



Finite Family Groups for Fermionic and Leptoquark Mixing Patterns

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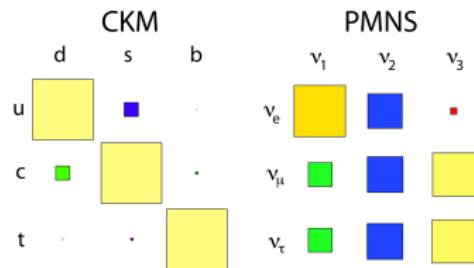
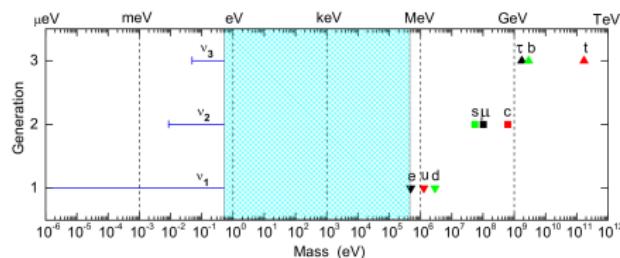
To be published

Outlook

- ① Discrete flavour symmetries
- ② Flavourful Leptoquarks
- ③ Model independent scan : Reconstructing flavour groups

Non-Abelian Discrete Flavour Symmetries

Motivations



The flavour problem

- Why such mass hierarchies ?
- Why 3 families ?
- Why these specific mixing patterns ?

⇒ Discrete non-abelian flavour symmetries !

An A_4 example for leptons [1002.0211]

Lepton mass terms

$$\mathcal{L}_{m_L} = \bar{E}_R m_\ell \ell_L + \frac{1}{2} \bar{\nu}_L^c m_\nu \nu_L + \text{h.c.}$$

Specific mixing patterns for leptons :

$$U_{PMNS} = \begin{pmatrix} 0.797 \rightarrow 0.842 & 0.518 \rightarrow 0.585 & 0.143 \rightarrow 0.156 \\ 0.233 \rightarrow 0.495 & 0.448 \rightarrow 0.679 & 0.639 \rightarrow 0.783 \\ 0.287 \rightarrow 0.532 & 0.486 \rightarrow 0.706 & 0.604 \rightarrow 0.754 \end{pmatrix} \sim \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} = U_{TBM}$$

Is it possible to reproduce U_{TBM} ?

In the basis where \mathbf{m}_ℓ is **diagonal**, m_ν^{TBM} is completely¹ specified by \mathbf{S}_{TBM}

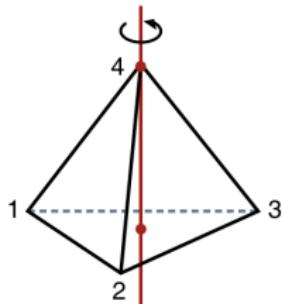
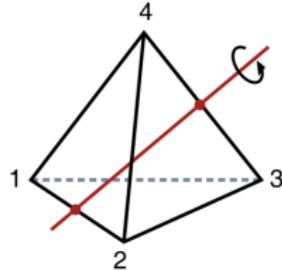
$$m_\nu^{TBM} = S_{TBM}^T m_\nu^{TBM} S_{TBM}$$

with $S_{TBM} = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$

1. Plus $\mu - \tau$ symmetry.

An A_4 example for leptons [1002.0211]

Group that leaves the tetrahedron invariant



$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$
$$S = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

Field	L	e^c	μ^c	τ^c	ν^c	ϕ_T	ϕ_S
A_4	3	1	1''	1'	3	3	3

Yukawas from flavons ϕ_i charged under A_4 and other "driving symmetries".

$$\begin{cases} \phi_T \rightarrow (\nu_T, 0, 0) \\ \phi_S \rightarrow (\nu_S, \nu_S, \nu_S) \end{cases} \quad m_\ell \text{ invariant under } T, m_\nu \text{ invariant under } S.$$

$$\implies U_{PMNS} = U_{TBM}$$

Discrete flavour symmetries : Summary

Motivations

- Explain why 3 families
- Recover mixing patterns
- Less free parameters

How it works

- Introduce flavon → break the symmetry in different directions
- The residual flavour symmetries drive the mass matrix
- The mixing matrices diagonalize the mass and residual symmetry matrices

Counterpart and subtleties

- Trade Yukawa parameters against new fields
- Usually EFT (possible to do UV)
- Difficult to realize correct vacuum alignment
- **Quark sector beyond Cabibbo mixing is much more complicated !**

Flavourful Leptoquarks

Motivations for LQs

Recent hints of LFNU :

$$R_{K^{(*)}, [a,b]} = \frac{\int_a^b dq^2 \left[d\Gamma(B \rightarrow K^{(*)} \mu^+ \mu^-) / dq^2 \right]}{\int_a^b dq^2 \left[d\Gamma(B \rightarrow K^{(*)} e^+ e^-) / dq^2 \right]}.$$

Ratio	Bin (GeV ²)	Data	Experimental Reference
R_K	[1, 6]	$0.745^{+0.090}_{-0.074} \pm 0.036$	LHCb
R_{K^*}	[1.1, 6.0]	$0.685^{+0.113}_{-0.069} \pm 0.047$	LHCb
	[0.045, 1.1]	$0.66^{+0.11}_{-0.07} \pm 0.03$	LHCb

Update Moriond 2019 : $R_K = 0.846^{+0.060}_{-0.054}$ (stat.) $^{+0.016}_{-0.014}$ (syst.)

⇒ Among many models, Leptoquark extensions rank among the most popular ones.

LQs models

Vector singlet, triplet and scalar triplet models

$$\Delta_3 \sim (\bar{3}, 3, 1/3), \quad \Delta_1^\mu \sim (3, 1, 2/3), \quad \Delta_3^\mu \sim (3, 3, 2/3),$$

We will focus on the scalar triplet (very similar for the other extensions).

$$\Delta_3 : \quad \mathcal{L} \supset y_{3,ij}^{LL} \bar{Q}_L^{C i,a} \epsilon^{ab} (\tau^k \Delta_3^k)^{bc} L_L^{j,c} + z_{3,ij}^{LL} \bar{Q}_L^{C i,a} \epsilon^{ab} ((\tau^k \Delta_3^k)^\dagger)^{bc} Q_L^{j,c} + \text{h.c.}$$

In the **fermion mass basis** (LQ redefinition + flavour rotations)

$$\begin{aligned} \mathcal{L}_{mass}^{LQ} \supset & \underbrace{-(U_d^T y_3^{LL} U_\nu)_{ij}}_{\lambda_{d\nu}} \bar{d}_L^{C i} \Delta_3^{1/3} \nu_L^j - \underbrace{\sqrt{2}(U_d^T y_3^{LL} U_l)_{ij}}_{\lambda_{dl}} \bar{d}_L^{C i} \Delta_3^{4/3} l_L^j \\ & + \underbrace{\sqrt{2}(U_u^T y_3^{LL} U_\nu)_{ij}}_{\lambda_{u\nu}} \bar{u}_L^{C i} \Delta_3^{-2/3} \nu_L^j - \underbrace{(U_u^T y_3^{LL} U_l)_{ij}}_{\lambda_{ul}} \bar{u}_L^{C i} \Delta_3^{1/3} l_L^j \\ & + \text{h.c.} \end{aligned}$$

The broad picture

We now assume that there exists a parent flavour symmetry \mathcal{G}_F . \mathcal{G}_F will then break to the different sectors, for e.g.

$$\mathcal{G}_F \rightarrow \begin{cases} \mathcal{G}_{\mathcal{L}} \rightarrow \left\{ \begin{array}{l} \mathcal{G}_{\nu} \\ \mathcal{G}_l \end{array} \right. \\ \mathcal{G}_{\mathcal{Q}} \rightarrow \left\{ \begin{array}{l} \mathcal{G}_u \\ \mathcal{G}_d \end{array} \right. \\ \mathcal{G}_{\mathcal{LQ}} \rightarrow \mathcal{G}'_{\mathcal{LQ}} \end{cases}$$

The LQ terms may exhibit an invariance under $\mathcal{G}_{\mathcal{LQ}}$

⇒ Controls the LQ coupling !

How do we find \mathcal{G}_F ? ⇒ we start from residual flavour symmetries.

The fermion mass matrices exhibit residual flavour symmetries in the mass basis. We take these residuals symmetries to be a smoking gun for the parent flavour group.

Residual flavour symmetries

- The ν SM fermion mass terms (in mass basis)

$$\mathcal{L}_{\text{mass}}^{\text{SM}} \supset \frac{1}{2} \bar{\nu}_L^C m_\nu \nu_L + \bar{E}_R m_I l_L + \bar{d}_R m_d d_L + \bar{u}_R m_u u_L + \text{h.c.}$$

is invariant under $U(1)_f^3$ for $f = l, q$ and $Z_2 \times Z_2$ for ν_L

$$\begin{aligned} \nu_L &\rightarrow T_{\nu_i} \nu_L, & \text{with } T_{\nu 1} = \text{diag}(1, -1, -1) & \text{and } T_{\nu 2} = \text{diag}(-1, 1, -1), \\ f &\rightarrow T_f f, & \text{with } T_f = \text{diag}\left(e^{i\alpha_f}, e^{i\beta_f}, e^{i\gamma_f}\right) & \text{for } f \in \{E_R, l_L, d_R, d_L, u_R, u_L\}. \end{aligned}$$

- The LQ interaction terms

$$\mathcal{L}_{\text{mass}}^{\text{LQ}} = \bar{d}_L^C \lambda_{dl} l_L \Delta_3^{4/3} + \bar{d}_L^C \lambda_{d\nu} \nu_L \Delta_3^{1/3} + \bar{u}_L^C \lambda_{ul} l_L \Delta_3^{1/3} + \bar{u}_L^C \lambda_{u\nu} \nu_L \Delta_3^{-2/3} + \text{h.c.}$$

Note that the λ_{QL} can be all expressed in terms of λ_{dl}

Hypothesis : We enforce that the LQ terms are invariant under the **same** symmetries as the fermion mass terms

$$T_Q^{(T,\dagger)} \lambda_{QL} T_L \stackrel{!}{=} \lambda_{QL}$$

Patterns from λ_{dl} symmetry constraints

We impose that

$$T_d^{(T,\dagger)} \lambda_{dl} T_I \stackrel{!}{=} \lambda_{dl},$$

which imposes phase equalities

$$\begin{pmatrix} e^{i(\alpha_d + \alpha_l)} \lambda_{de} & e^{i(\alpha_d + \beta_l)} \lambda_{d\mu} & e^{i(\alpha_d + \gamma_l)} \lambda_{d\tau} \\ e^{i(\beta_d + \alpha_l)} \lambda_{se} & e^{i(\beta_d + \beta_l)} \lambda_{s\mu} & e^{i(\beta_d + \gamma_l)} \lambda_{s\tau} \\ e^{i(\gamma_d + \alpha_l)} \lambda_{be} & e^{i(\gamma_d + \beta_l)} \lambda_{b\mu} & e^{i(\gamma_d + \gamma_l)} \lambda_{b\tau} \end{pmatrix} \stackrel{!}{=} \begin{pmatrix} \lambda_{de} & \lambda_{d\mu} & \lambda_{d\tau} \\ \lambda_{se} & \lambda_{s\mu} & \lambda_{s\tau} \\ \lambda_{be} & \lambda_{b\mu} & \lambda_{b\tau} \end{pmatrix}.$$

Different patterns :

- **Isolation** $\lambda_{dl}^{[e]} = \begin{pmatrix} \lambda_{de} & 0 & 0 \\ \lambda_{se} & 0 & 0 \\ \lambda_{be} & 0 & 0 \end{pmatrix}$, $\lambda_{dl}^{[\mu]} = \begin{pmatrix} 0 & \lambda_{d\mu} & 0 \\ 0 & \lambda_{s\mu} & 0 \\ 0 & \lambda_{b\mu} & 0 \end{pmatrix}$, $\lambda_{dl}^{[\tau]} = \begin{pmatrix} 0 & 0 & \lambda_{d\tau} \\ 0 & 0 & \lambda_{s\tau} \\ 0 & 0 & \lambda_{b\tau} \end{pmatrix}$
- **Two-columned** $\lambda_{dl}^{[e\mu]} = \begin{pmatrix} 0 & 0 & 0 \\ \lambda_{se} & \lambda_{s\mu} & 0 \\ \lambda_{be} & \lambda_{b\mu} & 0 \end{pmatrix}$, $\lambda_{dl}^{[e\tau]} = \begin{pmatrix} 0 & 0 & 0 \\ \lambda_{se} & 0 & \lambda_{s\tau} \\ \lambda_{be} & 0 & \lambda_{b\tau} \end{pmatrix}$, $\lambda_{dl}^{[\tau]} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda_{s\mu} & \lambda_{s\tau} \\ 0 & \lambda_{b\mu} & \lambda_{b\tau} \end{pmatrix}$
- **Three-columned ...**

Two environments

We can enforce additional constraints on the other terms : λ_{ul} , $\lambda_{d\nu}$, $\lambda_{u\nu}$, but ...

$$\lambda_{d\nu} = \frac{1}{\sqrt{2}} \lambda_{dl} U_{PMNS}, \quad \lambda_{ul} = \frac{1}{\sqrt{2}} U_{CKM}^* \lambda_{dl}, \quad \lambda_{u\nu} = -U_{CKM}^* \lambda_{dl} U_{PMNS}.$$

⇒ The symmetry of λ_{dl} transpose to the other terms !

Two different frameworks :

SE1 [1901.10484]

All mass symmetries acts in all LQ couplings

- The LQ couplings are fully reduced (1 free parameter)
- Model building probably challenging...

SE2

The residual mass symmetries holds only for λ_{dl} .

- Easier to build from a flavon approach
- Mimic models already in the market [1503.01084v3]
- Only partially reduced

A model independent scan

The strategy

We wish now to invert the breaking pattern arrow to recover \mathcal{G}_F , i.e.

$$\underbrace{\{\mathcal{G}_u, \mathcal{G}_d, \mathcal{G}_\nu, \mathcal{G}_l\}}_{\text{mass basis}} \xrightarrow{\text{SM mixing rotations}} \underbrace{\{\mathcal{G}'_u, \mathcal{G}'_d, \mathcal{G}'_\nu, \mathcal{G}'_l\}}_{\text{"flavour" basis}} \xrightarrow{\text{Close algebra}} \mathcal{G}_F$$

In the flavour basis, the generators "know" about the mixing matrices !

We need to include as well the "knowledge" about
the specific chosen LQ pattern !!!

To do so, we further move to a basis where λ_{dl} is diagonal by

$$\lambda'_{dl} = \Lambda_d^* \lambda_{dl} \Lambda_l^\dagger$$

The "LeptoFlavour" basis

$$\begin{aligned} l_L &\rightarrow \Lambda_I^\dagger l'_L, & d_L &\rightarrow \Lambda_d^\dagger d'_L, & \nu_L &\rightarrow U_{PMNS}^\dagger \Lambda_I^\dagger \nu'_L, & u_L &\rightarrow U_{CKM} \Lambda_d^\dagger u'_L, \\ E_R &\rightarrow \Lambda_E^\dagger E'_R, & d_R &\rightarrow \Lambda_D^\dagger d'_R, & \nu_R &\rightarrow \Lambda_R^\dagger \nu'_R, & u_R &\rightarrow \Lambda_U^\dagger u'_R, \end{aligned}$$

Therefore, the residual generators in the leptoflavour basis are

$$T'_I = \Lambda_I T_I \Lambda_I^\dagger, \quad T'_\nu = \Lambda_I U_{PMNS} T_\nu U_{PMNS}^\dagger \Lambda_I^\dagger, \quad T'_d = \Lambda_d T_d \Lambda_d^\dagger, \quad T'_u = \Lambda_d U_{CKM}^\dagger T_u U_{CKM} \Lambda_d^\dagger,$$

The CKM and PMNS approximation

We wish to recover specific patterns for the leading order PMNS and CKM matrices.

We set

$$U_{PMNS} \simeq U_{\mu\tau} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} \cos \theta_{\mu\tau} & \sqrt{2} \sin \theta_{\mu\tau} & 0 \\ -\sin \theta_{\mu\tau} & \cos \theta_{\mu\tau} & 1 \\ \sin \theta_{\mu\tau} & -\cos \theta_{\mu\tau} & 1 \end{pmatrix} + \mathcal{O}(\theta'_{13}),$$

$$U_{CKM} \simeq U_C \equiv \begin{pmatrix} \cos \theta_c & \sin \theta_c & 0 \\ -\sin \theta_c & \cos \theta_c & 0 \\ 0 & 0 & 1 \end{pmatrix} + \mathcal{O}(\theta_c^2, \theta_c^3).$$

$U_{\mu\tau}$ embed popular approximation for the PMNS matrix :

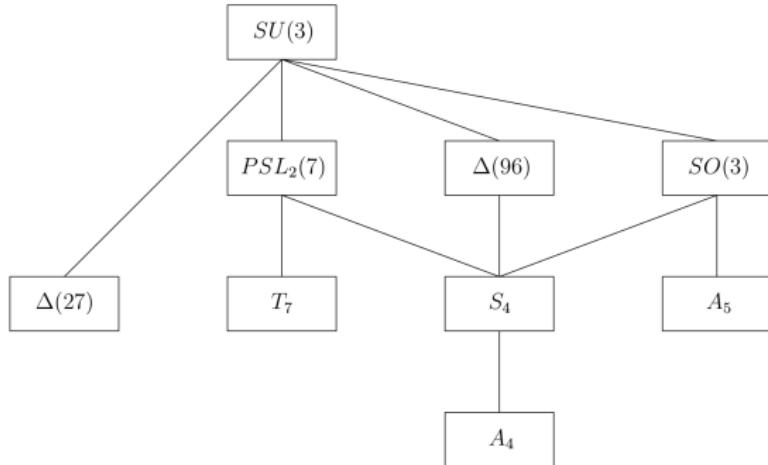
$$U_{\mu\tau}(\theta_{\mu\tau}) \rightarrow \begin{cases} U_{TBM} & \Leftrightarrow \tan \theta_{\mu\tau} = \frac{1}{\sqrt{2}} \\ U_{BM} & \Leftrightarrow \tan \theta_{\mu\tau} = 1 \text{ or } \theta_{\mu\tau} = \frac{\pi}{4} \\ U_{GR_i} & \Leftrightarrow \tan \theta_{\mu\tau} = \frac{2}{(1+\sqrt{5})}, \theta_{\mu\tau} = \frac{\pi}{5} \\ U_{HM} & \Leftrightarrow \tan \theta_{\mu\tau} = \frac{1}{\sqrt{3}} \text{ or } \theta_{\mu\tau} = \frac{\pi}{6} \end{cases} \quad (1)$$

Remark : In case of degenerate phases, the full mixing matrices cannot be predicted by the residual generators ! There is an additional freedom

$$T_a U = U_a T_a^{ii=jj} U_a^\dagger = U_a R_a^{ij} T_a^{ii=jj} R_a^{ij\dagger} U_a^\dagger.$$

⇒ Some angles will be left as free parameters.

Non-abelian discrete flavour symmetries



- Manifest as subgroup of continuous symmetries
- No Goldstone modes
- Can originate from extra-dimension
- Abelian flavour symmetries such $U(1)_{FN}$ disfavored for neutrino
- widely studied

Scan basic procedure

So far, we did not assume anything about the residual flavour symmetries (RFS). However, we want to close finite groups and we thus need to "discretize" the parameters.

First, assume that $\mathcal{G}_a \sim Z_{n_a}$ \Rightarrow Discretize phases in RFS

Discretize free parameters in mixing matrices

$$\text{angles } \theta_i = 2\pi \left(\frac{n}{m} \right)_i, \quad \text{phases } \Theta_i = 2\pi \left(\frac{n}{m} \right)_i, \quad \text{LQ couplings } \theta_i^{LQ} = \left(\pm \sqrt{\frac{n}{m}} \right)_i$$

Close algebra (GAP)

- | | |
|--|---|
| (SE1 & SE2 : Leptoquarks, PMNS, & CKM) : | $\mathcal{G}_F \sim \{T'_d, T'_I, T'_u, T'_\nu\}$ |
| (SE1 & SE2 : Leptoquarks, PMNS, & CKM) : | $\mathcal{G}_F \sim \{T'_d, T'_u\} \times \{T'_I, T'_\nu\}$ |
| (SE2 : Leptoquarks & PMNS) : | $\mathcal{G}_F \sim \{T'_I, T'_\nu\}$ |
| (SE2 : Leptoquarks & CKM) : | $\mathcal{G}_F \sim \{T'_u, T'_d\}$ |

(Preliminary !) Results

SE1 : e-isolation pattern

Electron Isolation and LO Fermionic Mixing						
$\{\theta_{13}^I, \theta_{23}^I, \theta_c\}$	T_l^{ii}	T_d^{ii}	T_u^{ii}	T_ν^{ii}	GAP-ID	\mathcal{G}_F
$\{0, \frac{\pi}{4}, \frac{\pi}{14}\}$	$[-1, -1, 1]$	$[1, -1, -1]$	$[1, -1, -1]$	$[-1, -1, 1]$	$[56, 5]$	D_{56}
$\{\theta_{13}^I, \theta_{23}^I, \theta_c\}$	T_l^{ii}	T_d^{ii}	T_u^{ii}	T_ν^{ii}	GAP-ID	$\mathcal{G}_Q \times \mathcal{G}_L$
$\{0, \frac{\pi}{4}, \frac{\pi}{15}\}$	$[-1, -\omega_4, -1]$	$[1, -1, -1]$	$[1, -1, -1]$	$[-1, -1, 1]$	$[(30, 3), (32, 11)]$	$D_{30} \times \Sigma(32)$
$\{0, \frac{\pi}{4}, \frac{\pi}{14}\}$	$[1, \omega_3, \omega_5^2]$	$[-1, 1, 1]$	$[-1, 1, 1]$	$[1, 1, -1]$	$[(28, 3), (6, 1)]$	$D_{28} \times S_3$
$\{0, \frac{\pi}{4}, \frac{\pi}{15}\}$	$[1, \omega_4, -\omega_4]$	$[-1, 1, 1]$	$[-1, 1, 1]$	$[1, 1, \omega_5^2]$	$[(30, 3), (100, 6)]$	$D_{30} \times (Z_5 \times (Z_5 \rtimes Z_4))$
...

SE1 : $\mu - \tau$ pattern

$\mu - \tau$ LQ Mixing and LO Fermionic Mixing						
$\{\tan \theta_{\mu\tau}, \theta_c\}$	T_l^{ii}	T_d^{ii}	T_u^{ii}	T_ν^{ii}	GAP-ID	Group Structure
$\{1, \frac{\pi}{14}\}$	$[-1, 1, 1]$	$[-1, 1, 1]$	$[-1, 1, 1]$	$[-1, 1, 1]$	$[56, 5]$	D_{56}
$\{\frac{1}{\sqrt{3}}, \frac{\pi}{14}\}$	$[-1, 1, 1]$	$[-1, 1, 1]$	$[-1, 1, 1]$	$[-1, 1, 1]$	$[84, 14]$	D_{84}
$\{\frac{1}{\sqrt{3}}, \frac{\pi}{15}\}$	$[-1, 1, 1]$	$[-1, 1, 1]$	$[-1, 1, 1]$	$[-1, 1, 1]$	$[60, 12]$	D_{60}
$\{\tan \theta_{\mu\tau}, \theta_c\}$	T_l^{ii}	T_d^{ii}	T_u^{ii}	T_ν^{ii}	GAP-ID	$\mathcal{G}_Q \times \mathcal{G}_L$
$\{1, \frac{\pi}{14}\}$	$[-1, 1, 1]$	$[-1, 1, 1]$	$[-1, 1, 1]$	$[-1, 1, 1]$	$[(28, 3), (8, 3)]$	$D_{28} \times D_8$
$\{1, \frac{\pi}{14}\}$	$[\omega_4, 1, 1]$	$[-1, 1, 1]$	$[-1, 1, 1]$	$[-1, 1, 1]$	$[(28, 3), (32, 11)]$	$D_{28} \times \Sigma(32)$
$\{1, \frac{\pi}{14}\}$	$[\omega_3^2, 1, 1]$	$[-1, 1, 1]$	$[-1, 1, 1]$	$[-1, 1, 1]$	$[(28, 3), (18, 3)]$	$D_{28} \times (Z_3 \times S_3)$
$\{\frac{1}{\sqrt{3}}, \frac{\pi}{14}\}$	$[-1, 1, 1]$	$[-1, 1, 1]$	$[-1, 1, 1]$	$[-1, 1, 1]$	$[(28, 3), (12, 4)]$	$D_{28} \times D_{12}$
...

(Preliminary !) Results

SE2 : e-isolation pattern & All mixing

Electron Isolation and LO Fermionic Mixing						
$\{x_e, \theta_{13,23}^q, \theta_{13}^l, \theta_{23}^l\}$	T_l^{ii}	T_d^{ii}	T_u^{ii}	T_ν^{ii}	GAP-ID	\mathcal{G}_F
$\{1, 0, 0, \frac{\pi}{4}\}$	$[-1, -1, 1]$	$[1, -1, -1]$	$[-1, -1, 1]$	$[-1, -1, 1]$	$[24, 12]$	S_4
$\{1, 0, 0, \frac{\pi}{4}\}$	$[1, 1, -1]$	$[-1, 1, 1]$	$[-1, -1, 1]$	$[1, 1, -1]$	$[48, 48]$	$Z_2 \times S_4$
$\{1, 0, \tan \theta_{\mu\tau} = \frac{1}{\sqrt{2}}\}$	$[1, \omega_3, 1]$	$[\omega_3^2, 1, 1]$	$[1, 1, \omega_3^2]$	$[1, \omega_3, 1]$	$[75, 25]$	$Z_3 \times SL(2, 3)$
$\{x_e, \tan \theta_{\mu\tau}, \theta_c\}$	T_l^{ii}	T_d^{ii}	T_u^{ii}	T_ν^{ii}	GAP-ID	$\mathcal{G}_Q \times \mathcal{G}_L$
$\{1, \frac{1}{\sqrt{2}}, \frac{\pi}{14}\}$	$[1, \omega_3, \omega_3^2]$	$[-1, 1, 1]$	$[-1, 1, -1]$	$[\omega_3^2, 1, \omega_3^2]$	$[(28, 3), (81, 7)]$	$D_{28} \times \Sigma(81)$
$\{1, \frac{1}{\sqrt{2}}, \frac{\pi}{14}\}$	$[1, \omega_3, \omega_3^2]$	$[-1, 1, 1]$	$[1, -1, 1]$	$[-1, 1, -1]$	$[(14, 1), (12, 3)]$	$D_{14} \times A_4$
$\{1, \frac{1}{\sqrt{2}}, \frac{\pi}{14}\}$	$[1, \omega_3, 1]$	$[-1, 1, 1]$	$[1, -1, 1]$	$[1, \omega_3, 1]$	$[(14, 1), (24, 3)]$	$D_{14} \times SL(2, 3)$
$\{\frac{1}{7}, \frac{1}{\sqrt{2}}, \frac{\pi}{14}\}$	$[1, \omega_3^2, \omega_3]$	$[-1, 1, 1]$	$[-1, 1, -1]$	$[1, -1, -1]$	$[(28, 3), (24, 12)]$	$D_{28} \times S_4$
many

SE2 : e-isolation pattern & Quark mixing

$\lambda^{[bs]} \text{ Mixing and Cabibbo Mixing}$				
$\{y_b, \theta_c\}$	T_d^{ii}	T_u^{ii}	GAP-ID	Group Structure
$\{1/6, \pi/15\}$	$[-1, 1, 1]$	$[-1, 1, -1]$	$[60, 12]$	D_{60}
$\{1/4, \pi/13\}$	$[-1, 1, 1]$	$[-1, 1, -1]$	$[52, 4]$	D_{52}
$\{3/8, \pi/14\}$	$[-1, 1, 1]$	$[-1, 1, -1]$	$[28, 3]$	D_{28}
$\{3/8, \pi/14\}$	$[-1, 1, 1]$	$[1, -1, 1]$	$[14, 1]$	D_{14}
$\{1, \pi/13\}$	$[-1, 1, 1]$	$[-1, 1, -1]$	$[52, 4]$	D_{52}
...

(Preliminary!) Results

SE2 : e -isolation pattern & Lepton mixing

Electron Isolation and Lepton Mixing				
$\tan \theta_{\mu\tau}$	T_l^{ii}	T_ν^{ii}	GAP-ID	Group Structure
$\sqrt{2/3}$	$[1, \omega_3, \omega_3^2]$	$[-1, -1, 1]$	$[6, 1]$	S_3
$\sqrt{2/3}$	$[1, -1, 1]$	$[-1, -1, 1]$	$[8, 3]$	D_8
$\sqrt{2/3}$	$[1, \omega_5^4, \omega_5]$	$[-1, -1, 1]$	$[10, 1]$	D_{10}
$\sqrt{2/3}$	$[1, \omega_3^2, \omega_3]$	$[\omega_4, \omega_4, -\omega_4]$	$[12, 1]$	$Z_3 \rtimes Z_4$
$\sqrt{2/3}$	$[1, -\omega_4, \omega_4]$	$[\omega_4, \omega_4, -\omega_4]$	$[16, 4]$	$Z_4 \rtimes Z_4$
$\sqrt{2/3}$	$[1, \omega_5^4, \omega_5]$	$[\omega_4, \omega_4, -\omega_4]$	$[20, 1]$	$Z_5 \rtimes Z_4$
$\sqrt{2/3}$	$[1, 1, -1]$	$[\omega_4, \omega_4, 1]$	$[32, 11]$	$\Sigma(32)$
$\sqrt{2/3}$	$[1, -\omega_4, 1]$	$[-\omega_4, -\omega_4, \omega_4]$	$[64, 20]$	$(Z_4 \times Z_4) \rtimes Z_4$
...	[...]	[...]	[...]	...
$1/\sqrt{2}$	$[1, \omega_3^2, \omega_3]$	$[-1, 1, -1]$	$[12, 3]$	A_4
$1/\sqrt{2}$	$[1, \omega_3, \omega_3^2]$	$[1, -1, -1]$	$[24, 12]$	S_4
$1/\sqrt{2}$	$[1, \omega_3, \omega_3^2]$	$[\omega_4, -1, \omega_4]$	$[48, 3]$	$\Delta(48)$
$1/\sqrt{2}$	$[1, \omega_3, \omega_3^2]$	$[\omega_5^2, \omega_5, \omega_5^2]$	$[75, 2]$	$\Delta(75)$
$1/\sqrt{2}$	$[1, \omega_3^2, \omega_3]$	$[\omega_3^2, 1, \omega_3^2]$	$[81, 7]$	$\Sigma(81)$
$1/\sqrt{2}$	$[1, \omega_3^2, \omega_3]$	$[-\omega_4, -1, \omega_4]$	$[96, 64]$	$\Delta(96)$
$\sqrt{3/5}$	[...]	[...]	[...]	...
$2/\sqrt{5}$	[...]	[...]	[...]	...
$1/\sqrt{3}$	[...]	[...]	[...]	...

Conclusions & perspectives

A model independent scan

- We derived specific patterns for LQs
- We combined these patterns with fermionic mixing
- We found various groups suitable for different frameworks

Under investigation

- Double check coherence of some results
- Search for more groups and other LQs

Moving forward

- Explore phenomenological implications for the LQs patterns
- Build an explicit model ?

Thanks !