

# Resummed photon spectrum from dark matter annihilation for intermediate and narrow energy resolution

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Based on  
Beneke, Broggio, Hasner, Urban, MV  
arXiv:[1903.08702](https://arxiv.org/abs/1903.08702)  
Beneke, Broggio, Hasner, MV  
arXiv:[1805.07367](https://arxiv.org/abs/1805.07367)

# Outline

- Motivation
- Gamma rays from DM annihilation
- Sommerfeld and Sudakov resummation
- Factorization formulas for the wino model
- Conclusions

# Outline

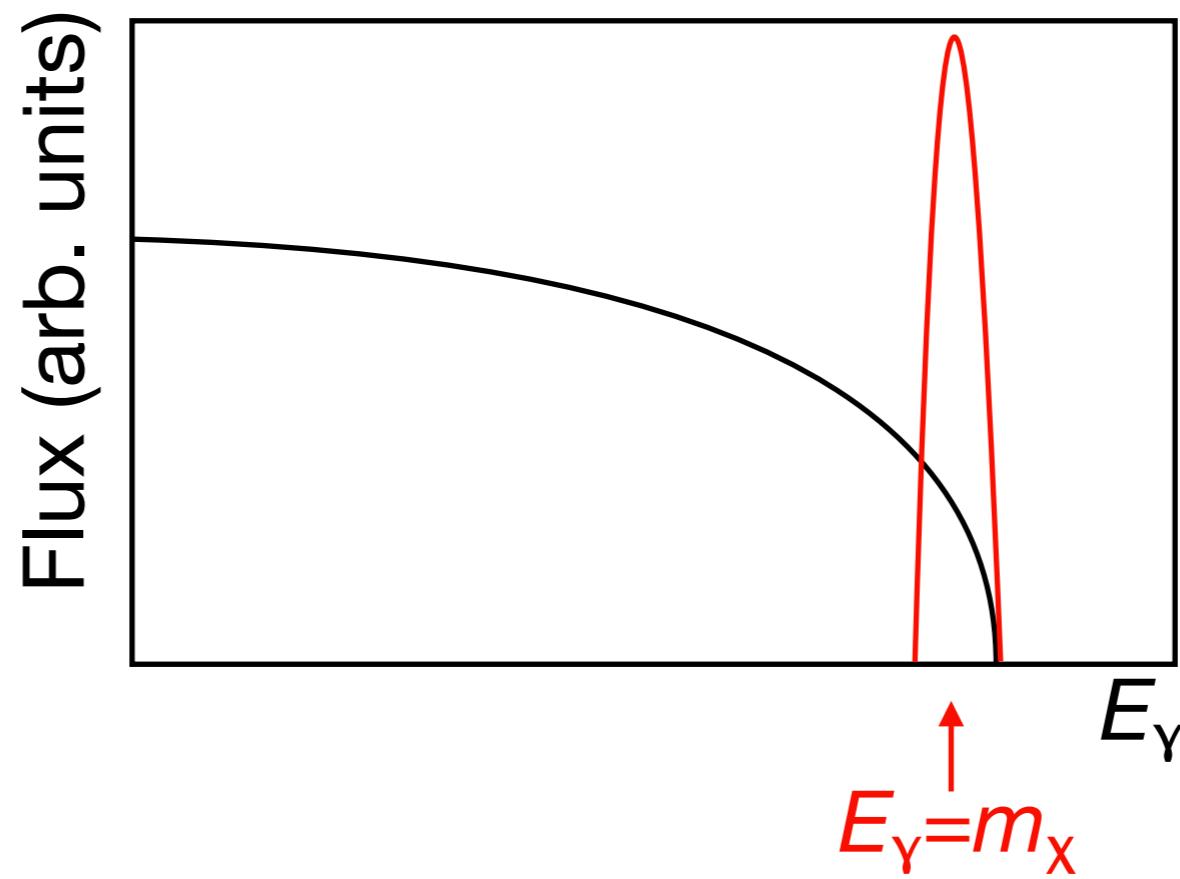
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# Motivation

- WIMP DM paradigm is very well motivated and scrutinized
- So far no discovery (un)fortunately
- Even though there are still viable options for WIMP model building for  $\mathcal{O}(1\text{-}100\text{GeV})$  masses, above-TeV WIMPs ‘start’ to become attractive

# Motivation

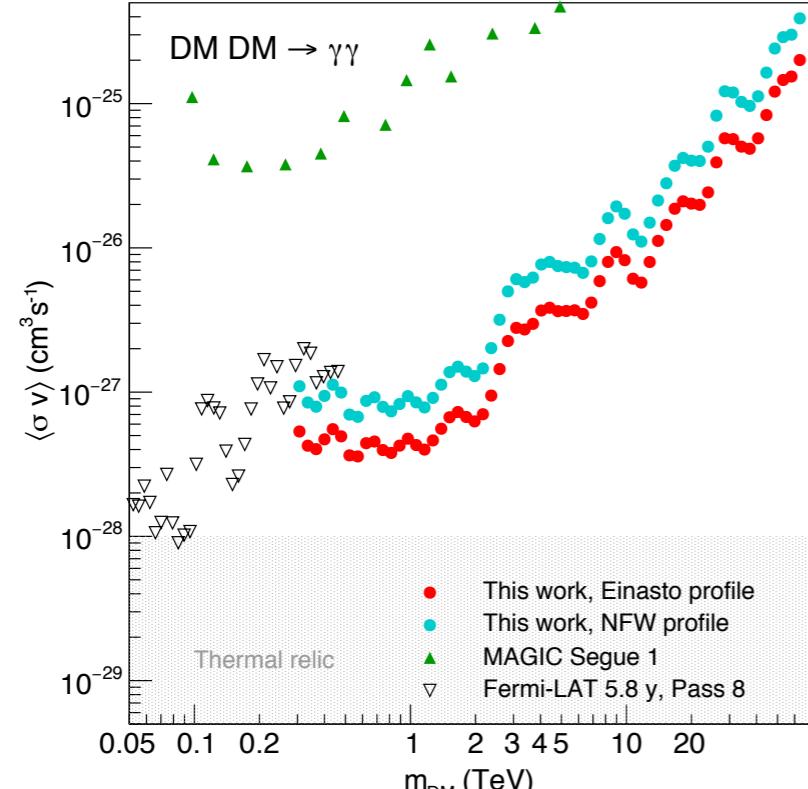
- Heavy ( $\mathcal{O}(1\text{-}100\text{TeV})$ ) DM  $\rightarrow$  Indirect detection
- Spectral-line feature in gamma ray spectrum is a smoking-gun signature of WIMP DM annihilation



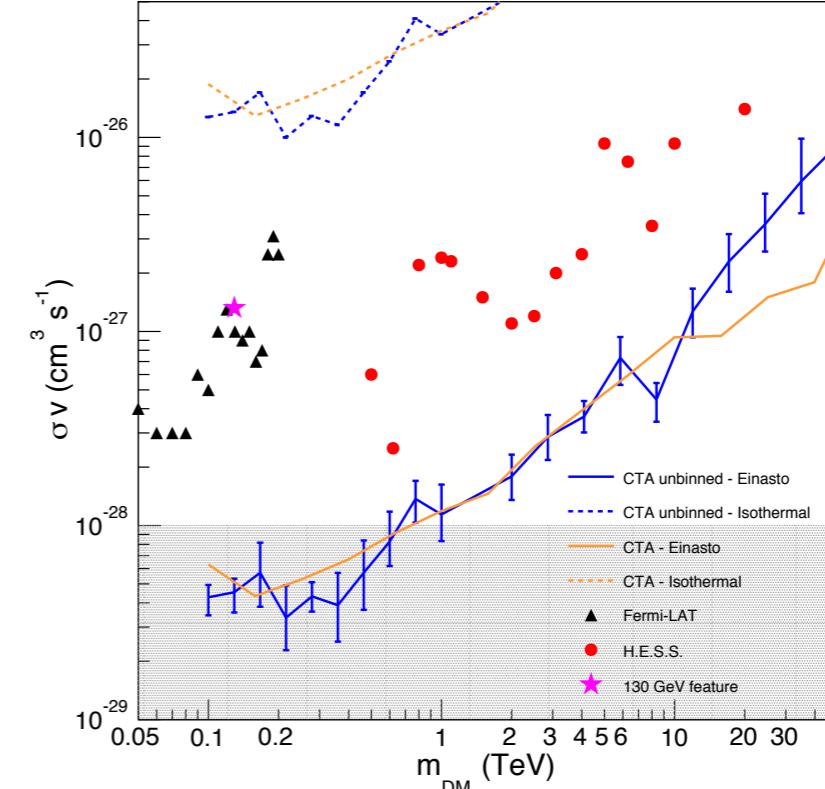
# Motivation

- Current- and next-generation gamma-ray telescopes will search for such spectral lines
- Particularly promising is the Cherenkov Telescope Array (**CTA**) with  $\sim 1$  order of magnitude improved sensitivity w.r.t. current technology

**State of the art**  
HESS (arXiv:1805.05741)



**Projected (500hrs)**  
CTA (arXiv:1709.07997)



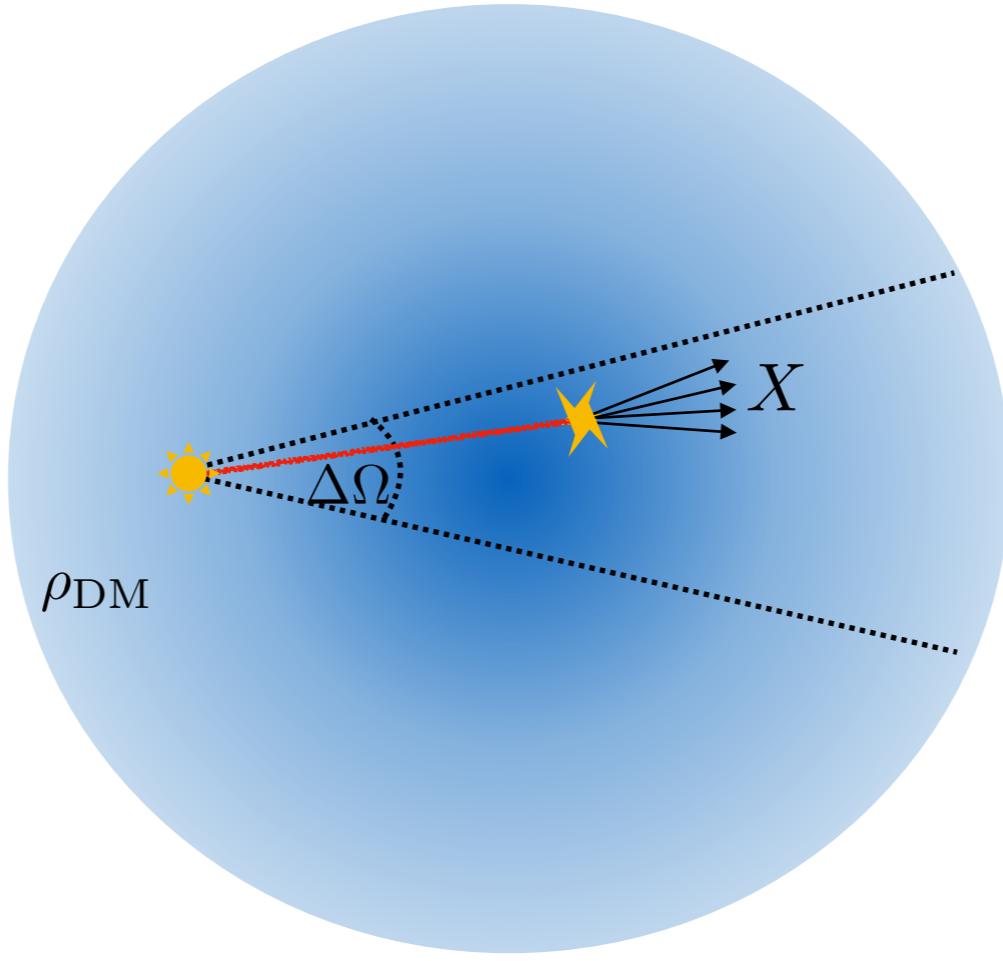
# Motivation

- Annihilation cross section computations for heavy wimps can be intricate
- Non-perturbative effects such as the *Sommerfeld* effect play a major role in their determination
- On top of this, large electroweak *Sudakov* double logarithms invalidate the perturbative expansion and need to be resummed
- In this work we focus on the latter (but also systematically treat the former)

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# Gamma rays from dark matter annihilation

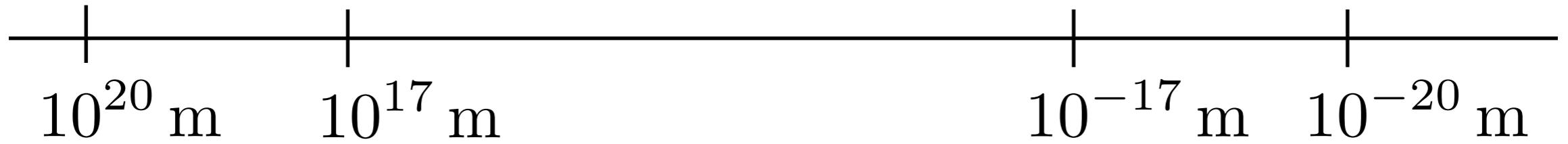


$$\Phi(E_\gamma) = \frac{1}{8\pi m_{\text{DM}}^2} \int_{\Delta\Omega} d\Omega \int_{\text{l.o.s.}} ds \rho_{\text{DM}}^2(\mathbf{r}(s)) \frac{d}{dE_\gamma} [\sigma v]_{\gamma+X}$$

# $\gamma$ rays from dark matter annihilation. Multi-scale problem

$$R_{\odot} \Delta\theta_{\text{obs}}$$

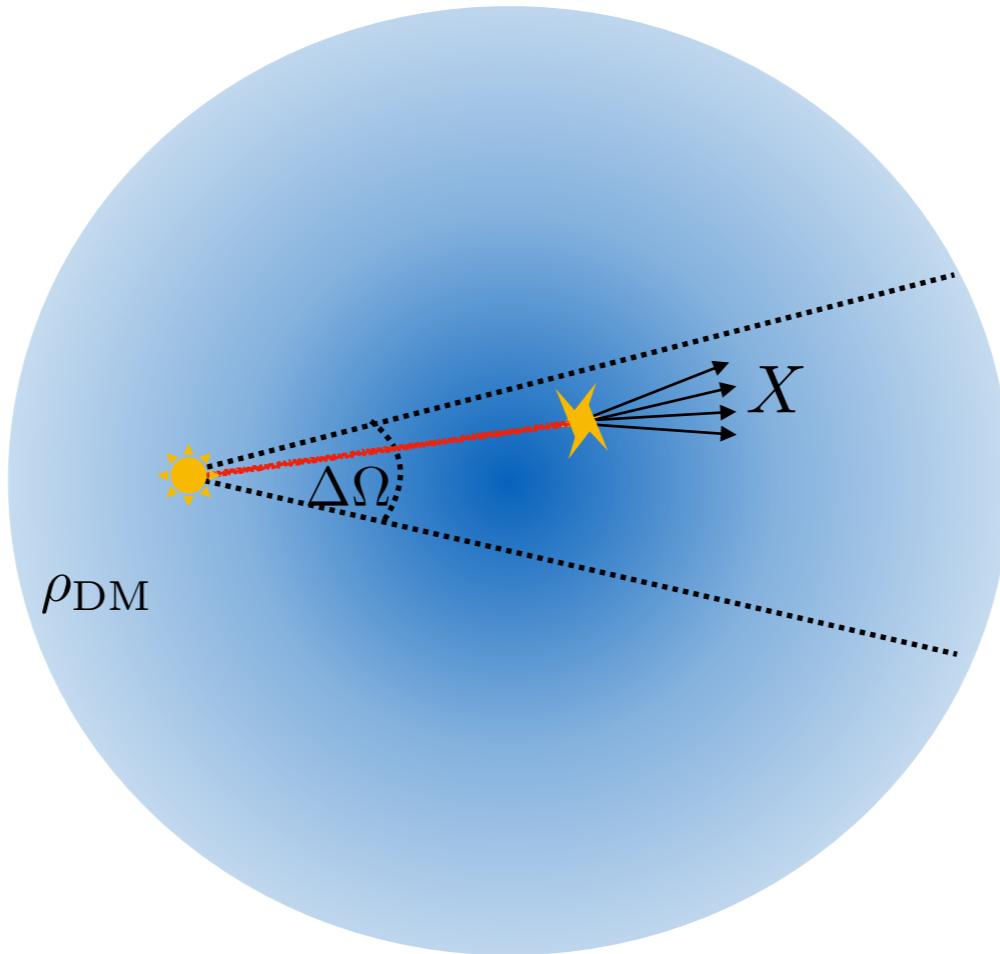
$$\lambda_{\text{soft } \gamma} \sim (\Delta E_{\text{obs}}^{\gamma})^{-1}$$



$$r_s, R_{\odot}$$

$$\lambda_{\text{DM}} \sim m_{\text{DM}}^{-1}$$

# $\gamma$ rays from dark matter annihilation. 1st factorization



$$\Phi(E_\gamma) = \frac{1}{8\pi m_{\text{DM}}^2} \underbrace{\int_{\Delta\Omega} d\Omega \int_{\text{l.o.s.}} ds}_{J(\Delta\Omega)} \rho_{\text{DM}}^2(r(s)) \frac{d}{dE_\gamma} [\sigma v]_{\gamma+X}$$

Astrophysical “ $J$ ” factor

independent of gamma-ray energy

# $\gamma$ rays from dark matter annihilation. 1st factorization

Propagation  
does not depend on  
any scale

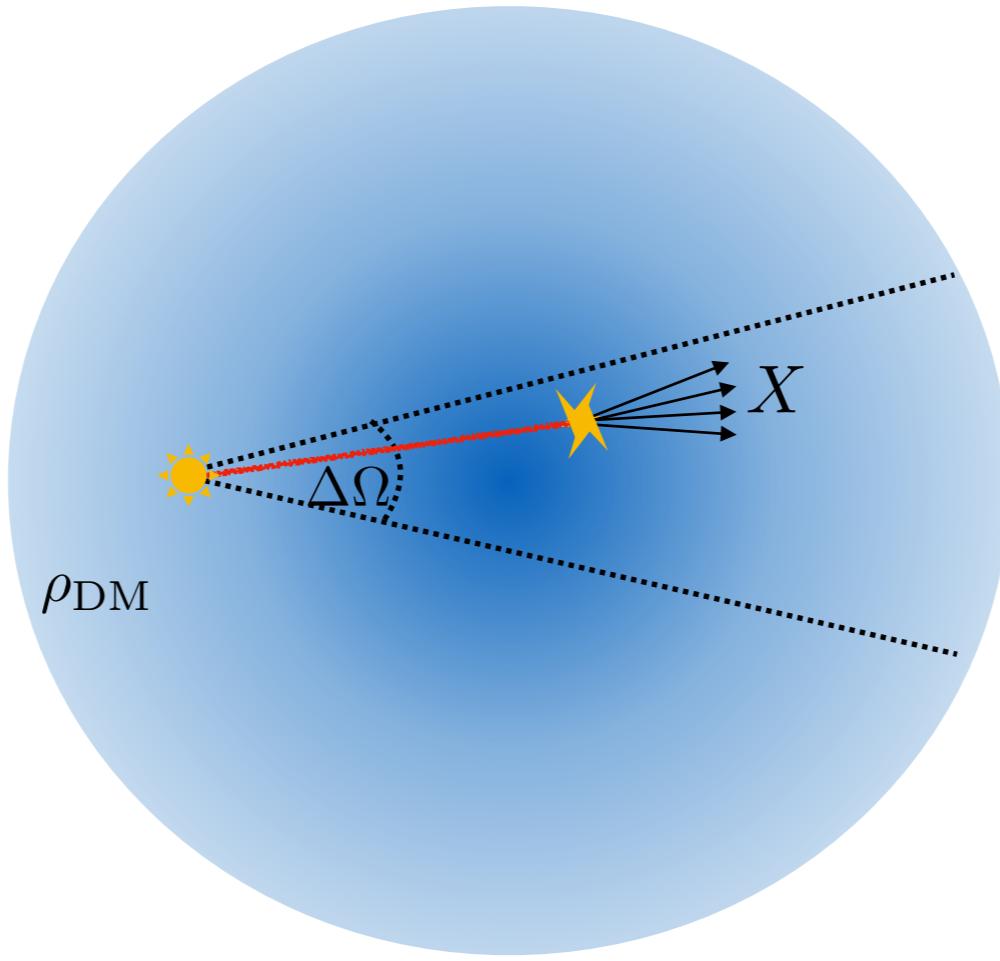
New-physics model  
 $m_W, m_\chi; \alpha_{EW} m_\chi, m_\chi v$ , etc.

Instrument/observation

$R_{\text{GC}}, r_s, \Delta\Omega, \Delta E_\gamma$

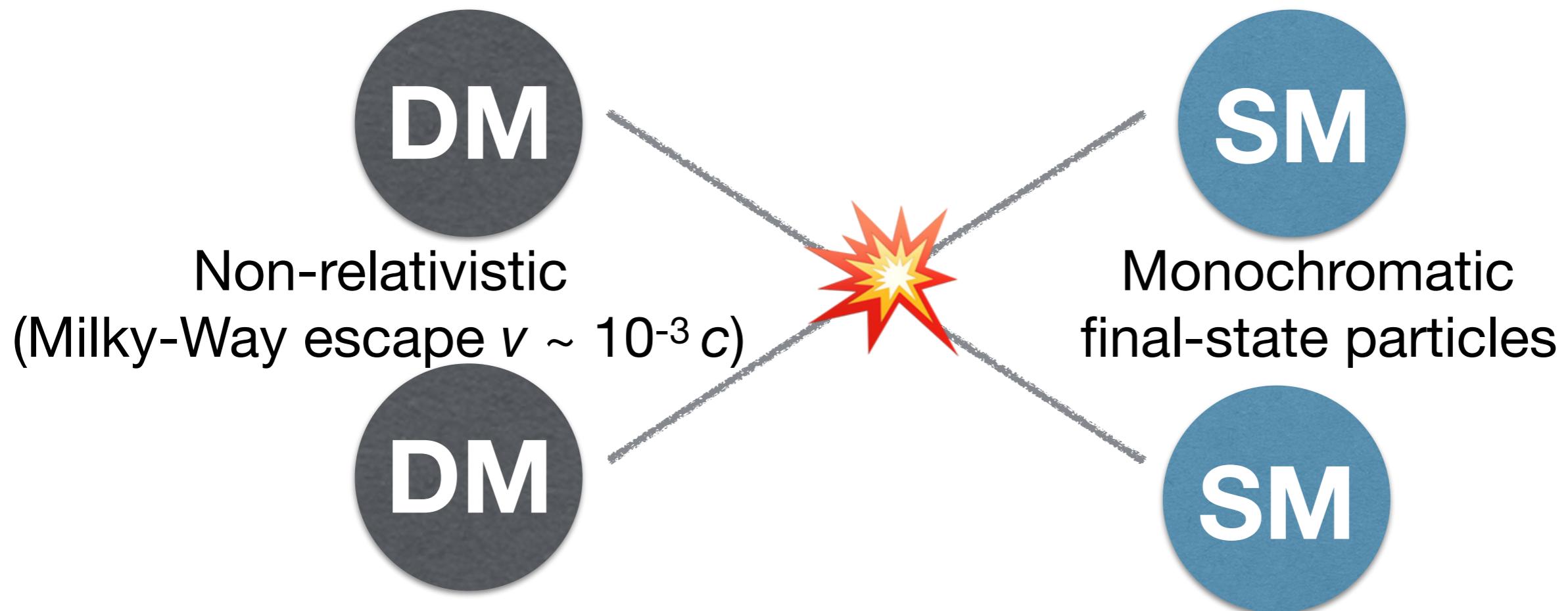
$$\Phi(E_\gamma) = \frac{1}{8\pi m_{\text{DM}}^2} \underbrace{\int_{\Delta\Omega} d\Omega \int_{\text{l.o.s.}} ds}_{J(\Delta\Omega)} \rho_{\text{DM}}^2(\mathbf{r}(s)) \frac{d}{dE_\gamma} [\sigma v]_{\gamma+X}$$

# $\gamma$ rays from dark matter annihilation. PP term



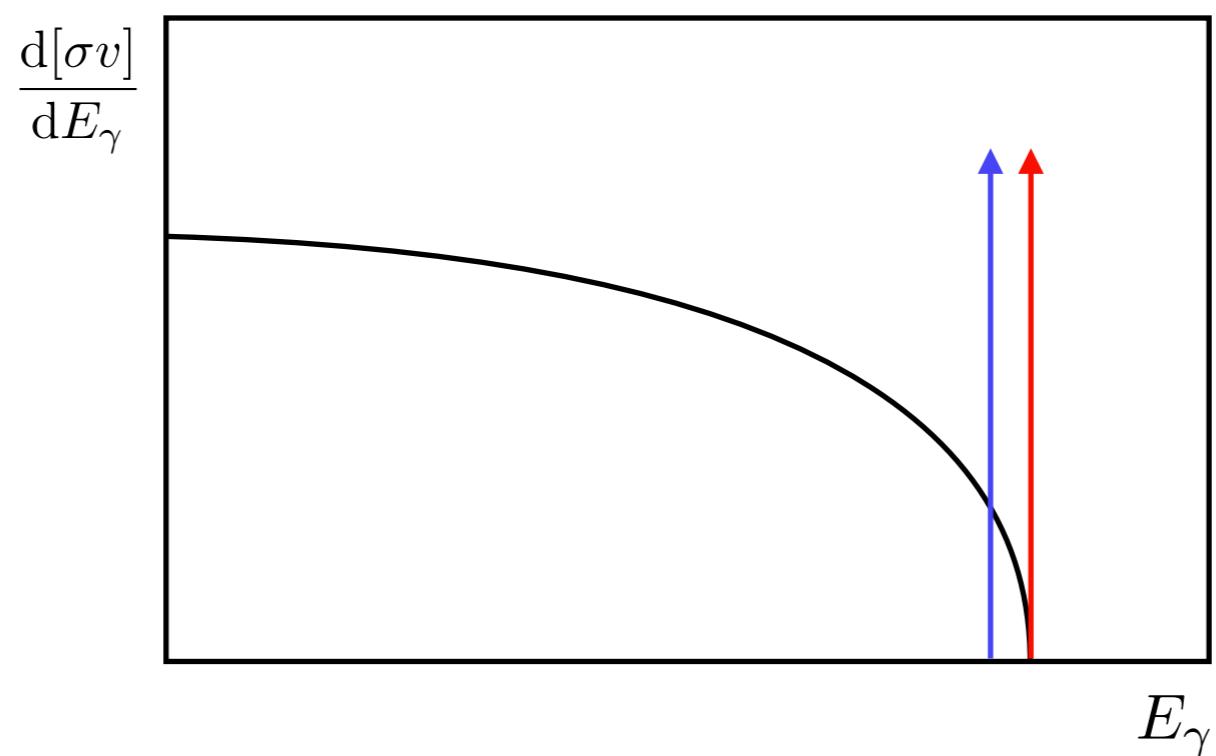
$$\Phi(E_\gamma) = \frac{1}{8\pi m_{\text{DM}}^2} \underbrace{\int_{\Delta\Omega} d\Omega \int_{\text{l.o.s.}} ds}_{J(\Delta\Omega)} \rho_{\text{DM}}^2(r(s)) \underbrace{\frac{d}{dE_\gamma} [\sigma v]_{\gamma+X}}_{}$$

# $\gamma$ rays from dark matter annihilation. Endpoint spectrum



# $\gamma$ rays from dark matter annihilation. Endpoint spectrum

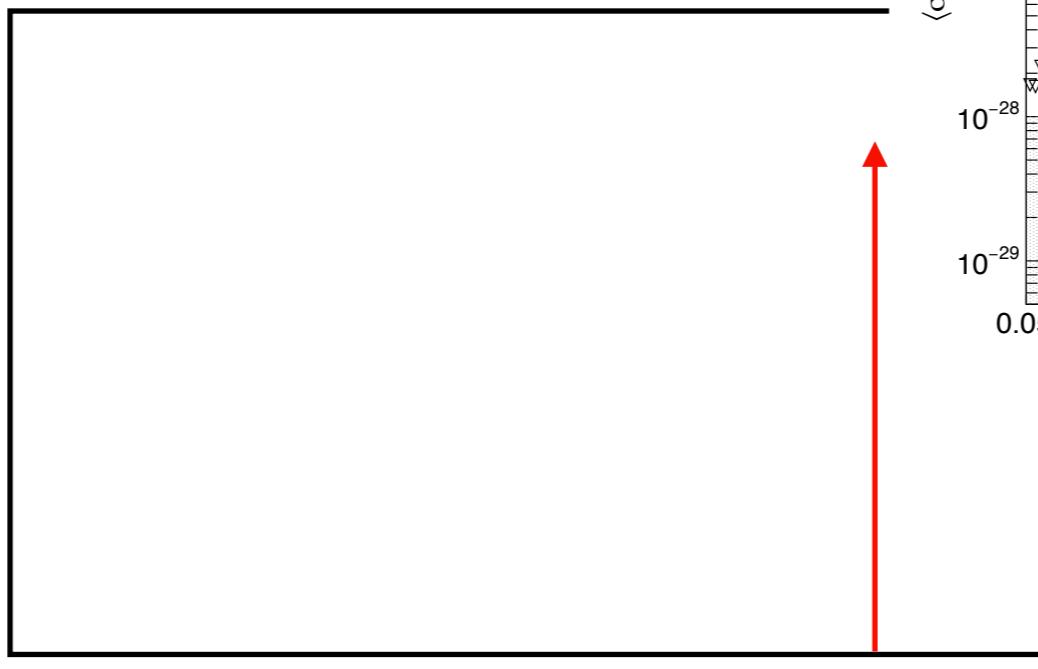
$$\begin{aligned} \frac{d}{dE_\gamma} [\sigma v]_{\gamma+X} = & 2[\sigma v]_{\gamma\gamma}\delta(E_\gamma - m_{\text{DM}}) + [\sigma v]_{\gamma Z}\delta(E_\gamma - E_0^{\gamma Z}) + \\ & + \frac{d}{dE_\gamma} [\sigma v]_{\gamma+N \geq 2-\text{bodies}} \end{aligned}$$



# $\gamma$ rays from dark matter annihilation. Endpoint spectrum

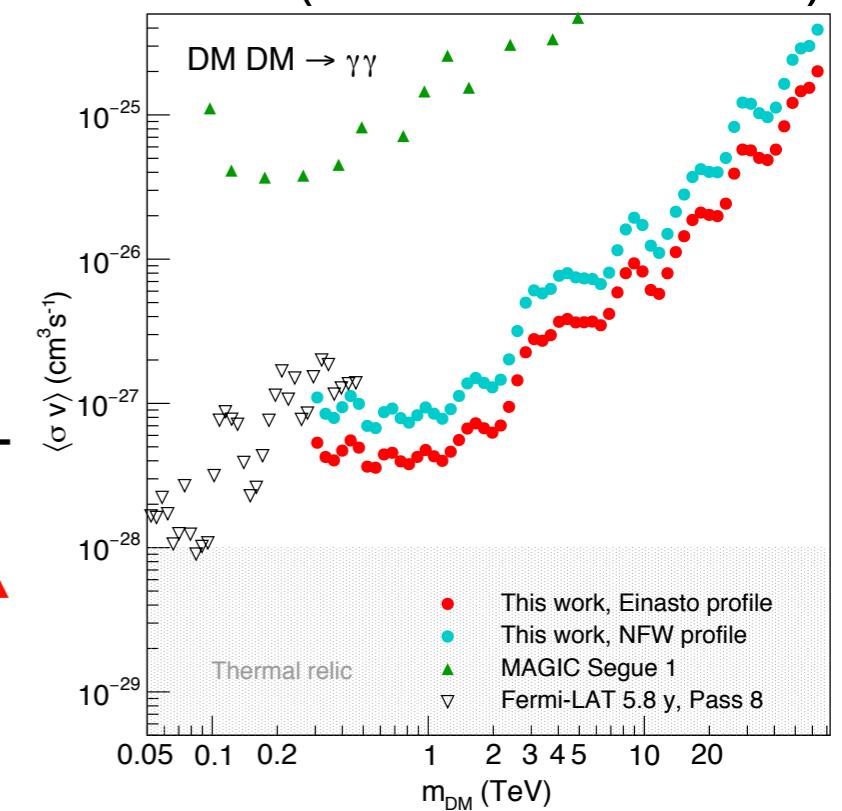
$$\frac{d}{dE_\gamma} [\sigma v]_{\gamma+X} = 2[\sigma v]_{\gamma\gamma} \delta(E_\gamma - m_{\text{DM}})$$

$$\frac{d[\sigma v]}{dE_\gamma}$$



**State of the art**

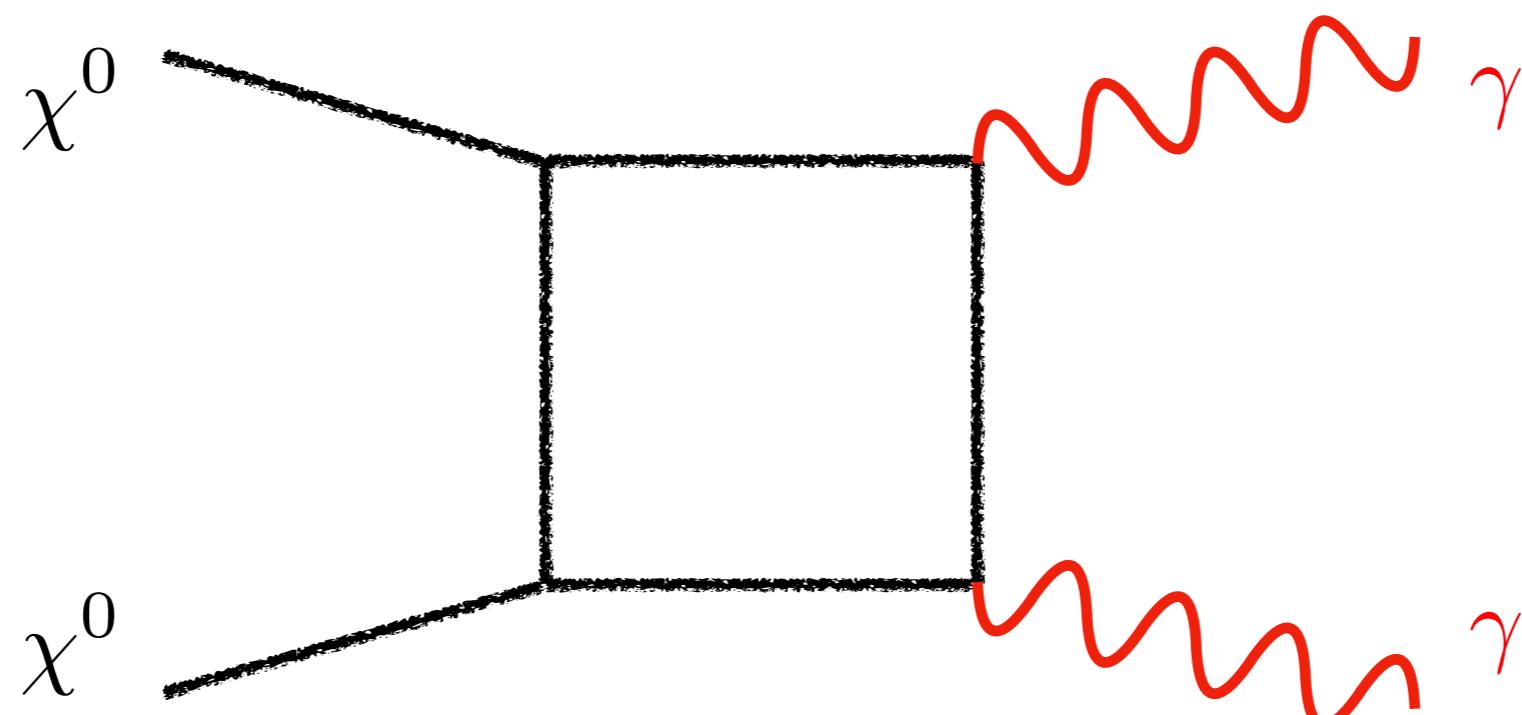
HESS (arXiv:1805.05741)



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# Naive computation of $\sigma V_{\gamma\gamma}$



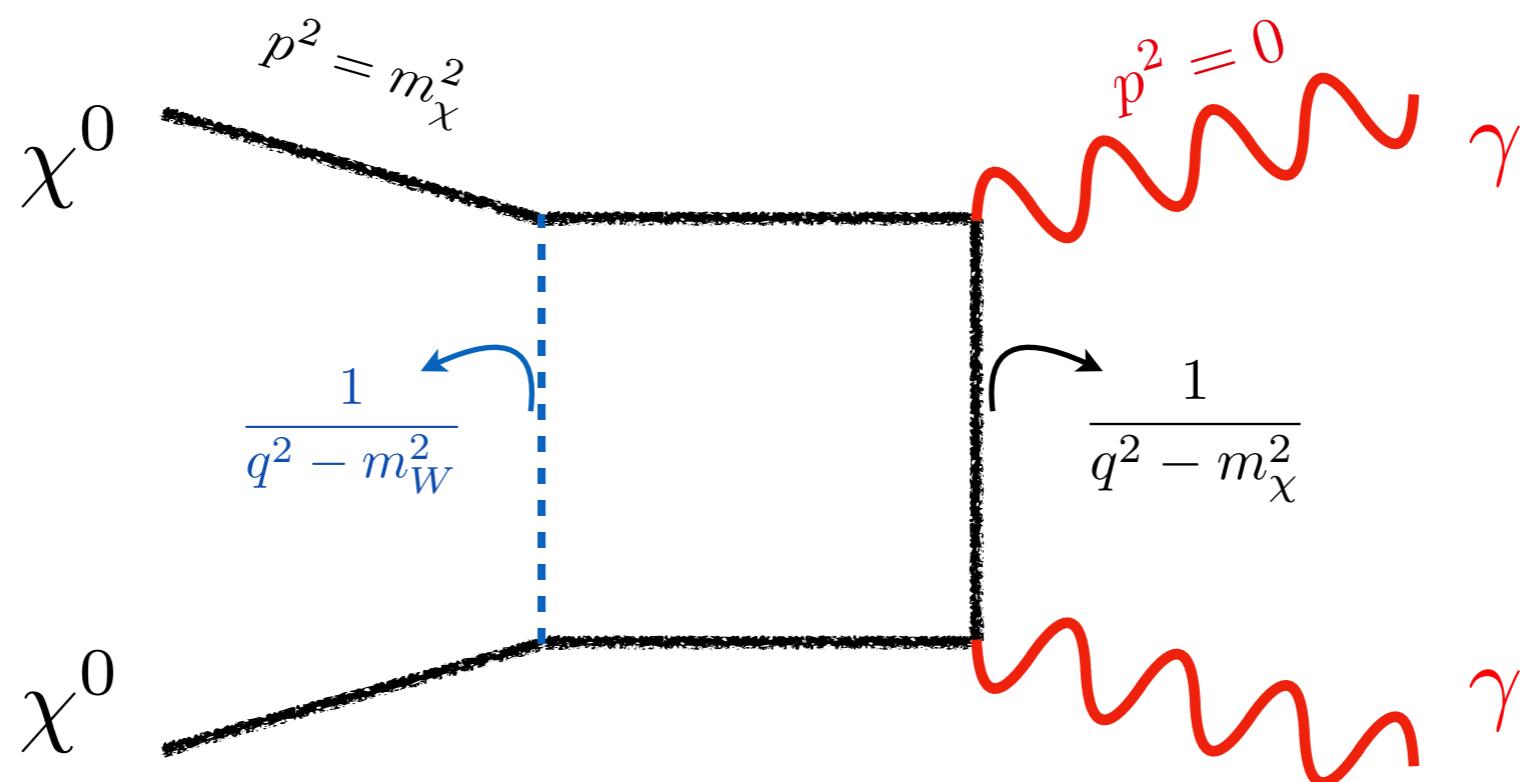
(naively) loop and  
mass suppressed

→

$$\Phi(E_\gamma) \sim \left( \frac{\alpha_{\text{EW}}}{m_{\text{DM}}} \right)^4$$

**(do not give up yet!!)**

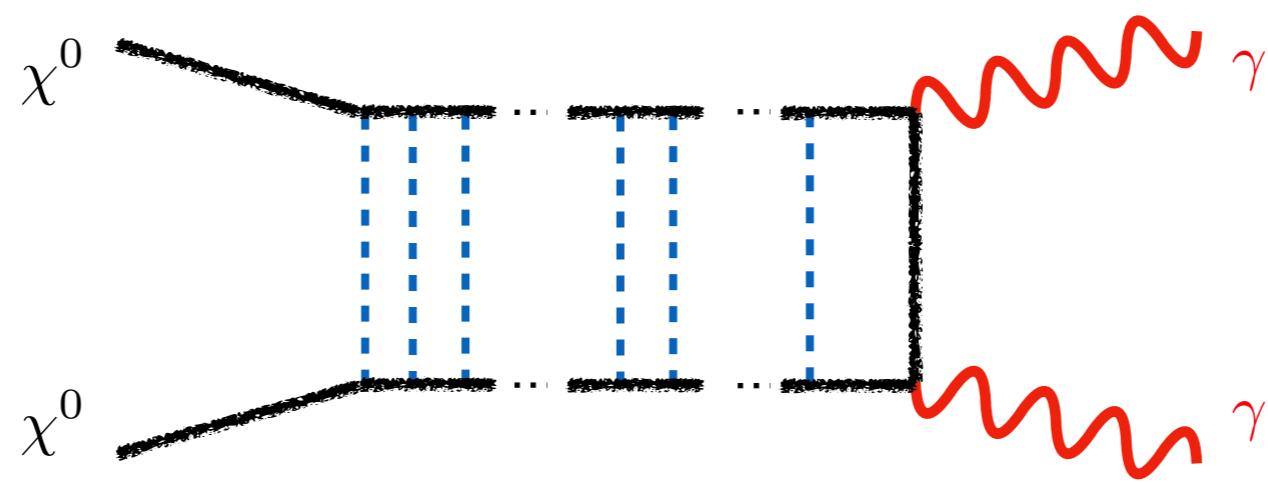
# Naive computation of $\sigma V_{\gamma\gamma}$



$$\mathcal{M}_{\text{So}} \sim \frac{g^4 m_\chi^2}{m_W^2} \gg g^2$$

# Naive computation of $\sigma V_{\gamma\gamma}$

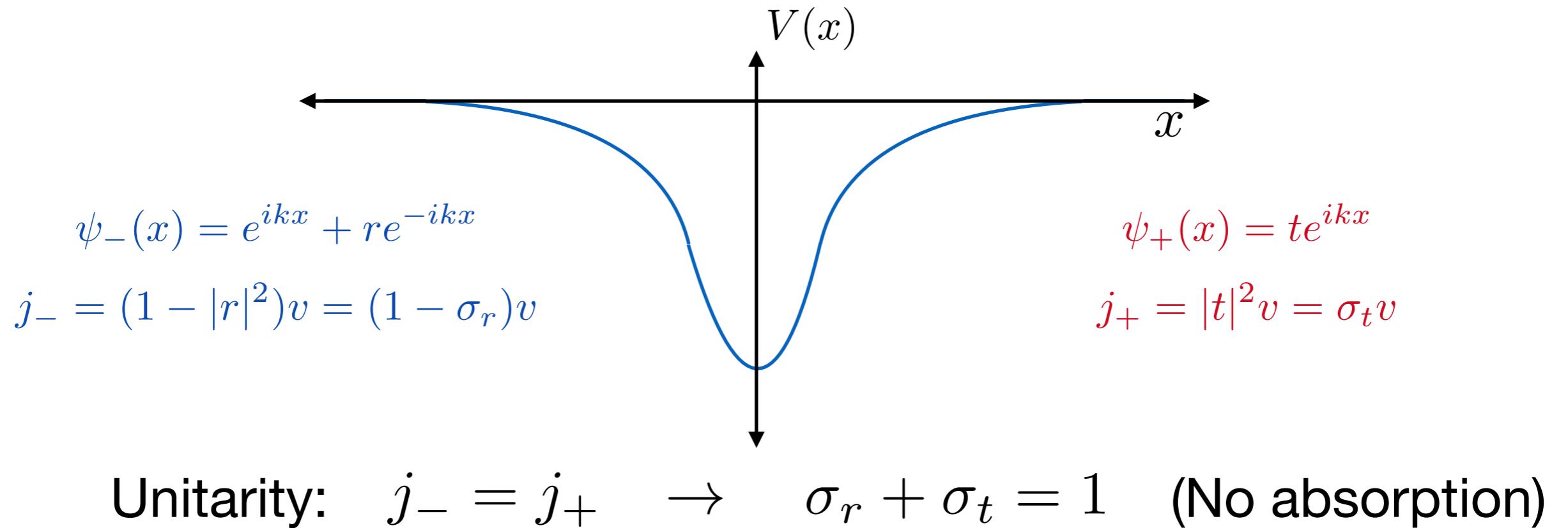
**Solution:** resum all ladder-like diagrams by matching onto a non-relativistic effective theory



# Sommerfeld effect (Scattering states in 1D QM)

$$\left( -\frac{1}{m_\chi} \frac{d^2}{dx^2} + V(x) \right) \psi(x) = E \psi(x)$$

$$j(x) = \frac{i}{m_\chi} [\psi(x)\psi'^*(x) - \psi^*(x)\psi'(x)] = \text{const.}$$



# Sommerfeld effect (Scattering states in 1D QM)

$$\left( -\frac{1}{m_\chi} \frac{d^2}{dx^2} + V(x) + \frac{i}{2} \sigma_a^{(0)} v \delta(x) \right) \psi(x) = E \psi(x)$$

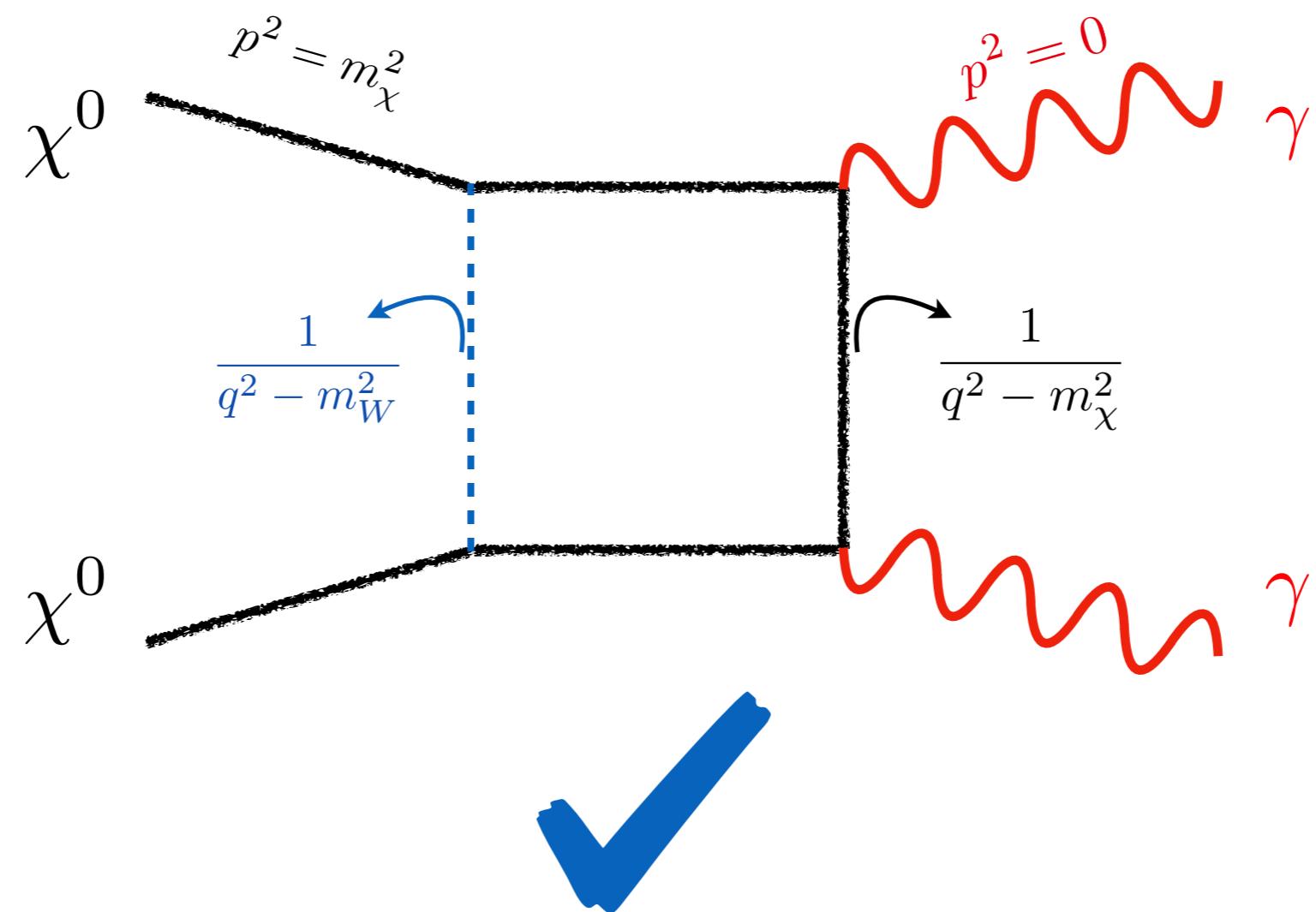
Unitarity-violating term  $\rightarrow j_+ = j_- + |\psi(0)|^2 \sigma_a v$

$$\sigma_r + \sigma_t + \sigma_a = 1$$

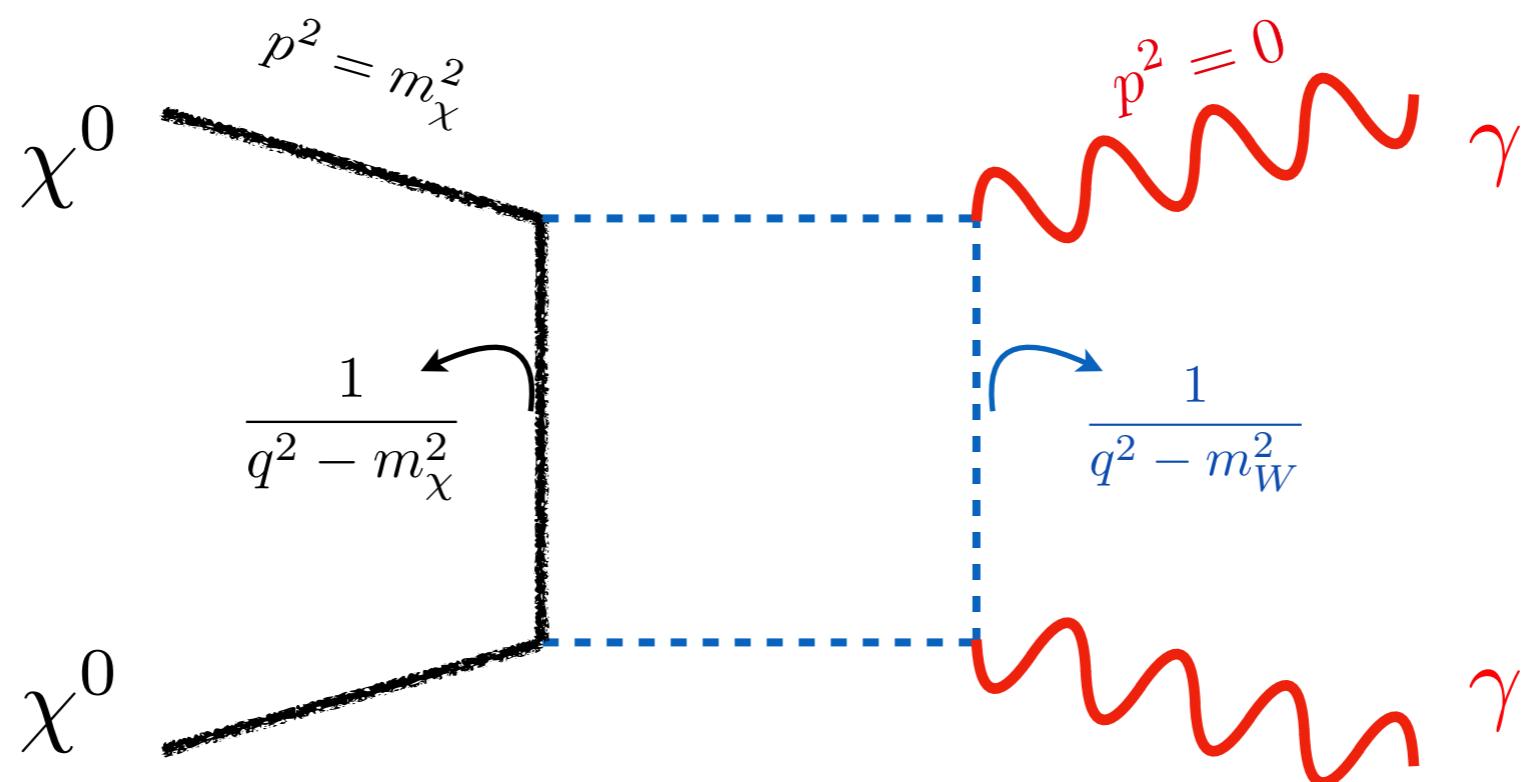
$$\sigma_a = |\psi(0)|^2 \sigma_a^{(0)}$$

**Resummed cross section** = Sommerfeld factor  $\times$  QFT cross section  
 $\times$  (long range physics)  $\times$  (short range physics)

# Naive computation of $\sigma V_{\gamma\gamma}$



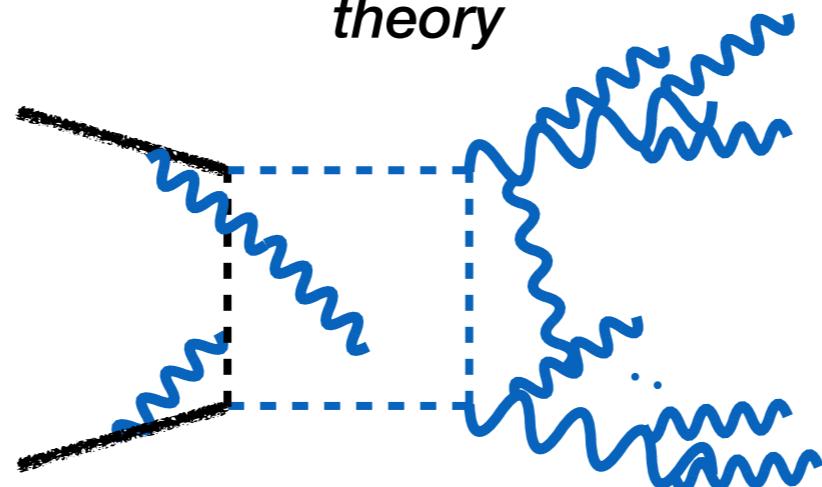
# Sudakov double logarithms



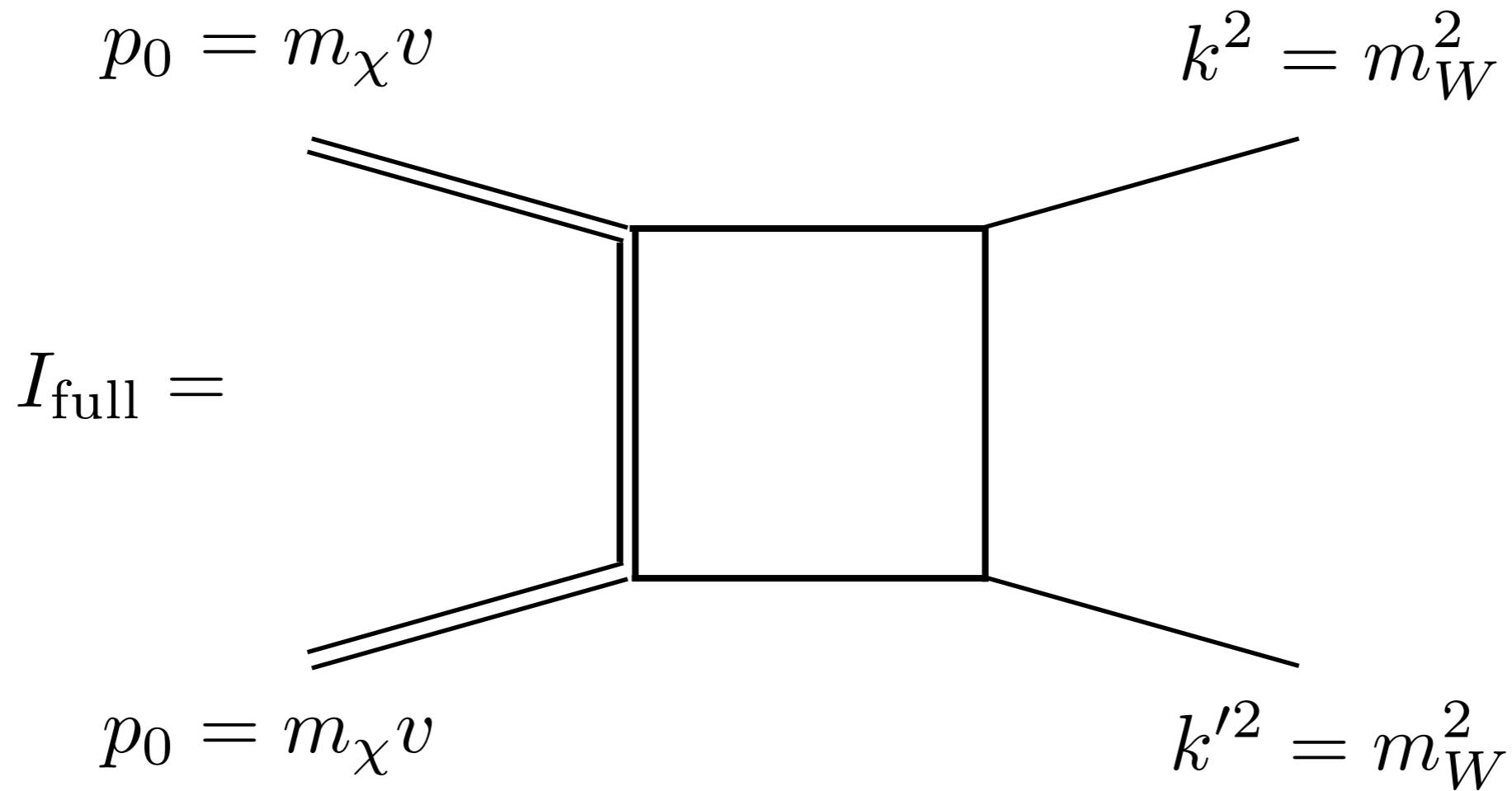
$$\mathcal{M} \sim g^4 \log^2 \frac{4m_\chi^2}{m_W^2} \gg g^2$$

# Sudakov-log resummation

Standard solution: resum soft virtual and real emissions by solving renormalization group eqs. in a *(soft-collinear) effective field theory*



# Soft-collinear effective theory (SCET). Method of regions



# SCET. Momentum regions

$$I_{\text{full}} = \text{Diagram} = \int \frac{d^D q}{(2\pi)^D} \frac{1}{(q+k-p_0)^2 - m_\chi^2} \frac{1}{(q+k)^2} \frac{1}{q^2} \frac{1}{(q-k')^2} \Big|_{k^2, k'^2 \sim m_W^2 \ll m_\chi^2}$$

Light-cone  
coordinates

$$q = q_c n + q_{\bar{c}} \bar{n} + q_\perp \rightarrow (q_c, q_{\bar{c}}, q_\perp)$$

## Momentum modes

$$q_h \sim m_\chi (1, 1, 1)$$

$$q_s \sim m_W (1, 1, 1)$$

$$q_{hc} \sim (m_W, m_\chi, \sqrt{m_\chi m_W})$$

$$q_{h\bar{c}} \sim (m_\chi, m_W, \sqrt{m_\chi m_W})$$

$$q_c \sim \left( \frac{m_W^2}{m_\chi}, m_\chi, m_W \right)$$

$$q_{\bar{c}} \sim \left( m_\chi, \frac{m_W^2}{m_\chi}, m_W \right)$$

# SCET. Momentum regions

$$I_{\text{full}} =$$
$$+ \qquad + \qquad + \qquad | \quad k^2 = 0$$

+ power corrections

# NRDM $\times$ SCET for DM annihilation

After several steps one can prove that:

$$\frac{d}{dE_\gamma} [\sigma v] = |\psi(0)|^2 \times |C|^2(\mu) \times Z_\gamma(\mu, \nu) \times J(\mu, \nu) \otimes W(\mu, \nu)$$

**Resummation is achieved by solving**

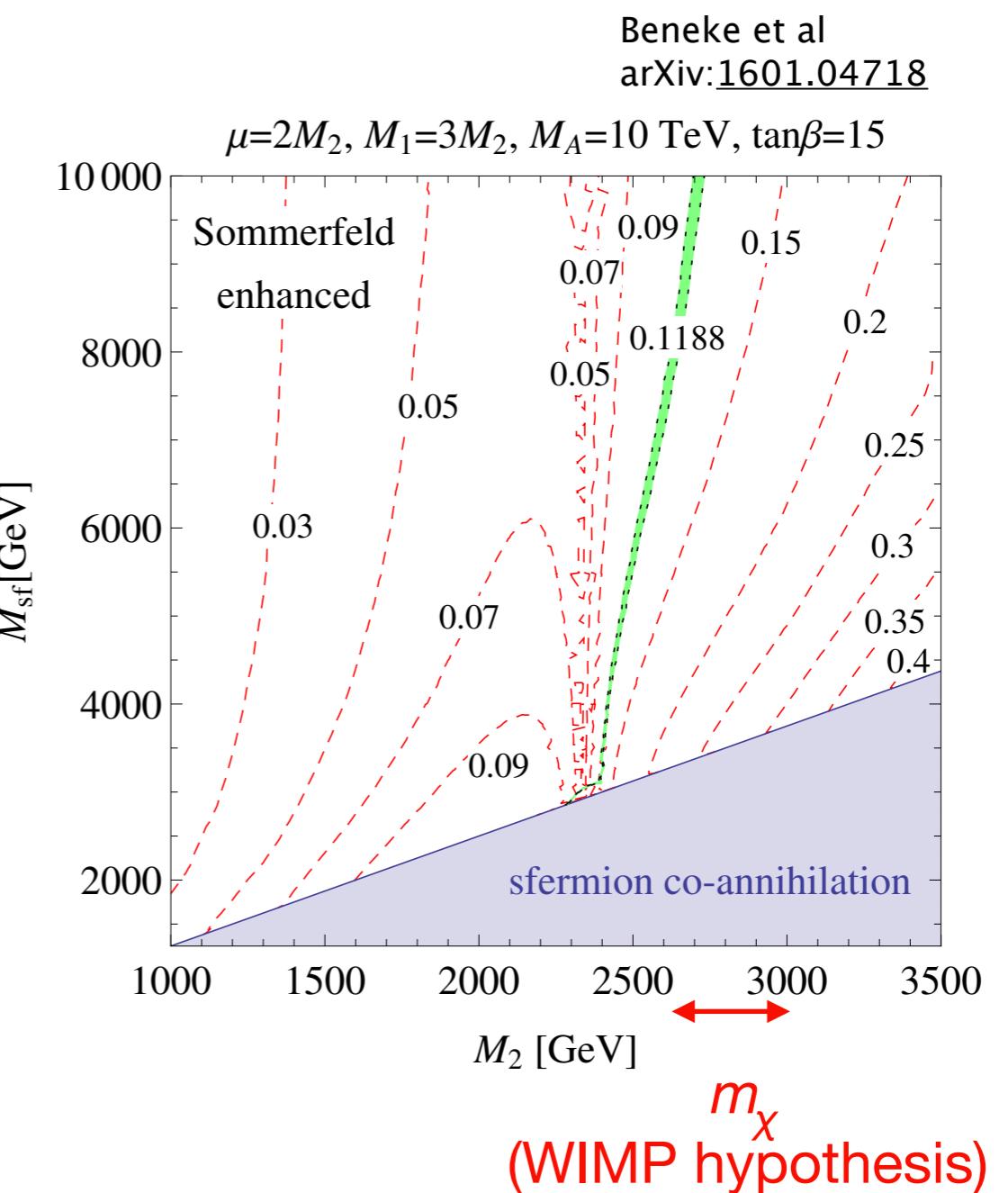
- an appropriate Schrödinger equation
- $\mu$  and  $\nu$  renormalization group equations for every piece of the factorization formula

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# The wino-like/MDM triplet model

- Add
$$\delta\mathcal{L}_{\text{Wino}} = \frac{1}{2}\bar{\chi}(i\gamma^\mu D_\mu - m_\chi)\chi$$
to the Lagrangian density of the standard model
- After electro-weak symmetry breaking: two mass eigenstates  $\chi^0$  (the DM) and  $\chi^+$
- Limiting case of the SUSY neutralino ( $M_{\text{sf}} > 20M_2$  in the plot)
- *Direct Detection:* below neutrino floor
- *LHC:* much too heavy to be observed



# Factorization theorem. Sommerfeld effect

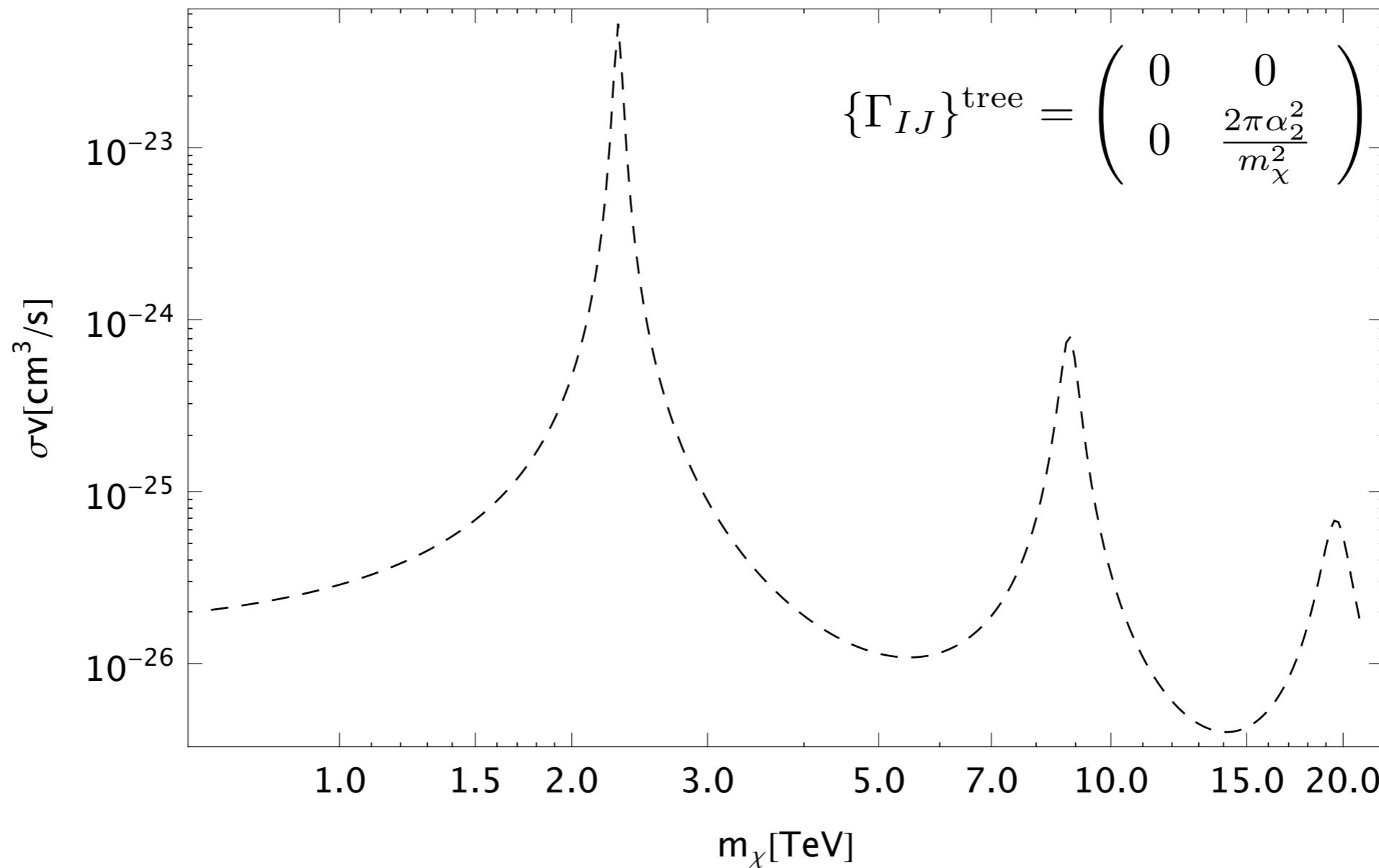
$$\frac{d(\sigma v_{\text{rel}})}{dE_\gamma} = \sum_{I,J} S_{IJ} \Gamma_{IJ}(E_\gamma)$$

Sommerfeld matrix  
 $I, J = (\chi^0 \chi^0) \text{ or } (\chi^+ \chi^-)$

$$V(r) = \begin{pmatrix} 0 & -\sqrt{2}\alpha_2 \frac{e^{-m_W r}}{r} \\ -\sqrt{2}\alpha_2 \frac{e^{-m_W r}}{r} & -\frac{\alpha}{r} - \alpha_2 c_W^2 \frac{e^{-m_Z r}}{r} \end{pmatrix}$$

see e.g. Beneke et al arXiv: [1411.6924](#)  
Hisano arXiv: [hep-ph/0412403](#)

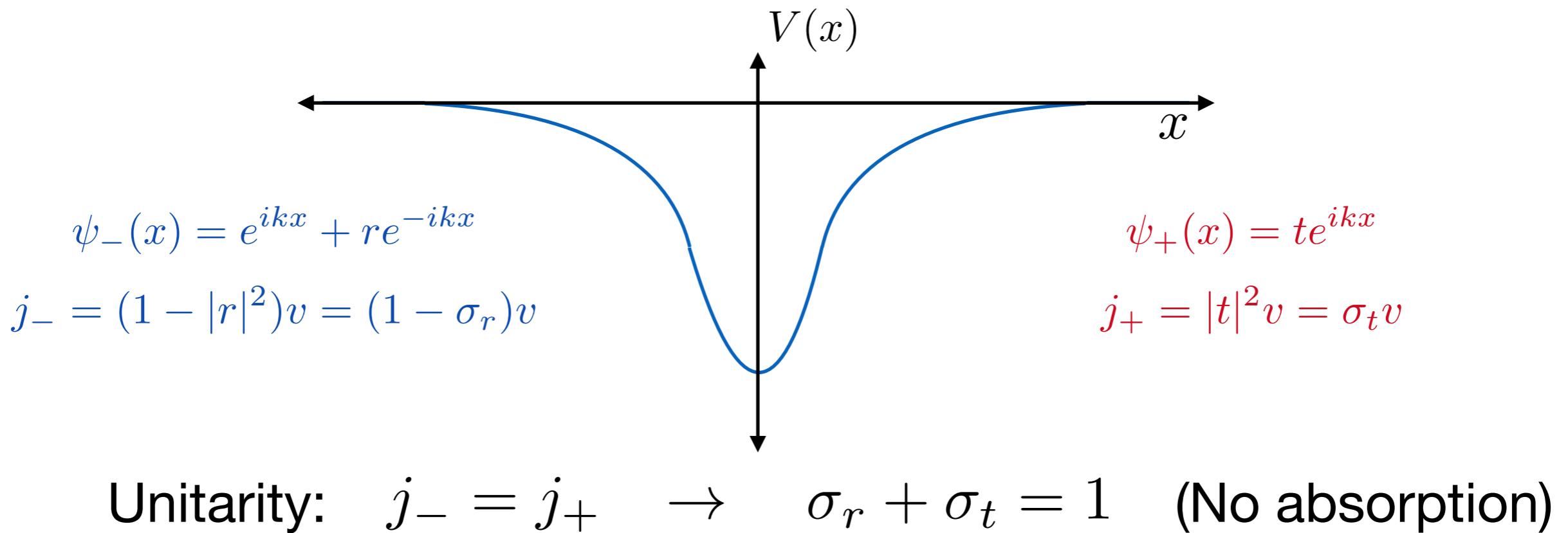
# Factorization theorem. Sommerfeld effect



# 1D QM (revisited)

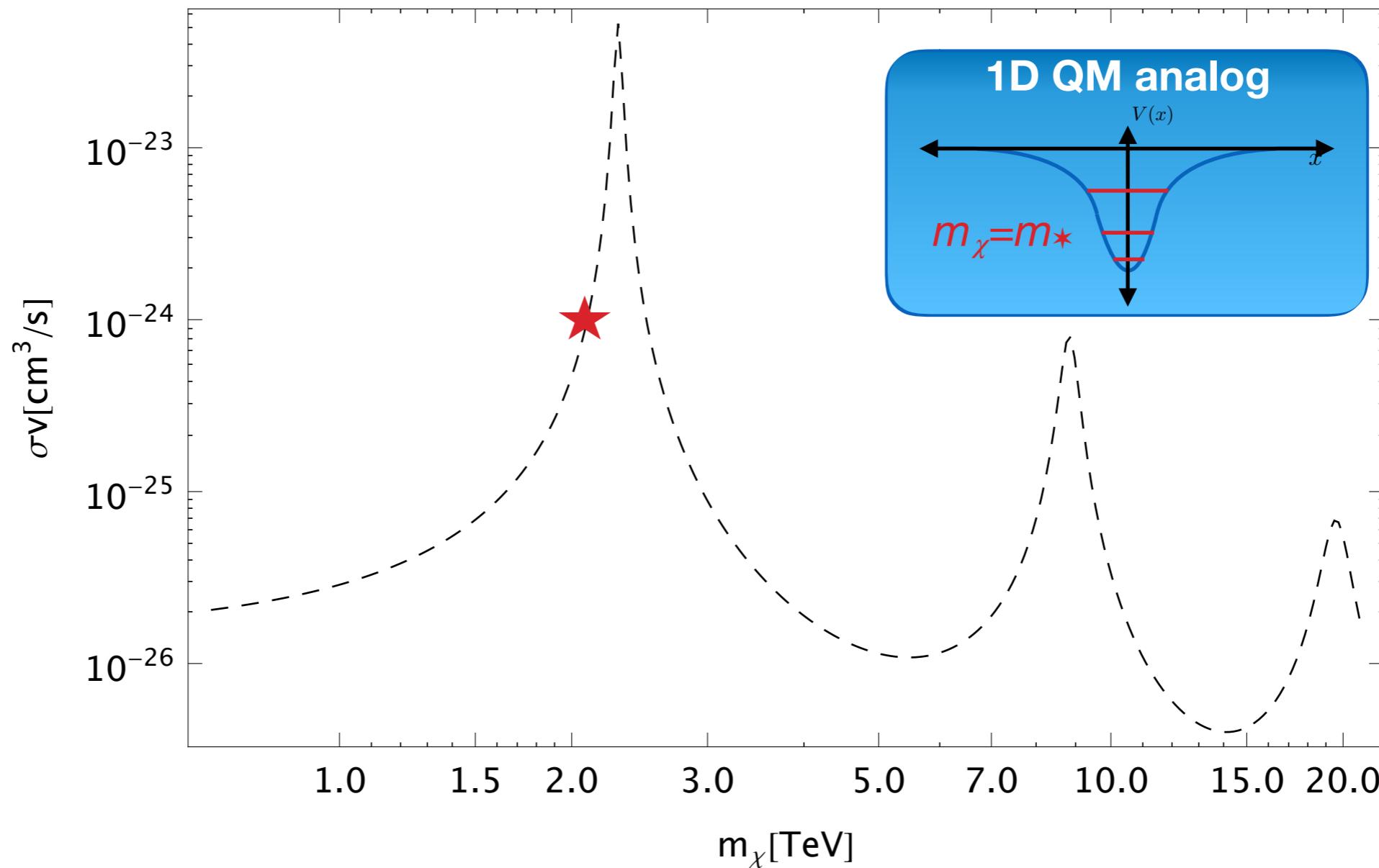
$$\left( -\frac{1}{m_\chi} \frac{d^2}{dx^2} + V(x) \right) \psi(x) = E \psi(x)$$

$$j(x) = \frac{i}{m_\chi} [\psi(x)\psi'^*(x) - \psi^*(x)\psi'(x)] = \text{const.}$$



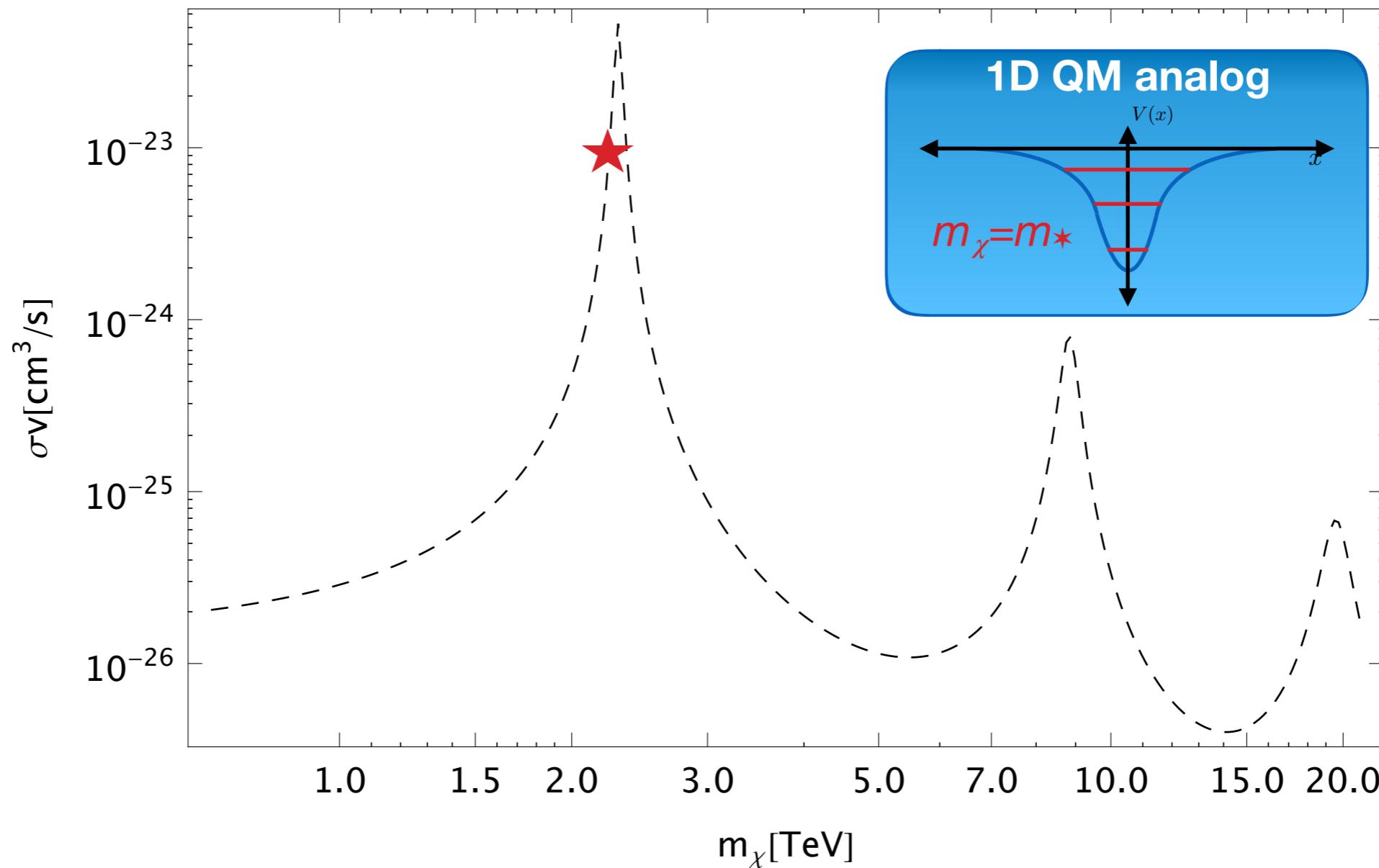
# Factorization theorem.

# Sommerfeld effect



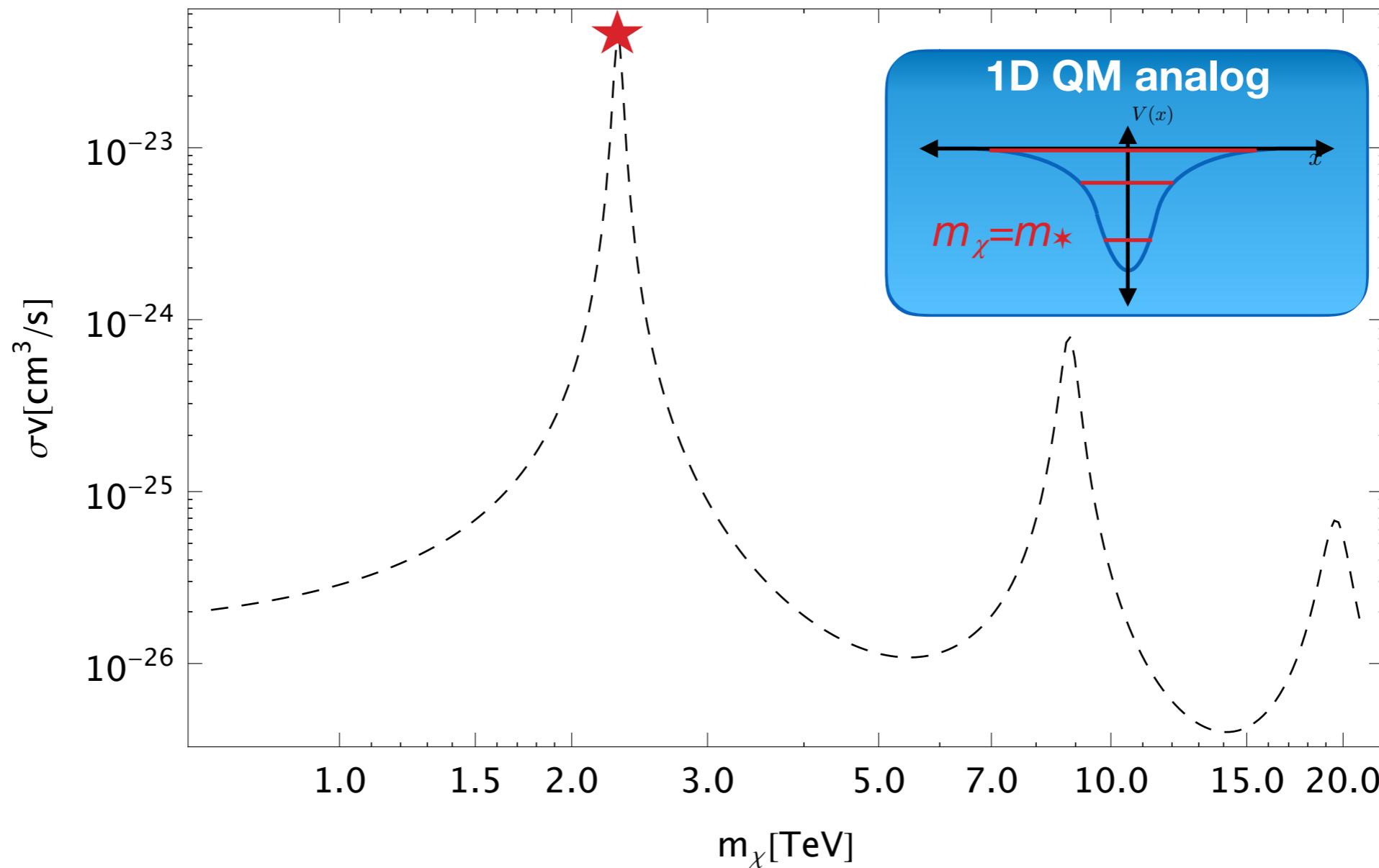
# Factorization theorem.

# Sommerfeld effect



# Factorization theorem.

# Sommerfeld effect



# Factorization theorem. Sommerfeld effect

$$\frac{d(\sigma v_{\text{rel}})}{dE_\gamma} = \sum_{I,J} S_{IJ} \Gamma_{IJ}(E_\gamma)$$

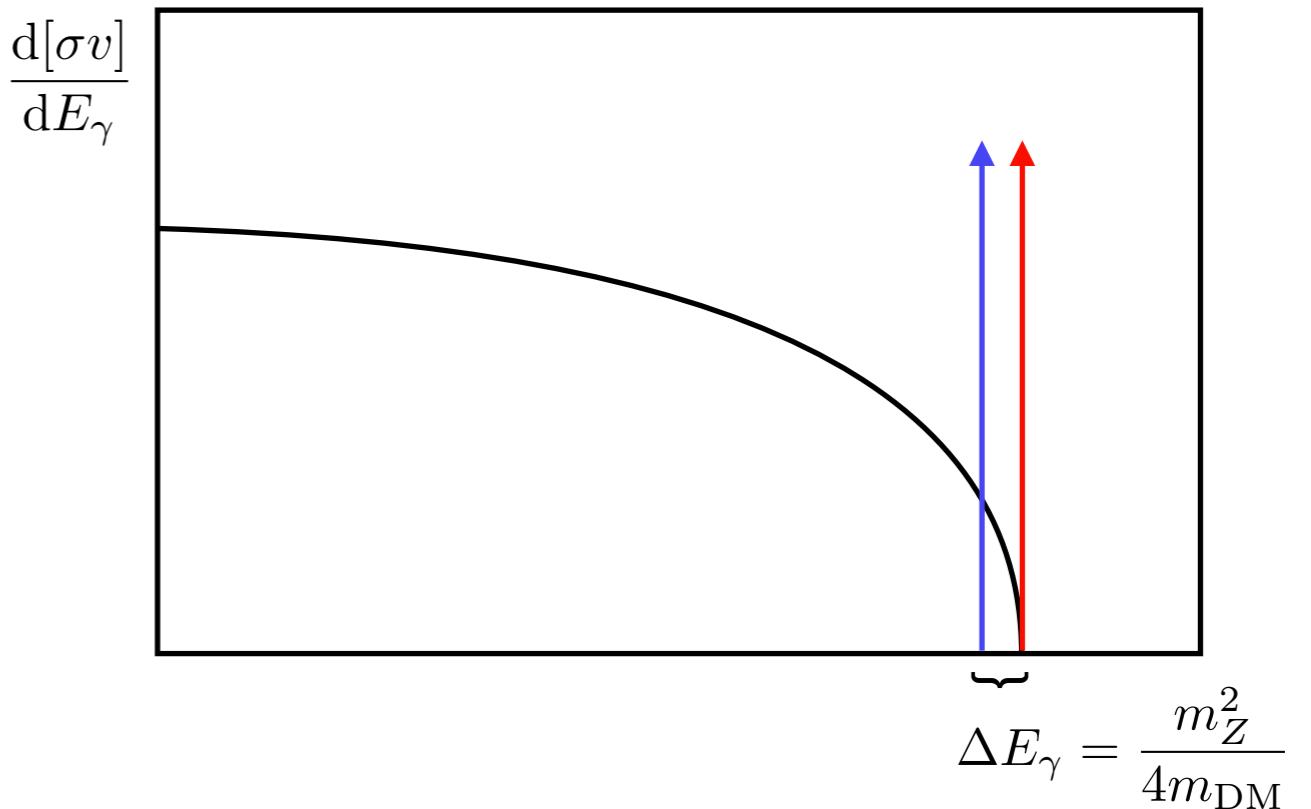
# Assumptions on the energy resolutions

The variable  $E_{\text{res}} = m_\chi - E_\gamma$  plays a decisive role in the factorization problem

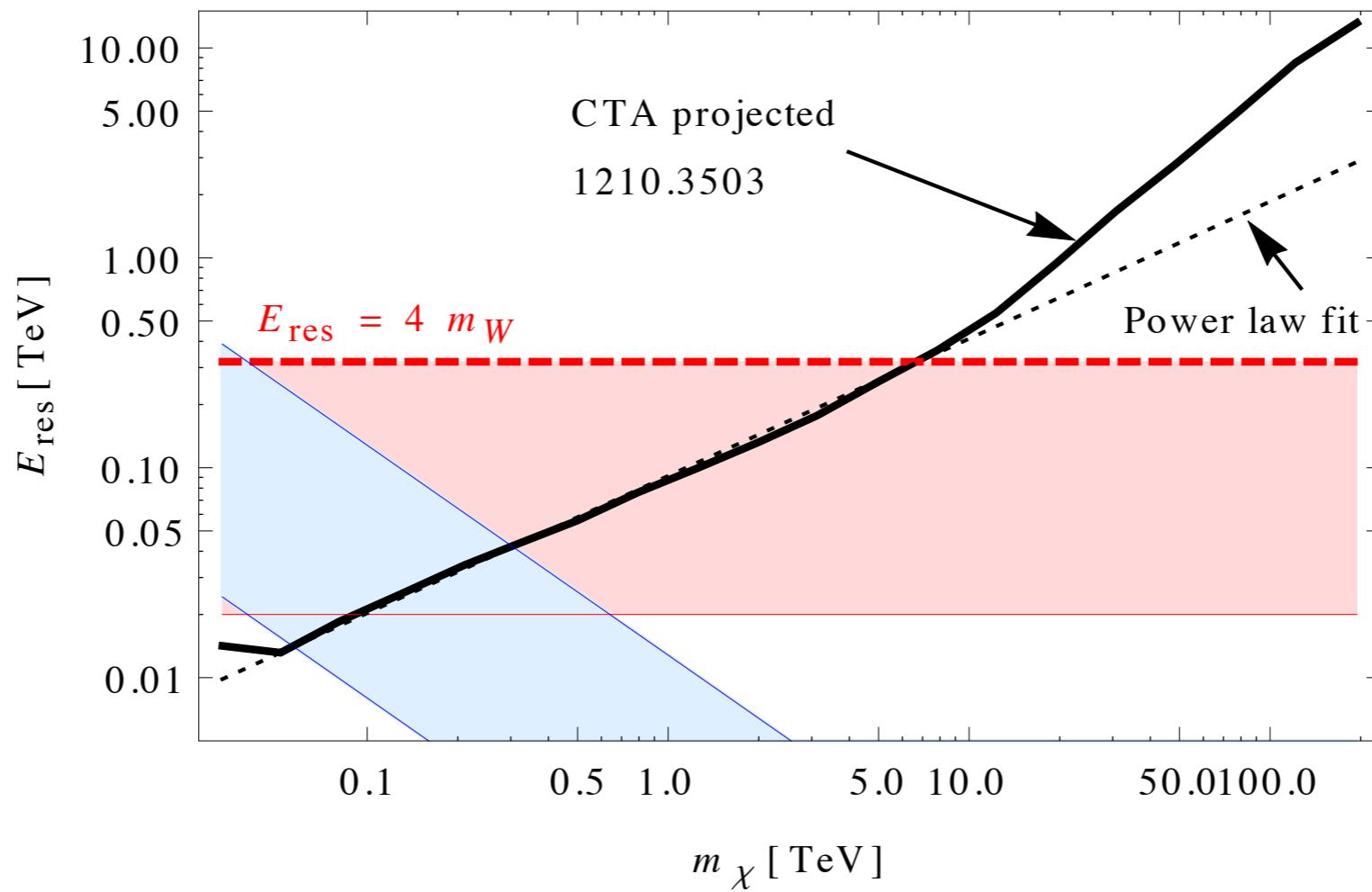
We investigated two situations

$E_{\text{res}} \sim m_W^2/m_\chi$  ([1805.07367](#))

$E_{\text{res}} \sim m_W$  ([1903.08702](#))



# Energy resolution



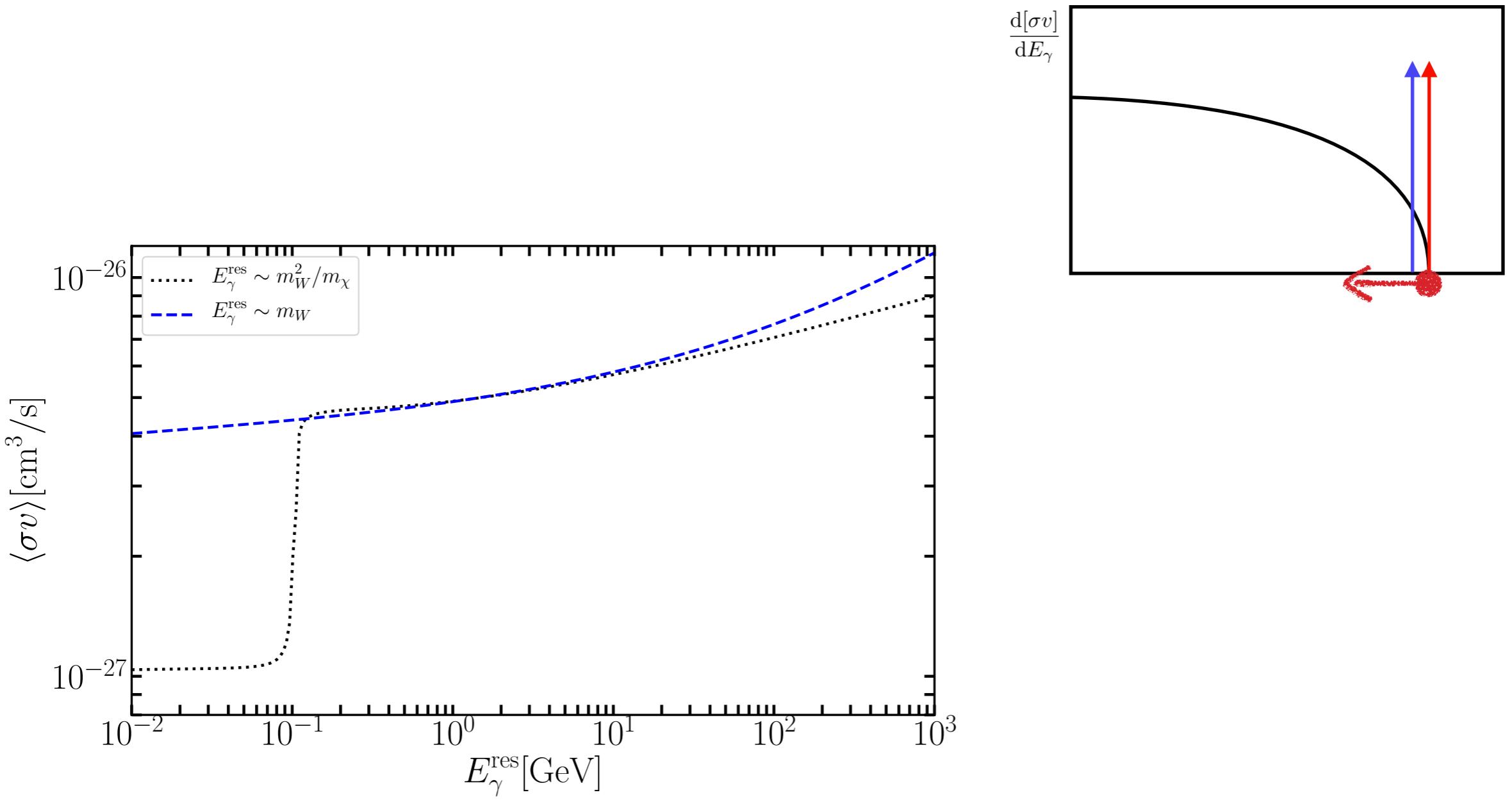
# Factorization theorem. Exclusive Wino $\chi\chi \rightarrow \gamma + X$ annihilation

$$\frac{d(\sigma v_{\text{rel}})}{dE_\gamma} = \sum_{I,J} S_{IJ} \Gamma_{IJ}(E_\gamma)$$

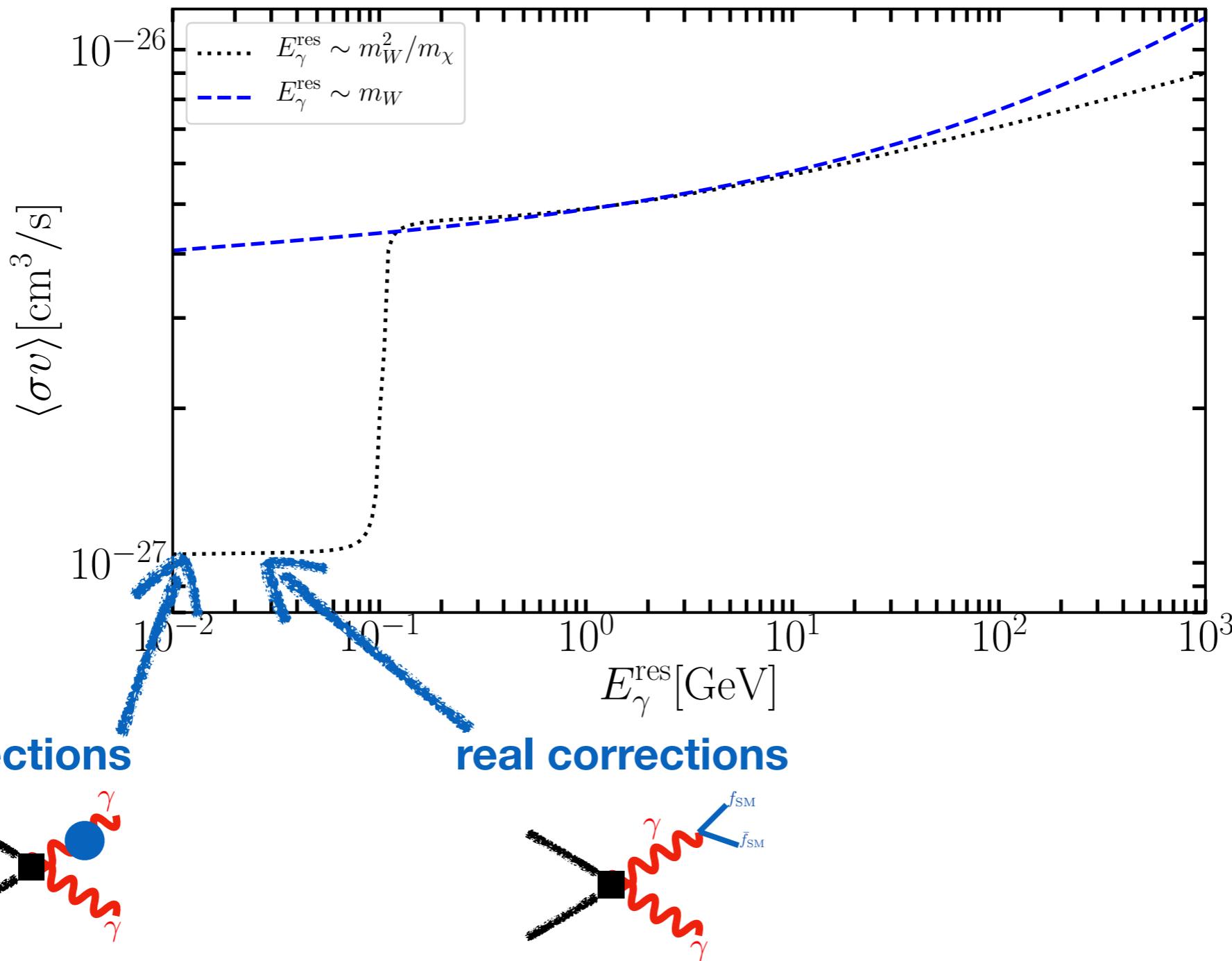
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$$\begin{aligned} \Gamma_{IJ}(E_\gamma) &= \frac{1}{4} \frac{2}{\pi m_\chi} \sum_{i,j=1,2} C_j^*(\mu_W) C_i(\mu_W) Z_\gamma(\mu_W, \nu_W) \\ &\times \int J(4m_\chi(m_\chi - E_\gamma - \omega/2), \mu_W) W_{IJ}^{ij}(\omega, \mu_W, \nu_W) \end{aligned}$$

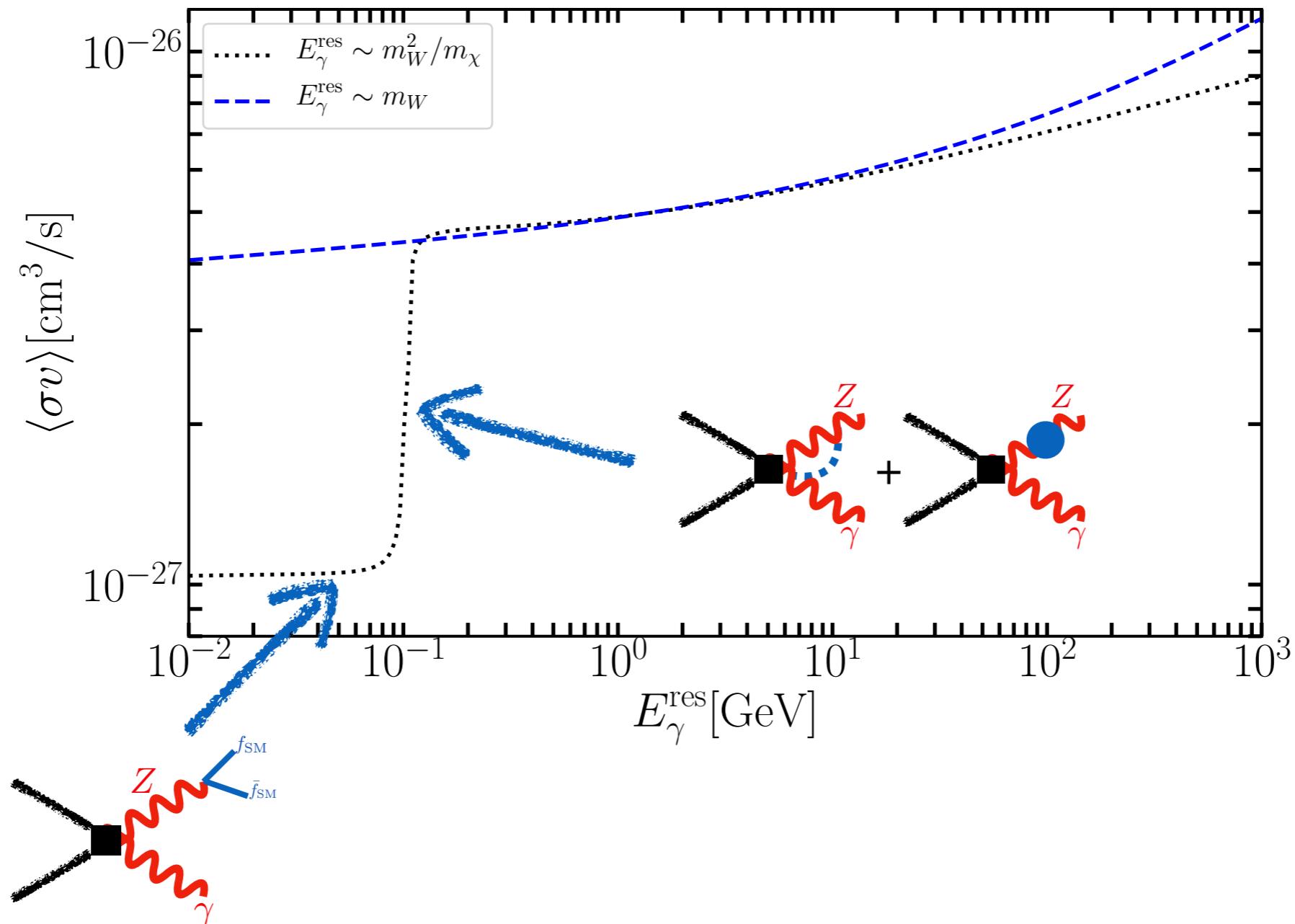
# Energy-integrated cross section



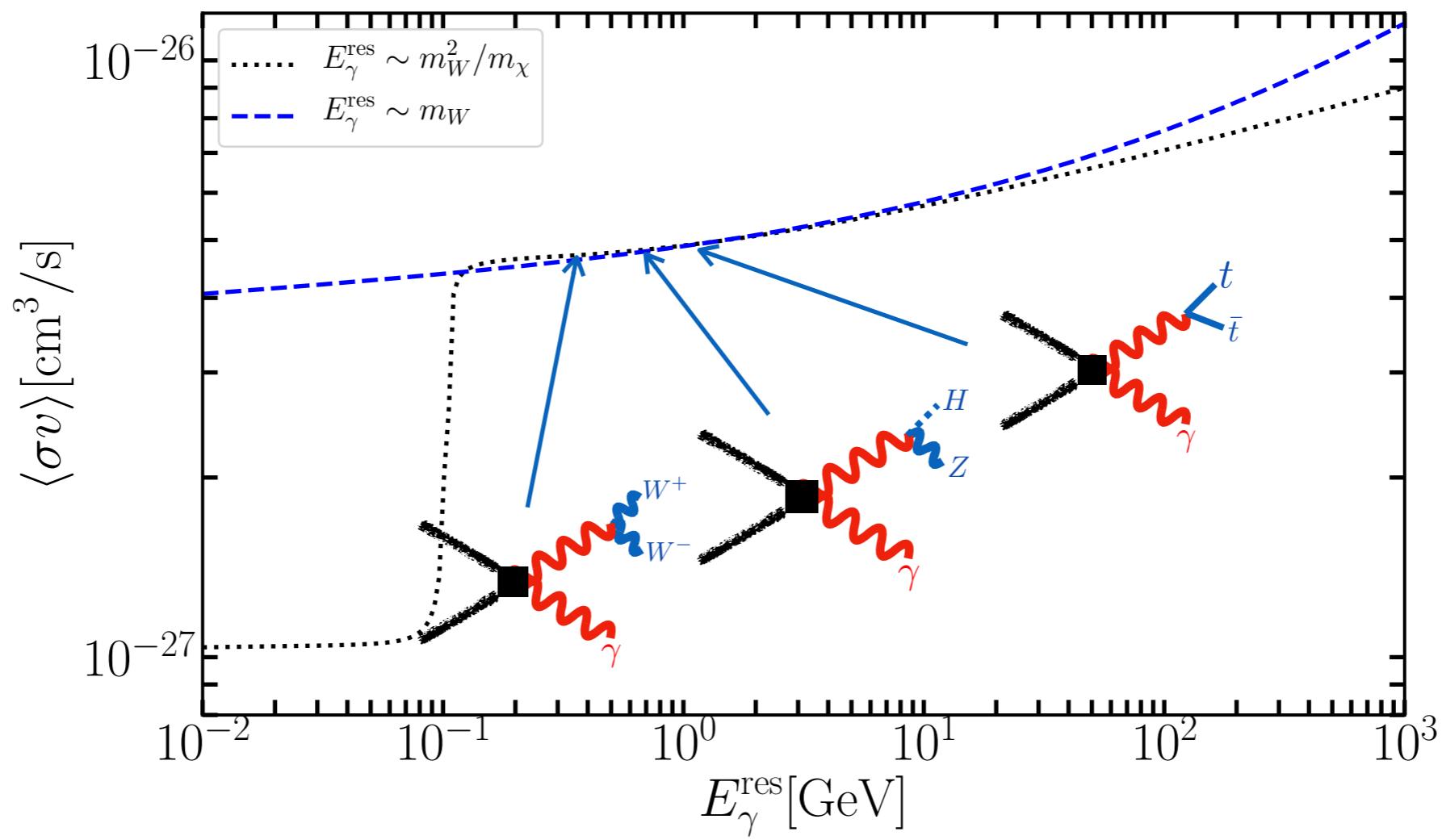
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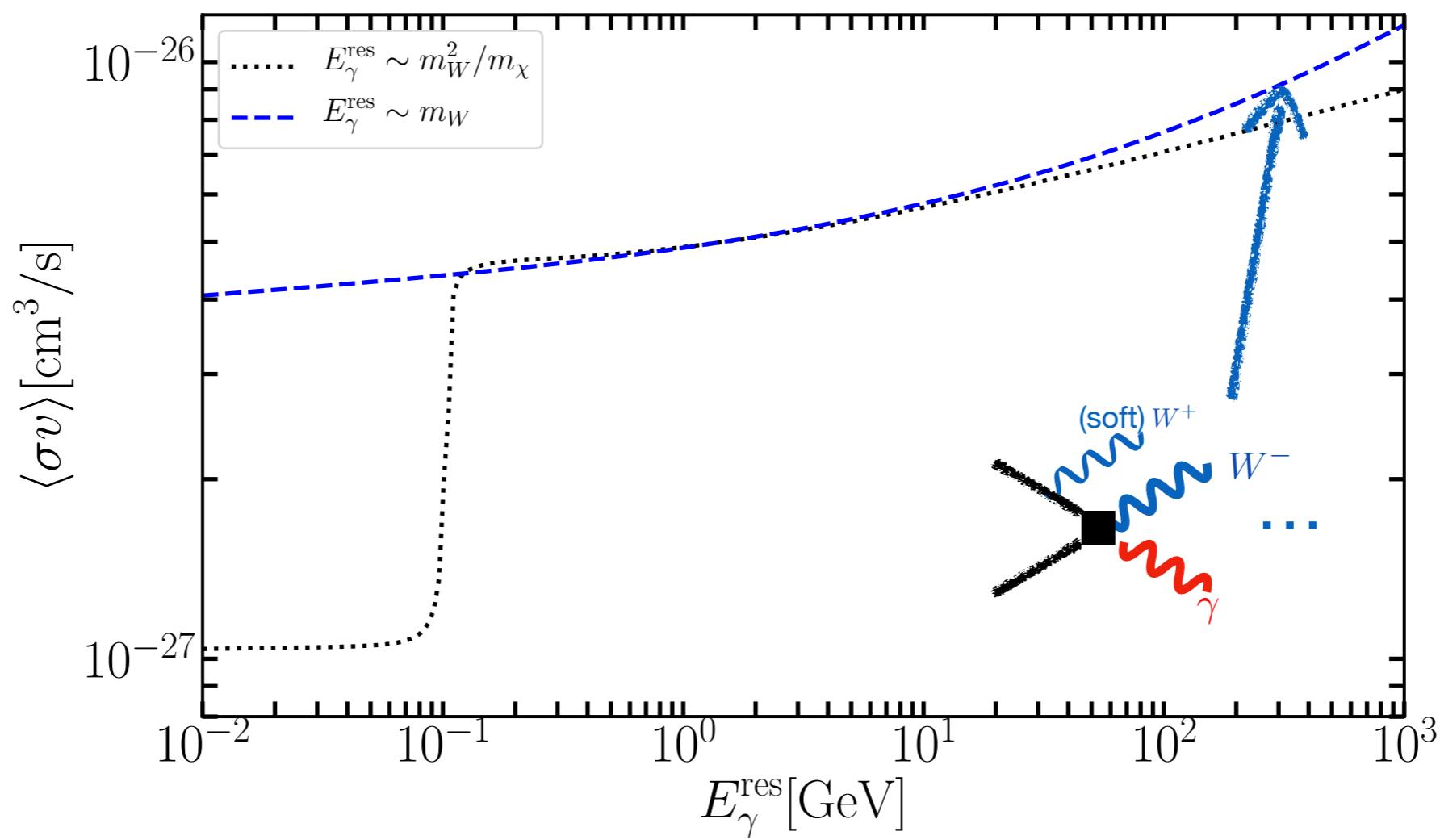
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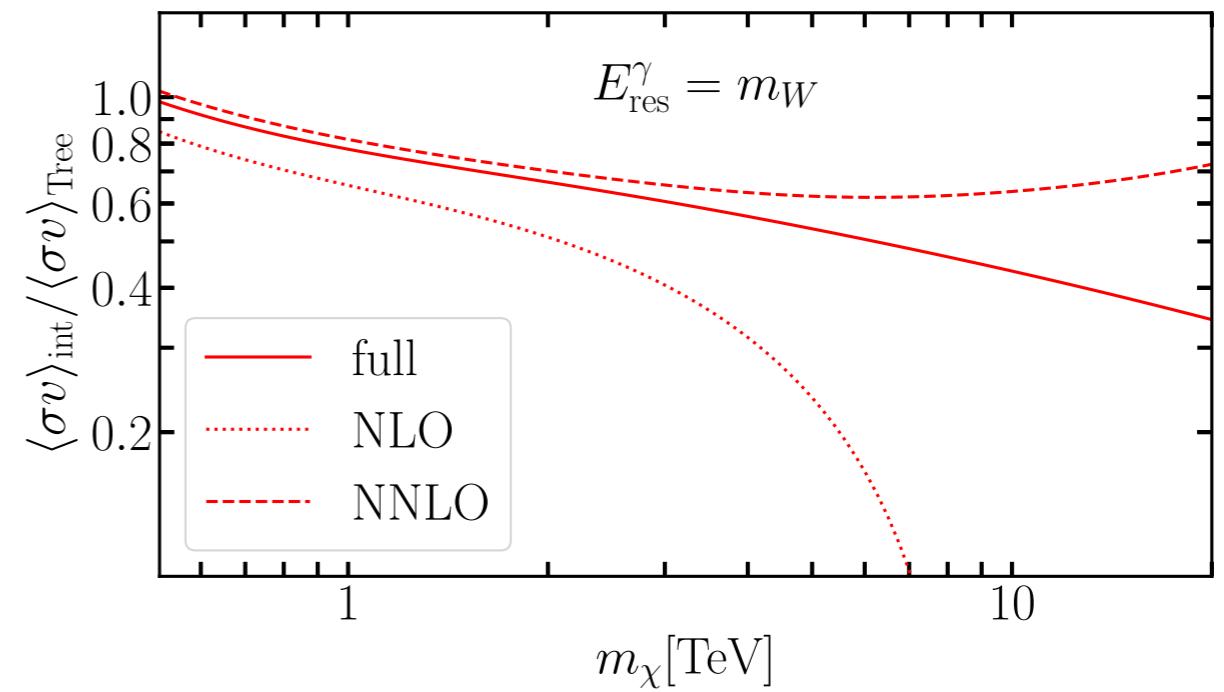
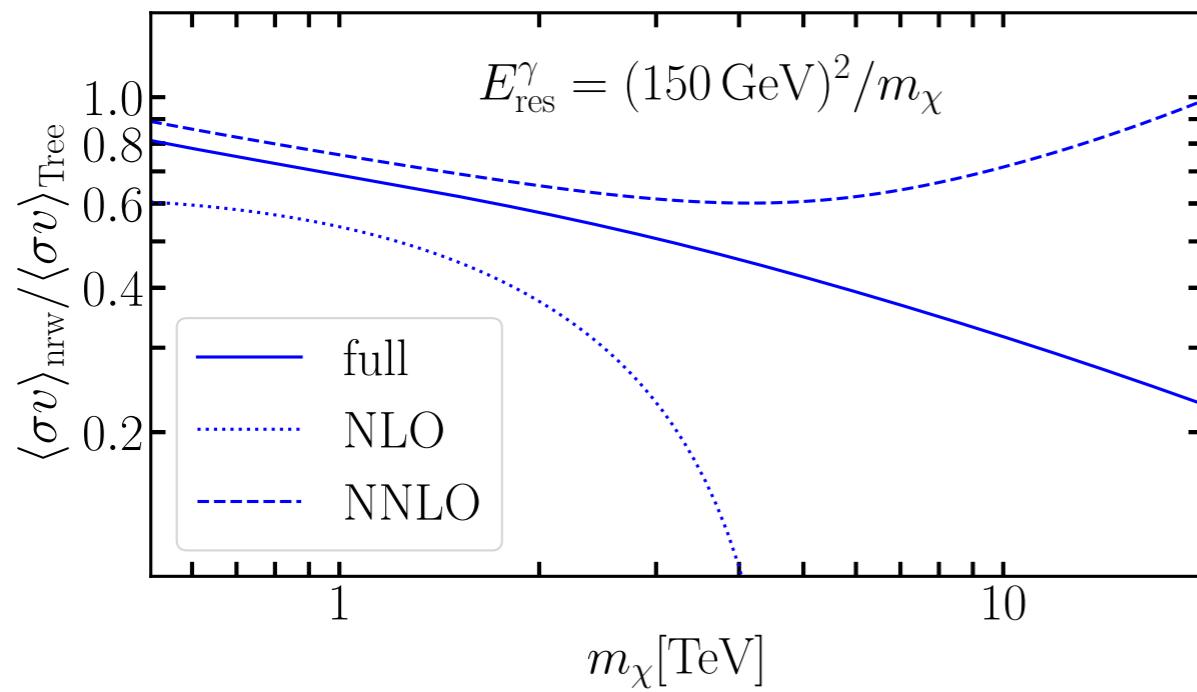
# Energy-integrated cross section



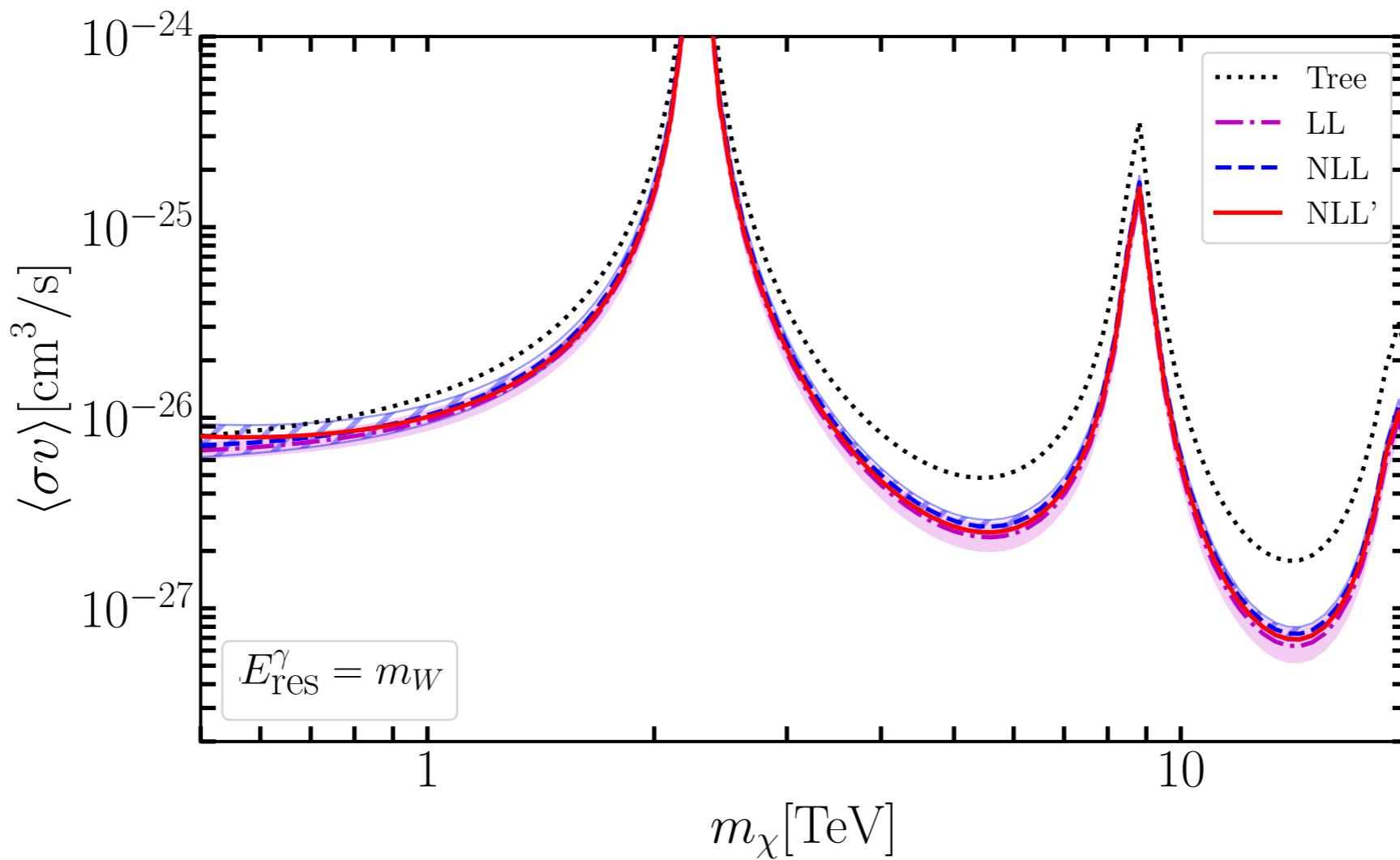
# Energy-integrated cross section



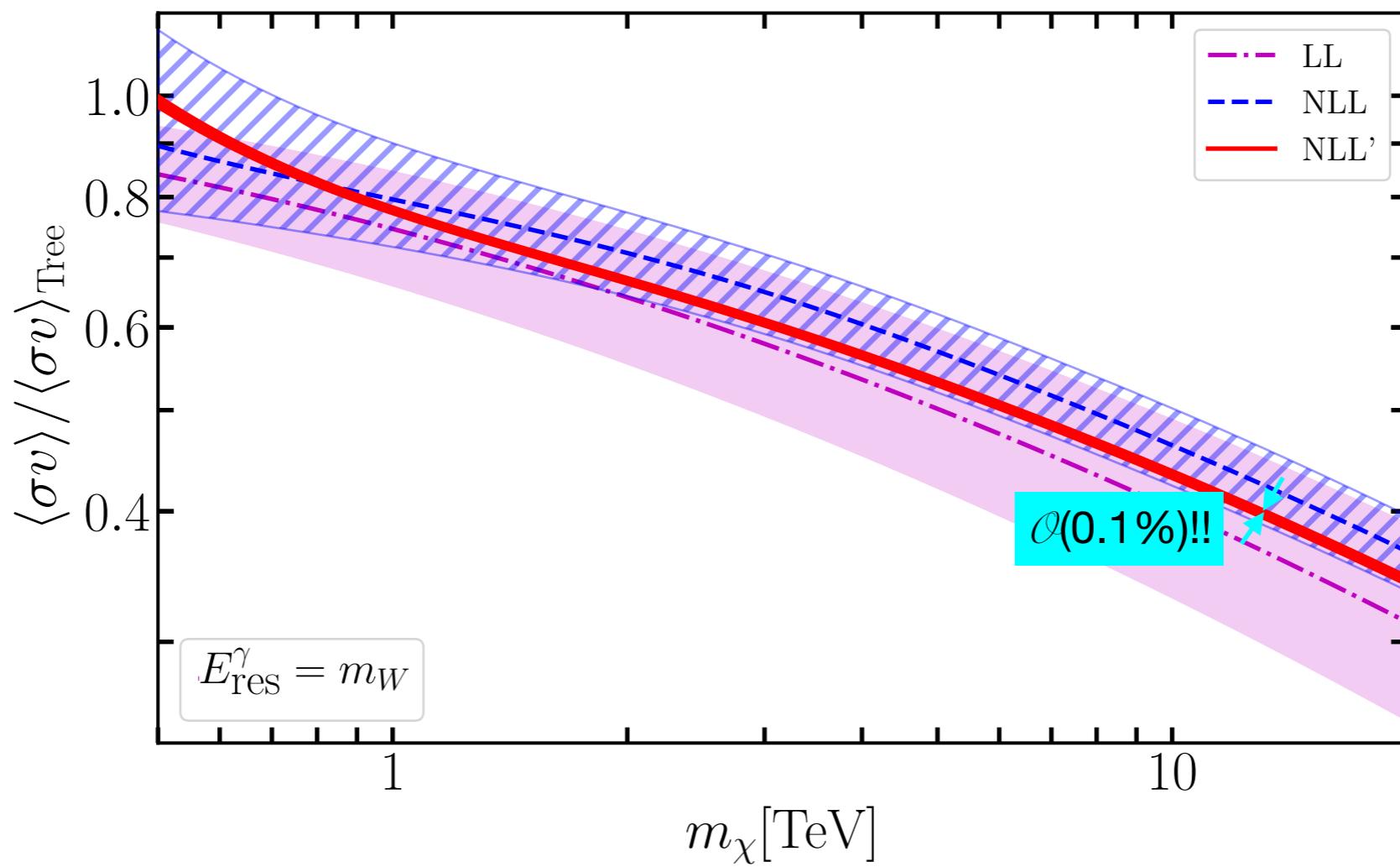
# Order-by-order “catastrophe”



# $\chi\chi \rightarrow \gamma + X$ cross sections ( $E_{\text{res}} \sim m_W$ )



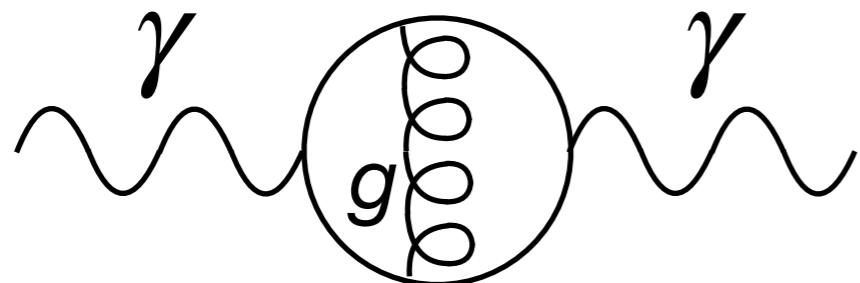
# Effect of the Sudakov resummation



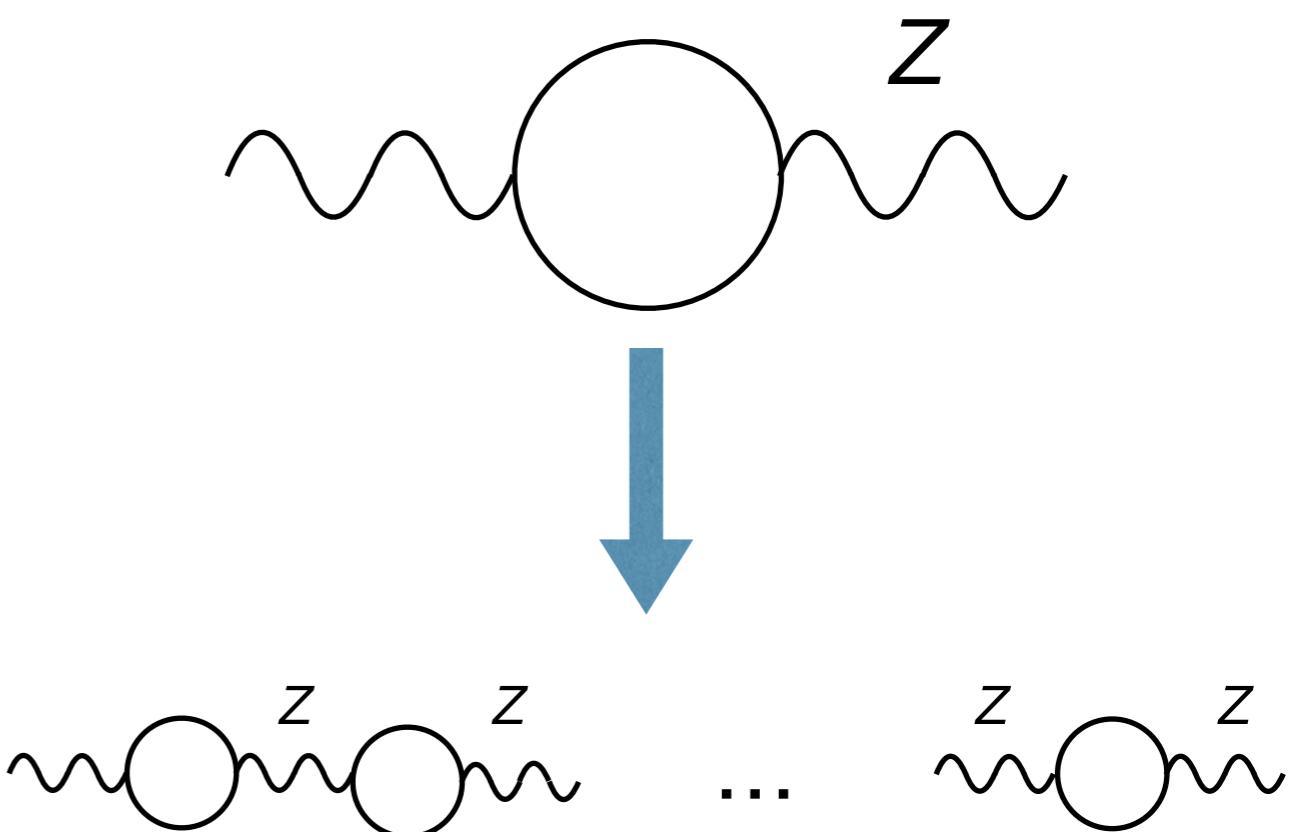
# Further technicalities addressed

**Photon jet function** (by definition) sensitive to the lowest energy scales in the SM

In particular, non-perturbative QCD effects are accounted-for using dispersion relations



**Z-pole singularity** can be cured by Dyson-resumming the Z propagator



# Conclusions

- Some heavy-WIMP theory region will be probed by indirect detection observations in the near future.
- We made accurate predictions for the spectral-line feature in gamma rays in the wino DM model
- The problem of resumming large Sudakov double logarithms was solved by means of a soft-collinear effective field theory. We also incorporate in a systematic way the Sommerfeld effect
- Our computations reduce theoretical uncertainties down to the **permille** level and they are suitable for the *energy resolutions of the CTA* at the interesting mass range of 1-10 TeV