Revisiting RGEs for general gauge theories

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Assumption of **a diagonal wave-function renormalization**

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We have studied both problems, corrected the expressions and provided detailed explanations.

I. Schienbein, F. Staub, T. Steudtner and K. S., Nuclear Physics B 939 (2019) 1–48 [arXiv:1809.06797 [hep-ph]]

RGEs in a general gauge theory

- ***Lagrangian depends on couplings***
- ***After renormalization, these couplings depend on the energy scale (running parameters)***
- ***This dependence is described by the β-function of the coupling***

The β-function of x_k:

$$\mu \frac{d x_k}{d \mu} \equiv \beta_{x_k}$$

– in $\overline{MS}$ scheme
(dimensional regularization with modified minimal subtraction)

μ - is an arbitrary mass scale parameter

RGEs in a general gauge theory

The Lagrangian for a general renormalizable gauge theory:

Gauge fields

$V_\mu^A(x)\ \ (A = 1,\ldots d)$

of a compact simple group $\boldsymbol{G}$ of dim. $\boldsymbol{d}$.

Real scalar fields

$\phi_a(x)\ \ (a = 1,\ldots N_\phi)$

transform under a reducible rep. of $\boldsymbol{G}$ with generators Θ_{ab}^A

Complex fermion fields

$\psi_j(x)\ \ (j = 1,\ldots N_\psi)$

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$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + (\text{gauge fixing} + \text{ghost terms})\,,$$

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$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + (\text{gauge fixing} + \text{ghost terms})\,,$$

where

$$\mathcal{L}_0 = -\frac{1}{4}F_A^{\mu\nu}F_{\mu\nu}^A + \frac{1}{2}D^\mu\phi_a D_\mu\phi_a + i\psi_j^\dagger\sigma^\mu D_\mu\psi_j - \frac{1}{2}\left(Y_{jk}^a\psi_j\zeta\psi_k\phi_a + Y_{jk}^{a*}\psi_j^\dagger\zeta\psi_k^\dagger\phi_a\right) - \frac{1}{4!}\lambda_{abcd}\phi_a\phi_b\phi_c\phi_d\,,$$

– contains no dimensional parameters

and

$$\mathcal{L}_1 = -\frac{1}{2}\left[(m_f)_{jk}\psi_j\zeta\psi_k + (m_f)_{jk}^*\psi_j^\dagger\zeta\psi_k^\dagger\right] - \frac{m_{ab}^2}{2!}\phi_a\phi_b - \frac{h_{abc}}{3!}\phi_a\phi_b\phi_c\,.$$

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– includes all terms with dimensional parameters.

The dummy field method [1]

The idea: **we introduced a scalar "dummy field" – non-propagating, with no gauge interactions, and rewrote the dimensionless part of the Lagrangian**

$$D_\mu \phi_{\hat{d}} = 0$$

$$\mathcal{L}_0 \subset -\frac{1}{2}\Big(\overset{\text{Yukawa coupling}}{\boxed{Y^{\hat{d}}_{jk}}}\psi_j\zeta\psi_k \underbrace{\phi_{\hat{d}}}_{\text{a dummy field}} + \text{h.c.}\Big) - 6\sum_{a,b=1}^{N_\phi}\frac{1}{4!}\overset{\text{Quartic coupling}}{\boxed{\lambda_{ab\hat{d}\hat{d}}}}\phi_a\phi_b\underbrace{\phi_{\hat{d}}\phi_{\hat{d}}}_{\text{dummy fields}} - 4\sum_{a,b,c=1}^{N_\phi}\frac{1}{4!}\overset{\text{Quartic coupling}}{\boxed{\lambda_{abc\hat{d}}}}\phi_a\phi_b\phi_c\underbrace{\phi_{\hat{d}}}_{\text{a dummy field}}$$

$$Y^{\hat{d}}_{jk}\phi_{\hat{d}} = (m_f)_{jk}$$

$$\lambda_{ab\hat{d}\hat{d}}\phi_{\hat{d}}\phi_{\hat{d}} = 2m^2_{ab}$$

$$\lambda_{abc\hat{d}}\phi_{\hat{d}} = h_{abc}$$

$$\mathcal{L}_1 = -\frac{1}{2}\left[\overset{\text{Fermion mass}}{\boxed{(m_f)_{jk}}}\psi_j\zeta\psi_k + (m_f)^*_{jk}\psi^\dagger_j\zeta\psi^\dagger_k\right]$$

$$-\frac{\overset{\text{Scalar mass}}{\boxed{m^2_{ab}}}}{2!}\phi_a\phi_b \qquad -\frac{\overset{\text{Trilinear coupling}}{\boxed{h_{abc}}}}{3!}\phi_a\phi_b\phi_c$$

[1] – *the idea, to our knowledge, was first mentioned by S.P. Martin and M.T. Vaughn, in "Two loop renormalization group equations for soft supersymmetry breaking couplings", Phys. Rev. D 50 (1994) 2282, arXiv: hep-ph/9311340*

The dummy field method

Example. **The β-function of the fermion mass term can be obtained from the expressions for the Yukawa couplings, using the following mappings**

$$a \to \hat{d}\,,\ Y^a \to Y^{\hat{d}} \to m_f\,,\ Y^{\dagger a} \to Y^{\dagger \hat{d}} \to m_f^\dagger\,,\ \lambda_{abcd} \to \lambda_{\hat{d}bcd} \to h_{bcd}$$

1-loop β-function for the Yukawa couplings:

$$\beta_a^I = \frac{1}{2}\left[Y_2^{+}(F)Y^a + Y^a Y_2(F)\right] + 2Y^b Y^{+a} Y^b + 2\kappa Y^b Y_2^{ab}(S) - 3g^2\{C_2(F), Y^a\}\,,$$

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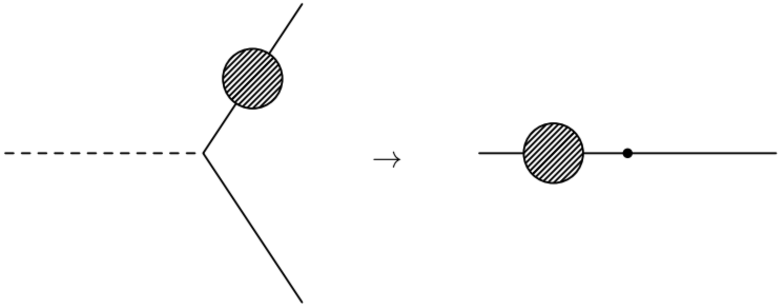

$$Y_2^{\dagger}(F)Y^a + Y^a Y_2(F) \to Y_2^{\dagger}(F)m_f + m_f Y_2(F)$$

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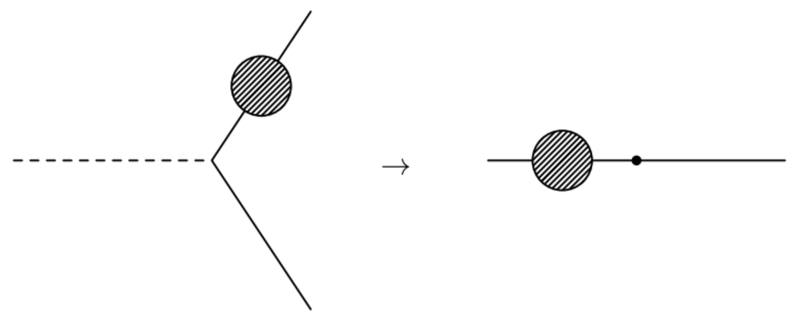

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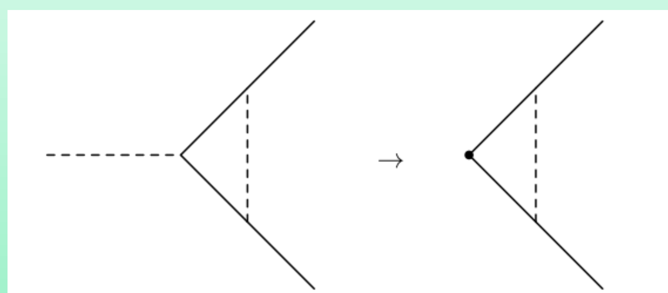

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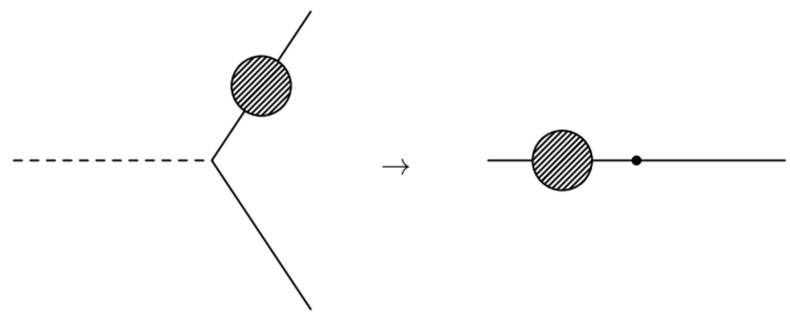

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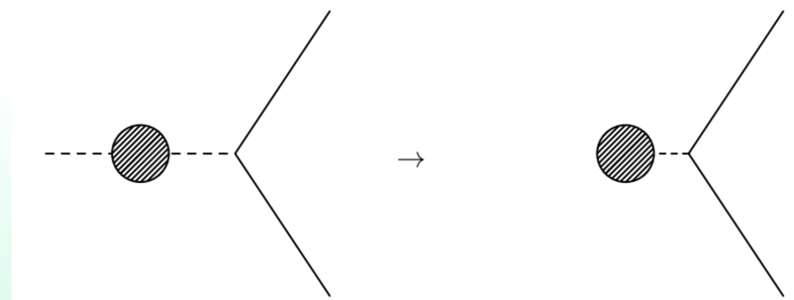
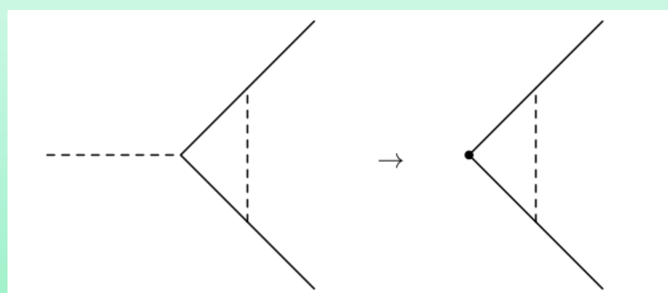


$$Y^b Y_2^{ab}(S) \quad \to \quad 0$$



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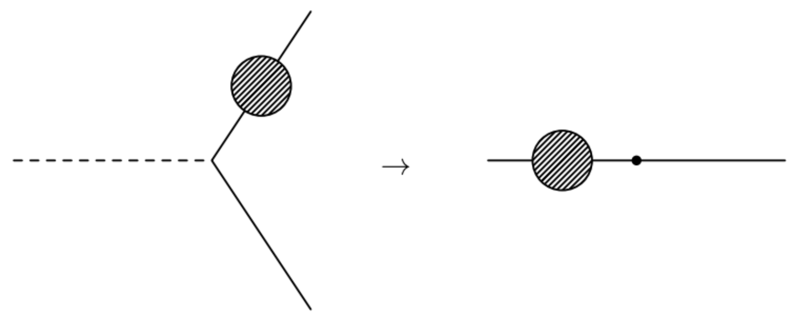

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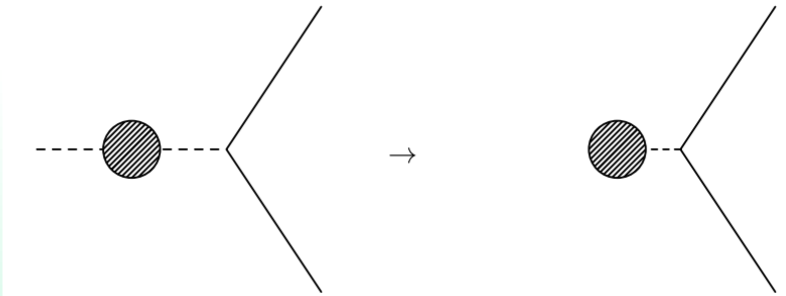
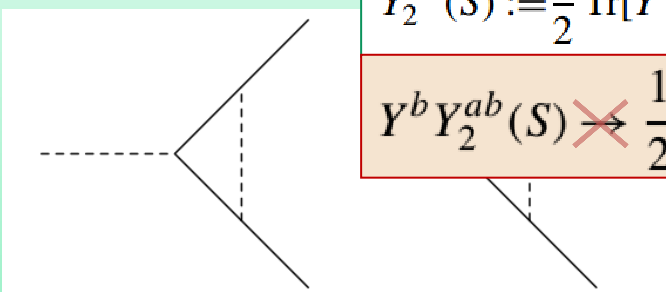


$$Y^b Y_2^{ab}(S) \quad \to \quad 0$$

$$Y_2^{ab}(S) := \frac{1}{2}\mathrm{Tr}[Y^{\dagger a}Y^b + Y^{\dagger b}Y^a]$$

$$Y^b Y_2^{ab}(S) \not\to \frac{1}{2}Y^b\,\mathrm{Tr}[m_f^\dagger Y^b + Y^{\dagger b} m_f]$$



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In this manner, the β-functions for the following parameters have been obtained:

Fermion mass:	$\beta_{m_f}^{1-loop}, \beta_{m_f}^{2-loop}$	**out of**	$\beta_a^{1-loop}, \beta_a^{2-loop}$	*(Yukawa c.)*
Trilinear sc.c.:	$\beta_{h_{abc}}^{1-loop}, \beta_{h_{abc}}^{2-loop}$	**out of**	$\beta_{\lambda_{abcd}}^{1-loop}, \beta_{\lambda_{abcd}}^{2-loop}$	*(quartic sc.c.)*
Scalar mass sq.:	$\beta_{m_{ab}^2}^{1-loop}, \beta_{m_{ab}^2}^{2-loop}$			

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We've reconsidered <u>diagrammatically</u> and corrected

The dummy field method (summarized)

The dummy field method allows to derive the β-functions for <u>dimensionful</u> parameters out of those for the <u>dimensionless</u> parameters

1. **Consider the Lagrangian in the presence of the same particle content + 1 extra scalar dummy field**
2. **Write down the β-functions for the dimensionless parameters**
3. **Substitute:** $Y^{\hat{d}}_{jk} = (m_f)_{jk}\,, \quad \lambda_{ab\hat{d}\hat{d}} = 2m^2_{ab}\,, \quad \lambda_{abc\hat{d}} = h_{abc}$
4. **Keep in mind that the dummy field – is a real scalar, non-propagating, with no gauge interactions, i.e.**
   - ***Expressions with 2 identical internal indices (≡ a propagating dummy field) <u>must vanish</u>***
   - ***Vertices <gauge boson-dummy scalar> <u>must vanish</u>***
   - ***<u>Tadpole diagrams</u> (if appear) must be also <u>dropped out</u>***
5. **Result: the β-functions for dimensionful parameters**

Numerical impact (I)

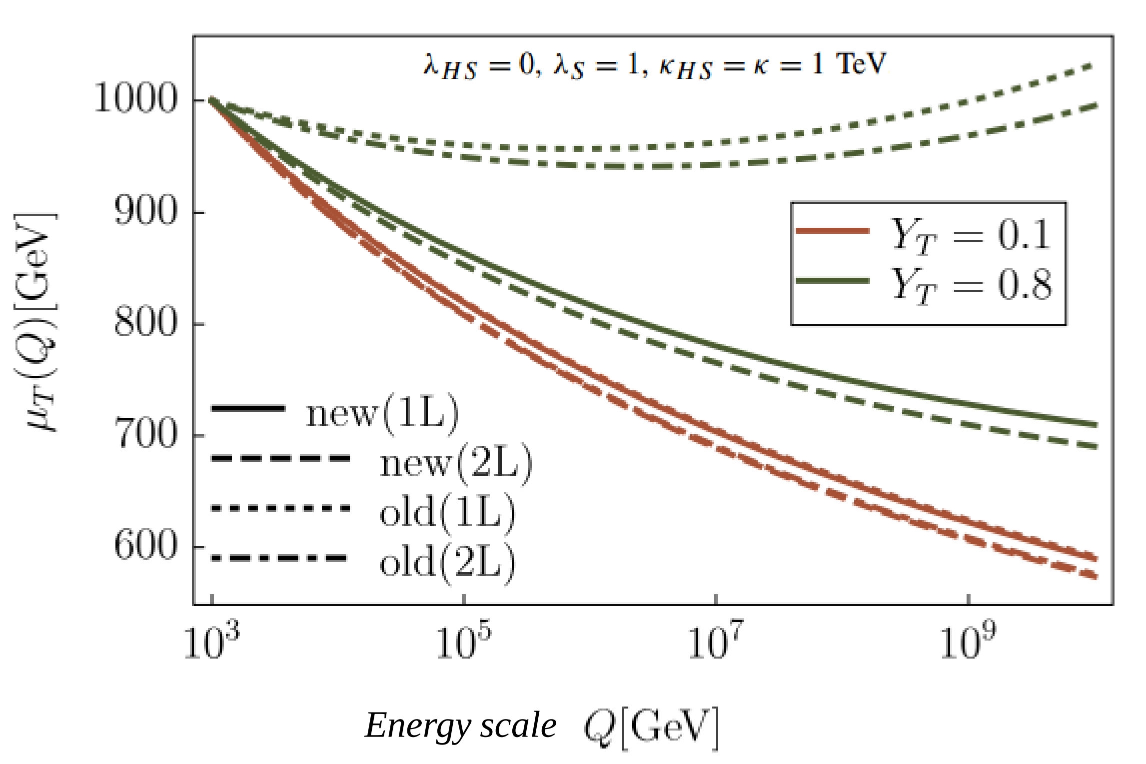
Running of fermion mass terms

$$\mathcal{L} \supset Y S f_1 f_2 + \mu f_1 f_2 + \text{h.c.}$$

For example: two heavy top-like states and a real singlet

$$T' : \quad (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}} \,, \qquad \bar{T}' : \quad (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} \,, \qquad S : \quad (\mathbf{1}, \mathbf{1})_0 \,,$$

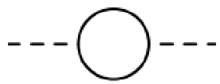
$$V = V_{SM} + \frac{1}{4}\lambda_S S^4 + \frac{1}{2}\lambda_{SH}|H|^2 S^2 + \kappa_{SH}|H|^2 S + \frac{1}{3}\kappa S^3 + \frac{1}{2}m_S^2 S^2 + \left(Y_T S \bar{T}' T' + \mu_T \bar{T}' T' + \text{h.c.}\right) .$$



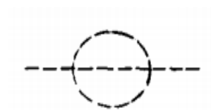
The running mass μ_T of the vector-like top partners at one- and two-loop level for two different choices of the Yukawa coupling Y_T

The discrepancy between the old and new results rapidly grows with increasing Y_T

Off-diagonal wave function renormalization



$$Y_2^{ab}(S) := \frac{1}{2}\,\mathrm{Tr}[Y^{\dagger a} Y^b + Y^{\dagger b} Y^a]\,,$$



$$\Lambda^2_{ab}(S) := \frac{1}{6} \sum_{c,d,e=1}^{N_\phi} \lambda_{acde}\lambda_{bcde}\,,$$

The assumption that

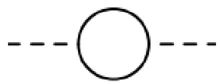
$Y_2^{ab}(S) = Y_2(S)\delta_{ab}$ **and** $\Lambda^2_{ab}(S) = \Lambda^2(S)\delta_{ab}$

is reasonable only if the considered model <u>does not contain several scalar particles with identical quantum numbers</u>

thus, in general, contributions from off-diagonal wave-function corrections must be included

(affects the results for the dimensionless parameters (the quartic scalar couplings), and ⇨ the trilinear coupling, the scalar mass)

Off-diagonal wave function renormalization



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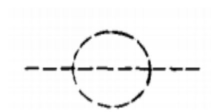

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thus, in general, contributions from off-diagonal wave-function corrections must be included

Corrected

(affects the results for the dimensionless parameters (the quartic scalar couplings), and $\Rightarrow$ the trilinear coupling, the scalar mass)

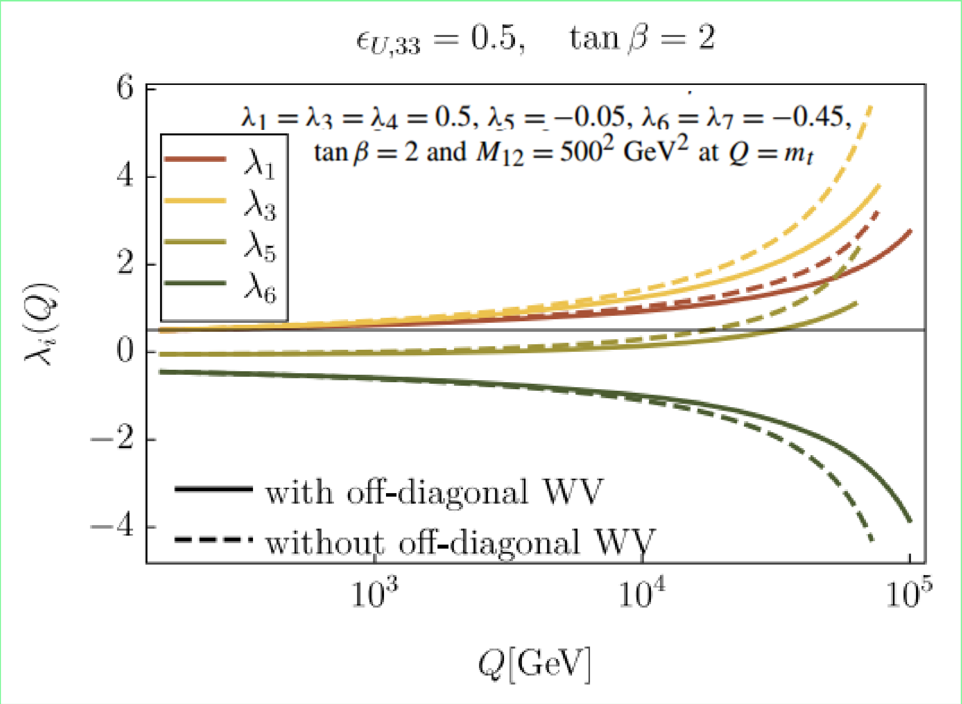
Numerical impact (II)

Example:
The general Two-Higgs-Doublet-Model type-III

$$V = m_1^2|H_1|^2 + m_2^2|H_2|^2 + \lambda_1|H_1|^4 + \lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 + \lambda_4|H_2^\dagger H_1|^2$$
$$+ \left(\frac{1}{2}\lambda_5(H_2^\dagger H_1) + \lambda_6|H_1|^2(H_1^\dagger H_2) + \lambda_7|H_2|^2(H_1^\dagger H_2) - M_{12}H_1^\dagger H_2 + \text{h.c.}\right)$$

$$\mathcal{L}_Y = -\left(Y_d H_1^\dagger dq + Y_e H_1^\dagger el - Y_u H_2 uq + \epsilon_d H_2^\dagger dq + \epsilon_e H_2^\dagger el - \epsilon_u H_1 uq + \text{h.c.}\right)$$

The additional one-loop contributions on the running of the quartic couplings lead to sizeable differences already for $\epsilon_{U,33}$= 0.5 and small tan $\beta = 2$



The running of different quartic couplings in the THDM-III with and without the contributions of off-diagonal wave-function renormalisation

Conclusions

- **We identified various mistakes in the literature for the β-functions of both dimensionless and dimensionful Lagrangian parameters**

- **The sources for these discrepancies: incorrect dummy field method application and assumption of a diagonal wave-function renormalization**

- **We obtained the correct expressions, cross-checked them and estimated the changes numerically**

- **We provided a detailed pedagogic discussion (of the dummy field method, in particular) and summarized all the correct expressions for the β-functions in one paper**

  I. Schienbein, F. Staub, T. Steudtner and K. S., Nuclear Physics B 939 (2019) 1–48 [arXiv:1809.06797 [hep-ph]]

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Thanks for your attention!