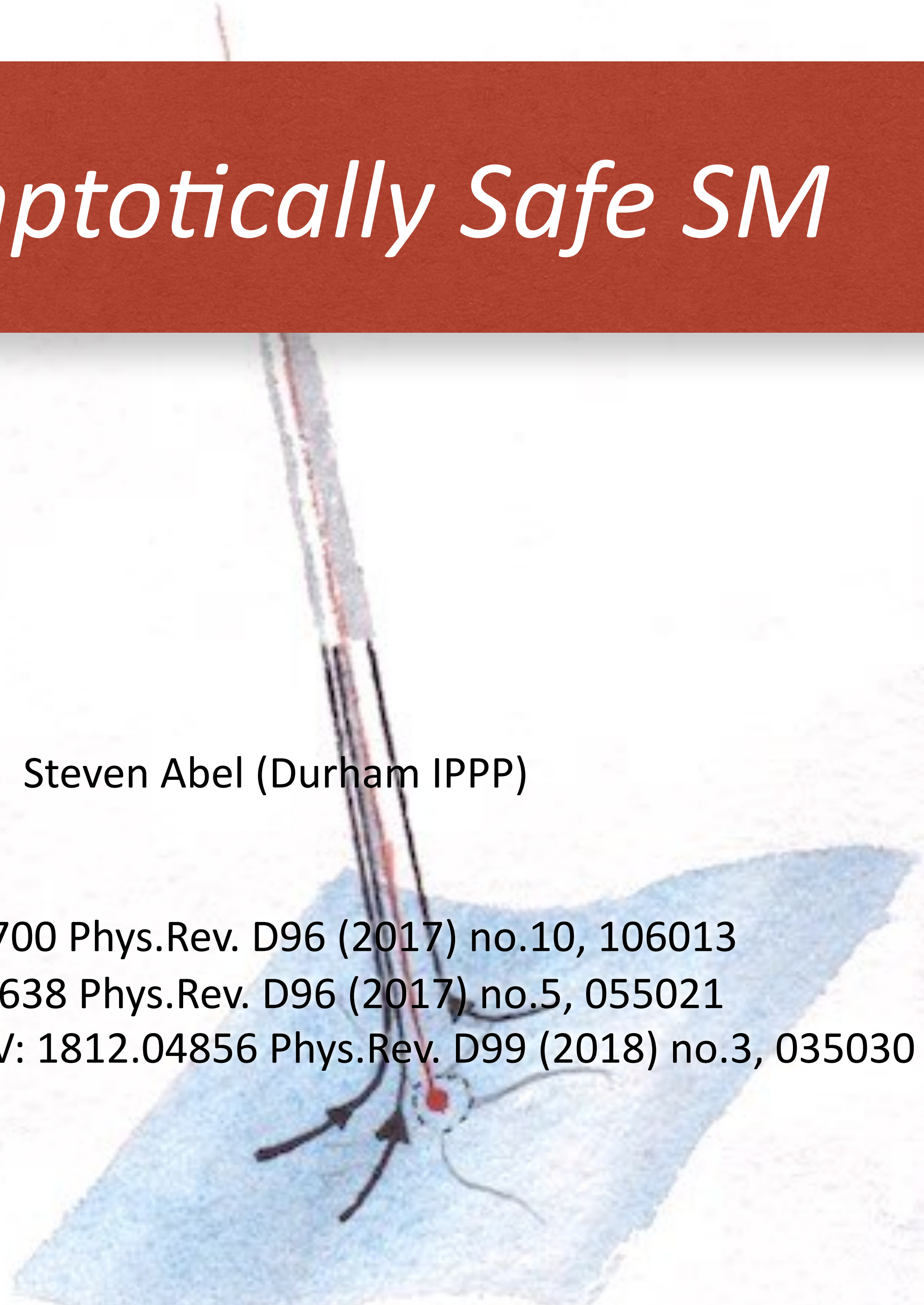
Asymptotically Safe SM

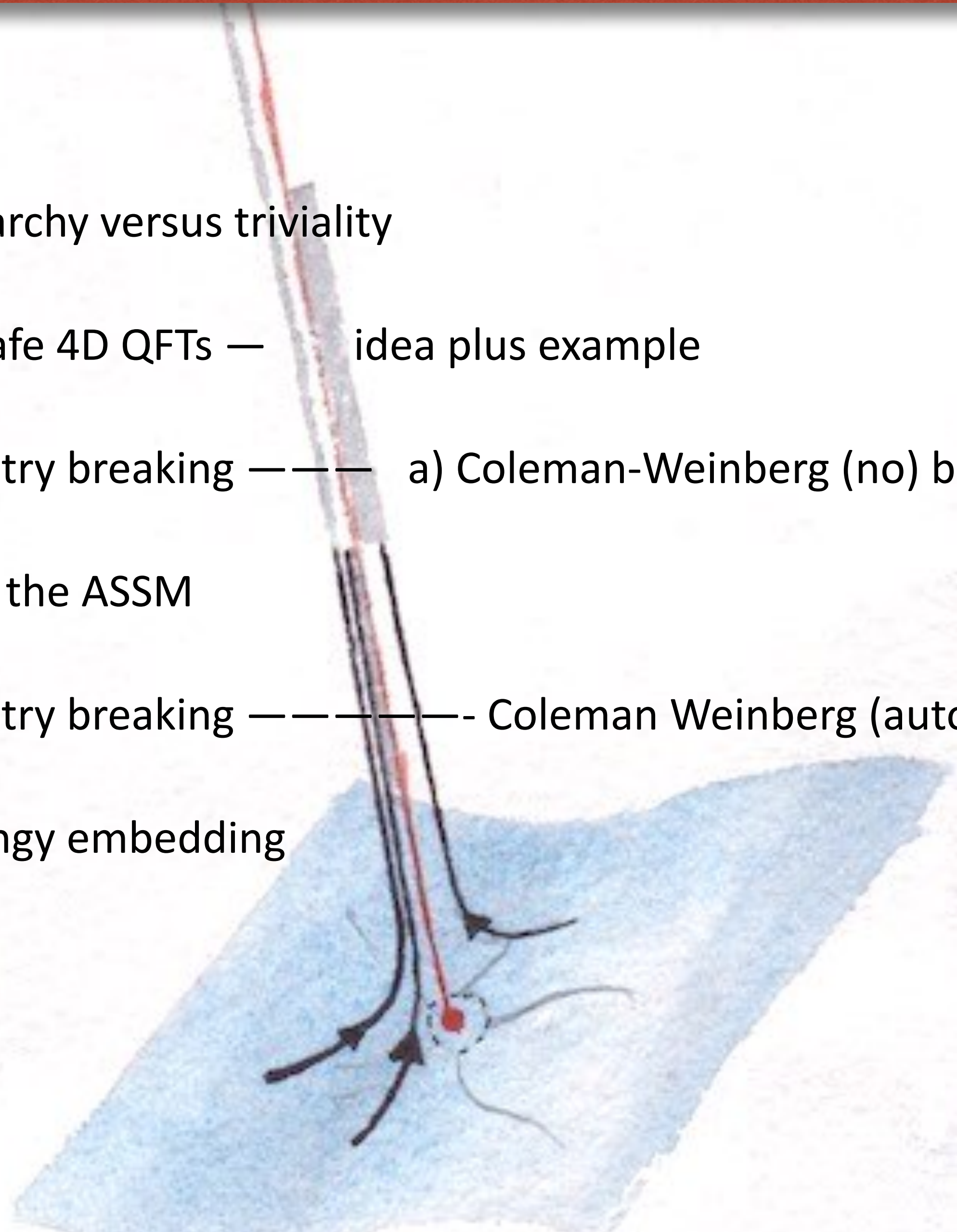


Steven Abel (Durham IPPP)

w/ Sannino, ArXiV:1704.00700 Phys.Rev. D96 (2017) no.10, 106013
w/ Sannino, ArXiV: 1707.06638 Phys.Rev. D96 (2017) no.5, 055021
w/ Molgaard,Sannino, ArXiV: 1812.04856 Phys.Rev. D99 (2018) no.3, 035030

Image by Kaća Bradonjić

Outline



- Motivation; hierarchy versus triviality
- Asymptotically safe 4D QFTs — idea plus example
- Radiative symmetry breaking ——— a) Coleman-Weinberg (no) b) Mass-squareds
- Tetrad model for the ASSM
- Radiative symmetry breaking —————- Coleman Weinberg (automatic!)
- Thoughts on stringy embedding

Image by Kaća Bradonjić

Hierarchy versus triviality

The hierarchy problem:

Why is the Weak Scale so much lower than the Planck Scale - and how is it protected?

More precisely perturbation theory with a higgs scalar is suspect: very “massive states” dominate any perturbative calculation to do with higgs physics.

Actually don’t even need a heavy resonance: this *can* be true for some other rapid change (in e.g. beta functions) at a high scale. e.g. at one-loop … suppose some physics comes in at a scale Λ_{UV} to *complete the theory*: then

$$\delta m_h^2 = \int_0^\infty \frac{dt}{t^2} f(\Lambda_{UV}^2 t)$$

$$= \Lambda_{UV}^2 \int_0^\infty \frac{dy}{y^2} f(y)$$

The hierarchy problem:

This integral might be small if there are some ***symmetries:***

- Higgs is a Goldstone mode of ***some broken global symmetry*** (like the pions in chiral symmetry breaking) with breaking scale of a few TeV: $$\delta m_h^2 \sim \frac{\Lambda_\chi^4}{\Lambda_{UV}^2}$$
- ***Supersymmetry*** - relates boson to fermions. Divergences cancel level by level. Phenomenology requires soft (a.k.a. dimensionful) breaking.
- ***Scaling symmetry*** **-** Higgs is the Goldstone mode of a broken scale invariance (a.k.a. dilaton) (a trivial perturbative example of this is the Standard Model with vanishing higgs mass, but it can occur in nonperturbative models based on AdS/CFT).
- ***Misaligned Supersymmetry*** - even non-supersymmetric non-tachyonic strings are finite. (stringy symmetry when you sum over entire tower of states) (Dienes, Moshe, Myers (90's), SAA+Dienes+Mavroudi)

The triviality problem:

Scalars lead to Landau poles: => the theory is UV incomplete

But trying to UV complete it results in the hierarchy problem back again (The longer I leave it the larger Λ_{UV} is by the time I have fixed the problem)

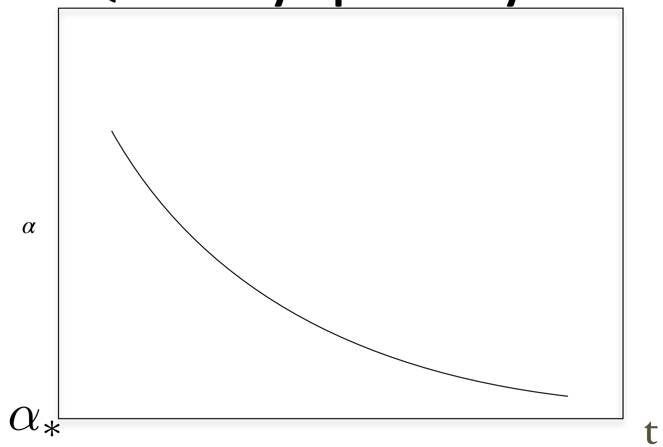
Hints from QCD about UV completness

QCD is (unlike SUSY) a UV complete theory. Why?

1. ***There is no hierarchy problem:*** quark masses are protected by chiral symmetry

2. ***There is no triviality problem:*** QCD is **asymptotically free**

$$\partial_t \alpha = -B\alpha^2$$



$$\boxed{\alpha_* = 0}$$

Note the philosophy of QCD: we do not mind masses running because they do not upset the Gaussian UV fixed point. We simply measure them and let them run. Or to put it another way: they are "relevant" operators that are effectively zero in the UV. They do not need to run to zero in the UV! (We also don't care too much about couplings blowing up in the IR.)

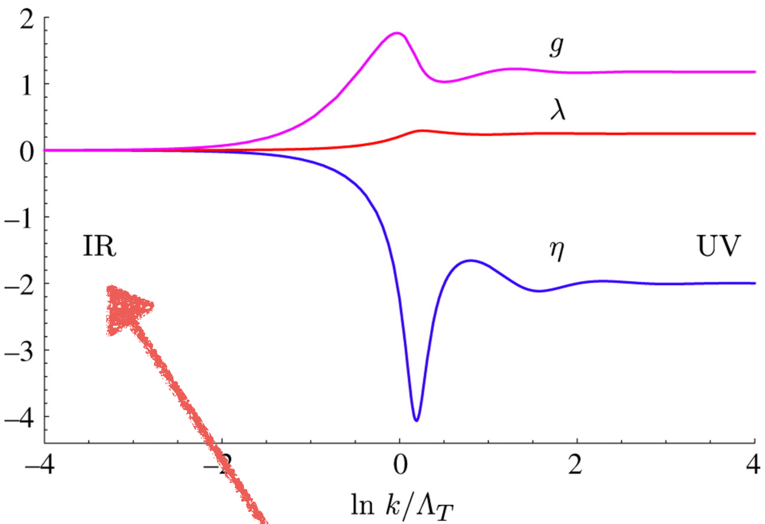
Asymptotic safety in 4D QFT

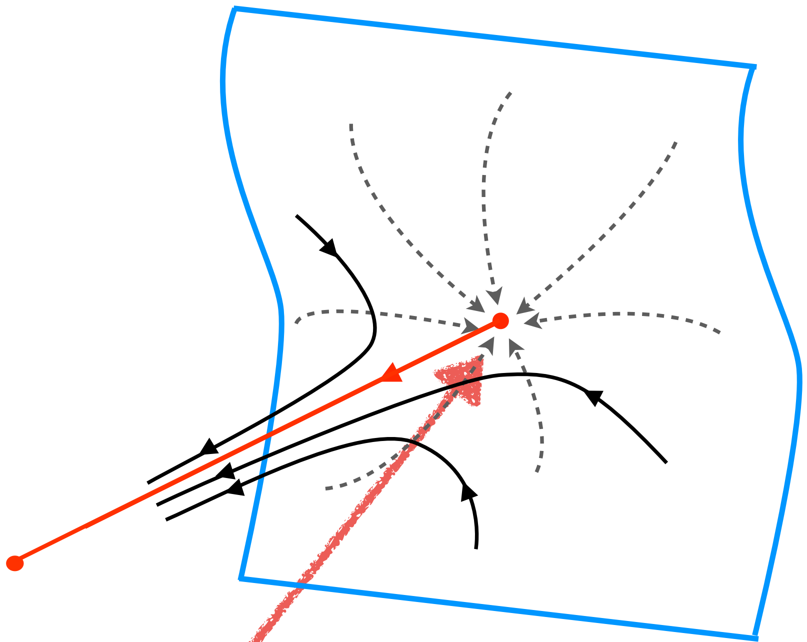
Philosophy: can we UV complete the SM?

The Basic idea of Asymptotic Safety

Gastmans et al ’78
Weinberg ’79
Peskin
Reuter, Wetterich
Gawedski, Kupiainen
Kawai et al,
de Calan et al ’,
Litim
Morris

Weinberg et als proposal for UV completing theories





Gaussian IR fixed point => perturbative

Interacting UV fixed point => finite anomalous dimensions
In a field theory replace $1/\epsilon$ with $1/\gamma$ => some divergences of marginal operators (which affect the fixed point), are cured and they stop running

Divide up the content of a theory as follows:

Irrelevant operators: like ϕ^6 would disrupt the fixed point - therefore asymptotically safe theories have to emanate precisely from UV fixed point where they are zero (exactly renormalizable trajectory)

Marginal operators: can be involved in determining the UV fixed point where they become *exactly* marginal. Or can be marginally relevant (asymptotically free) or irrelevant.

Relevant operators: become "irrelevant" in the UV but may determine the IR fixed point.

Dangerously irrelevant operators: grow in both the UV and IR (common in e.g. SUSY)

Harmless relevant operators: shrink in both the UV and IR

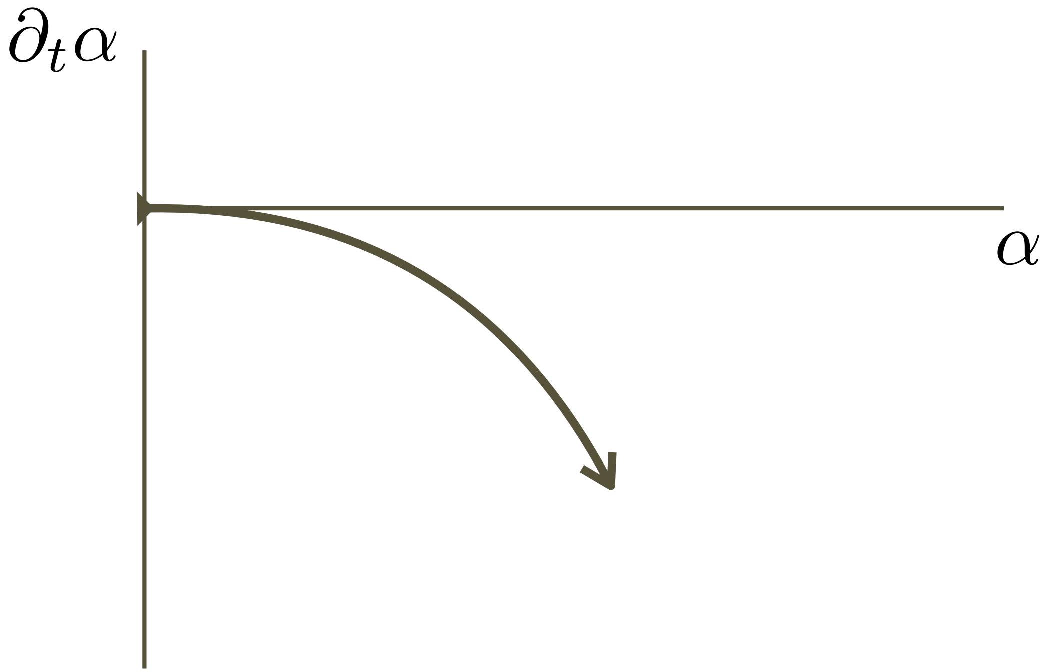
Note relevant or marginally relevant operators still have "infinities" at the FP - just as quark masses, they still run at the FP just like any other relevant operator: but being relevant they do not affect the FP. (By definition they become unimportant at in the UV.)

U.V. v. I.R. F.P.

Simple example of flow - normal QCD:

$$\partial_t \alpha = -B\alpha^2 \qquad t = \log \mu/\mu_0$$

This theory has *unstable* fixed point at $\alpha = 0$. Asymptotically free if $B > 0$

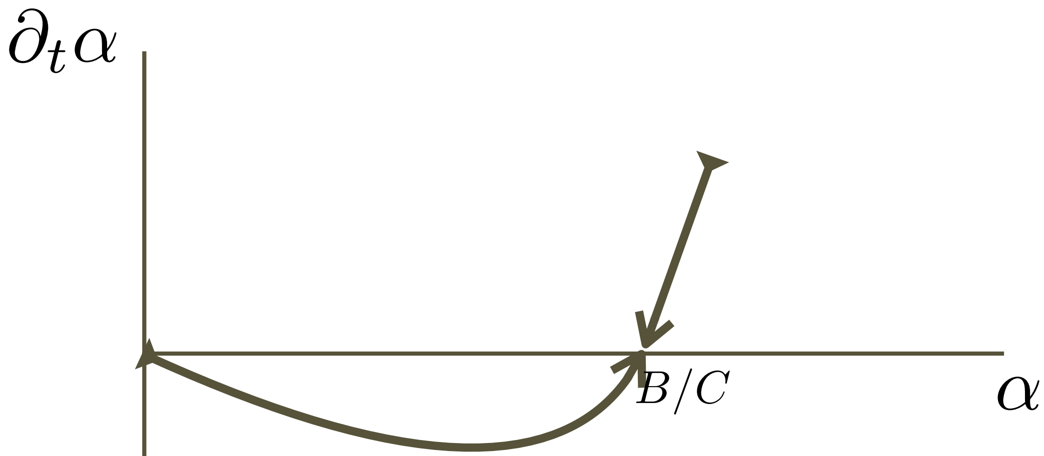


Caswell-Banks-Zaks fixed point: (Famously in Seiberg duality)

Take QCD with $SU(N_C)$ and N_F fermions but very large numbers of colours+flavours

$$\partial_t \alpha = -B\alpha^2 + C\alpha^3 \qquad \boxed{B \propto \epsilon = \frac{N_F}{N_C} - \frac{11}{2}}$$

Turns out C>0, B>0: theory has *stable* IR fixed point at α = B/C and *unstable* one in UV α = 0



Note perturbativity: $\Longrightarrow \; B \ll C$

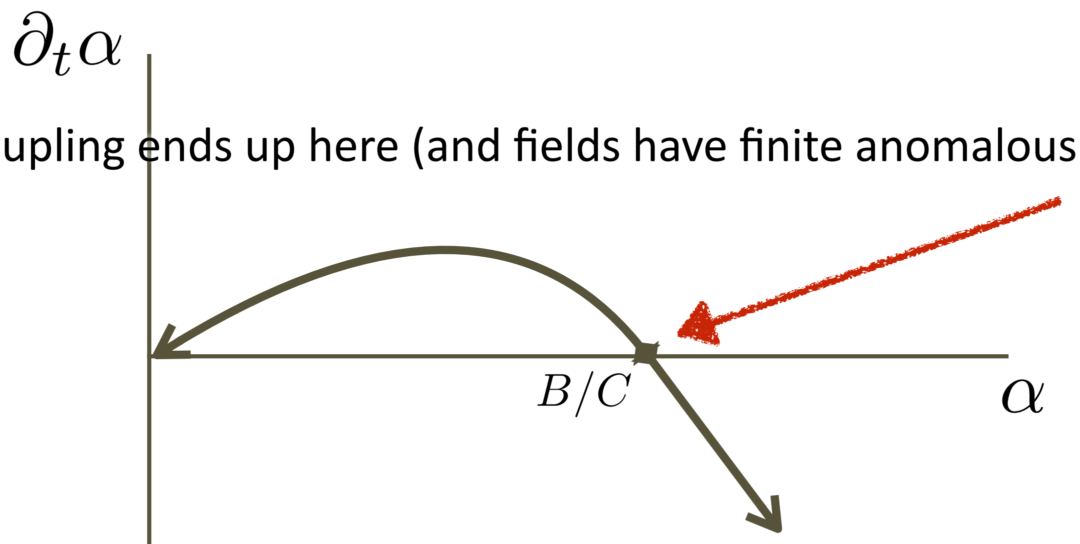
requires many fields (*Veneziano limit*) with $N_F \approx 11 N_C/2$

Familiar from weakly coupled supersymmetry where $N_F \lesssim 3N_C$ in $\mathcal{N} = 1$case

Cartoon of a would-be Interacting UV FP:

Again would have … $$\partial_t \alpha = -B\alpha^2 + C\alpha^3$$

But requires C<0, B<0, this theory has *stable* IR fixed point at α = 0 and *unstable* UV one at α = B/C



At t -> infinity the coupling ends up here (and fields have finite anomalous dimensions)

Again perturbativity would require $N_F \approx 11 N_C / 2$

Implementing Asymptotic Safety either requires strong coupling or many degrees of freedom

Asymptotic safety in 4D QFT (Example)

Real situation requires several couplings to realise

Litim & Sannino '14

Need to add **scalars** and **Yukawa couplings:**

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\, F^{\mu\nu}F_{\mu\nu} + \mathrm{Tr}\left(\overline{Q}\, i\not{D}\, Q\right) + y\,\mathrm{Tr}\left(\overline{Q}\, H\, Q\right) + \mathrm{Tr}\,(\partial_\mu H^\dagger\, \partial^\mu H)$$
$$-u\,\mathrm{Tr}\,[(H^\dagger H)^2] - v\,(\mathrm{Tr}\,[H^\dagger H])^2\,,$$

H is an $N_F \times N_F$ scalar

Initially have $U(N_F)_L \times U(N_F)_R$ flavour symmetry

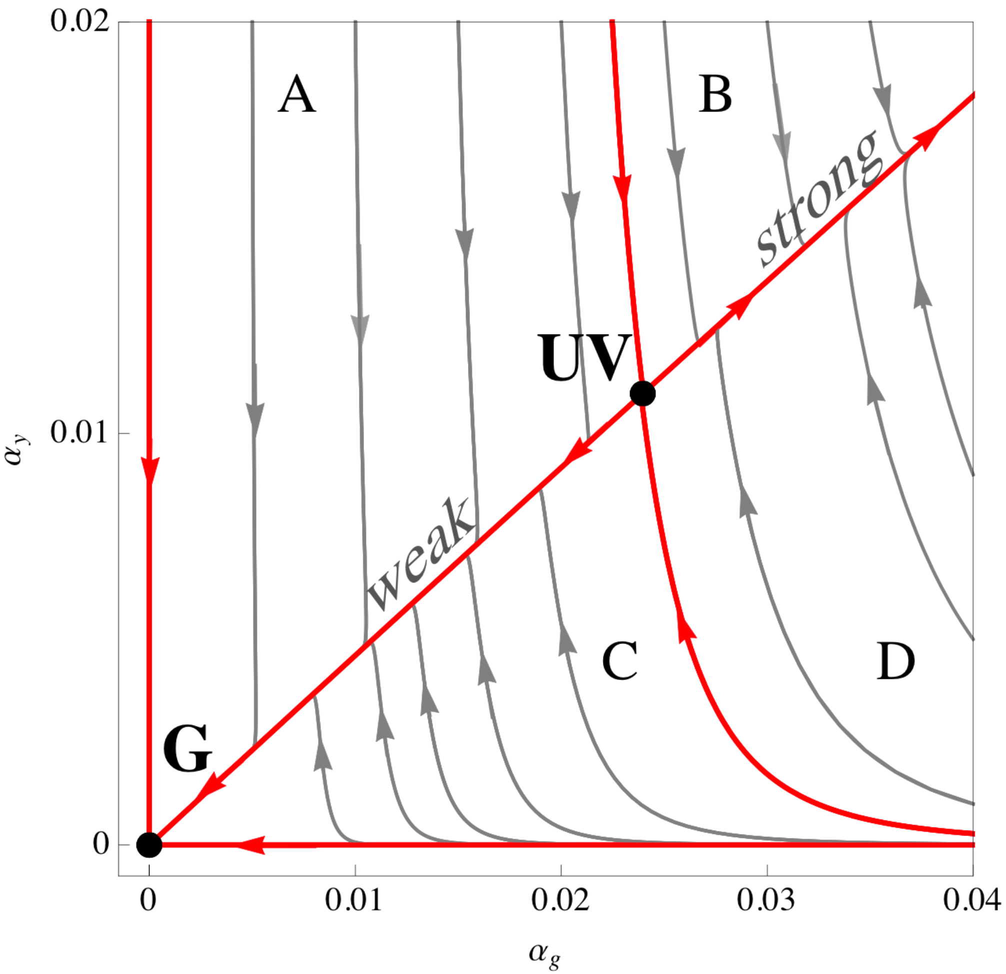
Effect of Yukawa ....

$$\left(\alpha_g = \frac{g^2\, N_C}{(4\pi)^2}\,, \quad \alpha_y = \frac{y^2\, N_C}{(4\pi)^2} \right)$$

$$\beta_g = \alpha_g^2 \left[\frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right)\alpha_g \boxed{- 2\left(\frac{11}{2} + \epsilon\right)^2 \alpha_y} \right]$$

$$\beta_y = \alpha_y \left[\boxed{(13 + 2\epsilon)\,\alpha_y - 6\,\alpha_g} \right]$$

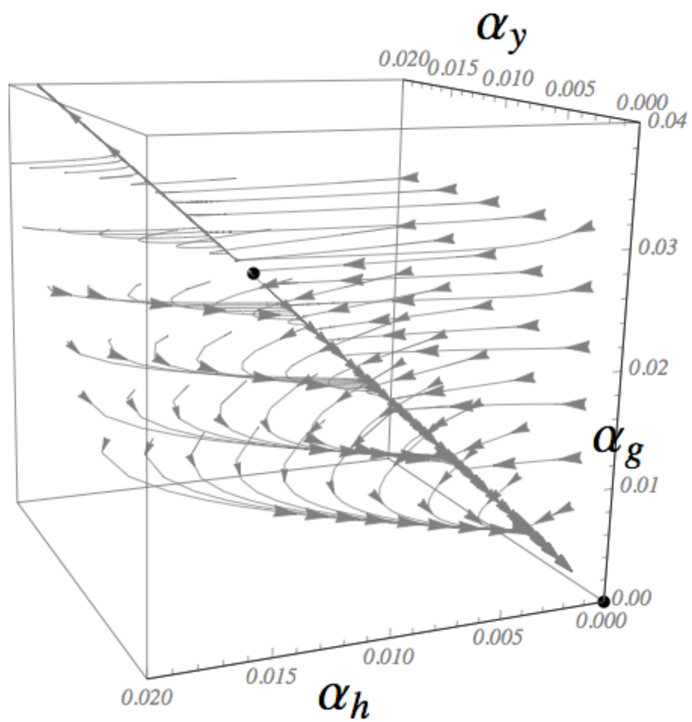
$$\epsilon = \frac{N_F}{N_C} - \frac{11}{2}$$



Four ’t Hooft-like couplings - flow could in principle be four dimensional

$$\alpha_g = \frac{g^2\, N_C}{(4\pi)^2}\,, \quad \alpha_y = \frac{y^2\, N_C}{(4\pi)^2}\,, \quad \alpha_h = \frac{u\, N_F}{(4\pi)^2}\,, \quad \alpha_v = \frac{v\, N_F^2}{(4\pi)^2}$$

but …



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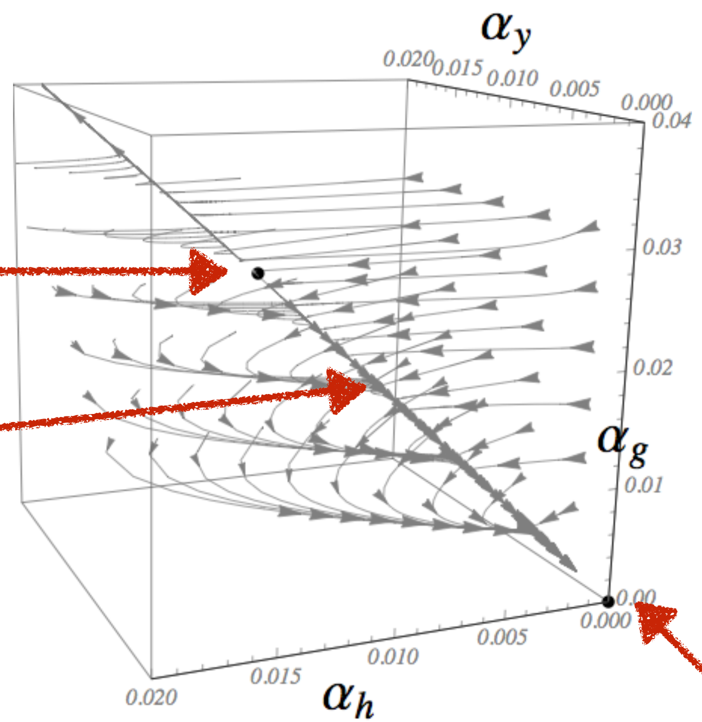


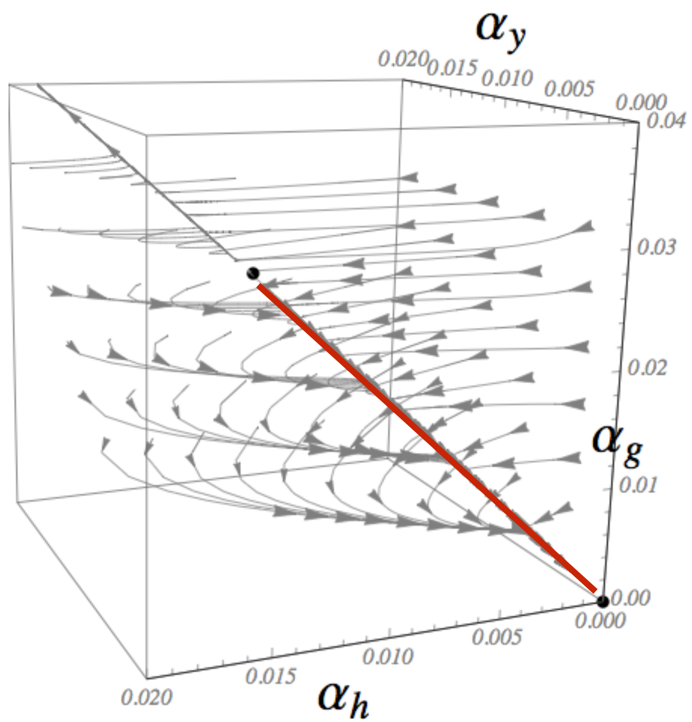
UV fixed point

1D exactly renormalisable trajectory!

Gaussian IR fixed point

Along the critical-curve/exact-trajectory can parameterise the flow in terms of $\alpha_g(t)$

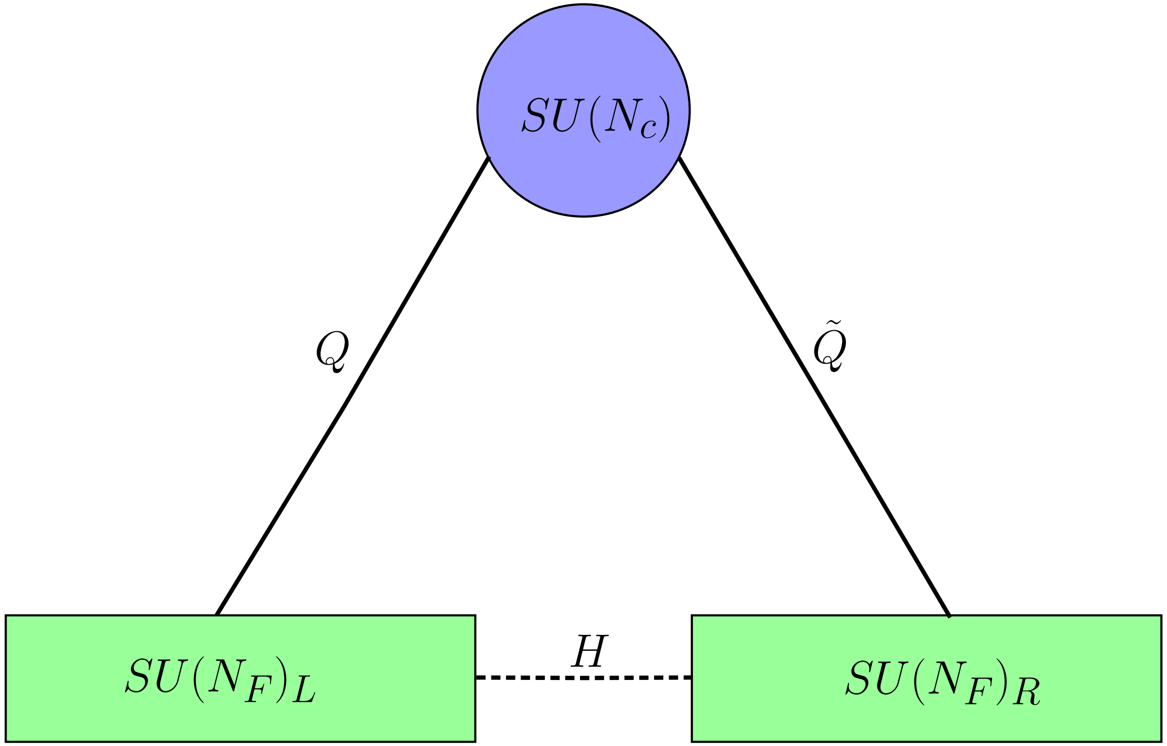
$$\alpha_y(t) = \frac{6}{13}\alpha_g(t)\ ,$$
$$\alpha_h(t) = 3\frac{\sqrt{23}-1}{26}\alpha_g(t)\ ,$$
$$\alpha_v(t) = \frac{3\sqrt{20+6\sqrt{23}}-6\sqrt{23}}{26}\alpha_g(t)\ ,$$



At the fixed point it is arbitrarily weakly coupled, $\alpha_g^* = \quad 0.4561\,\epsilon$, where $\epsilon = \dfrac{N_F}{N_C} - \dfrac{11}{2}$

Quiver diagram for this model:

	$SU(N_C)$	$SU(N_F)_L$	$SU(N_F)_R$	spin
Q_{ai}	$\square$	$\square$	1	$1/2$
$\tilde{Q}^{ia}$	$\tilde{\square}$	1	$\tilde{\square}$	$1/2$
H^i_j	1	$\tilde{\square}$	$\square$	0



Towards radiative symmetry breaking

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No Coleman-Weinberg mechanism

Recap of the idea

- ***The SM is "classically" scale invariant*** - tree level Lagrangian has no mass
- Coleman Weinberg mechanism leads to spontaneous breaking at a scale because the scale invariance is anomalous. (Huge amount of interest since 2012)
- Compute effective potential and renormalize it

$$V_{eff} = \frac{\lambda}{4!}|\phi|^4 + \frac{3g^4}{64\pi^2}|\phi|^4 \left(\log\frac{|\phi|}{\mu} - \frac{25}{6}\right) \qquad \frac{\partial^2 V}{\partial\phi^2}|_{\phi=0} = 0 \quad \frac{\partial^4 V}{\partial\phi^4}|_{\phi=\mu} = \lambda$$

We imposed by hand no generation of mass terms!

Minimization leads to dimensional transmutation

$$\langle\phi\rangle = \mu e^{\frac{11}{6} - \frac{4\pi^2\lambda}{9g^4}}$$

- ***Heuristically seems unlikely to work from a UV fixed point:*** CW is all about ***IR*** scale invariance where z=0 - which is why it is a strange starting point for solving the problems of large UV thresholds.
- Proof (already shown numerically by Litim, Mojaza, Sannino but can see it analytically): for example choose the real trace direction …

$$H = \frac{\phi}{\sqrt{2N_F}} \mathbb{1}_{N_F \times N_F} \quad \Longrightarrow \quad V^{(4)}_{class} = \frac{4\pi^2}{N_F^2} (\alpha_h + \alpha_v)\, \phi^4$$

- Effectively $$\lambda \;=\; 32\pi^2 \frac{3}{N_F^2}\left(\alpha_h + \alpha_v\right)$$
- Also define $$\kappa \;=\; 32\pi^2 \frac{1}{N_F^2}\left(3\alpha_h + \alpha_v\right)$$

$$\begin{aligned} V = \frac{\lambda}{4!}\phi^4 + \frac{m_\phi^2}{2}\phi^2 + \frac{1}{64\pi^2}\left(m_\phi^2 + \frac{\lambda}{2}\phi^2\right)^2 \left(\log \frac{m_\phi^2 + \frac{\lambda}{2}\phi^2}{\mu^2} - \frac{3}{2}\right) \\ - \;\; \frac{(4\pi)^2}{4N_F N_C}\alpha_y^2 \phi^4 \left(\log \frac{(4\pi)^2 \alpha_y \phi^2}{\sqrt{N_F N_C}\mu^2} - \frac{3}{2}\right) \\ + \;\; \frac{(N_F^2 - 1)}{64\pi^2}\left(\frac{\kappa}{2}\phi^2\right)^2 \left(\log \frac{\frac{\kappa}{2}\phi^2}{\mu^2} - \frac{3}{2}\right) + \frac{N_F^2}{64\pi^2}\left(\frac{\lambda}{6}\phi^2\right)^2 \left(\log \frac{\frac{\lambda}{6}\phi^2}{\mu^2} - \frac{3}{2}\right) \end{aligned}$$

- Effectively $$\lambda = 32\pi^2 \frac{3}{N_F^2} (\alpha_h + \alpha_v)$$
- Also define $$\kappa = 32\pi^2 \frac{1}{N_F^2} (3\alpha_h + \alpha_v)$$

Corrections all of order $\alpha\lambda$, so no perturbative minimum without a mass-squared for ϕ

$$V = \frac{\lambda}{4!}\phi^4 + \frac{m_\phi^2}{2}\phi^2 + \frac{1}{64\pi^2}\left(m_\phi^2 + \frac{\lambda}{2}\phi^2\right)^2 \left(\log\frac{m_\phi^2 + \frac{\lambda}{2}\phi^2}{\mu^2} - \frac{3}{2}\right)$$
$$- \frac{(4\pi)^2}{4N_F N_C}\alpha_y^2\phi^4\left(\log\frac{(4\pi)^2\alpha_y\phi^2}{\sqrt{N_F N_C}\mu^2} - \frac{3}{2}\right)$$
$$+ \frac{(N_F^2-1)}{64\pi^2}\left(\frac{\kappa}{2}\phi^2\right)^2\left(\log\frac{\frac{\kappa}{2}\phi^2}{\mu^2} - \frac{3}{2}\right) + \frac{N_F^2}{64\pi^2}\left(\frac{\lambda}{6}\phi^2\right)^2\left(\log\frac{\frac{\lambda}{6}\phi^2}{\mu^2} - \frac{3}{2}\right)$$

Adding relevant operators (e.g.mass-squareds)

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The non-predictive free parameters

Solve Callan Symanzik eqn for them as usual =>

- **warm-up**; first restrict ourselves to the diagonal direction where mass-squared term looks like the following operator:

$$V \supset \frac{m_\phi^2}{4N_F}\left(\mathrm{Tr}(H+H^\dagger)\right)^2$$

$$\bar{\beta} = \frac{d\lambda^{(n)}(t)}{dt} = \frac{\partial\lambda_{eff}^{(n)}}{\partial t} + n\bar{\gamma}\lambda^{(n)}$$

Anomalous dimension of fields

t-dependence in one-loop calculation of V

<u>*Solve Callan Symanzik eqn for them as usual =>*</u>

For mass-squareds, by dimensions have contributions from cross-terms only …

$$V \supset \frac{m_\phi^2}{2}\phi^2\left(1+\frac{\lambda t}{16\pi^2}\right)$$

Using the solutions along the separatrix:

$$\begin{aligned}\beta_{m_\phi^2} &= m_\phi^2\left(\frac{\lambda}{16\pi^2}+2\gamma\right),\\ \frac{1}{m_\phi^2}\beta_{m_\phi^2} &= 2\alpha_y+\frac{6}{N_F^2}(\alpha_v+\alpha_h)\\ &= f\alpha_g\,,\end{aligned}$$

$$\left(f = \frac{12}{13}\left[1+\frac{3}{4N_F^2}\left(\sqrt{20+6\sqrt{23}}-1-\sqrt{23}\right)\right]\right)$$

i.e. mass-squared **scales with the gauge coupling like all the marginal couplings …**

in the end …

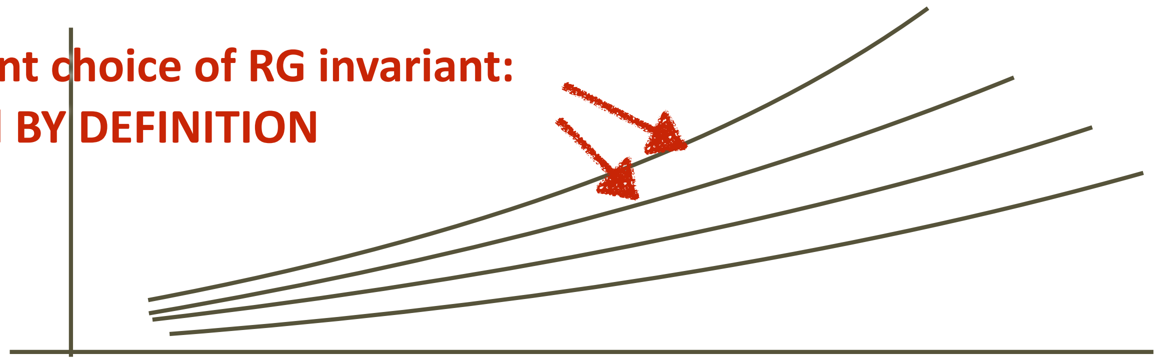
We find ***multiplicative*** renormalisation …

$$m_\phi^2(t) \;=\; m_*^2 \left(\frac{\alpha_g^*}{\alpha_g} - 1\right)^{-\frac{3f}{4\epsilon}} \qquad\qquad \alpha_g^* = \;\; 0.4561\,\epsilon$$

In principle … $m_*^2 = m_\phi^2(0)\left(\alpha_g^*/\alpha_g(0) - 1\right)^{\frac{3f}{4\epsilon}}$ but you should just think of it as an RG invariant that defines this particular trajectory. (Every relevant operator will have an associated invariant.)

It has the same status as the chiral quark masses.

Trajectories all correspond to different choice of RG invariant:
they cannot be determined BY DEFINITION



Radiative symmetry breaking by mass terms

Critique of that example…

- **Purely** multiplicative**:** Hence the mass-squared has to be negative along the whole trajectory
- **We cheated:** in the sense that we ignored all the orthogonal directions!! These also get contributions at one-loop even though their masses were zero at tree-level

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Instead organize everything in terms of the $U(N_F) \times U(N_F)$ **flavour symmetry that we break with the mass-squareds (operators must be closed under RG):**

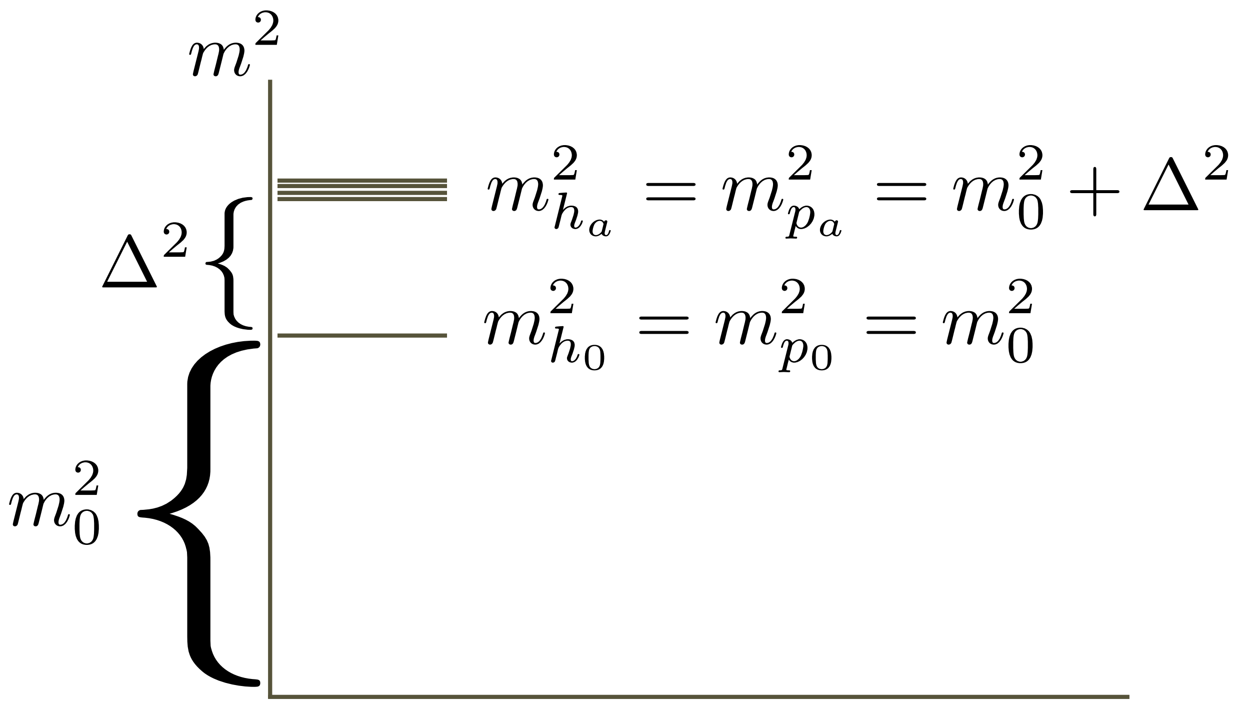
$$H = \frac{(h_0 + ip_0)}{\sqrt{2N_F}} \mathbb{1}_{N_F \times N_F} + (h_a + ip_a) T_a$$

$$\mathcal{L}_{Soft} \ = \ -m_{h_0}^2 \mathrm{Tr}\left[H^\dagger H\right] - \sum_{a=1}^{N_F^2-1} \Delta_a^2 \mathrm{Tr}\left[H T^a\right] \mathrm{Tr}\left[H^\dagger T^a\right]$$

Non-trivial simple example...

Consider the case where the trace component has a slightly smaller mass-squared:

$$V^{(2)}_{class} = m_0^2 \mathrm{Tr}(H^\dagger H) + 2\Delta^2 \sum_a \mathrm{Tr}(T_a H^\dagger)\mathrm{Tr}(T_a H)$$



Non-trivial simple example…

After some work find the following answer in terms of two RG invariants, one for each independent (non-predicted) relevant operator (where $\upsilon = (1 - 1/N_f^2)$):

$$m_0^2 = \tilde{m}_*^2 \left(\frac{\alpha_g^*}{\alpha_g} - 1\right)^{-\frac{3f_{m_0}}{4\epsilon}} - \Delta_*^2 \nu \left(\frac{\alpha_g^*}{\alpha_g} - 1\right)^{-\frac{3f_\Delta}{4\epsilon}},$$

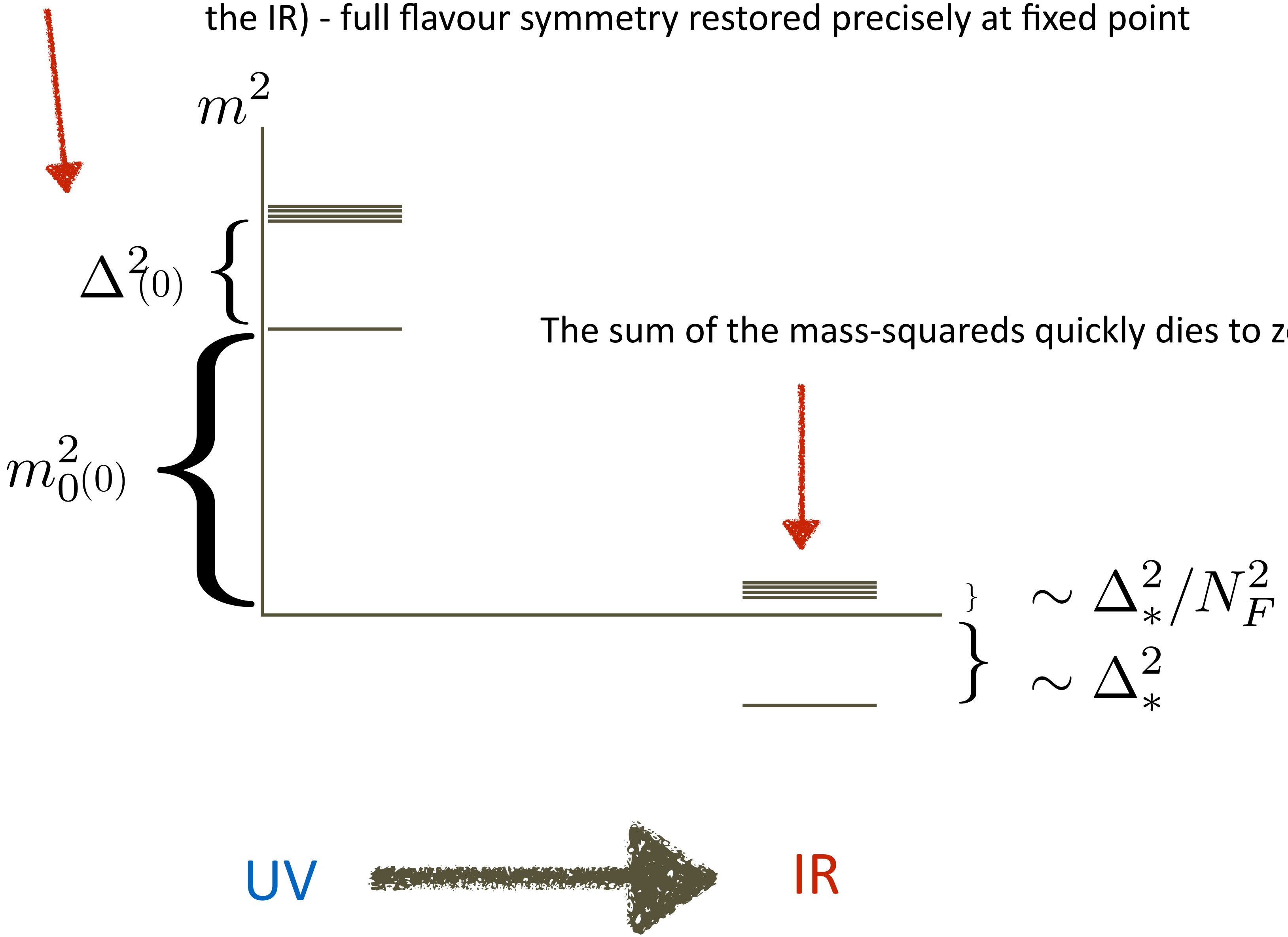
$$m_{a=1\ldots N_F^2-1}^2 = \tilde{m}_*^2 \left(\frac{\alpha_g^*}{\alpha_g} - 1\right)^{-\frac{3f_{m_0}}{4\epsilon}} + \Delta_*^2 (1-\nu) \left(\frac{\alpha_g^*}{\alpha_g} - 1\right)^{-\frac{3f_\Delta}{4\epsilon}}$$

$$f_{m_0} > f_\Delta$$

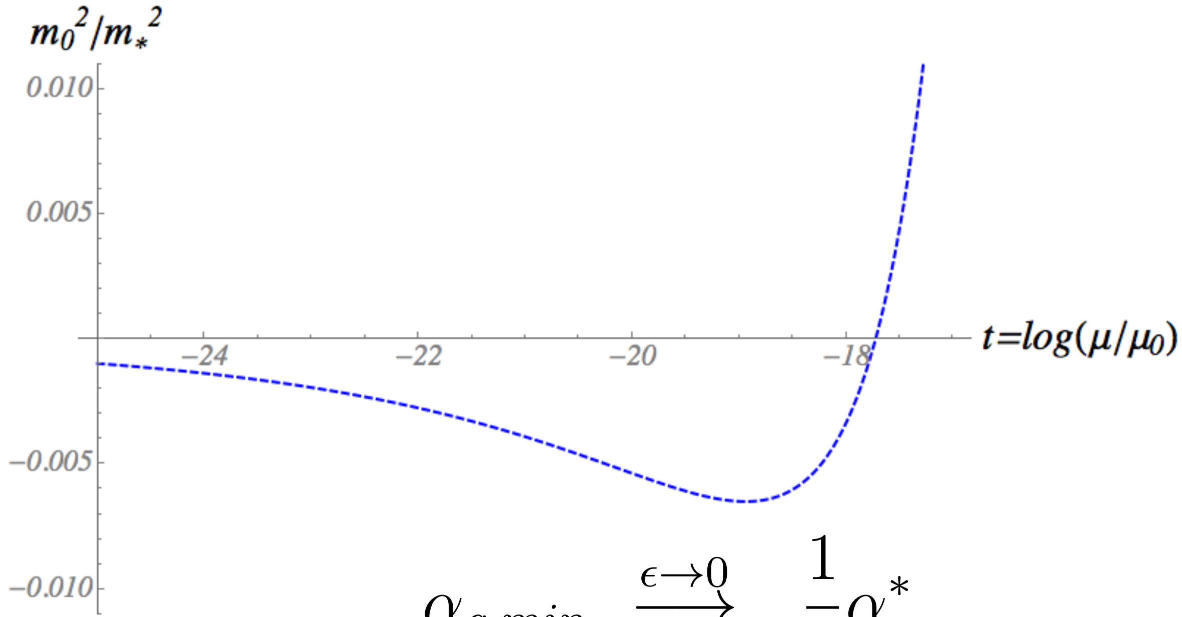
Dies away quickly *in the IR*

Dies away slowly *in the IR*

Starting values get relatively closer in UV (note the masses are all shrinking in absolute terms in the IR) - full flavour symmetry restored precisely at fixed point



Induces radiative breaking…



$$\alpha_{g,min} \xrightarrow{\epsilon\to 0} \frac{1}{2}\alpha_g^*$$

$$m_{0,min}^2 \sim -\tilde{m}_*^2$$

Pause to reflect: *No different from radiative breaking in SUSY (the masses have the same status as the quark masses of QCD). We do not need to protect them from anything — they just are what they are on this trajectory. (However we cannot do any GUT-ting or similar)*

The story with general flavour structure …

This gets complicated because we need to find the beta function for a set of operators that is closed under RG: useful to use a definition in terms of "hierarchical" nested SU(n) flavour factors :

$$H_{ij} = \frac{1}{\sqrt{2}}\left(h_{ij} + ip_{ij}\right) \qquad\qquad h_a + ip_a = \sqrt{2}T^a_{ij}(h_{ij} + ip_{ij})$$

$$SU(N_F) \supset SU(N_F-1)\ldots \supset SU(n)\ldots$$

$$T_{ij}^{(n^2-1)} = \frac{1}{\sqrt{2n(n-1)}}\begin{pmatrix} 1 & & & & & \\ & \ddots & & & & \\ & & 1 & & & \\ & & & 1-n & & \\ & & & & 0 & \\ & & & & & \ddots \end{pmatrix}$$

and define trace over $SU(n)$ block of generators $\qquad \mathrm{Tr}_n(\mathcal{O}_{ij}) = \sum_{i=1}^{n}\mathcal{O}_{ii}$

Then we have the usual dimensionless flavour symmetric Lagrangian (slight renaming) …

$$\mathcal{L}_{\rm UVFP} \supset \mathcal{L}_{\rm KE} + \frac{y}{\sqrt{2}}{\rm Tr}\left[(QH)\cdot\tilde{Q}\right] + \frac{\tilde{y}}{\sqrt{2}}{\rm Tr}\left[qH^\dagger\tilde{q}\right] - u_1{\rm Tr}\left[H^\dagger H\right]^2 - u_2{\rm Tr}\left[H^\dagger H\, H^\dagger H\right]$$

and consider adding all possible flavour breaking in the dimensionful operators …

$$\begin{aligned} V^{(2)} \;=\;& \frac{m_0^2}{2}{\rm Tr}_{N_F}\left(h^2+p^2\right) + \sum_{n=1}^{N_F-1}\frac{m_n^2}{2}\left[\frac{\left({\rm Tr}_n h\right)^2+\left({\rm Tr}_n p\right)^2}{n}\right] \\ & + \sum_{n=2}^{N_F}\frac{\Delta_n^2}{2}\left[{\rm Tr}_n\left(h^2+p^2\right) - \frac{\left({\rm Tr}_n h\right)^2+\left({\rm Tr}_n p\right)^2}{n}\right] \end{aligned}$$

Now we need to figure out the beta functions. This is big mess, but in the end you find …

coupl'g	Operator	Coefficient in $16\pi^2\partial_t V$
m_0^2	$Tr_{N_F}(h^2+p^2)$	$m_0^2\left\{2u_1\left[N_F^2+1\right]+4u_2N_F\right\}+\Delta_{N_F}^2\left(2u_1+\frac{4u_2}{N_F}\right)(N_F^2-1)$ $+\sum_n^{N_F-1}2u_1\left(m_n^2+\Delta_n^2\left(n^2-1\right)\right)$
$\Delta_{N_F}^2$	$\mathrm{Tr}_{N_F}\left(h^2+p^2\right)-\frac{\left(\mathrm{Tr}_{N_F}h\right)^2+\left(\mathrm{Tr}_{N_F}p\right)^2}{N_F}$	$2u_1\Delta_{N_F}^2$
Δ_n^2	$\mathrm{Tr}_n\left(h^2+p^2\right)-\frac{(\mathrm{Tr}_n h)^2+(\mathrm{Tr}_n p)^2}{n}$	$2u_1\Delta_n^2+\frac{4u_2}{n}\left(m_n^2+\Delta_n^2\left(n^2-1\right)\right)$
m_n^2	$\frac{(\mathrm{Tr}_n h)^2+(\mathrm{Tr}_n p)^2}{n}$	$2u_1m_n^2+\frac{4u_2}{n}\left(m_n^2+\Delta_n^2\left(n^2-1\right)\right)$

Then we find

$$V^{(2)} = \frac{m_0^2}{2}\mathrm{Tr}_{N_F}\left(h^2+p^2\right) + \sum_{n=1}^{N_F-1}\frac{m_n^2}{2}\left[\frac{\left(\mathrm{Tr}_n h\right)^2+\left(\mathrm{Tr}_n p\right)^2}{n}\right]$$
$$+ \sum_{n=2}^{N_F}\frac{\Delta_n^2}{2}\left[\mathrm{Tr}_n\left(h^2+p^2\right) - \frac{\left(\mathrm{Tr}_n h\right)^2+\left(\mathrm{Tr}_n p\right)^2}{n}\right]$$

In terms of $\tilde{\Omega}(t) = \left(\frac{\alpha_g^*}{\alpha_g}-1\right)^{-3/4\epsilon}$ which goes to zero in the IR, we have

$$m_0^2 = \left(\frac{\tilde{\Omega}(t)}{\tilde{\Omega}(0)}\right)^f \tilde{m}_*^2 - \frac{1}{N_F^2}\sum_n^{N_F}\frac{\sigma_{n*}^2}{1+2\frac{f_{u_2}}{f_{u_1}}(1-n/N_F)}\tilde{\Omega}^{f_\Delta+f_n}$$
$$\Delta_n^2 = \frac{1}{n^2}\left(\rho_{n*}^2\tilde{\Omega}^{f_\Delta} + \sigma_{n*}^2\tilde{\Omega}^{f_\Delta+f_n}\right),$$
$$m_n^2 = \frac{1}{n^2}\left(\rho_{n*}^2(1-n^2)\tilde{\Omega}^{f_\Delta} + \sigma_{n*}^2\tilde{\Omega}^{f_\Delta+f_n}\right).$$

Generally in IR find flavour hierarchies grow …

$$V \to \sum_{n>1} \Delta_n^2 \left[\mathrm{Tr}_n \left(h^2 + p^2\right) - n \left((\mathrm{Tr}_n h)^2 + (\mathrm{Tr}_n p)^2 \right) \right]$$

These bits all flow to zero faster

$$m_0^2 = \left(\frac{\tilde\Omega(t)}{\tilde\Omega(0)}\right)^f \tilde m_*^2 - \frac{1}{N_F^2} \sum_n^{N_F} \frac{\sigma_{n*}^2}{1 + 2\frac{f_{u_2}}{f_{u_1}}(1 - n/N_F)} \tilde\Omega^{f_\Delta + f_n}$$

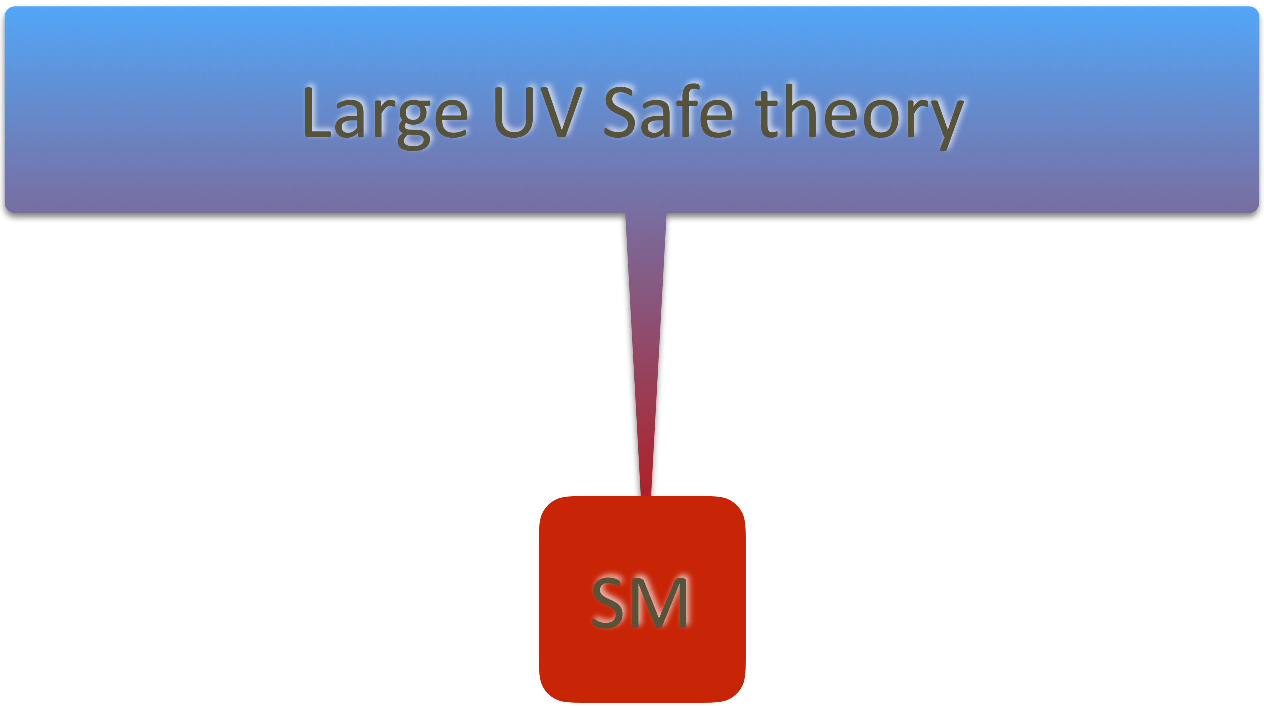
$$\Delta_n^2 = \frac{1}{n^2}\left(\rho_{n*}^2 \tilde\Omega^{f_\Delta} + \sigma_{n*}^2 \tilde\Omega^{f_\Delta + f_n}\right) ,$$

$$m_n^2 = \frac{1}{n^2}\left(\rho_{n*}^2 (1 - n^2) \tilde\Omega^{f_\Delta} + \sigma_{n*}^2 \tilde\Omega^{f_\Delta + f_n}\right) .$$

Also you could consider hierarchies generated by the Ω's themselves

Tetrad Model for the ASSM…

Tetrad Model for the ASSM…



Tetrad Model - focus on breaking SU(Nc) to SU(3) colour with new scalars …

c.f. Gies, Jaeckel, Wetterich '04; Bond, Litim; Bond, Hiller, Kowalska, Litim; Gies, Rechenberger, Scherer, Zambelli; Pelaggi, Plascencia, Salvio, Sannino, Smirnov; Molinaro, Sannino, Wang; Mann, Meffe, Sannino, Steele, Wang, Zhang,

$$SU(2)_R = [SU(2)_r \otimes SU(2)_S]_{\text{diag}}$$

	$SU(N_C)$	$SU(N_F)_L \supset SU(2)_L \otimes SU(n_g)_L$	$SU(N_F)_R \supset SU(2)_r \otimes SU(n_g)_r$	$SU(N_S) = SU(N_C-4)_S \oplus SU(2)_S$	spin
Q_{ai}	$\square$	$\square \supset (\square,\square)$	1	1	$1/2$
$\tilde{Q}^{ia}$	$\tilde{\square}$	1	$\tilde{\square} \supset (\tilde{\square},\tilde{\square})$	1	$1/2$
H^i_j	1	$\tilde{\square} \supset (\tilde{\square},\tilde{\square})$	$\square \supset (\square,\square)$	1	0
$\tilde{S}_{a,\ell=1..N_S}$	$\tilde{\square}$	1	1	$\tilde{\square} = \tilde{\square}_{N_C-4} \oplus \tilde{\square}_2$	0
$\tilde{q}^i_\ell$	1	$\tilde{\square} \supset (\tilde{\square},\tilde{\square})$	1	$\square = \square_{N_C-4} \oplus \square_2$	$1/2$
q^ℓ_j	1	1	$\square \supset (\square,\square)$	$\tilde{\square} = \tilde{\square}_{N_C-4} \oplus \tilde{\square}_2$	$1/2$

Extension of Pati-Salam - breaks to SU(3) if we choose $N_S = N_C - 2$

$$\frac{N_S}{N_C} \to 1\,; \quad \frac{N_F}{N_C} \to \frac{21}{4} + \epsilon$$

$$\tilde{S} = \left(\begin{array}{ccccc} \multicolumn{5}{c}{\overbrace{\hphantom{\begin{pmatrix}\tilde{d}^c\\ \tilde{u}^c\end{pmatrix} \quad \begin{pmatrix}\tilde{e}^c\\ \tilde{\nu}^c\end{pmatrix} \quad \begin{pmatrix}\tilde{\phi}_{-\frac{1}{2}}\\ \tilde{\phi}_{\frac{1}{2}}\end{pmatrix} \quad \dots \quad \begin{pmatrix}\tilde{\phi}_{-\frac{1}{2}}\\ \tilde{\phi}_{\frac{1}{2}}\end{pmatrix}}}^{N_C}} \\ \begin{pmatrix}\tilde{d}^c\\ \tilde{u}^c\end{pmatrix} & \begin{pmatrix}\tilde{e}^c\\ \tilde{\nu}^c\end{pmatrix} & \begin{pmatrix}\tilde{\phi}_{-\frac{1}{2}}\\ \tilde{\phi}_{\frac{1}{2}}\end{pmatrix} & \dots & \begin{pmatrix}\tilde{\phi}_{-\frac{1}{2}}\\ \tilde{\phi}_{\frac{1}{2}}\end{pmatrix} \\ \tilde{T}_{-\frac{1}{6}} & \tilde{\phi}_{\frac{1}{2}} & \tilde{\phi}_0 & \dots & \tilde{\phi}_0 \\ \vdots & \vdots & \vdots & & \vdots \\ \tilde{T}_{-\frac{1}{6}} & \tilde{\phi}_{\frac{1}{2}} & \tilde{\phi}_0 & \dots & \tilde{\phi}_0 \end{array}\right) \left.\vphantom{\begin{array}{c}1\\1\\1\\1\\1\\1\end{array}}\right\} N_S = N_C - 2$$

- Weak breaking must then occur along the H-Higgs directions:

$$H = \begin{pmatrix} \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{11} & \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{12} & \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{13} & \cdots \\ \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{21} & \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{22} & \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{23} & \cdots \\ \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{31} & \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{32} & \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{33} & \cdots \\ \vdots & \vdots & \vdots & H_0 \end{pmatrix}$$

- Assignment implies 9 pairs of Higgses one for each Yukawa coupling

- Explicit embedding looks like P-S with $SU(N_C) \times SU(2)_L \times SU(2)_R \longrightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$

$$
Q = \left.\begin{pmatrix} \overbrace{\begin{matrix}
q_1 & \ell_1 & \cdots \begin{pmatrix} \Psi_{\frac{1}{2}} \\ \Psi_{-\frac{1}{2}} \end{pmatrix} \cdots \\
q_2 & \ell_2 & \cdots \begin{pmatrix} \Psi_{\frac{1}{2}} \\ \Psi_{-\frac{1}{2}} \end{pmatrix} \cdots \\
q_3 & \ell_3 & \cdots \begin{pmatrix} \Psi_{\frac{1}{2}} \\ \Psi_{-\frac{1}{2}} \end{pmatrix} \cdots \\
\vdots & \vdots & \ddots
\end{matrix}}^{N_C} \end{pmatrix}\right\} N_F \; ; \quad
\tilde{Q} = \begin{pmatrix}
\begin{pmatrix} u^c \\ d^c \end{pmatrix} & \begin{pmatrix} \nu_e^c \\ e^c \end{pmatrix} & \cdots \begin{pmatrix} \tilde{\Psi}_{-\frac{1}{2}} \\ \tilde{\Psi}_{\frac{1}{2}} \end{pmatrix} \cdots \\
\begin{pmatrix} s^c \\ c^c \end{pmatrix} & \begin{pmatrix} \nu_\mu^c \\ \mu^c \end{pmatrix} & \cdots \begin{pmatrix} \tilde{\Psi}_{-\frac{1}{2}} \\ \tilde{\Psi}_{\frac{1}{2}} \end{pmatrix} \cdots \\
\begin{pmatrix} b^c \\ t^c \end{pmatrix} & \begin{pmatrix} \nu_\tau^c \\ \tau^c \end{pmatrix} & \cdots \begin{pmatrix} \tilde{\Psi}_{-\frac{1}{2}} \\ \tilde{\Psi}_{\frac{1}{2}} \end{pmatrix} \cdots \\
\vdots & \vdots & \ddots
\end{pmatrix}
$$

$$
q = \begin{pmatrix} \overbrace{\begin{matrix}
\begin{pmatrix} \psi_0 & \psi_1 \\ \psi_{-1} & \psi_0 \end{pmatrix} \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} \\
\begin{pmatrix} \psi_0 & \psi_1 \\ \psi_{-1} & \psi_0 \end{pmatrix} \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} \\
\begin{pmatrix} \psi_0 & \psi_1 \\ \psi_{-1} & \psi_0 \end{pmatrix} \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} \\
\vdots & \vdots & \vdots
\end{matrix}}^{N_S = N_C - 2} \end{pmatrix} ; \; \tilde{q} = \begin{pmatrix}
\begin{pmatrix} \tilde{\psi}_0 & \tilde{\psi}_{-1} \\ \tilde{\psi}_1 & \tilde{\psi}_0 \end{pmatrix} \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} \\
\begin{pmatrix} \tilde{\psi}_0 & \tilde{\psi}_{-1} \\ \tilde{\psi}_1 & \tilde{\psi}_0 \end{pmatrix} \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} \\
\begin{pmatrix} \tilde{\psi}_0 & \tilde{\psi}_{-1} \\ \tilde{\psi}_1 & \tilde{\psi}_0 \end{pmatrix} \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} \\
\vdots & \vdots & \vdots
\end{pmatrix}
$$

- Explicit embedding looks like P-S with $SU(N_C) \times SU(2)_L \times SU(2)_R \longrightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$

$$Q \;=\; \left.\overbrace{\begin{pmatrix} q_1 & \ell_1 & \cdots \begin{pmatrix}\Psi_{\frac{1}{2}} \\ \Psi_{-\frac{1}{2}}\end{pmatrix} \cdots \\ q_2 & \ell_2 & \cdots \begin{pmatrix}\Psi_{\frac{1}{2}} \\ \Psi_{-\frac{1}{2}}\end{pmatrix} \cdots \\ q_3 & \ell_3 & \cdots \begin{pmatrix}\Psi_{\frac{1}{2}} \\ \Psi_{-\frac{1}{2}}\end{pmatrix} \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}}^{N_C}\right\} N_F \;\; ; \;\; \tilde{Q} = \begin{pmatrix} \begin{pmatrix} u^c \\ d^c \end{pmatrix} & \begin{pmatrix} \nu_e^c \\ e^c \end{pmatrix} & \cdots \begin{pmatrix} \tilde{\Psi}_{-\frac{1}{2}} \\ \tilde{\Psi}_{\frac{1}{2}} \end{pmatrix} \cdots \\ \begin{pmatrix} s^c \\ c^c \end{pmatrix} & \begin{pmatrix} \nu_\mu^c \\ \mu^c \end{pmatrix} & \cdots \begin{pmatrix} \tilde{\Psi}_{-\frac{1}{2}} \\ \tilde{\Psi}_{\frac{1}{2}} \end{pmatrix} \cdots \\ \begin{pmatrix} b^c \\ t^c \end{pmatrix} & \begin{pmatrix} \nu_\tau^c \\ \tau^c \end{pmatrix} & \cdots \begin{pmatrix} \tilde{\Psi}_{-\frac{1}{2}} \\ \tilde{\Psi}_{\frac{1}{2}} \end{pmatrix} \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

$$q \;=\; \overbrace{\begin{pmatrix} \begin{pmatrix} \psi_0 & \psi_1 \\ \psi_{-1} & \psi_0 \end{pmatrix} & \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} \\ \begin{pmatrix} \psi_0 & \psi_1 \\ \psi_{-1} & \psi_0 \end{pmatrix} & \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} \\ \begin{pmatrix} \psi_0 & \psi_1 \\ \psi_{-1} & \psi_0 \end{pmatrix} & \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} \\ \vdots & & \vdots & \vdots \end{pmatrix}}^{N_S = N_C - 2} \; ; \; \tilde{q} \;=\; \begin{pmatrix} \begin{pmatrix} \tilde{\psi}_0 & \tilde{\psi}_{-1} \\ \tilde{\psi}_1 & \tilde{\psi}_0 \end{pmatrix} & \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} \\ \begin{pmatrix} \tilde{\psi}_0 & \tilde{\psi}_{-1} \\ \tilde{\psi}_1 & \tilde{\psi}_0 \end{pmatrix} & \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} \\ \begin{pmatrix} \tilde{\psi}_0 & \tilde{\psi}_{-1} \\ \tilde{\psi}_1 & \tilde{\psi}_0 \end{pmatrix} & \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} \\ \vdots & & \vdots & \vdots \end{pmatrix}$$

- Little q's required (by chiral symmetry) to remove the extra SU(2) doublets: (Nc-4) uncharged under SU(2)R

And the couplings that do this are as follows:

Standard Yukawas

masses remove 2 q's

removes excess quark colours:
S locks colour/flavour

$$\begin{aligned}\mathcal{L}_{\mathrm{UVFP}} \supset \mathcal{L}_{\mathrm{KE}} &+ \frac{y}{\sqrt{2}}\mathrm{Tr}\left[(QH)\cdot\tilde{Q}\right] + \frac{\tilde{y}}{\sqrt{2}}\mathrm{Tr}\left[qH^\dagger\tilde{q}\right] - \frac{\tilde{Y}}{\sqrt{2}}\mathrm{Tr}[\left(\tilde{S}\cdot Q\right)\tilde{q}] - \frac{Y}{\sqrt{2}}\mathrm{Tr}[\left(\tilde{Q}\cdot\tilde{S}^\dagger\right)q] \\ &- u_1\mathrm{Tr}\left[H^\dagger H\right]^2 - u_2\mathrm{Tr}\left[H^\dagger H\, H^\dagger H\right] - v_1\mathrm{Tr}\left[H^\dagger H\right]\mathrm{Tr}\left[\tilde{S}^\dagger\cdot\tilde{S}\right] \\ &- w_1\mathrm{Tr}\left[\tilde{S}^\dagger\cdot\tilde{S}\right]^2 - w_2\mathrm{Tr}\left[\tilde{S}^\dagger\cdot\tilde{S}\,\tilde{S}^\dagger\cdot\tilde{S}\right] \ ,\end{aligned}$$

Note expect relatively light (TeV scale) q-states looking like “higgsinos”

And the couplings that do this are as follows:

Standard Yukawas

masses remove 2 q's

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$$\mathcal{L}_{\rm UVFP} \supset \mathcal{L}_{\rm KE} + \frac{y}{\sqrt{2}} \mathrm{Tr}\left[(QH)\cdot\tilde{Q}\right] + \frac{\tilde{y}}{\sqrt{2}} \mathrm{Tr}\left[qH^\dagger\tilde{q}\right] - \frac{\tilde{Y}}{\sqrt{2}} \mathrm{Tr}[\left(\tilde{S}\cdot Q\right)\tilde{q}] - \frac{Y}{\sqrt{2}} \mathrm{Tr}[\left(\tilde{Q}\cdot\tilde{S}^\dagger\right) q]$$
$$- u_1 \mathrm{Tr}\left[H^\dagger H\right]^2 - u_2 \mathrm{Tr}\left[H^\dagger H\, H^\dagger H\right] - v_1 \mathrm{Tr}\left[H^\dagger H\right] \mathrm{Tr}\left[\tilde{S}^\dagger\cdot\tilde{S}\right]$$
$$- w_1 \mathrm{Tr}\left[\tilde{S}^\dagger\cdot\tilde{S}\right]^2 - w_2 \mathrm{Tr}\left[\tilde{S}^\dagger\cdot\tilde{S}\,\tilde{S}^\dagger\cdot\tilde{S}\right] ,$$

For later use define rescaled c'pgs:

$$\alpha_g = \frac{N_C g^2}{(4\pi)^2};\ \alpha_y = \frac{N_C y^2}{(4\pi)^2};\ \alpha_{\tilde{y}} = \frac{N_C \tilde{y}^2}{(4\pi)^2};\ \alpha_Y = \frac{N_C Y^2}{(4\pi)^2};\ \alpha_{\tilde{Y}} = \frac{N_C \tilde{Y}^2}{(4\pi)^2};$$
$$\alpha_{u_1} = \frac{N_F^2 u_1}{(4\pi)^2};\ \alpha_{u_2} = \frac{N_F u_2}{(4\pi)^2};\ \alpha_{v_1} = \frac{N_C^2 v_1}{(4\pi)^2};\ \alpha_{w_1} = \frac{N_C^2 w_1}{(4\pi)^2};\ \alpha_{w_2} = \frac{N_C w_2}{(4\pi)^2}$$

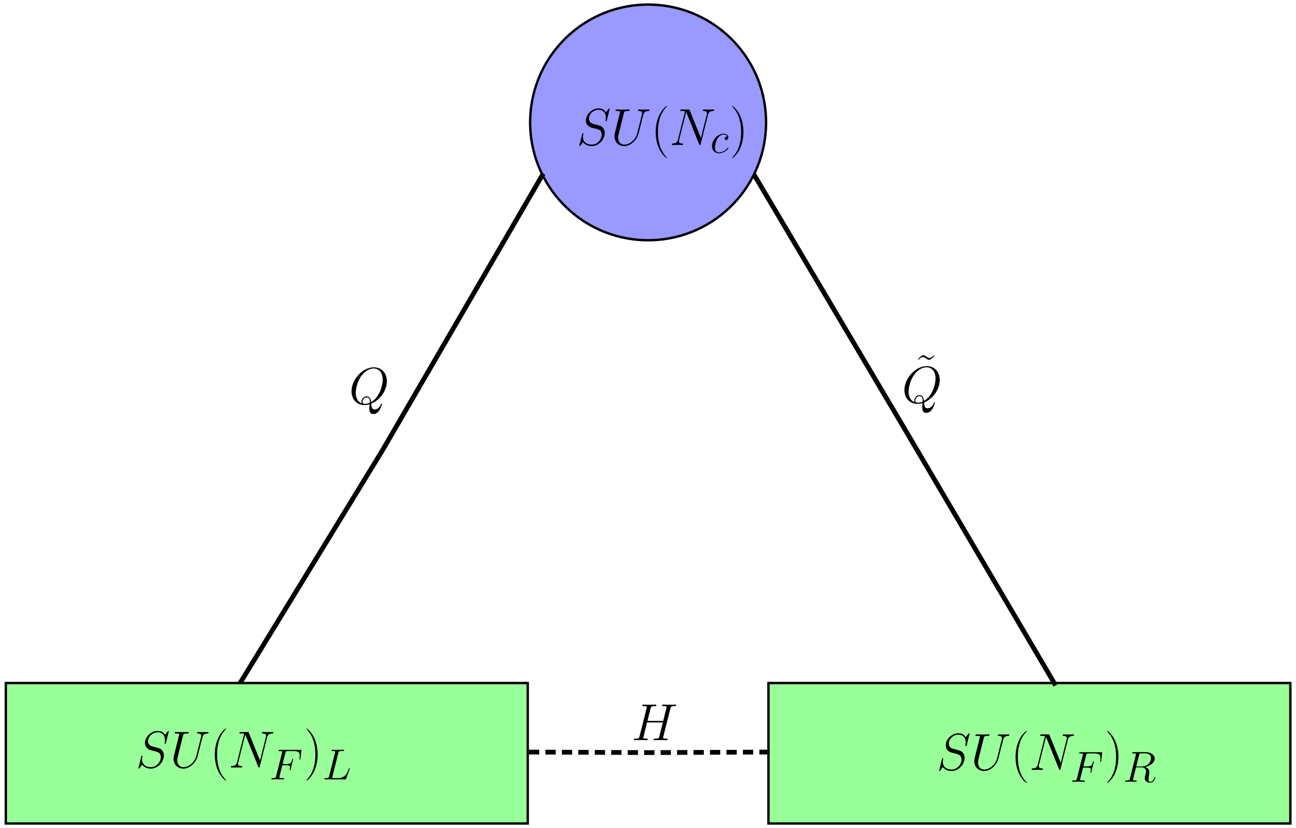
In case you're suffering from "expectation versus reality syndrome" …

Expectations VS Reality -

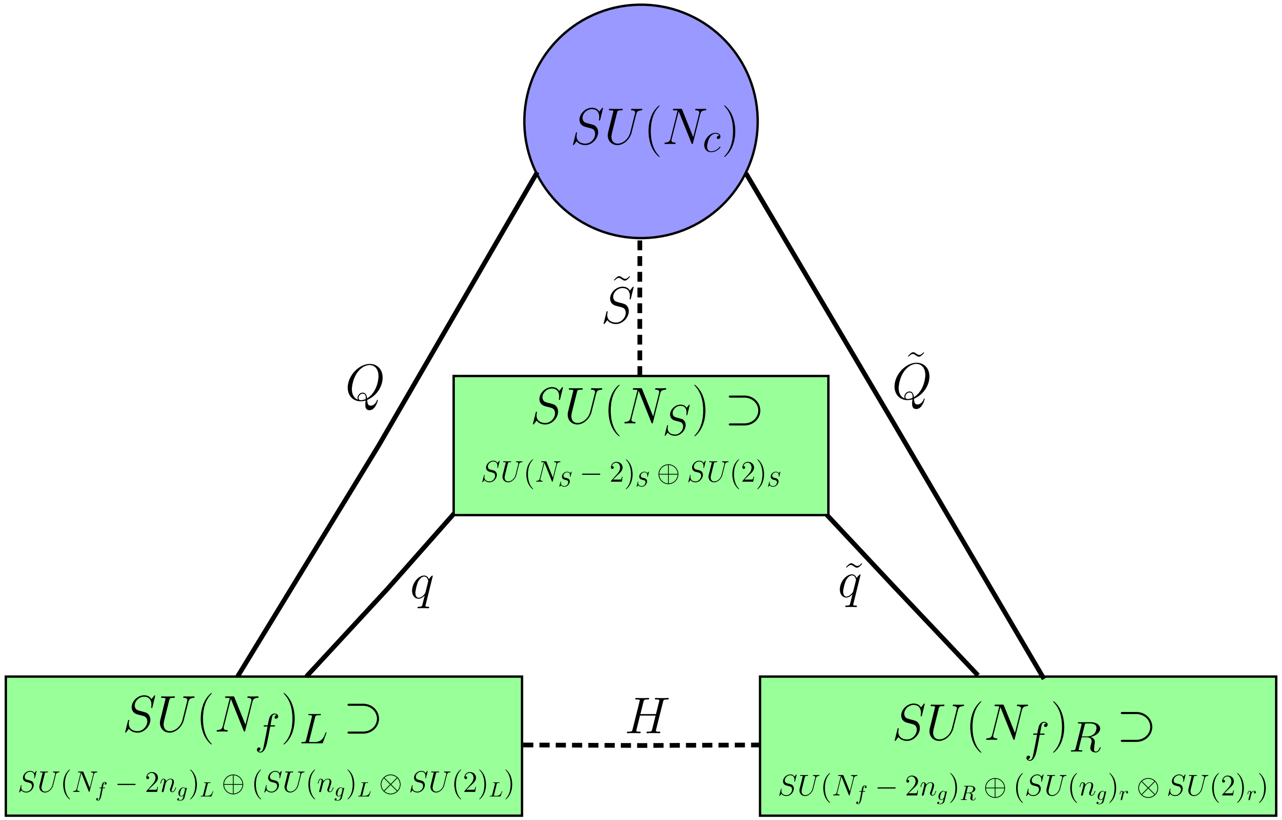
In case you're suffering from "expectation versus reality syndrome" …

A quiver diagram is useful to see (at least some of the structure of) what we did:

Before:



After: (hence the name Tetrad)



- **As this model is based on LS, the same UVFP applies (see later). But what about AS for the SU(2)xSU(2) electroweak gauge groups?**

  These see a large number of flavours (Nf (small f) of order order Nc)?

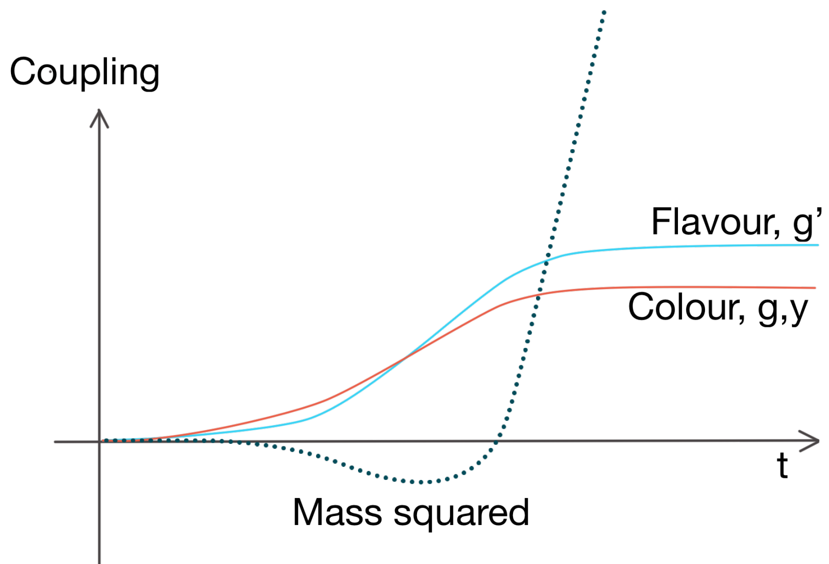
- This gives UVFP behaviour with a fixed point at ’t Hooft couple ~ 1 … if Nf >>16:

Palanques Mestre, Pascual; Gracey; Holdom; Shrock; Antipin, Pica, Sannino

Resum first terms gives

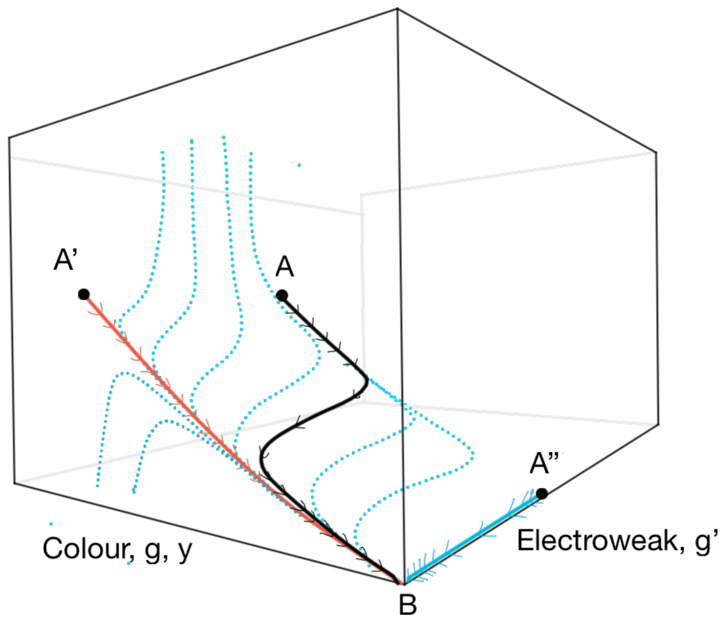
$$\frac{3}{4}\frac{\beta_{\tilde{\alpha}}}{\tilde{\alpha}^2} = 1 + \frac{H(\tilde{\alpha})}{N_f} + \mathcal{O}(N_f^{-2})$$

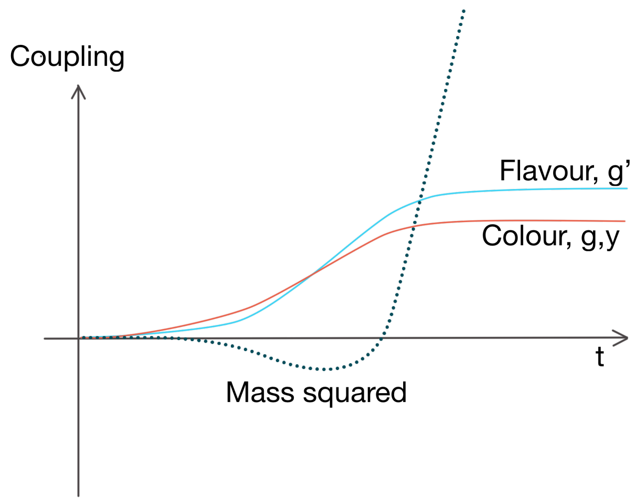
$$H(\tilde{\alpha}) \;=\; \frac{1}{4}\log|3-2\tilde{\alpha}| + \text{constant}$$



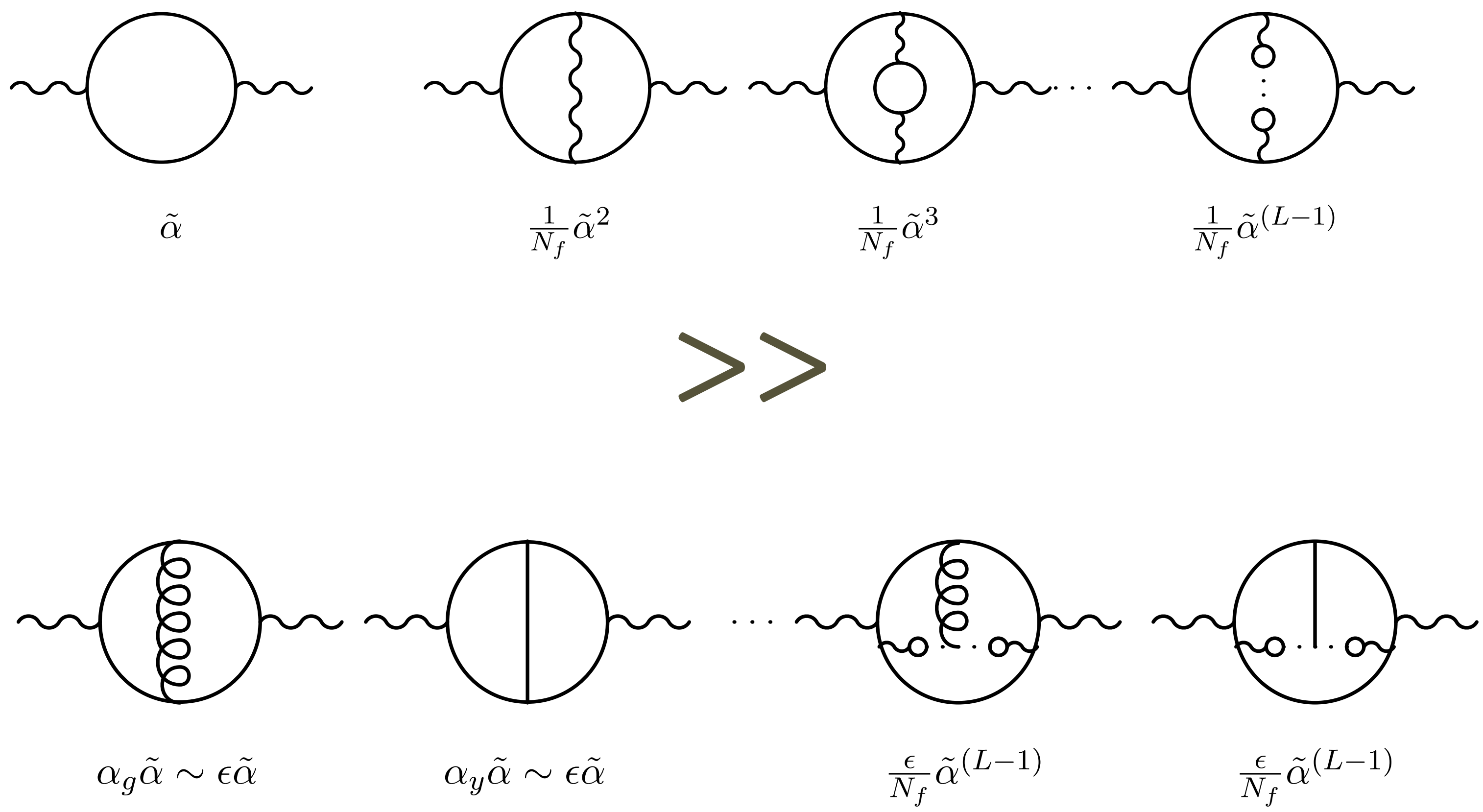
$$\tilde{\alpha}_* \;=\; \tfrac{3}{2} - Ce^{-4N_f}$$

- **Interpretation: the flow is on a hypersurface in g,y,g' that is independent of g'   (more later)**

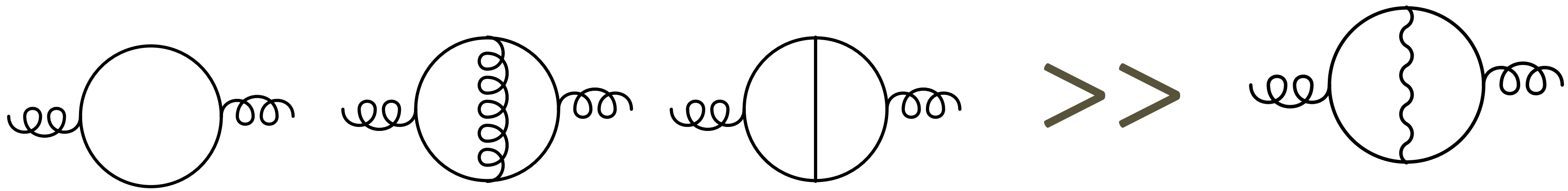




- Can show by power counting that the two kinds of UVFP decouple.
- In the Veneziano limit the corrections to the weak FP go like epsilon. Can neglect everything but SU(2) gauge couplings when determining the SU(2) fixed points…



- Conversely for the SU(Nc) fixed point …



$x_F\alpha_g$ $\qquad x_F\,\alpha_g^2 \sim \epsilon^2$ $\qquad x_F\,\alpha_y\alpha_g \sim \epsilon^2$ $\qquad \frac{1}{N_f N}\tilde{\alpha}\alpha_g \;\sim\; \frac{\epsilon}{N_f^2}\tilde{\alpha} \;\lesssim\; \epsilon^3\,\frac{\tilde{\alpha}}{25}$

C.W. Radiative symmetry breaking is automatic!

- Suppose that the classically relevant operators are negligible. (compared to the scales we are about to generate.)
- Then Coleman-Weinberg radiative symmetry breaking is induced along the flow.
- First look at Yukawas which run without caring about quartics:

$$\alpha_g = \frac{N_C g^2}{(4\pi)^2}\,;\ \alpha_y = \frac{N_C y^2}{(4\pi)^2};\ \alpha_{\tilde{y}} = \frac{N_C \tilde{y}^2}{(4\pi)^2};\ \alpha_Y = \frac{N_C Y^2}{(4\pi)^2};\ \alpha_{\tilde{Y}} = \frac{N_C \tilde{Y}^2}{(4\pi)^2}\,;$$

$$\alpha_{u_1} = \frac{N_F^2 u_1}{(4\pi)^2}\,;\ \alpha_{u_2} = \frac{N_F u_2}{(4\pi)^2}\,;\ \alpha_{v_1} = \frac{N_C^2 v_1}{(4\pi)^2}\,;\ \alpha_{w_1} = \frac{N_C^2 w_1}{(4\pi)^2}\,;\ \alpha_{w_2} = \frac{N_C w_2}{(4\pi)^2}\,.$$

- Suppose that the classically relevant operators are negligible. (compared to the scales we are about to generate.)
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$$\begin{aligned}
\beta_g &= \alpha_g^2 \left(\frac{4}{3}\epsilon + (\frac{26}{3}x_F - 20)\alpha_g - x_F^2 \alpha_y - x_F \alpha_Y - x_F \alpha_{\tilde{Y}} \right) , \\
\beta_y &= 4\Upsilon + \alpha_y \left((1 + x_F)\alpha_y + \alpha_{\tilde{y}} + \alpha_{\tilde{Y}} + \alpha_Y - 6\alpha_g \right) , \\
\beta_{\tilde{y}} &= 4\Upsilon + \alpha_{\tilde{y}} \left((1 + x_F)\alpha_{\tilde{y}} + \alpha_y + \alpha_{\tilde{Y}} + \alpha_Y \right) , \\
\beta_Y &= 2x_F\Upsilon + \alpha_Y \left(2(1 + x_F)\alpha_Y + x_F(\frac{1}{2}\alpha_y + \frac{1}{2}\alpha_{\tilde{y}} + 2\alpha_{\tilde{Y}}) - 3\alpha_g \right) , \\
\beta_{\tilde{Y}} &= 2x_F\Upsilon + \alpha_{\tilde{Y}} \left(2(1 + x_F)\alpha_{\tilde{Y}} + x_F(\frac{1}{2}\alpha_y + \frac{1}{2}\alpha_{\tilde{y}} + 2\alpha_Y) - 3\alpha_g \right) .
\end{aligned}$$

$$\Upsilon = \sqrt{\alpha_y \alpha_{\tilde{y}} \alpha_Y \alpha_{\tilde{Y}}}$$

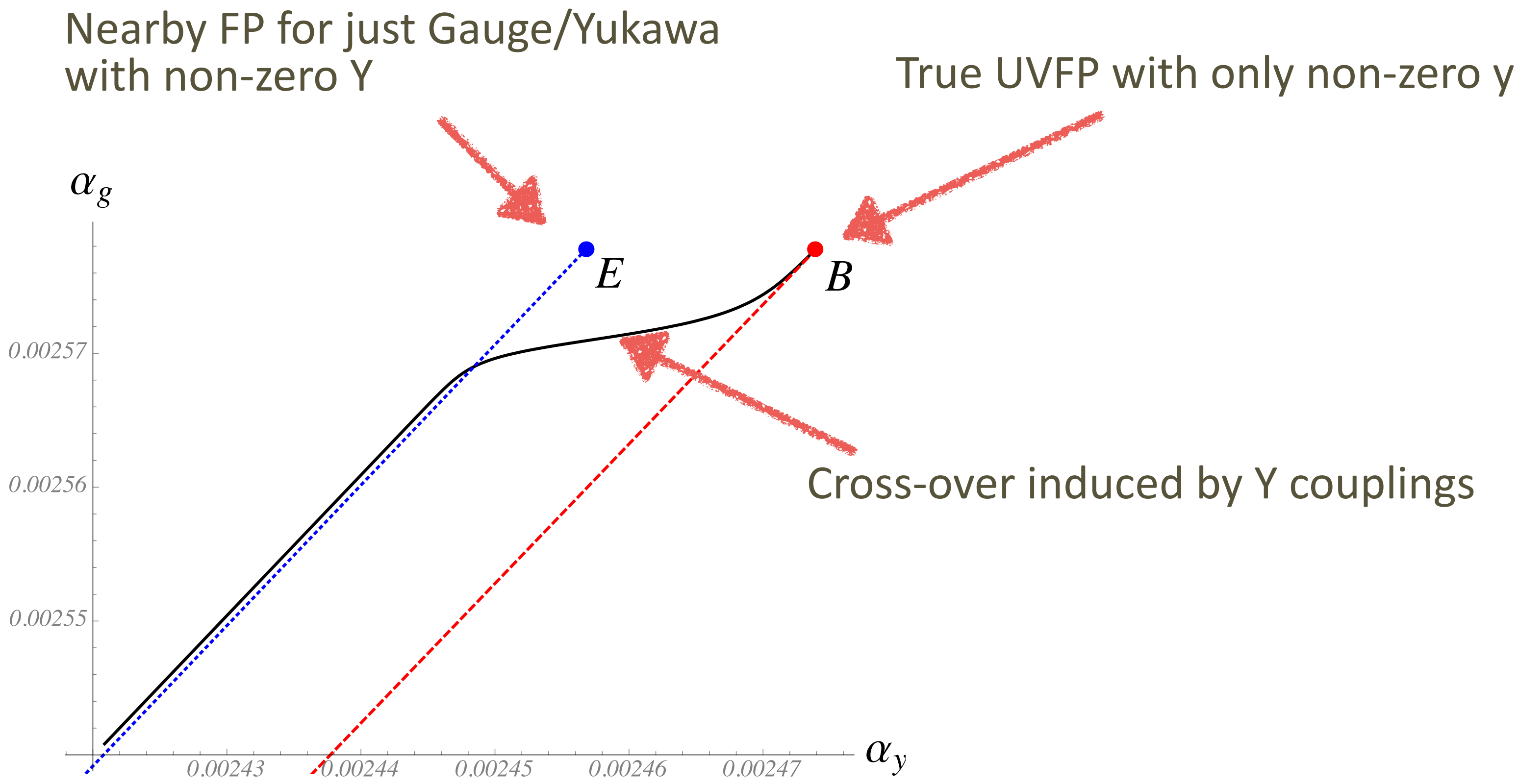
- Solve to find the following set of fixed points …

Label	α_g^*	$\alpha_{\tilde{y}}/\alpha_g$	α_y/α_g	α_Y/α_g	$\alpha_{\tilde{Y}}/\alpha_g$
A	0	0	0	0	0
B	$\frac{25}{18}\epsilon$	0	$\frac{6}{1+x_F} \to \frac{24}{25}$	0	0
C	$\frac{302}{225}\epsilon$	0	$\frac{6(3+4x_F)}{4+7x_F+4x_F^2} \to \frac{144}{151}$	$\frac{6}{4+7x_F+4x_F^2} \to \frac{6}{151}$	0
D	$\frac{302}{225}\epsilon$	0	$\frac{6(3+4x_F)}{4+7x_F+4x_F^2} \to \frac{144}{151}$	0	$\frac{6}{4+7x_F+4x_F^2} \to \frac{6}{151}$
E	$\frac{277}{207}\epsilon$	0	$\frac{6(1+4x_F)}{2+5x_F+4x_F^2} \to \frac{264}{277}$	$\frac{3}{2+5x_F+4x_F^2} \to \frac{6}{277}$	$\frac{3}{2+5x_F+4x_F^2} \to \frac{6}{277}$

$$A \to B \to C,D \to E$$

- A is the Gaussian fixed point — i.e. usual quartic theory
- B is the LS fixed point trajectory (so we know it leads to a true UVFP when we add quartics)
- C,D are unstable trajectories in both UV and IR directions
- E is an IR “fixed-trajectory” (sometimes called quasi-fixed point) in the absence of quartics

- The flow to the E-trajectory is induced by the Y couplings:



- This in turn induces a flow in the quartic couplings driving them negative! we essentially have Gildener-Weinberg breaking of the extended PS symmetry.

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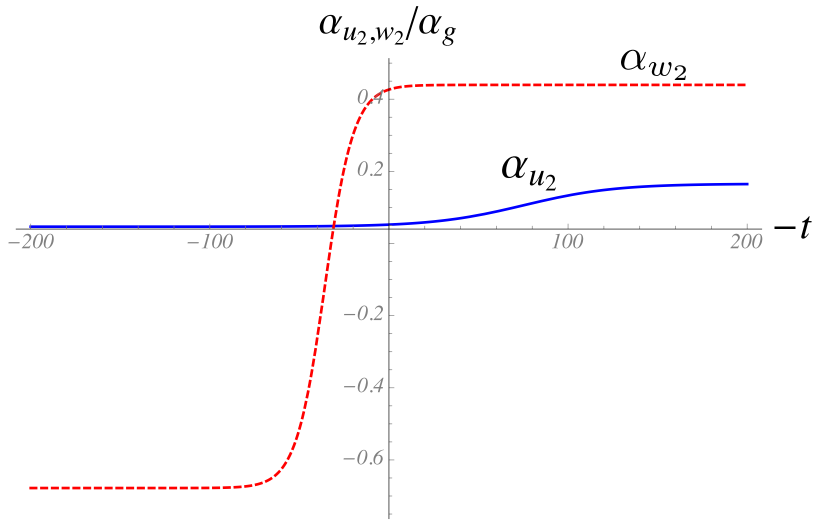

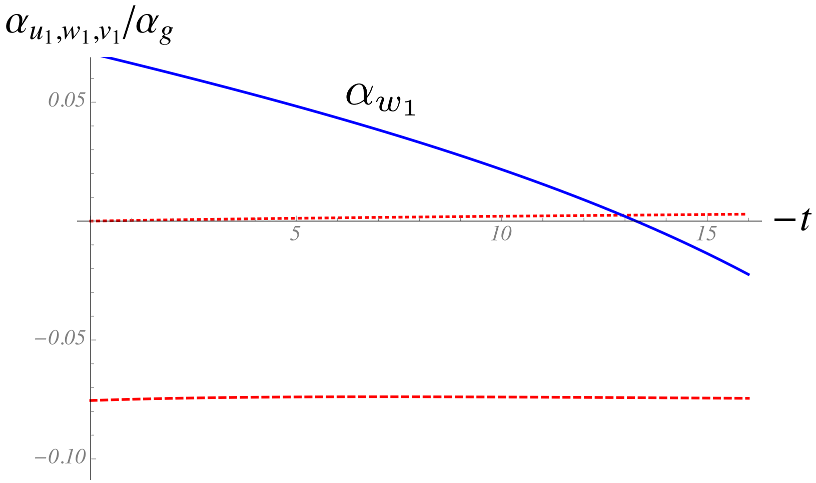

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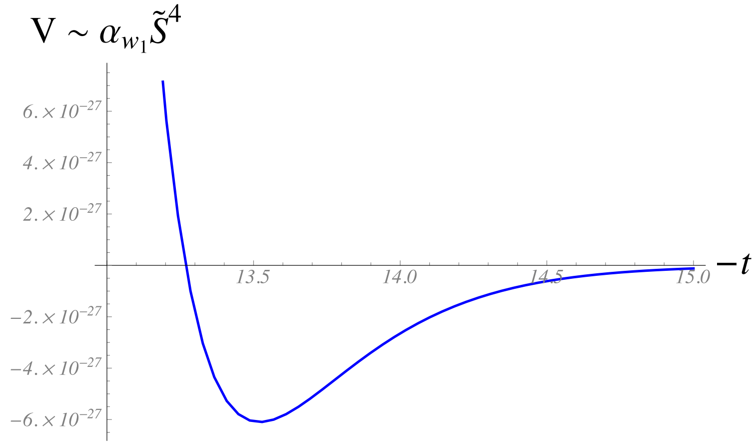

Thoughts on embedding in string theory

Normally try to think about such UV fixed point behaviour within field theory: but is string theory already asymptotically free?

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B) **Yes! (Wetterich)** String theory has only one dimensionful parameter (which goes into defining the units by which we measure energy). A second energy scale is needed to observe scale violation. This could be the Planck scale, or the dynamical scale of some field theory. But well above the physics at which this second scale is generated, the theory should return to scale invariance(a.k.a. a UV fixed point for operators)

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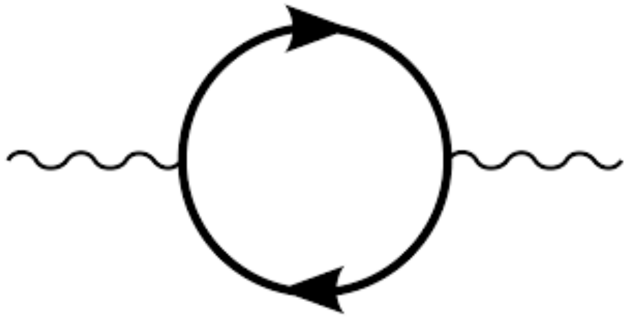
It would be interesting to know if it is B) and if so how string theory does it.

- ***A meaningful RG procedure with a messy UV: attempt 1)***



IR cut-off

$$\frac{16\pi^2}{g^2}\mathcal{A}^{(2)}_{\rm gauge}(s) = -\frac{22C_A}{3}(p_\mu p_\nu - p^2 g_{\mu\nu})\left(\frac{1}{\epsilon} - \gamma_E + \log 4\pi + \log\left(-\frac{\mu^2}{s}\right)\right) \ ,$$

$$\frac{16\pi^2}{g^2}\mathcal{A}^{(2)}_{\rm ferm}(s) = \frac{4N_f}{3}(p_\mu p_\nu - p^2 g_{\mu\nu})\left(\frac{1}{\epsilon} - \gamma_E + \log 4\pi + \log\frac{\mu^2}{m_f^2} + \left(1 + \frac{2m_s^2}{s}\right)\Lambda(s; m_f, m_f)\right) \ ,$$

$$\frac{16\pi^2}{g^2}\mathcal{A}^{(2)}_{\rm scalar}(s) = \frac{2N_s}{3}(p_\mu p_\nu - p^2 g_{\mu\nu})\left(\frac{1}{\epsilon} - \gamma_E + \log 4\pi + \log\frac{\mu^2}{m_s^2} + \left(1 - \frac{4m_s^2}{s}\right)\Lambda(s; m_s, m_s)\right) \ ,$$

Interested in s dependence at a particular mu. Normally count UV divergences

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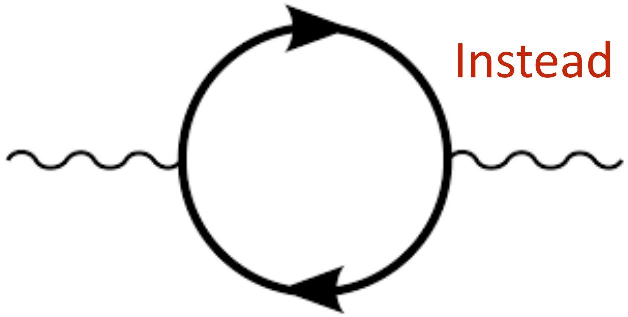

Instead count branch cuts as a function of s

$$\frac{16\pi^2}{g^2}\mathcal{A}^{(2)}_{\rm gauge}(s) = -\frac{22C_A}{3}(p_\mu p_\nu - p^2 g_{\mu\nu})\left(\frac{1}{\epsilon} - \gamma_E + \log 4\pi + \log\left(-\frac{\mu^2}{s}\right)\right) \ ,$$

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The most physical picture: Total s branch cuts just tell us how many states above threshold (s > 4m^2) (but hard to get without doing the actual integral)

$$\beta_{\frac{16\pi^2}{g^2}}(s) = -\frac{1}{\pi}\left[\frac{16\pi^2}{g^2}{\rm Im}\tilde{\mathcal{A}}^{(2)}(s)\right]$$

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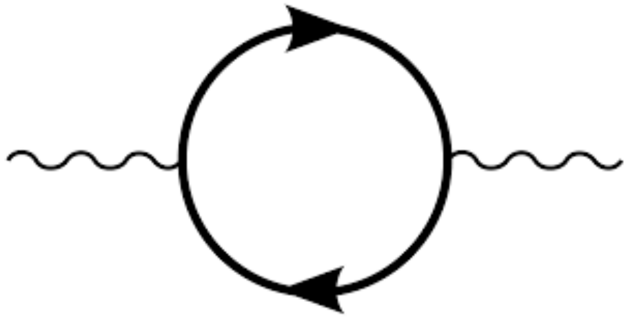

$$\frac{16\pi^2}{g^2}\mathcal{A}^{(2)}_{\text{gauge}}(s) = -\frac{22C_A}{3}(p_\mu p_\nu - p^2 g_{\mu\nu})\left(\frac{1}{\epsilon} - \gamma_E + \log 4\pi + \log\left(-\frac{\mu^2}{s}\right)\right) \,,$$
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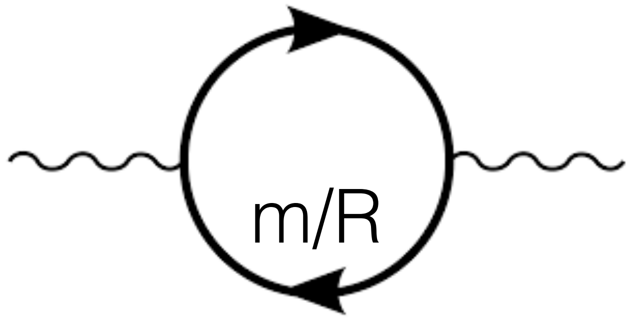
Or impose IR cut-off on Schwinger integral: equivalent to deep Euclidean s, and then..

$$\beta_{\frac{16\pi^2}{g^2}}(s) = \mathrm{Re}\frac{\partial\left(\frac{16\pi^2}{g^2}\tilde{A}^{(2)}\right)}{\partial \log s}$$

- ***Toy example: KK theory***



$$\beta_{\frac{8\pi^2}{g^2}}(s) = \beta^{(\text{non-KK})}_{\frac{8\pi^2}{g^2}} + \text{Im}\sum_{\vec{m}}\int_0^\infty\int_0^1 d\tau dx \tau^{-1}\,\Delta b \quad \exp\left(\tau(s\,x(1-x) - \frac{\vec{m}\cdot\vec{m}}{R^2}\right)$$

$$= \beta^{(\text{non-KK})}_{\frac{8\pi^2}{g^2}} + \text{Im}\int_0^\infty\int_0^1 d\tau dx \frac{1}{\tau^{1+\frac{d}{2}}}\,\Delta b\; \sum_{\vec{\ell}} R^d \pi^{d/2} \exp\left(\tau(s\,x(1-x) - \frac{\vec{\ell}\cdot\vec{\ell}}{\tau}\pi^2 R^2\right)$$

Poisson resum then to get the branch cut expand the exponential until you get the pole —> log —> power law running beta function:

$$\beta_{\frac{8\pi^2}{g^2}}(s) = \beta^{(\text{non-KK})}_{\frac{8\pi^2}{g^2}} + \frac{\Delta b}{\Gamma(3+d/2)}\frac{\pi^{(d+3)/2}}{2^{d+1}}\left(R\sqrt{s}\right)^d + \mathcal{O}\left(\left(R\sqrt{s}\right)^{d-1}\right)$$

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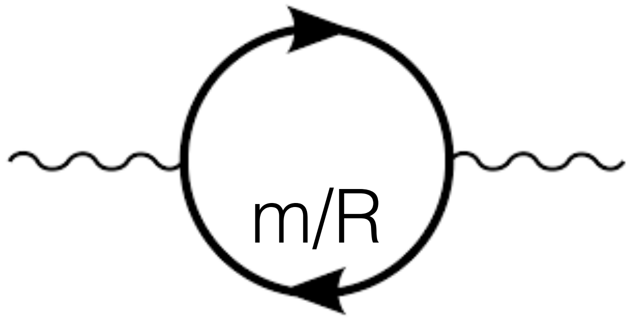
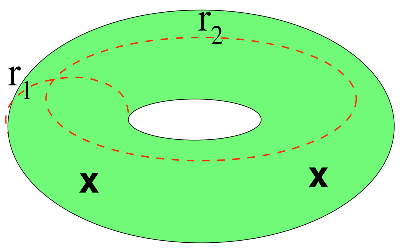
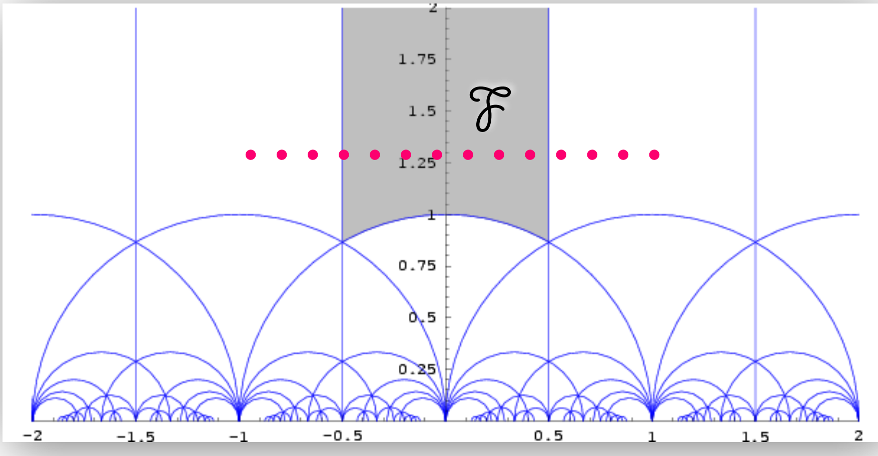


Note that the answer averages over the UV states and is *not the same as a naive rigid cut-off* at the scale s. (e.g. can introduce Scherk-Schwarz splitting of N=4 theory — the KK modes still give zero, even though the naive beta function would oscillate as ~ +- $\left(R\sqrt{s}\right)^d$)

RG in a messy UV: the string case

- *Can we do the same thing in a string theory?*
- *Kaplunovsky + \infty … calculate threshold corrections by doing the same diagram:*





$$\Pi^{\mu\nu} \approx \frac{g_{YM}^2}{16\pi^2}(k_1^\mu k_2^\nu - k_1.k_2\eta^{\mu\nu}) \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \frac{1}{4\pi^2|\eta(\tau)|^4} \sum_{\alpha,\beta,Z_2} \mathcal{Z}_{B_{int}}^{Z_2} \mathcal{Z}_F^{\alpha,\beta,Z_2}$$

$$\times \int \frac{d^2 z}{\tau_2} \left(4\pi i \partial_\tau \log(\frac{\vartheta_{\alpha\beta}(0|\tau)}{\eta(\tau)}\right) |\vartheta_1(z)|^{2k_1.k_2} \exp\left[-k_1.k_2 \frac{2\pi}{\tau_2} \Im(z)^2\right] \delta^{ab} \mathrm{Tr}\left[\frac{k}{4\pi^2}\partial_{\bar{z}}^2 \log\vartheta_1(\bar{z}) + Q^2\right]$$

$$\approx \frac{g_{YM}^2}{16\pi^2}\delta^{ab}(k_1^\mu k_2^\nu - k_1.k_2\eta^{\mu\nu}) \int \frac{d\tau_2}{\tau_2} e^{-\pi s \tau_2} \frac{1}{4\pi^2} \mathrm{Tr}\left(4\pi i \partial_\tau \log \frac{\vartheta_{\alpha\beta}(0|\tau)}{\eta(\tau)} \left[-\frac{1}{4\pi\tau_2} + Q^2\right]\right)$$

This is the scale s — the answer will go like log(s) — so this gives the correct running in the field theory limit (s << 1) where the cut-off is at tau_2 >> 1.

$$
\begin{aligned}
\Pi^{\mu\nu} \approx\ & \frac{g_{YM}^2}{16\pi^2}(k_1^\mu k_2^\nu - k_1.k_2\eta^{\mu\nu}) \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \frac{1}{4\pi^2|\eta(\tau)|^4} \sum_{\alpha,\beta,Z_2} \mathcal{Z}_{B_{int}}^{Z_2} \mathcal{Z}_F^{\alpha,\beta,Z_2} \\
& \times \int \frac{d^2 z}{\tau_2} \left(4\pi i \partial_\tau \log(\frac{\vartheta_{\alpha\beta}(0|\tau)}{\eta(\tau)}\right) |\vartheta_1(z)|^{2k_1.k_2} \exp\left[-k_1.k_2 \frac{2\pi}{\tau_2}\Im(z)^2\right] \delta^{ab}\mathrm{Tr}\left[\frac{k}{4\pi^2}\partial_{\bar z}^2 \log\vartheta_1(\bar z) + Q^2\right] \\
\approx\ & \frac{g_{YM}^2}{16\pi^2}\delta^{ab}(k_1^\mu k_2^\nu - k_1.k_2\eta^{\mu\nu}) \int \frac{d\tau_2}{\tau_2} e^{-\pi s \tau_2} \frac{1}{4\pi^2}\mathrm{Tr}\left(4\pi i\partial_\tau \log\frac{\vartheta_{\alpha\beta}(0|\tau)}{\eta(\tau)}\left[-\frac{1}{4\pi\tau_2} + Q^2\right]\right)
\end{aligned}
$$

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This is the one you want:

$$G(z|\tau) = \sum_{(m,n)\neq(0,0)} \frac{\tau_2}{\pi|m\tau+n|^2} e^{2\pi i(mu-nv)}$$
$$\equiv \sum_{(m,n)\neq(0,0)} \frac{\tau_2}{\pi|m\tau+n|^2} e^{2\pi i(m(z_1-\tau_1 z_2/\tau_2)-nz_2/\tau_2)}$$
$$\equiv \sum_{(m,n)\neq(0,0)} \frac{\tau_2}{\pi|m\tau+n|^2} e^{\frac{\pi}{\tau_2}(\bar z(m\tau+n)-z(m\bar\tau+n))}$$

$$G(z|\tau) = -\log\left|\frac{\theta_1(z|\tau)}{\theta_1'(\tau)}\right|^2 + 2\pi\frac{z_2^2}{\tau_2}$$

$$G(z|\tau) = \frac{2\pi z_2^2}{\tau_2} - \log(\left|\frac{\sin(\pi z)}{\pi}\right|^2) - 4\sum_{m=1}^{\infty}\left\{\frac{q^m}{1-q^m}\frac{\sin^2(\pi m z)}{m} + c.c.\right\}$$

$$G(z|\tau) = -2\left(\sum_{n,m\in\mathbb{Z}} \log|z+m+n\tau| - \sum_{(m,n)\neq(0,0)} \log|m+n\tau|\right) + \frac{2\pi z_2^2}{\tau_2}$$

$$\hat G(z|\tau) = \sum_{p=1}^{\infty}\frac{1}{p^2}\sum_{\gamma\in\Gamma_\infty\backslash\Gamma} \psi(\gamma(z),\gamma(\tau)), \qquad \text{with } \psi(z,\tau) = \frac{\tau_2}{\pi}e^{-2\pi i p z_2/\tau_2}$$

$$\hat G(z|\tau) = \underbrace{\frac{\tau_2}{\pi}\sum_{n\neq 0}\frac{1}{n^2}e^{2\pi i n z_2/\tau_2}}_{=\hat G^\infty(z|\tau)=2\pi\tau_2(z_2^2/\tau_2^2-|z_2/\tau_2|+\frac{1}{6})} + \sum_{\substack{m\neq 0\\ k\in\mathbb{Z}}}\frac{1}{|m|}e^{2\pi i m(k\tau_1+z_1)}e^{-2\pi\tau_2|m||k-z_2/\tau_2|}$$

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$$\boxed{\approx 2\pi\tau_2(z_2^2/\tau_2^2 - |z_2/\tau_2| + \frac{1}{6}) + e^{-2\pi\tau_2} + \dots}$$

c.f. the the factor $e^{\tau(sx(1-x)-m^2)}$ that appeared in the field theory two-point fn.

Takes the form of the one-loop ***world-line*** Green's function + stringy corrections.

However: string theory is defined on-shell — can use tricks but probably not very meaningful at scales well above s>>1.

- ***A meaningful RG procedure with a messy UV: attempt 2)***

Instead focus on amplitudes we can calculate on-shell: 4pt gluon amplitude in the Euclidean region s>>1, t,u<0 and add contributions from t channel and u channel. Also gives corrections to the Yang-Mills action, but can now put gluons on-shell.

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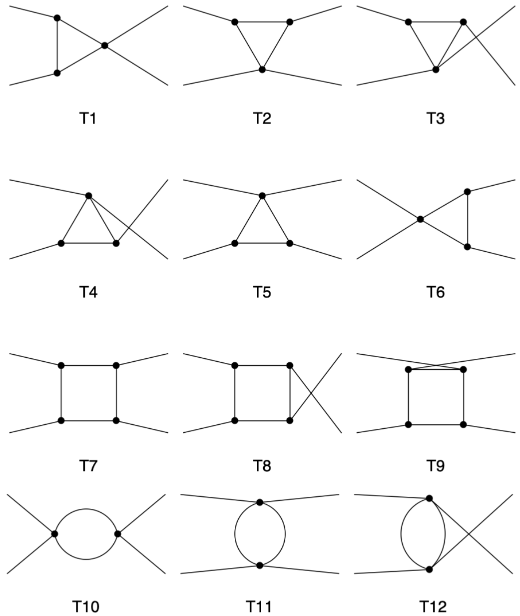
In field theory: in principle we need to calculate about 1000 diagrams. However can use various tricks to extract the divergences, or branch-cuts. e.g. only need to populate these topologies …

Adding the diagrams in s,t,u channel gives correct answer!



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In string theory: The fixed angle scattering amplitude and region of phase space was done by Gross-Mende: dominated by saddle at

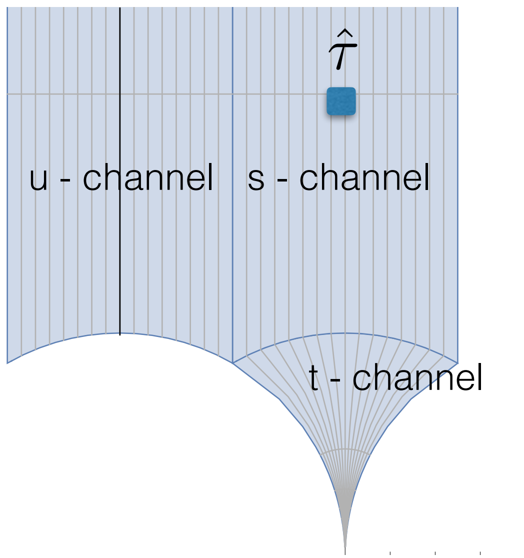
$$\left(\frac{\theta_2}{\theta_3}\right)^4 = -\frac{t}{s} \simeq \sin^2\phi/2 \; ,$$
$$\left(\frac{\theta_4}{\theta_3}\right)^4 = -\frac{u}{s} \simeq \cos^2\phi/2 \; .$$

• ***A meaningful RG procedure with a messy UV: attempt 2)***

$$\left(\frac{\theta_2}{\theta_3}\right)^4 = -\frac{t}{s} \simeq \sin^2\phi/2 \ ,$$

$$\left(\frac{\theta_4}{\theta_3}\right)^4 = -\frac{u}{s} \simeq \cos^2\phi/2 \ .$$

$$\hat{\tau} = \frac{F(\frac{1}{2},\frac{1}{2};1;\cos^2(\phi/2))}{F(\frac{1}{2},\frac{1}{2};1;\sin^2(\phi/2))}$$



$\hat{\tau} \longrightarrow i\infty$ in the zero angle limit logarithmically ... $\exp(-\pi\hat{\tau}_2) = -\frac{t}{s}$

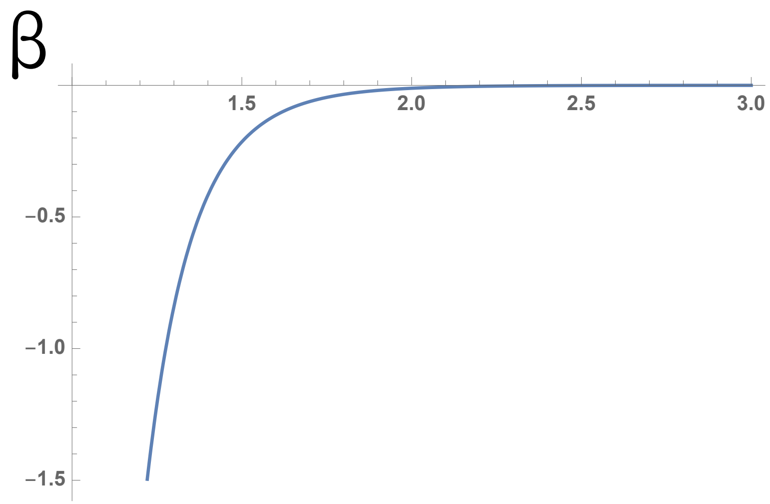
We now see that if we add the s,t,u parts equally, the definition is modular invariant !

- ***A meaningful RG procedure with a messy UV: attempt 2)***

The integrand has a well defined saddle point which gives the amplitude

$$g^4 2^{10} \pi^{-24} (stu)^{-8/3} e^{-(s\log s + t\log t + u\log u)/8} \left| \prod_{\alpha=2}^{4} \frac{\vartheta''_\alpha}{\vartheta_\alpha} \left(\frac{\vartheta''_\alpha}{\vartheta_\alpha} + \frac{2\pi}{\Im(\hat\tau)} \right) \right|^{-\frac{1}{2}} \Im(\hat\tau)^{-13} \left(\frac{\vartheta'_1}{\pi} \right)^{40/3}$$

Adding the 3 channels we get a “beta function” that goes to zero in the UV:



Summary

- Adapted perturbative asymptotically safe QFTs (gauge-Yukawa theories)
- A minimal embedding of the SM within this set-up straightforward within an extended PS structure
- Radiative symmetry breaking can be driven by Coleman-Weinberg or running mass-terms
- Overall now has the "feel of" other RG systems with large numbers of degrees of freedom in the UV: simpler dual way to understand this type of theory?
- It would be very nice to have a better lattice handle on large Nf UV fixed points
- It would be nice to think about flavour hierarchies in this set-up.