

Asymptotically safe quantum gravity

Astrid Eichhorn,

CP3-Origins, SDU (Odense) & Heidelberg University

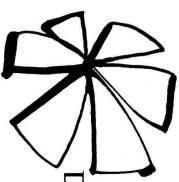
GGL mini school “From freedom to safety”,

May 13, 2019



CP3-Origins

University of
Southern Denmark



Die Junge Akademie



RUPRECHT-KARLS-
UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

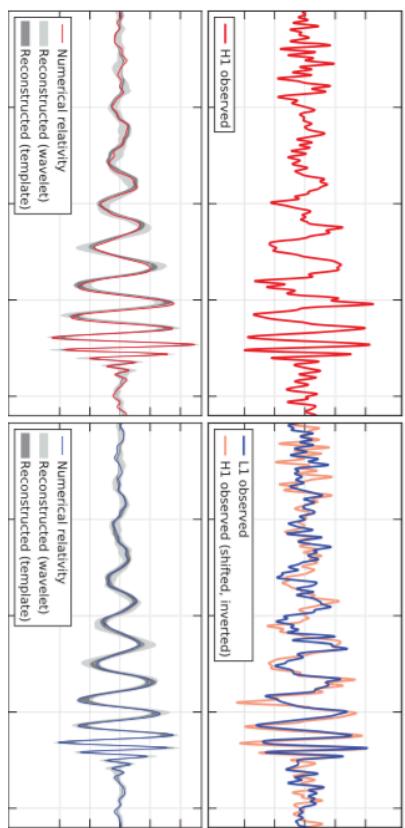


DFG

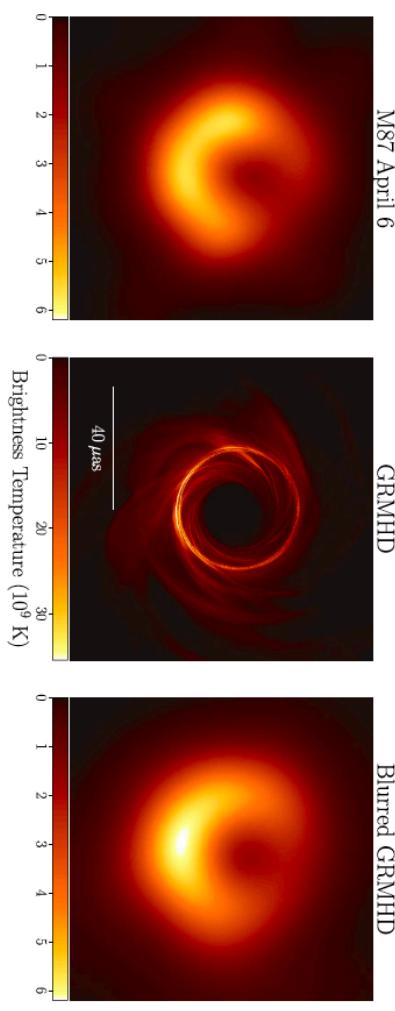
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Observations in the strong-gravity regime

The Astrophysical Journal Letters, 875:L5 (31pp), 2019 April 10
The EHT Collaboration et al.



LIGO 2015

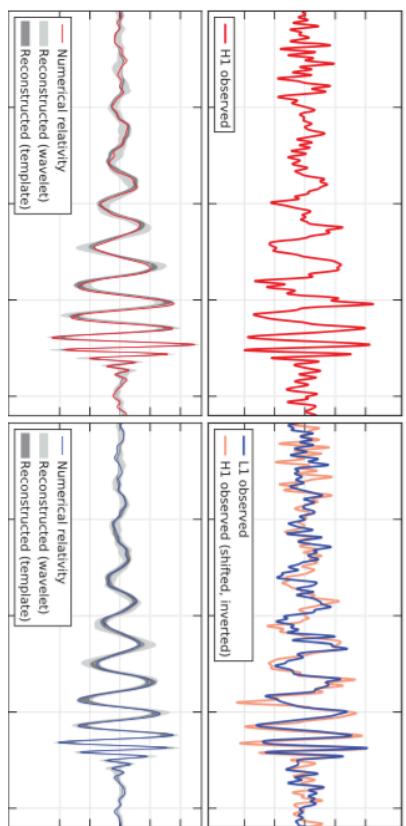


EHT 2019

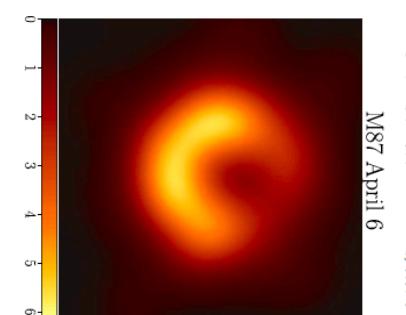
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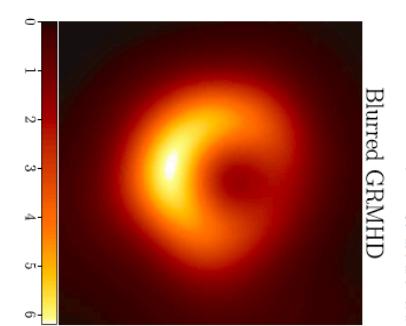
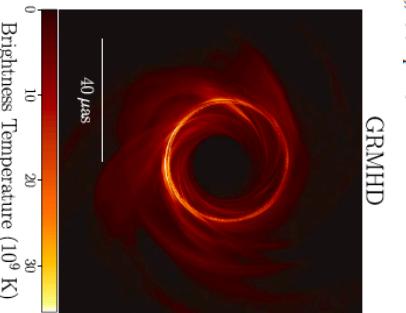
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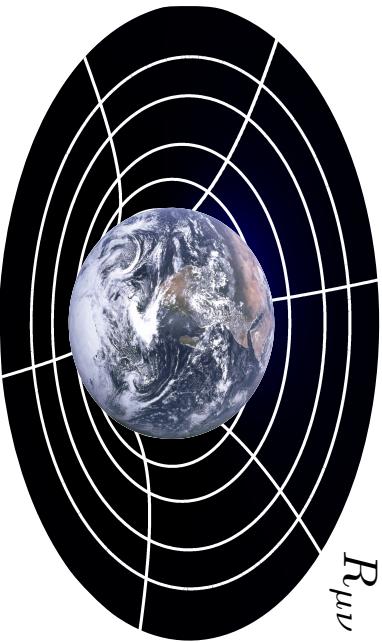
EHT 2019



observations compatible with GR predictions

Gravity = Spacetime geometry

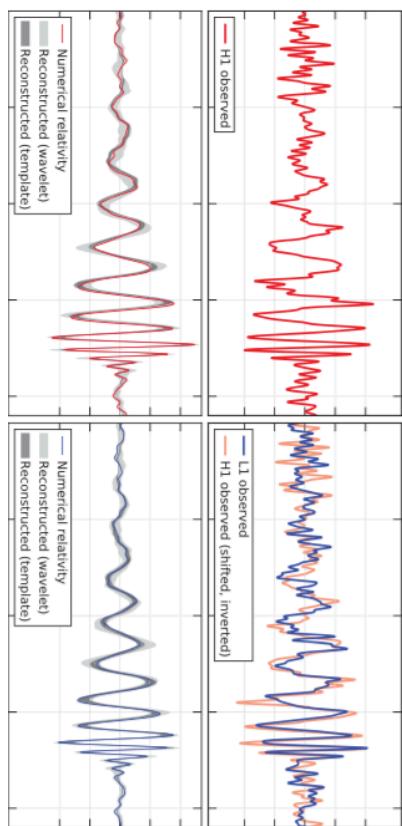
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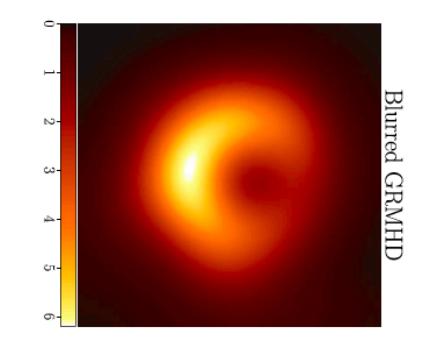
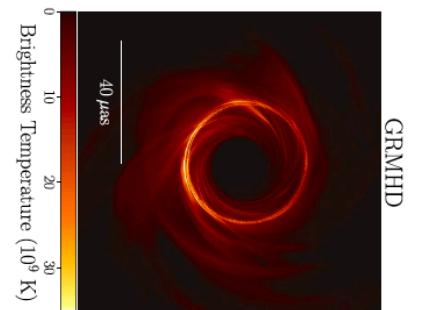
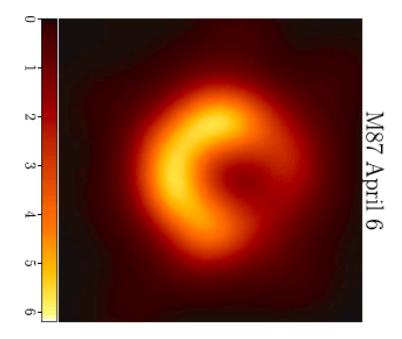
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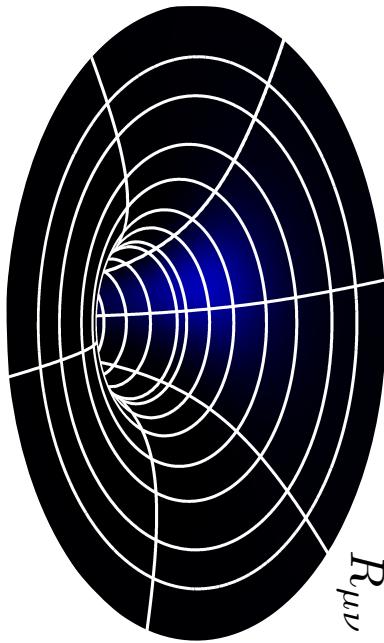
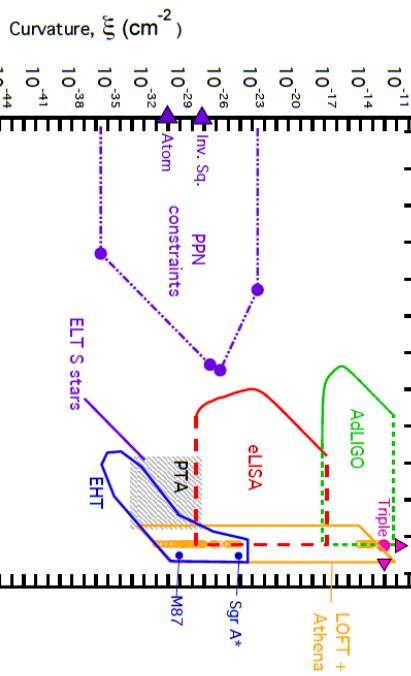
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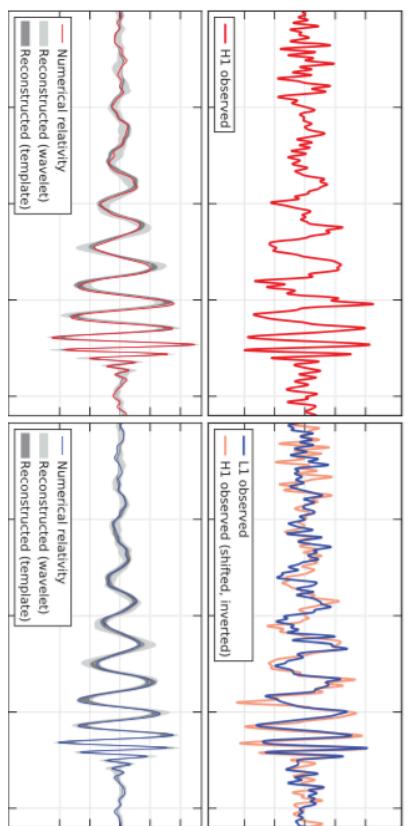
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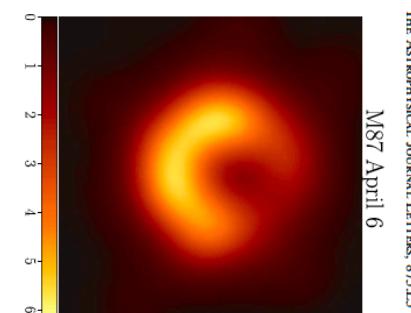
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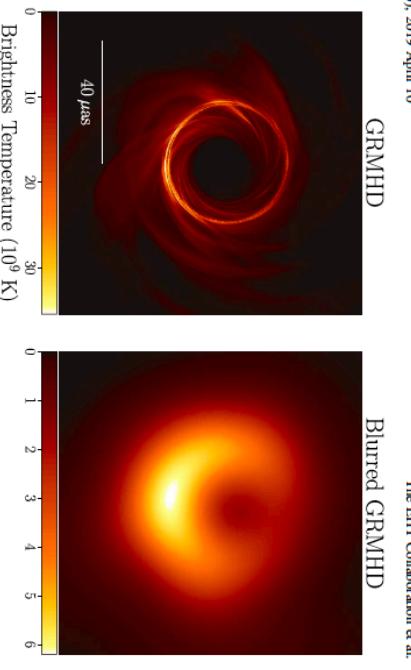
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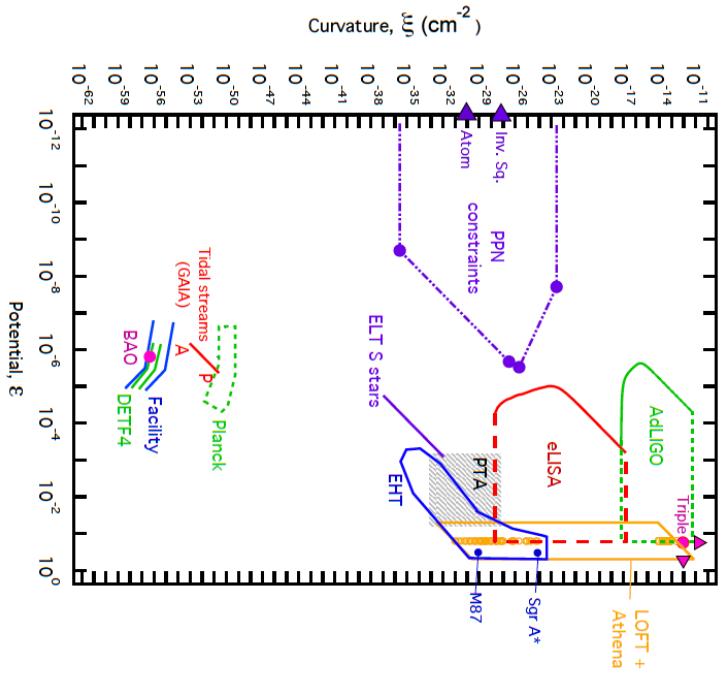


observations compatible with GR predictions

but: observed objects compatible with being GR black holes

→ curvature singularities!

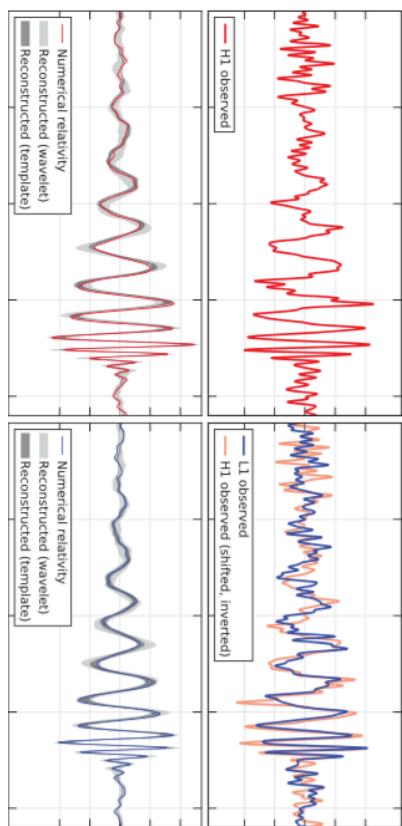
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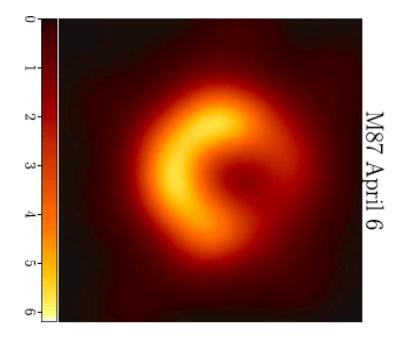
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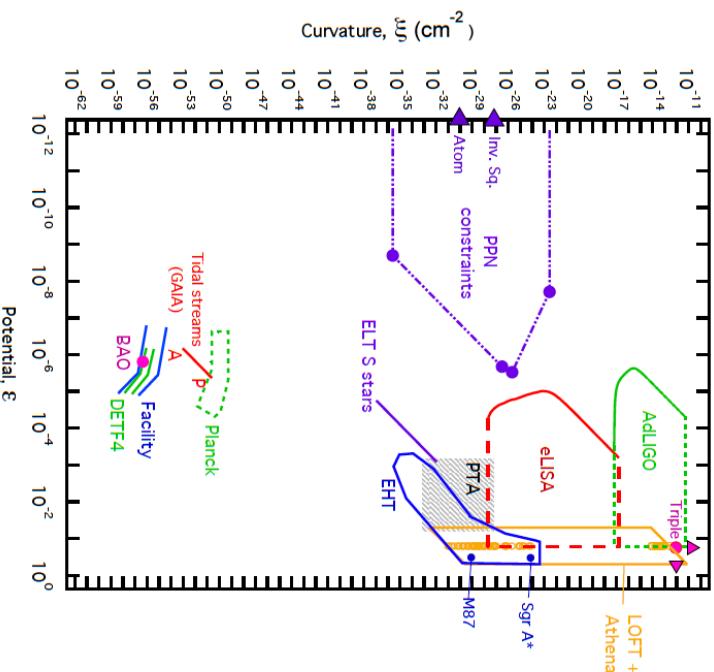
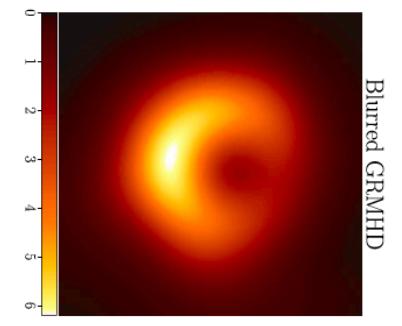
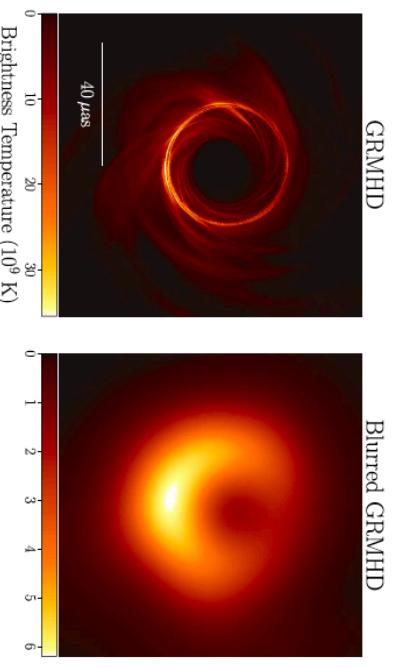
$$R = 0, \quad R_{\mu\nu} = 0$$

$$\text{Schwarzschild } K = R_{\mu\nu\kappa\lambda}R^{\mu\nu\kappa\lambda} = \frac{G_N^2 M^2}{r^6}$$



EHT 2019

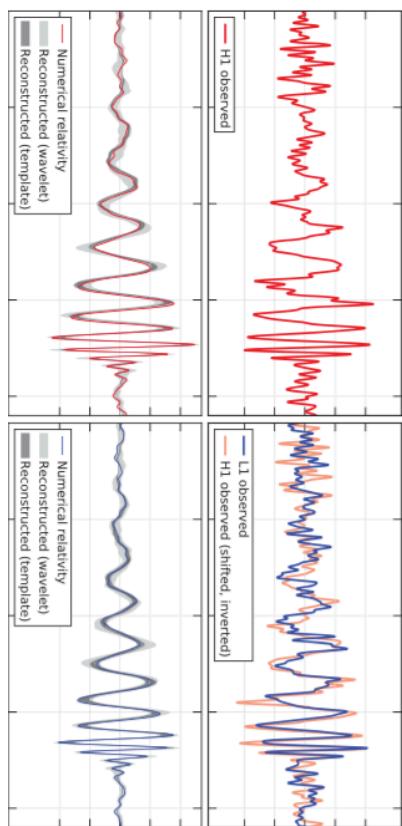
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LIGO 2015

observations highlight the necessity to go beyond GR

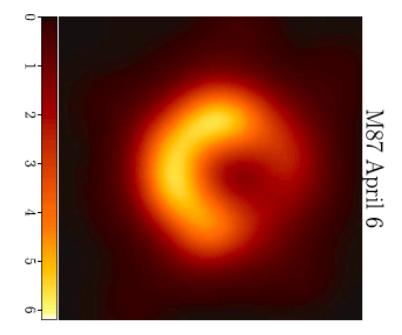
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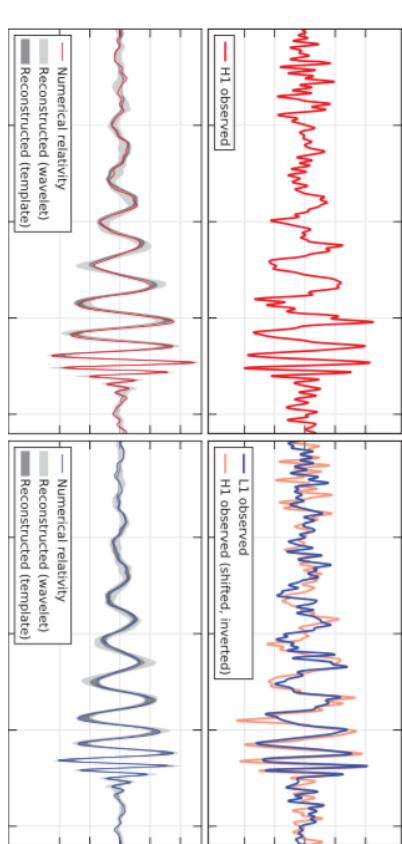
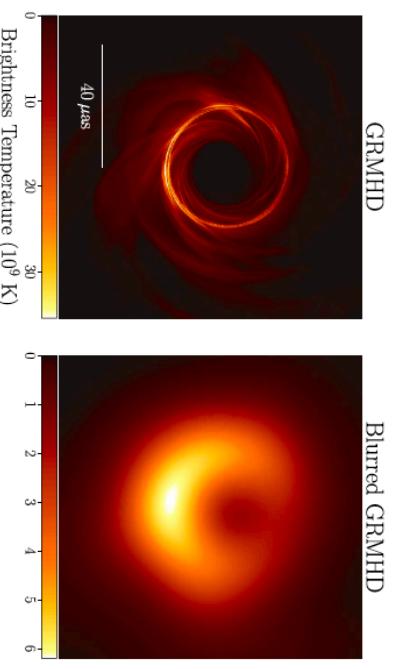
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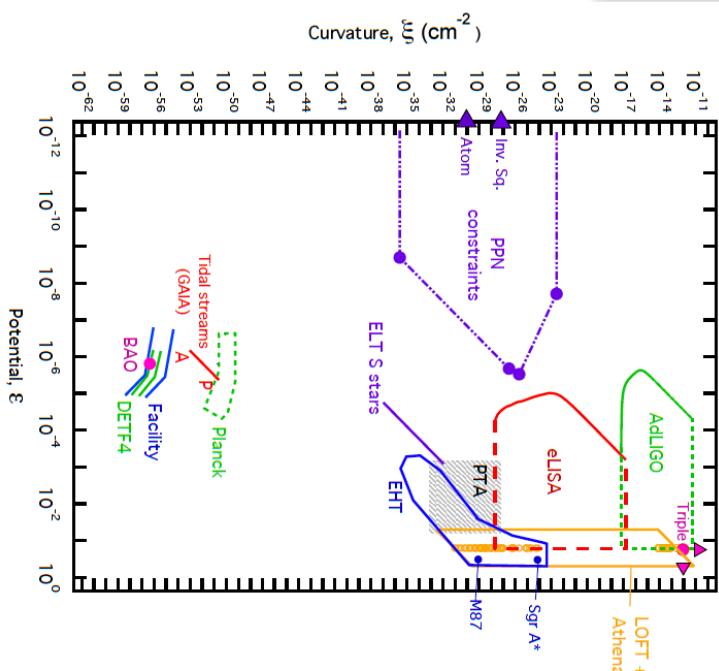
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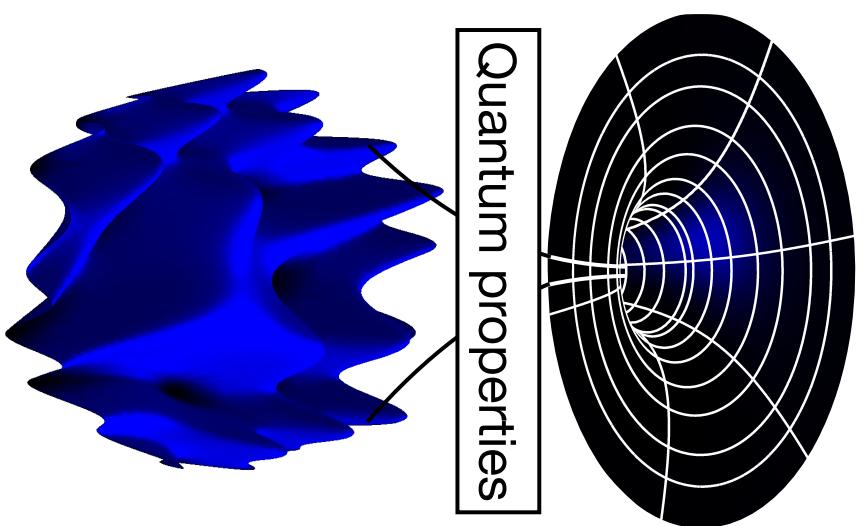


Singularity resolution expected from Quantum Gravity

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at $r \leq l_{\text{Planck}} = \sqrt{\frac{G_N \hbar}{c^3}} = 10^{-35} m$

$$M_{\text{pl}} = \sqrt{\frac{\hbar c}{G_N}} = 10^{19} \text{ GeV} = 10^{15} E_{\text{LHC}}$$



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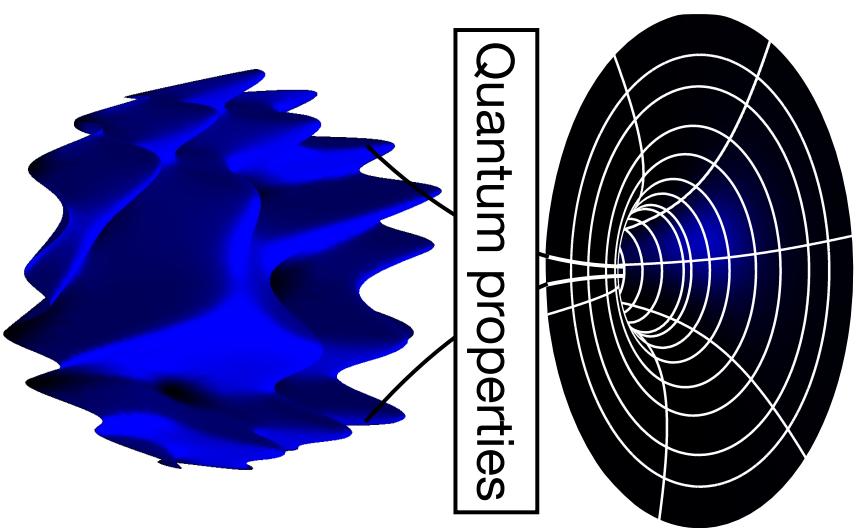
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quantum fluctuations of spacetime:

$$Z = \int \mathcal{D}g_{\mu\nu} e^{iS[g_{\mu\nu}]}$$



Quantum field theory for gravity

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Which microscopic action?

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spin-2-field on flat background

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counterterms:

1-loop: R^2 , $R_{\mu\nu}R^{\mu\nu}$

*t Hooft, Veltman '74;
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(can eliminate using e.o.m
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breakdown of predictivity

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counterterms:

also:	R^2 , $R_{\mu\nu}R^{\mu\nu}$	<small>t Hooft, Veltman '74; Deser, Nieuwenhuizen '74</small>
perturbative renormalizability	$C_{\mu\nu\kappa\lambda}C^{\kappa\lambda\rho\sigma} C_{\rho\sigma}^{\mu\nu}$	<small>Goroff, Sagnotti '86; Van de Ven '92</small>
neither necessary nor sufficient for "fundamentality" (cf. QED: Landau pole: trivial theory)		

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breakdown of predictivity

consistent choice of \mathbf{S} with finite number of free parameters?

Asymptotic safety

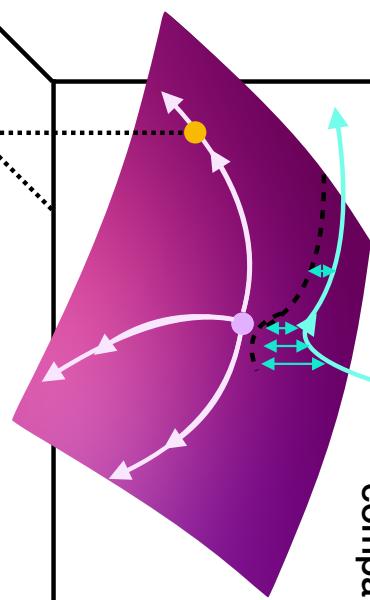
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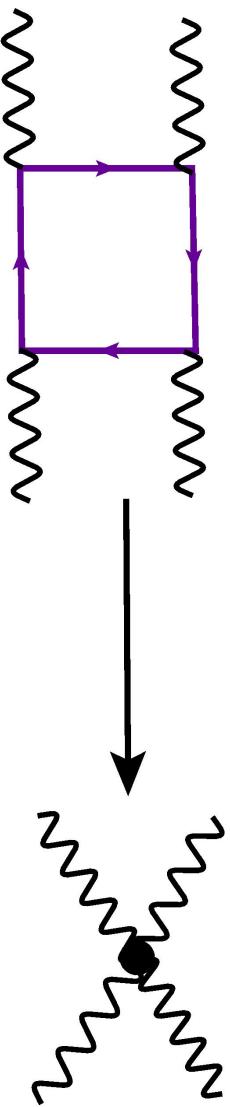
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$\sigma \cdot k^2$
 $G_N \cdot k^2$
 $\Lambda \cdot k^{-2}$

theory space:
all couplings
compatible w. symmetries



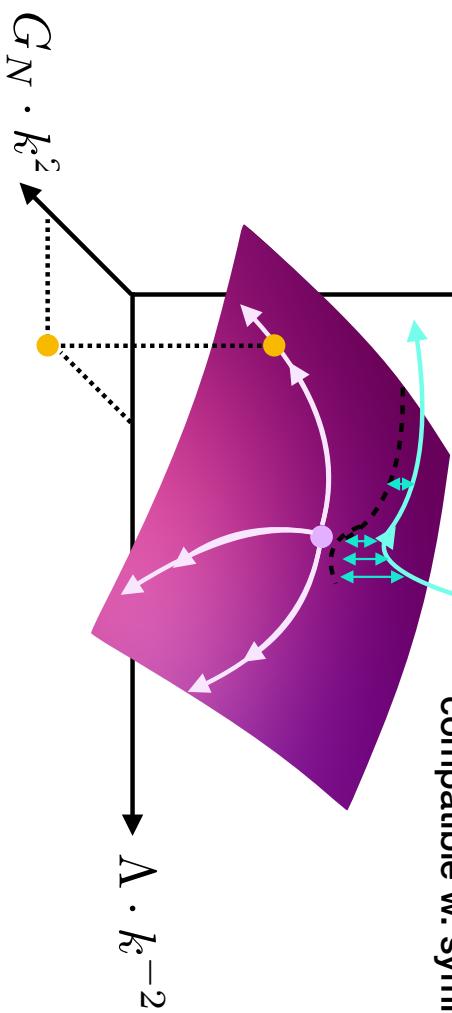
qm fluctuations generate ALL couplings compatible with symmetries



Asymptotic safety

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Theory space features an interacting fixed point

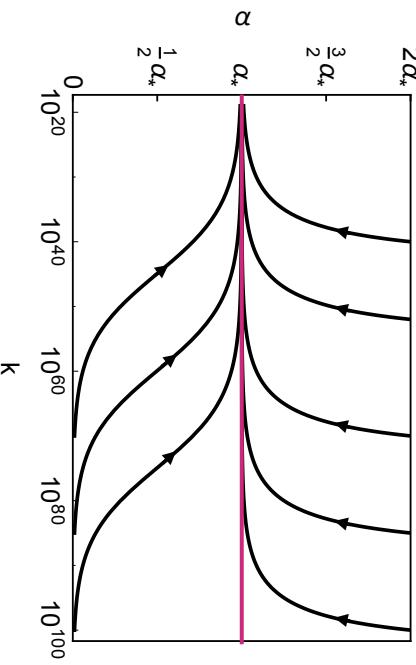
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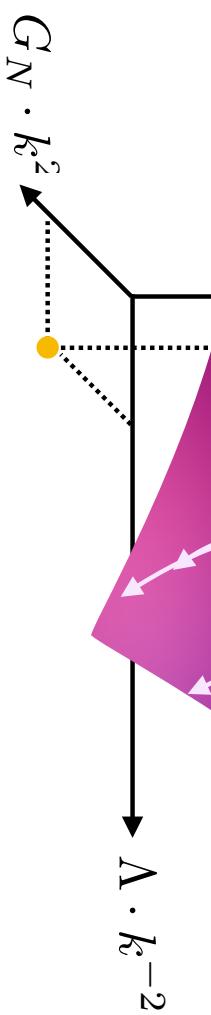
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Irrelevant directions:
Predictions from asymptotic safety



**predictive power of new symmetry
(scale symmetry)**



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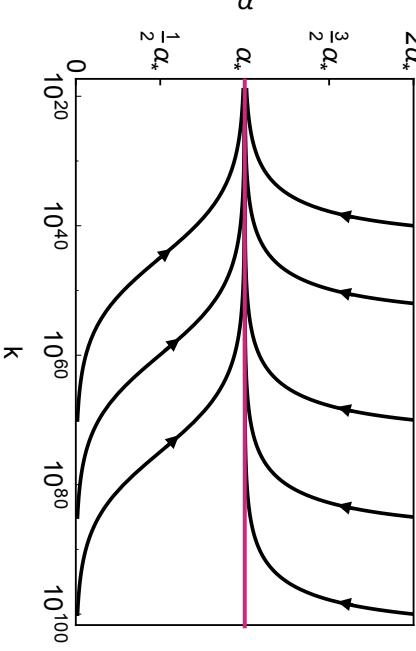
with a finite number of relevant directions.

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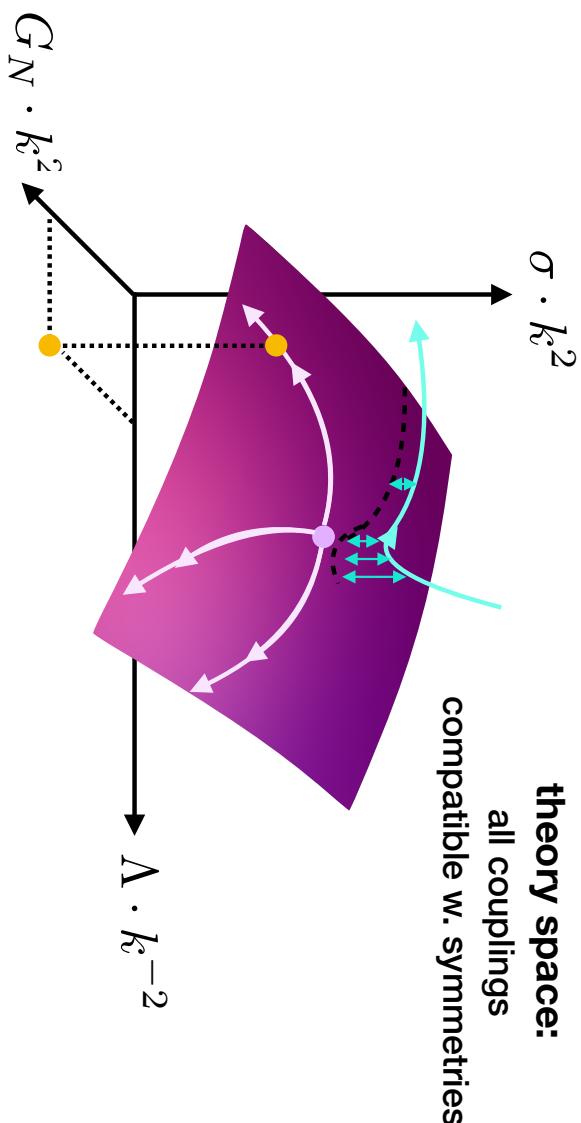
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$\theta = d_{g_i} + \eta_{g_i} \rightarrow$ at ASFP canonical dimension does not determine (ir)relevance

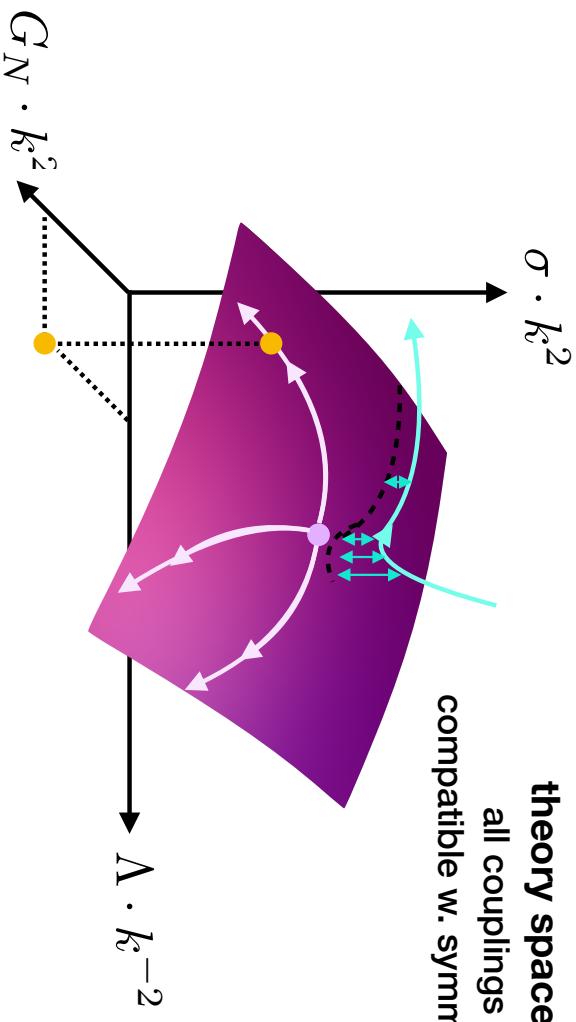
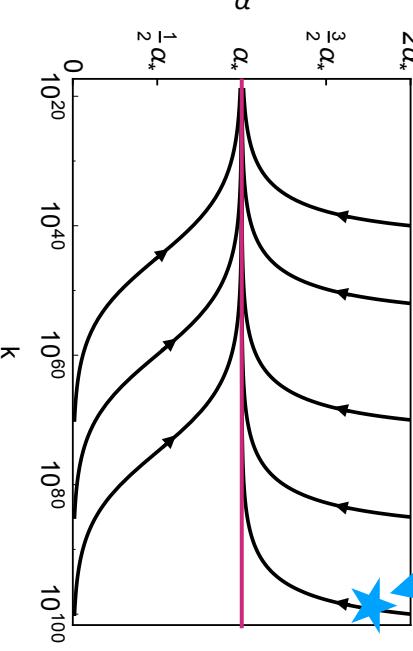
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“non-fundamental”
asymptotic safety

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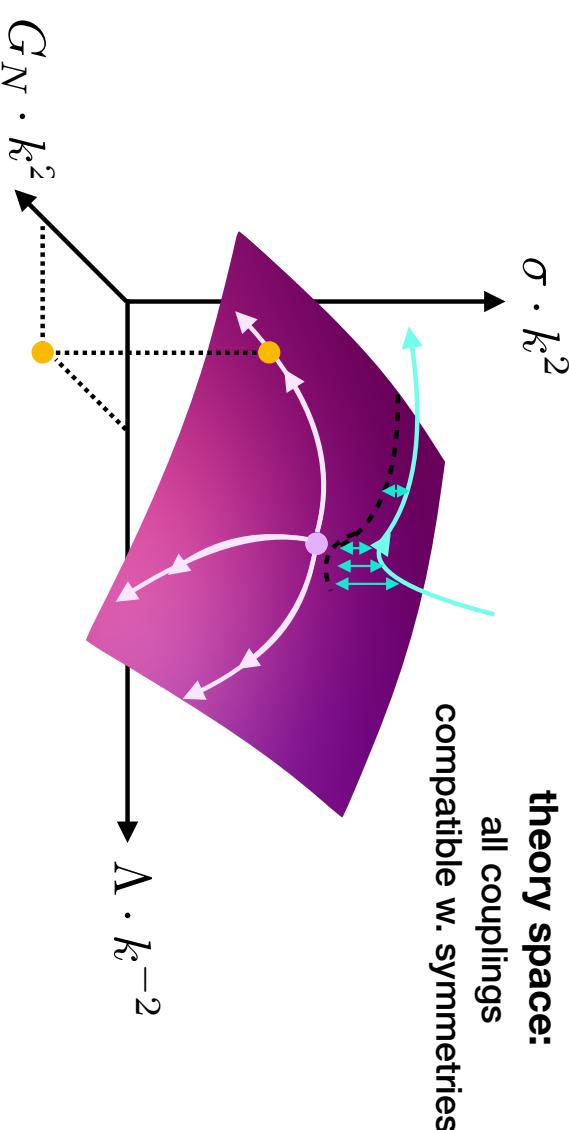
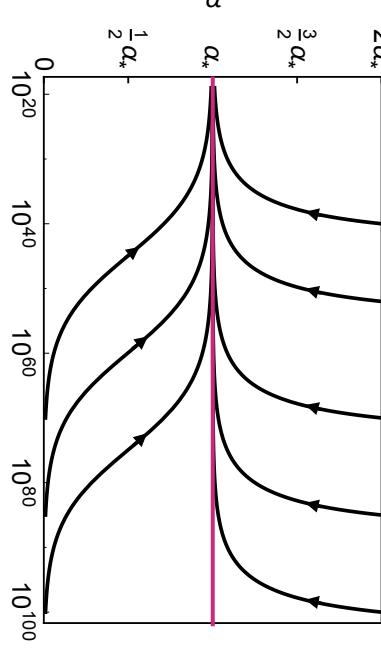
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Theory space features an interacting fixed point

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(At least) one trajectory emanating from the fixed point reaches a phenomenologically viable IR regime.

- UV complete $\beta_i = 0 \forall i$
- lattice: universal continuum limit
- predictive
(finite # free parameters)
- predictions for irrelevant
couplings (= IR attractive)
match observations

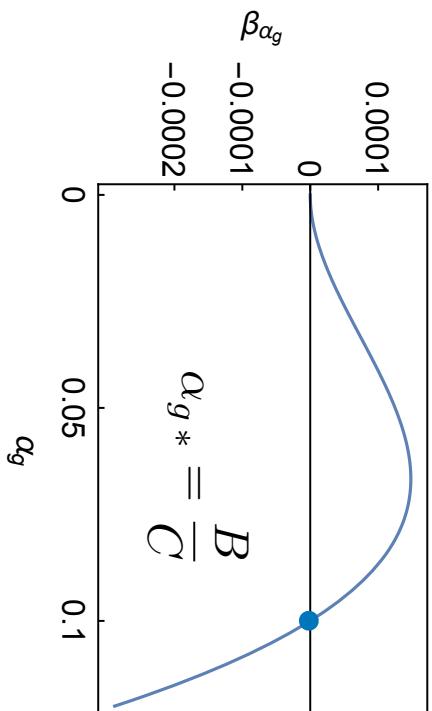
Mechanisms for asymptotic safety

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- perturbative: one-loop versus two-loop

**blueprint for gauge-Yukawa models
in d=4 dimensions:**
[Litim, Sannino '14]

$$\beta_{\alpha_g} = (-B + C\alpha_g) \alpha_g^2 + \mathcal{O}(\alpha_g^4)$$



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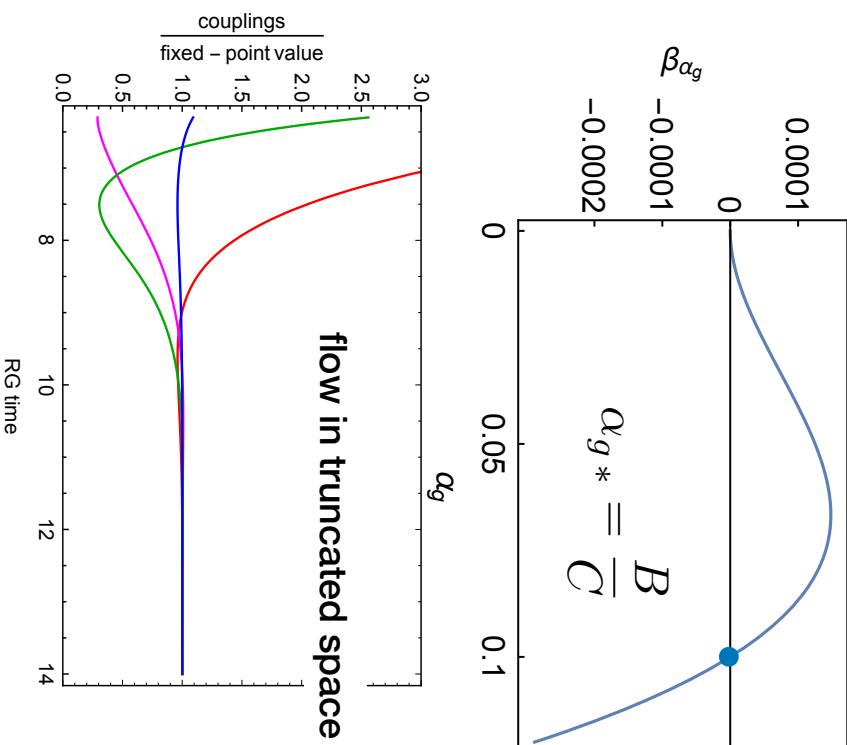
$$\beta_\lambda = \beta^{(\text{bosonic})} - \beta^{(\text{fermionic})}$$

first tentative hints in fermionic Higgs portal

[AE, Held, Vander Griend HEP 1808 (2018) 147]

analysis of Higgs stability:

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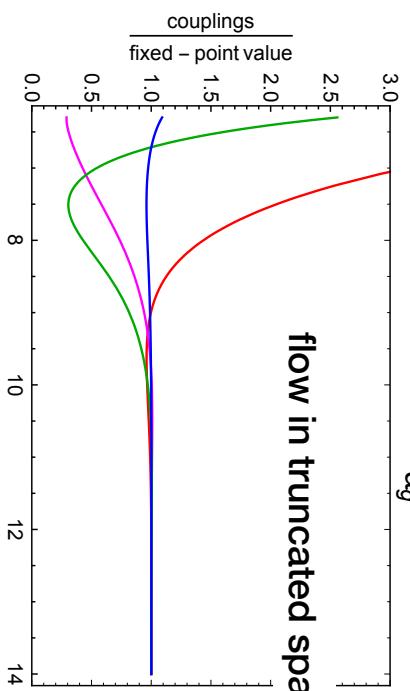
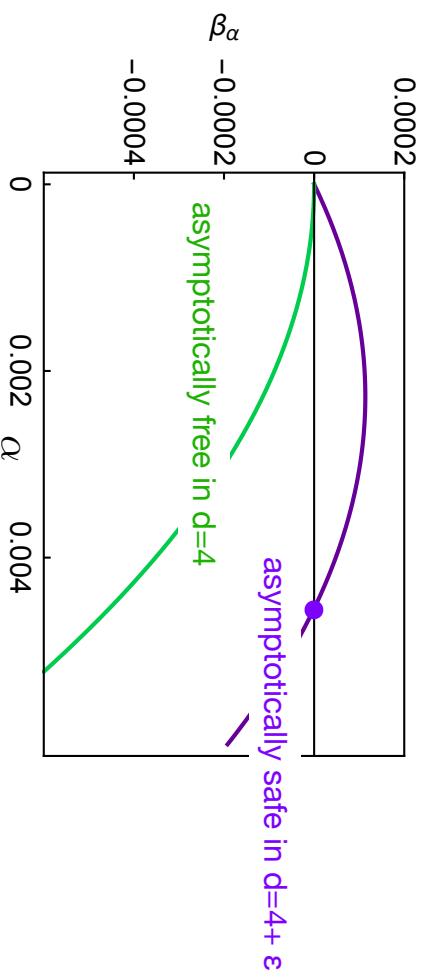
- canonical vs. quantum

example: Yang Mills in d=4 + ϵ

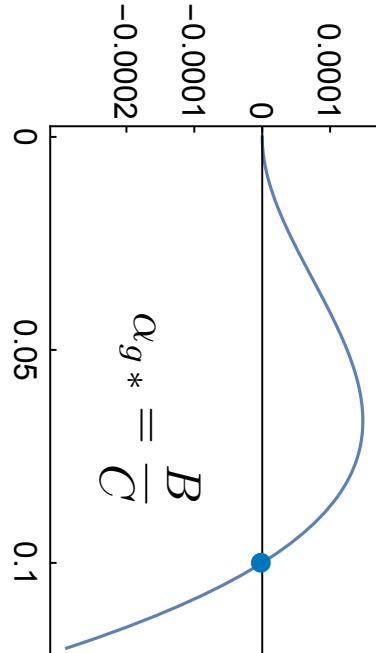
[Peskin '80; Gies '03; Morris '04; Knetchli, Rinaldi '16]

$$\beta_{\alpha_g} = \epsilon \alpha_g - B \alpha_g^2 + \dots$$

negative canonical dimensionality



flow in truncated space of couplings



Mechanisms for asymptotic safety

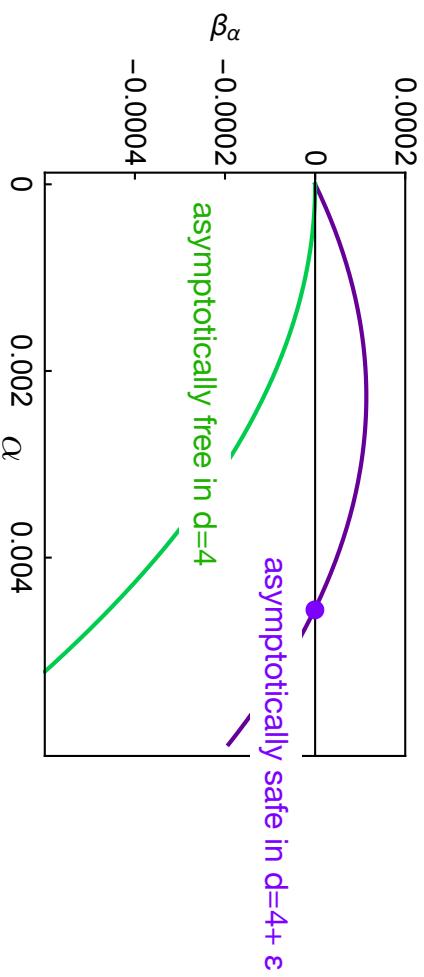
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example: Yang Mills in $d=4 + \epsilon$

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negative canonical dimensionality

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asymptotically safe gravity in $d > 2$

[Reuter '96]

ϵ -expansion [Weinberg '86; Gastmans, Kallosh, Truffin '78; Christensen, Duff '78...]

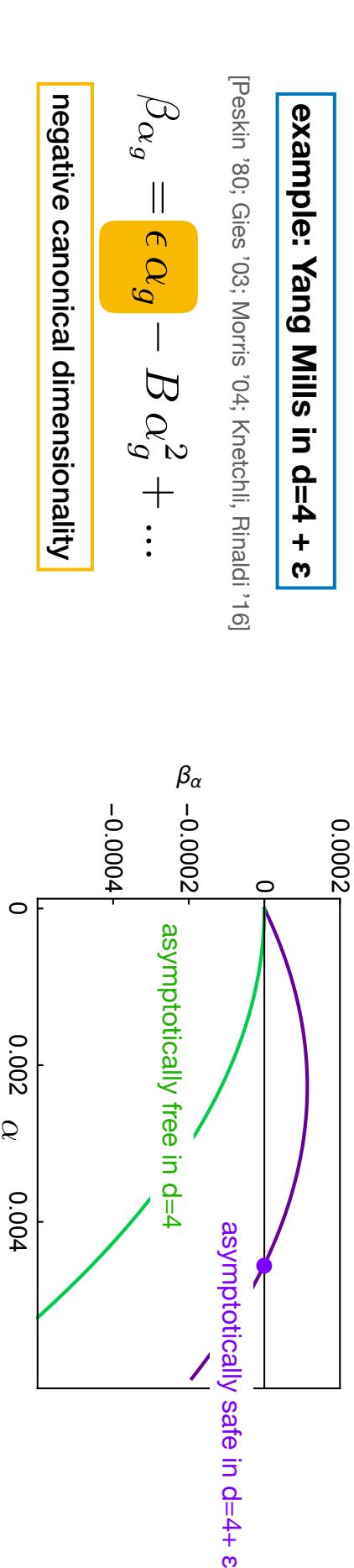
$$\beta_G = \epsilon G - \frac{38}{3} G^2$$

- canonical vs. quantum

example: Yang Mills in $d=4 + \epsilon$

[Peskin '80; Gies '03; Morris '04; Knetchli, Rinaldi '16]

$$\beta_{\alpha_g} = \epsilon \alpha_g - B \alpha_g^2 + \dots$$



negative canonical dimensionality

Functional Renormalization Group

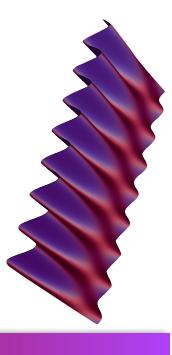
probe scale dependence of QFT

Functional Renormalization Group

probe scale dependence of QFT

Wetterich '93, Reuter '96

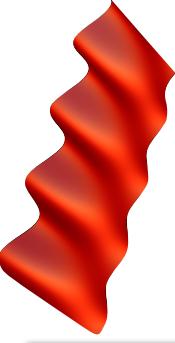
$$e^{-\Gamma_k[\phi]} = \int \mathcal{D}\varphi e^{-S[\varphi] - \frac{1}{2} \int \varphi(-p) R_k(p) \varphi(p)}$$



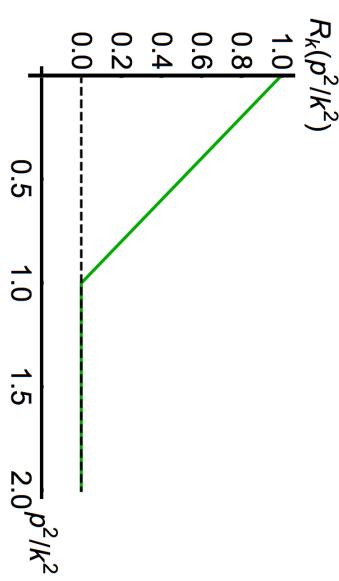
S

scale- and momentum-dependent “mass”

Γ_k contains effect of quantum fluctuations above k



$\Gamma_{k \rightarrow 0}$

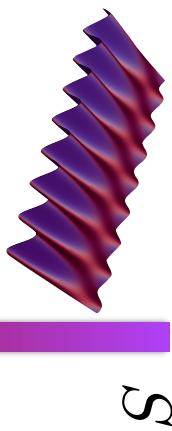


Functional Renormalization Group

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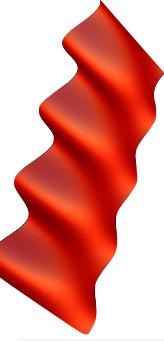
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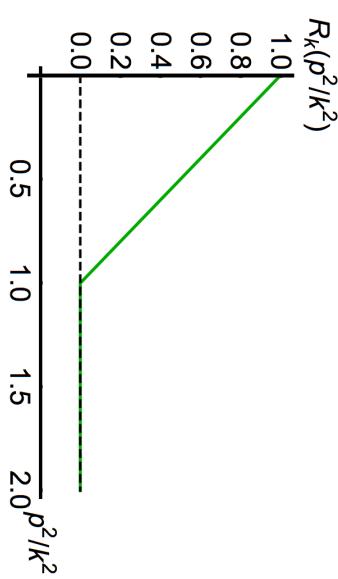
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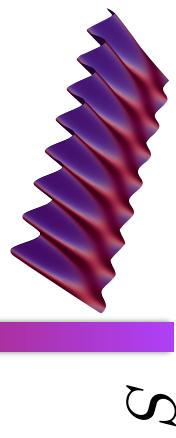
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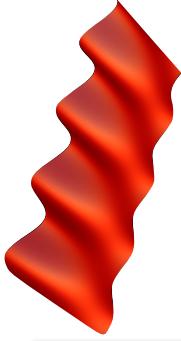
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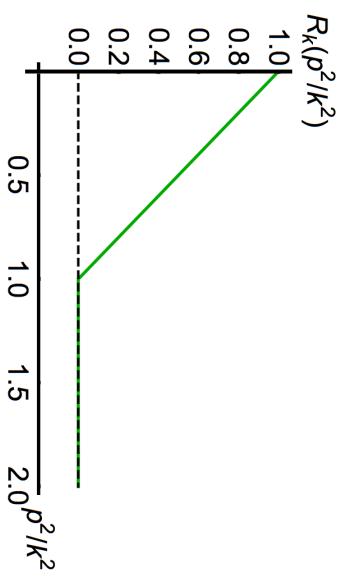


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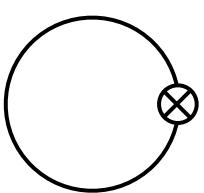
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- exact
- UV- and IR- finite

$$\text{Wetterich equation: } \partial_k \Gamma_k = \frac{1}{2} \text{STr} \left(\Gamma_k^{(2)} + R_k \right)^{-1} \partial_k R_k =$$

Truncations

$$\partial_k \Gamma_k = \frac{1}{2} \text{STr} \left(\Gamma_k^{(2)} + R_k \right)^{-1} \partial_k R_k = \circlearrowleft$$

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→ infinite tower of coupled differential equations

e.g. vertex expansion: $\Gamma_k = \sum_n \int V_k(x_1, \dots, x_n) \phi(x_1) \dots \phi(x_n)$
flow of n-point fct depends on up to (n+2)-point fct:

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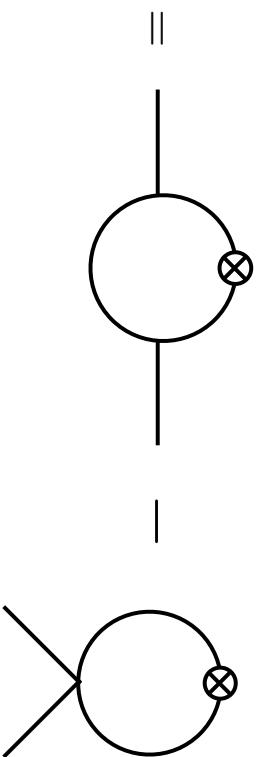
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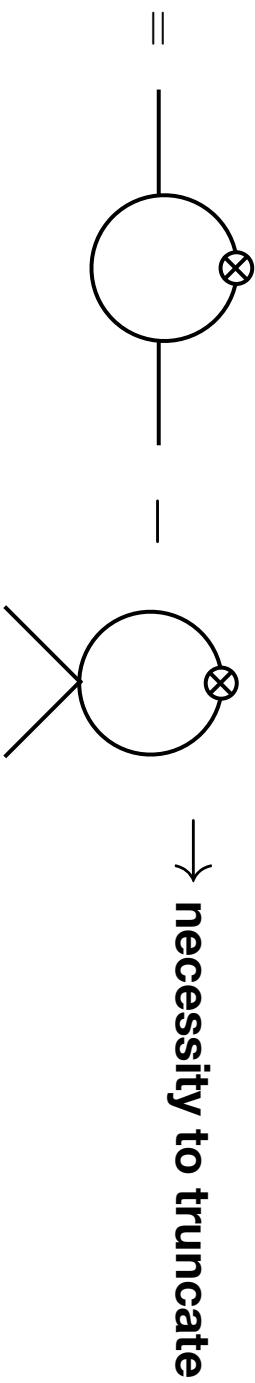
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Truncations

- truncate effective dynamics according to an expansion principle
(e.g., derivative expansion, vertex expansion, near-canonical scaling....)
- calculate beta-functions, search for fixed point, evaluate universal quantities (critical exponents)
- extend truncation and look for apparent convergence of universal fixed-point properties

Quantitative characterization of universality classes from the FRG

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example: Ising model

local potential approximation (LPA $_N$): $\Gamma_k = \int (Z_k \partial_\mu \phi \partial^\mu \phi + V_k(\phi^2))$.

$$V_k = \sum_{i=1}^N \lambda_i \phi^{2i}$$

truncation	$\nu = 1/\theta_1$	$\omega = -\theta_2$	η
LPA 2	1/2	1/3	0
LPA 3	0.729	1.07	0
LPA 4	0.651	0.599	0
LPA 5	0.645	0.644	0
LPA 6	0.65	0.661	0
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<hr/>			
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LPA' 3	0.684	1.33	0.0387
LPA' 4	0.64	0.703	0.0433
LPA' 5	0.634	0.719	0.0445
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fourth order derivative expansion:

$$\nu = 0.632, \quad \eta = 0.033$$

[Canet et al, '04; Litim, Zappala '10]

cf. 7-loop pert. theory

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- ..similar for other universality classes
(with fermions, scalars, vectors, in arbitrary dimension)

Functional Renormalization Group for gravity

$$Z = \int \mathcal{D}g_{\mu\nu} e^{iS[g_{\mu\nu}]} \rightarrow Z = \int \mathcal{D}g_{\mu\nu} e^{-S[g_{\mu\nu}]}$$

**note: no Wick-rotation in quantum gravity;
relation between Euclidean & Lorentzian QG unknown**

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local coarse graining: $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ linear split

cutoff: $\Delta_k S \sim \int d^4x \sqrt{\bar{g}} h^{\mu\nu} R_{k\mu\nu\kappa\lambda} [-\bar{D}^2/k^2] h_{\kappa\lambda}$

$$\rightarrow \Gamma_k = \Gamma_k [g_{\mu\nu}, \bar{g}_{\mu\nu}]$$

background independence (“there is no preferred metric”):
quantize “on all backgrounds simultaneously” (M. Reuter)

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truncation: (e.g., Einstein-Hilbert)

$$\Gamma_k = -\frac{1}{16\pi G_N} \int d^4x \sqrt{g} (R - 2\Lambda) + S_{\text{gauge-fixing}} \quad G_N \rightarrow G_N(k), \Lambda \rightarrow \Lambda(k)$$

$$\lambda(k) = \Lambda(k) k^{-2} \quad g(k) = G_N(k) k^2$$

$$\rightarrow \beta_g = 2g + \eta_N(g, \lambda)g \quad \beta_\lambda = -2\lambda + \eta_\lambda(g, \lambda)$$

Perturbative nonrenormalizability: FRG viewpoint

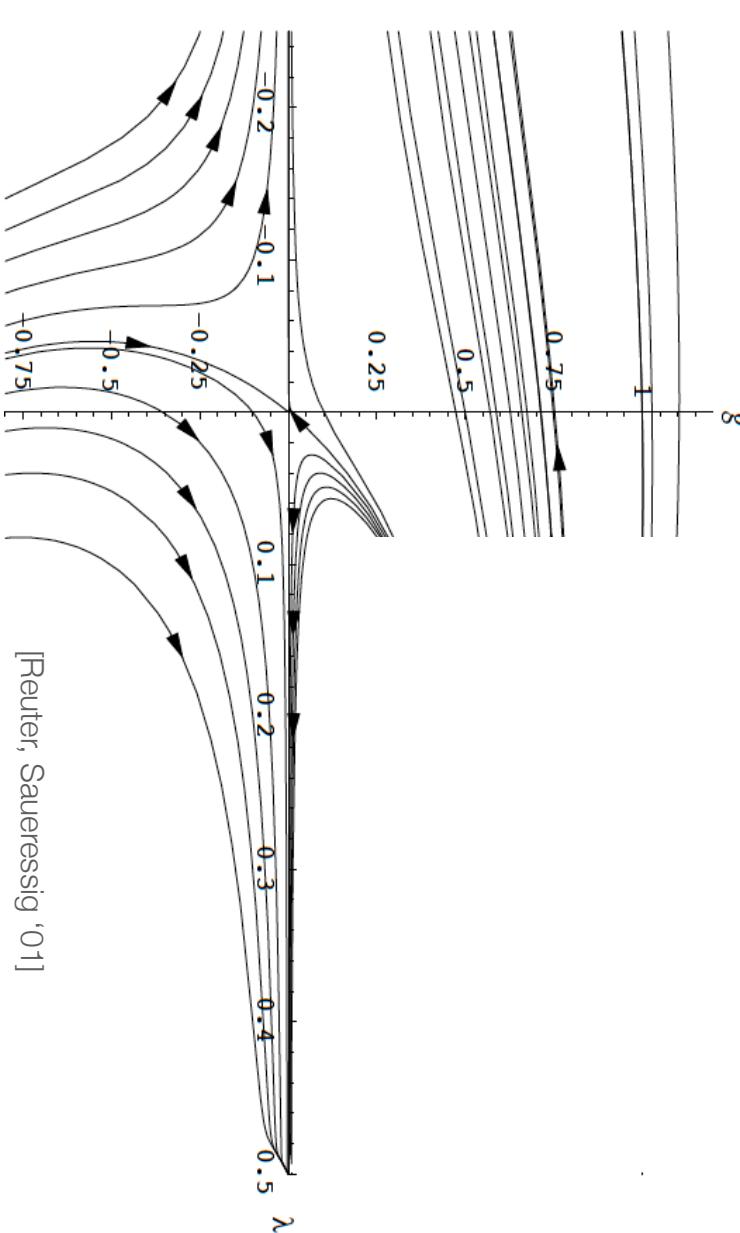
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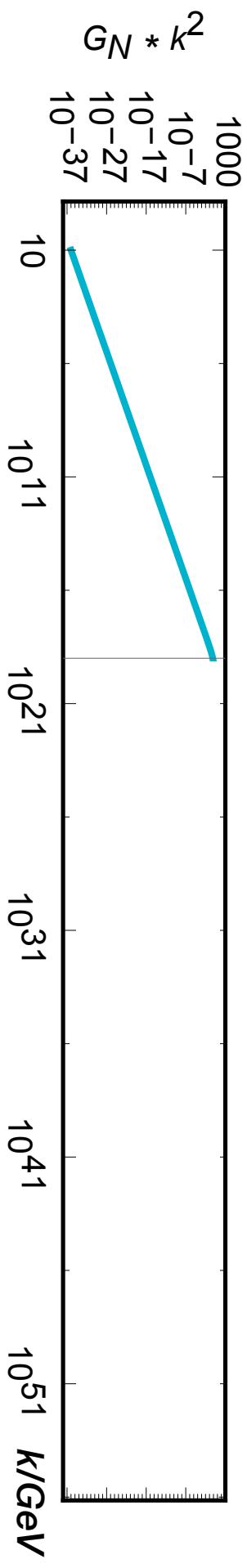
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\Rightarrow higher-order counterterms

\Rightarrow no asymptotic freedom



[Reuter, Saueressig '01]



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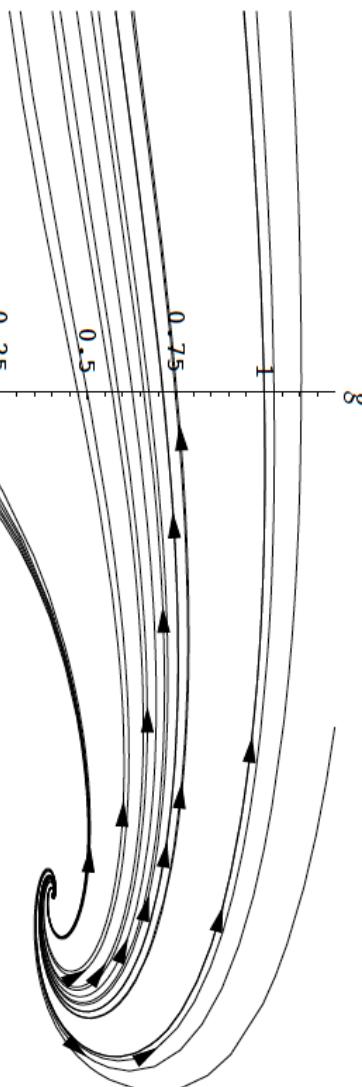
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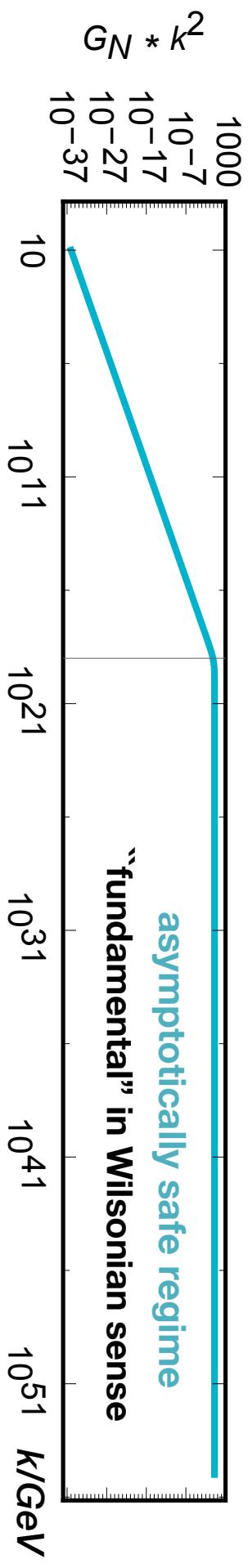
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$$G_N * k^2$$



asymptotically safe regime
"fundamental" in Wilsonian sense

$$\rightarrow \beta_g = 2g + \eta_N(g, \lambda)g$$

[Reuter, Saueressig '01]