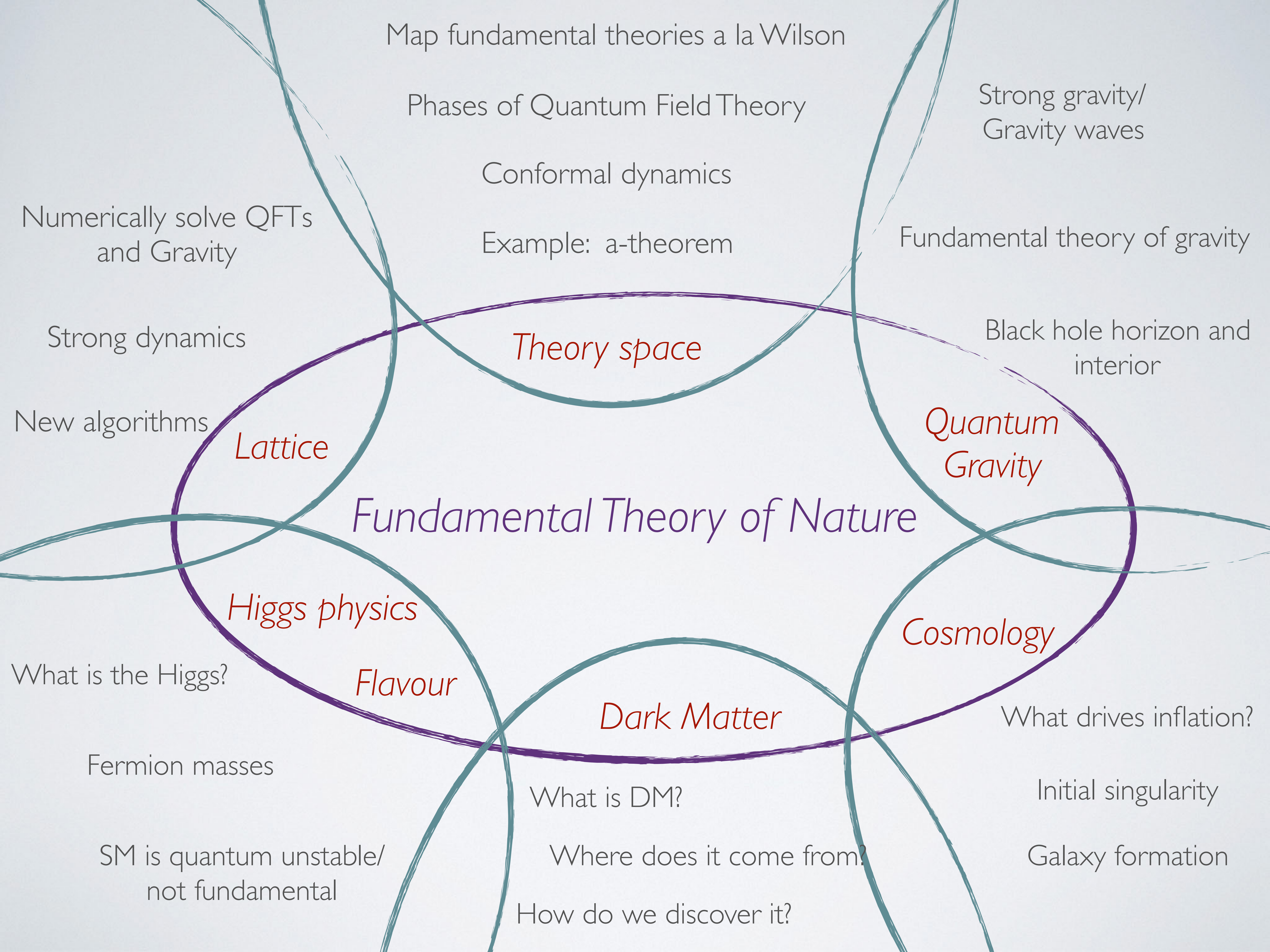
Safe Interactions

Francesco Sannino


Map fundamental theories a la Wilson
Phases of Quantum Field Theory
Conformal dynamics
Example: a-theorem
Strong gravity/
Gravity waves
Fundamental theory of gravity
Black hole horizon and
interior
Numerically solve QFTs
and Gravity
Strong dynamics
New algorithms
Theory space
Lattice
Quantum
Gravity
Fundamental Theory of Nature
Higgs physics
Flavour
Cosmology
Dark Matter
What is the Higgs?
Fermion masses
SM is quantum unstable/
not fundamental
What is DM?
Where does it come from?
How do we discover it?
What drives inflation?
Initial singularity
Galaxy formation

Bridging Theory and Experiments

$\boxed{\text{Effective}}$ $\boxed{\text{Fundamental}}$

Effective

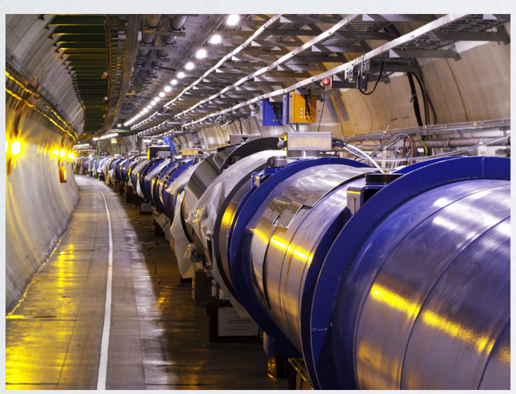
Bottom-up

Energy

M_{Planck} Gravity scale

Λ

Unknown fundamental theory



$$\boxed{\mathrm{SM}} + \sum_{p\geq 1} \frac{\boxed{c_p}\mathcal{O}_p}{\Lambda^p}$$

Standard Model

Experiments fix $\{c_p\}$ coefficients

Fundamental

Top - down I

Energy

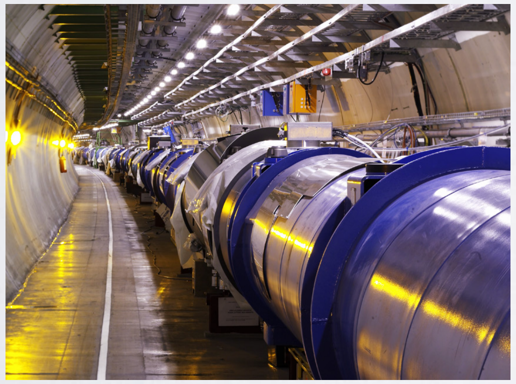
M_{Planck} Gravity scale

Λ

Fundamental theory (up to Gravity)

- Grand unified theories
- Safe theories
- Composite (Goldstone) Higgs
- Extra dimensions
- ….

Theory fixes $\{c_p\}$



Standard Model

$$\mathrm{SM} + \sum_{p\geq 1} \frac{\boxed{c_p}\mathcal{O}_p}{\Lambda^p}$$

Exp tests of theory via $\{c_p\}$

Top - down 2

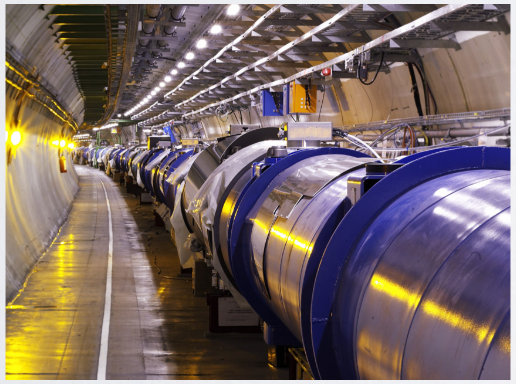
Energy

M_{Planck} Gravity scale

Dream of a theory of everything

- String theory
- Alternative quantum gravity approaches
- No Lagrangian approaches
- ….

Weaker predictive power for $\{c_p\}$



Standard Model

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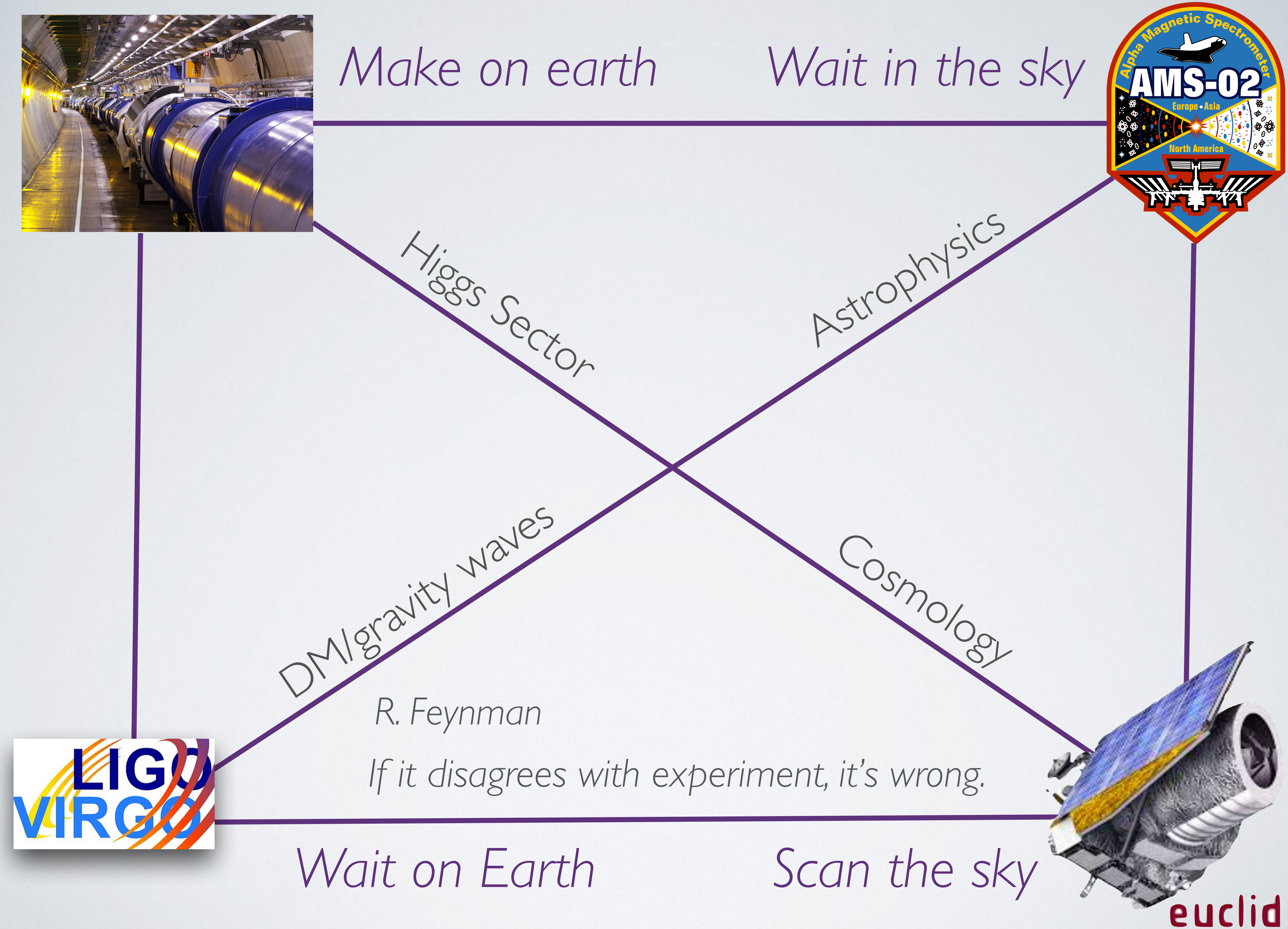

Make on earth
Wait in the sky
AMS-02
Alpha Magnetic Spectrometer
Europe • Asia
North America
Higgs Sector
Astrophysics
DM/gravity waves
Cosmology
R. Feynman
If it disagrees with experiment, it's wrong.
LIGO
VIRGO
Wait on Earth
Scan the sky
euclid

Fundamentality from scale independence

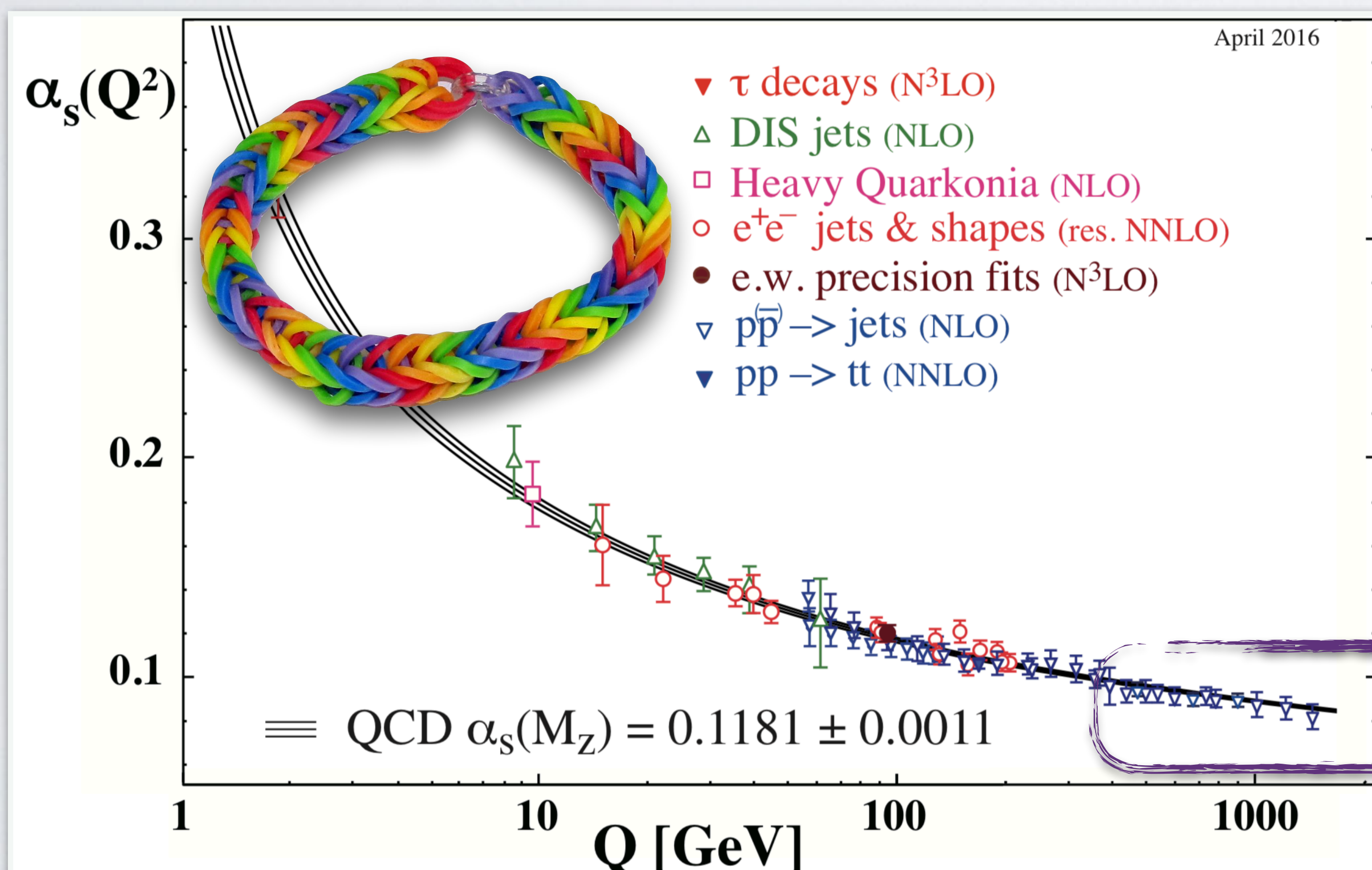
S. Weinberg and K. Wilson

Tindàr *Tryptichon Manuscriptca* (2012)

Freedom

The elementary constituents are interaction "free"

Quantum Chromo Dynamics (QCD)

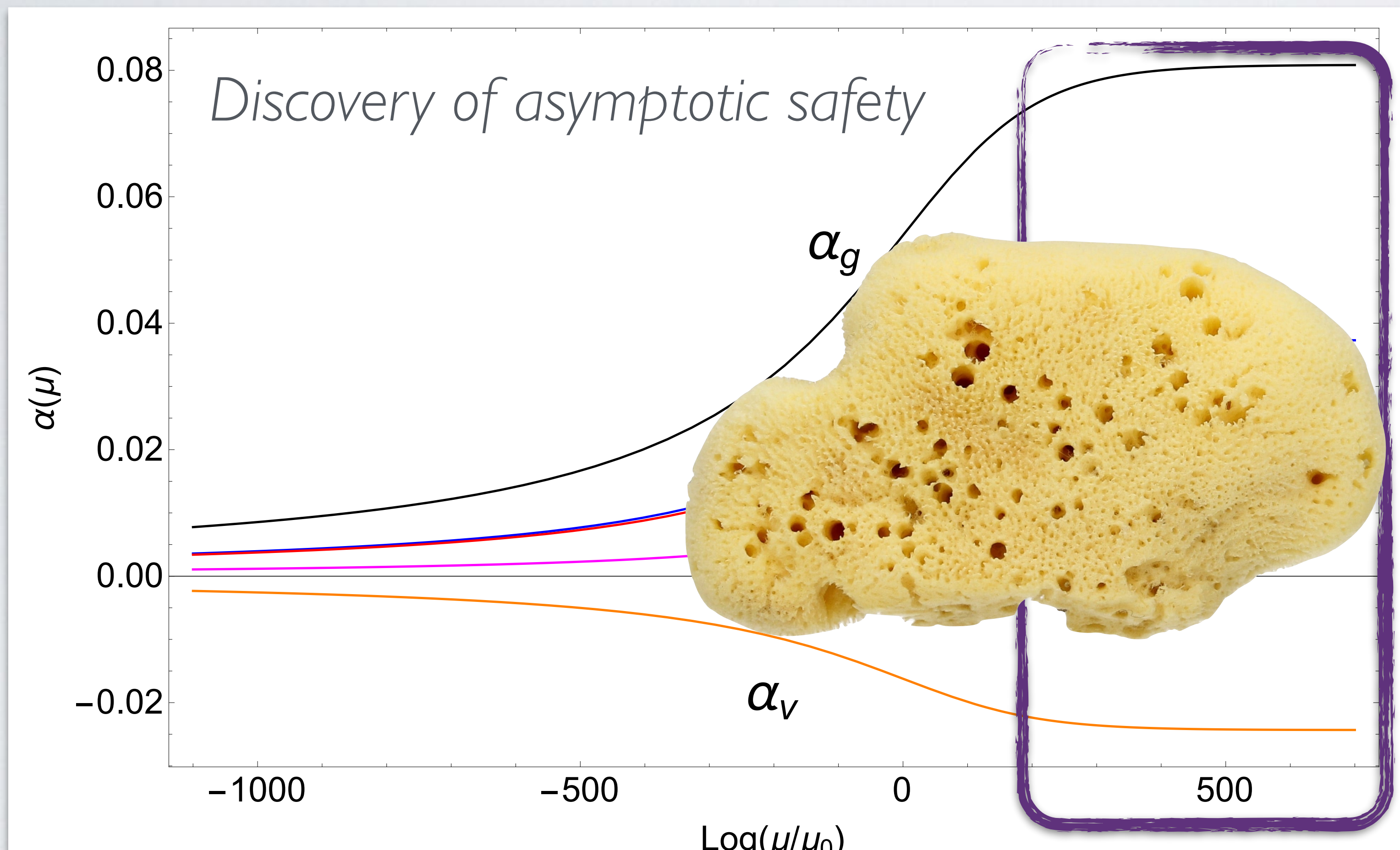


Gross Politzer Wilczek

Exp untested at very high energies

Safety

The elementary constituents have "safe" interactions



JHEP PUBLISHEI

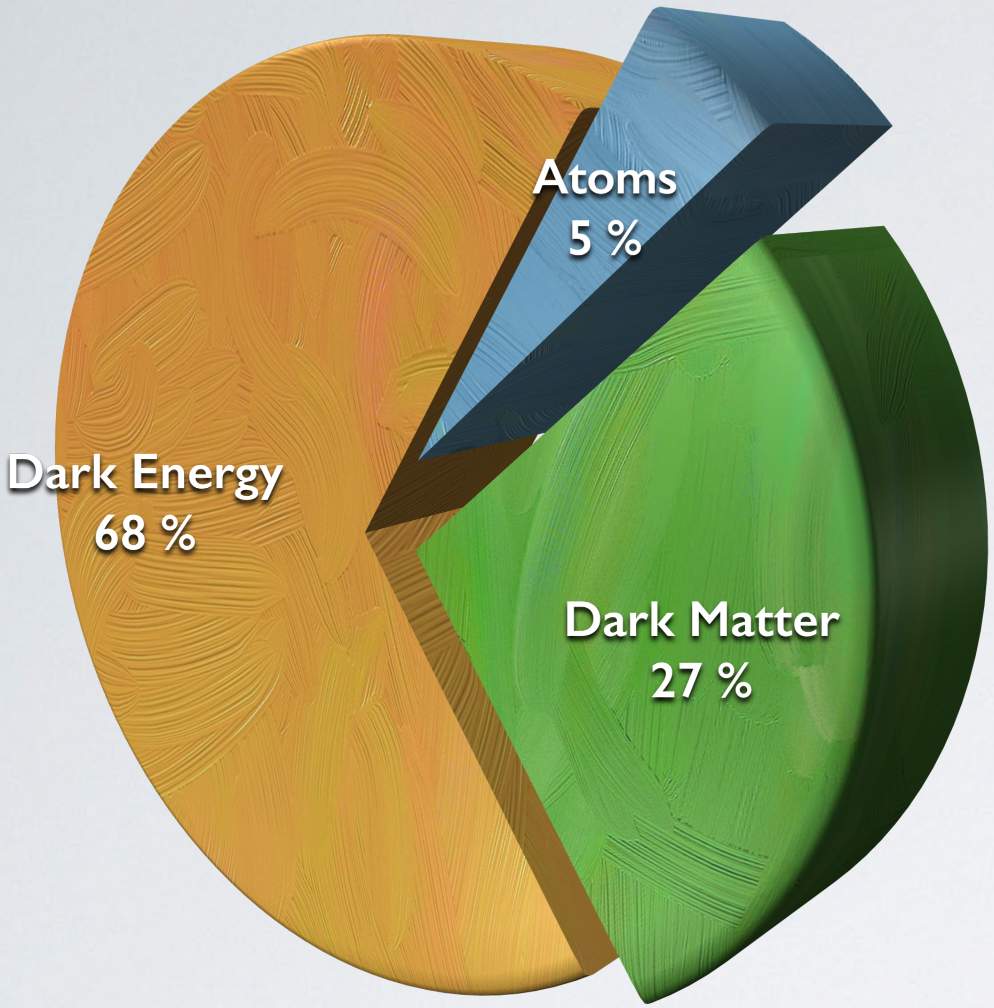
Asymptotic safety guaranteed

Daniel F. Litim[a] and Francesco Sannino[b]

Fields	$[SU(N_c)]$	$SU_L(N_f)$	$SU_R(N_f)$	$U_V(1)$
G_μ	Adj	1	1	0
Q_L	$\square$	$\overline{\square}$	1	1
Q_R^c	$\overline{\square}$	1	$\square$	-1
H	1	$\square$	$\overline{\square}$	0

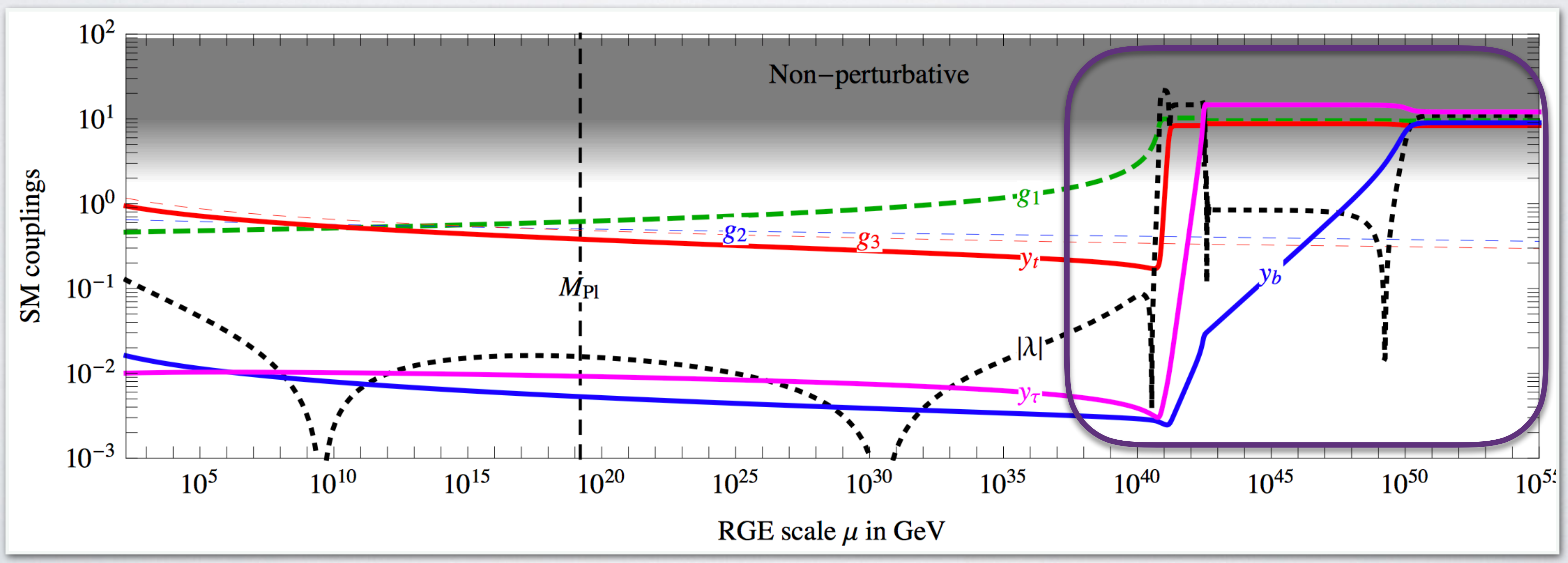
"Better be asymptotically safe than asymptotically sorry"

A. Bond



SM is incomplete

- Dark matter & energy
- Matter over antimatter
- Neutrino masses*
- Quantum Gravity



Fundamental interactions

Wilson: A fundamental theory has an UV fixed point

- Short distance conformality
- Continuum limit well defined
- Complete UV fixed point
- Smaller critical surface dim. = more IR predictiveness
- Mass operators relevant only for IR

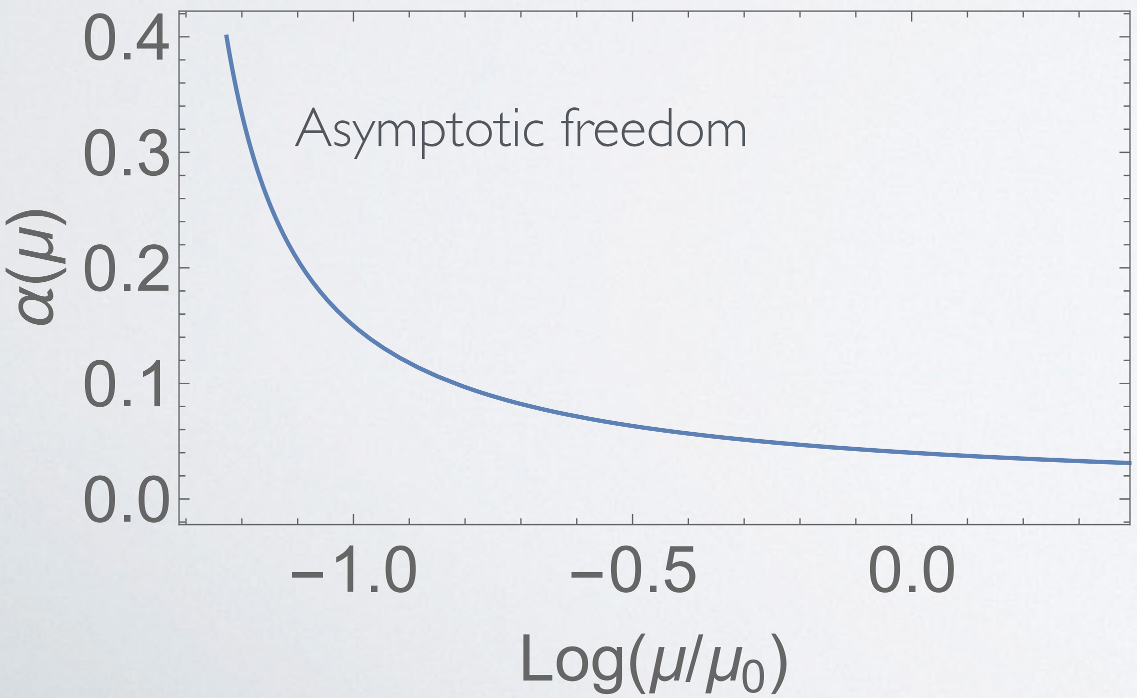
The Standard Model is not a fundamental theory

Free versus Safe

Wilson: A fundamental theory has an UV fixed point

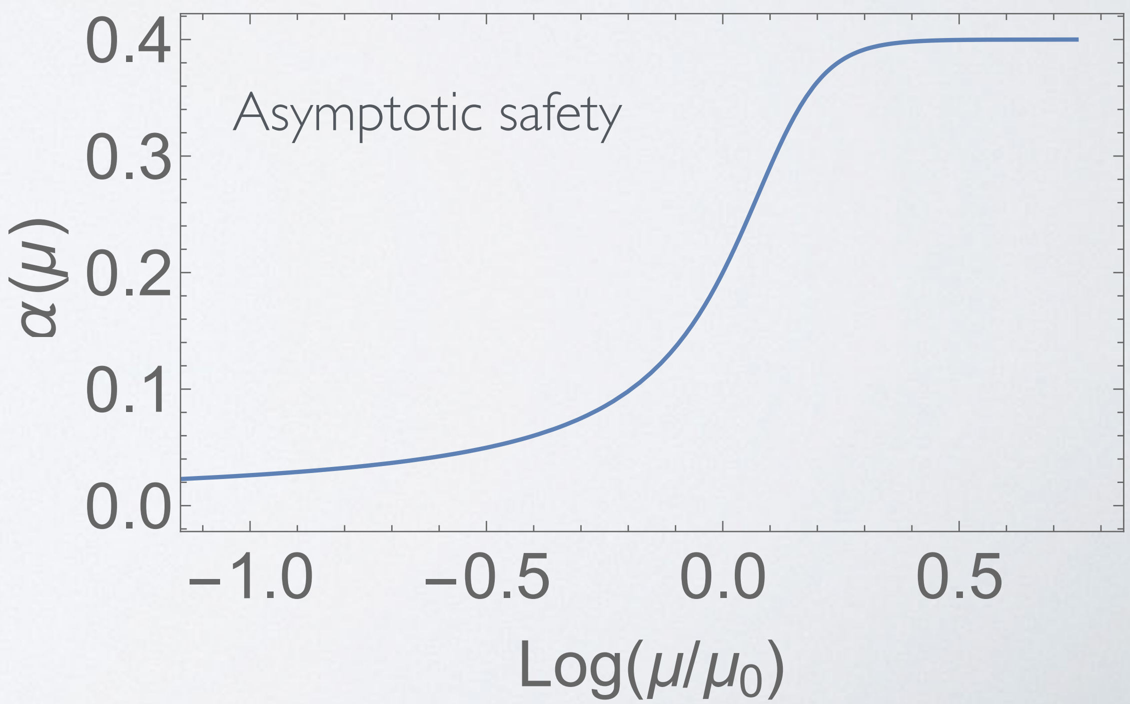
Trivial fixed point

- Non-interacting in the UV
- Logarithmic scale depend.



Interacting fixed point

- Integrating in the UV
- Power law



Constructing a safe theory

Precise and/or nonperturbative exact results for UV interacting fixed points

Exact 4D Interacting UV Fixed Point

Antipin, Gillioz, Mølgaard, Sannino 1303.1525 PRD

Litim and Sannino, 1406.2337, JHEP

Pelaggi, Sannino, Strumia, Vigiani, 1701.01453

$$L = -F^2 + i\overline{Q}\gamma \cdot DQ + y(\overline{Q}_L H Q_R + \text{h.c.}) +$$

$$\mathrm{Tr}\left[\partial \mathrm{H}^\dagger \partial \mathrm{H}\right] - \mathrm{u}\mathrm{Tr}\left[(\mathrm{H}^\dagger \mathrm{H})^2\right] - \mathrm{v}\mathrm{Tr}\left[(\mathrm{H}^\dagger \mathrm{H})\right]^2$$

Fields	$[SU(N_c)]$	$SU_L(N_f)$	$SU_R(N_f)$	$U_V(1)$
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H	1	$\square$	$\overline{\square}$	0

Veneziano Limit

Normalised couplings

$$\alpha_g = \frac{g^2\, N_C}{(4\pi)^2}\,, \quad \alpha_y = \frac{y^2\, N_C}{(4\pi)^2}\,, \quad \alpha_h = \frac{u\, N_F}{(4\pi)^2}\,, \quad \alpha_v = \frac{v\, N_F^2}{(4\pi)^2}$$

$$\frac{v}{u} = \frac{\alpha_v}{\alpha_h N_F}$$

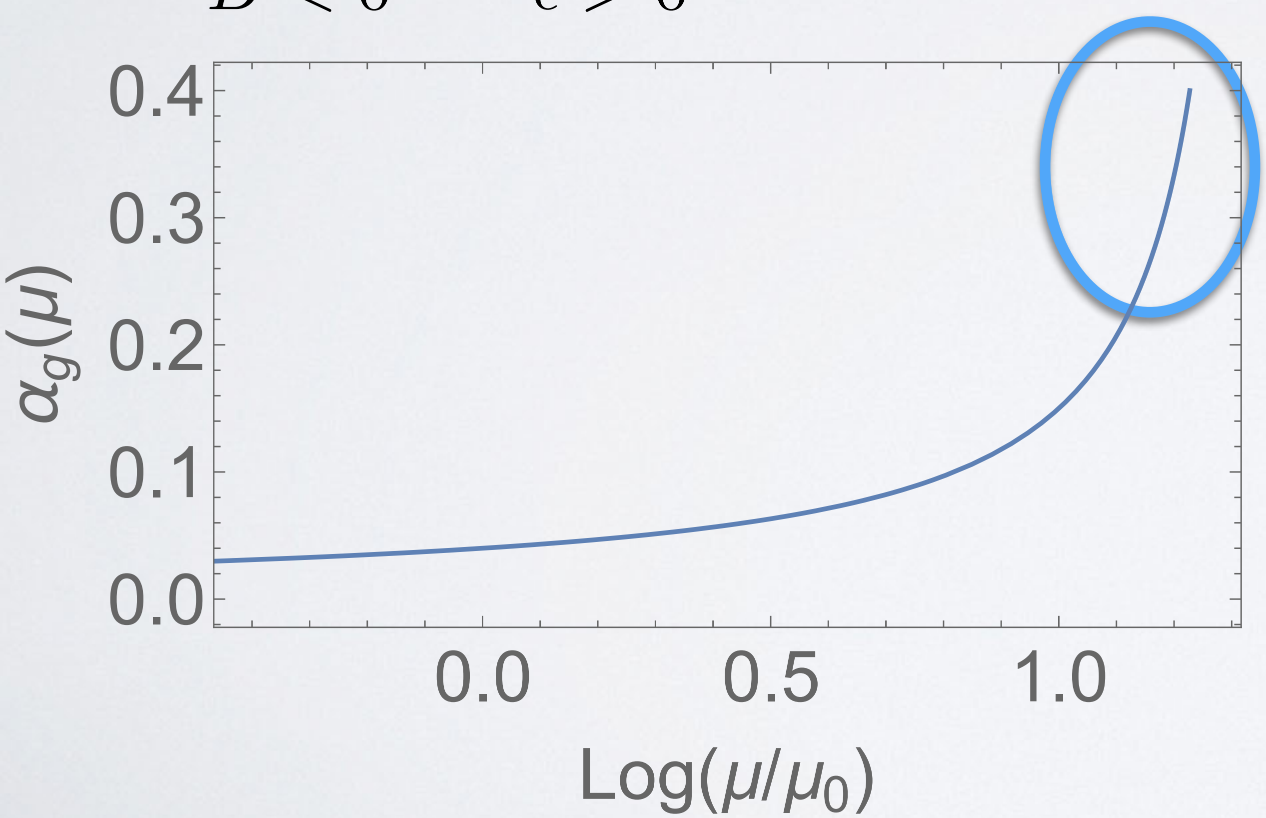
At large N $\quad \dfrac{N_F}{N_C} \in \Re^+$

Small parameters

$$B = -\frac{4}{3}\epsilon$$

$$\epsilon = \frac{N_F}{N_C} - \frac{11}{2}$$

$B < 0 \qquad \epsilon > 0$



$$0 \leq \epsilon \ll 1$$

Landau Pole ?

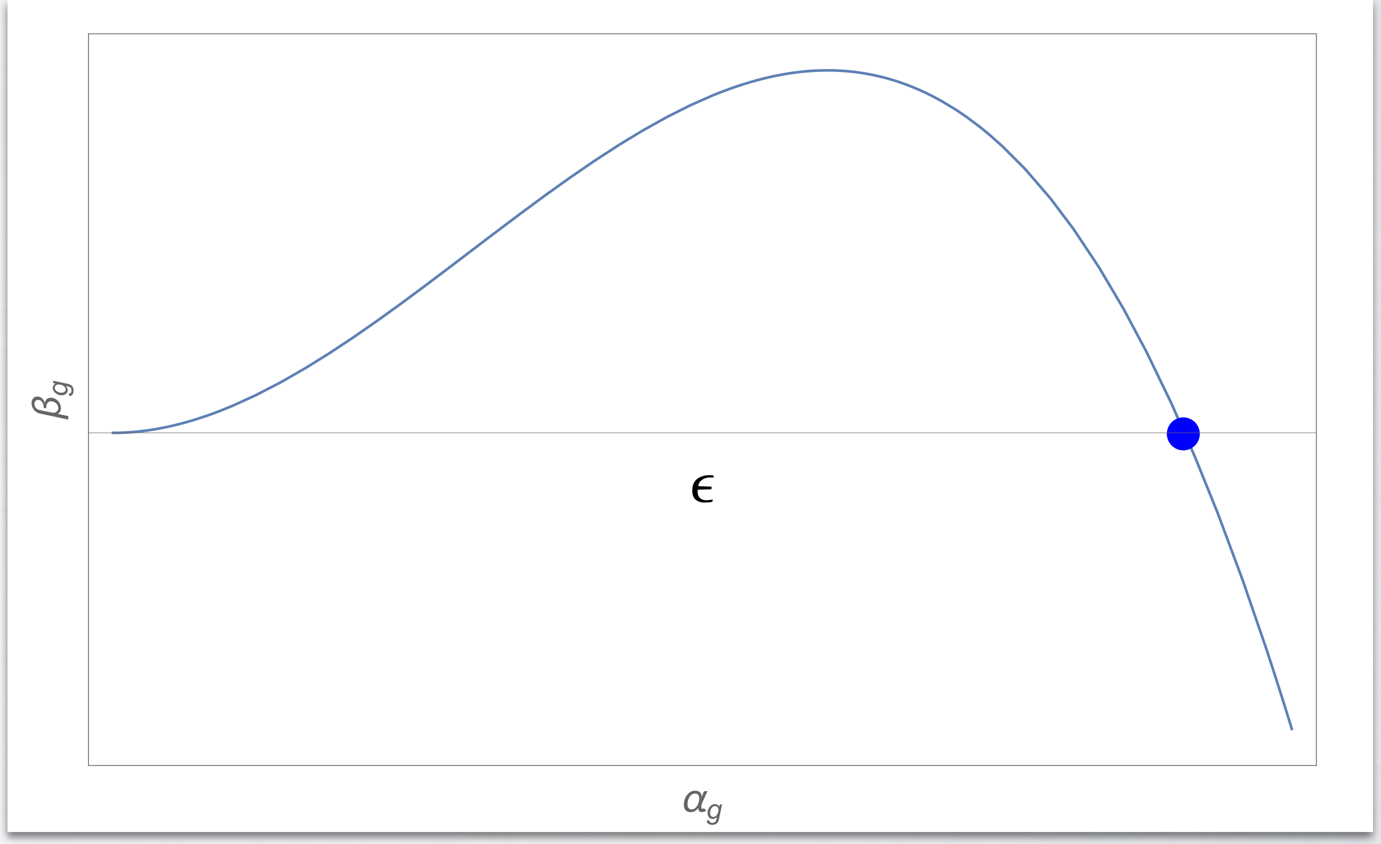
Can NL help?

$$\beta_g = -B\alpha_g^2 + C\alpha_g^3$$

$$B = -\frac{4}{3}\epsilon$$

$$0 \leq \alpha_g^* \ll 1 \quad \text{iff} \quad C < 0$$

$$\alpha_g^* = \frac{B}{C} \propto \epsilon$$



Impossible in Gauge Theories with Fermions alone

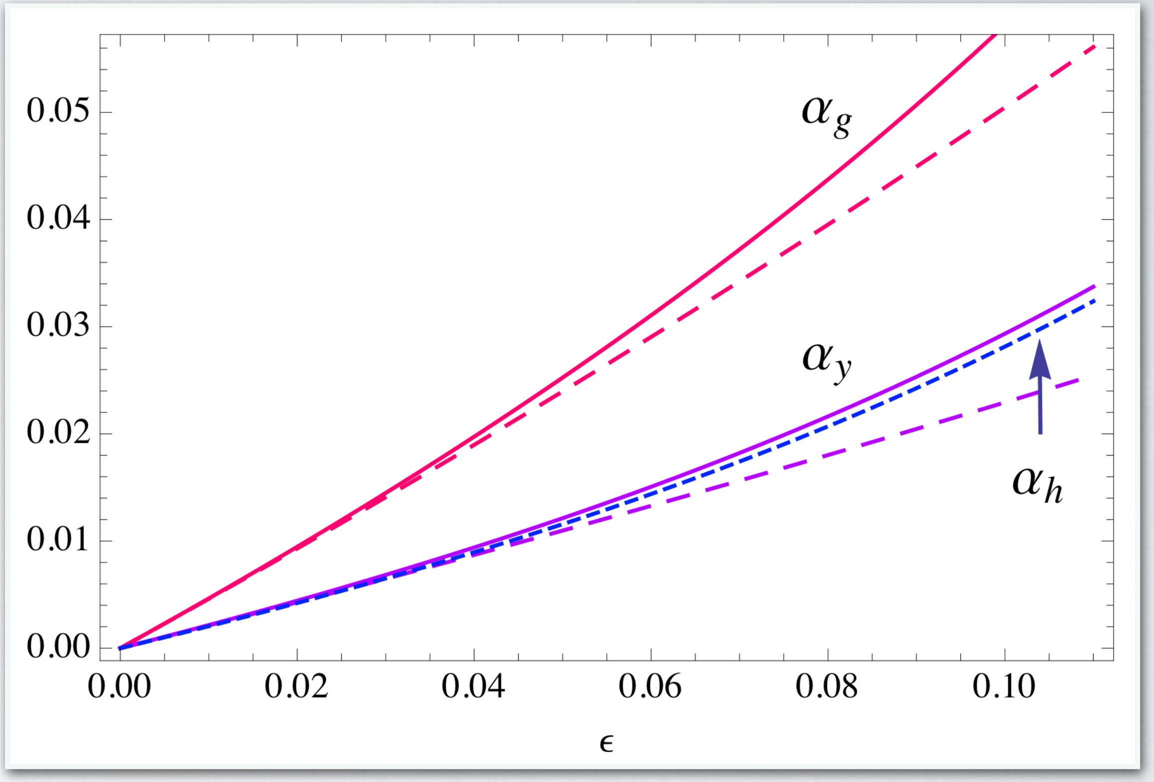
Caswell, PRL 1974

Add Yukawa

$$\beta_g = \alpha_g^2 \left[\frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon \right) \alpha_g \boxed{- 2 \left(\frac{11}{2} + \epsilon \right)^2 \alpha_y} \right]$$

$$\beta_y = \alpha_y \; \left[\boxed{(13 + 2\epsilon)\, \alpha_y - 6\, \alpha_g} \right]$$

NLO - Fixed Points



Gaussian fixed point

$$(\alpha_g^*, \alpha_y^*) = (0, 0)$$

Interacting fixed point

$$\alpha_g^* = \frac{26\epsilon + 4\epsilon^2}{57 - 46\epsilon - 8\epsilon^2} = \frac{26}{57}\epsilon + \frac{1424}{3249}\epsilon^2 + \frac{77360}{185193}\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$\alpha_y^* = \frac{12\epsilon}{57 - 46\epsilon - 8\epsilon^2} = \frac{4}{19}\epsilon + \frac{184}{1083}\epsilon^2 + \frac{10288}{61731}\epsilon^3 + \mathcal{O}(\epsilon^4)\,.$$

Linearised RG Flow

$$\delta\alpha = (\alpha - \alpha_*) \propto \left(\frac{\mu}{\Lambda_c}\right)^{\vartheta}$$

$$\vartheta = \partial\beta/\partial\alpha|_*$$

Stability Matrix

$$\beta_i = \sum_j M_{ij}\left(\alpha_i - \alpha_j^*\right) + \text{subleading}$$

$$M_{ij} = \partial\beta_i/\partial\alpha_j|_*$$

$$i = (g, y)$$

Scaling exponents: UV completion

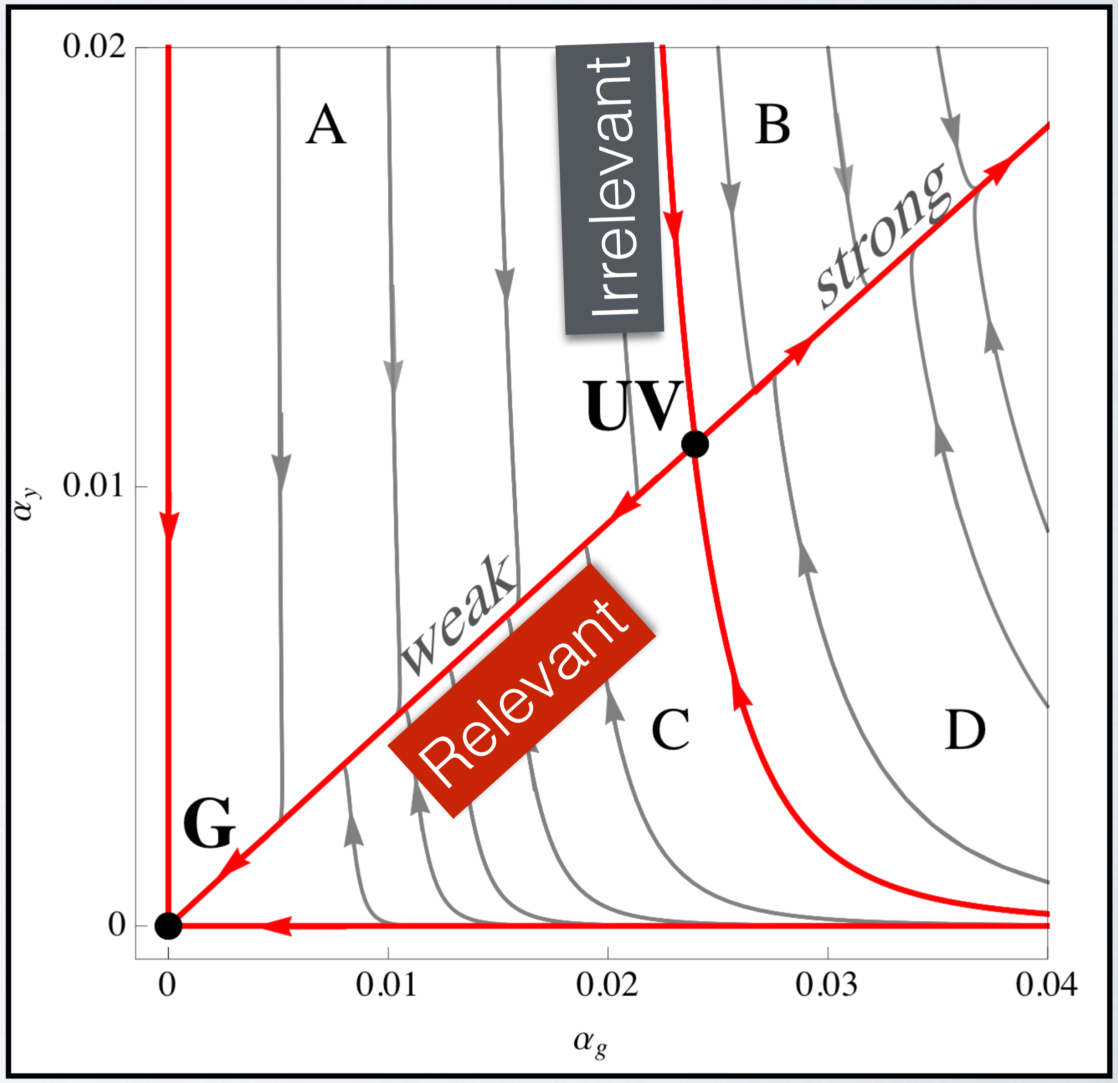
Eigen values of M

$$\vartheta_1 = -\frac{104}{171}\epsilon^2 + \frac{2296}{3249}\epsilon^3 + \frac{1387768}{1666737}\epsilon^4 + \mathcal{O}(\epsilon^4)$$
$$\vartheta_2 = \frac{52}{19}\epsilon + \frac{9140}{1083}\epsilon^2 + \frac{2518432}{185193}\epsilon^3 + \mathcal{O}(\epsilon^4)\,.$$

$\vartheta_1 < 0$ Relevant direction

$\vartheta_2 > 0$ Irrelevant direction

A true UV fixed point to this order



NNLO - The scalars

The scalar self-couplings

$$\beta_h = -(11+2\epsilon)\,\alpha_y^2 + \boxed{4\alpha_h(\alpha_y + 2\alpha_h)}$$

Single trace

$$\beta_v = 12\alpha_h^2 + 4\alpha_v\,(\alpha_v + 4\alpha_h + \alpha_y)$$

Double trace

Only single trace effect on Yukawa

$$\Delta\beta_y^{(2)} = \alpha_y\left\{\frac{20\epsilon-93}{6}\alpha_g^2 + (49+8\epsilon)\alpha_g\alpha_y - \left(\frac{385}{8}+\frac{23}{2}\epsilon+\frac{\epsilon^2}{2}\right)\alpha_y^2 - \boxed{(44+8\epsilon)\alpha_y\alpha_h + 4\alpha_h^2}\right\}$$

$$\Delta\beta_g^{(3)} = \alpha_g^2\left\{\left(\frac{701}{6}+\frac{53}{3}\epsilon-\frac{112}{27}\epsilon^2\right)\alpha_g^2 - \frac{27}{8}(11+2\epsilon)^2\alpha_g\alpha_y + \frac{1}{4}(11+2\epsilon)^2(20+3\epsilon)\alpha_y^2\right\}.$$

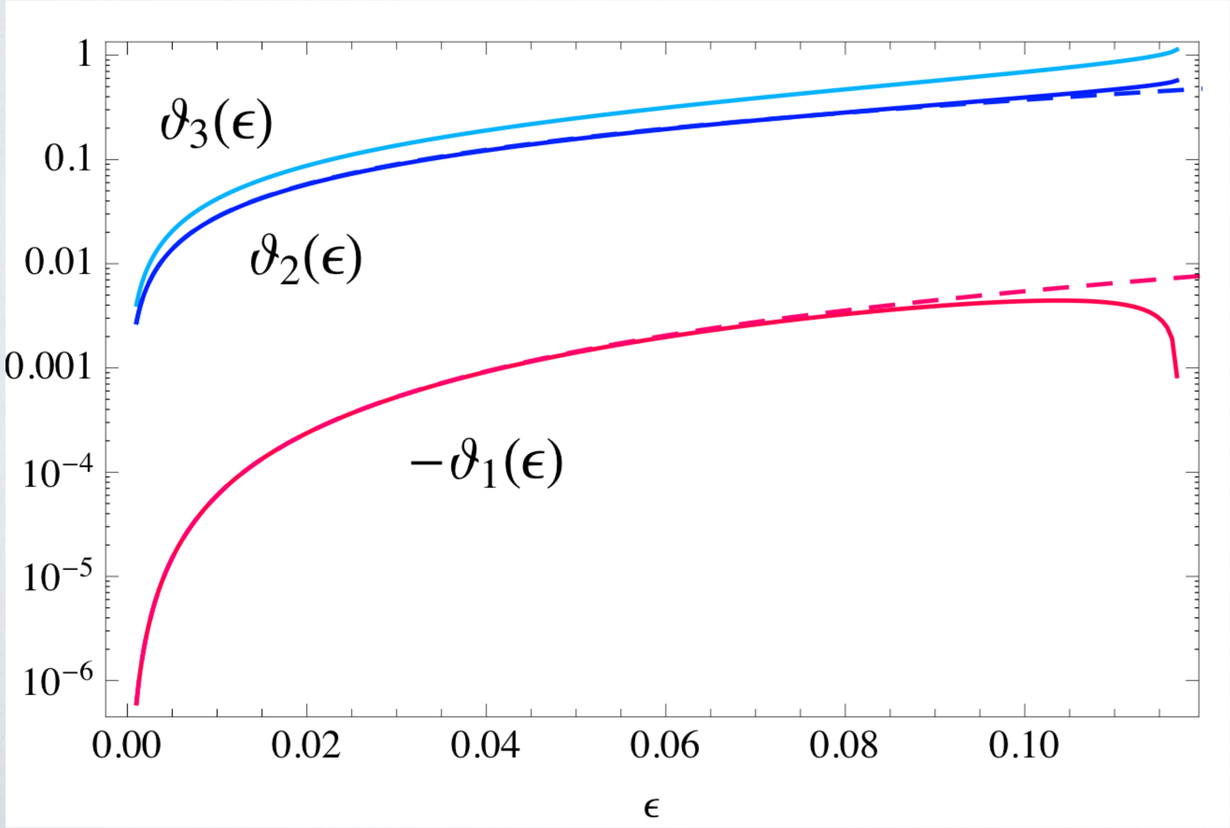
Double-trace coupling is a spectator

NNLO - All direction UV Stable FP

Fixed point

$$
\begin{aligned}
\alpha_g^* &= \quad 0.4561\,\epsilon + 0.7808\,\epsilon^2 + 3.112\,\epsilon^3 + \mathcal{O}(\epsilon^4) \\
\alpha_y^* &= \quad 0.2105\,\epsilon + 0.5082\,\epsilon^2 + 2.100\,\epsilon^3 + \mathcal{O}(\epsilon^4) \\
\alpha_h^* &= \quad 0.1998\,\epsilon + 0.5042\,\epsilon^2 + 2.045\,\epsilon^3 + \mathcal{O}(\epsilon^4)
\end{aligned}
$$

$$\alpha_h^* \equiv \alpha_{h1}^* > 0$$



Scaling exponents

$$
\begin{aligned}
\vartheta_1 &= -0.608\,\epsilon^2 + 0.707\,\epsilon^3 + 2.283\,\epsilon^4 + \cdots \\
\vartheta_2 &= \quad 2.737\,\epsilon \;\; + 6.676\,\epsilon^2 + \cdots \\
\vartheta_3 &= \quad 4.039\,\epsilon \;\; + 14.851\,\epsilon^2 + \cdots \,.
\end{aligned}
$$

$$i = (g, y, h)$$

Double - trace and stability

$$\alpha^*_{v1,v2} = \boxed{-\frac{1}{19}}\left(2\sqrt{23} \mp \sqrt{20 + 6\sqrt{23}}\right)\,\epsilon + \mathcal{O}(\epsilon^2)$$

Is the potential stable at FP?

- Which FP survives?

Moduli

Classical moduli space

$$V = u\,\mathrm{Tr}\,(H^\dagger H)^2 + v\,(\mathrm{Tr}\,H^\dagger H)^2$$

Use $U(N_f) \times U(N_f)$ symmetry $\qquad H_c = \mathrm{diag}(h_1, \ldots, h_{N_F})$

$$V = u \sum_{i=1}^{N_F} h_i^4 + v \left(\sum_{i=1}^{N_F} h_i^2 \right)^2 - 2\lambda(\textstyle\sum_i h_i^2 - 1)$$

If V vanishes on H_c it will vanish for any multiple of it

Litim, Mojaza, Sannino 1501.03061 JHEP

Ground state conditions at any Nf

$$\boxed{\alpha_h > 0 \quad \text{and} \quad \alpha_h + \alpha_v \geq 0} \qquad H_c \propto \delta_{ij}$$

$$\alpha_h < 0 \quad \text{and} \quad \alpha_h + \alpha_v / N_F \geq 0 \qquad H_c \propto \delta_{i1}$$

$$V_\phi = (4\pi)^2 \boxed{(\alpha_h + \alpha_v)} \phi^4 \qquad \alpha_h^* + \alpha_{v_2}^* < 0 < \alpha_h^* + \alpha_{v_1}^*$$

Stability for $\alpha_{v_1}^*$

UV critical surface

(Ir)relevant directions implies UV lower dim. critical

$$\alpha_i = F_i(\alpha_g) \qquad\qquad \alpha_i(\mu) = \alpha_i^* + \sum_n c_n\, V_i^n \left(\frac{\mu}{\Lambda_c}\right)^{\vartheta_n} + \text{subleading}$$

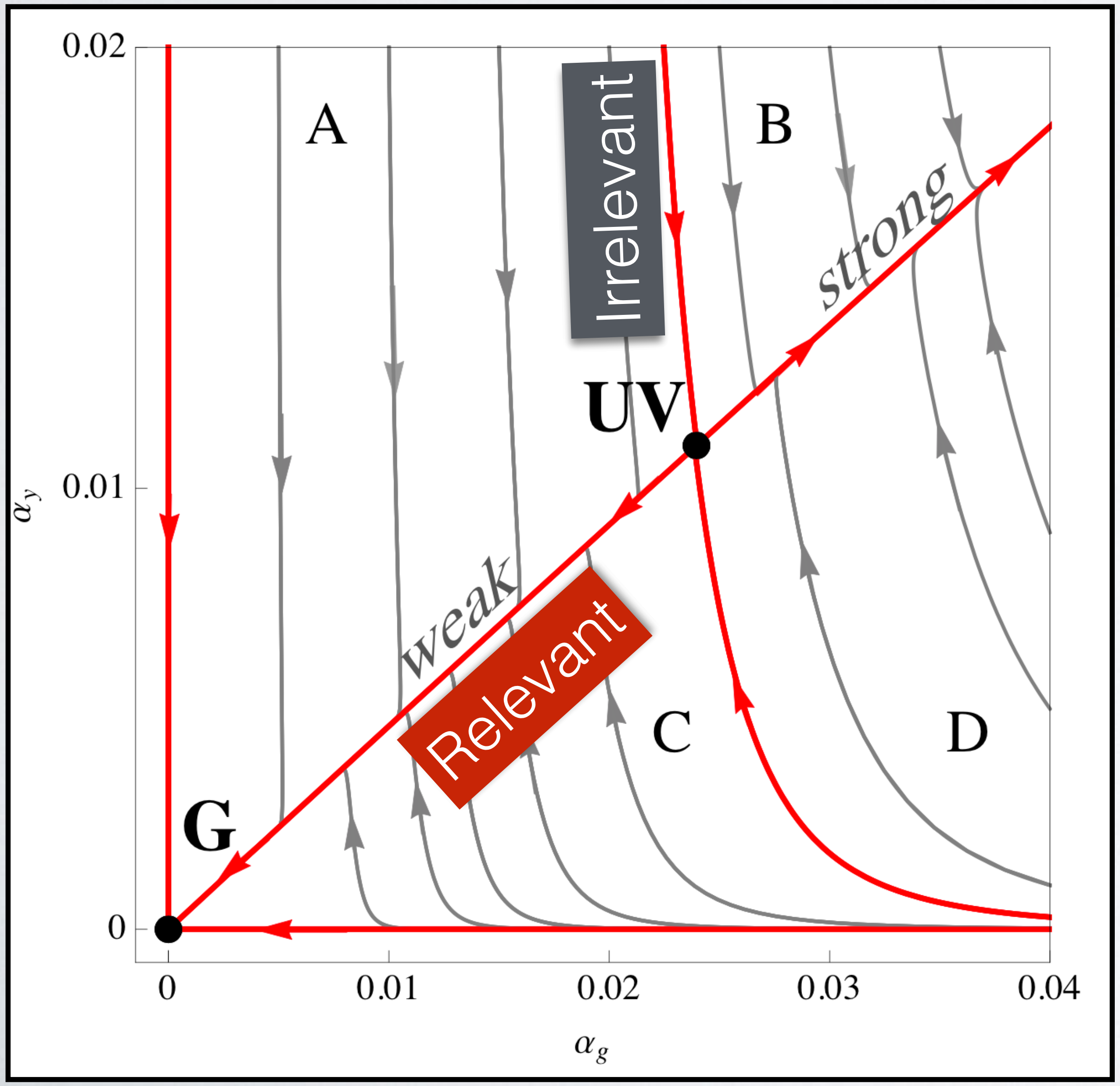
$$\begin{aligned} F_y(\alpha_g) &= \quad (0.4615 + 0.6168\,\epsilon)\,\alpha_g \\ F_h(\alpha_g) &= \quad (0.4380 + 0.5675\,\epsilon)\,\alpha_g \\ F_v(\alpha_g) &= -(0.3009 + 0.3241\,\epsilon)\,\alpha_g \end{aligned}$$

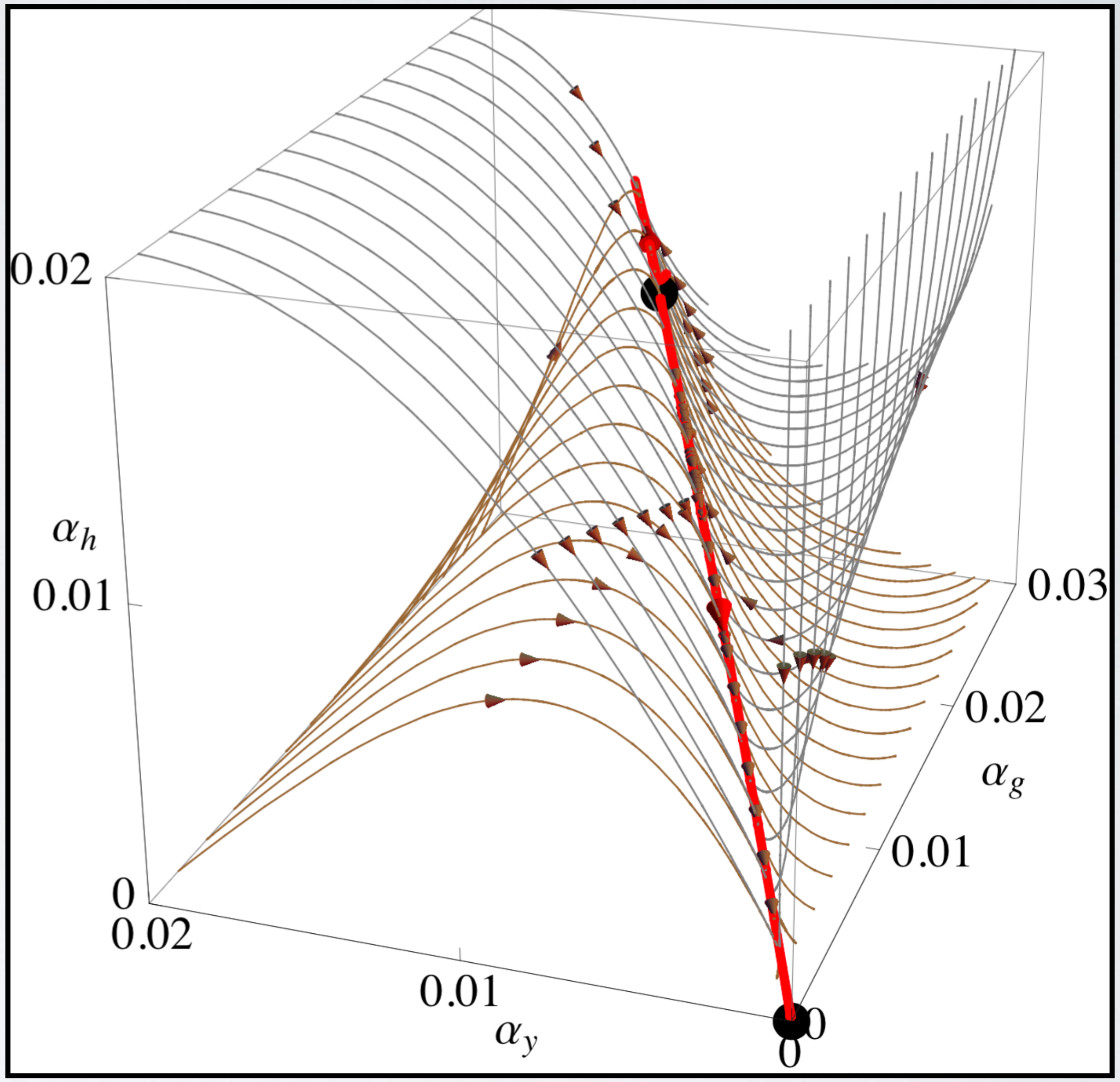
Near the fixed point

$$\alpha_g(\mu) = \alpha_g^* + \Big(\alpha_g(\Lambda_c) - \alpha_g^*\Big)\left(\frac{\mu}{\Lambda_c}\right)^{\vartheta_1(\epsilon)}$$

Phase Diagram

$$\vartheta_1 = -0.608\,\epsilon^2 + \mathcal{O}(\epsilon^3)$$
$$\vartheta_2 = \quad 2.737\,\epsilon + \mathcal{O}(\epsilon^2)$$
$$\vartheta_3 = \quad 4.039\,\epsilon + \mathcal{O}(\epsilon^2)$$
$$\vartheta_4 = \quad 2.941\,\epsilon + \mathcal{O}(\epsilon^2)$$





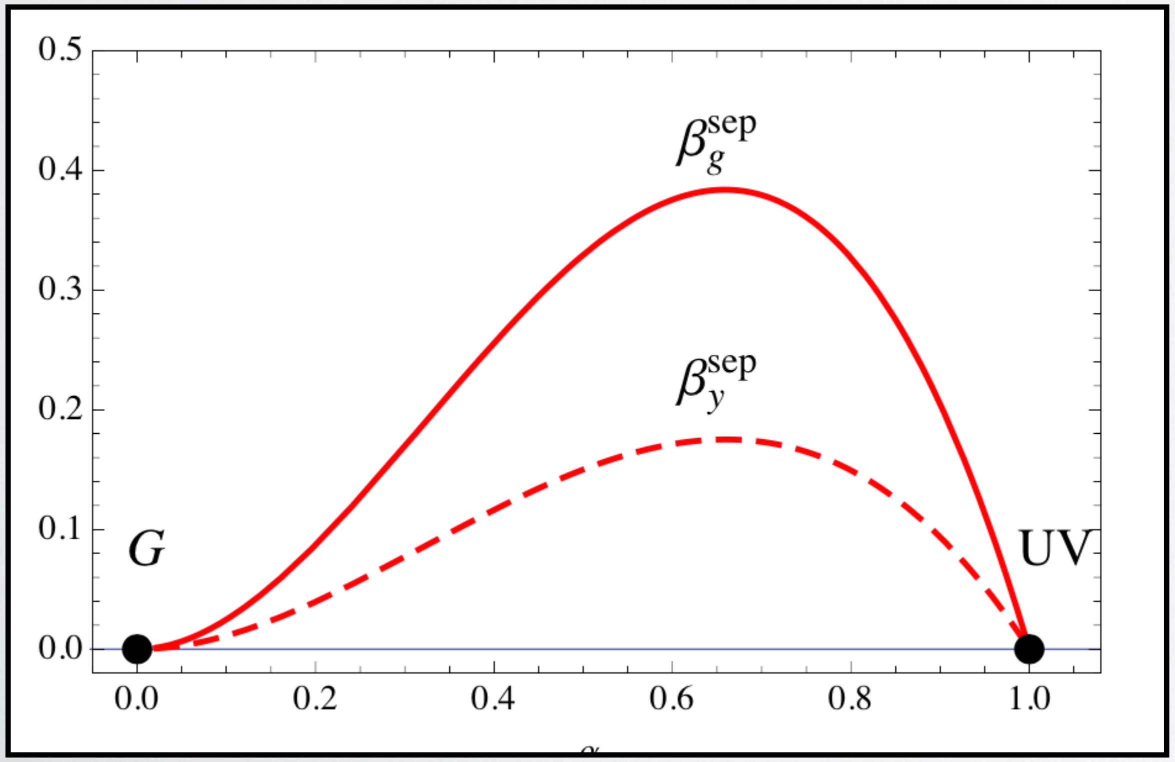
$$\vartheta_1 < 0 < \vartheta_2 < \vartheta_4 < \vartheta_3$$

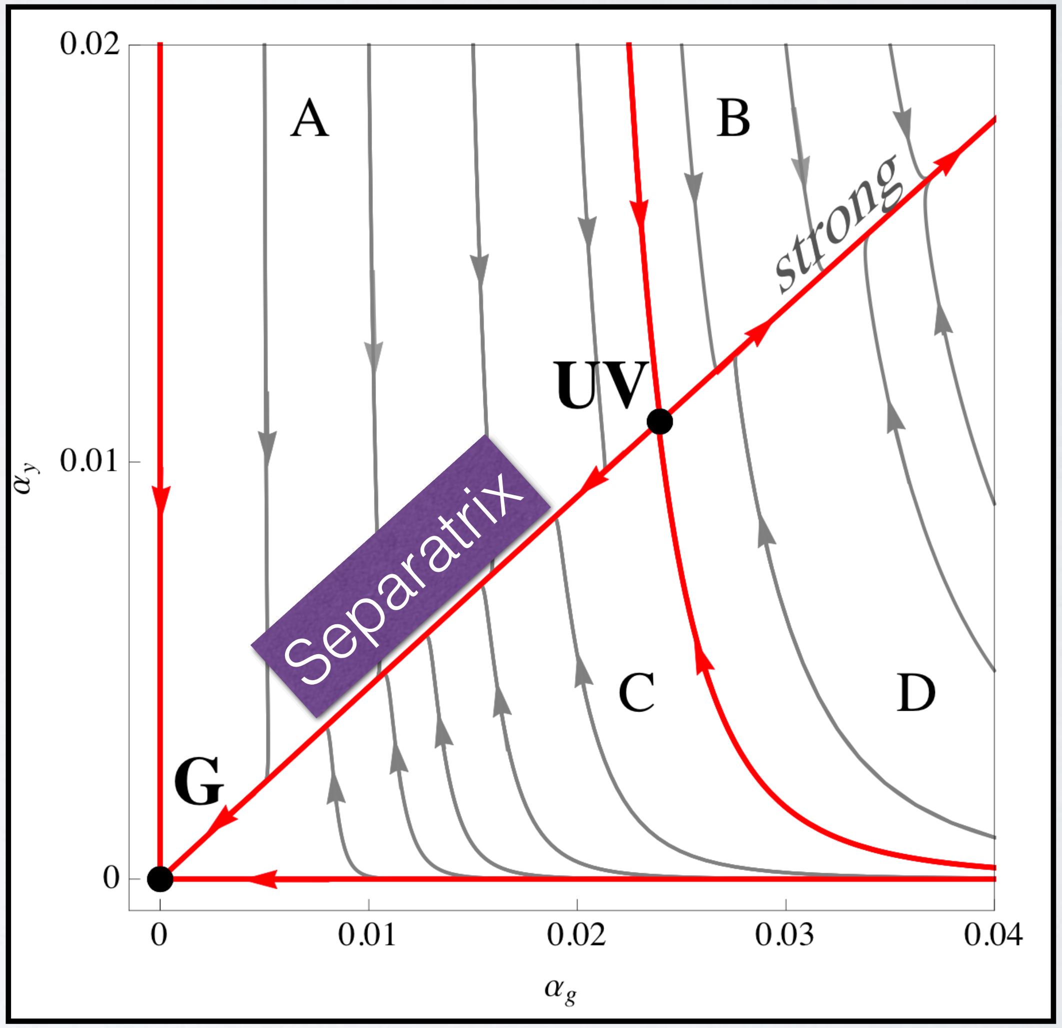
Separatrix = Line of Physics

Globally defined line connecting two FPs

$$\beta_g^{\rm sep}(\alpha_g) \equiv \beta_g(\alpha_g, \alpha_y = F_y(\alpha_g))$$

$$\beta_y^{\rm sep}(\alpha_g) \equiv \beta_y(\alpha_g, \alpha_y = F_y(\alpha_g))$$

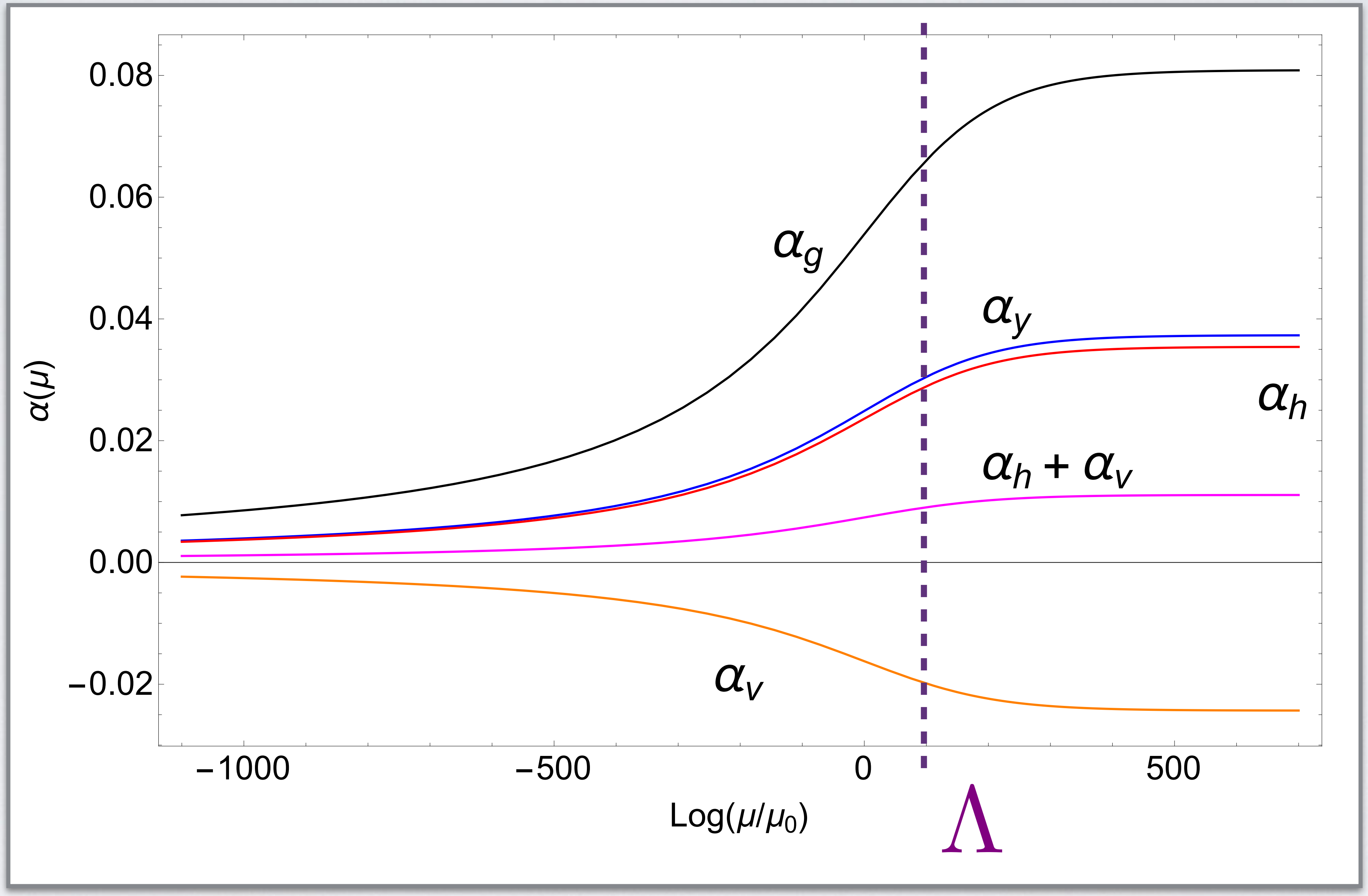




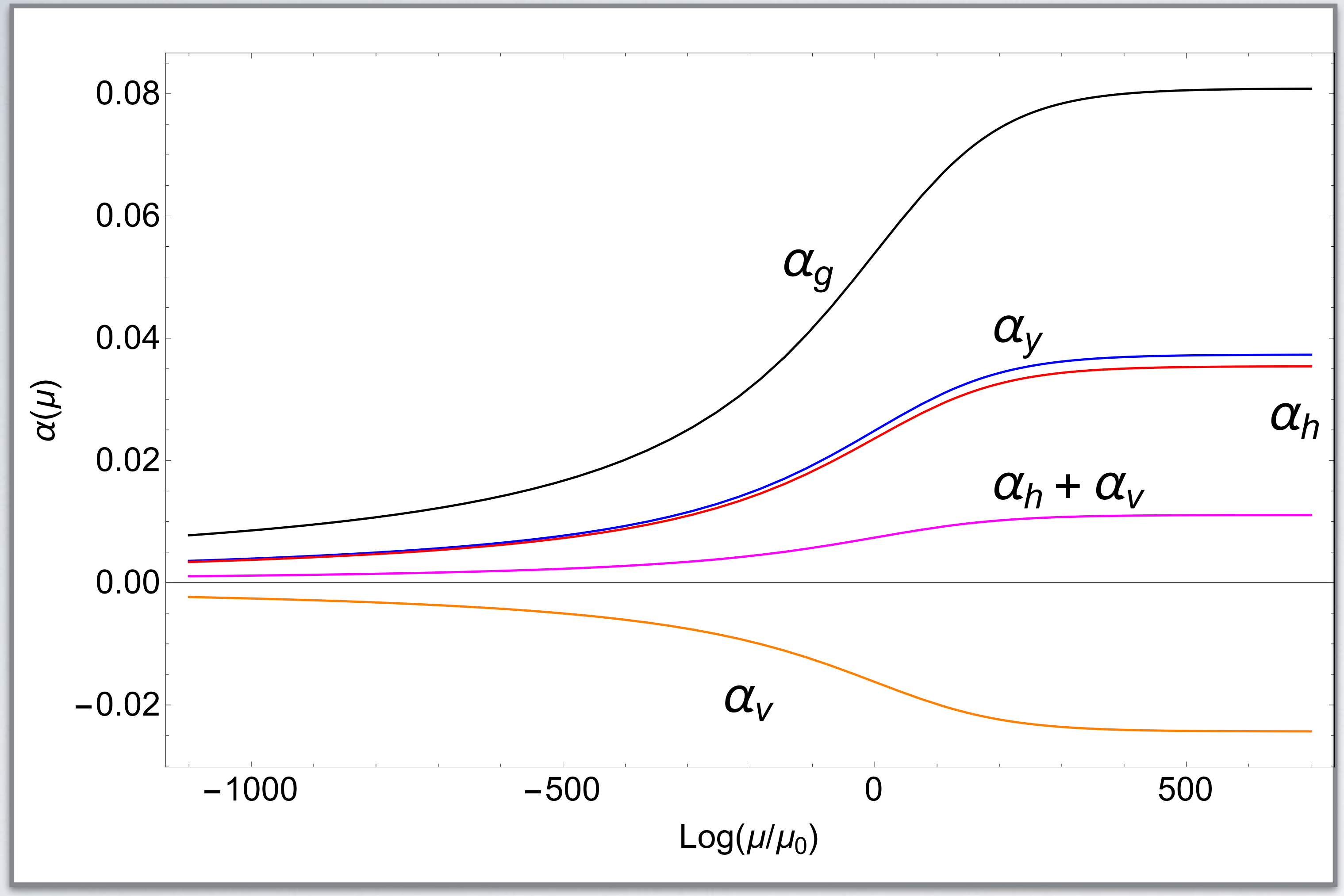
Complete asymptotic safety

Litim and Sannino, 1406.2337, JHEP

Gauge + fermion + scalars theories can be fund. at any energy scale

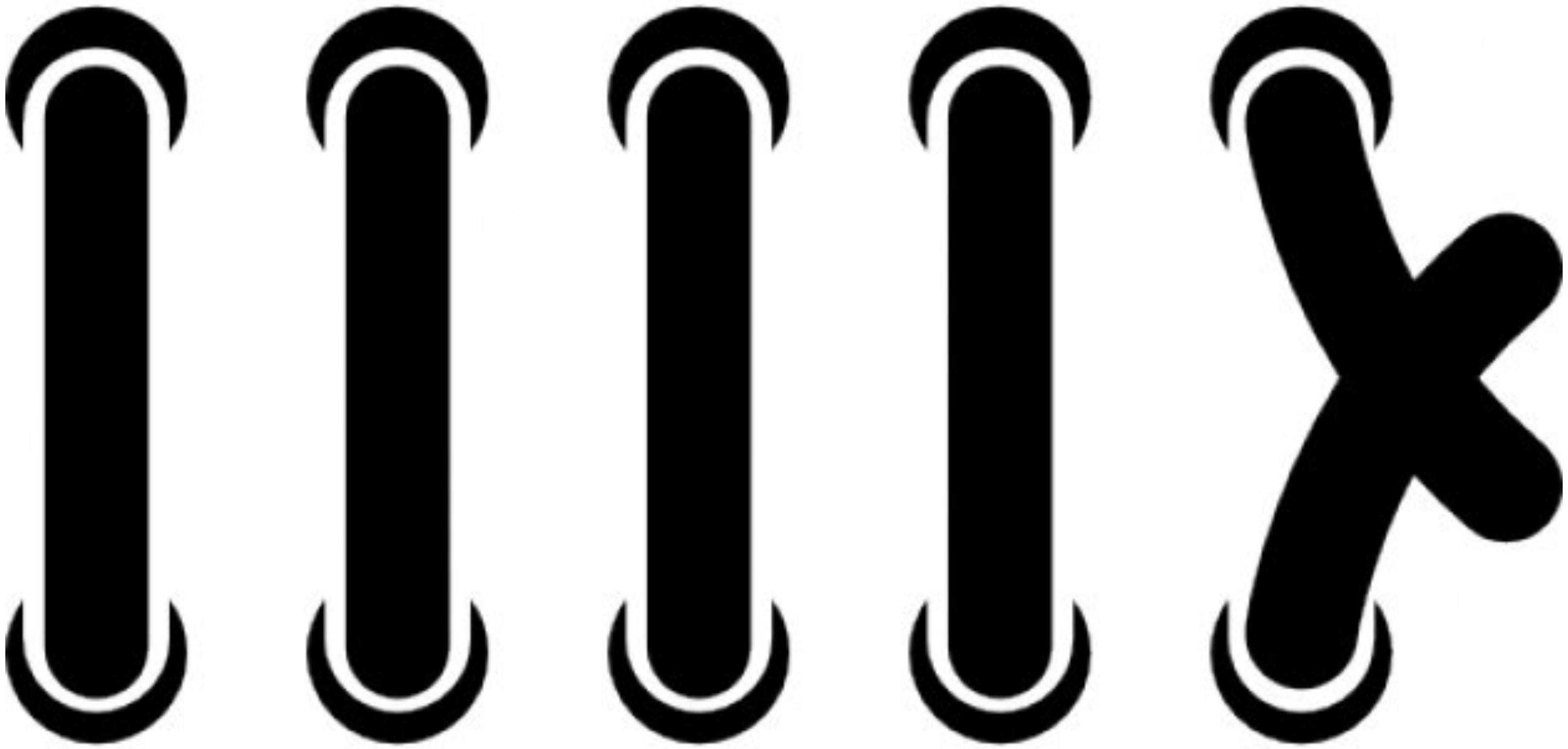


Scalars are needed perturbatively to make the theory fundamental



Condensed matter type unification across interactions

First 4D realisation of Wilson and Weinberg's safe paradigm



Higgs as shoelace

Large charge limit

State - operator correspondence

Cardy 84, 88

Δ on $\mathbb{R}^{d+1}$ $\Longleftrightarrow$ $\Delta = r_0 \, E_{S^d}$

E_{S^d} Energy of states living on a d-sphere of radius r_0

Reason

$\mathbb{R}^{d+1}$ is conformally equivalent to $\mathbb{R} \times S^d(r_0)$

Large charge safety

Orlando, Reffert, Sannino 2019

$$\mathcal{L}_H = \mathrm{Tr}[\partial H^\dagger \partial H] - u\, \mathrm{tr}[(H^\dagger H)^2] - v\, \mathrm{tr}[H^\dagger H]^2 - \frac{R}{6}\mathrm{Tr}[H^\dagger H]$$

$$J_L := i\, dH\, H^\dagger \qquad J_R = -i\, H^\dagger dH$$

$$Q_{L/R} = \int d^3x\, J^0_{L/R} \qquad H \to L H R^\dagger \Rightarrow Q_L \to L Q_L L^\dagger$$

Due to global symmetry

$$\mathrm{spec}(Q_{L/R}) = \{ J_1^{L/R}, J_2^{L/R}, \dots, J_{N_f}^{L/R} \}$$

Minimal energy solution is space - homogeneous

Imposing charge conservation

$$\boxed{H_0 = e^{2i\mathcal{M}t} B} \leftarrow \text{Ansatz}$$

$\mathcal{M} \in$ Cartan subalgebra of $SU(N_f)$

B is a self-adjoint $N_f \times N_f$ matrix

$$\boxed{Q_L = -Q_R = -2V\mathcal{M}B^2} \qquad V = Vol(M_3)$$

Equation of motion

$$\boxed{2M^2 = u B^2 + v \,\mathrm{Tr}[B^2] - \frac{R}{12}}$$

With $\underbrace{M^2 = \mu^2 \,\mathbb{1}_{N_f \times N_f}}$ $\qquad \underbrace{B = b \,\mathbb{1}_{N_f \times N_f}}$

$M \propto$ charge matrix $\subset$ Algebra of $SU(N_f)$

$$\Rightarrow \mathrm{tr}[M] = 0 \Rightarrow M = \mu \begin{bmatrix} \mathbb{1}_{N_f/2 \times N_f/2} & \\ & -\mathbb{1}_{N_f/2 \times N_f/2} \end{bmatrix}$$

$$\Downarrow$$

$$\boxed{2\mu^2 = (u + v N_f) b^2 - \frac{R}{12}} \quad \text{and} \quad \boxed{J = |J_i| = |Q_{L\,ii}| = 2 V b^2 \mu}$$

Large charge expansion

Def: $\mathcal{J} = 2 J_{TOT} \frac{\alpha_h + \alpha_v}{N_f^2}$

$J_{TOT} = J N_f$ recall $J = 2 V b^2 \mu$

$\Rightarrow \quad \mu\left(\mu^2 + \frac{R}{24}\right) = \left(\frac{2\pi^2}{V}\right) \mathcal{J}$ Taking $\mathcal{J} \gg 1$

$$\mu = \left(\frac{2\pi^2}{V}\right)^{1/3} \mathcal{J}^{1/3} + \frac{R}{72}\left(\frac{V}{2\pi^2}\right)^{1/3} \mathcal{J}^{-1/3} + \mathcal{O}\left(\mathcal{J}^{-5/3}\right)$$

Def charge density

$\rho = \frac{2\pi^2}{V} \mathcal{J}$

$$\mu = \rho^{1/3} + \frac{R}{72} \rho^{-1/3} + \mathcal{O}\left(\rho^{-5/3}\right)$$

Ground state energy

Legendre transform of the Lagrangian

$$\frac{E}{V} = \sum_{i=1}^{N_f} \mu_i \frac{\delta \mathcal{L}_H}{\delta \mu_i} - \mathcal{L}_H$$

On a 3-sphere $V = 2\pi^2 r_0^3$; $R = \frac{6}{r_0^2}$

$$E = \frac{3}{2r_0} \frac{N_f^2}{\alpha_n + \alpha_v} \left[\mathcal{J}^{4/3} + \frac{1}{6} \mathcal{J}^{2/3} - \frac{1}{144} \mathcal{J}^0 + \mathcal{O}\left(\mathcal{J}^{-2/3}\right) \right]$$

$\rho^{4/3}$ $\rho^{2/3}$ ρ^0

First term dictated by dimensional analysis

Symmetry breaking

$$H_0 = e^{2iMt} B$$

$$SU(N_f)\times SU(N_f)\times U(1) \xrightarrow[M]{\text{exp.}} C(M)\times \underbrace{SU(N_f) \xrightarrow[B]{\text{spont.}} C(M)}_{N_f^2-1 \text{ spontaneously broken generators}}$$

$$C(M) \subset SU(N_f)\times U(1)$$

In our case

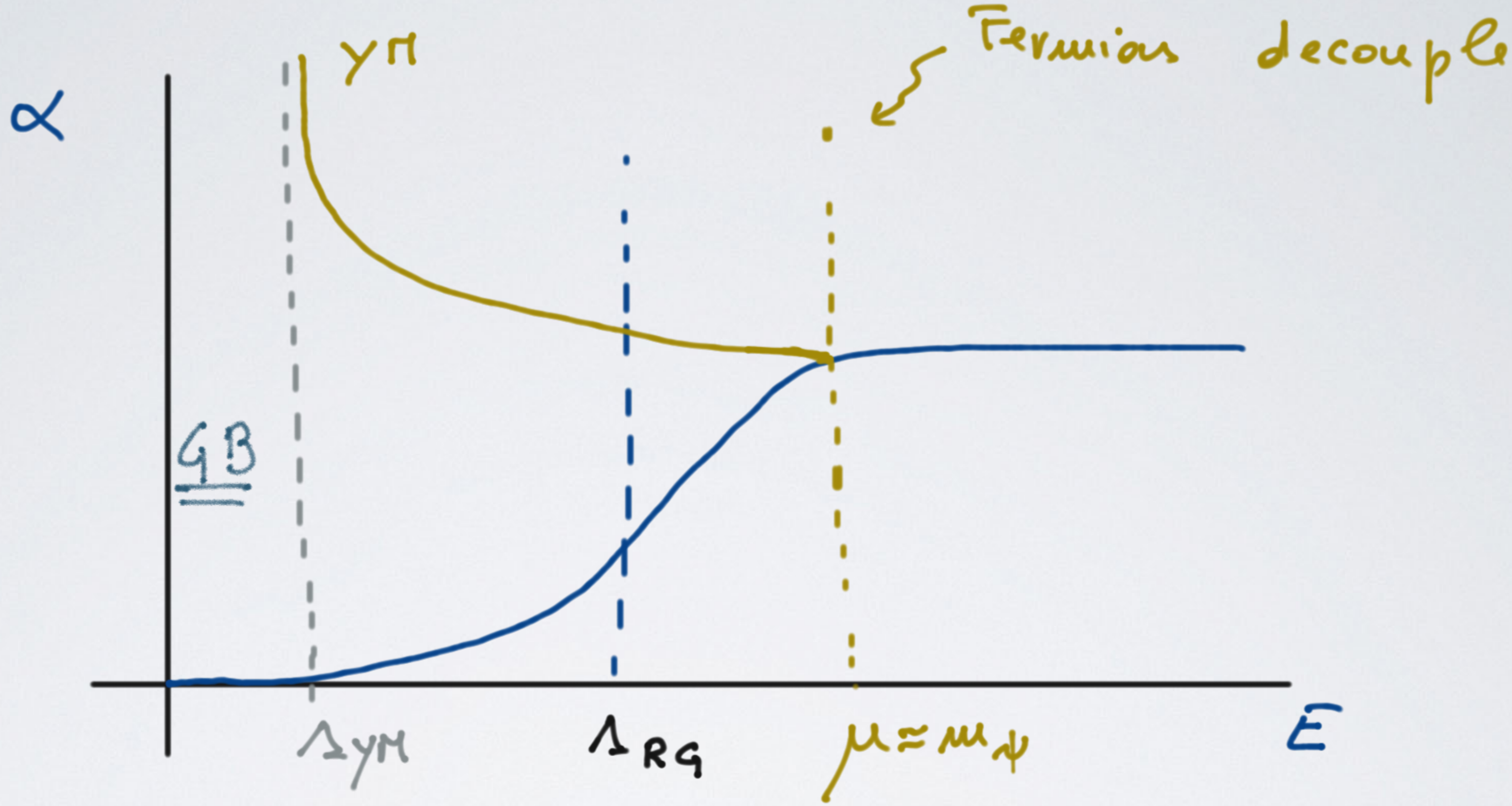
$$Q_L = J\left(\begin{array}{c|c} \mathbb{1} & 0 \\ \hline 0 & -\mathbb{1} \end{array}\right) \qquad M = \mu\left(\begin{array}{c|c} \mathbb{1} & 0 \\ \hline 0 & -\mathbb{1} \end{array}\right) \qquad B = b\left(\begin{array}{c|c} \mathbb{1} & 0 \\ \hline 0 & \mathbb{1} \end{array}\right)$$

$$\Rightarrow \quad C(M) = SU(N_f/2)\times SU(N_f/2)\times U(1)^2$$

How many Goldstone Bosons?

What kind of dispersion relations?

Where are we ?



Fermions decouple

$$m_\psi = \left(1 + 2\frac{N_f}{N_c}\frac{\alpha_y}{\alpha_h+\alpha_v}\right)^{-1/2} \rho^{1/3} + \mathcal{O}(\rho^{-1/3})$$

The last gap

$$\Lambda_{YM} = m_\psi \exp\left[-\frac{3}{22\,\alpha_g(m_\psi)}\right] \qquad \alpha_g(m_\psi) \approx \alpha_g(\mathrm{F.P.}) \propto \epsilon$$

Goldstone Spectrum

$N_f^2 - 1$ broken generators

Naively $N_f^2 - 1$ G.B.

$$H(t,x) = \exp\left[2i\mu\begin{pmatrix}\mathbb{1} & 0\\ 0 & -\mathbb{1}\end{pmatrix}\right]\left(b\begin{pmatrix}\mathbb{1} & 0\\ 0 & \mathbb{1}\end{pmatrix} + \Phi(t,x)\right)$$

Substitute back in the $\mathcal{L}_H$

solve for dispersion relations

	$SU(N_f/2)$	$\times SU(N_f/2)$	Velocity	
Conformal GB	1	1	$1/\sqrt{3}$	$\omega = \sigma p + \cdots$
Type I	$\frac{N_f^2}{4} - 1 = \text{Adj}$	1	$\sqrt{\frac{\alpha_n}{3\alpha_n + 2\alpha_v}}$	$\omega = v p + \cdots$
count as ② I	1	$\frac{N_f^2}{4} - 1 = \text{Adj}$	$\sqrt{\frac{\alpha_n}{3\alpha_n + 2\alpha_v}}$	$\omega = \delta p + \cdots$
2×Type II	$\square$	$\square$	0	$\omega = p^2/4\mu + \cdots$

Generalized GB theorem.

$$\#\,\text{Brok. Gen.} = \#\,\text{GB I} + 2\#\,\text{GB II} = 1 + 2\left(\frac{N_f^2}{4} - 1\right) + 2\,\frac{N_f^2}{4} = -1 + \frac{N_f^2}{2} + \frac{N_f^2}{2} = N_f^2 - 1$$

Type I vacuum energy

Leading quantum correction to the energy formula

- Zero point energy of type I GB

$$S_G = \int_{\mathbb{R}\times M_3} dt\, d\vec{r} \left[\frac{1}{2}(\partial_t \phi)^2 + \frac{c^2}{2}(\nabla\phi)^2 \right]$$

$$E_G = \frac{1}{2}\, \mathrm{tr}\left[\log\left(-\partial_t^2 - c^2\Delta\right)\right] = \frac{1}{4\pi}\int_{-\infty}^{+\infty} d\omega \sum_p \log\left(\omega^2 + c^2 E(p)^2\right) = \cdots = v\, \zeta(-1/2 \,|\, M_3)$$

$$\omega = vp + \cdots$$

$E(p)$ = eigenvalues of the Laplacian on M_3

$$\Delta f_p(\vec{r}) + E^2(p) f_p(\vec{r}) = 0$$

$$E_0 = \frac{1}{2}\left(2\times\left(\frac{N_f^2}{4} - 1\right)\sqrt{\frac{\alpha_n}{3\alpha_n + 2\alpha_r}} + \frac{1}{\sqrt{3}}\right) \underbrace{\zeta\left(-1/2 \,|\, S^3\right)}_{-\frac{0.414}{r_0}}$$

Conformal Dimension for the lowest State

$$\Delta(\mathcal{J}) = r_0 E(S^3) = \frac{3}{2}\frac{N_f^2}{\alpha_h+\alpha_v}\left[\mathcal{J}^{4/3} + \frac{1}{6}\mathcal{J}^{2/3} - \frac{1}{144}\mathcal{J}^0 + \mathcal{O}(\mathcal{J}^{-2/3})\right]$$

$$-\left[\left(\frac{N_f^2}{2}-2\right)\sqrt{\frac{\alpha_h}{3\alpha_h+2\alpha_v}} + \frac{1}{\sqrt{3}}\right]\times 0.212 + \dots$$

Triple expansion in $\frac{1}{\epsilon}$, N_f^2 and $\mathcal{J}$

$$\Delta(\mathcal{J}) = \frac{N_f^2}{\epsilon}\left[\left(c_{4/3}+\mathcal{O}(N_f^{-2})\right)\mathcal{J}^{4/3} + \left(c_{2/3}+\mathcal{O}(N_f^{-2})\right)\mathcal{J}^{2/3} + \left(c_0+\mathcal{O}(N_f^{-2})\right) + \mathcal{O}(\mathcal{J}^{-2/3})\right]$$

$$-\left(\underbrace{\left(\frac{N_f^2}{2}-2\right)d_1}_{\text{Symmetry Breaking}} + \underbrace{\frac{1}{\sqrt{3}}}_{\text{Conformal GB}}\right)\times 0.212 + \mathcal{O}(\mathcal{J}^{-2/3}) + \mathcal{O}(\epsilon)$$

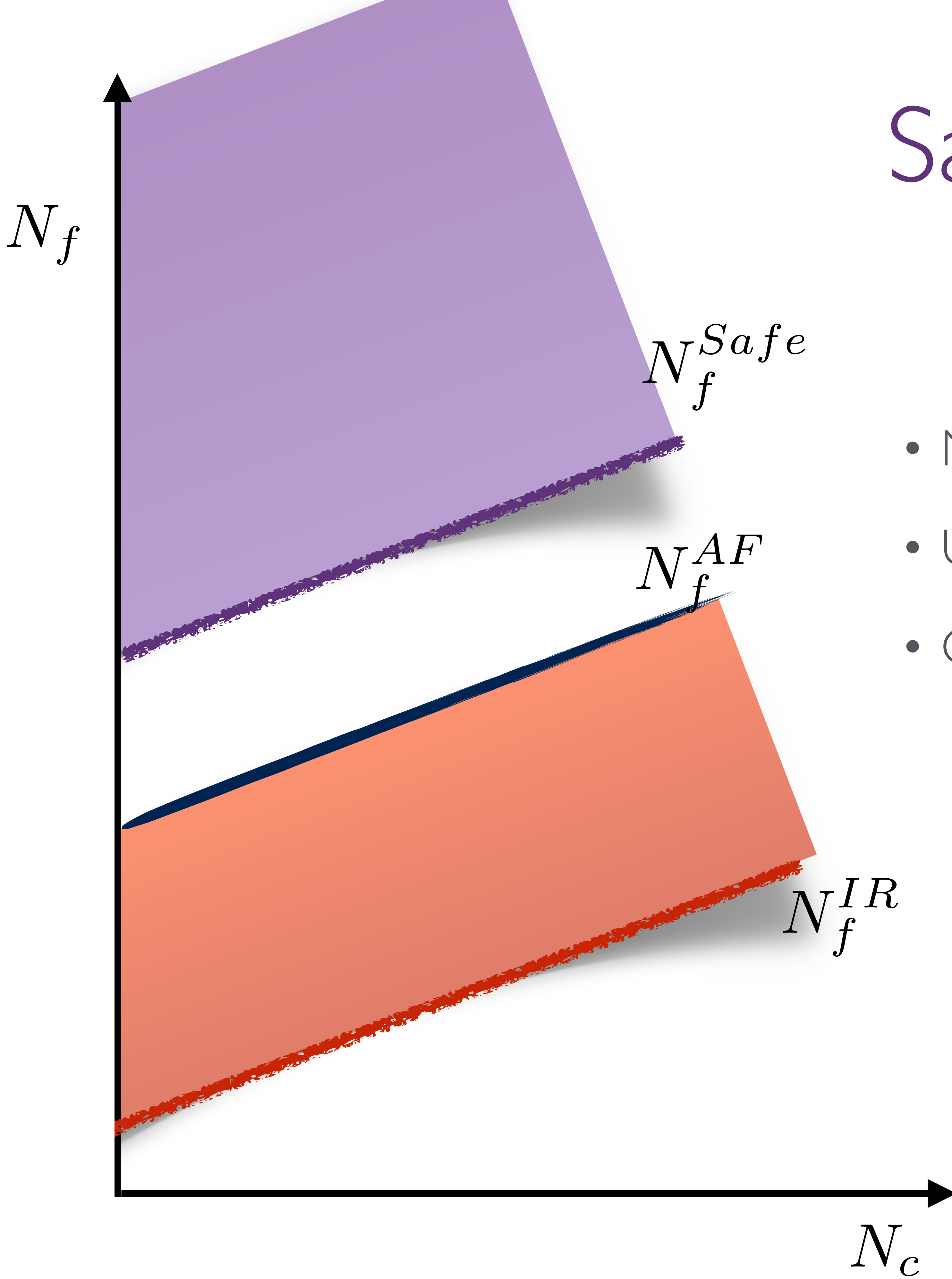
- c_i: coefficients are calculable here

Beyond

- Non perturbative CFT regime at $\mathcal{T} \gg 1$
- Finite N_c and N_f can be investigated
- Similar IR - CFT theories
- Walking dynamics

Lot of fun ahead !!

Safe QCD

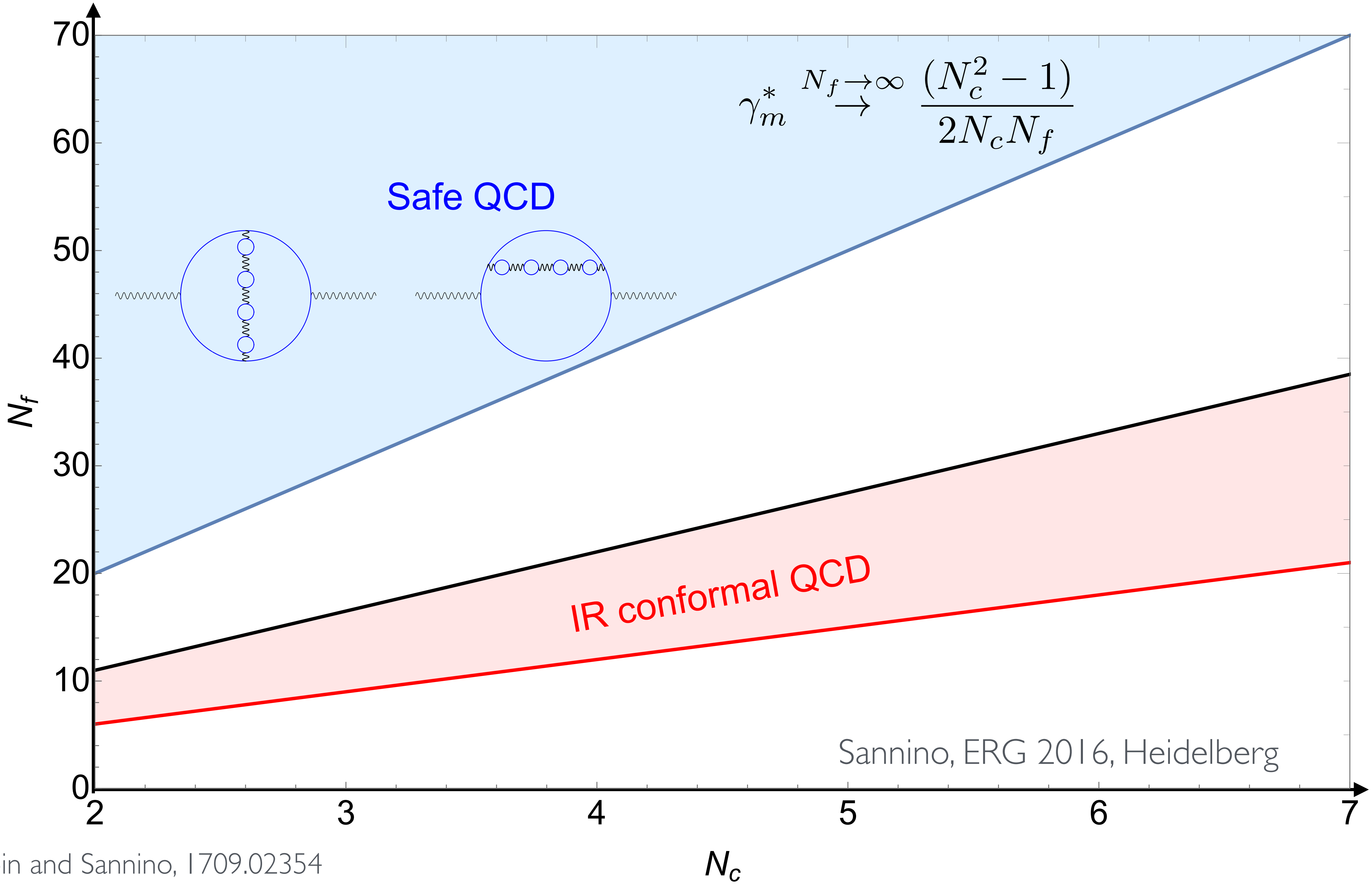


- Must exist a critical Safe Nf
- Unsafe region in Nf-Nc
- Continuous (Walking) transition?

Sannino, ERG 2016, Trieste

Antipin and Sannino, 1709.02354
Pica and Sannino 1011.5917, PRD

Safe QCD: Conformal Window 2.0



Antipin and Sannino, 1709.02354
Palanques-Mestre, Pascual, Commun. Math. Phys. 84
Gracey, PLB, 96, Holdom PLB 2011
Pica and Sannino 1011.5917, PRD

Scalars are not needed at large Nf

Analytic Coupling Structure of Large N_f (Super) QED and QCD

Nicola Andrea Dondi[♡], Gerald V. Dunne[△], Manuel Reichert[♡], and Francesco Sannino[♡◇]
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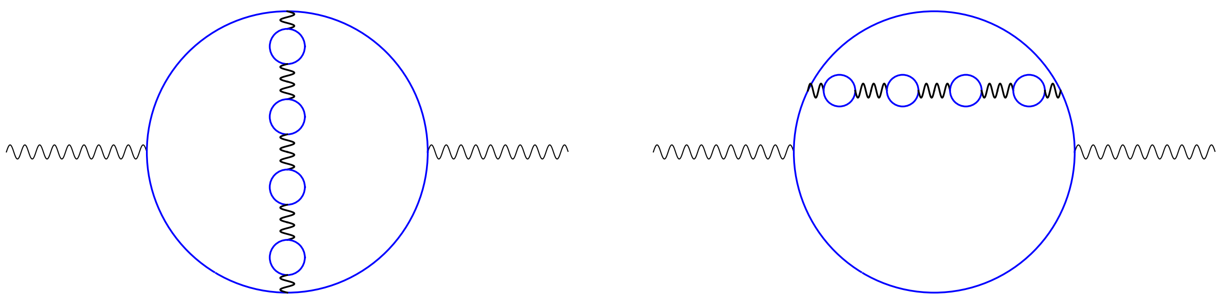
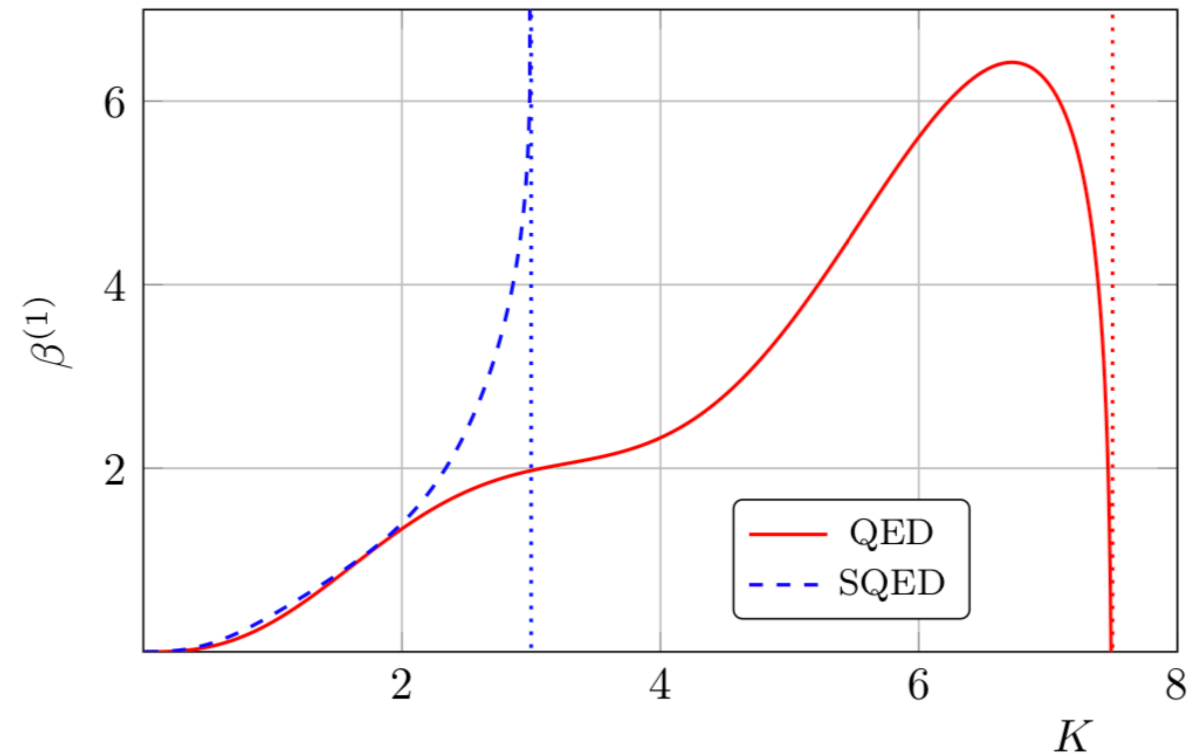


$$K = \frac{g^2 N_f S_2(R)}{4\pi^2}$$



$$\beta^{(1)}_{\mathrm{QED}}(K) \sim \frac{14K^2}{45\pi^2} \ln\left(\frac{15}{2} - K\right) + \dots\ , \quad K \to \frac{15}{2}$$

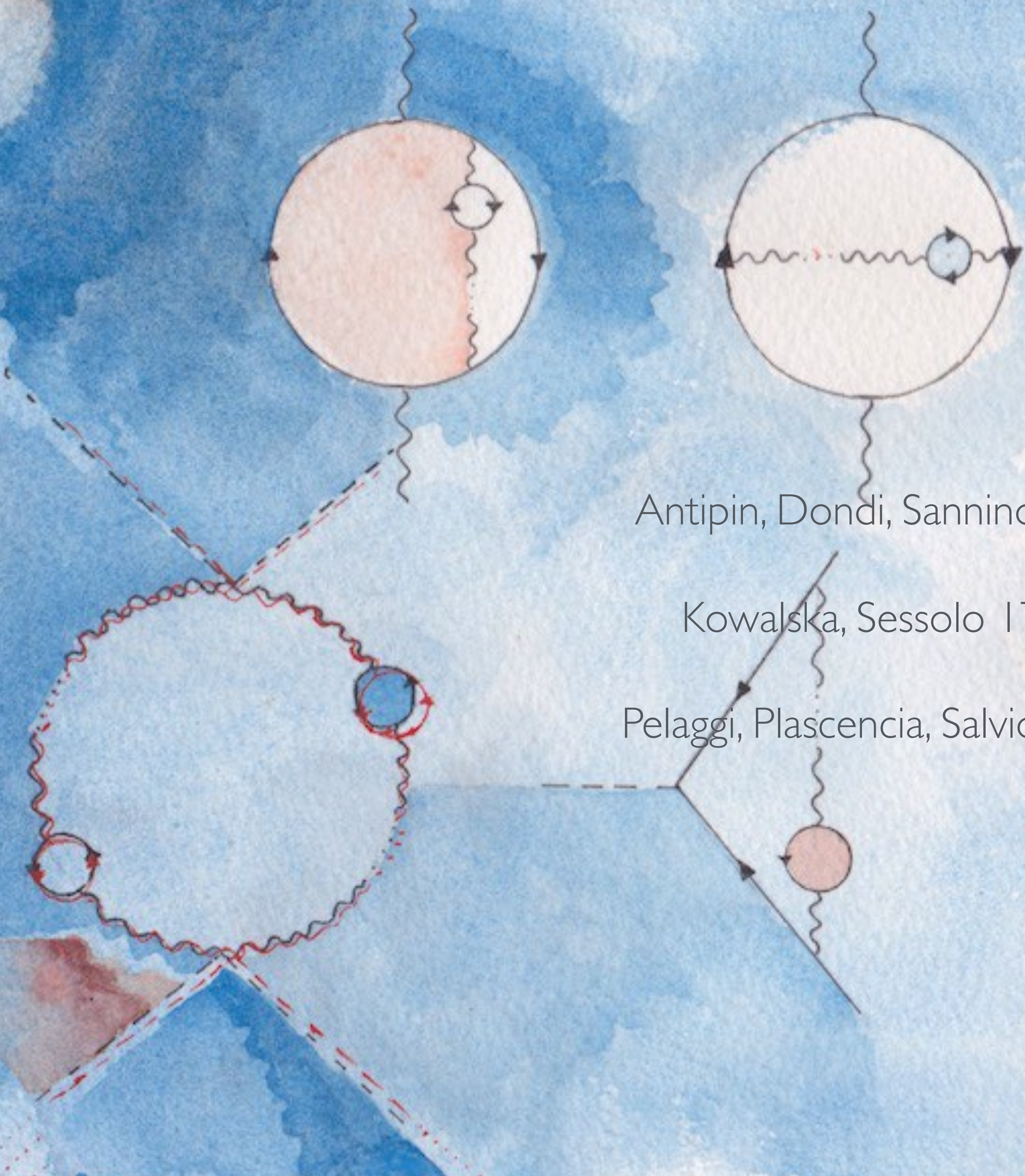
Non-abelian gauge theories

$$\beta^{(1)}_{\mathrm{QCD}}(K) \sim \frac{K^2}{24}\frac{C_2(G)}{S_2(R)} \ln(3-K) + \dots\ , \quad K \to 3$$

$$K_*^{\mathrm{np}} = 3 - \exp\left[-16\frac{S_2(R)}{C_2(G)} N_f\right]$$

30 orders in PT to reproduce the first singularity

Gauge-Yukawa beta functions at large Nf



Antipin, Dondi, Sannino, Thomsen, Zhi-Wei Wang 1803.09770

Kowalska, Sessolo 1712.06859   Alanne, Blasi 1806.06954

Pelaggi, Plascencia, Salvio, Sannino, Smirnov, Strumia 1708.00437

Artist: Kaća Bradonjić

Novel avenues

- Safe extensions of the Standard Model
- Solving large Nf QFT
- Gauge-gravity & gauge-gauge duality
- Conformal window 2.0
- Safe QCD on the lattice
- Safe amplitudes
- Quantising gravity at large Nf
- ….

Thank you

"All theories are equals but some theories are more equal than others."

Inspired by George Orwell's "Animal Farm"

Supersymmetric (un)safety

Intriligator and Sannino, 1508.07413

Bajc and Sannino, 1610.09681

Bajc, Dondi, Sannino, 1709.07436

Exact results beyond perturbation theory

Central charges

- Positivity of coefficients related to the stress-energy trace anomaly
- 'a(R)' Conformal anomaly of SCFT = $U(1)_R$ 't Hooft anomalies
  [proportional to the square of the dual of the Rieman Curvature]

$$a(R) = 3\mathrm{Tr}U(1)_R^3 - \mathrm{Tr}U(1)_R$$

- 'c(R)'
  [proportional to the square of the Weyl tensor]

$$c(R) = 9\mathrm{Tr}U(1)_R^3 - 5\mathrm{Tr}U(1)_R$$

- 'b(R)'
  [proportional to the square of the flavor symmetry field strength]

$$b(R) = \mathrm{Tr}U(1)_R F^2$$

a-theorem

For any super CFT besides positivity we have, following Cardy

$$\Delta a \equiv a_{UV} - a_{IR} > 0$$

$$\Delta a = a_{\rm UV} - a_{\rm IR} = \pm\frac{1}{9}\sum_i |r_i| \left[(3R_i - 2)^2 (3R_i - 5)\right] > 0$$

r_i = dim. of matter rep.

+(-) for asymptotic safety (freedom)

Stronger constraint for asymp. safety, since at least one large R > 5/3

SQCD & related theories are unsafe

Can be ruled out via a-theorem $a(R) = 3\mathrm{Tr}U(1)_R^3 - \mathrm{Tr}U(1)_R.$

$$a_{\mathrm{UV-safe}} - a_{\mathrm{IR-safe}} < 0$$

Safe SUSY QCD does not exist

Generalisation to several susy theories using a-maximisation*

Intriligator and Sannino, 1508.07413, JHEP

SUSY GUTs + R charge are challenging

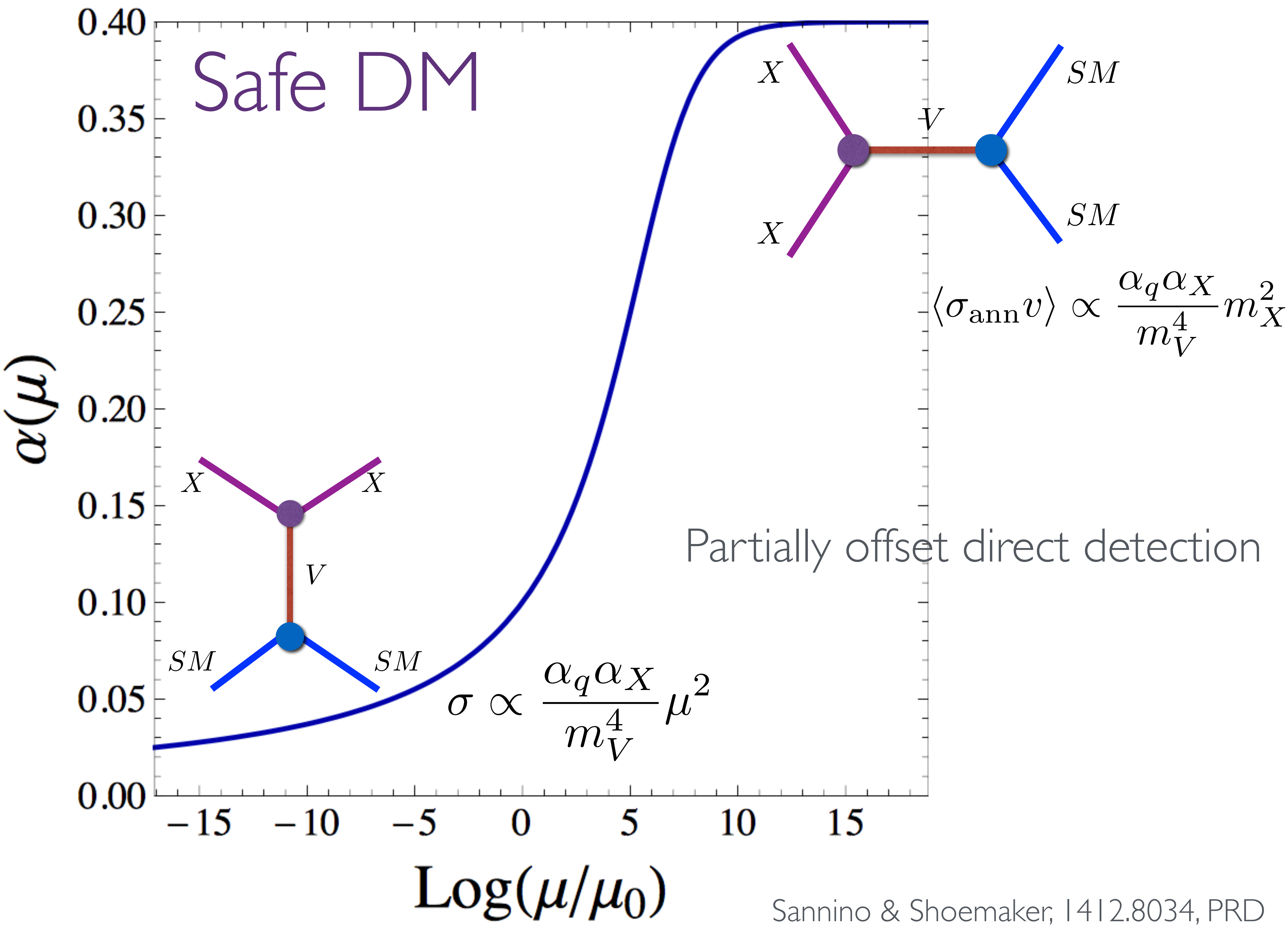
Bajc and Sannino, 1610.09681, JHEP

Safe SUSY mechanics

To avoid the Intriligator-Sannino constraints

- At least one large R-charge for some of the fields
- UV-Interacting to IR-Interacting can be achieved with moderate R-charges [First non-SUSY example Esbensen-Ryttov-Sannino 1512.04402]
- Adding IR/UV relevant operators to modify the flow

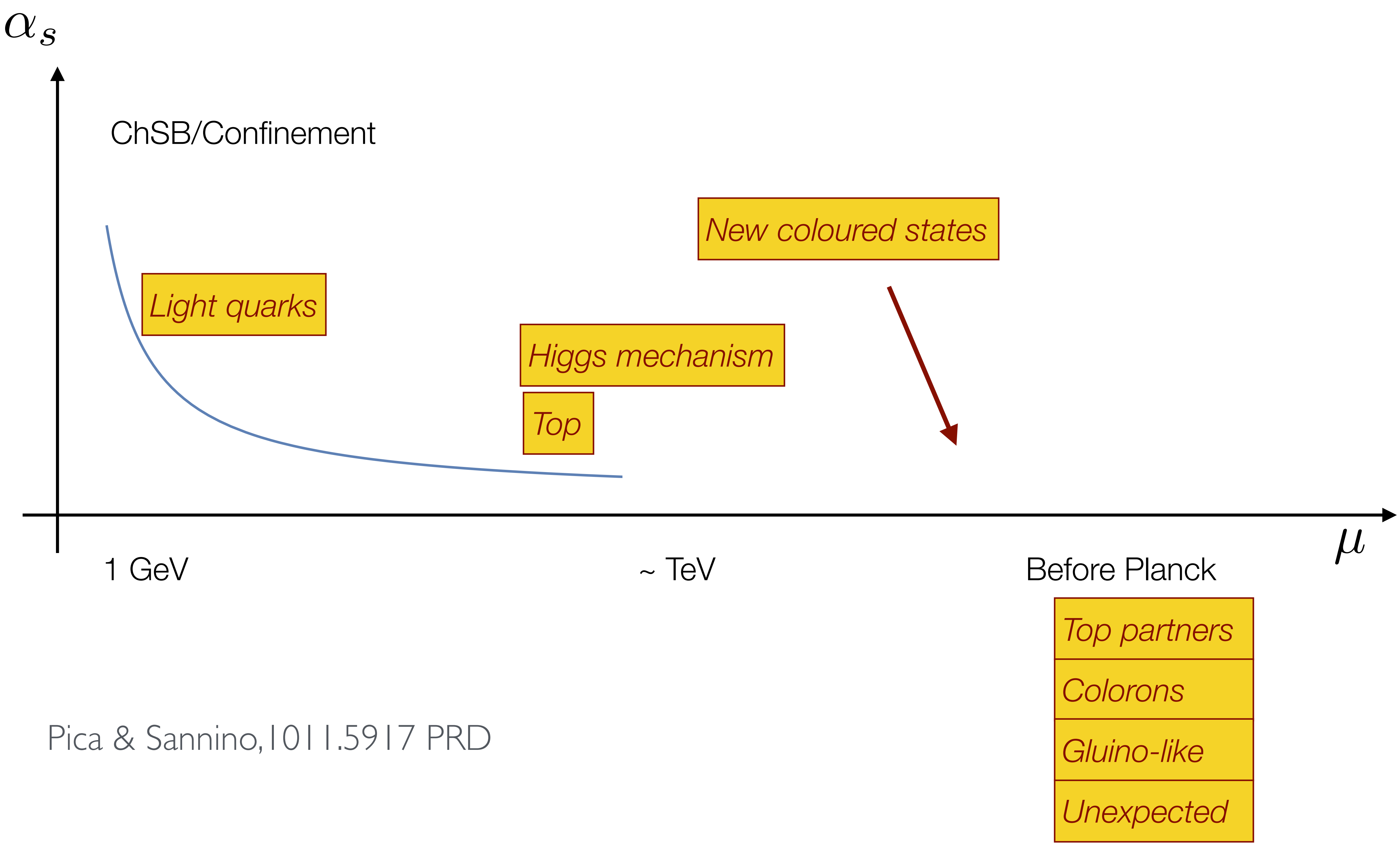
Intriligator and Sannino, JHEP 1508.07411
Bajc and Sannino, JHEP 1610.09681
Bajc, Dondi, Sannino, JHEP 1709.07436
Bond, Litim, PRL1709.06953



Sannino & Shoemaker, 1412.8034, PRD

Safe QCD

Sannino, 1511.09022



Pica & Sannino,1011.5917 PRD