

Relic density at one-loop with gauge boson pair production

CHALONS Guillaume

in collaboration with N. Baro, F. Boudjema, Sun Hao
Laboratoire d'Annecy-le-Vieux de Physique THéorique

October 15th, 2009

RELIC DENSITY OF DARK MATTER

- WMAP : $0.098 < \Omega_{DM} h^2 < 0.114$ (10% precision)
- PLANCK : 2% precision

RELIC DENSITY OF DARK MATTER

- WMAP : $0.098 < \Omega_{DM} h^2 < 0.114$ (10% precision)
- PLANCK : 2% precision

POSSIBLE CANDIDATE : NEUTRALINO (SUPERSYMMETRY)

- At tree-level : $m_h < m_Z$ but we never saw the Higgs
- At one loop : the Higgs receives huge corrections
- More generally SUSY processes are known to give large radiative corrections.
- Models for the relic abundance valid at tree-level can be excluded at one-loop and vice-versa.

RELIC DENSITY OF DARK MATTER

- WMAP : $0.098 < \Omega_{DM} h^2 < 0.114$ (10% precision)
- PLANCK : 2% precision

POSSIBLE CANDIDATE : NEUTRALINO (SUPERSYMMETRY)

- At tree-level : $m_h < m_Z$ but we never saw the Higgs
- At one loop : the Higgs receives huge corrections
- More generally SUSY processes are known to give large radiative corrections.
- Models for the relic abundance valid at tree-level can be excluded at one-loop and vice-versa.

⇒ RADIATIVE CORRECTIONS ARE IMPORTANT

RELIC DENSITY IN STANDARD SCENARIO

$$\Omega_{DM} h^2 \simeq \frac{3 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1}}{\langle \sigma(\chi\chi \rightarrow SM)v \rangle}$$

RELIC DENSITY IN STANDARD SCENARIO

$$\Omega_{DM} h^2 \simeq \frac{3 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1}}{\langle \sigma(\chi\chi \rightarrow SM)\nu \rangle}$$

PRECISION

- Need to know precisely $\sigma \Rightarrow$ **one-loop** calculations
- Parameters **reconstruction** at the LHC/LC
- Check the underlying cosmological **scenario**

EW + QCD corrections

- $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow \gamma\gamma, Z\gamma, gg$: Boudjema, Semenov, Temes, *Phys. Rev.* **D72**, 055024 (2005), hep-ph/0507127
- $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow ZZ, W^+W^-$: Baro, Boudjema, Semenov, *Phys. Lett.* **B660** (2008) 550, hep-ph/0710.1821
- $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow \tau^+\tau^-, b\bar{b}$: Baro, Boudjema, Semenov, *Phys. Lett.* **B660** (2008) 550, hep-ph/0710.1821
- **Co-annihilation with $\tilde{\tau}$** : Baro, Boudjema, Semenov, *Phys. Lett.* **B660** (2008) 550, hep-ph/0710.1821

QCD corrections

- **Co-annihilation with $\tilde{\tau}$** Freitas *Phys. Lett.* **B652** (2007) 280
- **Annihilation into massive quarks** Hermann, Klasen, Kovarik *Phys. Rev.* **D79** (2009), hep-ph/0901.0481

PRESENT STUDY : ANNIHILATION INTO FINAL STATES DOMINATED BY GAUGE BOSONS

- Need substantial **higgsino/wino** component
 - **Concomitant** channels : **coannihilation** (with $\tilde{\chi}_1^\pm$), final state into $q\bar{q}'$ also present (interesting for **colliders**).
-
- Calculating the relic density including **coannihilation** effects can significantly change the results.
 - Regions of parameters difficult to probe (in mSUGRA) in colliders are regions where coannihilation comes into account for the relic density.
 - Can push the relic density **in** or **out** the cosmologically interesting region, as one-loop calculations do.

To deal with radiative corrections we need :

To deal with radiative corrections we need :

A coherent **renormalisation scheme** and a choice of **input parameters**

To deal with radiative corrections we need :

A coherent **renormalisation scheme** and a choice of **input parameters**

To generate **counter-terms**, for SUSY **gigantic** task

To deal with radiative corrections we need :

A coherent **renormalisation scheme** and a choice of **input parameters**

To generate **counter-terms**, for SUSY **gigantic** task

Loop Integrals to handle **Gram determinant** when $v \rightarrow 0$

To deal with radiative corrections we need :

A coherent **renormalisation scheme** and a choice of **input parameters**

To generate **counter-terms**, for SUSY **gigantic** task

Loop Integrals to handle **Gram determinant** when $v \rightarrow 0$

To deal with **IR** and **collinear divergencies** \rightarrow include bremsstrahlung.

OVERVIEW

- Mass matrix

$$Y = \begin{pmatrix} M_1 & 0 & -c_\beta s_W M_Z & s_\beta s_W M_Z \\ 0 & M_2 & c_\beta c_W M_Z & -s_\beta c_W M_Z \\ -c_\beta s_W M_Z & c_\beta c_W M_Z & 0 & -\mu \\ s_\beta s_W M_Z & -s_\beta c_W M_Z & -\mu & 0 \end{pmatrix}, X = \begin{pmatrix} M_2 & \sqrt{2}s_\beta M_W \\ \sqrt{2}c_\beta M_W & \mu \end{pmatrix}$$

- Diagonalisation and Decomposition

$$\hookrightarrow \tilde{\chi}_1^0 = N_{11} \tilde{B} + N_{12} \tilde{W} + N_{13} \tilde{H}_1^0 + N_{14} \tilde{H}_2^0$$

- At one-loop : 3 counterterms $(\delta M_1, \delta M_2, \delta \mu) \rightarrow 3$ renormalisation conditions
- Our choice : **ON-SHELL** scheme with 3 masses $m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_1^+}, m_{\tilde{\chi}_2^+}$.
- Other** schemes/definitions possible
- Remaining counter-terms are determined in the gauge and higgs sector $(\alpha(0), M_W, M_Z, \delta t_\beta)$

t_β doesn't represent a **physical** observable \rightarrow **several** definitions :

t_β doesn't represent a **physical** observable \rightarrow **several** definitions :

\overline{DR} : δt_β is a pure divergence

t_β doesn't represent a **physical** observable \rightarrow **several** definitions :

\overline{DR} : δt_β is a pure divergence

$DCPR$: δt_β is defined with $\hat{\Sigma}_{A^0 Z^0}(m_A^2) = 0$

t_β doesn't represent a **physical** observable \rightarrow **several** definitions :

\overline{DR} : δt_β is a pure divergence

$DCPR$: δt_β is defined with $\hat{\Sigma}_{A^0 Z^0}(m_A^2) = 0$

MH : δt_β is defined from the measure of the mass m_H

t_β doesn't represent a **physical** observable \rightarrow **several** definitions :

\overline{DR} : δt_β is a pure divergence

$DCPR$: δt_β is defined with $\hat{\Sigma}_{A^0 Z^0}(m_A^2) = 0$

MH : δt_β is defined from the measure of the mass m_H

$A^0_{\tau\tau}$: δt_β is defined from the decay $A^0 \rightarrow \tau^+ \tau^-$ (vertex $\propto m_\tau t_\beta$)

t_β doesn't represent a **physical** observable \rightarrow **several** definitions :

\overline{DR} : δt_β is a pure divergence

$DCPR$: δt_β is defined with $\hat{\Sigma}_{A^0 Z^0}(m_A^2) = 0$

MH : δt_β is defined from the measure of the mass m_H

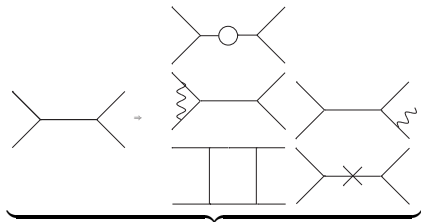
$A^0_{\tau\tau}$: δt_β is defined from the decay $A^0 \rightarrow \tau^+ \tau^-$ (vertex $\propto m_\tau t_\beta$)

Only $MH, A^0_{\tau\tau}$ are gauge independent

FROM TREE TO LOOPS : NEED FOR AUTOMATION

At tree-level we have for process involving W's 7 diagrams

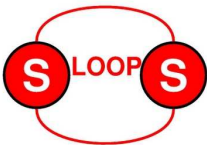
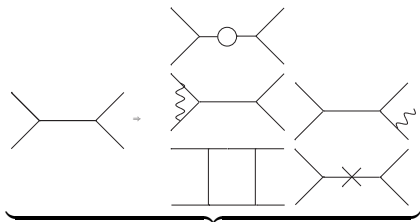
At one-loop we have $\simeq 7000$ diagrams \rightarrow need for automation



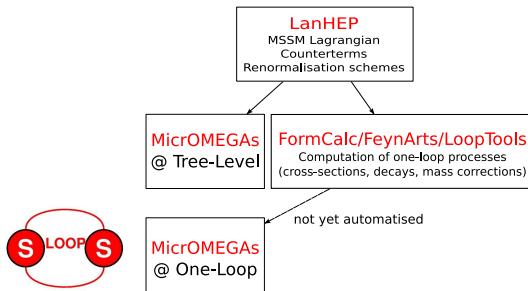
FROM TREE TO LOOPS : NEED FOR AUTOMATION

At tree-level we have for process involving W 's 7 diagrams

At one-loop we have $\simeq 7000$ diagrams \rightarrow need for automation



A code for the calculation of loop diagrams in the MSSM with application to collider physics, astrophysics and cosmology.



- Evaluation of one-loop diagrams including a **complete** and **coherent** renormalisation of **each sector** of the MSSM with an **OS** scheme.
- Modularity between different renormalisation schemes.

PARTICLE PHYSICS (SloopS)

GAUGE BOSON PRODUCTION

- $SU(2)_L$ type couplings
- Channels contributions $> 5\%$ to Ωh^2 at TL corrected at one-loop
- Coannihilation channels

PARTICLE PHYSICS (SloopS)

GAUGE BOSON PRODUCTION

- $SU(2)_L$ type couplings
- Channels contributions $> 5\%$ to Ωh^2 at TL corrected at one-loop
- Coannihilation channels

$\tilde{\chi}_1^0 \tilde{\chi}_1^\pm W^\pm$, $\tilde{\chi}_1^0 \tilde{\chi}_2^0 Z^0$ vertices

1-loop corrections to $\sigma_0 v = a_0 + b_0 v^2$

Also light $q\bar{q}'$ production

PARTICLE PHYSICS (SloopS)

GAUGE BOSON PRODUCTION

- $SU(2)_L$ type couplings
- Channels contributions $> 5\%$ to Ωh^2 at TL corrected at one-loop
- Coannihilation channels

$\tilde{\chi}_1^0 \tilde{\chi}_1^\pm W^\pm$, $\tilde{\chi}_1^0 \tilde{\chi}_2^0 Z^0$ vertices

1-loop corrections to $\sigma_0 v = a_0 + b_0 v^2$

Also light $q\bar{q}'$ production

COSMOLOGY (MicrOmegas)

THERMAL RELIC DENSITY

- Solve the Boltzmann equation
- $\Omega h^2 \simeq \left(\frac{10}{\sqrt{g_*(x_F)}} \frac{x_F}{24} \right) \frac{0.237 \times 10^{-26} \text{ cm}^3 \cdot \text{s}^{-1}}{x_F \text{ J}}$

PARTICLE PHYSICS (SloopS)

GAUGE BOSON PRODUCTION

- $SU(2)_L$ type couplings
- Channels contributions $> 5\%$ to Ωh^2 at TL corrected at one-loop
- Coannihilation channels

$\tilde{\chi}_1^0 \tilde{\chi}_1^\pm W^\pm, \tilde{\chi}_1^0 \tilde{\chi}_2^0 Z^0$ vertices

1-loop corrections to $\sigma_0 v = a_0 + b_0 v^2$

Also light $q\bar{q}'$ production

COSMOLOGY (MicrOmegas)

THERMAL RELIC DENSITY

- Solve the Boltzmann equation
- $\Omega h^2 \simeq \left(\frac{10}{\sqrt{g_*(x_F)}} \frac{x_F}{24} \right) \frac{0.237 \times 10^{-26} \text{ cm}^3 \cdot \text{s}^{-1}}{x_F \boxed{J}}$

$$J = \int_{x_F}^{\infty} \langle \sigma v \rangle_{\text{eff}} dx / x^2$$

$$\langle \sigma v \rangle_{\text{eff}} = \sum_{ij} \frac{g_{i,\text{eff}} g_{j,\text{eff}}}{g_{\text{eff}}^2} \langle \sigma_{ij} v_{ij} \rangle$$

$$g_{j,\text{eff}} = \frac{g_1}{g_{\tilde{\chi}_1^+}} \left(1 + \frac{\delta m}{m_1} \right)^{\frac{3}{2}} e^{-x \frac{\delta m}{m_1}}$$

Parameter	M_1	M_2	μ	t_β	M_3	$M_{\tilde{L}, \tilde{Q}}$	A_i	M_{A^0}
Value	110	134.5	-245	10	600	600	0	600

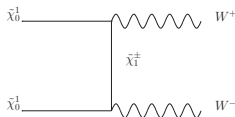
$$\tilde{\chi}_1^0 = 0.94\tilde{B} - 0.20\tilde{W} - 0.27\tilde{H}_1^0 - 0.10\tilde{H}_2^0$$

	Tree-Level	
$\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow W^+ W^-$ [44%]	a	+0.81
	b	+1.219
$\tilde{\chi}_1^0 \tilde{\chi}_1^+ \rightarrow ud/c\bar{s}$ [8%]	a	+15.61
	b	-5.81
$\tilde{\chi}_1^0 \tilde{\chi}_1^+ \rightarrow Z^0 W^+$ [5%]	a	+8.26
	b	+1.42
$\tilde{\chi}_1^0 \tilde{\chi}_2^0 \rightarrow W^+ W^-$ [5%]	a	+17.81
	b	+11.86
Ωh^2		0.108

- $m_{\tilde{\chi}_1^0} = 107$ GeV
- $\delta(m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_1^0}) = 18$ GeV

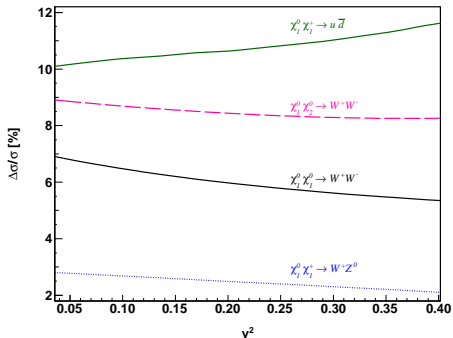
- $\sigma v^2 = a + bv^2$ works well

- Annihilations driven by the s-wave coefficient



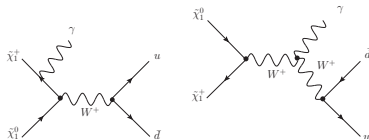
Parameter	M_1	M_2	μ	t_β	M_3	$M_{\tilde{L},\tilde{Q}}$	A_i	M_{A^0}
Value	110	134.5	-245	10	600	600	0	600

$$\tilde{\chi}_1^0 = 0.94\tilde{B} - 0.20\tilde{W} - 0.27\tilde{H}_1^0 - 0.10\tilde{H}_2^0$$



	$A_{\tau\tau}$	$\overline{\text{DR}}$	MH
$\delta\Omega h^2/\Omega h^2$	-2.8%	-5.6%	-10.2%

- Bulk of corrections to the **s-wave coefficient**
- **Large** δt_β scheme dependence
- **Careful treatment** of **hard radiation** during integration of photon momenta \rightarrow Internal W boson **ON-SHELL**
- **QCD corrections** to $u\bar{d} \simeq 2.5\%$



Parameter	M_1	M_2	μ	t_β	M_3	$M_{\tilde{L},\tilde{Q}}$	A_i	M_{A^0}
Value	400	350	-250	4	1000	650	0	800

$$\tilde{\chi}_1^0 = 0.11\tilde{B} - 0.31\tilde{W} - 0.70\tilde{H}_1^0 - 0.63\tilde{H}_2^0$$

	Tree	
$\tilde{\chi}_1^0\tilde{\chi}_1^0 \rightarrow W^+W^-$ [26%]	a	+11.84
	b	+4.17
$\tilde{\chi}_1^0\tilde{\chi}_1^+ \rightarrow u\bar{d}/c\bar{s}$ [12%]	a	+15.28
	b	-5.31
$\tilde{\chi}_1^0\tilde{\chi}_1^0 \rightarrow Z^0Z^0$ [9%]	a	+4.28
	b	+1.83
$\tilde{\chi}_1^0\tilde{\chi}_1^+ \rightarrow Z^0W^+$ [6%]	a	+6.99
	b	-0.51
$\Omega_\chi h^2$	0.00931	

- $m_{\tilde{\chi}_1^0} \simeq m_{\tilde{\chi}_1^+} \simeq |\mu|$

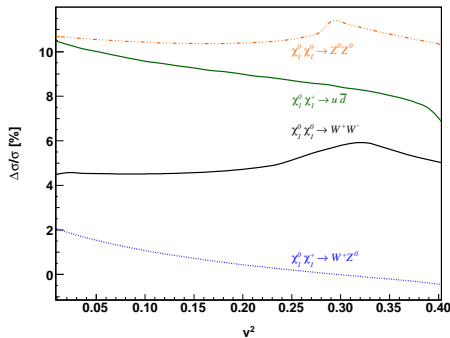
- $m_{\tilde{\chi}_1^0} = 234 \text{ GeV}$

- $\delta(m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_1^0}) = 9 \text{ GeV}$

- Annihilations driven by the **s-wave coefficient** and too efficient because of strong $\tilde{\chi}_1^0\tilde{\chi}_1^\pm W^\pm$, $\tilde{\chi}_1^0\tilde{\chi}_2^0 Z^0$ vertices.

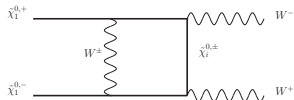
Parameter	M_1	M_2	μ	t_β	M_3	$M_{\tilde{L},\tilde{Q}}$	A_i	M_{A^0}
Value	400	350	-250	4	1000	650	0	800

$$\tilde{\chi}_1^0 = 0.11\tilde{B} - 0.31\tilde{W} - 0.70\tilde{H}_1^0 - 0.63\tilde{H}_2^0$$



	$A_{\tau\tau}$	\overline{DR}	MH
$\delta\Omega h^2 / \Omega h^2$	-2.4%	-2.5%	-3.3%

- Bulk of corrections to the **s-wave coefficient**
- Small δt_β scheme dependence
- QCD corrections to $u\bar{d} \simeq 3\%$
- Bump = $\tilde{\chi}_1^\pm$ threshold in boxes, **not present** at Tree-Level
- $a + bv^2$ expansion **doesn't work** anymore at 1-L



Parameter	M_1	M_2	μ	t_β	M_3	$M_{\tilde{L}, \tilde{Q}}$	A_i	M_{A^0}
Value	565	1000	550	4	1200	1700	0	1350

$$\tilde{\chi}_1^0 = 0.58\tilde{B} - 0.11\tilde{W} + 0.58\tilde{H}_1^0 - 0.56\tilde{H}_2^0$$

	Tree	
$\tilde{\chi}_1^0\tilde{\chi}_1^0 \rightarrow W^+W^-$ [19%]	a	+0.81
	b	+0.28
$\tilde{\chi}_1^0\tilde{\chi}_1^0 \rightarrow Z^0Z^0$ [13%]	a	+0.56
	b	+0.15
$\tilde{\chi}_1^0\tilde{\chi}_1^+ \rightarrow ud/c\bar{s}$ [9%]	a	+1.89
	b	-0.61
$\Omega_\chi h^2$	0.0814	

- $m_{\tilde{\chi}_1^0} \simeq m_{\tilde{\chi}_1^+} \simeq |\mu| \lesssim M_1$

- $m_{\tilde{\chi}_1^0} = 514 \text{ GeV}$

- $\delta(m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_1^0}) = 27 \text{ GeV}$

- Annihilation driven by the \tilde{B}/\tilde{H}_i^0 fraction

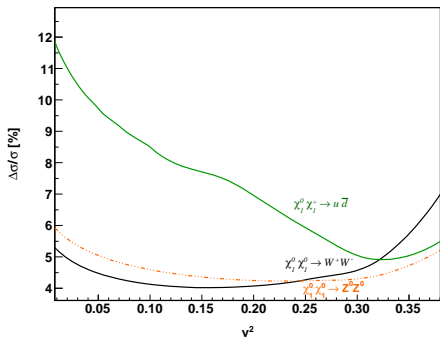
- \tilde{B}/\tilde{H}_i^0 mixing needed to stay within the WMAP range

- Only 50% of the relic density corrected

MIXED HIGGSINO-BINO NEUTRALINO

Parameter	M_1	M_2	μ	t_β	M_3	$M_{\tilde{L}, \tilde{Q}}$	A_i	M_{A^0}
Value	565	1000	550	4	1200	1700	0	1350

$$\tilde{\chi}_1^0 = 0.58\tilde{B} - 0.11\tilde{W} + 0.58\tilde{H}_1^0 - 0.56\tilde{H}_2^0$$



- Bulk of corrections to the **s-wave coefficient**
- Very **small** δt_β scheme dependence
- **QCD corrections** to $u\bar{d} \simeq 5\%$
- Bump = $\tilde{\chi}_1^\pm$ threshold in boxes, **not present** at Tree-Level
- Strong velocity dependence of the corrections due to **box diagrams**

	$A_{\tau\tau}$	\overline{DR}	MH
$\delta\Omega h^2 / \Omega h^2$	-1.2%	-1.0%	-0.5%

Parameter	M_1	M_2	μ	t_β	M_3	$M_{\tilde{u}_L}$	$M_{\tilde{e}_L}$	$M_{\tilde{u}_R, \tilde{e}_R}$	A_i	M_{A^0}
Value	550	210	-600	30	1200	387	360	800	0	700

$$\tilde{\chi}_1^0 = 0.005\tilde{B} - 0.99\tilde{W} - 0.15\tilde{H}_1^0 - 0.05\tilde{H}_2^0$$

	Tree	
$\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow W^+ W^-$ [13%]	a	+161.8
	b	+53.52
$\tilde{\chi}_1^+ \tilde{\chi}_1^+ \rightarrow W^+ W^+$ [12%]	a	+80.75
	b	+26.83
$\tilde{\chi}_1^0 \tilde{\chi}_1^+ \rightarrow Z^0 W^+$ [12%]	a	+37.50
	b	+10.15
$\tilde{\chi}_1^0 \tilde{\chi}_1^+ \rightarrow u\bar{d}/c\bar{s}$ [7%]	a	+24.44
	b	-12.62
$\tilde{\chi}_1^+ \tilde{\chi}_1^- \rightarrow Z^0 Z^0$ [7%]	a	+47.08
	b	+17.71
$\tilde{\chi}_1^+ \tilde{\chi}_1^- \rightarrow W^+ W^-$ [7%]	a	+46.49
	b	+14.01
$\Omega_\chi h^2$		0.00215

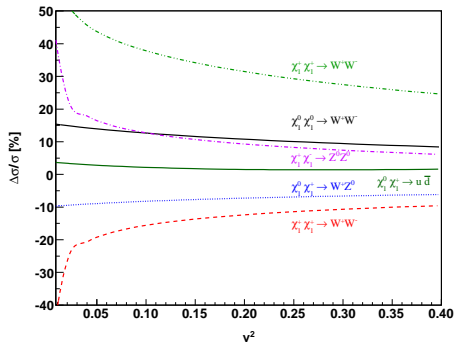
- $m_{\tilde{\chi}_1^0} = 206.6$ GeV
- $\delta(m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_1^0}) = 0.05$ GeV

- $m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_1^\pm}$ almost **degenerate**
- Coannihilation very important
- Degeneracy** between processes $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow W^+ W^-$ and $\tilde{\chi}_1^+ \tilde{\chi}_1^+ \rightarrow W^+ W^+$
- Relic density too low \rightarrow **non-thermal scenarios**
- Claim that a wino could explain positron excess (PAMELA)

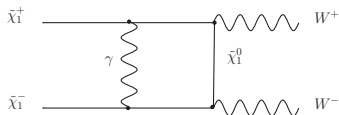
LIGHT-WINO NEUTRALINO

Parameter	M_1	M_2	μ	t_β	M_3	$M_{\tilde{u}_L}$	$M_{\tilde{e}_L}$	$M_{\tilde{u}_R, \tilde{e}_R}$	A_i	M_{A^0}
Value	550	210	-600	30	1200	387	360	800	0	700

$$\tilde{\chi}_1^0 = 0.005\tilde{B} - 0.99\tilde{W} - 0.15\tilde{H}_1^0 - 0.05\tilde{H}_2^0$$



- At 1-L new feature appear for $v \rightarrow 0$: **Coulomb effect**
- Possible to capture its **one-loop manifestation**
- Degeneracy **lifted** between processes
- Large** corrections
- Almost **no** δt_β scheme dependence
- Strong** cancellations between QCD/EW corrections

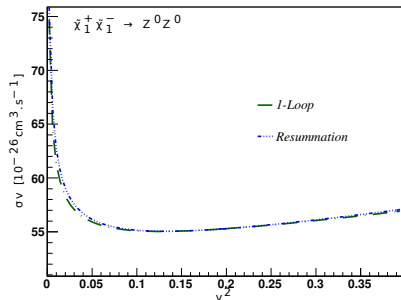
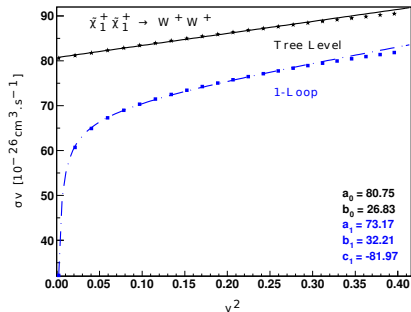


	$A_{\tau\tau}$	$\overline{\text{DR}}$	MH
$\delta\Omega h^2 / \Omega h^2$	-1.9%	-1.9%	-1.9%

LIGHT-WINO NEUTRALINO : ONE-LOOP vs RESUMMATION

Parameter	M_1	M_2	μ	t_β	M_3	$M_{\tilde{u}_L}$	$M_{\tilde{e}_L}$	$M_{\tilde{u}_R, \tilde{e}_R}$	A_i	M_{A^0}
Value	550	210	-600	30	1200	387	360	800	0	700

$$\tilde{\chi}_1^0 = 0.005\tilde{B} - 0.99\tilde{W} - 0.15\tilde{H}_1^0 - 0.05\tilde{H}_2^0$$

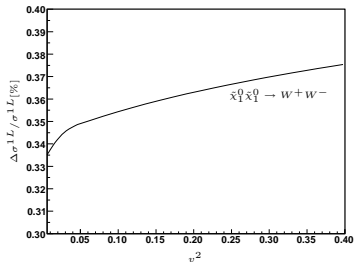


$$\sigma_{1\nu} = a_1 + b_1 v^2 - \pi \alpha a_0 Q_i Q_j / v$$

$$S_{nr} = X_{nr} / (1 - e^{-X_{nr}}) \quad X_{nr} = -2\pi \alpha Q_i Q_j / v$$

- Complete renormalisation of the MSSM and modularity between different schemes
- Investigation of scenarios where gauge boson pair production dominant
- A lot of processes to correct to have complete $\mathcal{O}(\alpha)$ corrections
- For light-wino case no need for resummation
- Future study of a scenario with a heavy wino \rightarrow QED+EW Sommerfeld enhancement
- Influence of the gaugino scheme

Comparison between two chargino/neutralino schemes $(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm})$ and $(m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm})$



Masses [GeV]		$m_{\tilde{\chi}_2^0}$	$m_{\tilde{\chi}_3^0}$	$m_{\tilde{\chi}_4^0}$
Tree Level		125.3	258.1	270.4
One Loop	- $A_{\tau\tau}$ scheme	125.13	258.58	270.42
	- MH scheme	125.31	258.05	270.65
	- \overline{DR} scheme	125.17	258.46	270.47