α_s and m_b with three-loop accuracy

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Outline:

- Motivation
- Running and decoupling in the MSSM
- Phenomenology

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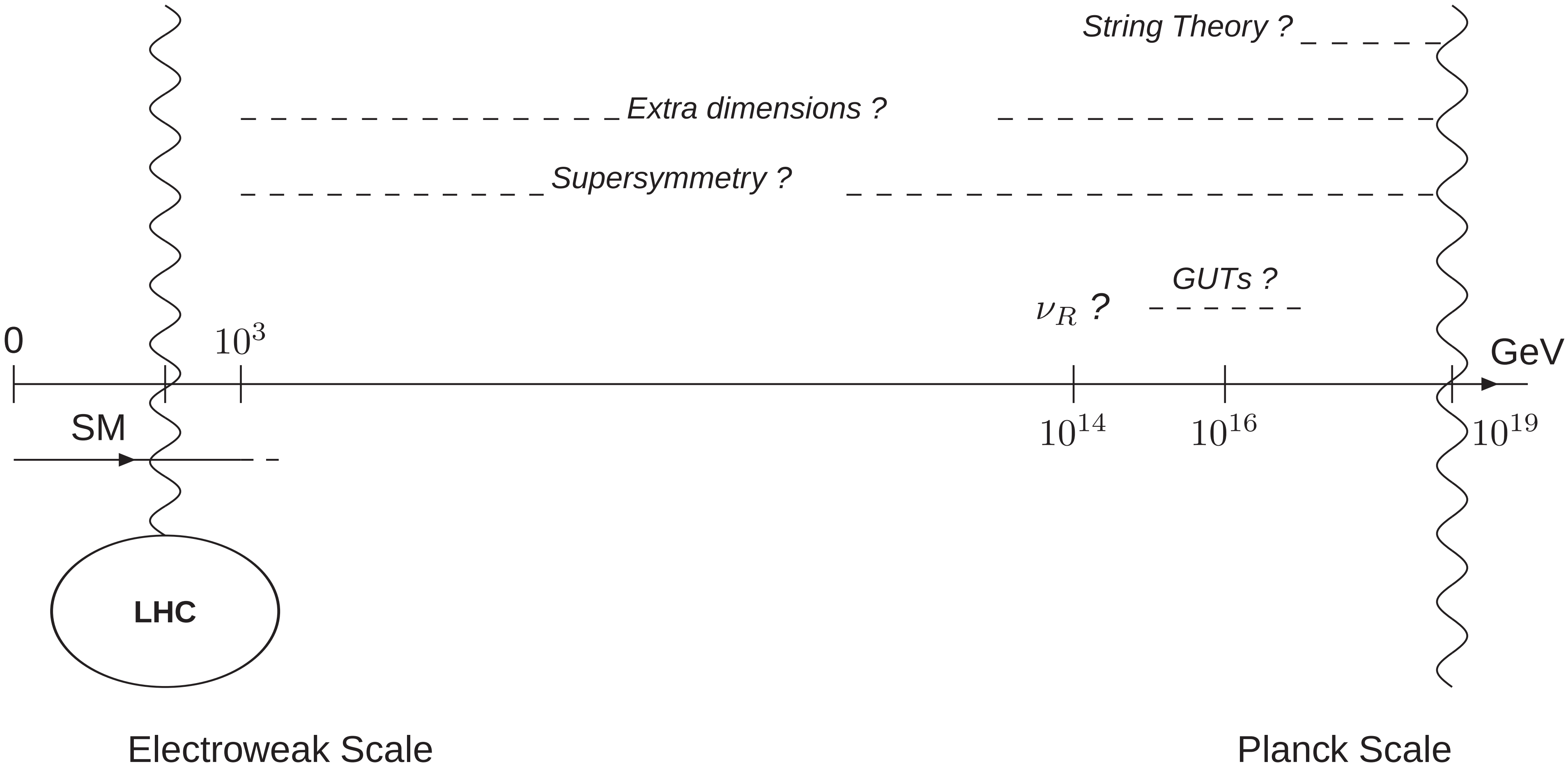

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  - Our aim: 3-loop RGEs for SUSY-QCD *I. Jack, T. Jones and C. North '96, R. Harlander, L. M., M. Steinhauser '09*
    2-loop threshold corrections *R. Harlander, L. M., M. Steinhauser '05, A. Bauer, L. M.,J. Salomon '08*

$\Rightarrow \alpha_s^{\overline{\rm DR}}(\mu)$ and $m_{\rm b}^{\overline{\rm DR}}(\mu)$, $\mu = M_{\rm GUT}, M_{\rm SUSY}$ with 3-loop accuracy

Running of couplings

Running = variation of coupling strength with the energy.

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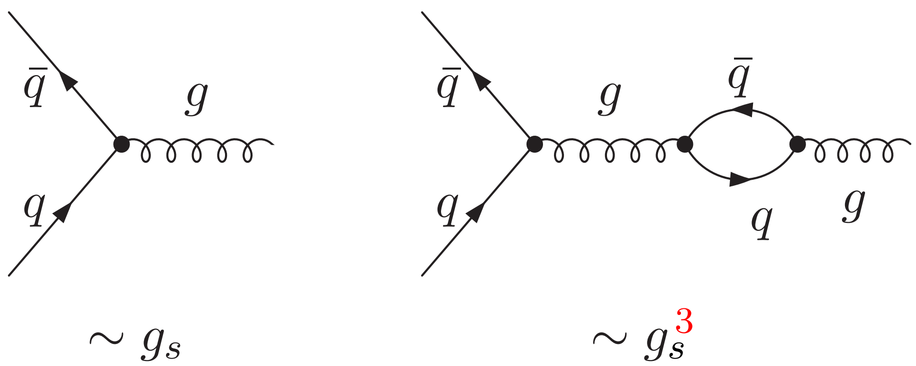

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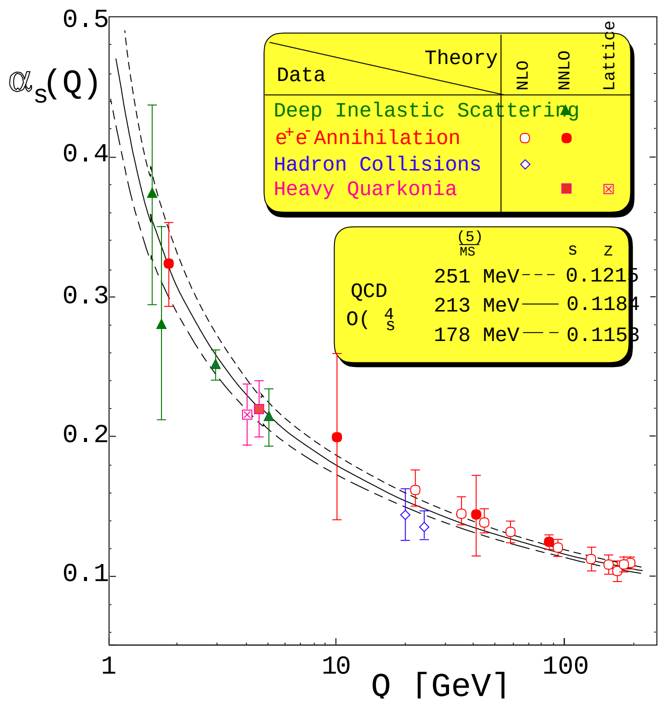
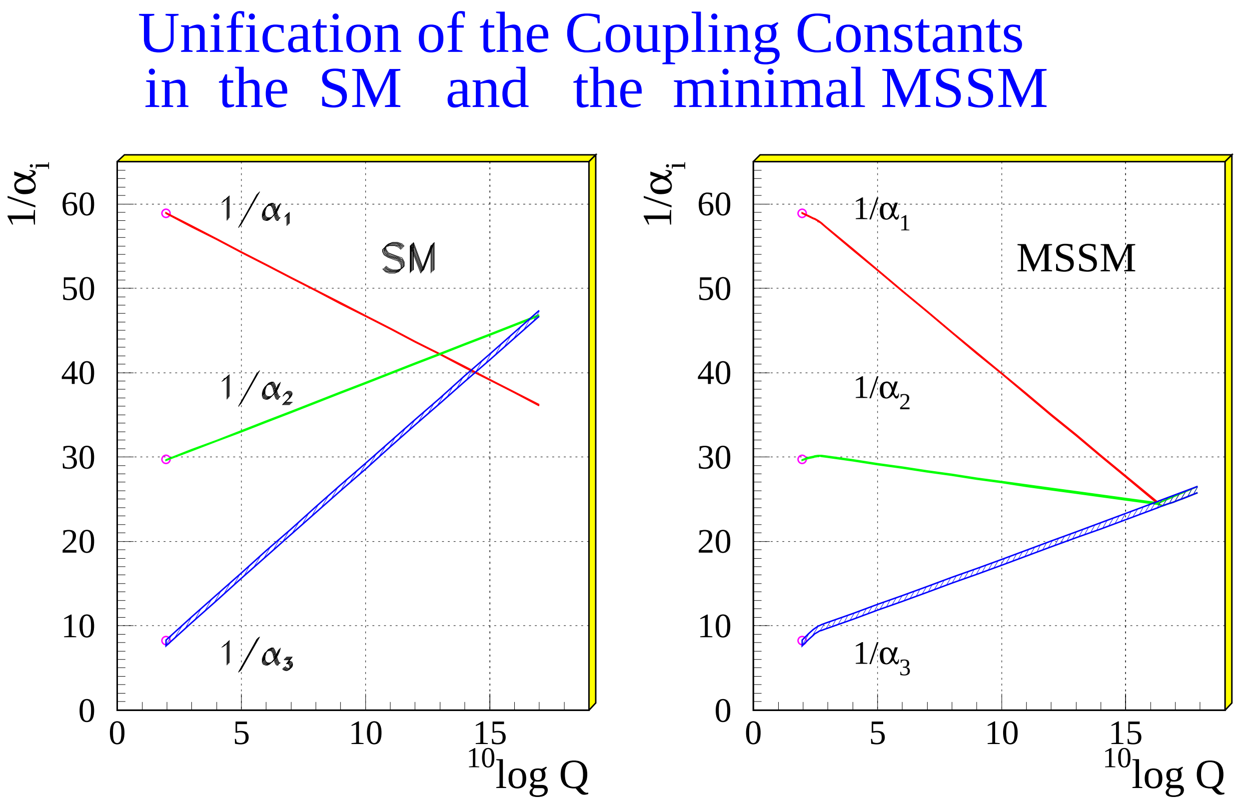
  - Vacuum can screen or anti-screen the gauge charges.
  - Anti-screening gives rise to the asymptotic freedom of strong interactions.



MSSM and LEP data



[Amaldi, Furstenau, de Boer]

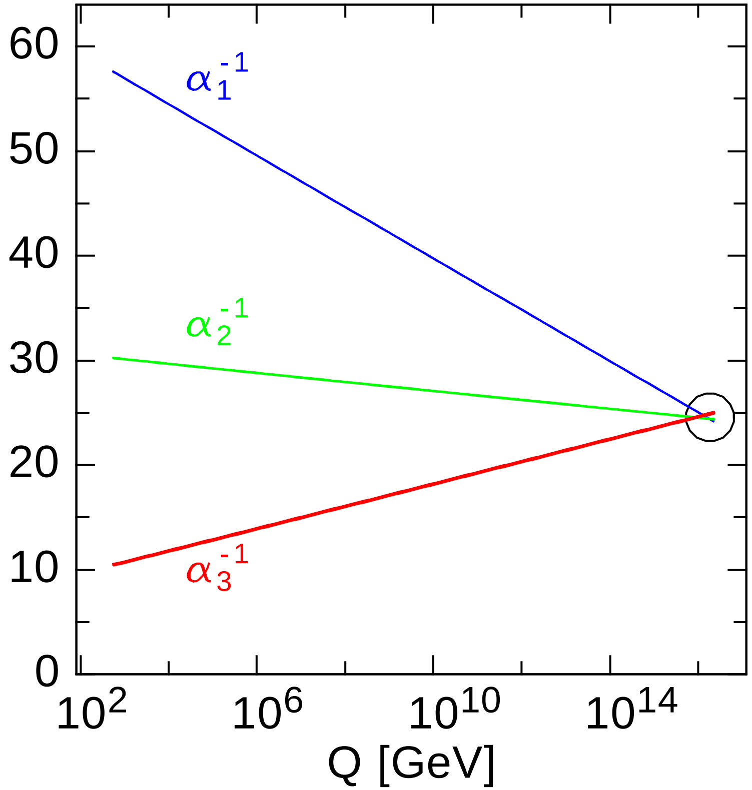
[Langacker, Luo]

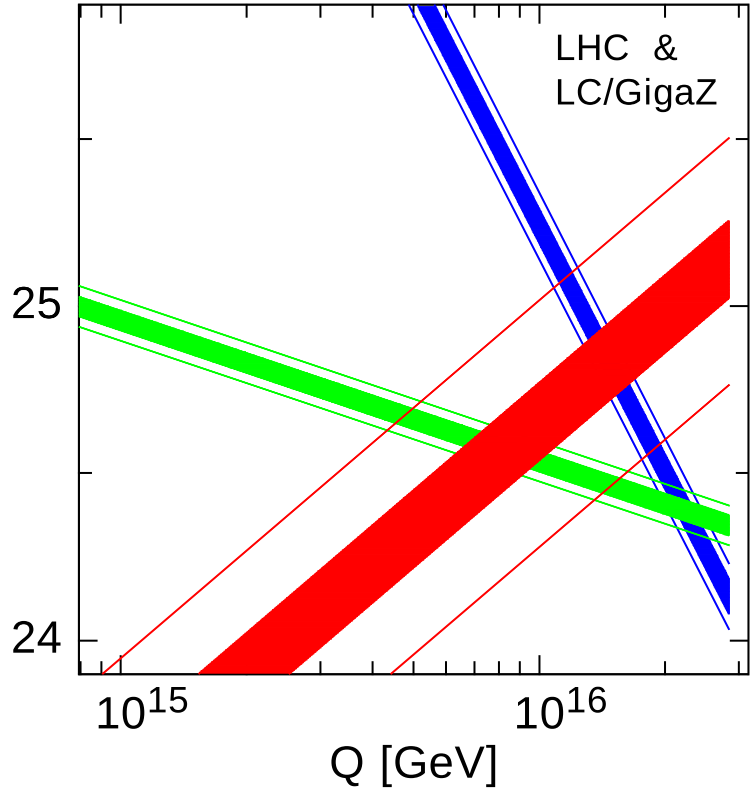
[Ellis, Kelley, Nanopoulos]

- Gauge Coupling unification within SM excluded by about $12\,\sigma$.
- Gauge coupling Unification within SUSY GUTs works extremely well: it fits within 3σ the present low energy data.

High precision data

[Blair, Porod, Zerwas '04]

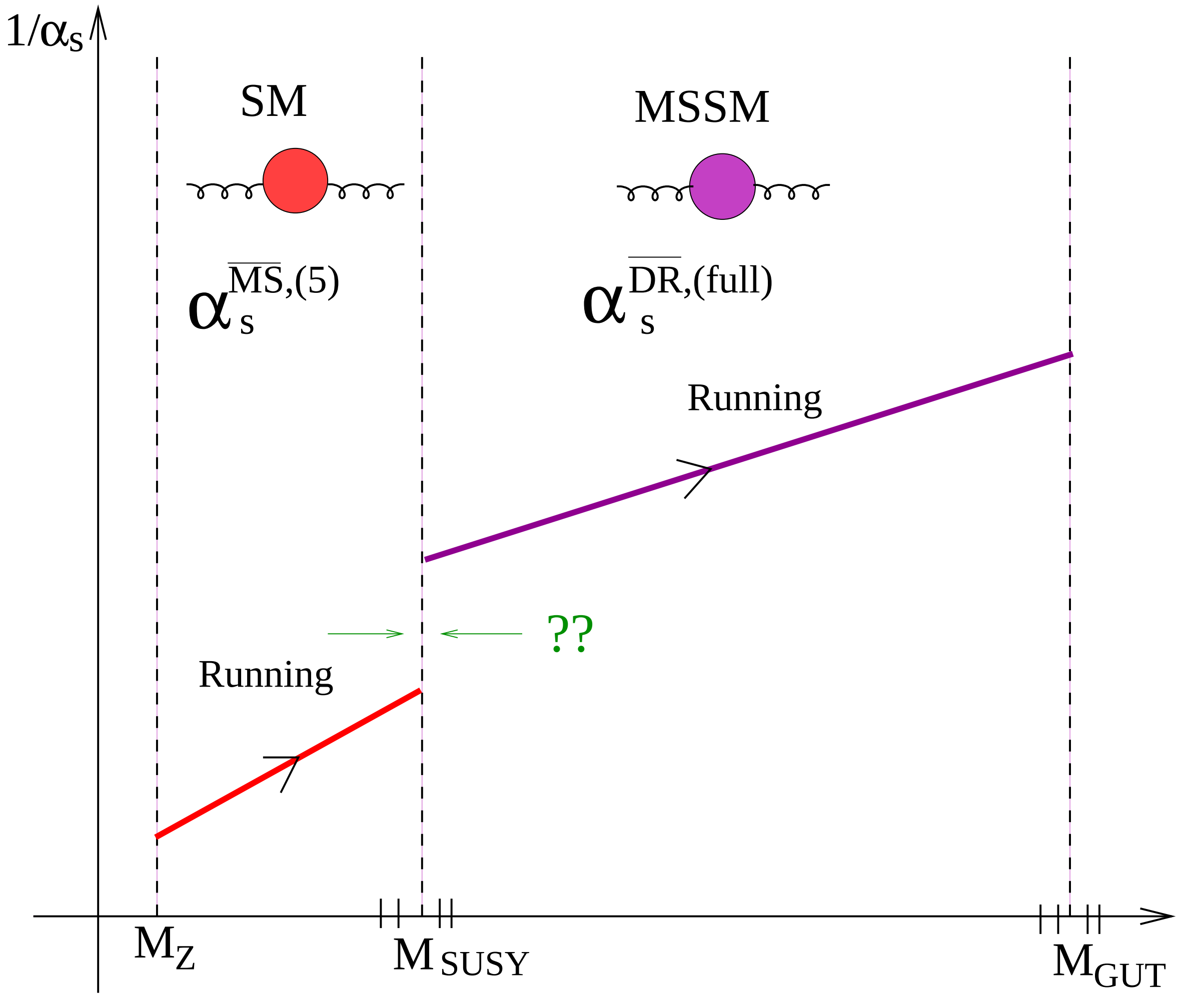




- Computation: 2-loop Renormalization Group Running 1-loop threshold corrections at the weak scale (M_Z)

Evolution of the strong coupling

- Input parameter: $\alpha_s^{\overline{MS},(5)}(M_Z)$ $\Rightarrow$ Output parameter: $\alpha_s^{\overline{DR},(full)}(M_{GUT})$



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$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \alpha_s(\mu) = \beta(\alpha_s)$$

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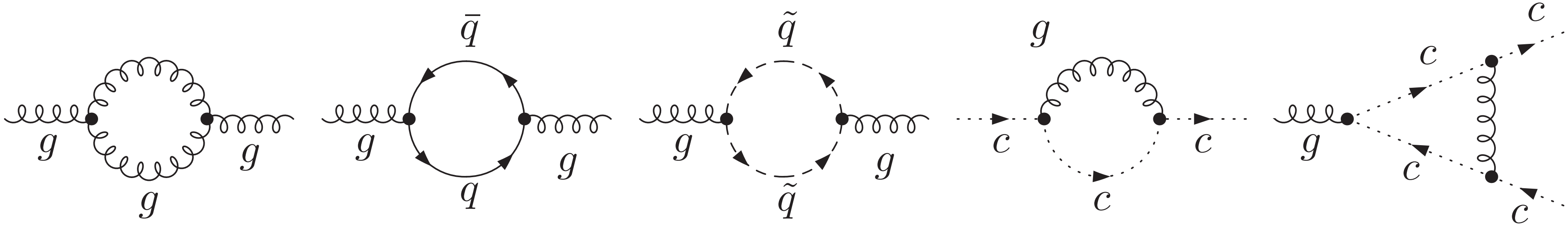

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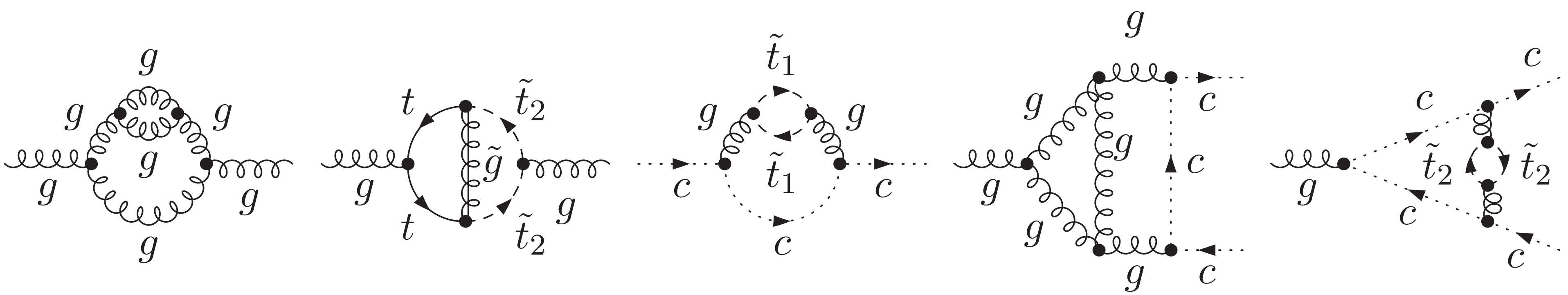

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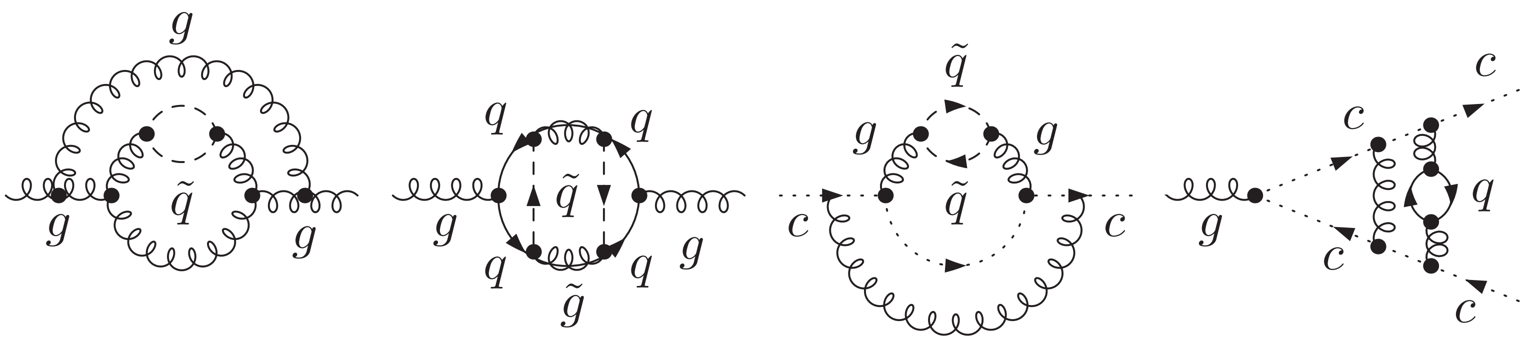

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- 3-loop β_s in the MSSM
  - $\simeq$ 100.000 diagrams
  - Computer programs: QGRAF, FORM, MINCER, MATAD, EXP, ...

    [Noguiera; Vermaseren; Larin, Tkachov; Steinhauser; Seidensticker, Harlander; ...]

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$$\beta(\alpha_s) = -\sum_{n \geq 0} \left(\frac{\alpha_s}{\pi}\right)^{n+2} \beta_n \,,$$

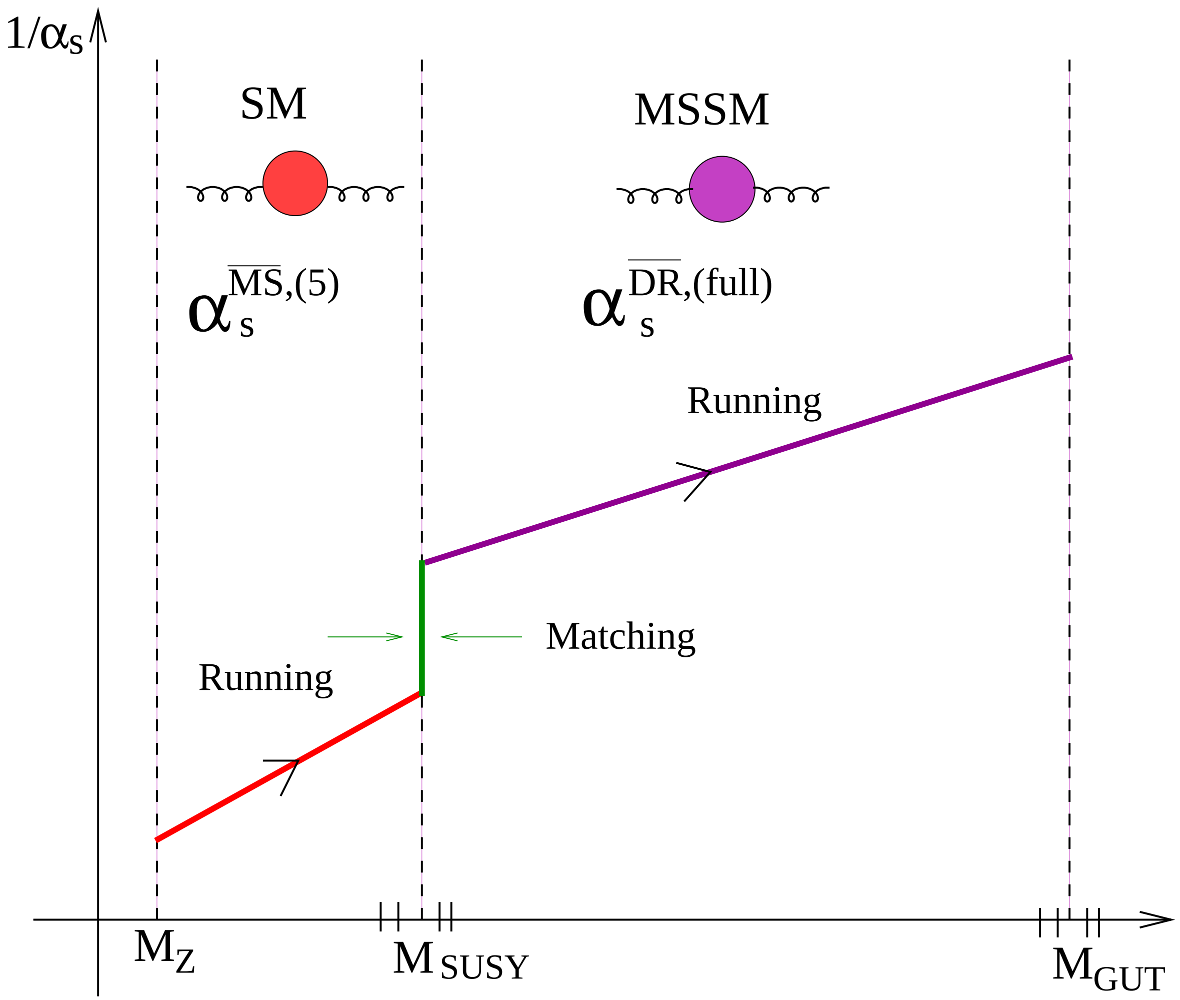
$$\beta_0 = \frac{3}{4} C_A - \frac{1}{2} T_f \,, \qquad C_F = 4/3,\, C_A = 3,\, 2T_f = n_f,$$

$$\beta_1 = \frac{3}{8} C_A^2 - T_f \left(\frac{1}{2} C_F + \frac{1}{4} C_A \right) ,$$

$$\beta_2 = \frac{21}{64} C_A^3 + T_f \left(\frac{1}{4} C_F^2 - \frac{13}{16} C_A C_F - \frac{5}{16} C_A^2 \right) + T_f^2 \left(\frac{3}{8} C_F + \frac{1}{16} C_A \right) .$$

Matching

- Effective Field Theory:



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$$\mathcal{L}_{\rm MSSM}(\alpha_s^{(\rm full)}, \ldots) \quad \to \quad \mathcal{L}(\alpha_s^{(5)}, \ldots) \quad \text{at energy } \mu$$

- "Matching" : low energy physics must be unchanged !!

$$\begin{aligned} \alpha_s^{(5)} &= \zeta_s\, \alpha_s^{(\rm full)} \\ m_q^{(5)} &= \zeta_m\, m_q^{(\rm full)} \qquad q = u, d, s, c, b \\ &\vdots \\ \zeta_{s,m} &= \zeta_{s,m}(\alpha_s, M_{\rm SUSY}, m_t, \mu) \\ \zeta_s &= \text{matching coefficients} \end{aligned}$$

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  - μ not predicted by theory
  - Physical quantities must be independent of μ
  - Quantum corrections improve stability

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Relate Green functions computed in the full and effective theory.

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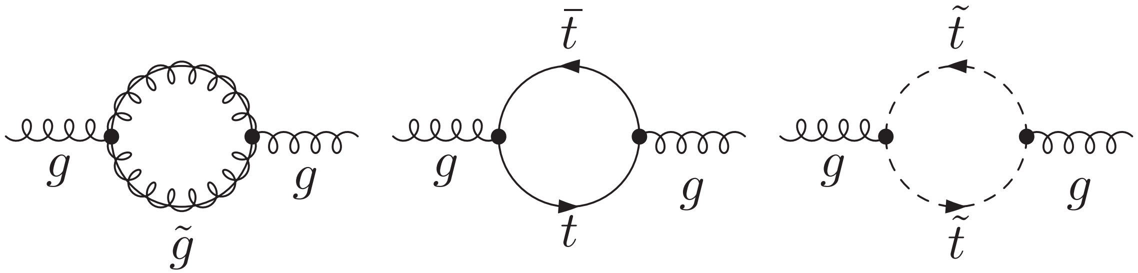

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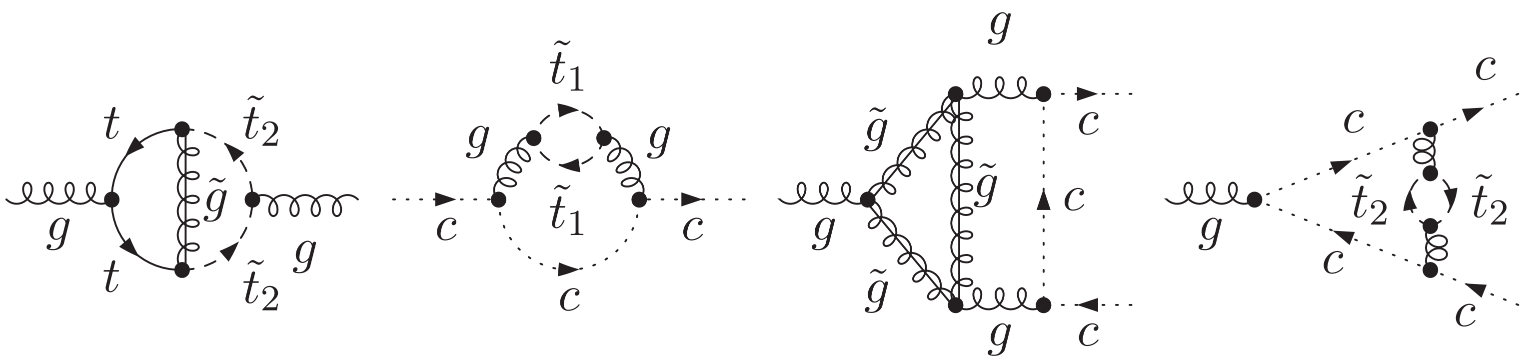

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$$\zeta_{s1}^{(5)} = -\frac{1}{6}\ln\frac{\mu^2}{m_t^2} - \ln\frac{\mu^2}{\tilde{M}^2}, \qquad \tilde{M} = M_{\rm SUSY}$$

$$\begin{aligned}\zeta_{s2}^{(5)} = & -\frac{215}{96} - \frac{19}{24}\ln\frac{\mu^2}{m_t^2} - \frac{5}{2}\ln\frac{\mu^2}{\tilde{M}^2} + \left[\frac{1}{6}\ln\frac{\mu^2}{m_t^2} + \ln\frac{\mu^2}{\tilde{M}^2}\right]^2 \\ & + \left(\frac{m_t}{\tilde{M}}\right)^2\left(\frac{5}{48} + \frac{3}{8}\ln\frac{m_t^2}{\tilde{M}^2}\right) - \frac{7\pi}{36}\left(\frac{m_t}{\tilde{M}}\right)^3 \\ & + \left(\frac{m_t}{\tilde{M}}\right)^4\left(\frac{881}{7200} - \frac{1}{80}\ln\frac{m_t^2}{\tilde{M}^2}\right) + \frac{7\pi}{288}\left(\frac{m_t}{\tilde{M}}\right)^5\end{aligned}$$

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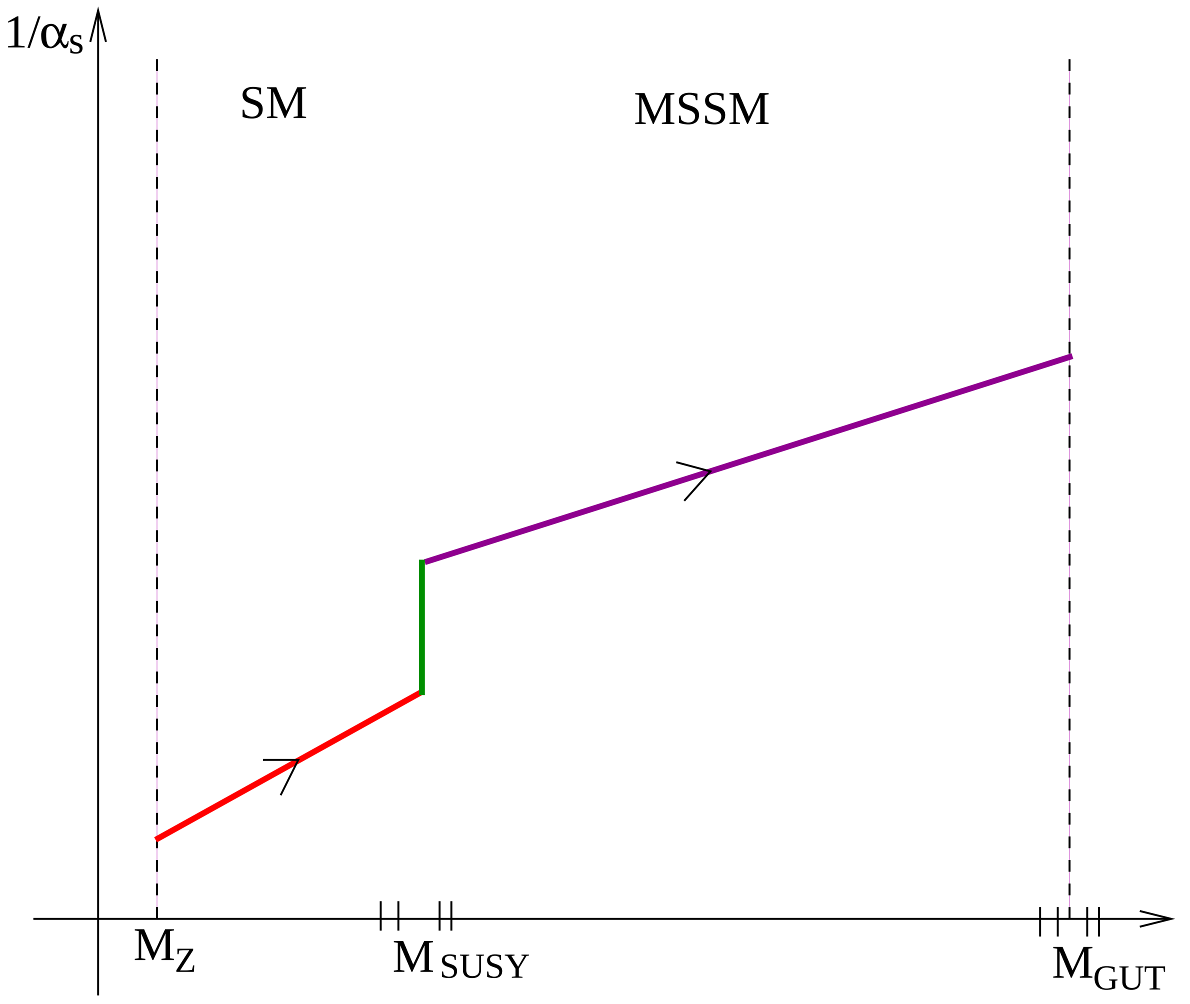

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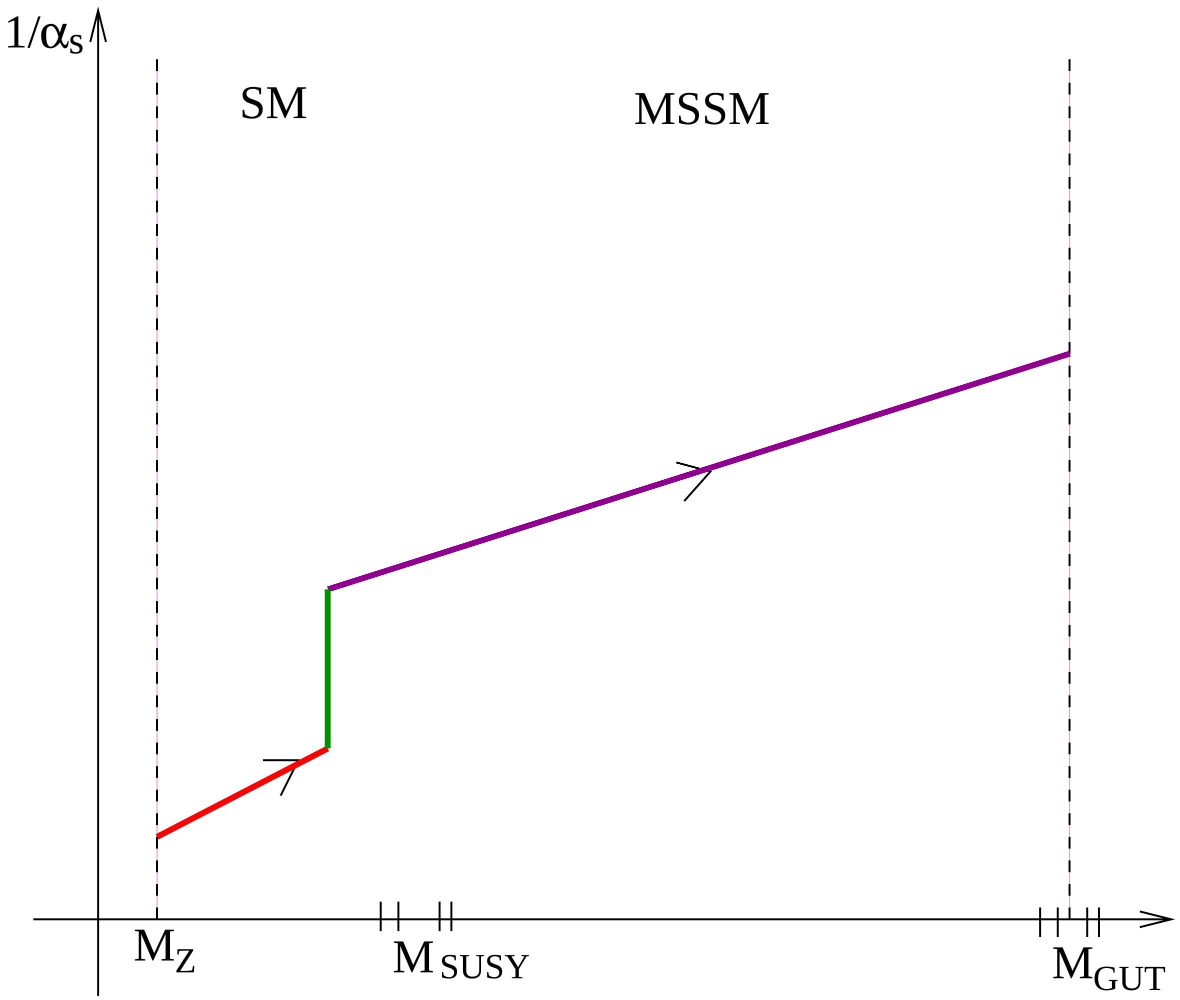

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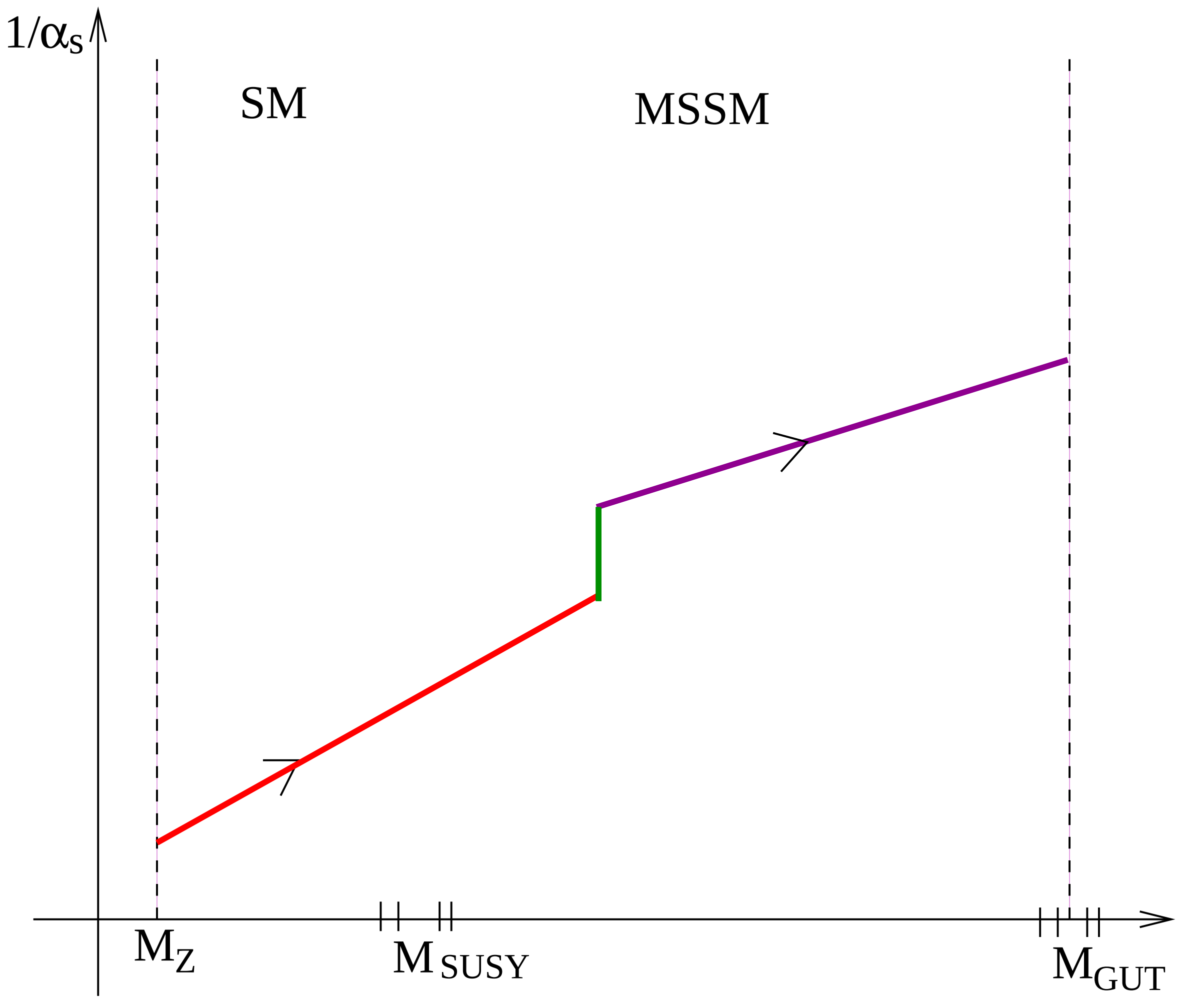

$\alpha_s(M_{\rm GUT})$

Input: $\alpha_s^{\overline{\rm MS},(5)}(M_Z) = 0.1189 \pm 0.001$ [Bethke '06], $M_Z = 91.1876$ GeV ,

MSSM parameters: SPS1a' scenario

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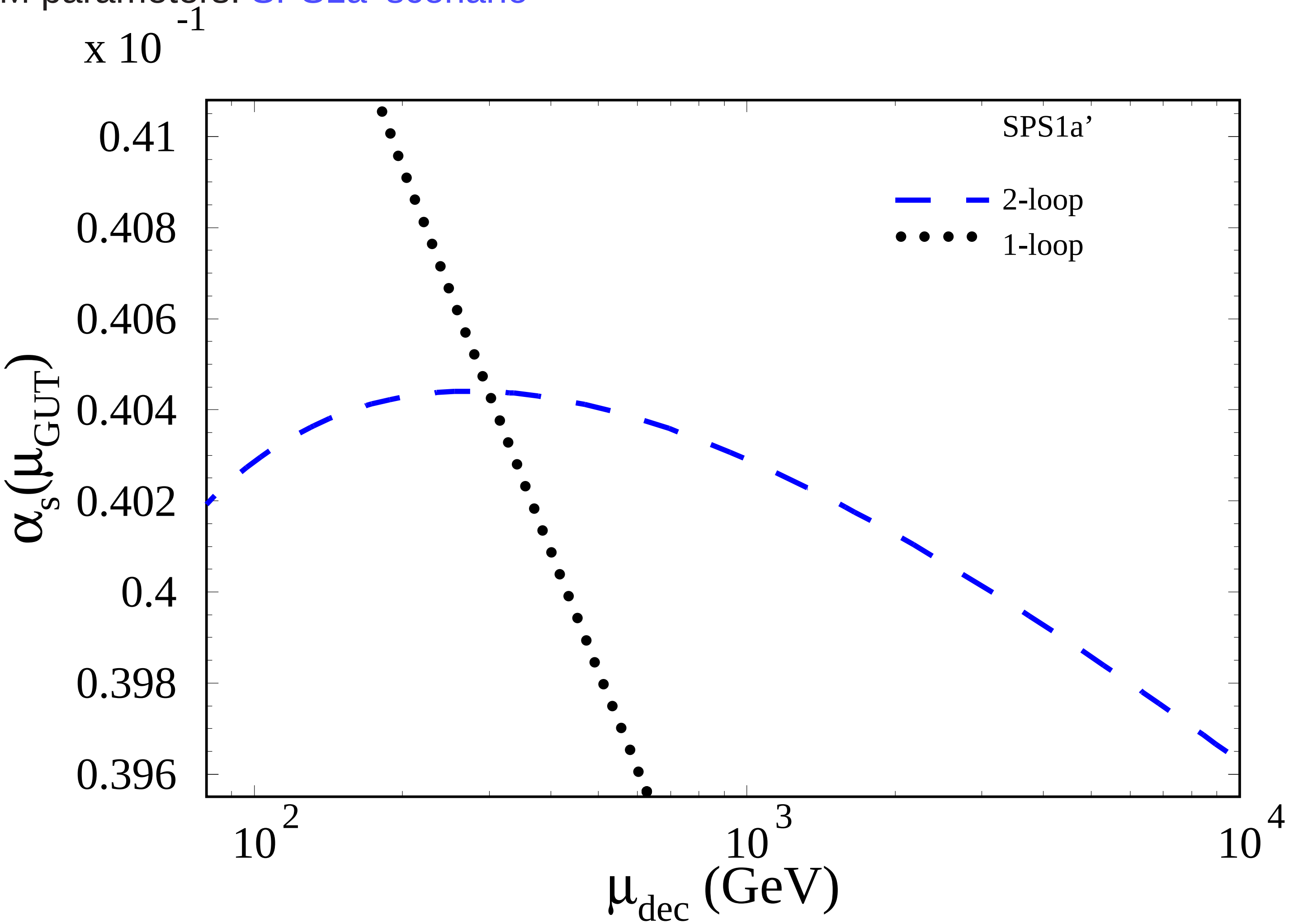

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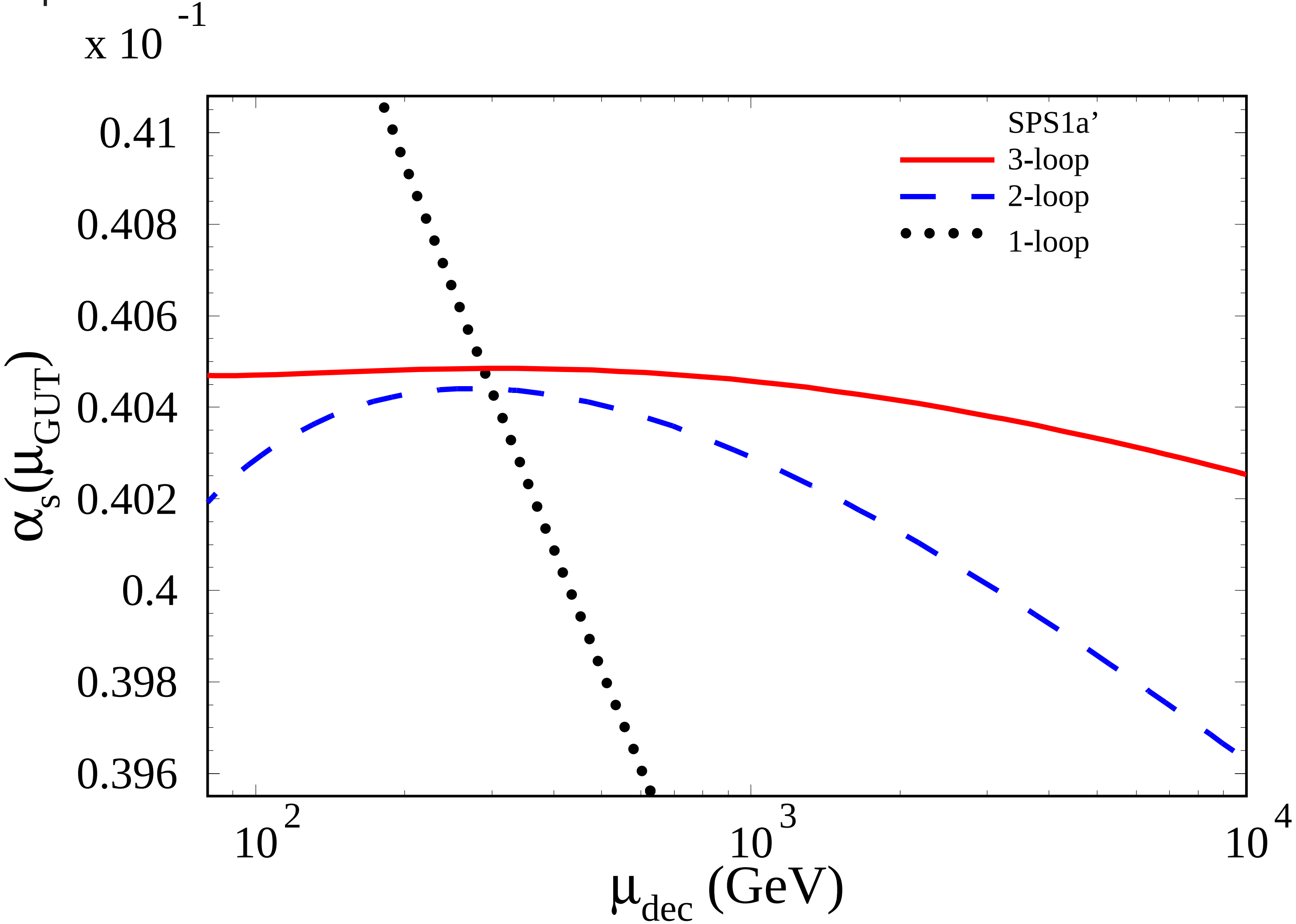

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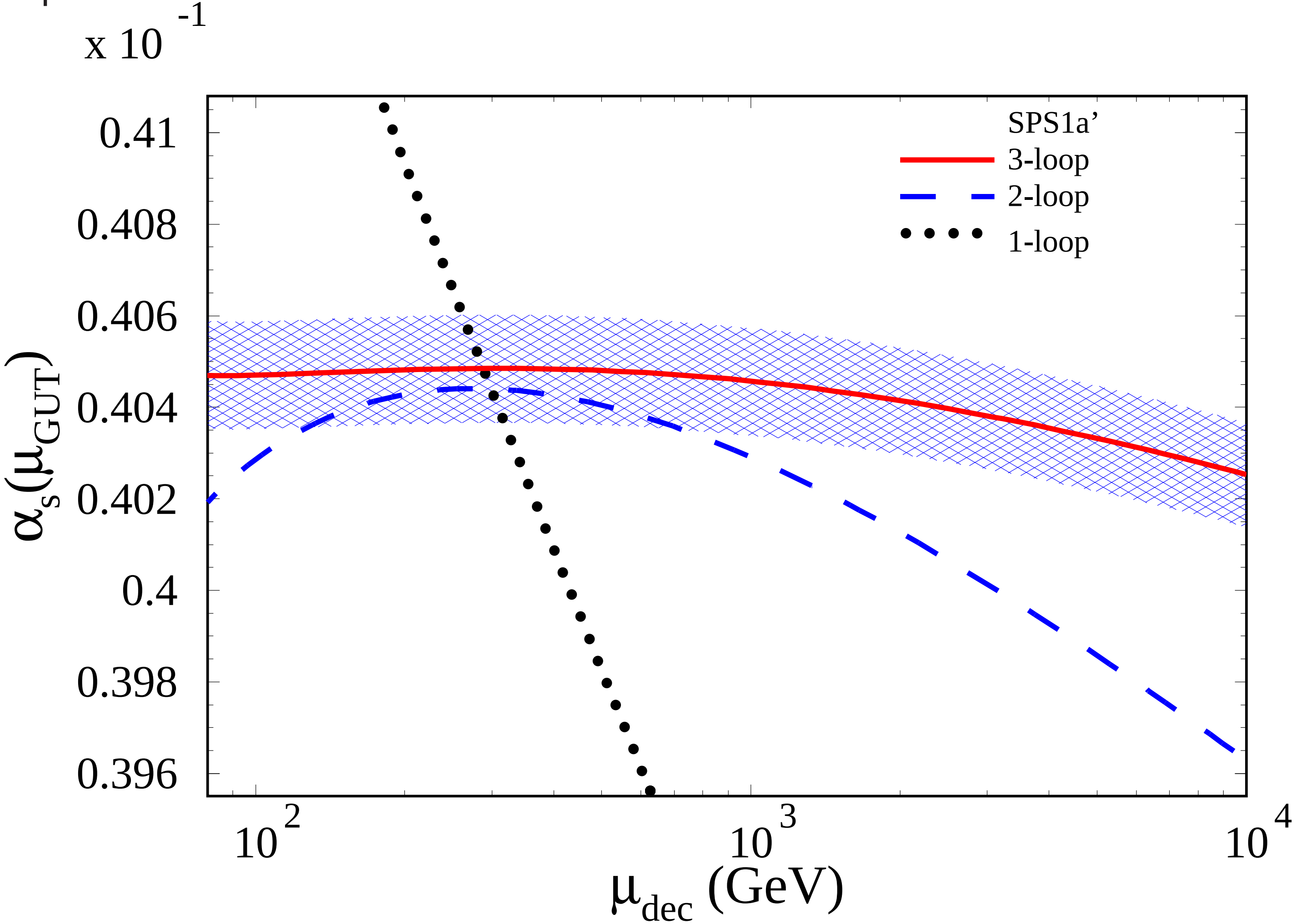

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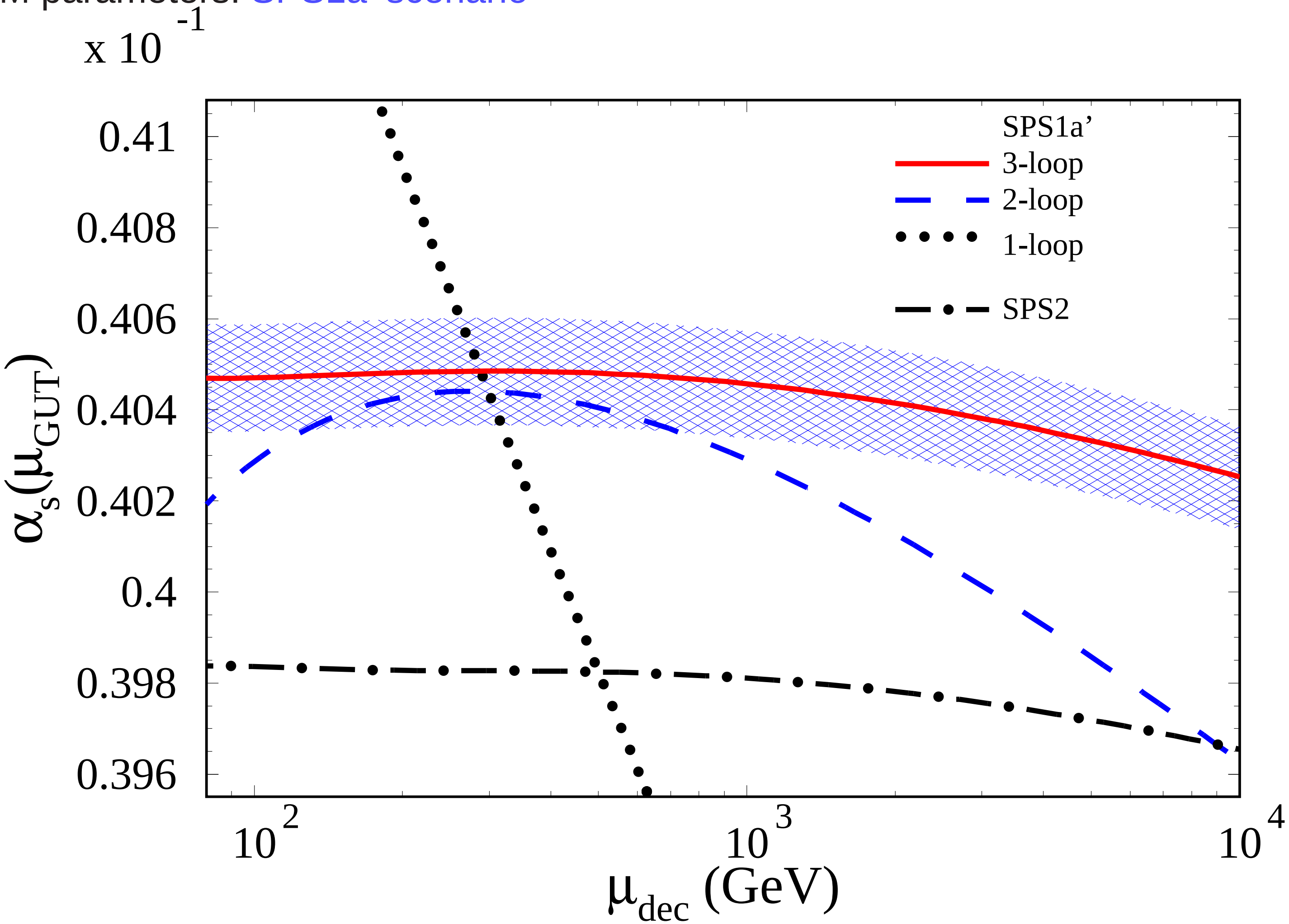
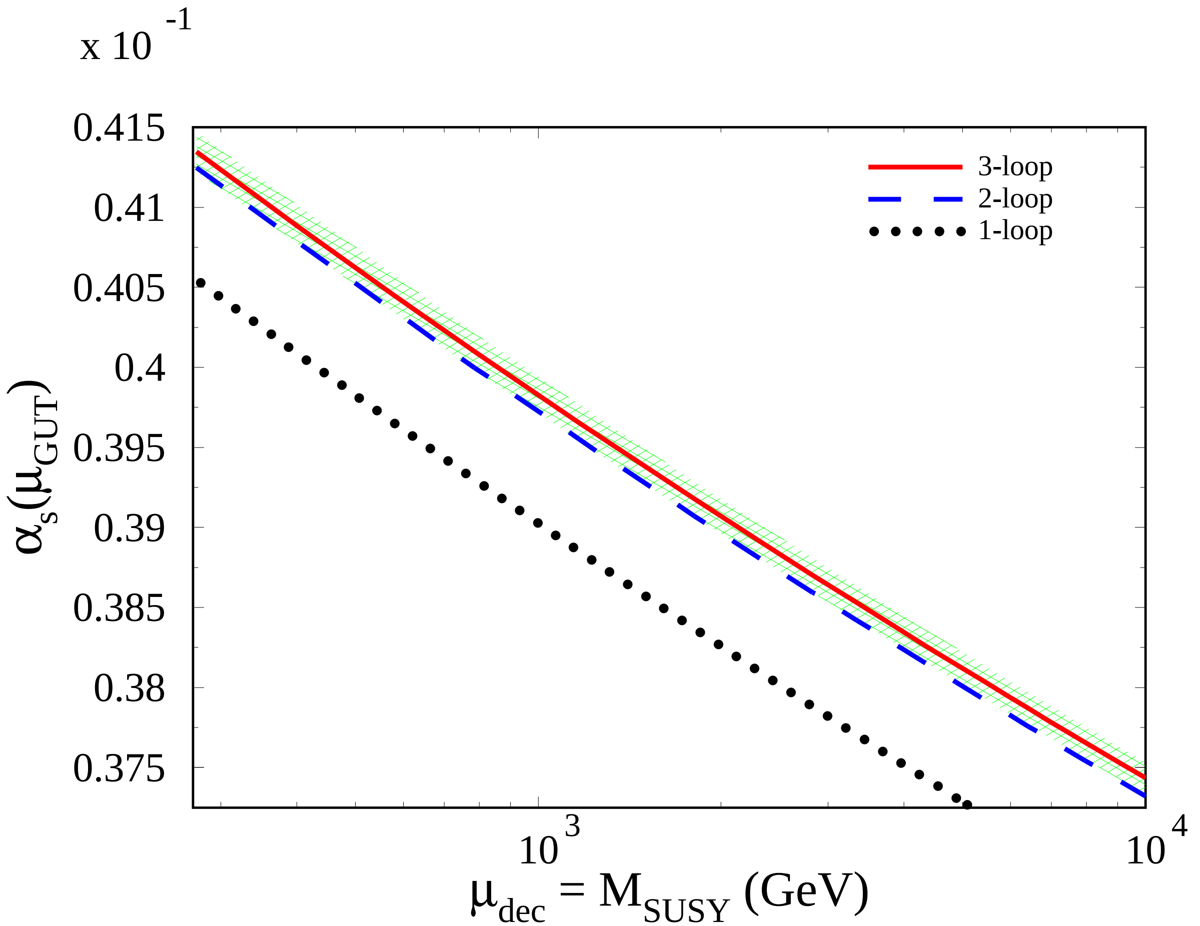


M_{SUSY}

- Sensitivity of $\alpha_s(M_{\mathrm{GUT}})$ to SUSY-mass scale:



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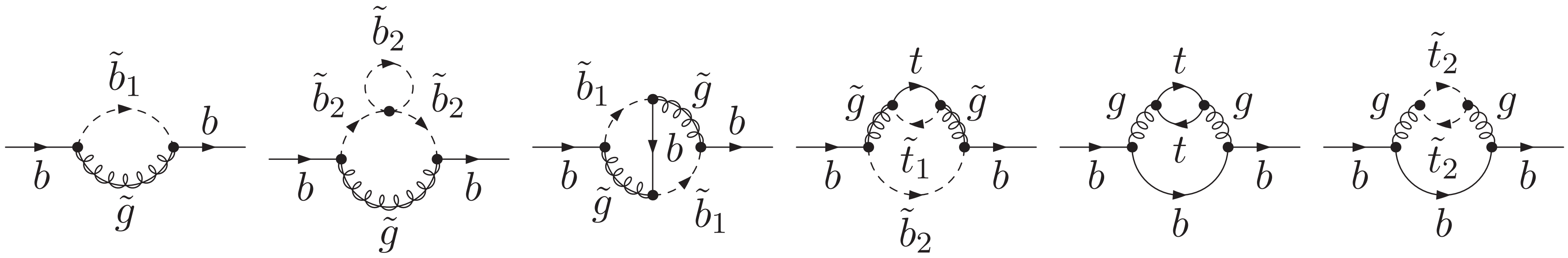

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1-loop

$$\zeta_{m_b} = 1 + \frac{\alpha_s^{(5)}}{\pi} C_F \sum_{i=1,2} \Bigg\{ -\frac{(1+L_{\tilde{b}_i})}{4} \frac{m_{\tilde{b}_i}^2}{(m_{\tilde{b}_i}^2 - m_{\tilde{g}}^2)} + \frac{(3+2L_{\tilde{b}_i}) m_{\tilde{b}_i}^4 - (3+2L_{\tilde{g}}) m_{\tilde{g}}^4}{16(m_{\tilde{b}_i}^2 - m_{\tilde{g}}^2)^2}$$
$$- (A_b - \mu_{\rm SUSY} \tan\beta) \frac{(-1)^i \, m_{\tilde{g}}}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2} \frac{m_{\tilde{b}_i}^2 L_{\tilde{b}_i} - m_{\tilde{g}}^2 L_{\tilde{g}}}{2(m_{\tilde{b}_i}^2 - m_{\tilde{g}}^2)} \Bigg\} \qquad L_i = \ln \frac{\mu^2}{m_i^2}$$

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$$\zeta_{m_b} = 1 + \delta\zeta_{m_b}^{\tan\beta} + \delta\zeta_{m_b}^{\rm rest}\,, \qquad \delta\zeta_{m_b}^{\tan\beta} = 1 + \sum_n \alpha_s^n (A_b - \mu_{\rm SUSY}\tan\beta) C_n$$

large $\tan\beta$: $\delta\zeta_{m_b}^{\tan\beta}$ has to be resummed !!

1-loop: [Hempfling '94; Hall, Ratazzi, Sarid '94; Carena, Garcia, Nierste, Wagner '01]

2-loop: [Noth, Spira '08, Bauer, L. M, Salomon '08]

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$m_b^{\overline{\rm MS}}(m_b) = 4.164 \pm 0.025$ GeV [Kühn, Steinhauser, Sturm '07]

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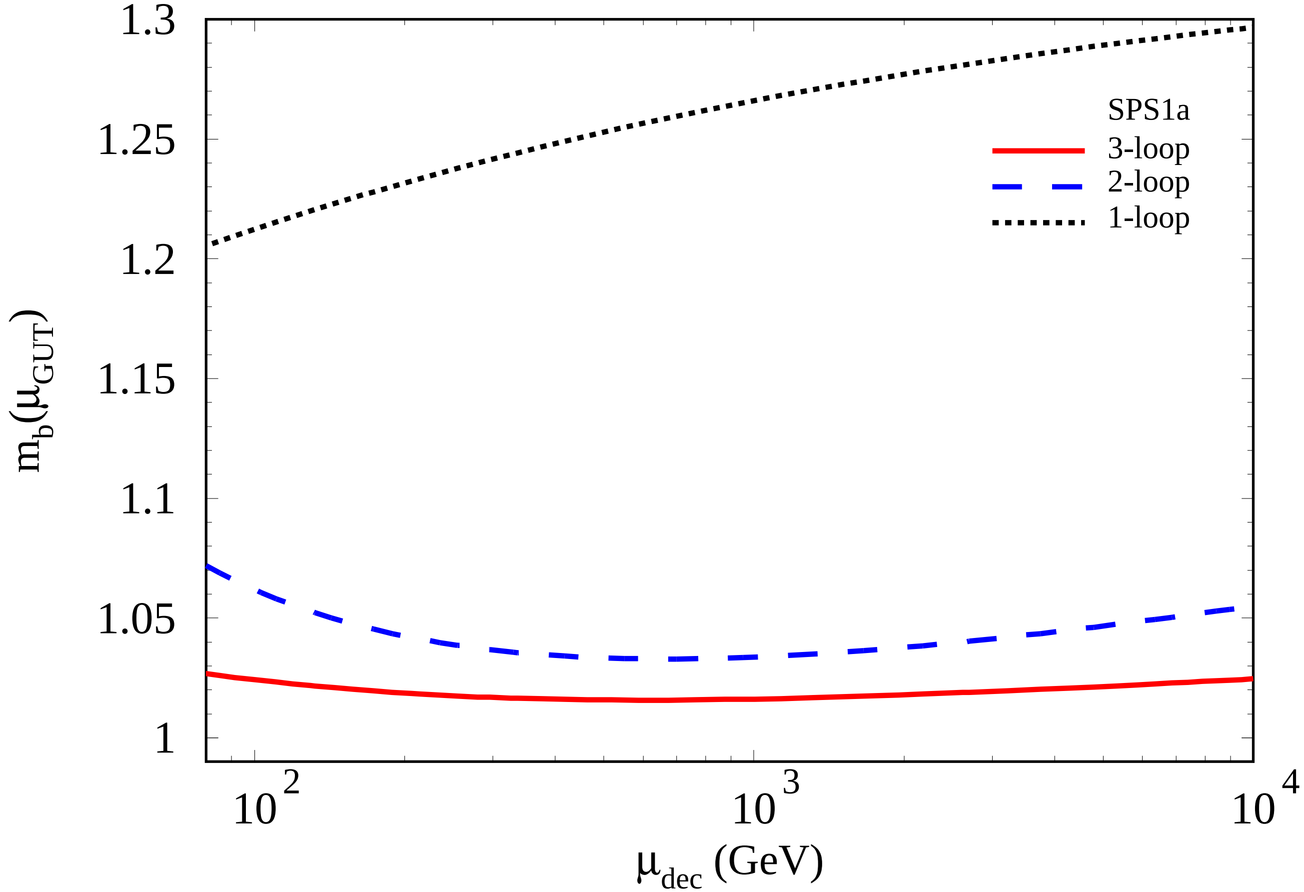
$m_b^{\overline{\rm MS}}(m_b) = 4.164 \pm 0.025$ GeV [Kühn, Steinhauser, Sturm '07]

MSSM parameters: SPS1a' scenario



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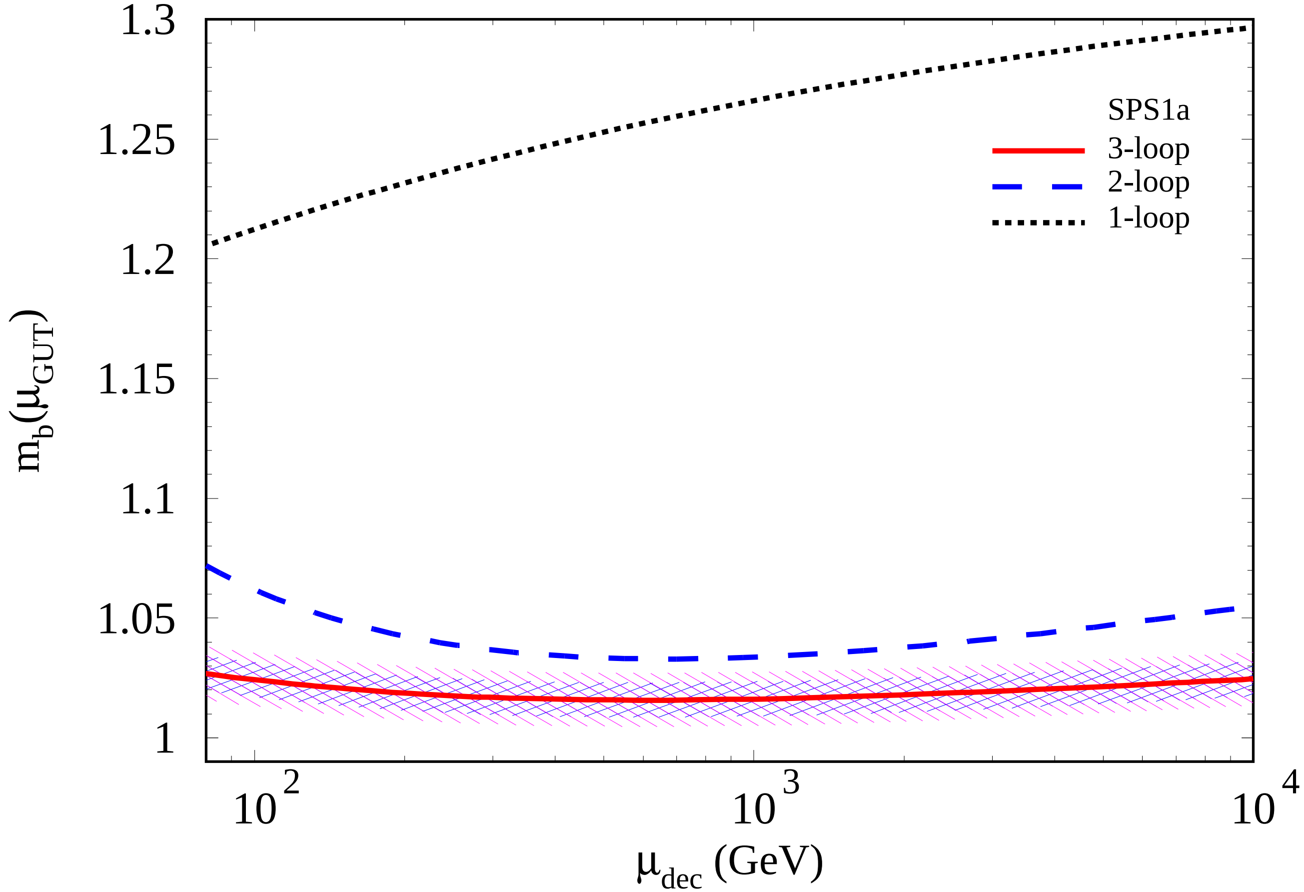

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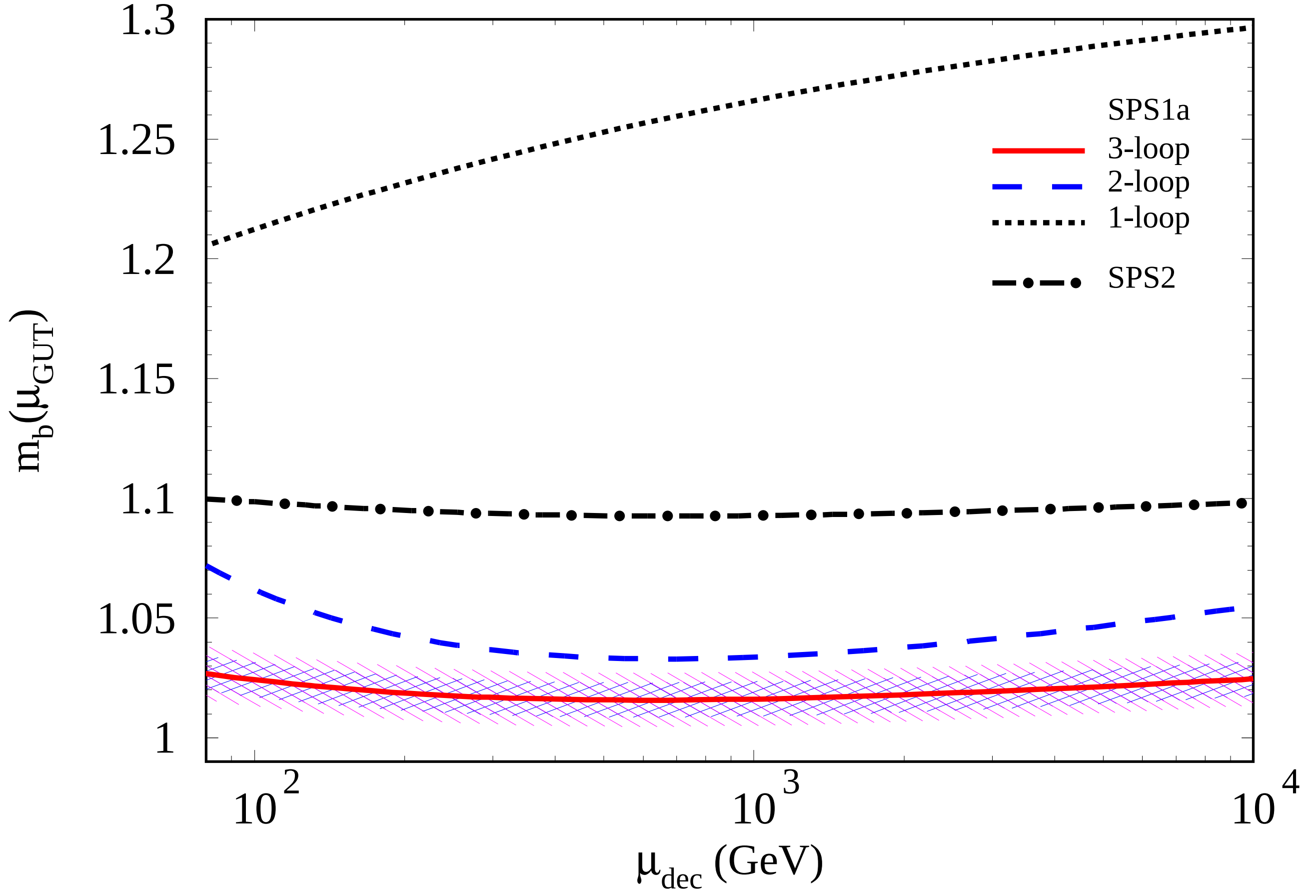

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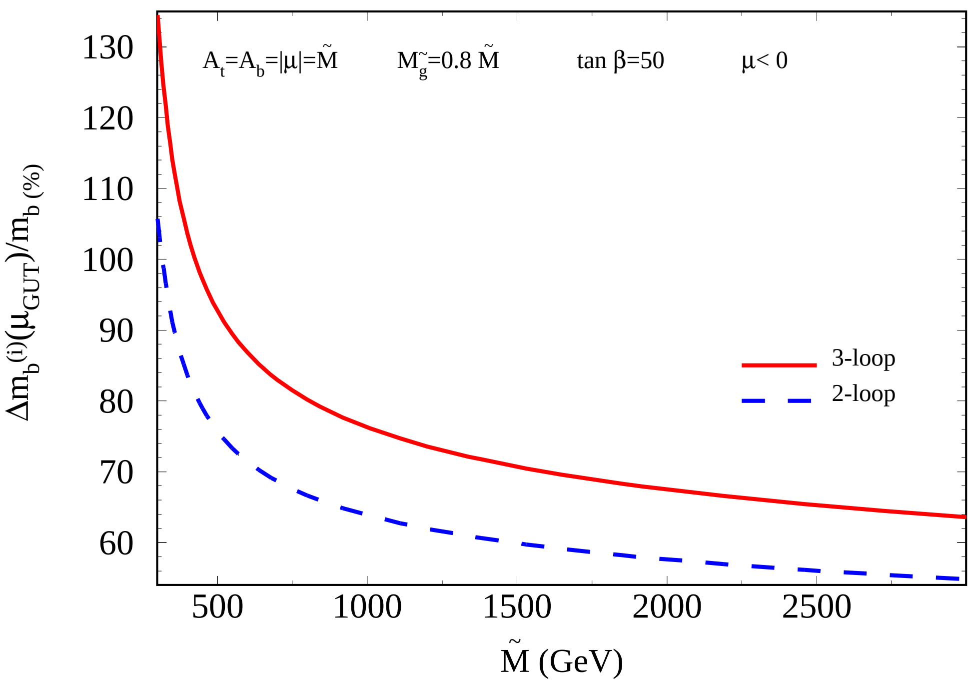
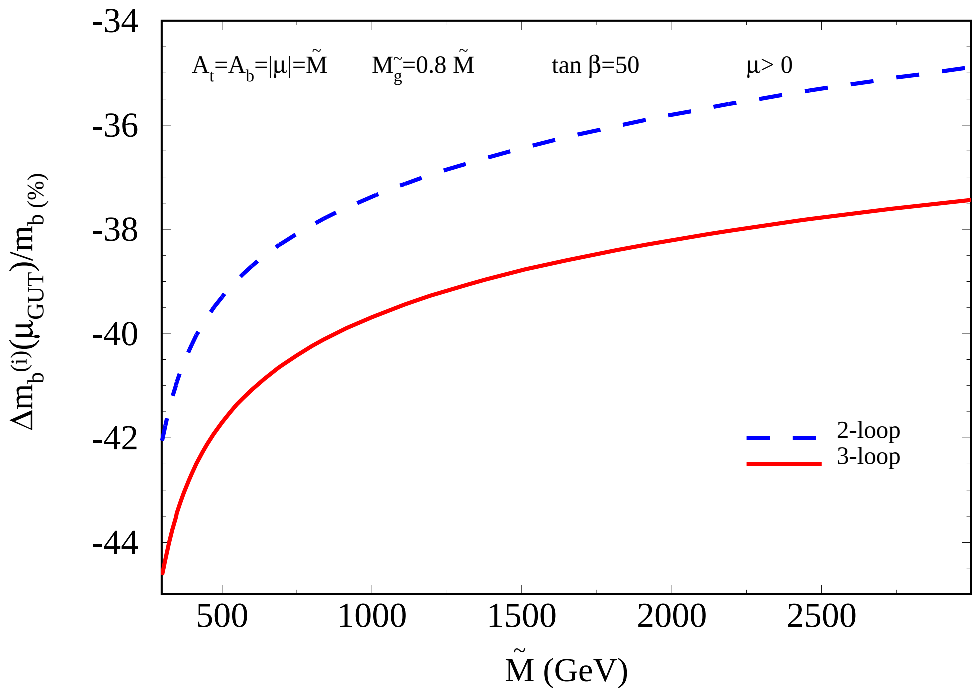
MSSM parameters: SPS1a' scenario

[A. Bauer, L. M., J. Salomon '08]





Conclusions

- $\alpha_s^{\overline{\mathrm{DR}}}(M_{\mathrm{GUT}})$
  - The 3-loop effects comparable with the experimental accuracy for α_s
  - $\alpha_s(M_{\mathrm{GUT}})$ very sensitive to SUSY-mass scale
- $m_b^{\overline{\mathrm{DR}}}(M_{\mathrm{GUT}})$
  - MSSM with large $\tan\beta$: 3-loop effects reach up to 30%

- ToDo:
  - combine 3-loop running analysis for SUSY-QCD with SUSY-EW
  - extend analysis to SUSY-GUT models