The effective action for gauge bosons

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*J. Quevillon, C. Smith and **S. Touati**, Phys.Rev. D99 (2019) no.1, 013003*

Overview

- Introduction: The Euler-Heisenberg Lagrangian
- Photon effective interactions
- Gluon effective interactions
- SU(N) effective interactions
- Mixed effective interactions
- Conclusion

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The Euler-Heisenberg Lagrangian

❖ Maxwell Theory:

Classical Lagrangian → Maxwell's equations in vacuum $\xrightarrow[\text{principle}]{\text{Superposition}}$ No interactions between electromagnetic waves

$$\mathcal{L}_{Maxwell} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

❖ Dirac's Theory:

e^-e^+ pairs created in the vacuum → Electrodynamics becomes **non-linear** even in vacuum → 2 waves can interact indirectly

« Consequences of Dirac's Theory of the Positron », W. Heisenberg & H. Euler (1936)

❖ Estimation of non-linear interactions among photons induced by an electron loop in a constant electromagnetic field

Effective Lagrangian:

$$\mathcal{L}_{\text{EH}} = -\mathcal{F} + \frac{8}{45}\left(\frac{\alpha^2}{m_e^4}\right)\mathcal{F}^2 + \frac{14}{45}\left(\frac{\alpha^2}{m_e^4}\right)\mathcal{G}^2$$

Dim-8 operators

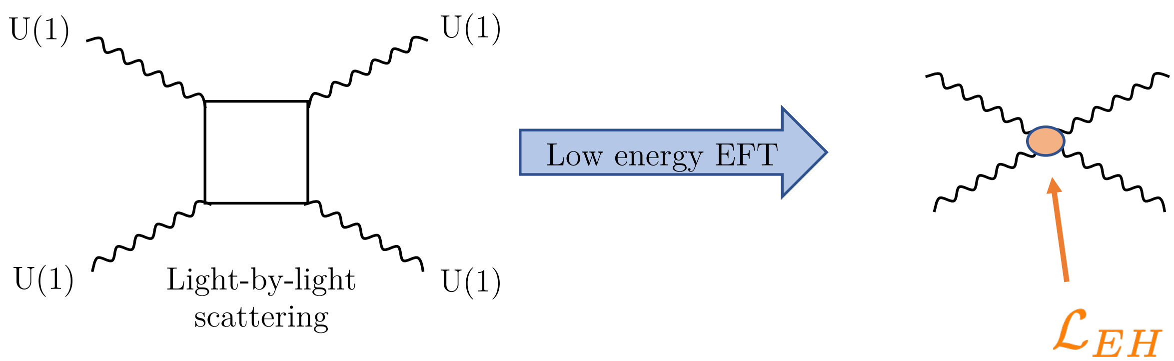
Maxwell + Corrections

where $\mathcal{F} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{2}(\mathbf{B}^2 - \mathbf{E}^2)$ and $\mathcal{G} = \frac{1}{8} \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} F^{\lambda\rho} = \mathbf{E} \cdot \mathbf{B}$

The Euler-Heisenberg Lagrangian

❖ In modern language → archetype of an Effective Field Theory (EFT)

Diagrammatic approach:



Purpose of the paper:

- **Generalize** the Euler-Heisenberg result for photons to the **gauge bosons** of an **arbitrary gauge group**
- Effective interactions induced by loops of heavy fields in **generic representations** and of **spin 0, ½ or 1**

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- ❖ Introduction: The Euler-Heisenberg Lagrangian
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Photon effective interactions

❖ QED generating functional:

$$Z_{QED}\left[J^{\mu},\eta,\overline{\eta}\right]=\int DA^{\mu}D\psi D\overline{\psi}\ \exp i\int dx(\mathcal{L}_{QED}+\overline{\eta}\psi+\overline{\psi}\eta+J^{\mu}A_{\mu})\ ,$$

❖ Integrate out the fermion field

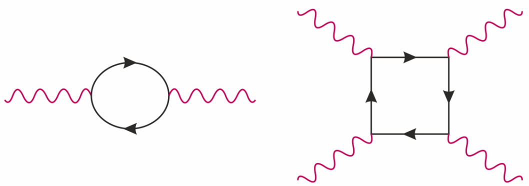
$$\begin{aligned}Z_{QED}\left[J^{\mu},0,0\right]&=\int DA^{\mu}\ \exp i\int dx\left\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+J^{\mu}A_{\mu}\right\}\times\det(i\not{D}-m)\\&\equiv\int DA^{\mu}\ \exp i\int dx(\mathcal{L}_{eff}+J^{\mu}A_{\mu})\end{aligned}$$

→ QED effective Lagrangian: $\mathcal{L}_{eff}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-iTr\ \ln(i\not{D}-m)$

Perturbative 1/m expansion:

$$\mathcal{L}_{eff}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+i\underbrace{\sum_{n=1}^{\infty}\frac{e^{n}}{n}Tr\left(\frac{1}{i\not{\partial}-m}\not{A}\right)^{n}}_{\text{Tower of one-loop 1PI diagrams}}$$

Up to dim-8 operators:



(a) (b)

Photon effective interactions

❖ Most general basis up to $\mathcal{O}(m^{-4})$:

$$\mathfrak{L}_{eff} = -\frac{1}{4}\left\{1+\alpha_0\frac{e^2}{4!\pi^2}\right\}F_{\mu\nu}F^{\mu\nu} + \alpha_2\frac{e^2}{5!\pi^2 m^2}\partial^\mu F_{\mu\nu}\partial_\rho F^{\rho\nu} + \alpha_4\frac{e^2}{6!\pi^2 m^4}\partial^\mu F_{\mu\nu}\Box\partial_\rho F^{\rho\nu}$$
$$+\gamma_{4,1}\frac{e^4}{6!\pi^2 m^4}(F_{\mu\nu}F^{\mu\nu})^2 + \gamma_{4,2}\frac{e^4}{6!\pi^2 m^4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2 + \mathcal{O}(m^{-6})$$

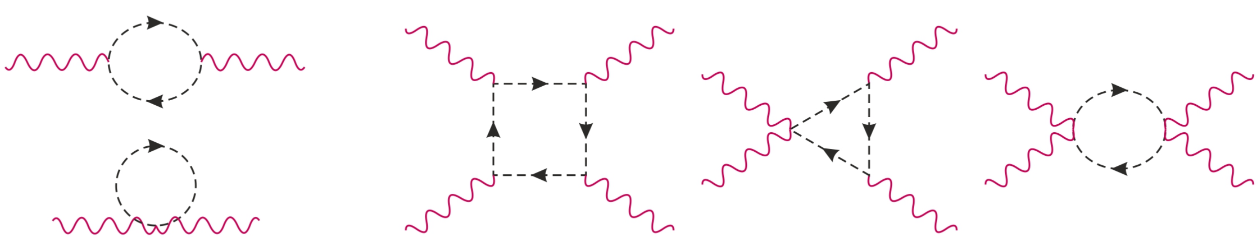
❖ Matching this effective Lagrangian to QED:

$$\boxed{\mathcal{A}_{UV}\big|_{m\to\infty} \overset{!}{=} \mathcal{A}_{EFT}\ (\text{Wilson Coeffs})}$$

What if a scalar or a vector circulates in the loop?

❖ Scalar case:

Scalar one-loop 1PI amplitudes generating the QED effective action up to dim-8 operators



Photon effective interactions

❖ Vector case: Integrating out vectors is far more challenging!

→ **Problem**: the gauge fixing procedure

- In 't Hooft Feynman gauge: Violates QED Ward identities when photons are off-shell!

  4-photons amplitude matches onto effective operators **only for on-shell photons**

  → usual procedure to construct the effective action breaks down

- In linear R_ξ gauge: $\mathfrak{L}^{R_\xi, linear}_{gauge-fixing} = -\frac{1}{\xi}|\partial^\mu W^+_\mu + \xi M_W \phi^+|^2$ ⟶ Explicitly breaks $U(1)_{QED}$

- In unitary gauge:

$$\mathcal{L}^{Unitary}_{gauge-fixing} = -\frac{1}{2}(D_\mu W^+_\nu - D_\nu W^+_\nu)(D^\mu W^{-\nu} - D^\nu W^{-\mu}) + ieF^{\mu\nu}W^+_\mu W^-_\nu + M_W^2 W^+_\mu W^{-\mu}$$

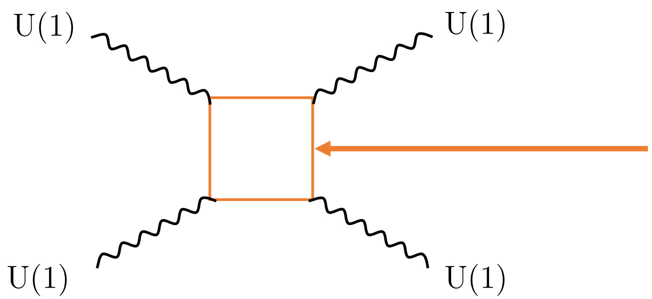
⟶ Matching fails again

- Solution ⟶ Use a non-linear gauge *F. Boudjema, Phys. Lett. B* ***187*** *(1987) 362*

$$\partial^\mu \to D^\mu$$

$$\boxed{\mathfrak{L}^{non-linear}_{gauge-fixing} = -\frac{1}{\xi}|(\partial^\mu + i\kappa e A^\mu)W^+_\mu + \xi M_W \phi^+|^2}$$

Photon effective interactions



Either spin 0, ½ or 1 and electric charge Q circulating in the loop

❖ Effective Lagrangian:

$$\mathfrak{L}_{eff} = -\frac{1}{4}\left\{1+\alpha_0\frac{e^2}{4!\pi^2}\right\}F_{\mu\nu}F^{\mu\nu} + \alpha_2\frac{e^2}{5!\pi^2 m^2}\partial^\mu F_{\mu\nu}\partial_\rho F^{\rho\nu} + \alpha_4\frac{e^2}{6!\pi^2 m^4}\partial^\mu F_{\mu\nu}\Box\partial_\rho F^{\rho\nu}$$
$$+\boxed{\gamma_{4,1}}\frac{e^4}{6!\pi^2 m^4}(F_{\mu\nu}F^{\mu\nu})^2 + \boxed{\gamma_{4,2}}\frac{e^4}{6!\pi^2 m^4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2 + \mathcal{O}(m^{-6})$$

❖ Matching QED at low energy to the EFT for each case:

	α_0	α_2	α_4	$\gamma_{4,1}$	$\gamma_{4,2}$
Scalar	$\frac{1}{2}D_\varepsilon Q^2$	$-\frac{1}{8}Q^2$	$\frac{3}{56}Q^2$	$\frac{7}{32}Q^4$	$\frac{1}{32}Q^4$
Fermion	$2D_\varepsilon Q^2$	$-Q^2$	$\frac{9}{14}Q^2$	$\frac{1}{2}Q^4$	$\frac{7}{8}Q^4$
Vector	$-\frac{21D_\varepsilon+2}{2}Q^2$	$\frac{37}{8}Q^2$	$-\frac{159}{56}Q^2$	$\frac{261}{32}Q^4$	$\frac{243}{32}Q^4$

Euler-Heisenberg

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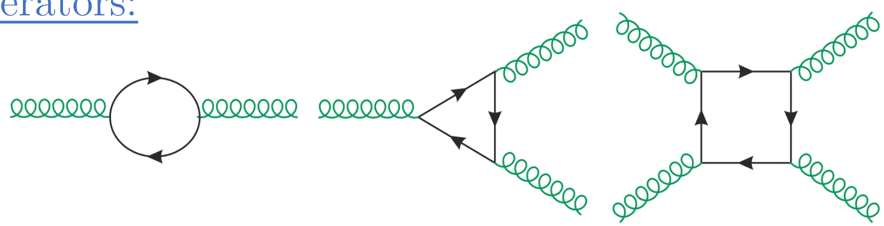
Gluon effective interactions

❖ Construct the EFT by integrating out a heavy fermion:

$$\begin{aligned}\mathcal{L}_{eff} &= -\frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu} - iTr\ \ln(i\not{D} - m) \\ &= -\frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu} + i\underbrace{\sum_{n=1}^{\infty}\frac{e^n}{n}Tr\left(\frac{1}{i\not{\partial} - m}\not{G}^a T^a\right)^n}_{\text{Tower of one-loop 1PI diagrams}}\end{aligned}$$

Up to dim-8 operators:



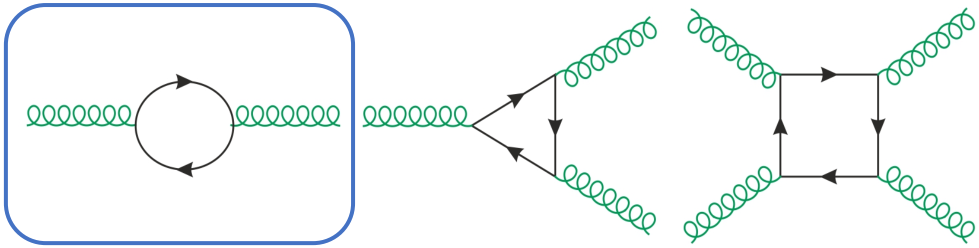
Do not vanish

- Non-abelian nature of QCD → basis quite different from the QED case

$$D_\rho G^a_{\mu\nu} = (\partial_\rho \delta^{ac} + g f^{abc} G^b_\rho) G^c_{\mu\nu}$$

- It blurs the relationship between **number of gluon fields** and **inverse mass dimension**
- **3-gluons** amplitude forbidden kinematically → necessarily **off-shell**
- **3-gluons** and 4-gluons divergences renormalize the same operator

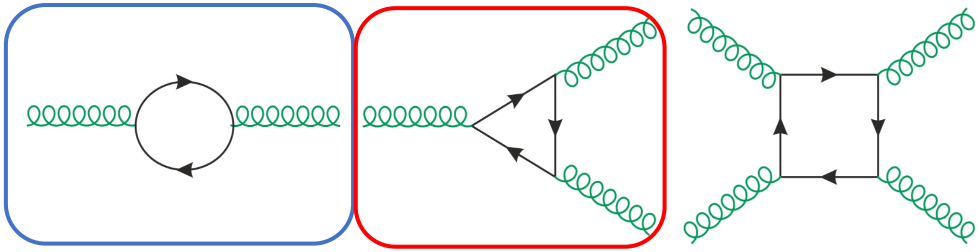
Gluon effective interactions



❖ Effective Lagrangian:

$$\mathfrak{L}_{eff}^{(0+2)} = -\frac{1}{4}\left\{1+\alpha_0\frac{g_S^2}{4!\pi^2}\right\}G^a_{\mu\nu}G^{a,\mu\nu}$$
$$+\alpha_2\frac{g_S^2}{5!\pi^2 m^2}D^\nu G^a_{\nu\mu}D_\rho G^{a,\rho\mu}+\alpha_4\frac{g_S^2}{6!\pi^2 m^4}D^\nu G^a_{\nu\mu}D^2 D_\rho G^{a,\rho\mu}$$

Gluon effective interactions

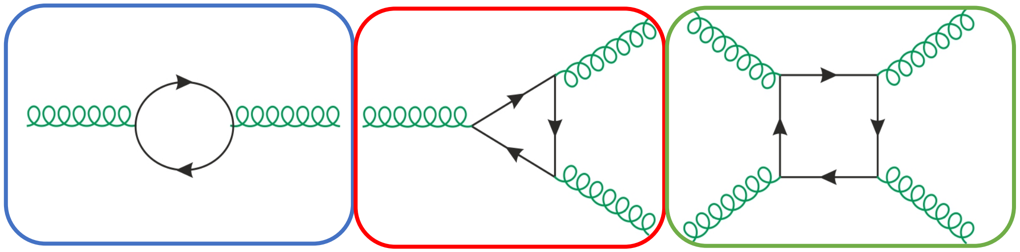


❖ Effective Lagrangian:

$$\mathfrak{L}_{eff}^{(0+2)} = -\frac{1}{4}\left\{1+\alpha_0\frac{g_S^2}{4!\pi^2}\right\}G^a_{\mu\nu}G^{a,\mu\nu} + \alpha_2\frac{g_S^2}{5!\pi^2 m^2}D^\nu G^a_{\nu\mu}D_\rho G^{a,\rho\mu} + \alpha_4\frac{g_S^2}{6!\pi^2 m^4}D^\nu G^a_{\nu\mu}D^2 D_\rho G^{a,\rho\mu}$$

$$\mathfrak{L}_{eff}^{(3)} = \beta_2\frac{g_S^3}{5!\pi^2 m^2}f^{abc}G^{a\ \nu}_{\mu}G^{b\ \rho}_{\nu}G^{c\ \mu}_{\rho} + \beta_{4,1}\frac{g_S^3}{6!\pi^2 m^4}f^{abc}G^{a,\mu\nu}D^\alpha G^b_{\mu\nu}D^\beta G^c_{\alpha\beta} + \beta_{4,2}\frac{g_S^3}{6!\pi^2 m^4}f^{abc}G^{a,\mu\nu}D^\alpha G^b_{\alpha\mu}D^\beta G^c_{\beta\nu}$$

Gluon effective interactions



❖ Effective Lagrangian:

No E.O.M used to reduce the basis

$$\mathfrak{L}_{eff}^{(0+2)} = -\frac{1}{4}\left\{1+\alpha_0\frac{g_S^2}{4!\pi^2}\right\}G^a_{\mu\nu}G^{a,\mu\nu}$$
$$+\alpha_2\frac{g_S^2}{5!\pi^2m^2}D^\nu G^a_{\nu\mu}D_\rho G^{a,\rho\mu}+\alpha_4\frac{g_S^2}{6!\pi^2m^4}D^\nu G^a_{\nu\mu}D^2D_\rho G^{a,\rho\mu}$$

$$\mathfrak{L}_{eff}^{(3)} = \beta_2\frac{g_S^3}{5!\pi^2m^2}f^{abc}G^{a\ \nu}_{\mu}G^{b\ \rho}_{\nu}G^{c\ \mu}_{\rho}$$
$$+\beta_{4,1}\frac{g_S^3}{6!\pi^2m^4}f^{abc}G^{a,\mu\nu}D^\alpha G^b_{\mu\nu}D^\beta G^c_{\alpha\beta}+\beta_{4,2}\frac{g_S^3}{6!\pi^2m^4}f^{abc}G^{a,\mu\nu}D^\alpha G^b_{\alpha\mu}D^\beta G^c_{\beta\nu}$$

$$\mathfrak{L}_{eff}^{(4)} = \gamma_{4,1}\frac{g_S^4}{6!\pi^2m^4}G^a_{\mu\nu}G^{a,\mu\nu}G^b_{\rho\sigma}G^{b,\rho\sigma}+\gamma_{4,2}\frac{g_S^4}{6!\pi^2m^4}G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}G^b_{\rho\sigma}\tilde{G}^{b,\rho\sigma}$$
$$+\gamma_{4,3}\frac{g_S^4}{6!\pi^2m^4}G^a_{\mu\nu}G^{b,\mu\nu}G^a_{\rho\sigma}G^{b,\rho\sigma}+\gamma_{4,4}\frac{g_S^4}{6!\pi^2m^4}G^a_{\mu\nu}\tilde{G}^{b,\mu\nu}G^a_{\rho\sigma}\tilde{G}^{b,\rho\sigma}$$
$$+\gamma_{4,5}\frac{g_S^4}{6!\pi^2m^4}f^{abe}f^{cde}G^a_{\mu\nu}G^{c,\mu\nu}G^b_{\rho\sigma}G^{d,\rho\sigma}+\gamma_{4,6}\frac{g_S^4}{6!\pi^2m^4}f^{abe}f^{cde}G^a_{\mu\nu}\tilde{G}^{c,\mu\nu}G^b_{\rho\sigma}\tilde{G}^{d,\rho\sigma}$$

Gluon effective interactions

❖ For fermions and scalars circulating in the loop → rather straightforward

→ Computation done using the SM and MSSM FeynArts models

→ Quarks or squarks in the fundamental representation as representative particles

❖ Vectors in the loop → calculation far more challenging!

1. We need a consistent model with a massive vector field in the fundamental representation of QCD

2. Unitary gauge does not work!

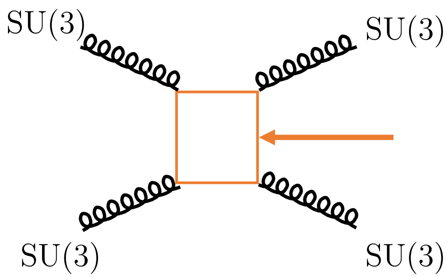
➢ Need to **generalize the non-linear gauge** to preserve QCD symmetry otherwise 1PI off-shell amplitudes cannot be **matched onto gauge invariant operators**

➢ We used a custom minimal SU(5) GUT model quantized using a non-linear gauge

$$\mathcal{L}_{\rm gf} = -\frac{1}{\xi}|D^\mu X_\mu^{k+} - i\xi M_{XY} H_X^{k+}|^2 - \frac{1}{\xi}|D^\mu Y_\mu^{k+} - i\xi M_{XY} H_Y^{k+}|^2 + \ldots.$$

➢ Non-linear gauge drastically reduce the number of diagrams to compute (4-gluons diagrams: 207 → 84)

Gluon effective interactions



Either spin 0, ½ or 1 circulating in the loop

❖ Matching the EFT to QCD:

- **Off-shell gluons**
  → Some Wilson coefficients are gauge dependent

- **For a physical 4-gluons process**
  → Gauge dependent parts cancels!

Checked for gluon-gluon scattering ✓

J. Quevillon, C. Smith and ***S.T****, Phys.Rev. D99 (2019) no.1, 013003*

	α_0	α_2	α_4	β_2	$\beta_{4,1}$	$\beta_{4,2}$
Scalar	$\frac{1}{4}D_\varepsilon$	$-\frac{1}{16}$	$\frac{3}{112}$	$\frac{1}{48}$	$-\frac{1}{28}$	0
Fermion	D_ε	$-\frac{1}{2}$	$\frac{9}{28}$	$-\frac{1}{24}$	$\frac{1}{14}$	$-\frac{3}{4}$
Vector	$-\frac{21D_\varepsilon+2}{4}$	$\frac{37}{16}$	$-\frac{159}{112}$	$\frac{1}{16}$	$-\frac{3}{28}$	3
	$\gamma_{4,1}$	$\gamma_{4,2}$	$\gamma_{4,3}$	$\gamma_{4,4}$	$\gamma_{4,5}$	$\gamma_{4,6}$
Scalar	$\frac{7}{768}$	$\frac{1}{768}$	$\frac{7}{384}$	$\frac{1}{384}$	$\frac{1}{96}$	$\frac{1}{672}$
Fermion	$\frac{1}{48}$	$\frac{7}{192}$	$\frac{1}{24}$	$\frac{7}{96}$	$\frac{1}{96}$	$\frac{19}{672}$
Vector	$\frac{87}{256}$	$\frac{81}{256}$	$\frac{87}{128}$	$\frac{81}{128}$	$-\frac{3}{32}$	$-\frac{27}{224}$

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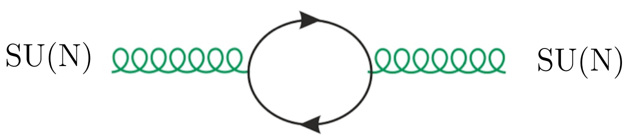
SU(N) effective interactions

❖ Extend the QCD case to arbitrary representations of other Lie groups

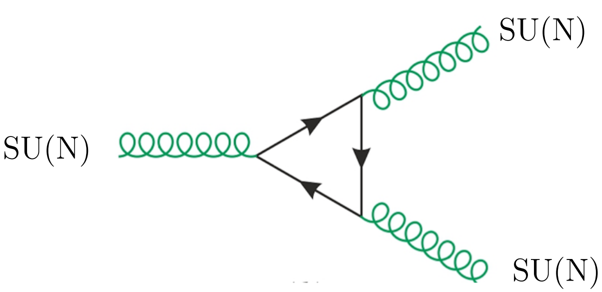
Tr over the fundamental generators of SU(3) → Tr over generators in some generic rep. R of SU(N)

Wilson coeffs for SU(3) → normalization → Wilson coeffs for SU(N)

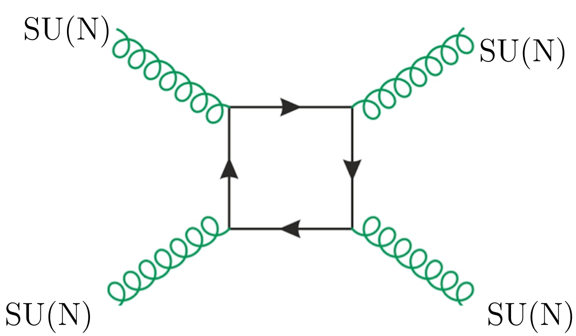


$$\propto \mathrm{Tr}(T^a_{\mathbf{R}} T^b_{\mathbf{R}}) = I_2(\mathbf{R})\delta^{ab}$$

Quadratic Casimir Invariant ensures a proper matching (divergences absorbed in the same operator)



$$\propto \mathrm{Tr}(T^a_{\mathbf{R}}[T^b_{\mathbf{R}}, T^c_{\mathbf{R}}]) = i I_2(\mathbf{R}) f^{abc}$$



$$\mathcal{M}^{abcd} = C_1^{abcd}\mathcal{M}_1 + C_2^{abcd}\mathcal{M}_2 + C_3^{abcd}\mathcal{M}_3$$

$$\begin{cases} C_1^{abcd} = \mathrm{Tr}(T^a_{\mathbf{R}} T^b_{\mathbf{R}} T^d_{\mathbf{R}} T^c_{\mathbf{R}}) + \mathrm{Tr}(T^a_{\mathbf{R}} T^c_{\mathbf{R}} T^d_{\mathbf{R}} T^b_{\mathbf{R}}) \\ C_2^{abcd} = \mathrm{Tr}(T^a_{\mathbf{R}} T^b_{\mathbf{R}} T^c_{\mathbf{R}} T^d_{\mathbf{R}}) + \mathrm{Tr}(T^a_{\mathbf{R}} T^d_{\mathbf{R}} T^c_{\mathbf{R}} T^b_{\mathbf{R}}) \\ C_3^{abcd} = \mathrm{Tr}(T^a_{\mathbf{R}} T^c_{\mathbf{R}} T^b_{\mathbf{R}} T^d_{\mathbf{R}}) + \mathrm{Tr}(T^a_{\mathbf{R}} T^d_{\mathbf{R}} T^b_{\mathbf{R}} T^c_{\mathbf{R}}) \end{cases}$$

SU(N) effective interactions

1/m expansion of $\mathcal{M}^{abcd}$

- At $\mathcal{O}(m^0)$ and $\mathcal{O}(m^{-2})$: only 2 independent combinations of Tr expressed in terms of $I_2(\mathbf{R})$

$$
\begin{aligned}
D_1^{abcd} &= 2C_1^{abcd} - C_2^{abcd} - C_3^{abcd} = I_2(\mathbf{R})(2f^{ace}f^{bde} - f^{ade}f^{bce})\,, \\
D_2^{abcd} &= 2C_2^{abcd} - C_1^{abcd} - C_3^{abcd} = I_2(\mathbf{R})(2f^{ade}f^{bce} - f^{ace}f^{bde})\,, \\
D_3^{abcd} &= 2C_3^{abcd} - C_1^{abcd} - C_2^{abcd} = I_2(\mathbf{R})(-f^{ade}f^{bce} - f^{ace}f^{bde}) = -D_1^{abcd} - D_2^{abcd}
\end{aligned}
$$

- At $\mathcal{O}(m^{-4})$: $\boxed{D_{1,2,3}^{abcd}}$ induce operators tuned by $\gamma_{4,5}$ and $\gamma_{4,6}$

The rest is **proportional** to the **fully symmetrized Tr**

$$
\begin{aligned}
D_0^{abcd} = C_1^{abcd} + C_2^{abcd} + C_3^{abcd} &= \frac{1}{4} S\,\mathrm{Tr}(T_{\mathbf{R}}^a T_{\mathbf{R}}^b T_{\mathbf{R}}^c T_{\mathbf{R}}^d) \\
&= 6\boxed{I_4(\mathbf{R})}d^{abcd} + 6\boxed{\Lambda(\mathbf{R})}(\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc})
\end{aligned}
$$

$$
\begin{aligned}
\mathfrak{L}_{eff}^{(4)} = &\boxed{\gamma_{4,1}}\frac{g_S^4}{6!\pi^2 m^4}G_{\mu\nu}^a G^{a,\mu\nu}G_{\rho\sigma}^b G^{b,\rho\sigma} + \boxed{\gamma_{4,2}}\frac{g_S^4}{6!\pi^2 m^4}G_{\mu\nu}^a \tilde{G}^{a,\mu\nu}G_{\rho\sigma}^b \tilde{G}^{b,\rho\sigma} \\
+ &\boxed{\gamma_{4,3}}\frac{g_S^4}{6!\pi^2 m^4}G_{\mu\nu}^a G^{b,\mu\nu}G_{\rho\sigma}^a G^{b,\rho\sigma} + \boxed{\gamma_{4,4}}\frac{g_S^4}{6!\pi^2 m^4}G_{\mu\nu}^a \tilde{G}^{b,\mu\nu}G_{\rho\sigma}^a \tilde{G}^{b,\rho\sigma} \\
+ &\boxed{\gamma_{4,5}}\frac{g_S^4}{6!\pi^2 m^4}f^{abe}f^{cde}G_{\mu\nu}^a G^{c,\mu\nu}G_{\rho\sigma}^b G^{d,\rho\sigma} + \boxed{\gamma_{4,6}}\frac{g_S^4}{6!\pi^2 m^4}f^{abe}f^{cde}G_{\mu\nu}^a \tilde{G}^{c,\mu\nu}G_{\rho\sigma}^b \tilde{G}^{d,\rho\sigma} \\
+ &\boxed{\gamma_{4,7}}\frac{g_S^4}{6!\pi^2 m^4}d^{abcd}G_{\mu\nu}^a G^{b,\mu\nu}G_{\rho\sigma}^c G^{d,\rho\sigma} + \boxed{\gamma_{4,8}}\frac{g_S^4}{6!\pi^2 m^4}d^{abcd}G_{\mu\nu}^a \tilde{G}^{b,\mu\nu}G_{\rho\sigma}^c \tilde{G}^{d,\rho\sigma}
\end{aligned}
$$

Need to extend the basis

SU(N) effective interactions

❖ Wilson coefficients of the effective operators for SU(N) gauge bosons

*J. Quevillon, C. Smith and **S.T**, Phys.Rev. D99 (2019) no.1, 013003*

	α_0	α_2	α_4	β_2	$\beta_{4,1}$	$\beta_{4,2}$
Scalar	$\frac{1}{2}I_2(\mathbf{R})D_\varepsilon$	$-\frac{1}{8}I_2(\mathbf{R})$	$\frac{3}{56}I_2(\mathbf{R})$	$\frac{1}{24}I_2(\mathbf{R})$	$-\frac{1}{14}I_2(\mathbf{R})$	0
Fermion	$2I_2(\mathbf{R})D_\varepsilon$	$-I_2(\mathbf{R})$	$\frac{9}{14}I_2(\mathbf{R})$	$-\frac{1}{12}I_2(\mathbf{R})$	$\frac{1}{7}I_2(\mathbf{R})$	$-\frac{3}{2}I_2(\mathbf{R})$
Vector	$-\frac{21D_\varepsilon+2}{2}I_2(\mathbf{R})$	$\frac{37}{8}I_2(\mathbf{R})$	$-\frac{159}{56}I_2(\mathbf{R})$	$\frac{1}{8}I_2(\mathbf{R})$	$-\frac{3}{14}I_2(\mathbf{R})$	$6I_2(\mathbf{R})$
	$\gamma_{4,1}=\gamma_{4,3}/2$	$\gamma_{4,2}=\gamma_{4,4}/2$	$\gamma_{4,5}$	$\gamma_{4,6}$	$\gamma_{4,7}$	$\gamma_{4,8}$
Scalar	$\frac{7}{32}\Lambda(\mathbf{R})$	$\frac{1}{32}\Lambda(\mathbf{R})$	$\frac{1}{48}I_2(\mathbf{R})$	$\frac{1}{336}I_2(\mathbf{R})$	$\frac{7}{32}I_4(\mathbf{R})$	$\frac{1}{32}I_4(\mathbf{R})$
Fermion	$\frac{1}{2}\Lambda(\mathbf{R})$	$\frac{7}{8}\Lambda(\mathbf{R})$	$\frac{1}{48}I_2(\mathbf{R})$	$\frac{19}{336}I_2(\mathbf{R})$	$\frac{1}{2}I_4(\mathbf{R})$	$\frac{7}{8}I_4(\mathbf{R})$
Vector	$\frac{261}{32}\Lambda(\mathbf{R})$	$\frac{243}{32}\Lambda(\mathbf{R})$	$-\frac{3}{16}I_2(\mathbf{R})$	$-\frac{27}{112}I_2(\mathbf{R})$	$\frac{261}{32}I_4(\mathbf{R})$	$\frac{243}{32}I_4(\mathbf{R})$

$$\boxed{\begin{aligned}\gamma_{4,1}&=\frac{1}{2}\gamma_{4,3}\\ \gamma_{4,2}&=\frac{1}{2}\gamma_{4,4}\end{aligned}}$$

No matter the rep. R or spin in the loop

SU(N), SO(N) effective interactions

❖ Reduction to SM gauge groups:

SU(N) effective action
(8 operators)

Removing quartic invariant ops
$\gamma_{4,7} = \gamma_{4,8} = 0$

$f^{abe} f^{cde} \rightarrow \varepsilon^{abe} \varepsilon^{cde} = \delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bc}$

$U(1)_\alpha \subset SU(N)$

SU(3)
effective action
(6 operators)

SU(2) effective action
(4 operators)

U(1) effective action
(2 operators)

$$
\begin{aligned}
\mathfrak{L}^{(4)}_{eff,SU(2)_L} = \; & \frac{(\gamma_{4,1}+\gamma_{4,3})g^4}{6!\pi^2 m^4}(W^3_{\mu\nu}W^{3,\mu\nu})^2 + \frac{(\gamma_{4,2}+\gamma_{4,4})g^4}{6!\pi^2 m^4}(W^3_{\mu\nu}\tilde{W}^{3,\mu\nu})^2 \\
&+ \frac{4(\gamma_{4,1}+\gamma_{4,5})g^4}{6!\pi^2 m^4} W^3_{\mu\nu}W^{3,\mu\nu}W^+_{\rho\sigma}W^{-,\rho\sigma} + \frac{4(\gamma_{4,2}+\gamma_{4,6})g^4}{6!\pi^2 m^4} W^3_{\mu\nu}\tilde{W}^{3,\mu\nu}W^+_{\rho\sigma}\tilde{W}^{-,\rho\sigma} \\
&+ \frac{4(\gamma_{4,3}-\gamma_{4,5})g^4}{6!\pi^2 m^4}|W^3_{\mu\nu}W^{+,\mu\nu}|^2 + \frac{4(\gamma_{4,4}-\gamma_{4,6})g^4}{6!\pi^2 m^4}|W^3_{\mu\nu}\tilde{W}^{+,\mu\nu}|^2 \\
&+ \frac{2(2\gamma_{4,1}+\gamma_{4,3}+\gamma_{4,5})g^4}{6!\pi^2 m^4}(W^+_{\mu\nu}W^{-,\mu\nu})^2 + \frac{2(\gamma_{4,4}-\gamma_{4,6})g^4}{6!\pi^2 m^4}|W^+_{\mu\nu}\tilde{W}^{+,\mu\nu}|^2 \\
&+ \frac{2(\gamma_{4,3}-\gamma_{4,5})g^4}{6!\pi^2 m^4}|W^+_{\mu\nu}W^{+,\mu\nu}|^2 + \frac{2(2\gamma_{4,2}+\gamma_{4,4}+\gamma_{4,6})g^4}{6!\pi^2 m^4}(W^+_{\mu\nu}\tilde{W}^{-,\mu\nu})^2
\end{aligned}
$$

$$
\begin{aligned}
\mathfrak{L}^{(4)}_{eff}(U(1)_\alpha \subset SU(N)) = \; & (\gamma_{4,1}+\gamma_{4,3}+d^{\alpha\alpha\alpha\alpha}\gamma_{4,7})\frac{g_S^4}{6!\pi^2 m^4}G^\alpha_{\mu\nu}G^{\alpha,\mu\nu}G^\alpha_{\rho\sigma}G^{\alpha,\rho\sigma} \\
&+ (\gamma_{4,2}+\gamma_{4,4}+d^{\alpha\alpha\alpha\alpha}\gamma_{4,8})\frac{g_S^4}{6!\pi^2 m^4}G^\alpha_{\mu\nu}\tilde{G}^{\alpha,\mu\nu}G^\alpha_{\rho\sigma}\tilde{G}^{\alpha,\rho\sigma}
\end{aligned}
$$

→ Euler-Heisenberg operators

SU(N) effective interactions

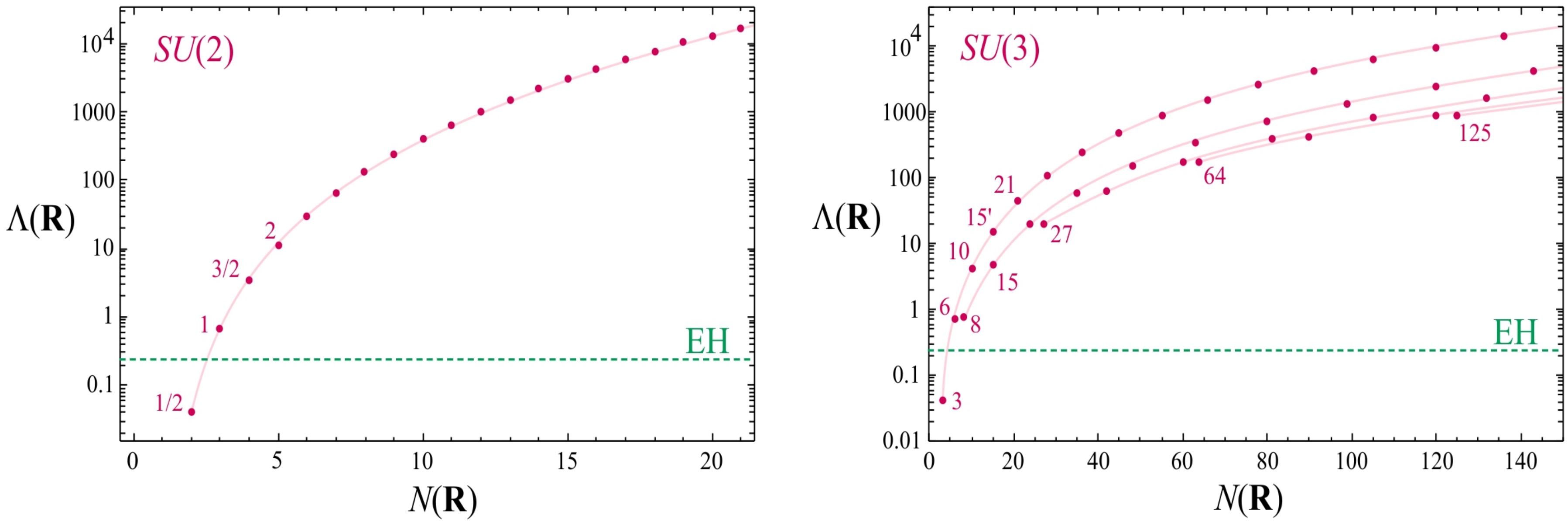
$$\frac{1}{4} S\,\mathrm{Tr}(T^a_{\mathbf{R}} T^b_{\mathbf{R}} T^c_{\mathbf{R}} T^d_{\mathbf{R}}) = 6 I_4(\mathbf{R}) d^{abcd} + 6\Lambda(\mathbf{R})(\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc})$$

*S. Okubo, J. Math. Phys **23** (1982) 8*

For SU(2) and SU(3), $d^{abcd}=0$ ⇒ $$\Lambda(\mathbf{R}) = \left(\frac{N(\mathbf{A}) I_2(\mathbf{R})}{N(\mathbf{R})} - \frac{I_2(\mathbf{A})}{6}\right)\frac{I_2(\mathbf{R})}{2+N(\mathbf{A})}$$

❖ Evolution of $\Lambda(\mathbf{R})$ as a function of the dimension $N(\mathbf{R})$ for SU(2) and SU(3)

*J. Quevillon, C. Smith and **S.T**, Phys.Rev. D99 (2019) no.1, 013003*



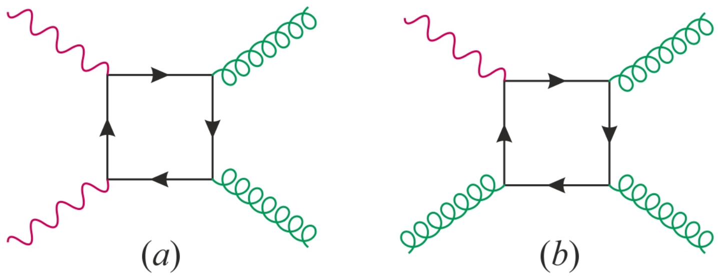
$\Lambda(\mathbf{R}) \sim N(\mathbf{R})^3$ $\quad$ $\Lambda(\mathbf{R}) \sim N(\mathbf{R})^5$

Overview

- Introduction: The Euler-Heisenberg Lagrangian
- Photon effective interactions
- Gluon effective interactions
- SU(N) effective interactions
- Mixed effective interactions
- Conclusion

Mixed effective interactions

❖ 1PI diagrams:



❖ Effective Lagrangian:

$$\mathfrak{L}^{(4)}_{eff}(U(1)\otimes SU(N)) = \alpha_1\frac{g_1^2g_n^2}{6!\pi^2m^4}F_{\mu\nu}F^{\mu\nu}G^a_{\rho\sigma}G^{a,\rho\sigma} + \alpha_2\frac{g_1^2g_n^2}{6!\pi^2m^4}F_{\mu\nu}\tilde{F}^{\mu\nu}G^a_{\rho\sigma}\tilde{G}^{a,\rho\sigma}$$
$$+\alpha_3\frac{g_1^2g_n^2}{6!\pi^2m^4}F_{\mu\nu}G^{a,\mu\nu}F_{\rho\sigma}G^{a,\rho\sigma} + \alpha_4\frac{g_1^2g_n^2}{6!\pi^2m^4}F_{\mu\nu}\tilde{G}^{a,\mu\nu}F_{\rho\sigma}\tilde{G}^{a,\rho\sigma}$$
$$+\beta_1\frac{g_1g_n^3}{6!\pi^2m^4}d^{abc}F_{\mu\nu}G^{a,\mu\nu}G^b_{\rho\sigma}G^{c,\rho\sigma} + \beta_2\frac{g_1g_n^3}{6!\pi^2m^4}d^{abc}F_{\mu\nu}\tilde{G}^{a,\mu\nu}G^b_{\rho\sigma}\tilde{G}^{c,\rho\sigma}$$

$$\mathfrak{L}^{(4)}_{eff}(SU(M)\otimes SU(N)) = \alpha_1\frac{g_m^2g_n^2}{6!\pi^2m^4}W^i_{\mu\nu}W^{i,\mu\nu}G^a_{\rho\sigma}G^{a,\rho\sigma} + \alpha_2\frac{g_m^2g_n^2}{6!\pi^2m^4}W^i_{\mu\nu}\tilde{W}^{i,\mu\nu}G^a_{\rho\sigma}\tilde{G}^{a,\rho\sigma}$$
$$+\alpha_3\frac{g_m^2g_n^2}{6!\pi^2m^4}W^i_{\mu\nu}G^{a,\mu\nu}W^i_{\rho\sigma}G^{a,\rho\sigma} + \alpha_4\frac{g_m^2g_n^2}{6!\pi^2m^4}W^i_{\mu\nu}\tilde{G}^{a,\mu\nu}W^i_{\rho\sigma}\tilde{G}^{a,\rho\sigma}$$

❖ Matching:

*J. Quevillon, C. Smith and **S.T**, Phys.Rev. D99 (2019) no.1, 013003*

	$\alpha_1=\alpha_3/2$	$\alpha_2=\alpha_4/2$	β_1	β_2
Scalar	$\frac{7}{16}Q(\mathbf{R})^2I_2(\mathbf{R})$	$\frac{1}{16}Q(\mathbf{R})^2I_2(\mathbf{R})$	$\frac{7}{32}Q(\mathbf{R})I_3(\mathbf{R})$	$\frac{1}{32}Q(\mathbf{R})I_3(\mathbf{R})$
Fermion	$Q(\mathbf{R})^2I_2(\mathbf{R})$	$\frac{7}{4}Q(\mathbf{R})^2I_2(\mathbf{R})$	$\frac{1}{2}Q(\mathbf{R})I_3(\mathbf{R})$	$\frac{7}{8}Q(\mathbf{R})I_3(\mathbf{R})$
Vector	$\frac{261}{16}Q(\mathbf{R})^2I_2(\mathbf{R})$	$\frac{243}{16}Q(\mathbf{R})^2I_2(\mathbf{R})$	$\frac{261}{32}Q(\mathbf{R})I_3(\mathbf{R})$	$\frac{243}{32}Q(\mathbf{R})I_3(\mathbf{R})$

Conclusion

❖ Using the diagrammatic approach,

- Construction of **gauge bosons effective interactions** up to **dimension-8** operators
- Computation of their **Wilson coefficients** as induced by loops of **heavy particles of spin 0, ½ or 1**

❖ Photon EFT:

- Spin 0 and ½ → straightforward (recover Euler-Heisenberg result)
- Spin 1: usual procedure to construct effective action **breaks down**

➢ Quantize the SM in the **non-linear gauge:** matching consistent **off-shell**

❖ **Generalization to QCD gluons, SU(N), SO(N), U(1)xSU(N) and SU(N)xSU(M) gauge bosons and heavy particle in arbitrary representation:**

➢ **Quantization** of the minimal **SU(5) GUT model** using a **non-linear gauge**
→ It works as expected

❖ At one-loop, **some operators are redundant**, **no matter the representation or spin** of the particle circulating in the loop

Thank you for your attention !