y}{z}\,, \qquad \lambda = \sqrt{(1-u-v)^2 - 4\,u\,v}\,, \qquad x_\pm = \frac{1}{2}\,[1 \pm (u-v) - \lambda]\,. \qquad \text{(D6)}
$$

The definition (D5) is valid for the case $x/z < 1$ and $y/z < 1$. The other branches of Φ can be obtained using the symmetry properties:

$$
\Phi\,(x,y,z) = \Phi\,(y,x,z)\,, \qquad x\,\Phi\,(x,y,z) = z\,\Phi\,(z,y,x)\,. \qquad \text{(D7)}
$$

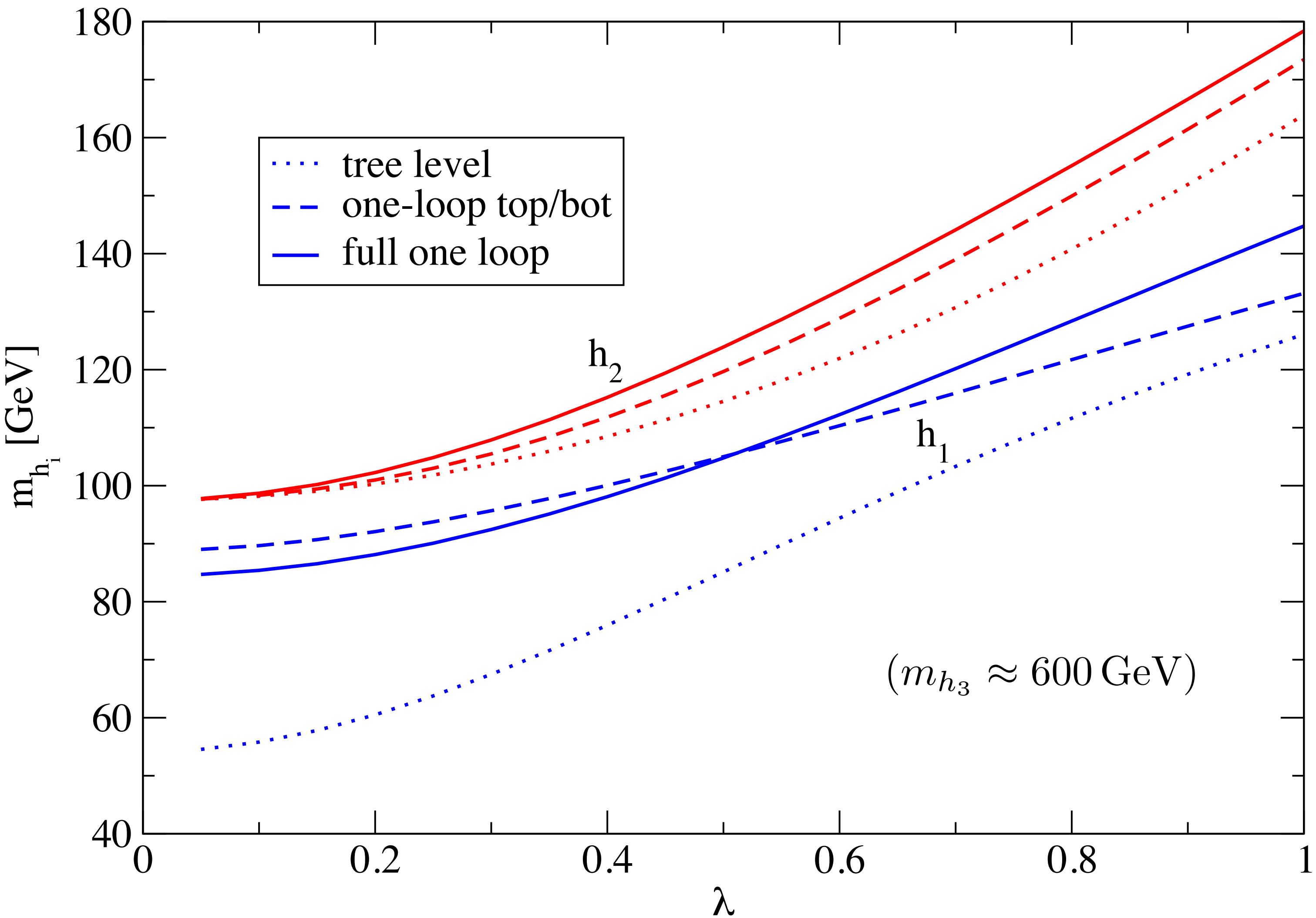
Finally, the following recursive relation for the derivatives of Φ proves very useful for obtaining compact analytical results:

$$
\Delta(x,y,z)\,\frac{\partial\,\Phi(x,y,z)}{\partial\,x} = (y+z-x)\,\Phi(x,y,z) + \frac{z}{x}\left[(y-z)\,\ln\frac{z}{y} + x\left(\ln\frac{x}{y} + \ln\frac{x}{z}\right)\right]. \qquad \text{(D8)}
$$

The derivatives of Φ with respect to y and z can be obtained from the above equation with the help of the symmetry properties of Eq. (D7).

Numerical examples

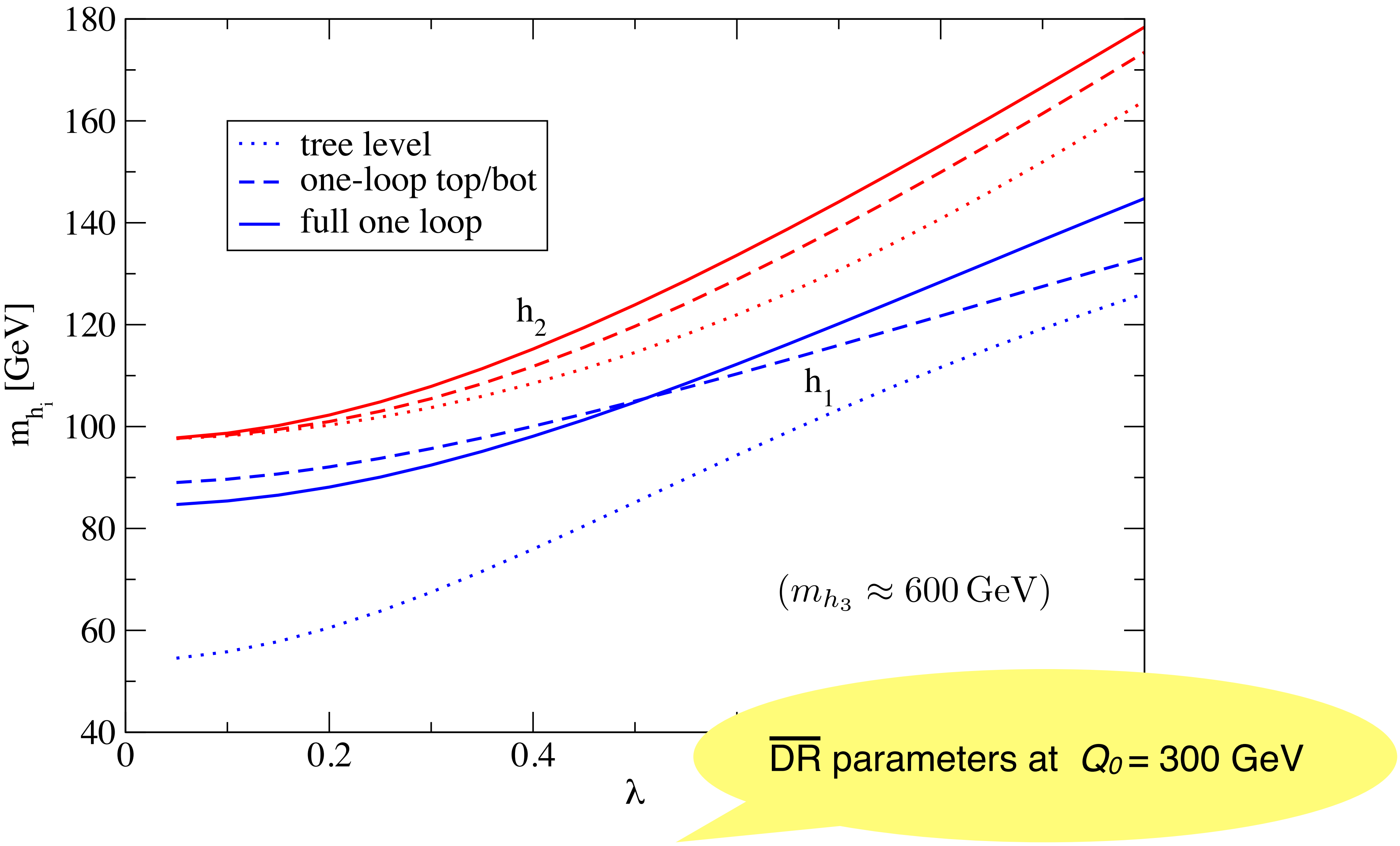
One-loop corrections to the two lightest scalar masses



$$\kappa = \lambda/5\,, \quad \tan\beta = 2\,, \quad A_\lambda = 500 \text{ GeV}, \quad A_\kappa = -10 \text{ GeV}, \quad \mu = 250 \text{ GeV}, \quad M_S = 300 \text{ GeV}$$

$$A_t = A_b = A_\tau = -450\,\text{GeV}\;, \qquad M_3 = 600\,\text{GeV}\;, \qquad M_2 = 200\,\text{GeV}\;, \qquad M_1 = 100\,\text{GeV}$$

One-loop corrections to the two lightest scalar masses



$$\kappa = \lambda/5\,, \quad \tan\beta = 2\,, \quad A_\lambda = 500 \text{ GeV}, \quad A_\kappa = -10 \text{ GeV}, \quad \mu = 250 \text{ GeV}, \quad M_S = 300 \text{ GeV}$$

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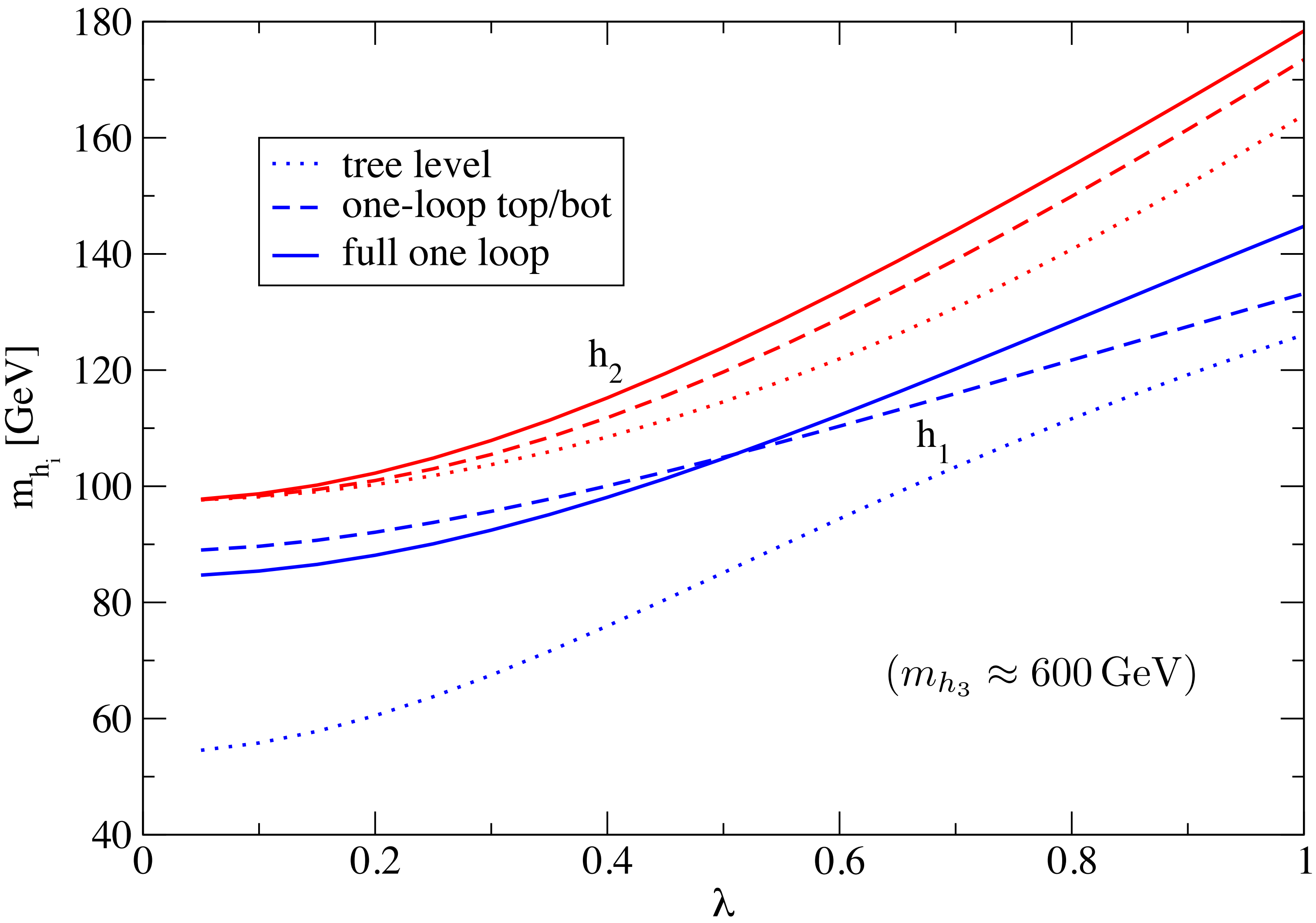
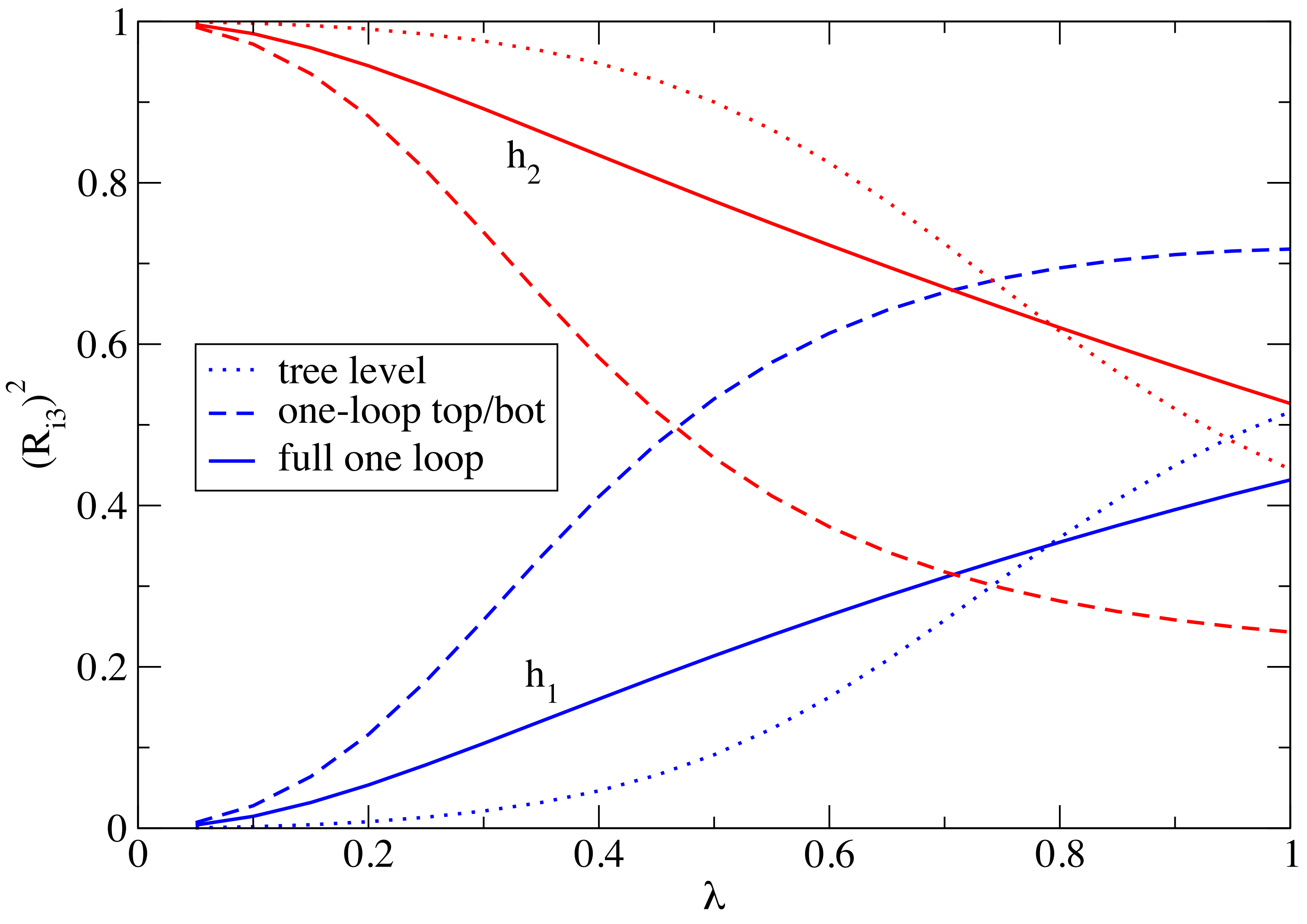
One-loop corrections to the two lightest scalar masses



$$\kappa = \lambda/5\,, \quad \tan\beta = 2\,, \quad A_\lambda = 500\ \mathrm{GeV}, \quad A_\kappa = -10\ \mathrm{GeV}, \quad \mu = 250\ \mathrm{GeV}, \quad M_S = 300\ \mathrm{GeV}$$

$$A_t = A_b = A_\tau = -450\,\mathrm{GeV}\ , \quad M_3 = 600\,\mathrm{GeV}\ , \quad M_2 = 200\,\mathrm{GeV}\ , \quad M_1 = 100\,\mathrm{GeV}$$

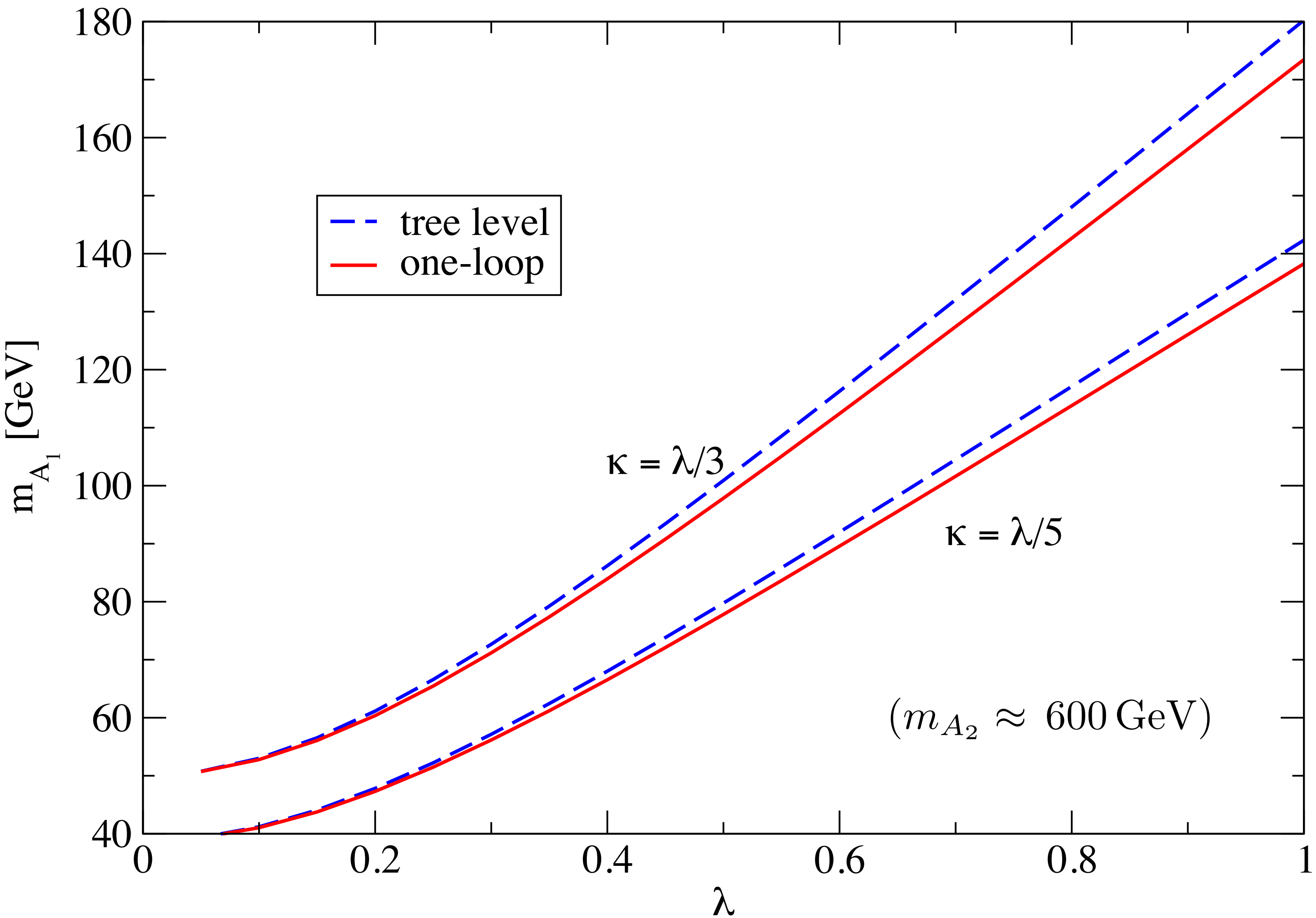
Effect of the one-loop corrections on the CP-even mixing



$\kappa = \lambda/5\,, \quad \tan\beta = 2\,, \quad A_\lambda = 500\ \text{GeV}, \quad A_\kappa = -10\ \text{GeV}, \quad \mu = 250\ \text{GeV}, \quad M_S = 300\ \text{GeV}$

$A_t = A_b = A_\tau = -450\,\text{GeV}\ , \quad M_3 = 600\,\text{GeV}\ , \quad M_2 = 200\,\text{GeV}\ , \quad M_1 = 100\,\text{GeV}$

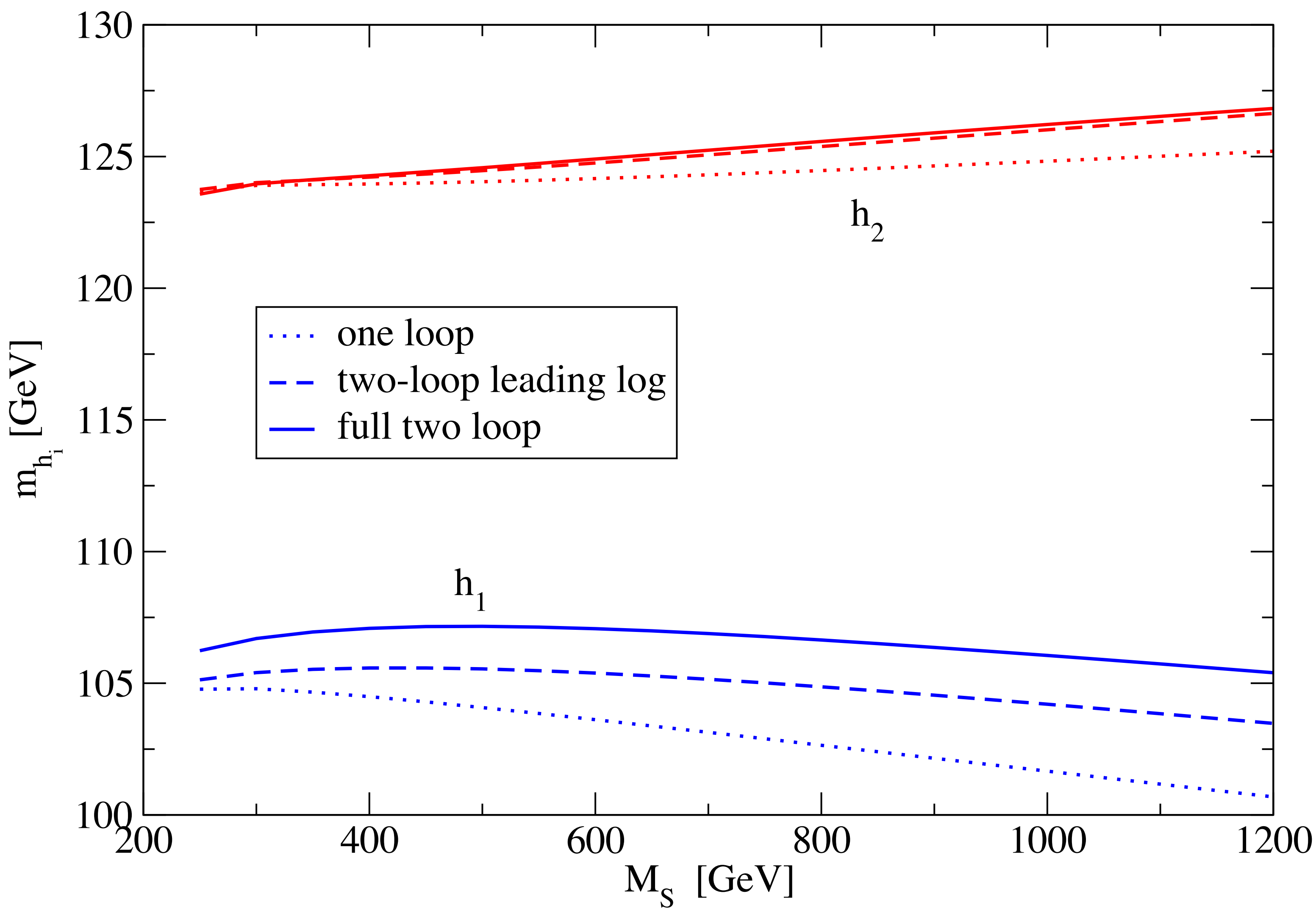
One-loop corrections to the lightest pseudoscalar mass



$$\tan\beta = 2\ , \quad A_\lambda = 500\ \text{GeV}\ , \quad A_\kappa = -10\ \text{GeV}\ , \quad \mu = 250\ \text{GeV}\ , \quad M_S = 300\ \text{GeV}\ ,$$

$$A_t = A_b = A_\tau = -450\,\text{GeV}\ , \quad M_3 = 600\,\text{GeV}\ , \quad M_2 = 200\,\text{GeV}\ , \quad M_1 = 100\,\text{GeV}$$

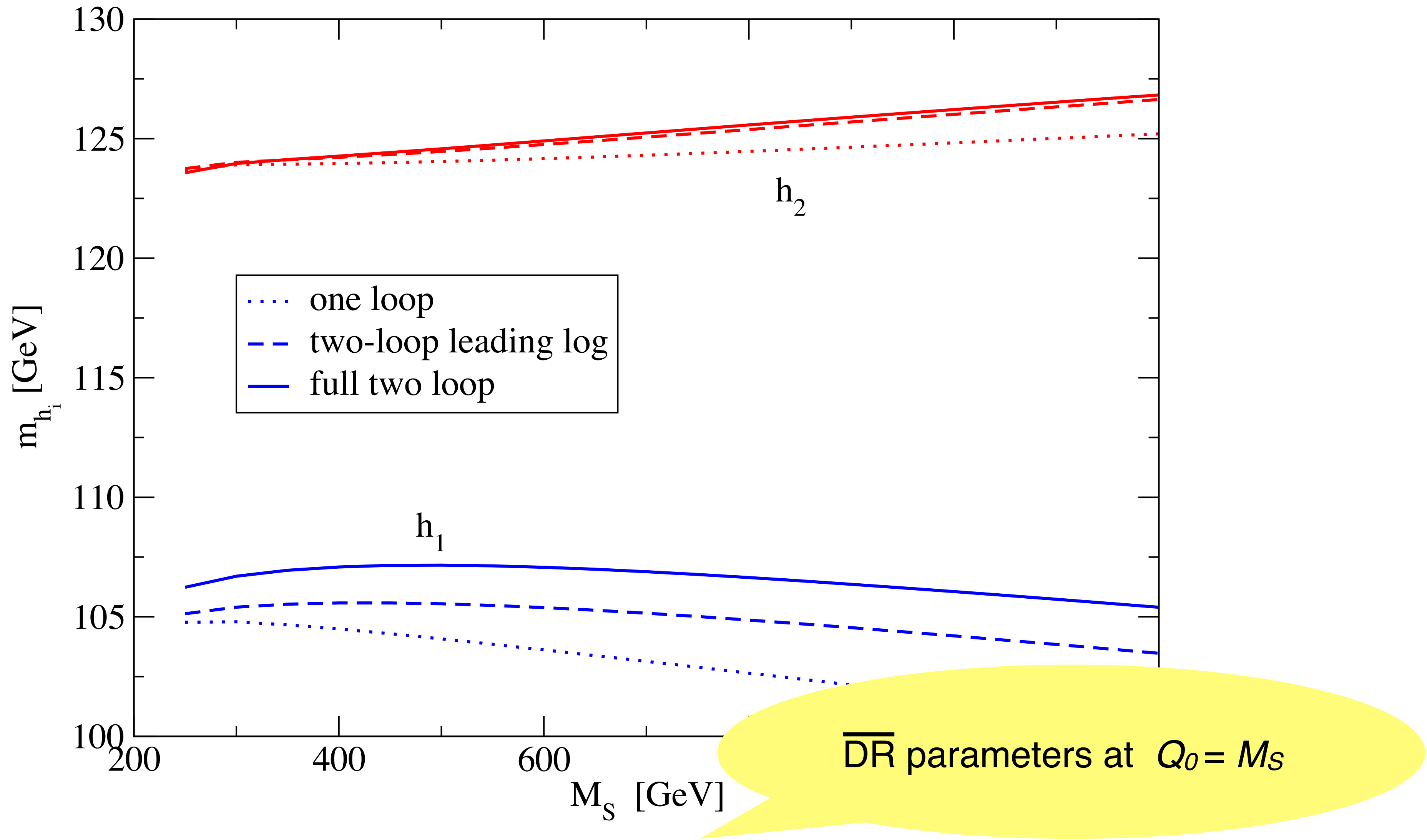
Two-loop corrections to the two lightest scalar masses



$$\lambda = 0.5\ , \quad \kappa = 0.1\ , \quad \tan\beta = 2\ , \quad A_\lambda = 500\ \text{GeV}\ , \quad A_\kappa = -10\ \text{GeV}\ , \quad \mu = 250\ \text{GeV}$$

$$A_t = A_b = A_\tau = -1.5\, M_S\ , \quad M_3 = 2\, M_S\ , \quad M_2 = 2/3\, M_S\ , \quad M_1 = M_S/3$$

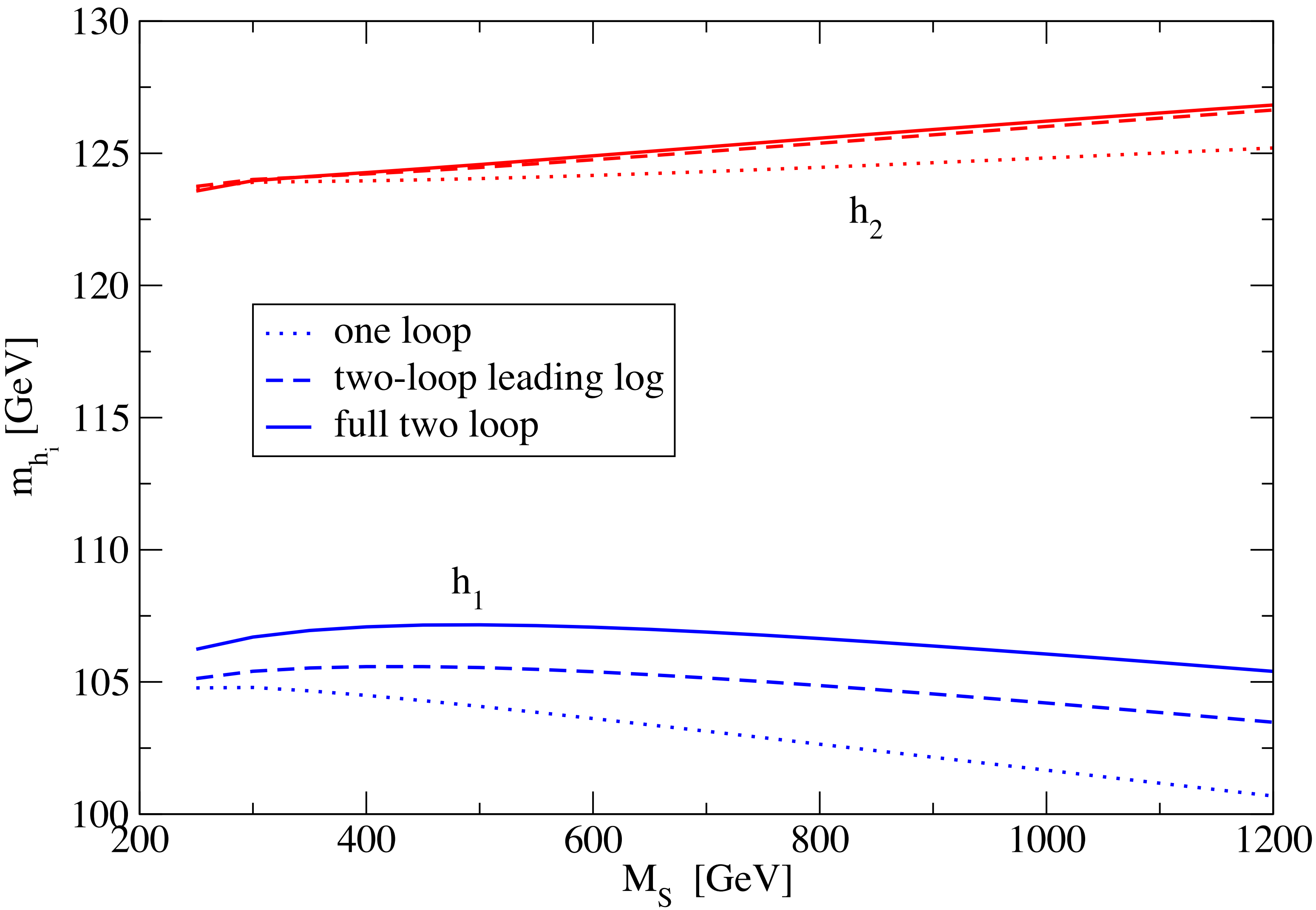
Two-loop corrections to the two lightest scalar masses



$$\lambda = 0.5 \ , \quad \kappa = 0.1 \ , \quad \tan\beta = 2 \ , \quad A_\lambda = 500 \text{ GeV} \ , \quad A_\kappa = -10 \text{ GeV} \ , \quad \mu = 250 \text{ GeV}$$

$$A_t = A_b = A_\tau = -1.5\, M_S \ , \quad M_3 = 2\, M_S \ , \quad M_2 = 2/3\, M_S \ , \quad M_1 = M_S/3$$

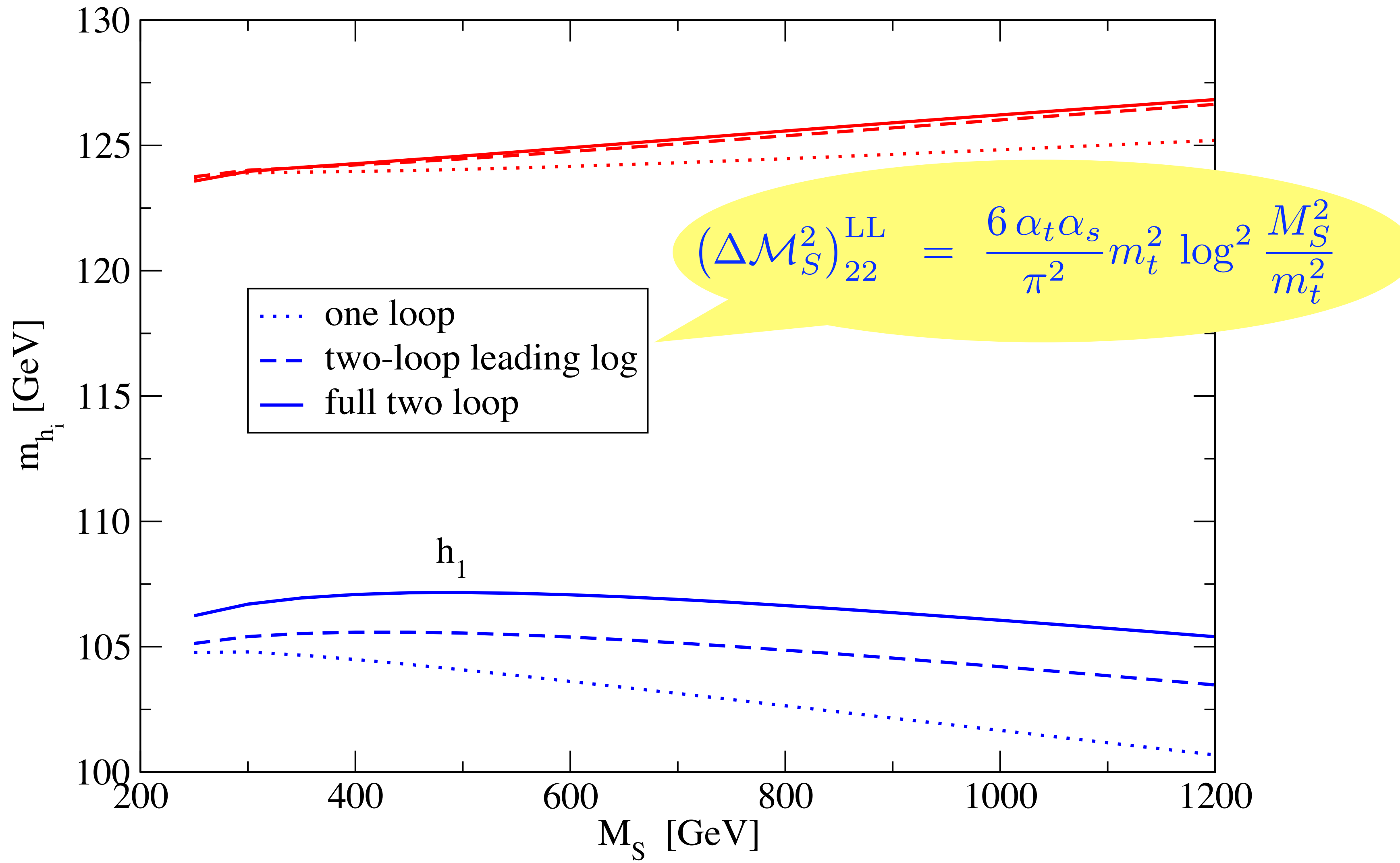
Two-loop corrections to the two lightest scalar masses



$$\lambda = 0.5\ , \quad \kappa = 0.1\ , \quad \tan\beta = 2\ , \quad A_\lambda = 500\ \text{GeV}\ , \quad A_\kappa = -10\ \text{GeV}\ , \quad \mu = 250\ \text{GeV}$$

$$A_t = A_b = A_\tau = -1.5\, M_S\ , \quad M_3 = 2\, M_S\ , \quad M_2 = 2/3\, M_S\ , \quad M_1 = M_S/3$$

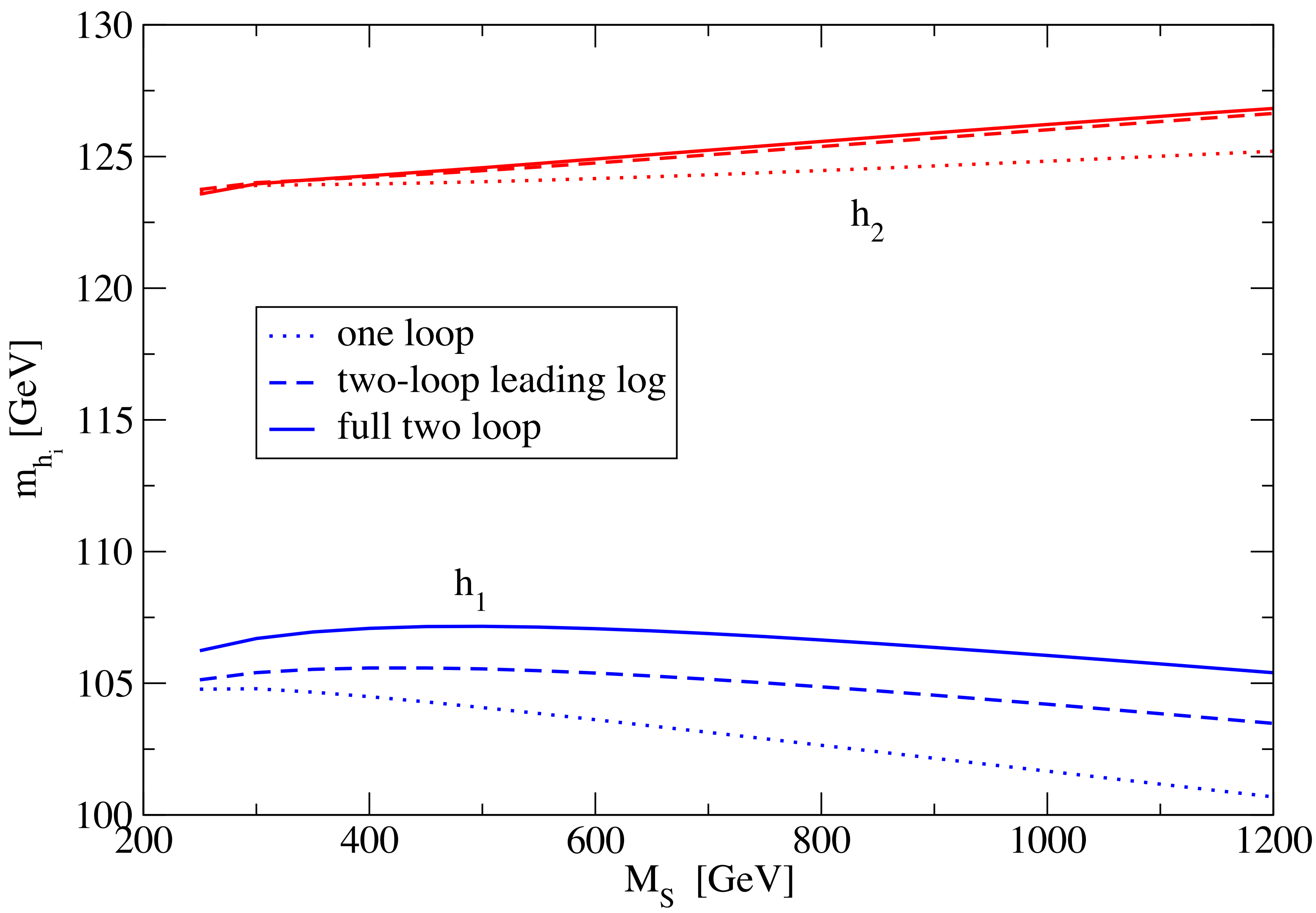
Two-loop corrections to the two lightest scalar masses



$$\lambda = 0.5\ , \quad \kappa = 0.1\ , \quad \tan\beta = 2\ , \quad A_\lambda = 500\ \text{GeV}\ , \quad A_\kappa = -10\ \text{GeV}\ , \quad \mu = 250\ \text{GeV}$$

$$A_t = A_b = A_\tau = -1.5\,M_S\ , \quad M_3 = 2\,M_S\ , \quad M_2 = 2/3\,M_S\ , \quad M_1 = M_S/3$$

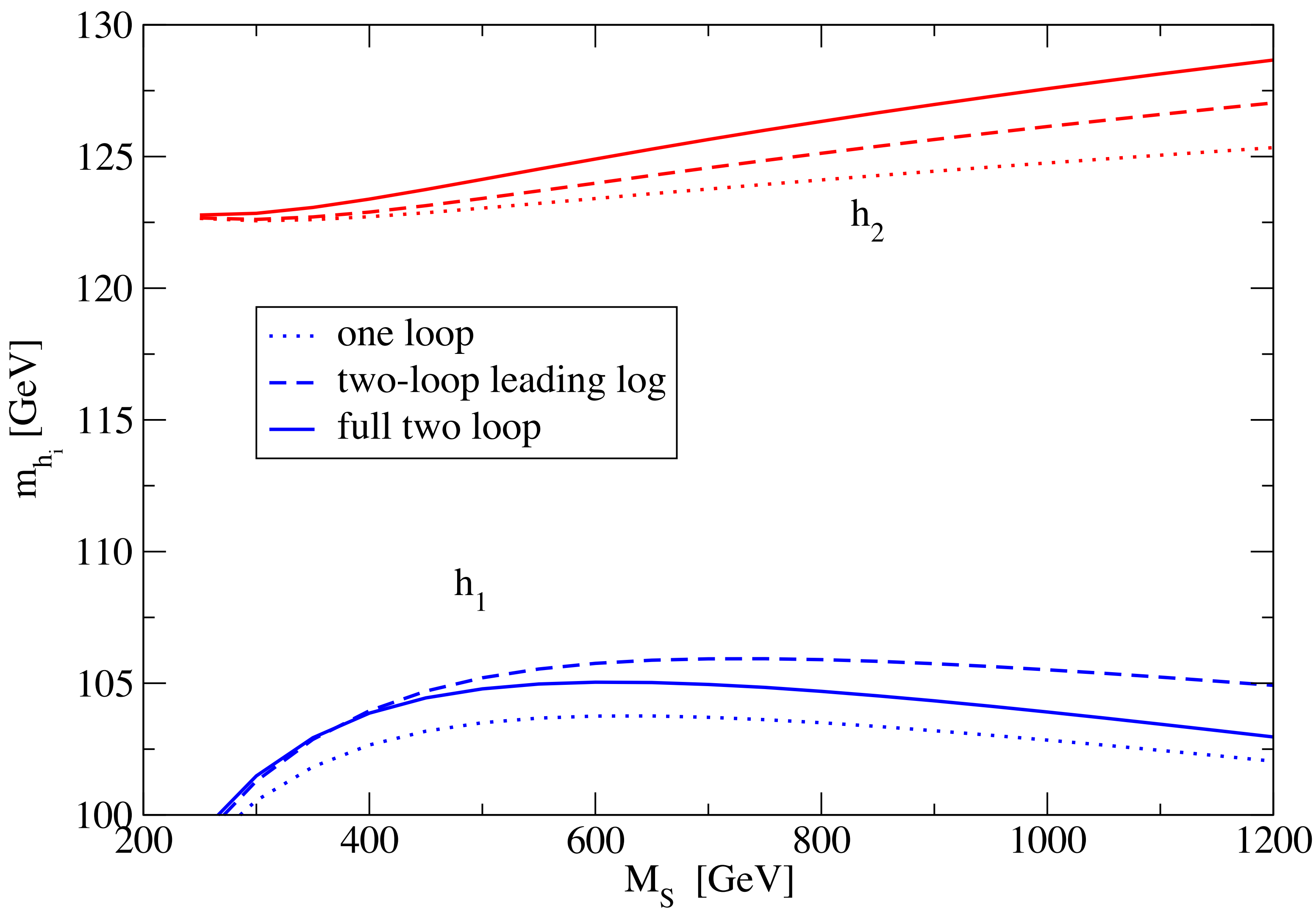
Two-loop corrections to the two lightest scalar masses



$$\lambda = 0.5 \ , \quad \kappa = 0.1 \ , \quad \tan\beta = 2 \ , \quad A_\lambda = 500 \text{ GeV} \ , \quad A_\kappa = -10 \text{ GeV} \ , \quad \mu = 250 \text{ GeV}$$

$$A_t = A_b = A_\tau = -1.5\, M_S \ , \quad M_3 = 2\, M_S \ , \quad M_2 = 2/3\, M_S \ , \quad M_1 = M_S/3$$

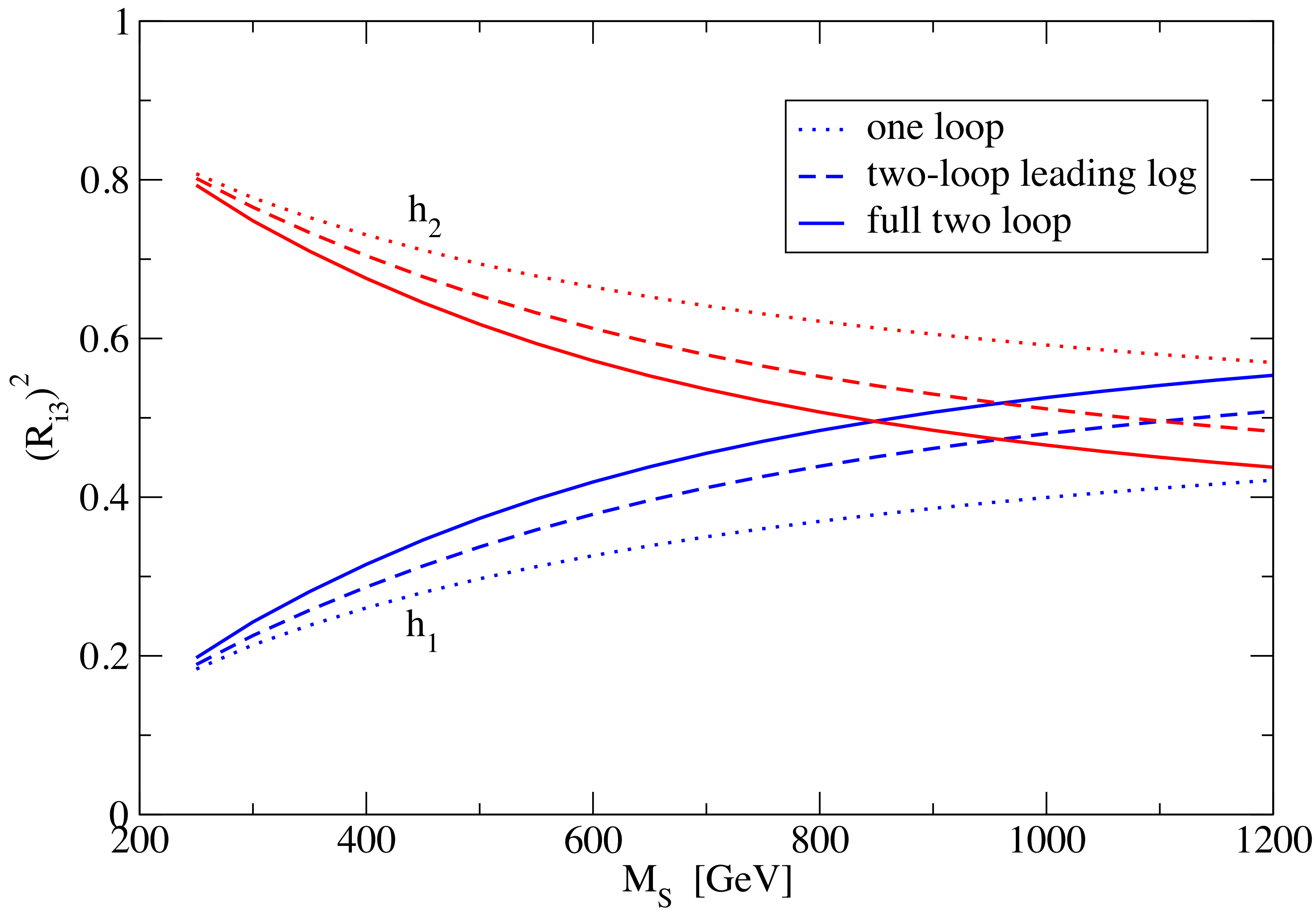
Two-loop corrections to the two lightest scalar masses (II)



$$\lambda = 0.5 \ , \quad \kappa = 0.1 \ , \quad \tan\beta = 2 \ , \quad A_\lambda = 500 \text{ GeV} \ , \quad A_\kappa = -10 \text{ GeV} \ , \quad \mu = 250 \text{ GeV}$$

$$A_t = A_b = A_\tau = +1.5\, M_S \ , \quad M_3 = 2\, M_S \ , \quad M_2 = 2/3\, M_S \ , \quad M_1 = M_S/3$$

Effect of the two-loop corrections on the CP-even mixing



$\lambda = 0.5$, $\kappa = 0.1$, $\tan\beta = 2$, $A_\lambda = 500$ GeV , $A_\kappa = -10$ GeV , $\mu = 250$ GeV

$A_t = A_b = A_\tau = -1.5\, M_S$, $M_3 = 2\, M_S$, $M_2 = 2/3\, M_S$, $M_1 = M_S/3$

Conclusions

- The NMSSM is a simple and attractive extension of the MSSM:
  - elegant solution to the μ problem
  - interesting collider phenomenology
  - reduced fine-tuning in the EWSB conditions

- The accuracy of the Higgs mass calculation in the NMSSM was until now stuck to the level that for the MSSM had been achieved in the mid-1990s

- The newly computed one-loop and two-loop corrections can account for shifts of several GeV in the light scalar and pseudoscalar masses, and sizeably affect the mixing between singlet and MSSM-like Higgs scalars

- These corrections must be taken into account for a meaningful comparison between the MSSM and NMSSM predictions for the Higgs sector

Thank you!!!

To-do list

- Implement the new corrections in NMHDECAY?

- Include the corrections to the *charged* Higgs mass
  - the 1-loop corrections are straightforward
  - the 2-loop strong corrections are the same as in the MSSM

- To fully match the accuracy of the existing MSSM codes, we should also compute the 2-loop *Yukawa* corrections to both neutral and charged Higgses. They cannot be easily adapted from the MSSM result and require a dedicate calculation *(is it worth the pain?)*