

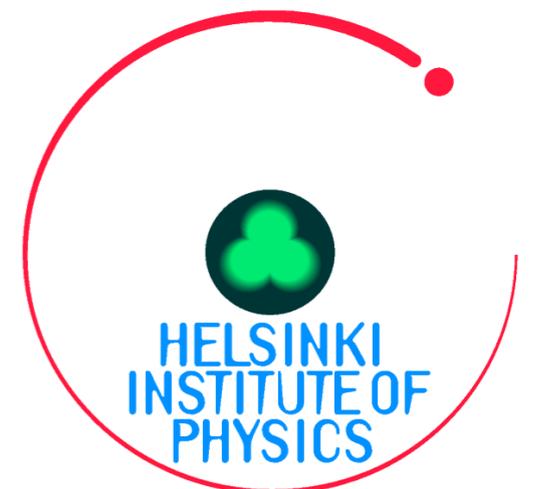
Asymptotic safety on the lattice ?

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Outline

- Some details of lattice gauge theory
- Example: IRFP
- **SU(2) gauge with many flavors: what to expect and some preliminary results**

Based on the work of HELSINKI Lattice group

J. Rantaharju, T. Rantalaiho, V. Leino, T. Rindlisbacher, K. Rummukainen,
J. Suorsa, K. Tuominen, S. Tähtinen

ArXiv: 1701.04666, 1707.04722, 1804.02319, 1806.07154

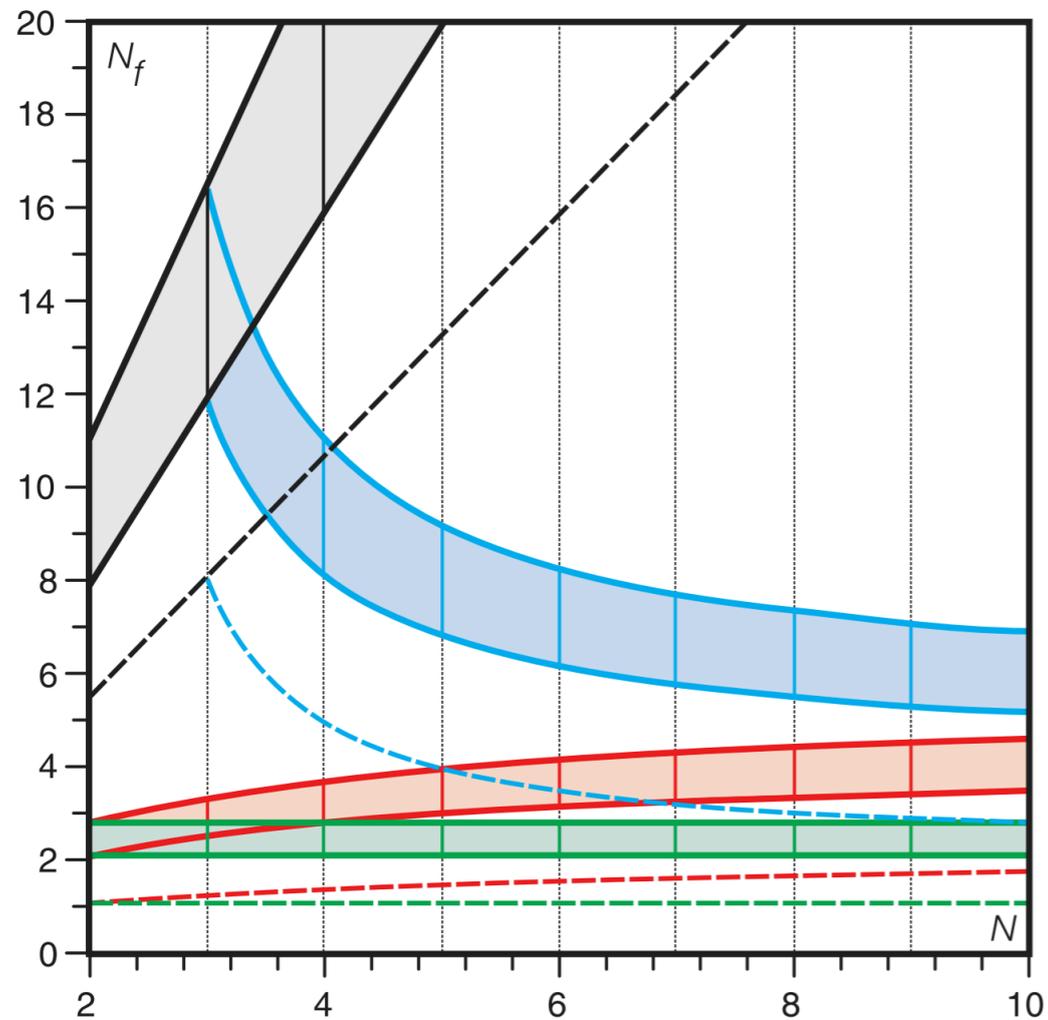
and in collaboration with

T. Rytto, F. Sannino

ArXiv: 1903.09089, 1906.****

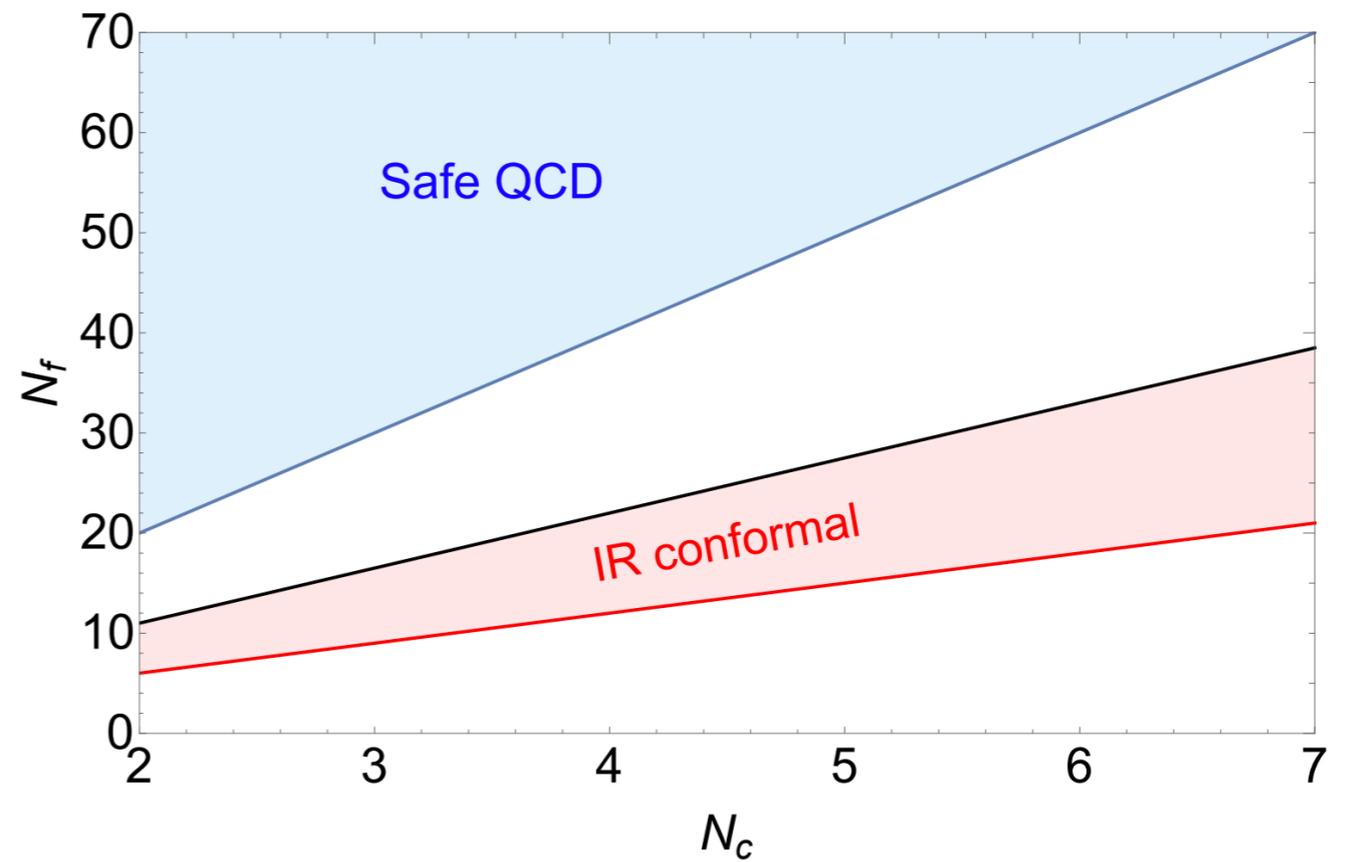
Motivation

Conformal window 1.0



F. Sannino, K. Tuominen '04
F. Sannino, D.D. Dietrich '06

Conformal window 2.0

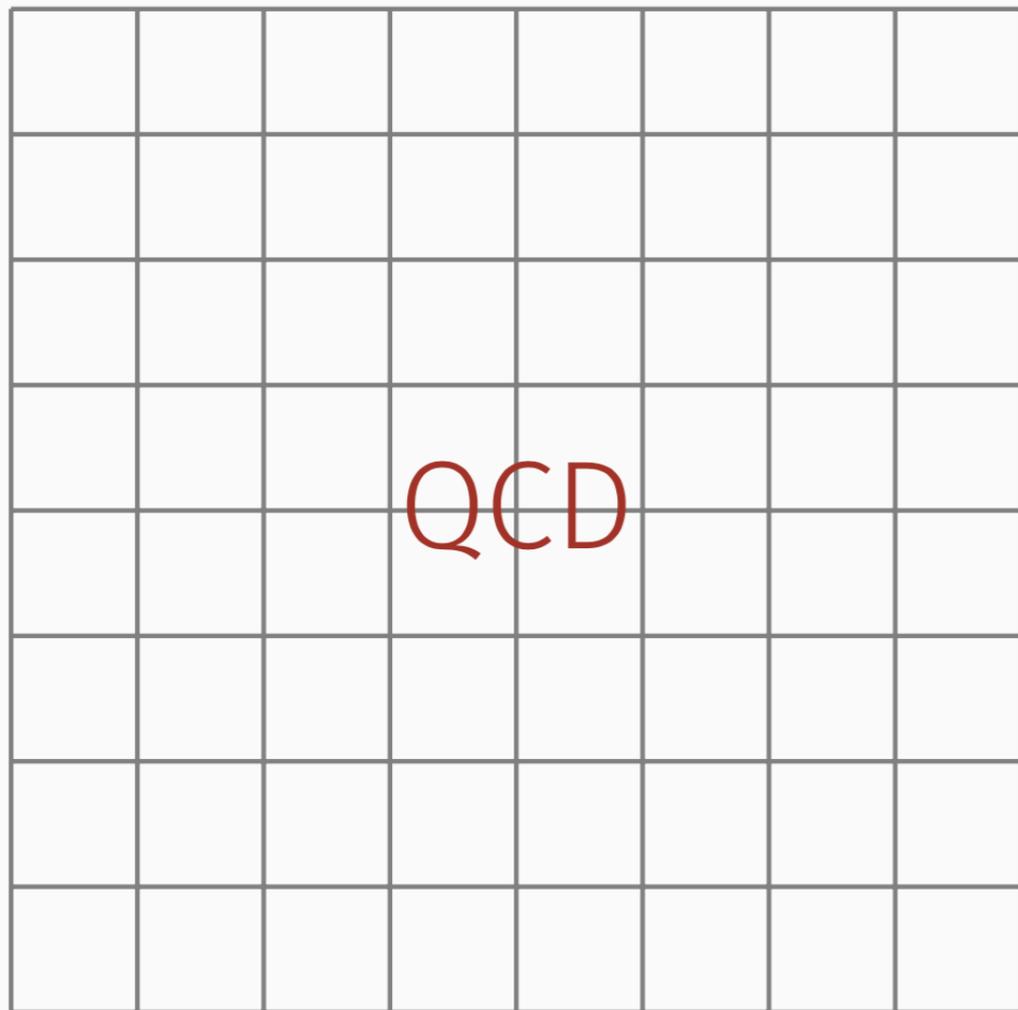


F. Sannino, O. Antipin '17

nonperturbative studies needed.

Some details of lattice gauge theory

Coupling is large here...



If near fixed point,
coupling is large everywhere.

Life at strong coupling:
improved lattice actions

Anonymous referee 2009:
"I have never seen this
[measurement of the coupling]
done with unimproved fermions.."



...but small here.

The lattice action as an effective action

e.g. the Wilson plaquette action:

see L. del Debbio's lecture

$$\frac{2N}{g^2} \sum_x \sum_{\mu < \nu} \left(1 - \frac{1}{N} \text{Tr} U_p\right) = \frac{a^4}{4} \sum_x \sum_{\mu, \nu} F_{\mu\nu} F^{\mu\nu} + \mathcal{O}(a^6)$$

contributes here..

$$S = \int d^4x \left[\mathcal{L}_0 + a\mathcal{L}_1 + a^2\mathcal{L}_2 + \dots \right]$$

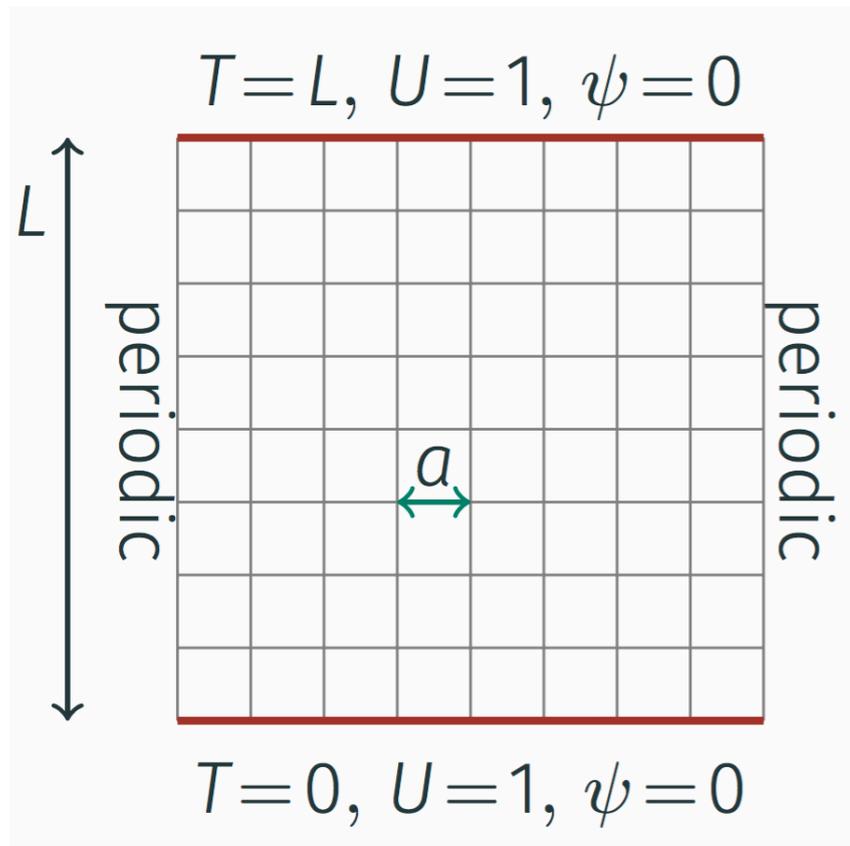
...but can add dim
6 (irrelevant)
operators so that
 $\mathcal{O}(a^2)$
artifacts cancel.

see e.g. R. Gupta hep-lat/9807028

Similarly, Wilson fermions at $\mathcal{O}(a)$:

$$\text{"Clover"}: \quad \delta\mathcal{L}_1 = \frac{ic_{sw}}{4} a^5 \sum_x \bar{\psi} \sigma_{\mu\nu} F^{\mu\nu} \psi(x)$$

Details of the lattice setup: general



$$S = (1 - c_g)S_G(U) + c_g S_G(V) + S_F(V)$$

- HEX smearing to simulate at strong coupling
- Smearred (V) and unsmeared (U) gauge fields mixed with c_g
- Clover improved Wilson action

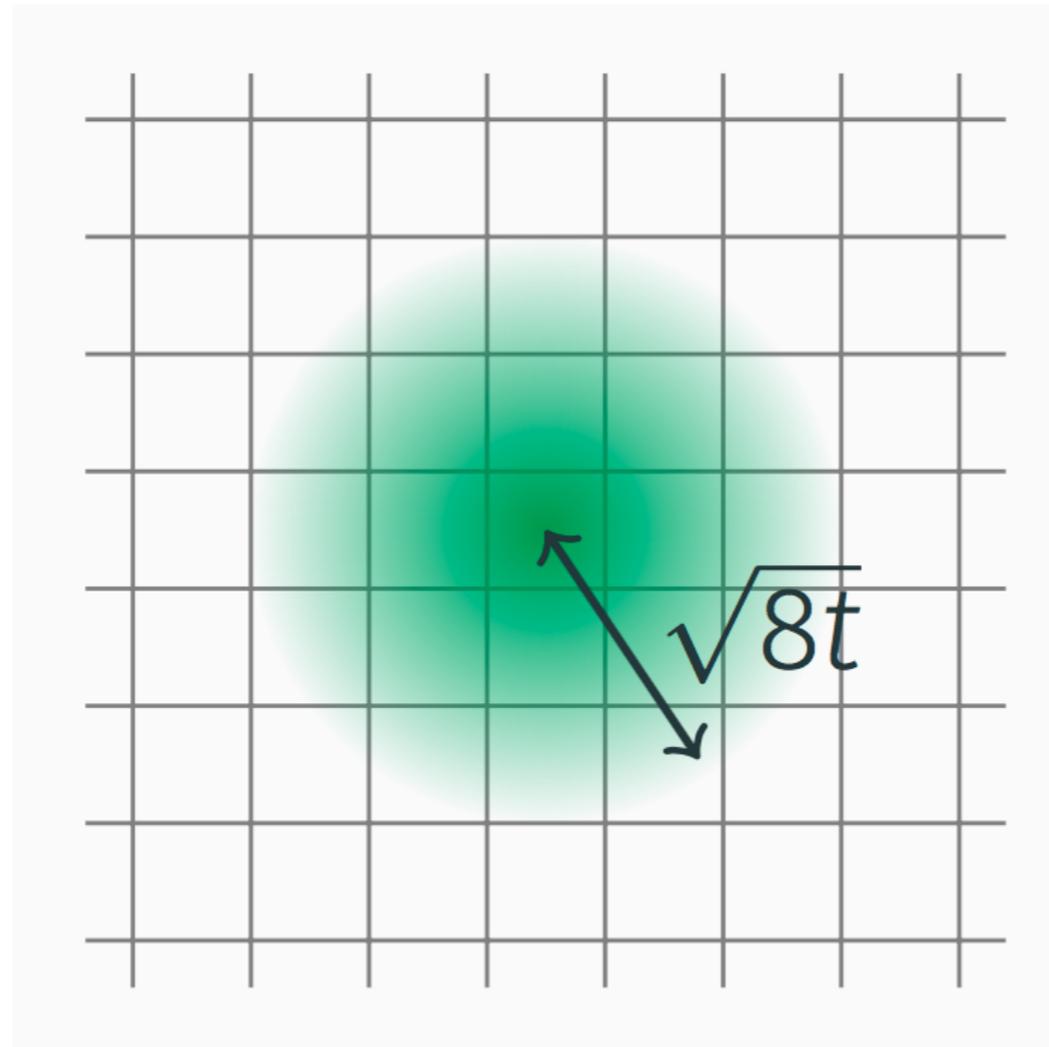
Simulation parameters:

$$\mathbf{N}_f = \mathbf{6} : L = 8^4 - 30^4 \text{ and } \beta_L = 8 - 0.5$$

$$\mathbf{N}_f = \mathbf{8} : L = 6^4 - 32^4 \text{ and } \beta_L = 8 - 0.4$$

$$\beta_L \equiv \frac{2N_c}{g^2}$$

Details of the lattice setup: gradient flow



- Set up a flow equation for the gauge field.
- Evolved along a fictitious time coordinate.
- Initial gauge field diffuses with radius $\sqrt{8t}$
 - ▶ UV divergences removed
 - ▶ Allows for nonperturbative definition of the coupling

Let's try to understand these features

the gradient flow

M. Luscher '10

(1006.4518)

Add extra time coordinate t , and a gauge field $B_\mu(x, t)$

$$G_{\mu\nu}(x, t) = \partial_\mu B_\nu(x, t) - \partial_\nu B_\mu(x, t) + [B_\nu(x, t), B_\mu(x, t)]$$

$$\frac{dB_\mu(x, t)}{dt} = D_\nu G_{\nu\mu}(x, t); \quad B_\mu(x, 0) = A_\mu(x)$$

A new length scale: $\sim \sqrt{t}$ (Note the dimensions: $[t] = -2$)

the gradient flow: perturbation theory analysis I

M. Luscher & P. Weisz '11 (1101.0963)

$$\frac{dB_\mu(x, t)}{dt} = D_\nu G_{\nu\mu}(x, t); \quad B_\mu(x, 0) = A_\mu(x)$$

Rescale: $A_\mu \rightarrow g_0 A_\mu$

$$\text{Expand: } B_\mu(x, t) = \sum_{n=1}^{\infty} B_{\mu,n}(x, t) g_0^n; \quad B_{\mu,n}(x, 0) = \delta_{n,1} A_\mu(x)$$

$$\frac{dB_\mu(x, t)}{dt} = g_0 \frac{dB_{\mu,1}(x, t)}{dt} + \mathcal{O}(g_0^2)$$

$$G_{\mu\nu}(x, t) = g_0 (\partial_\mu B_{\nu,1}(x, t) - \partial_\nu B_{\mu,1}(x, t) + \mathcal{O}(g_0^2))$$

$$\text{Leading order: } \frac{dB_{\mu,1}(x, t)}{dt} = \partial^2 B_{\mu,1}(x, t) - \partial_\mu \partial_\nu B_{\nu,1}(x, t)$$

Heat equation

Gauge dependent part

Exercise 1: 1+1 dimensional heat equation

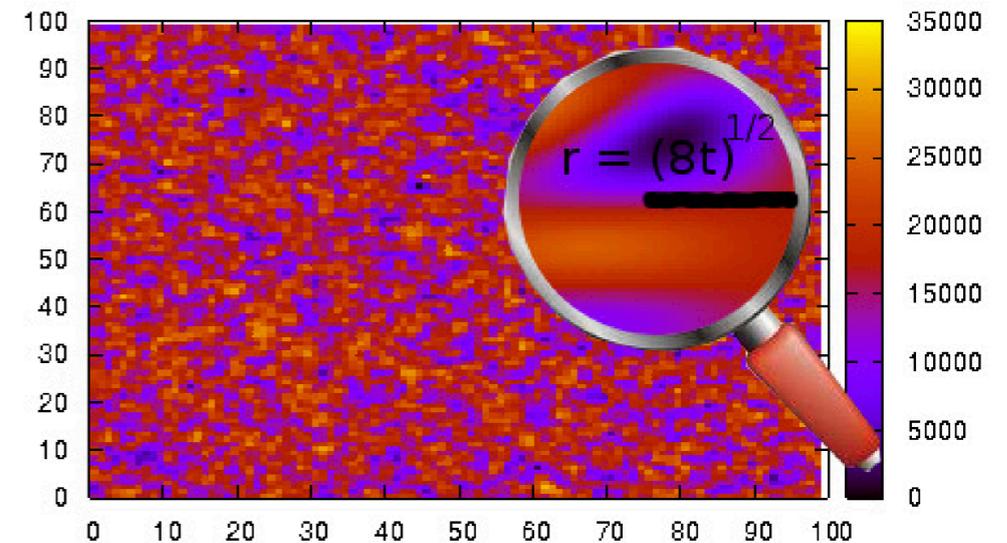
$$\frac{\partial \Phi}{\partial t} = K \frac{\partial^2 \Phi}{\partial x^2}$$

First, show that if $\Phi(x, t)$ is a solution, so are $\bar{\Phi}(x, t) = \Phi(x - x_0, t - t_0)$,
 $\tilde{\Phi}(x, t) = A\Phi(\lambda x, \lambda^2 t)$

We set $A = \lambda$. (Why?)

Second, show that the solution is $\Phi(x, t) = G(x, t) = \frac{1}{\sqrt{4\pi Kt}} e^{-x^2/(4Kt)}$
(Normalized to unity)

Heat flow: smoothing over radius $\sqrt{8Kt}$



In $n+1$ dimensions the heat kernel is:

$$G(\mathbf{x} - \mathbf{x}_0, t - t_0) = \frac{1}{\left(\sqrt{4\pi K(t - t_0)}\right)^{n/2}} e^{-\frac{|\mathbf{x} - \mathbf{x}_0|^2}{4K(t - t_0)}}$$

Same story in momentum space $B_{\mu,1}(x, t) = \int_p e^{ipx} \tilde{B}_{\mu,1}(p, t)$ $\int_p \equiv \int_{-\infty}^{+\infty} \frac{d^4p}{(2\pi)^4}$

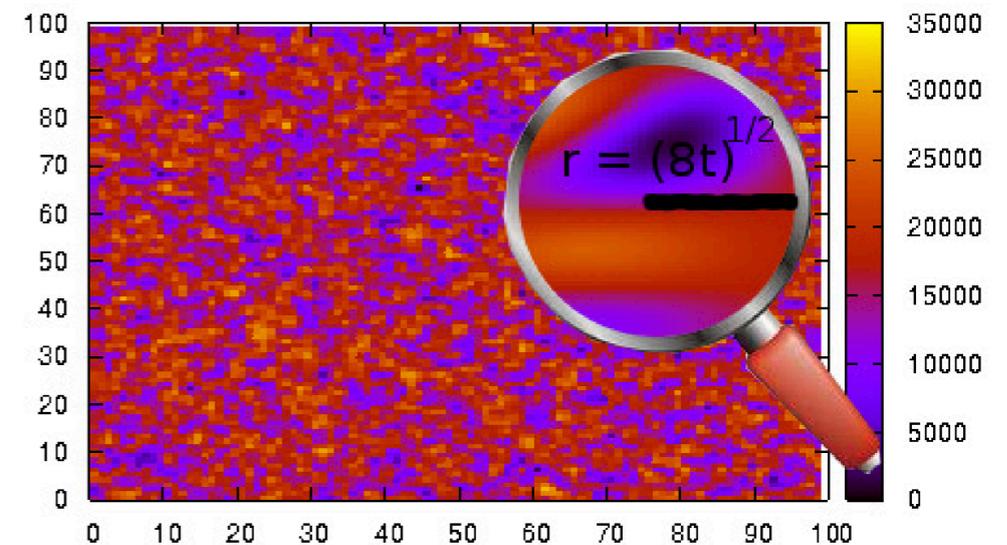
Flow equation becomes:

$$\frac{d\tilde{B}_{\mu,1}(p, t)}{dt} = - (p^2 - p_\mu p_\nu) \tilde{B}_{\mu,1}(p, t)$$

Solution:

$$\tilde{B}_{\mu,1}(p, t) = e^{-tp^2} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \tilde{A}_\nu(p)$$

Damping of high momentum modes.



Heat flow: smoothens temperature fluctuations over distances $\sqrt{8t}$

Gradient flow: smoothens quantum fluctuations over distances $\sqrt{8t}$

the gradient flow: perturbation theory analysis II

We show that $\langle E(t) \rangle = \frac{1}{2} \langle \text{Tr} G_{\mu\nu} G_{\mu\nu} \rangle$ is a finite quantity.

Exercise 2:

Express $G_{\mu\nu}$ in terms of $\tilde{B}_\mu(p, t)$ to obtain

$$G_{\mu\nu} G_{\mu\nu} = -2g_0^2 \int_{p,q} e^{i(p+q)x} \left[p_\mu q_\nu \tilde{B}_{\nu,1}(p) \tilde{B}_{\nu,1}(q) - p_\nu q_\mu \tilde{B}_{\mu,1}(p) \tilde{B}_{\mu,1}(q) \right]$$

Then integrate over $\int d^4x$ to obtain $\delta(p+q)$ on RHS.

Insert the explicit formula for $\tilde{B}_\mu(p, t)$,

and use the gluon propagator $\langle \text{Tr} \tilde{A}_\mu(p) \tilde{A}_\nu(-p) \rangle = \frac{1}{2} \frac{\delta_{ab} \delta_{\mu\nu}}{p^2}$.

You should get $\langle E(t) \rangle = \frac{3 \times 8}{128\pi^2 t^2} g_0^2$

for SU(3) gauge group.

What do you get for SU(2)?

Definition of the nonperturbative coupling:

$$g_{\text{GF}}^2(\mu) = \frac{128\pi^2}{9} t^2 \langle E(t) \rangle \Big|_{\mu=1/\sqrt{8t}}$$

- Agrees with pert theory @ weak coupling,
- Defines the coupling @ any scale

Remark:

- @ NLO $t^2 \langle E(t) \rangle = \#g_0^2 [1 + c_1 g_0^2 + \dots]$ finite.

M. Luscher & P. Weisz '11 (1101.0963)

Measurement of the coupling:

Idea:

Measure how much the coupling changes as $\mu \rightarrow \mu/2$ by changing the lattice size L .

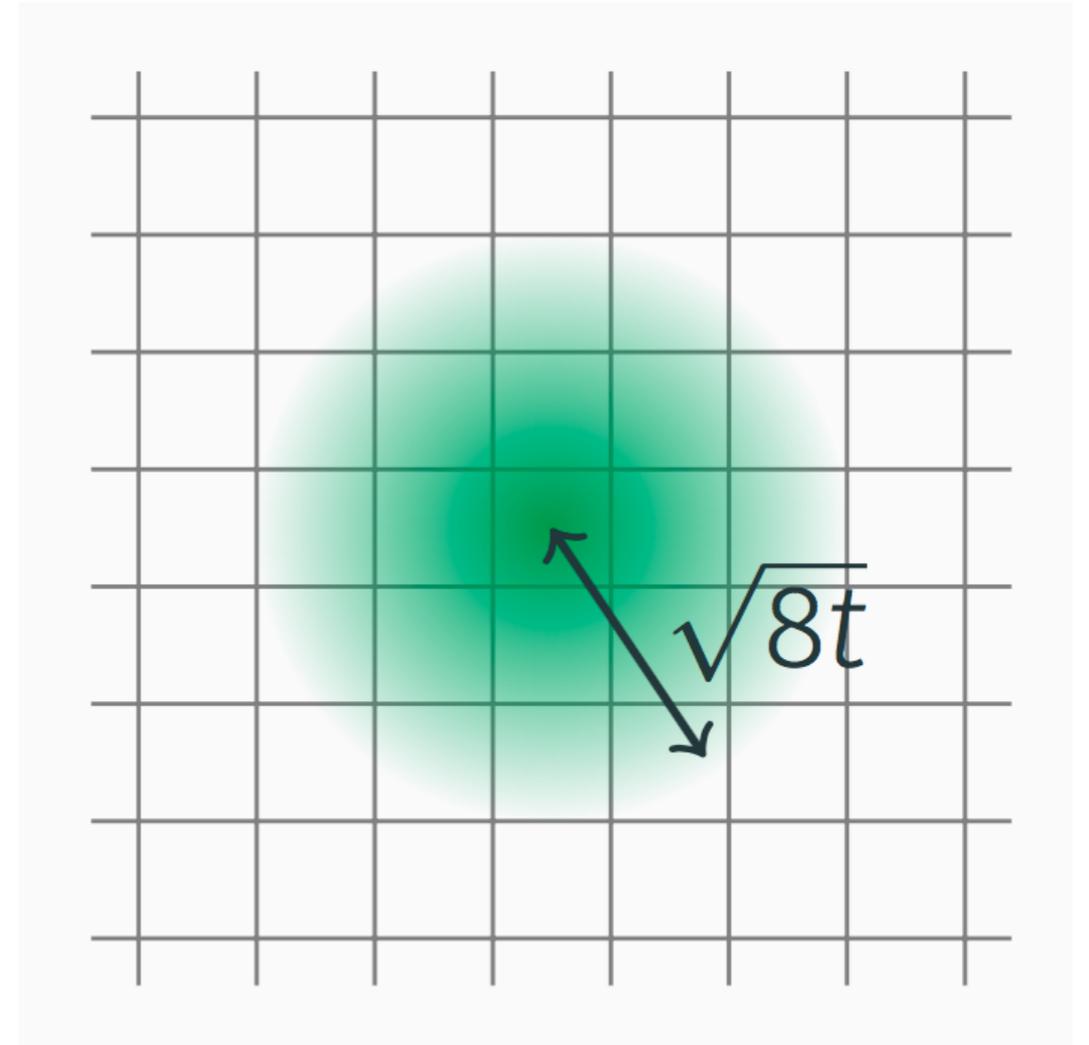
The step scaling function

$$\sigma(u) = \alpha(\mu/2) \Big|_{\alpha(\mu)=u}$$

Details of the lattice setup: gradient flow

$$\begin{aligned}\langle E(t) \rangle &= \frac{1}{4} \langle G_{\mu\nu}(t) G_{\mu\nu}(t) \rangle \\ &= \frac{3(N^2 - 1)g_0^2}{128\pi^2 t^2} + \mathcal{O}(g_0^4),\end{aligned}$$

$$g_{\text{GF}}^2(\mu) \equiv \mathcal{N}^{-1} \langle E(t) \rangle \Big|_{x_0=L/2, t=1/(8\mu^2)}$$



Flow equation evolved to time t

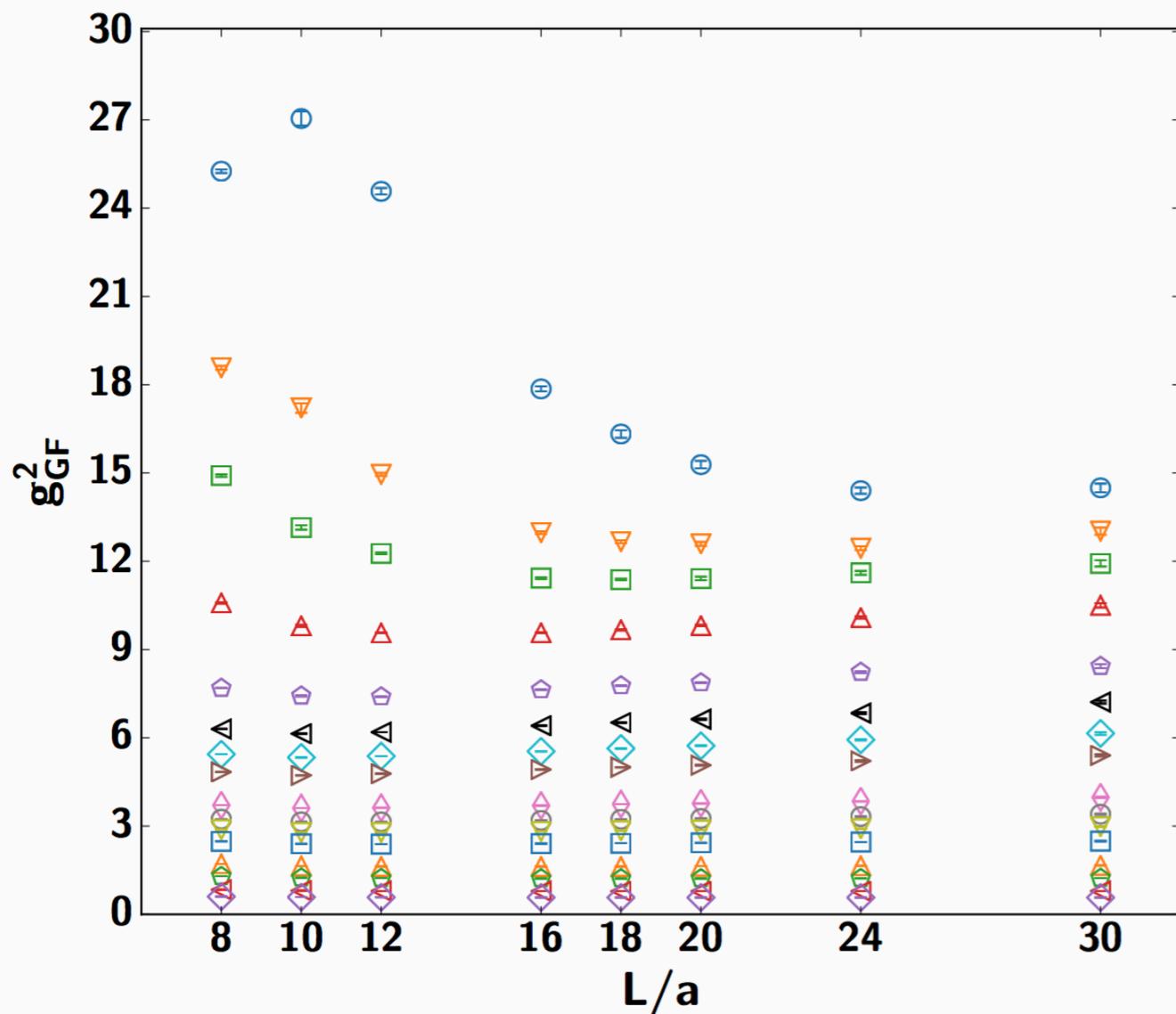
Coupling at scale $\mu^{-1} = \sqrt{8t} = cL$

Each c defines a scheme.

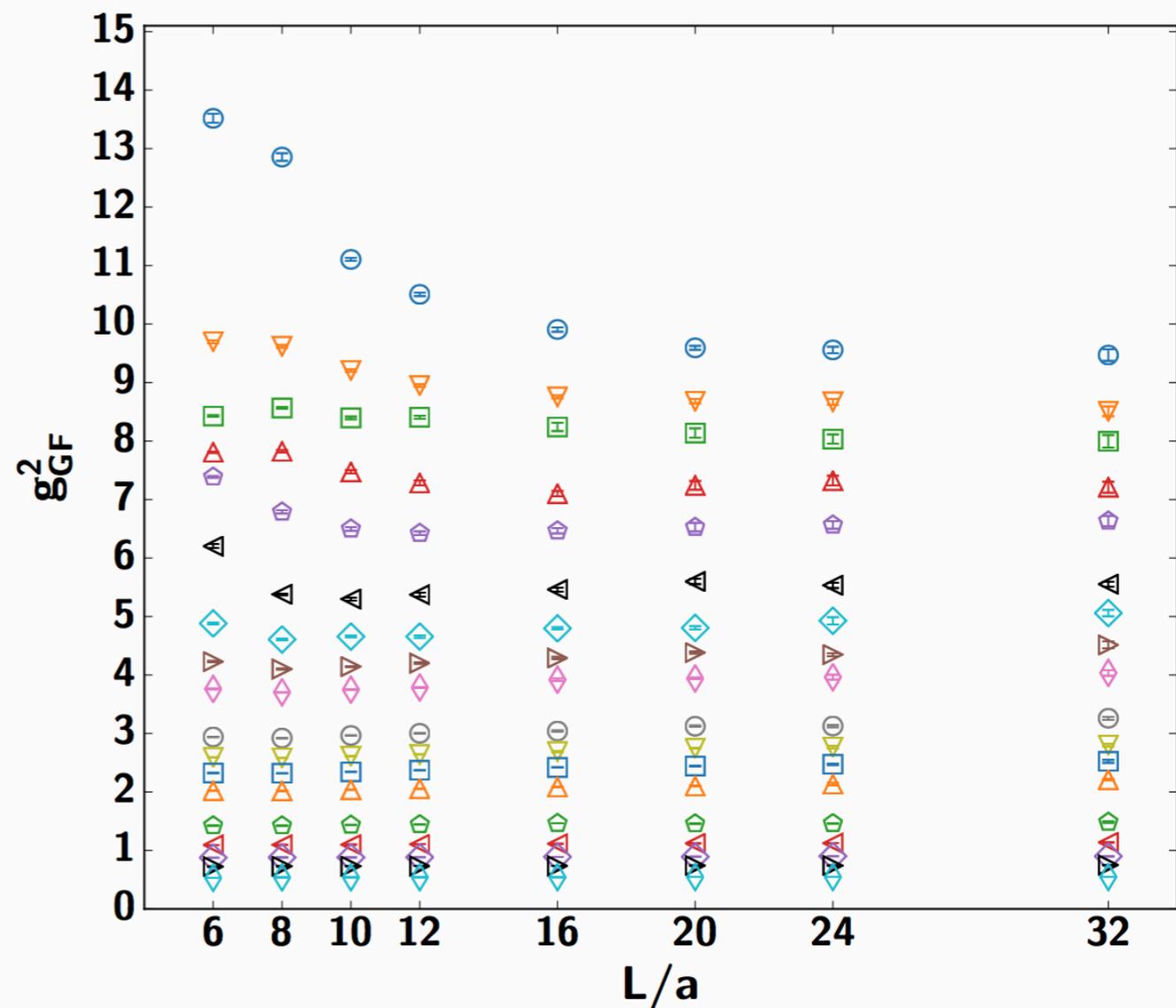
We use $c = 0.4$ ($N_f = 8$) and $c = 0.3$ ($N_f = 6$)

Raw data

$N_f = 6:$



$N_f = 8:$



Strong finite size effects on small lattices.

Use only size 10 or bigger in the analysis.

Step scaling function:

$$\Sigma(s, u, a/L) = g_{\text{GF}}^2(g_0, s \frac{L}{a}) \Big|_{g_{\text{GF}}^2(g_0, \frac{L}{a})=u}$$

$$\sigma(u, s) = \lim_{a/L \rightarrow 0} \Sigma(u, s, a/L)$$

Do the limit as:

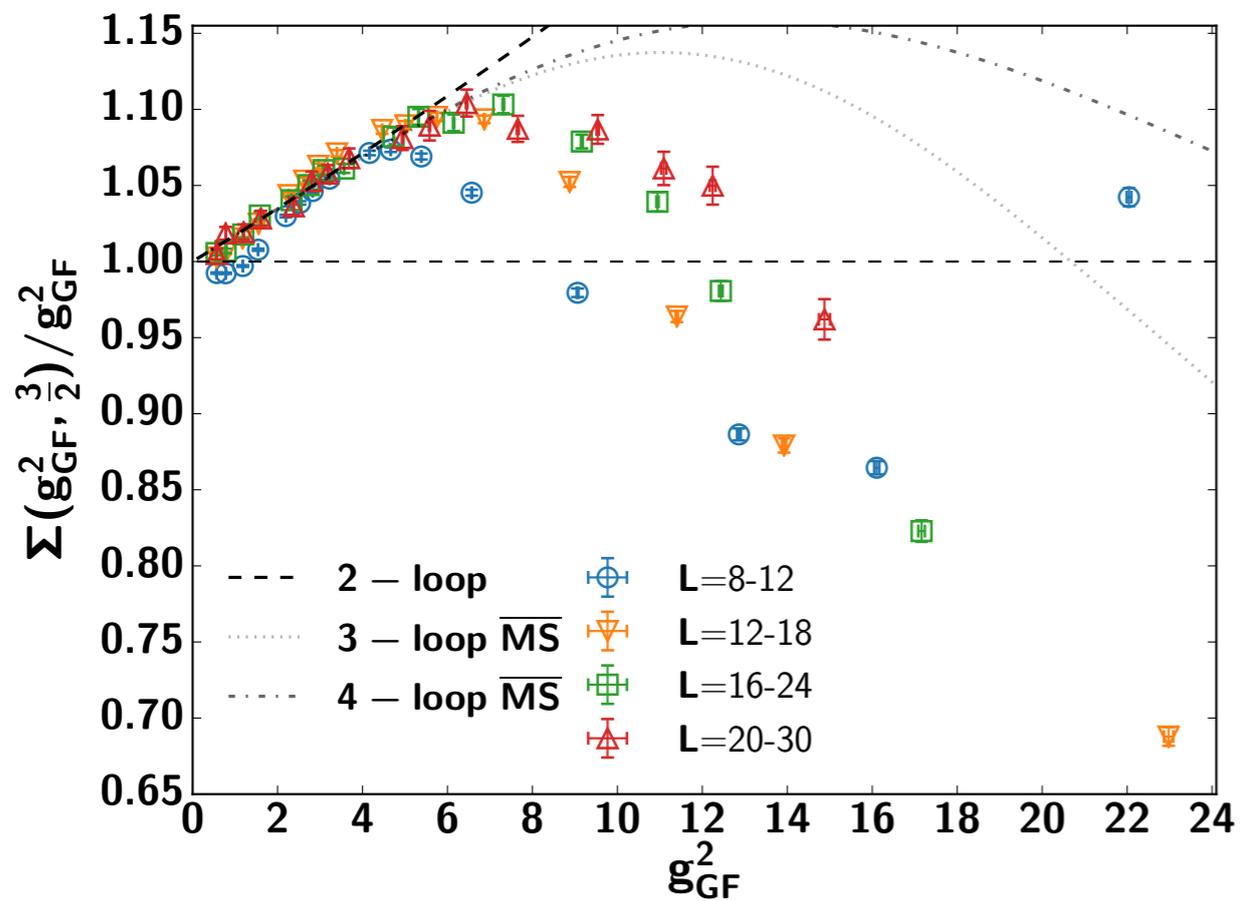
$$\Sigma(u, s, a/L) = \sigma(u, s) + c(u) \left(\frac{L}{a} \right)^{-2}$$

Fixed point: $\sigma(u)/u = 1$

$$N_f = 6 : s = 1.5$$

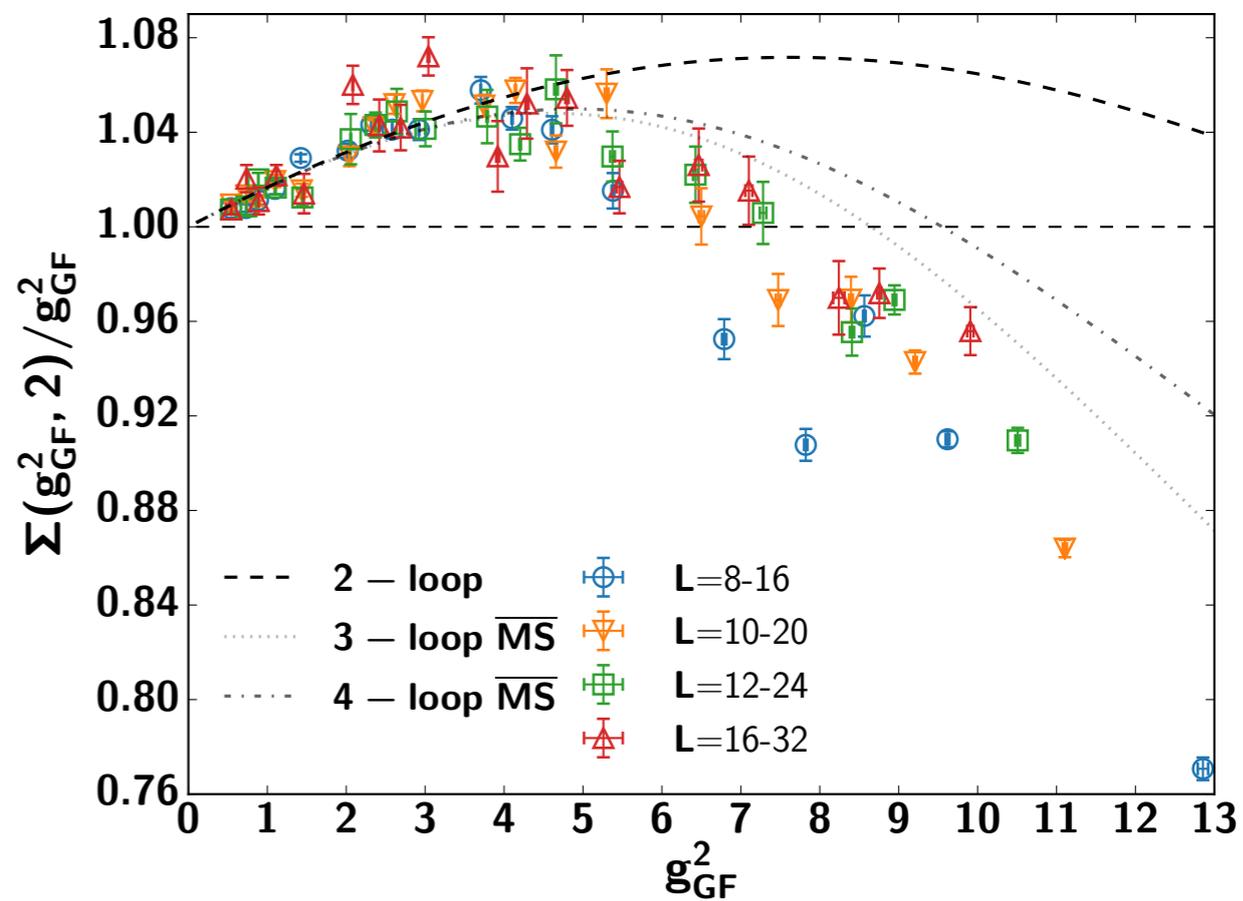
$$N_f = 8 : s = 2$$

Raw step scaling function



$$N_f = 6$$

$$s = 3/2, c = 0.3$$

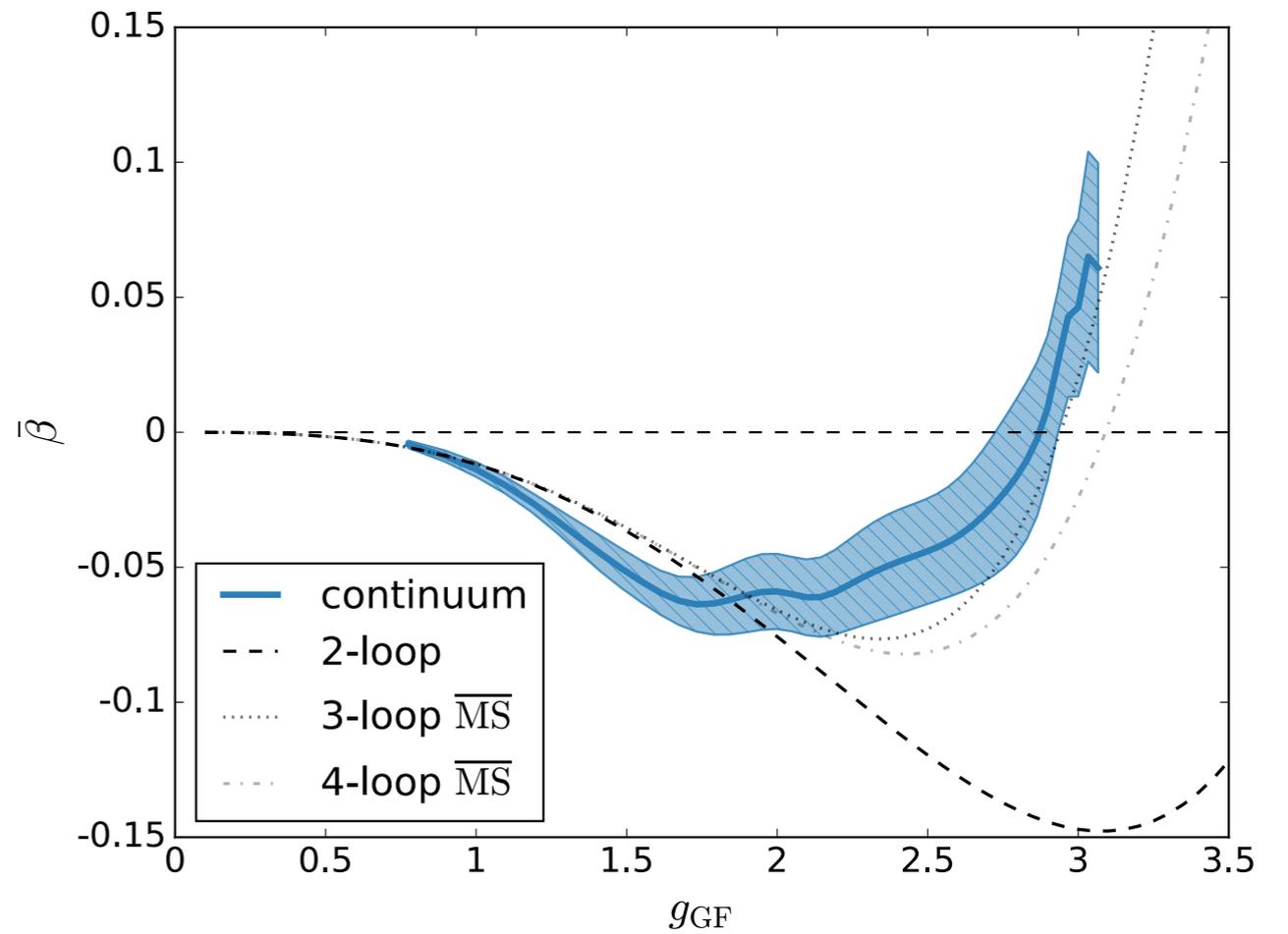
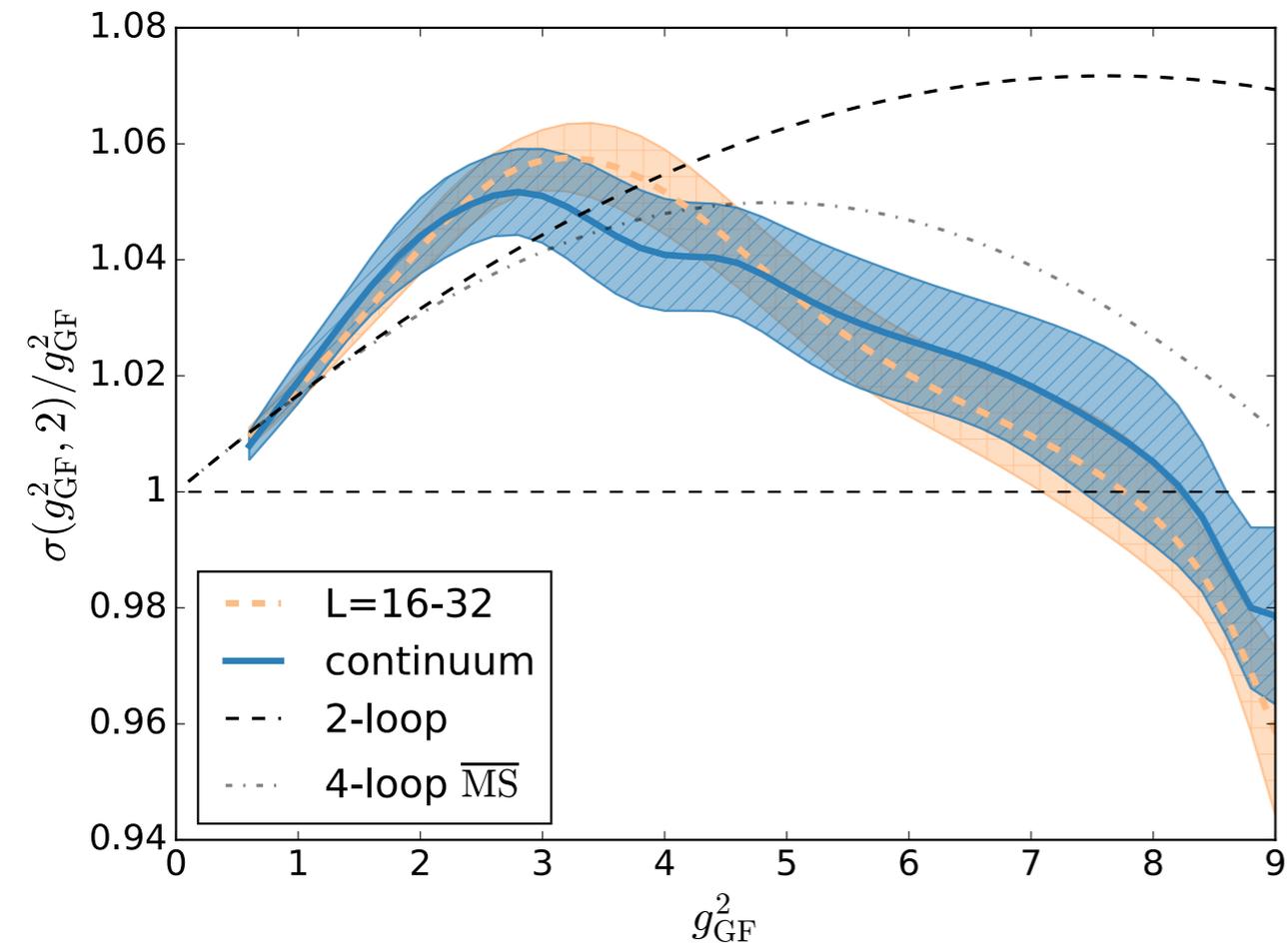


$$N_f = 8$$

$$s = 2, c = 0.4$$

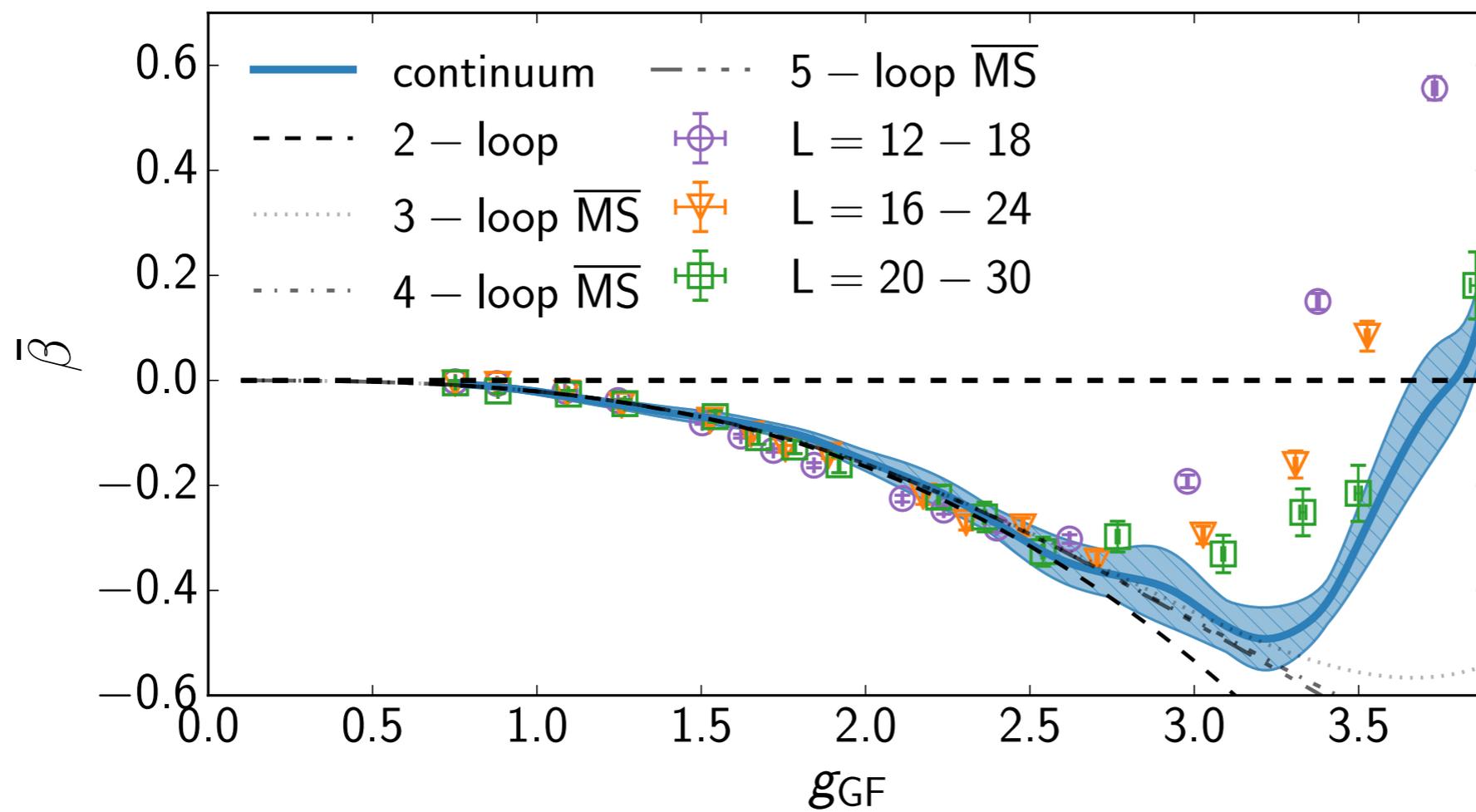
Continuum extrapolation

$N_f = 8$



Fixed point: $g_{\text{GF}}^2 \approx 8$

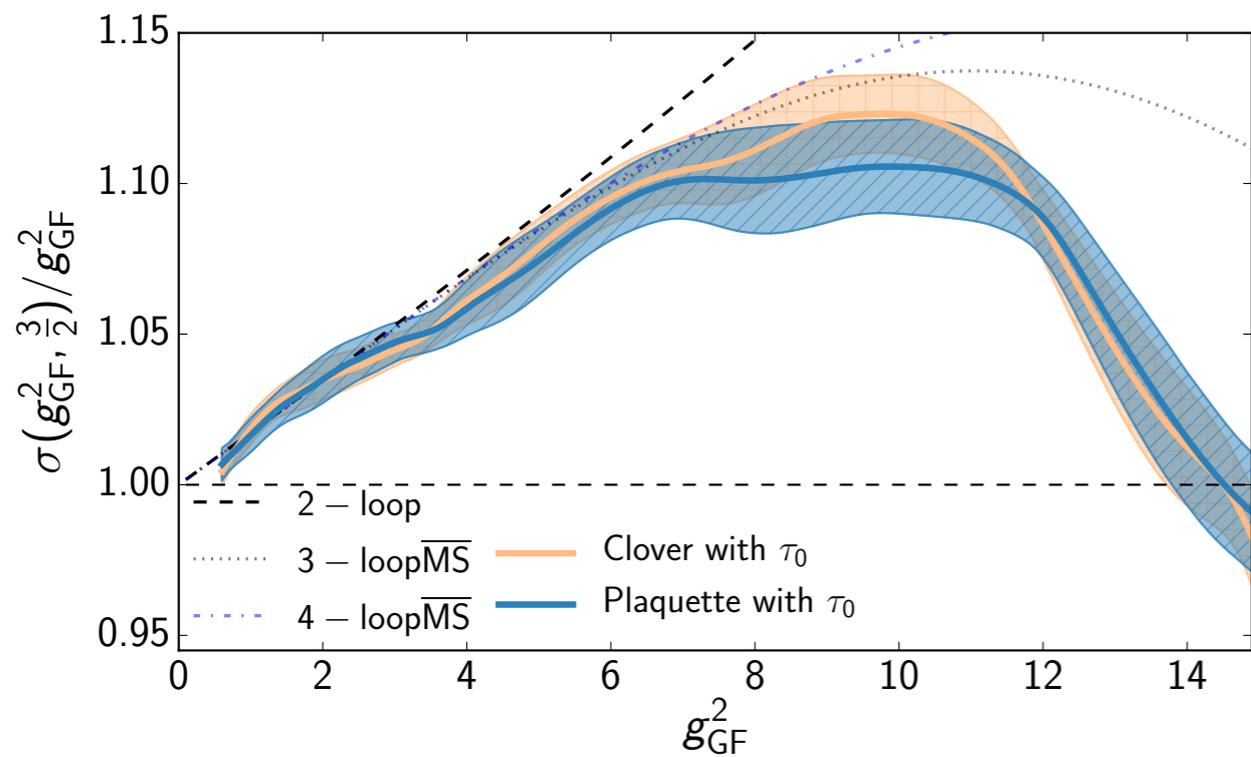
$$N_f = 6$$



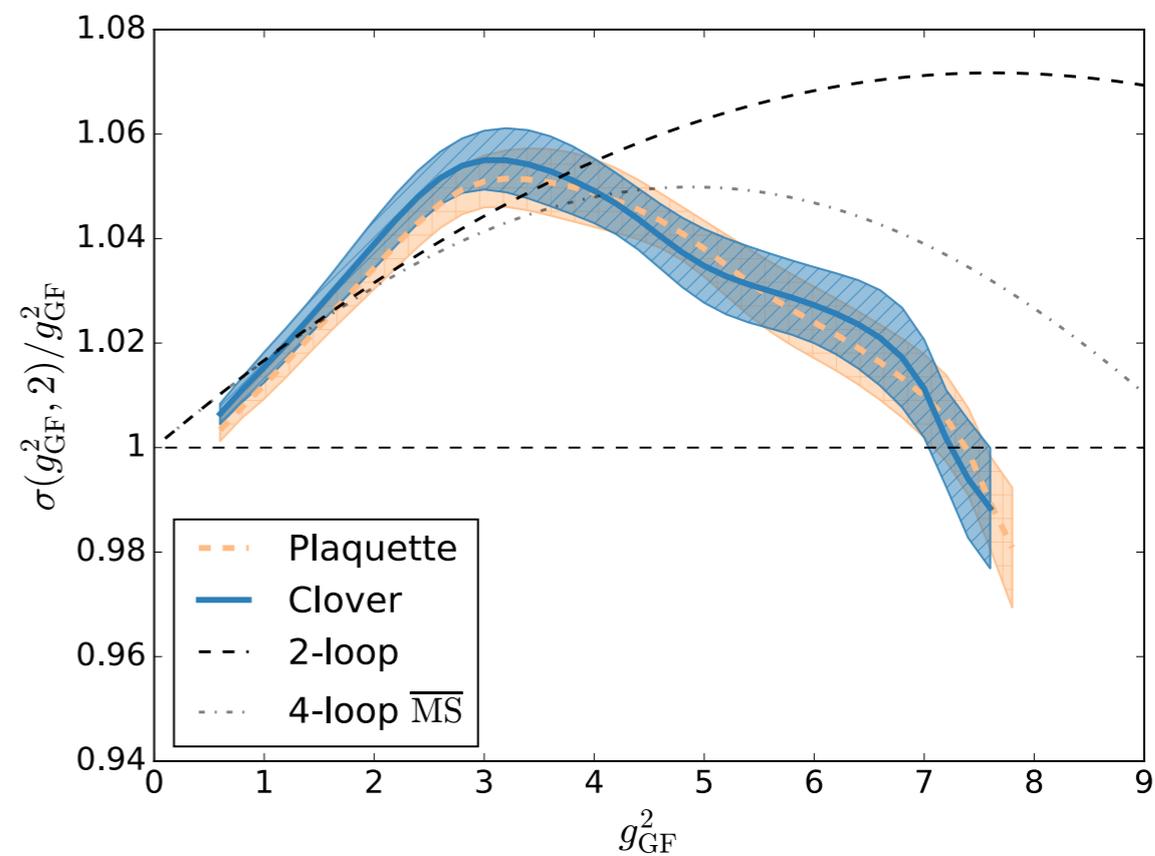
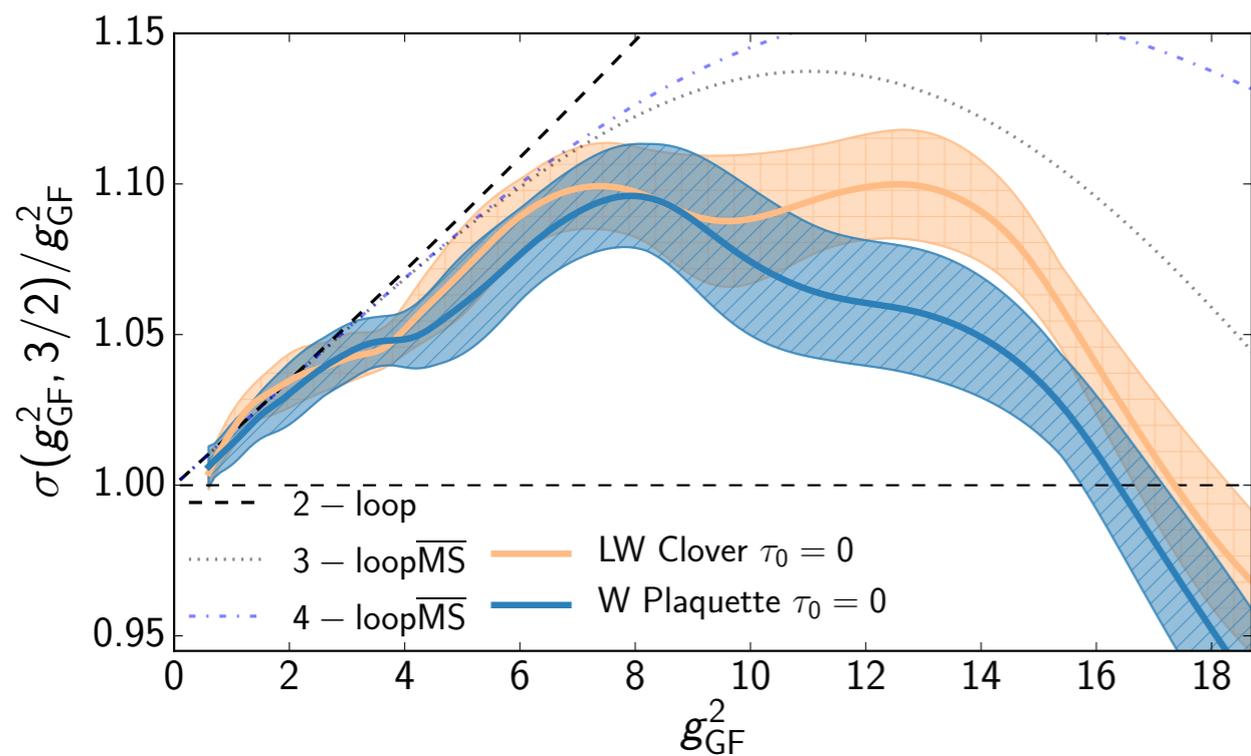
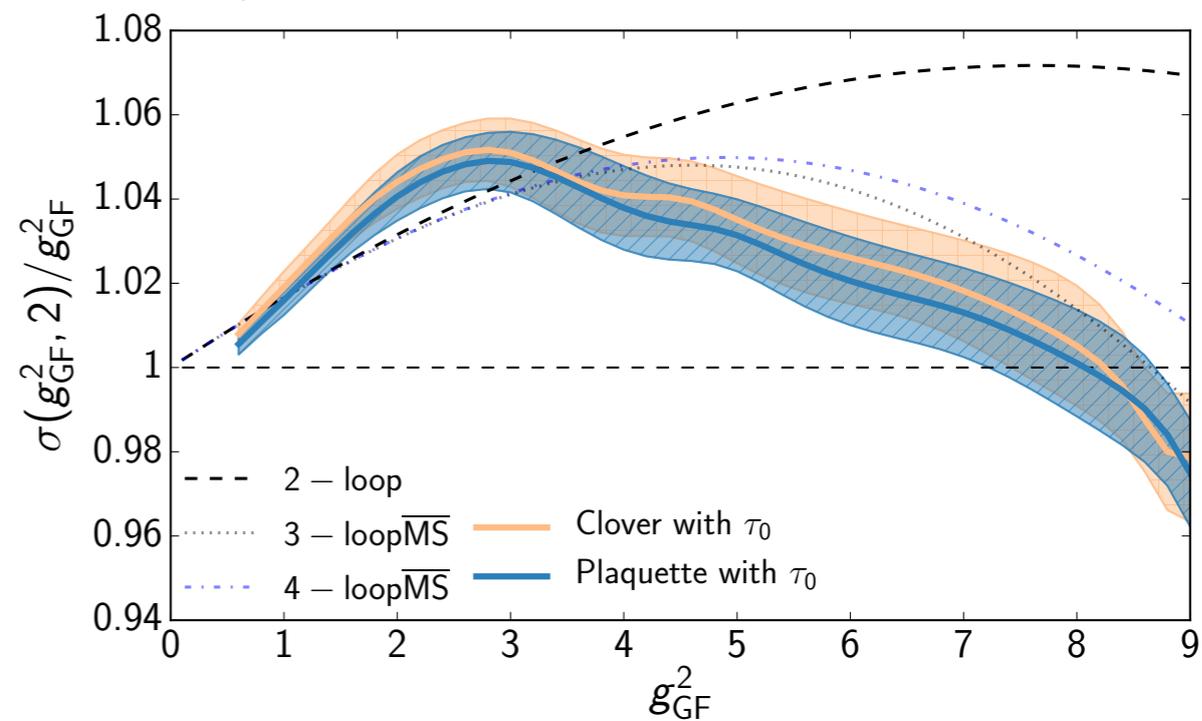
Fixed point: $g_{GF}^2 \approx 14$

Robustness wrt different discretizations

$N_f = 6$



$N_f = 8$



Summary I

Current state-of-the-art:

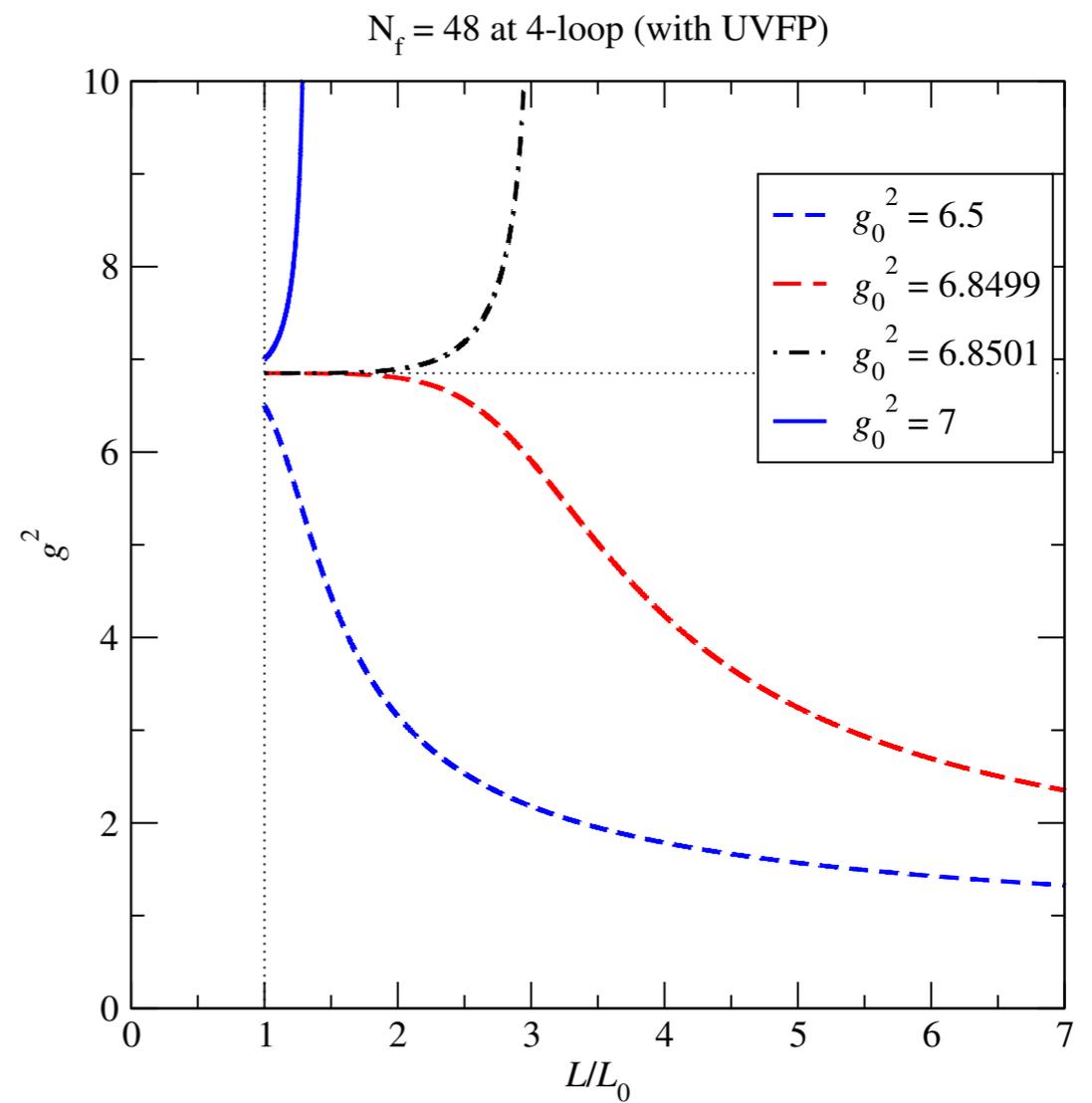
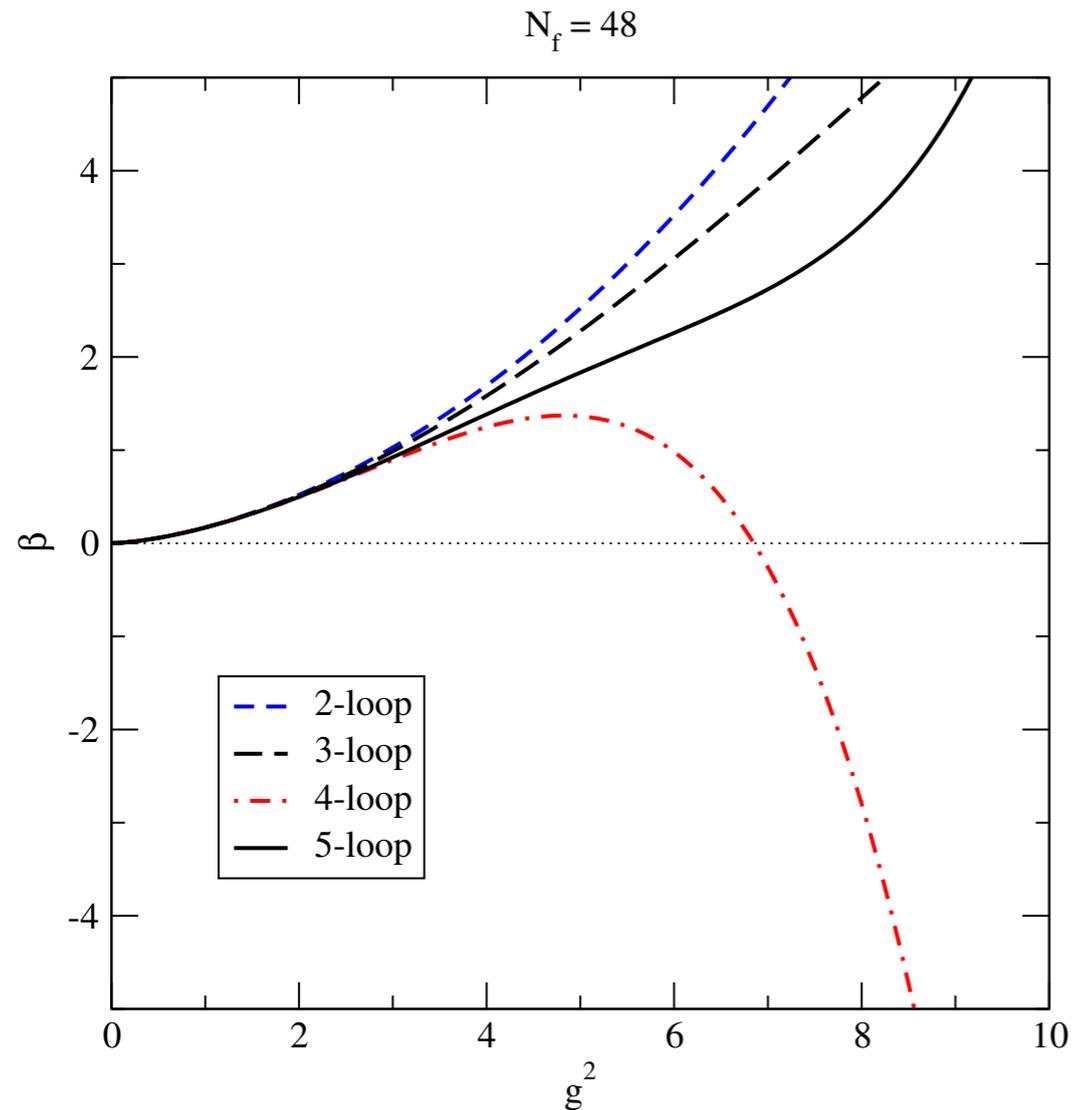
- Improved actions
- Coupling from gradient flow

Well suited for studies of IRFP:

- Conformal window 1.0 charted for $SU(2)$
- $SU(3)$ by other groups
- Also studies for other N and other reps.

Next: conformal window 2.0

Perturbative beta-function



Similar strategy as for IRFP: try to see runnings on both sides of the fixed point

Challenge: UVFP is repulsive, so likely need heavy fine tuning to see it.

Large Nf beta-function

$$\beta(\alpha) = \mu \frac{d\alpha}{d\mu} = -b_1 \frac{\alpha^2}{\pi} - b_2 \frac{\alpha^3}{\pi} - b_3 \frac{\alpha^4}{\pi} - \dots$$

Define $A = T_r N_f \frac{\alpha}{\pi}$

Then $\beta(A) = \mu \frac{dA}{d\mu} = \frac{2}{3} A^2 \left(1 + \sum_{i=1}^{\infty} \frac{H_i(A)}{N_f^i} \right)$

Exercise 3: use known 5-loop beta-function to determine perturbative formulas for

$$H_1(A), H_2(A), H_3(A), H_4(A), H_5(A)$$

$H_1(A)$ known to all orders:

J. Gracey '96 (hep-ph/9602214)

$$H_1(A) = -\frac{11}{4} \frac{C_A}{T_r} + \int_0^{A/3} I_1(x) I_2(x) dx$$

$$I_1(x) = \frac{(1+x)(2x-1)^2(2x-3)^2 \sin(\pi x)^3 \Gamma(x-1)^2 \Gamma(-2x)}{(x-2)\pi^3}$$

$$I_2(x) = \frac{C_r}{T_r} + \frac{(20 - 43x + 32x^2 - 14x^3 + 4x^4) C_A}{4(2x-1)(2x-3)(1+x)(1-x) T_r}$$

O. Antipin, F Sannino '17 (1709.02354)

Exercise 4: Zero of the beta-function.

Solve $1 + \frac{H_1(A_{UV})}{N_f} = 0$ by expanding the integrand in $H_1(A)$ around $x = 1$. This should give

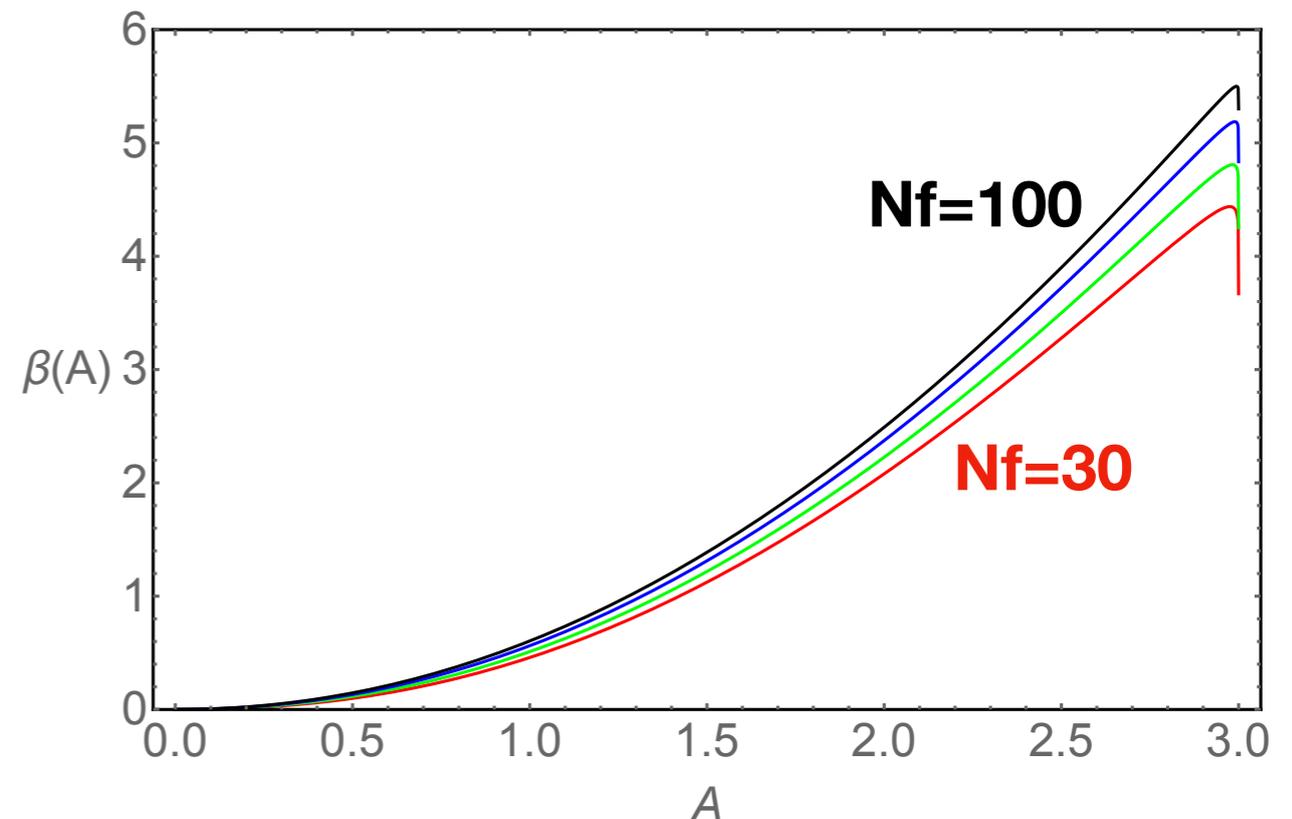
$$A_{UV} = 3 - \delta \quad \text{where} \quad \delta = \exp \left[-16 \frac{T_r}{C_A} N_f + 18.49 - 5.26 \frac{C_r}{C_A} \right]$$

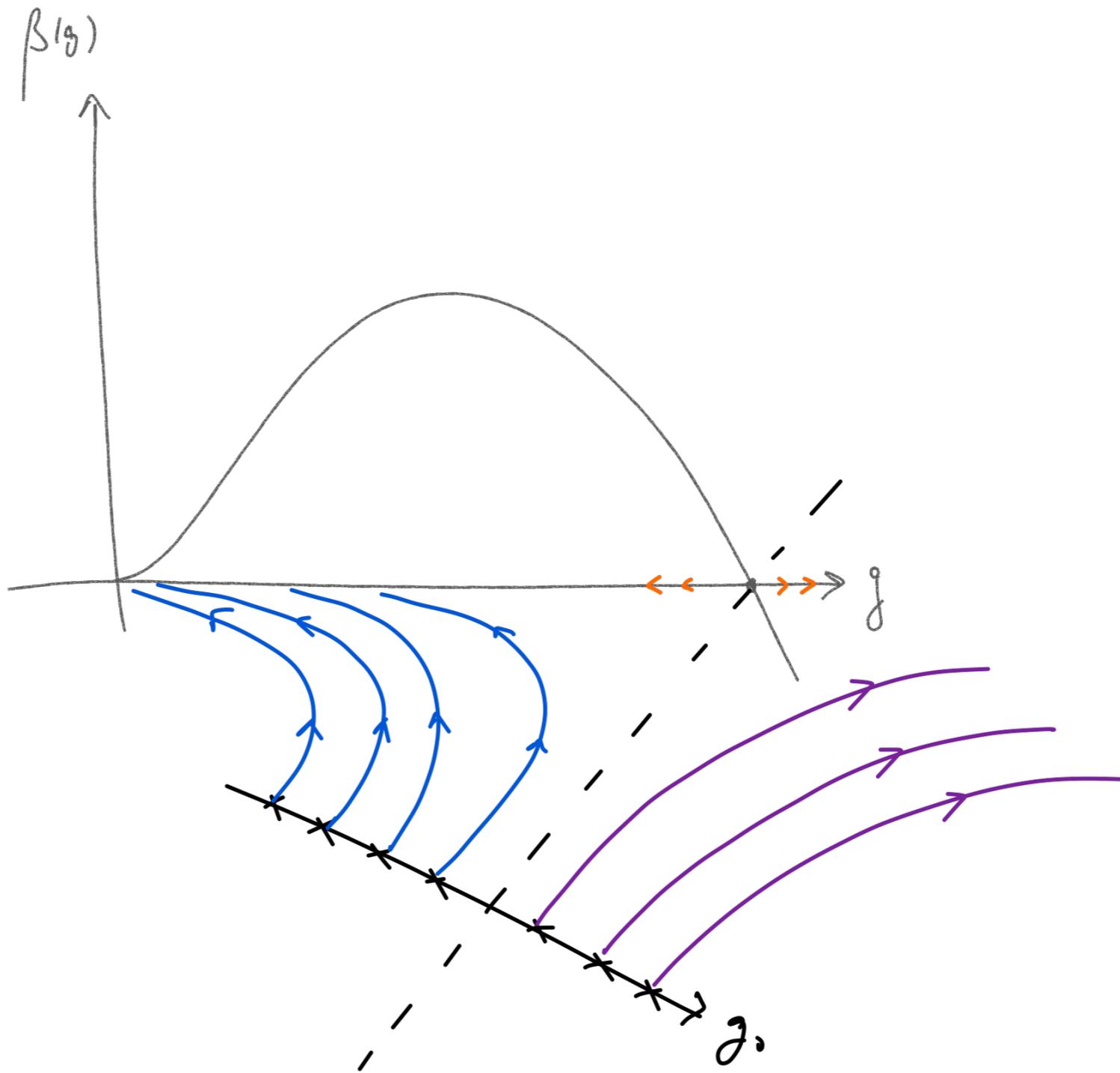
According to these results:

- Beta-function changes very rapidly near the FP
- May be difficult to see in simulations!
- Need to reach couplings $A \sim 3$

In other words (for fund. rep.) $g^2 N_f = 24\pi^2$

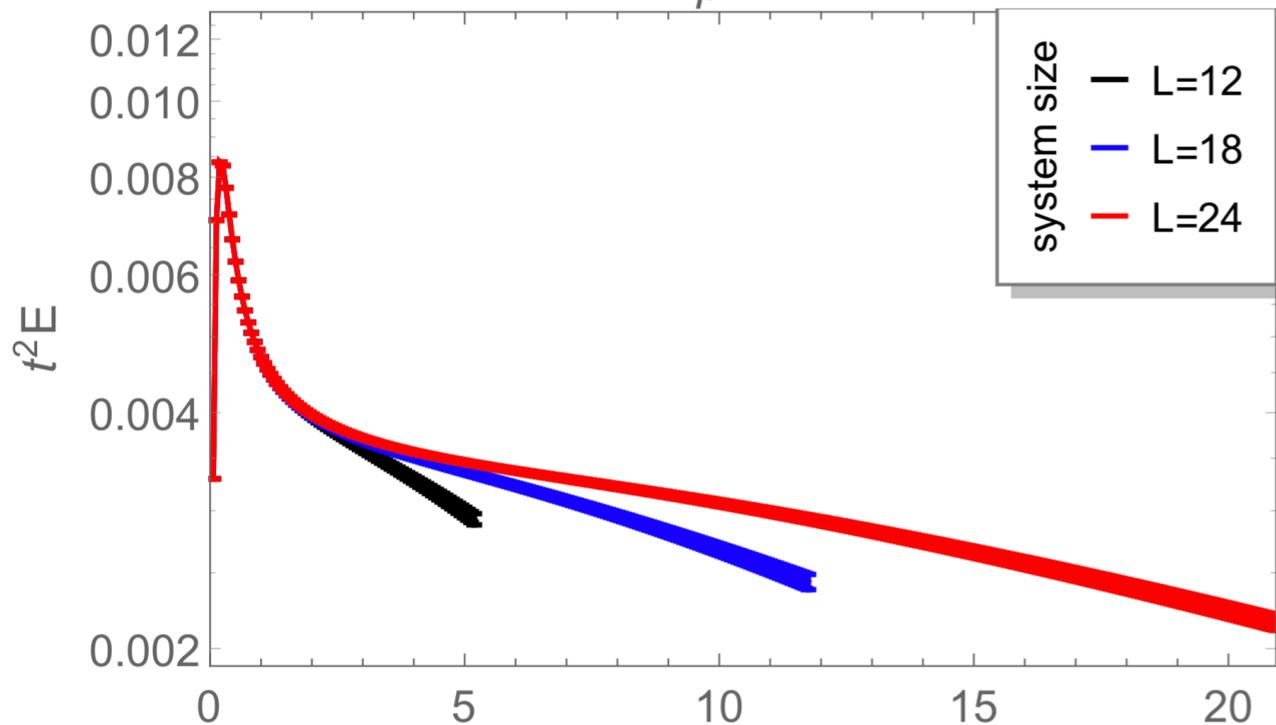
For 48 flavors this is $g^2 \sim 5$.



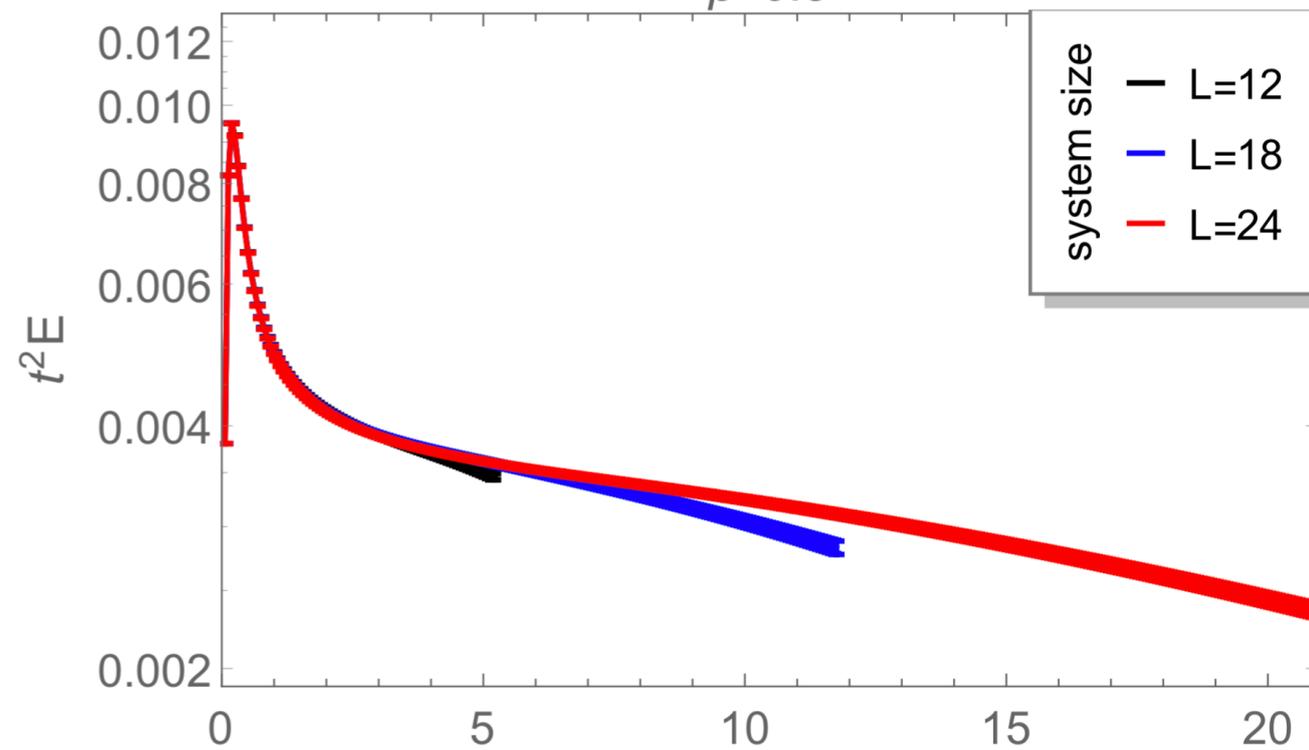


SU(2) gauge and 48 fermions

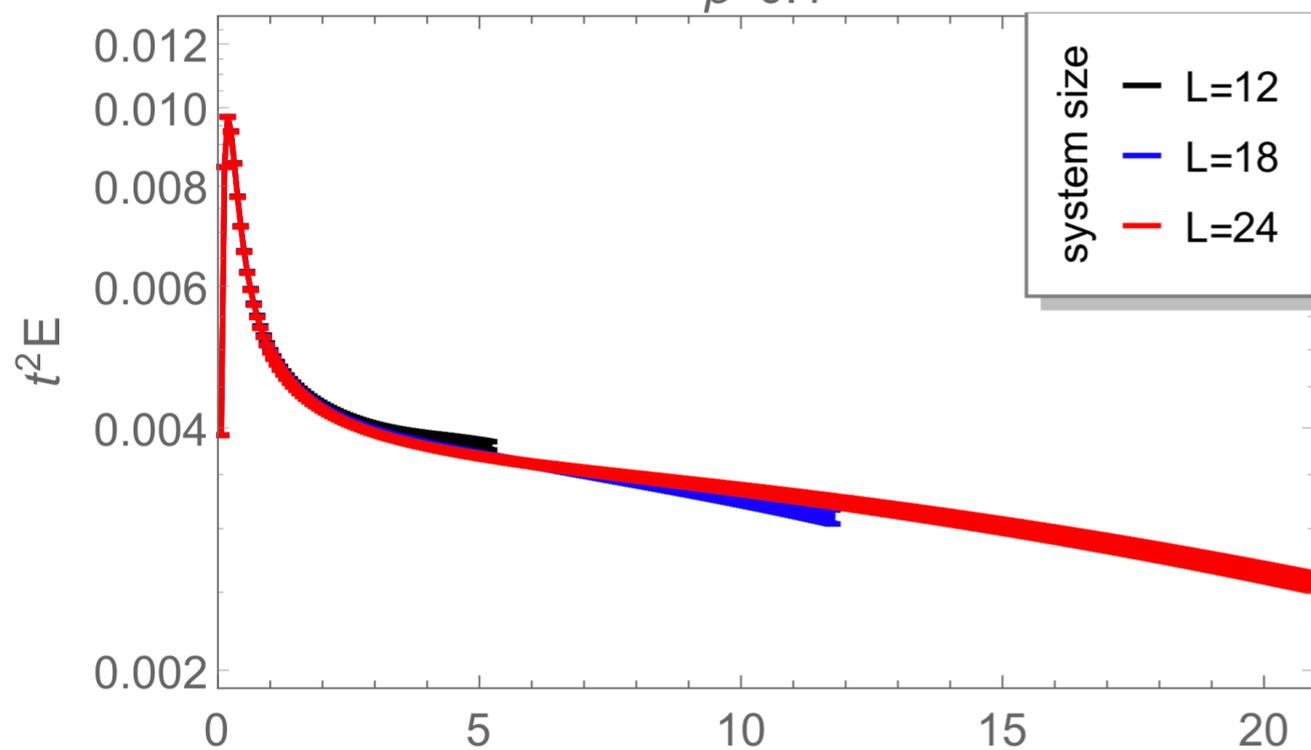
$\beta=1.$



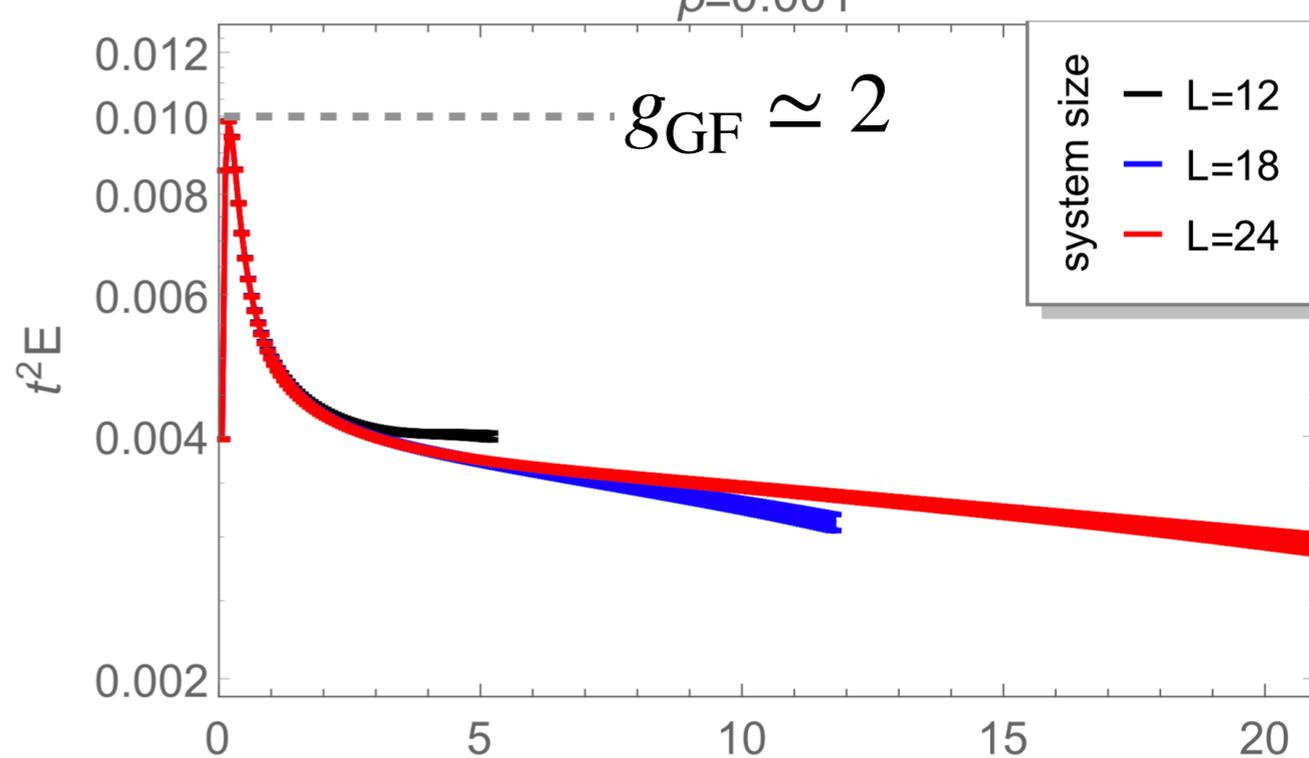
$\beta=0.3$



$\beta=0.1$



$\beta=0.001$



Summary II

Conformal window 2.0

- Predicted at leading order in $1/N_f$
- UVFP around $g^2 \sim 24\pi^2/N_f$ (for large N_f)

First lattice calculations

- @ $N_f=48$ for $SU(2)$ gauge
- No UVFP observed
- But confined to small couplings, $g_{GF}^2 < 2$

Further developments in progress.