



# $CP$ asymmetries in $b \rightarrow s(d)ll$ transitions

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# Outline & introduction

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- CP asymmetries in angular observables
  - CP asymmetries in rates
  - CP asymmetries in triple products (baryons)
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- Experimental challenges (LHCb specific)
    - Detectors are made of matter, not CP symmetric...
      - *Reconstruction eff. asymmetries of ~1% at LHCb*
    - Initial  $pp$  state not CP symmetric, particle/anti-particle production asymmetric
      - *Can measure this, or construct observables which are less sensitive (angular, triple products)*

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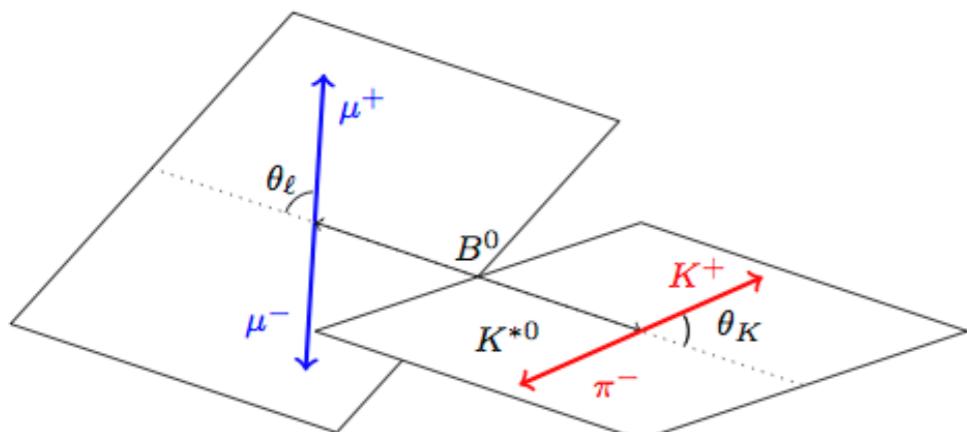
CP asymmetries in angular  
observables

# Example: $B^0 \rightarrow K^{*0} [\rightarrow K^+ \pi^-] \mu^+ \mu^-$

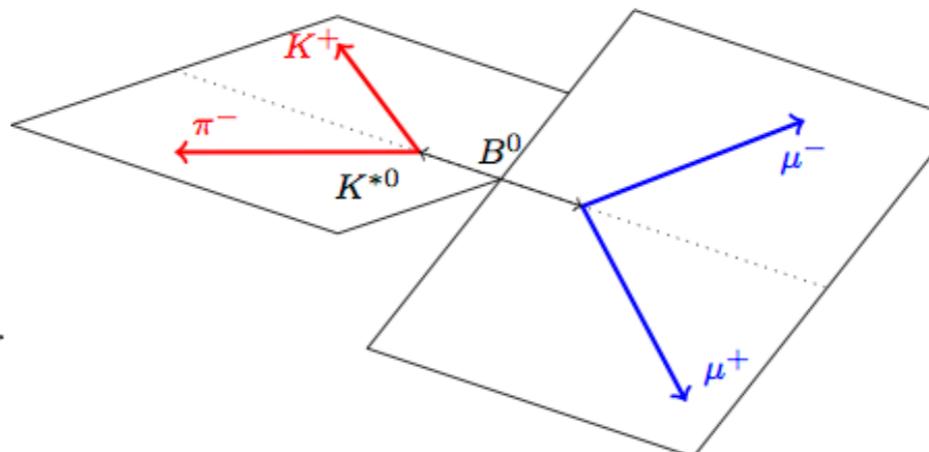
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- Decay fully described by:

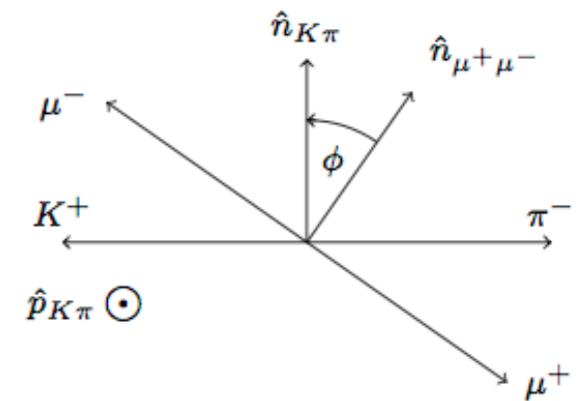
$$\vec{\Omega} = (\cos \theta_l, \cos \theta_k, \phi), q^2$$



(a)  $\theta_K$  and  $\theta_\ell$  definitions for the  $B^0$  decay



(b)  $\phi$  definition for the  $B^0$  decay



- $K^{*0}$  has 6 complex amplitudes  $A_{0,\perp\parallel}^{L,R}$ , which are dependent on different Wilson coefficients [ $C_{i=7,9,10}$ ]
  - Access to  $A_{0,\perp\parallel}^{L,R}$  via angular coefficients
-

# CP asymmetries in angular observables

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- Full angular description of the decays via the 4-differential distribution

Angular coefficients

$$\frac{d^4\Gamma[\bar{B}^0 \rightarrow \bar{K}^{*0} \mu^+ \mu^-]}{dq^2 d\vec{\Omega}} = \frac{9}{32\pi} \sum_i [I_i(q^2) f_i(\vec{\Omega})] \text{ and}$$

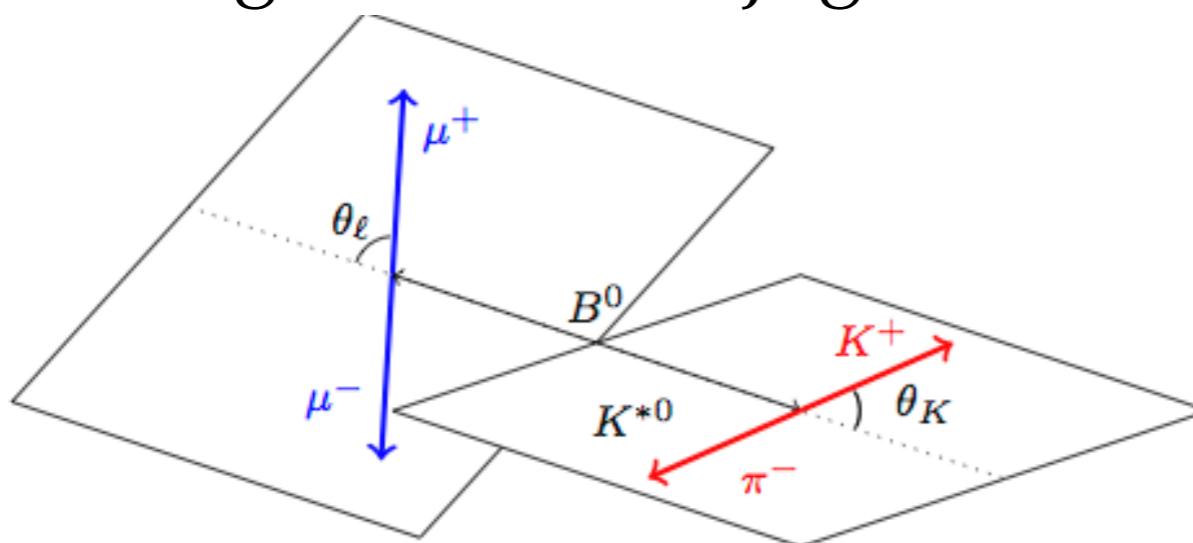
$$\frac{d^4\bar{\Gamma}[B^0 \rightarrow K^{*0} \mu^+ \mu^-]}{dq^2 d\vec{\Omega}} = \frac{9}{32\pi} \sum_i \bar{I}_i(q^2) f_i(\vec{\Omega}) ,$$

- Expanding:

$$\begin{aligned} I(q^2, \theta_l, \theta_{K^*}, \phi) = & I_1^s \sin^2 \theta_{K^*} + I_1^c \cos^2 \theta_{K^*} + (I_2^s \sin^2 \theta_{K^*} + I_2^c \cos^2 \theta_{K^*}) \cos 2\theta_l \\ & + I_3 \sin^2 \theta_{K^*} \sin^2 \theta_l \cos 2\phi + I_4 \sin 2\theta_{K^*} \sin 2\theta_l \cos \phi \\ & + I_5 \sin 2\theta_{K^*} \sin \theta_l \cos \phi \\ & + (I_6^s \sin^2 \theta_{K^*} + I_6^c \cos^2 \theta_{K^*}) \cos \theta_l + I_7 \sin 2\theta_{K^*} \sin \theta_l \sin \phi \\ & + I_8 \sin 2\theta_{K^*} \sin 2\theta_l \sin \phi + I_9 \sin^2 \theta_{K^*} \sin^2 \theta_l \sin 2\phi . \end{aligned}$$

# CP asymmetries in angular observables

- Swapping  $B^0 \rightarrow K^{*0} \mu\mu \leftrightarrow \bar{B}^0 \rightarrow \bar{K}^{*0} \mu\mu$   
 $I_{1,2,3,4,7}^{(a)} \rightarrow \bar{I}_{1,2,3,4,7}^{(a)}, \quad I_{5,6,8,9}^{(a)} \rightarrow \square \bar{I}_{5,6,8,9}^{(a)}$ ,  $a = s, c$
- Why the **minus** sign?  
➤  $\theta_l$  is defined always relative to the **positive muon**  
➤  $\theta_K$  is defined always relative to the **kaon**, which changes charge in the conjugate



(a)  $\theta_K$  and  $\theta_\ell$  definitions for the  $B^0$  decay

$$\begin{array}{l} \theta_l \rightarrow \theta_l - \pi \\ \phi \rightarrow -\phi \end{array}$$

# CP asymmetries in angular observables

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$$S_i = (I_i + \bar{I}_i) / \left( \frac{d\Gamma}{dq^2} + \frac{d\bar{\Gamma}}{dq^2} \right)$$

CP-averaged

$$A_i = (I_i - \bar{I}_i) / \left( \frac{d\Gamma}{dq^2} + \frac{d\bar{\Gamma}}{dq^2} \right)$$

CP-asymmetries

- The difference between charge conjugates in the angular observables gives sensitivity to CP violation affects
- The T-parity of  $A_i$  is relevant for CP-violation sensitivity

*The T-operator reverses the sign of all particles spin and momenta*

# Observable sensitivity

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- CP violation: seen in difference between weak phases  $\Delta_W$
- Also have difference between (CP conserving) strong phases  $\Delta_S$

## T-odd

[odd under  $\phi \rightarrow -\phi$ ]

$$\propto \boxed{\cos \Delta_S} \sin \Delta_W$$

Asymmetries associated with

$$I_{7,8,9}$$

## T-even

[even under  $\phi \rightarrow -\phi$ ]

$$\propto \boxed{\sin \Delta_S} \sin \Delta_W$$

Asymmetries associated with

$$I_{1-6}$$

- Low  $q^2$ , **strong phase** predicted to be small
- **T-odd observables thus much more sensitive to CP effects**

# Observable sensitivity

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A non-zero imaginary part for a Wilson coefficient corresponds to CP violation beyond the CKM phase (“non-standard CP violation”)

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \boxed{V_{tb}V_{ts}^*} \frac{e^2}{16\pi^2} \sum_i (\boxed{C_i} O_i + C'_i O'_i) + \text{h.c.}$$

T-odd

$\mathcal{O}(1)$  in case of non-standard CP violation

T-even

$\mathcal{O}(0.1)$  in case of non-standard CP violation

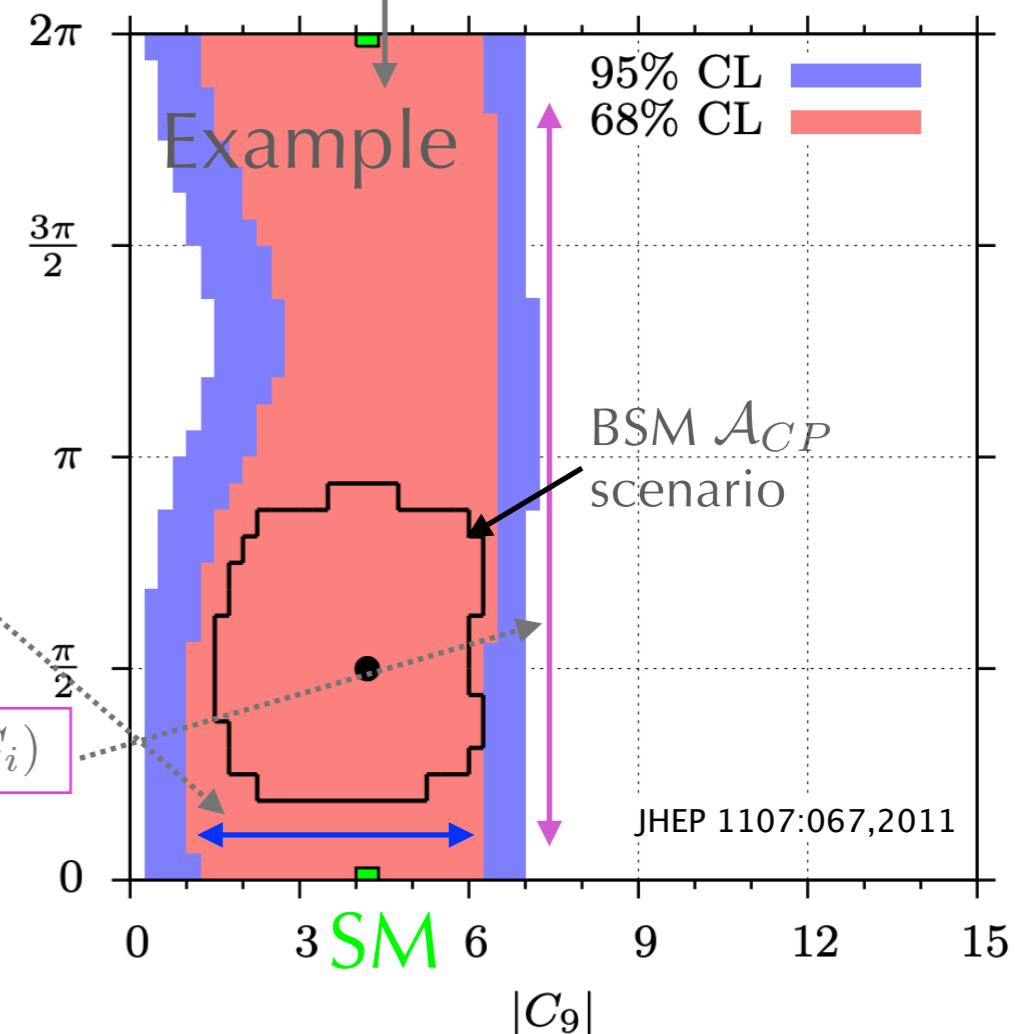
# Observable sensitivity

Most interesting observables:

Obs.	low $q^2$	high $q^2$
BR	$C_{7,9,10}, C'_{7,9,10}$	$C_{9,10}, C'_{9,10}$
$F_L$	$C_{7,9}, C'_{7,9,10}$	$C'_{9,10}$
$S_3$	$C'_{7,10}$	$C'_{9,10}$
$S_4$	$C_{7,10}, C'_{7,10}$	$C'_{9,10}$
$S_5$	$C_{7,9}, C'_{7,10}$	$C_{9,10}, C'_{9,10}$
$A_{\text{FB}}$	$C_7, C_9$	$C_{9,10}, C'_{9,10}$
$A_7$	$C_{7,10}, C'_{7,10}$	—
$A_8$	$C_{7,9}, C'_{7,9,10}$	$C'_{9,10}$
$A_9$	$C'_{7,10}$	$C'_{9,10}$

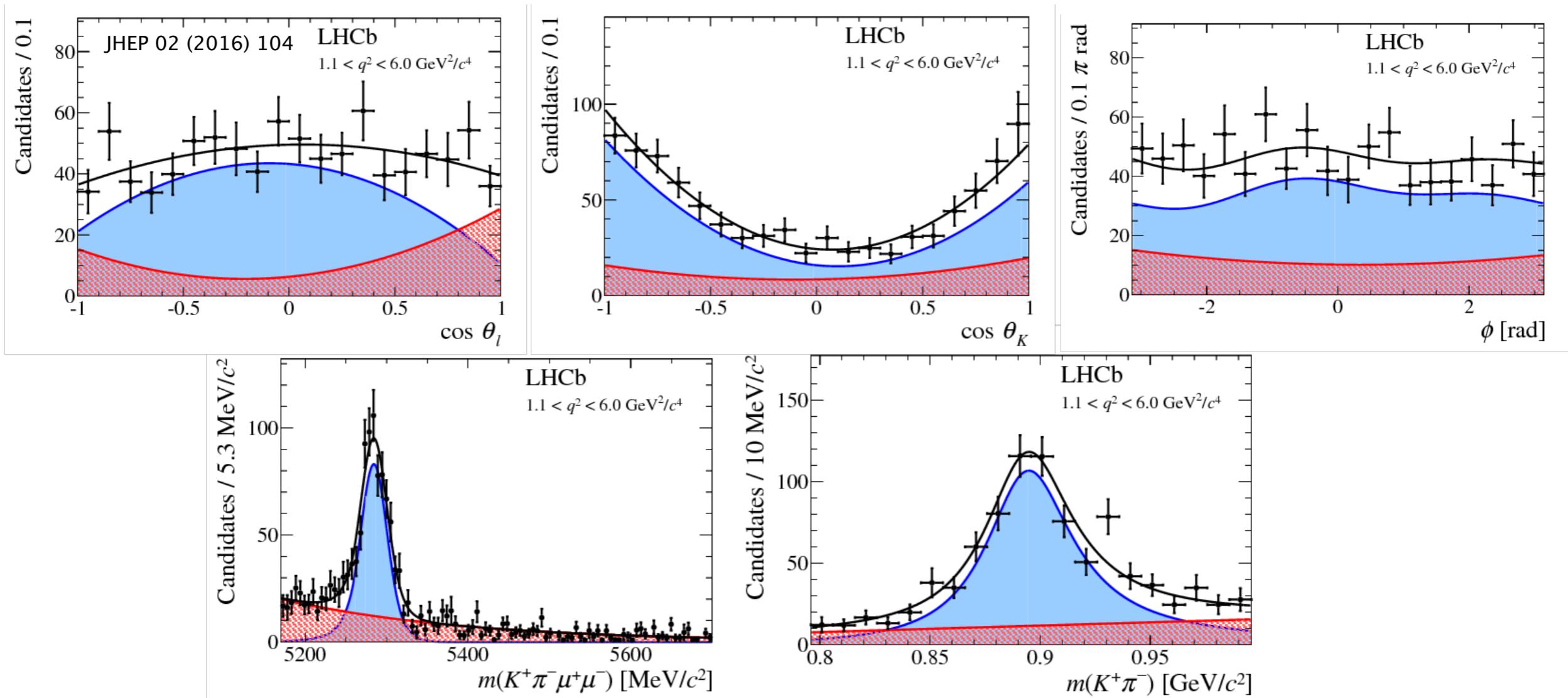
T-odd

CP-conserving data



# Extraction of $\mathcal{A}_i$ : $B^0 \rightarrow K^{*0} [\rightarrow K^+ \pi^-] \mu^+ \mu^-$

- Perform unbinned maximum likelihood fit to determine angular observables
- Use reconstructed B mass for signal/background separation
- Use reconstructed  $m_{K\pi}$  mass to constrain non-resonant S-wave



# Aside: angular acceptance

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- The reconstruction and selection efficiency must be calculated as a function of angles and  $q^2$ .
- Efficiency can be parametrise using Legendre polynomials

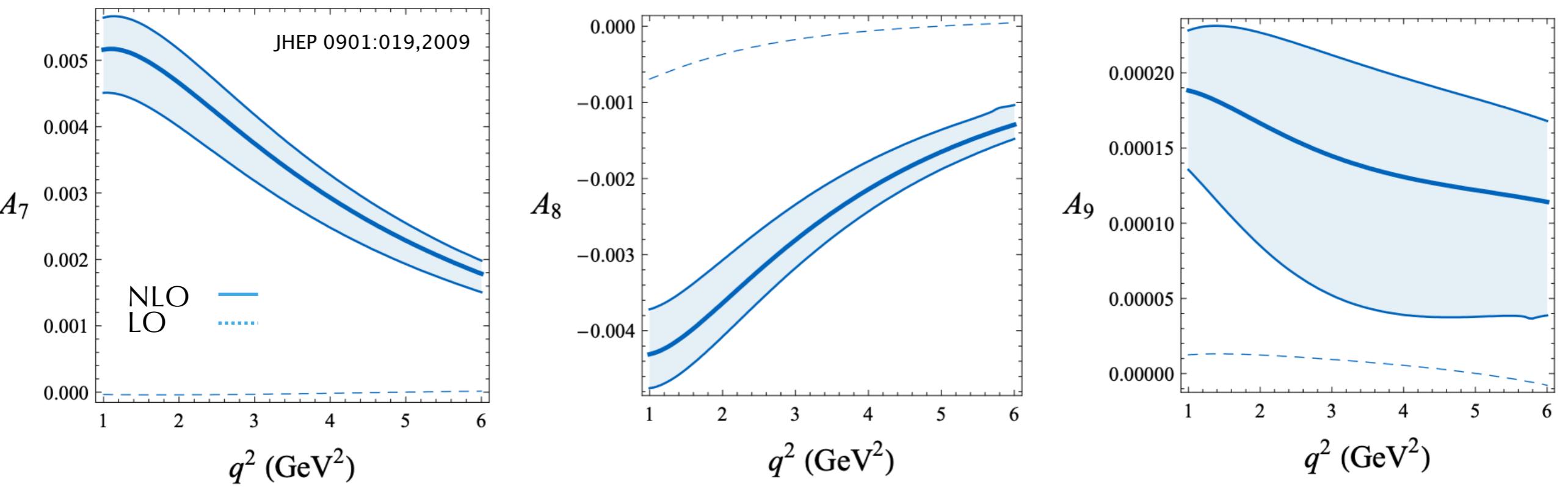
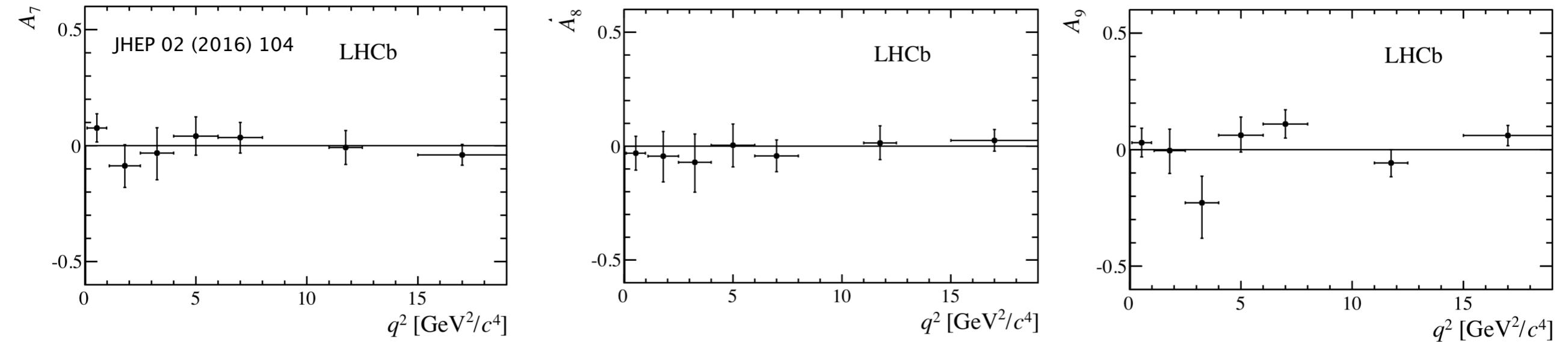
$$\varepsilon(\cos \theta_\ell, \cos \theta_K, \phi, q^2) = \sum_{k,l,m,n} c_{k,l,m,n} P(\cos \theta_\ell, k) P(\cos \theta_K, l) P(\phi, m) P(q^2, n)$$

- The coefficients  $c_{k,l,m,n}$  are calculated via the method of moments using large statistic MC samples

$$c_{k,l,m,n} = \frac{1}{N'} \sum_{i=1}^N w_i \left[ \left( \frac{2k+1}{2} \right) \left( \frac{2l+1}{2} \right) \left( \frac{2m+1}{2} \right) \left( \frac{2n+1}{2} \right) \right. \\ \left. \times P(\cos \theta_\ell, k) P(\cos \theta_K, l) P(\phi, m) P(q^2, n) \right]$$

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$$B^0 \rightarrow K^{*0} [ \rightarrow K^+ \pi^- ] \mu^+ \mu^-$$



$$B^0 \rightarrow K^{*0} [\rightarrow K^+ \pi^-] \mu^+ \mu^-$$


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JHEP 02 (2016) 104

Source	$F_L$	$S_3-S_9$	$A_3-A_9$	$P_1-P'_8$	$q_0^2$	$\text{GeV}^2/c^4$
Acceptance stat. uncertainty	< 0.01	< 0.01	< 0.01	< 0.01		0.01
Acceptance polynomial order	< 0.01	< 0.02	< 0.02	< 0.04		0.01–0.03
Data-simulation differences	0.01–0.02	< 0.01	< 0.01	< 0.01		< 0.02
Acceptance variation with $q^2$	< 0.01	< 0.01	< 0.01	< 0.01		–
$m(K^+\pi^-)$ model	< 0.01	< 0.01	< 0.01	< 0.03		< 0.01
Background model	< 0.01	< 0.01	< 0.01	< 0.02		0.01–0.05
Peaking backgrounds	< 0.01	< 0.01	< 0.01	< 0.01		0.01–0.04
$m(K^+\pi^-\mu^+\mu^-)$ model	< 0.01	< 0.01	< 0.01	< 0.02		< 0.01
Det. and prod. asymmetries	–	–	< 0.01	< 0.02		–

Systematic effects very small compared to stat.

Statistical errors  $\sim 0.05\text{--}0.150$

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# Example: $B_s^0 \rightarrow \phi(\rightarrow KK)\mu\mu$

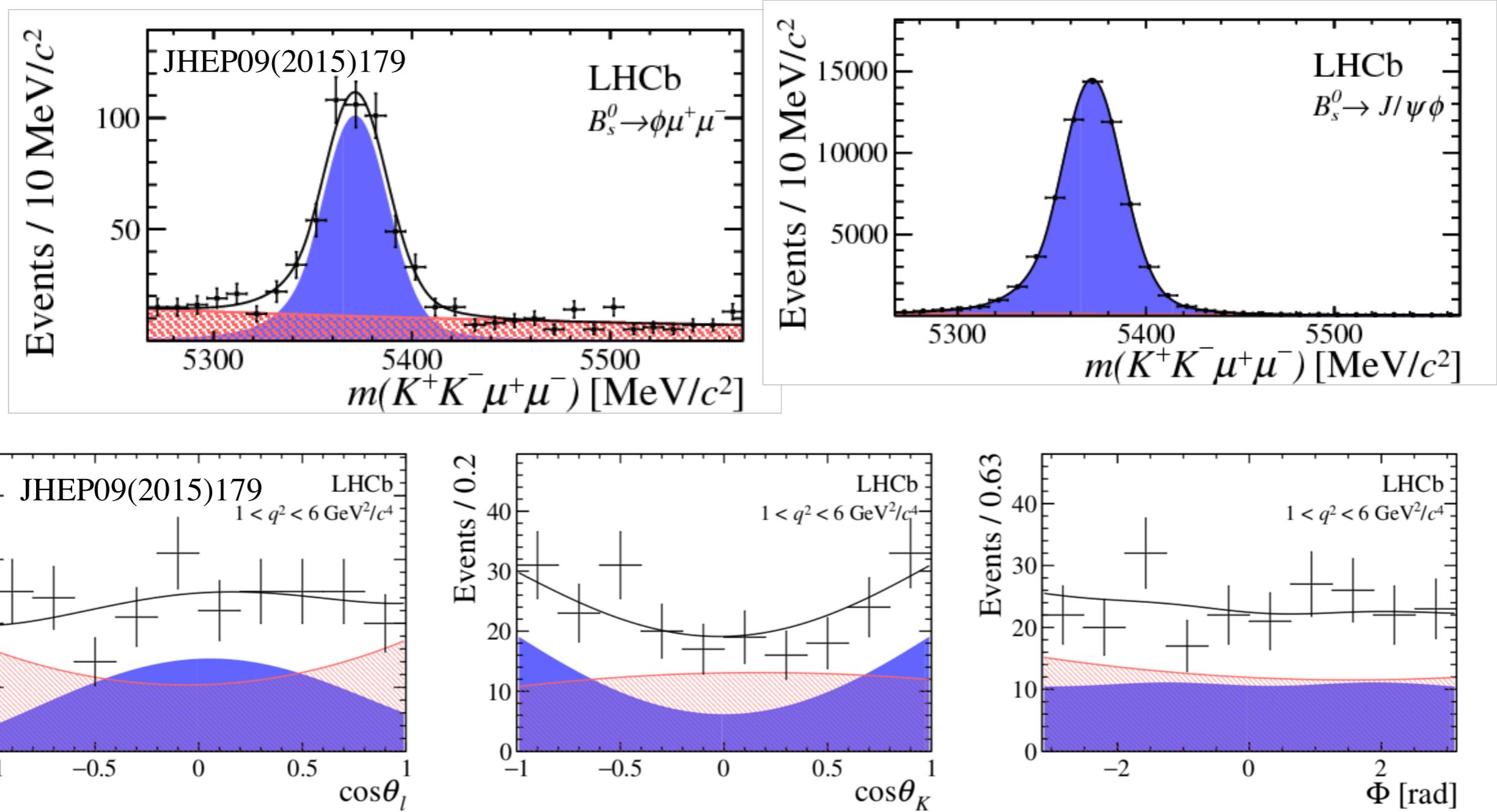
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- Not self tagging (hadrons same type) so can only access CP-averaged when no sign swap, CP- asymmetries otherwise

$$I_{1,2,3,4,7}^{(a)} \longrightarrow \bar{I}_{1,2,3,4,7}^{(a)}, \quad I_{5,6,8,9}^{(a)} \longrightarrow -\bar{I}_{5,6,8,9}^{(a)},$$

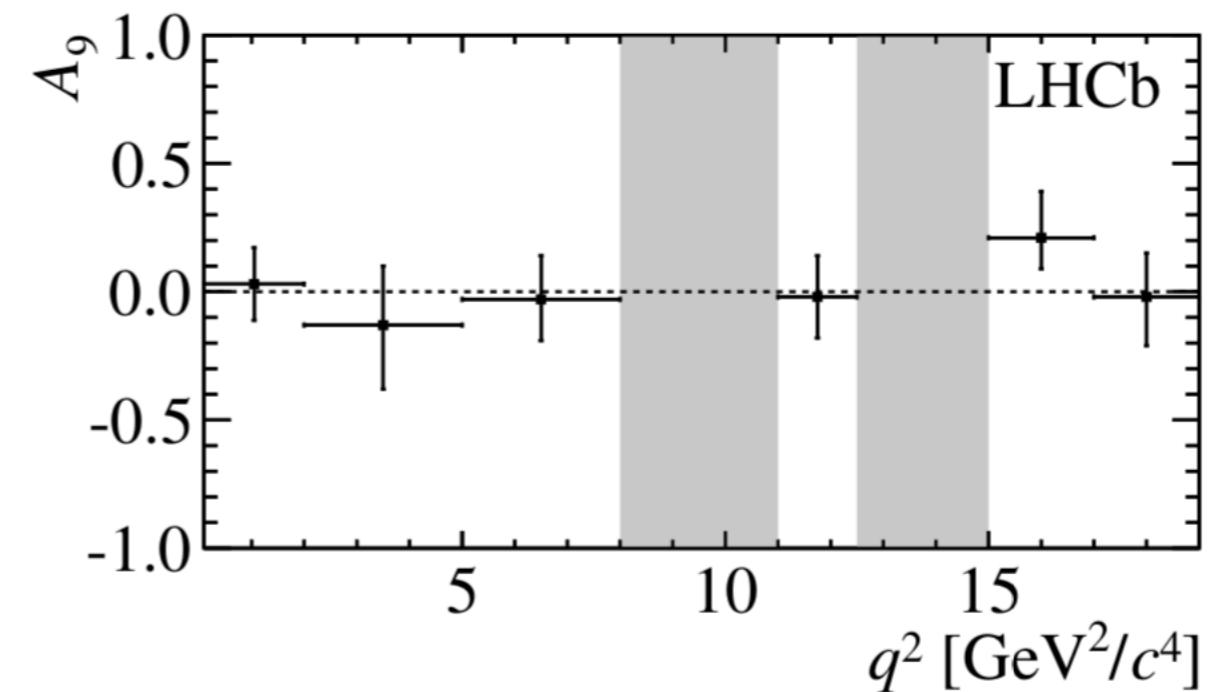
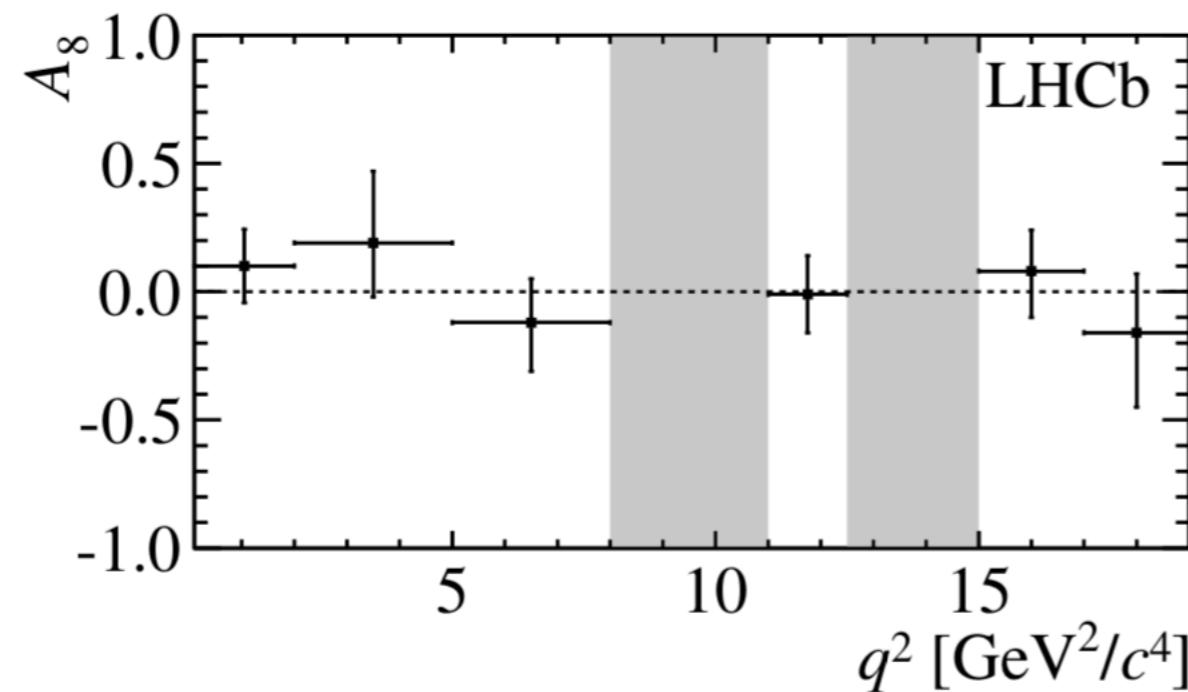
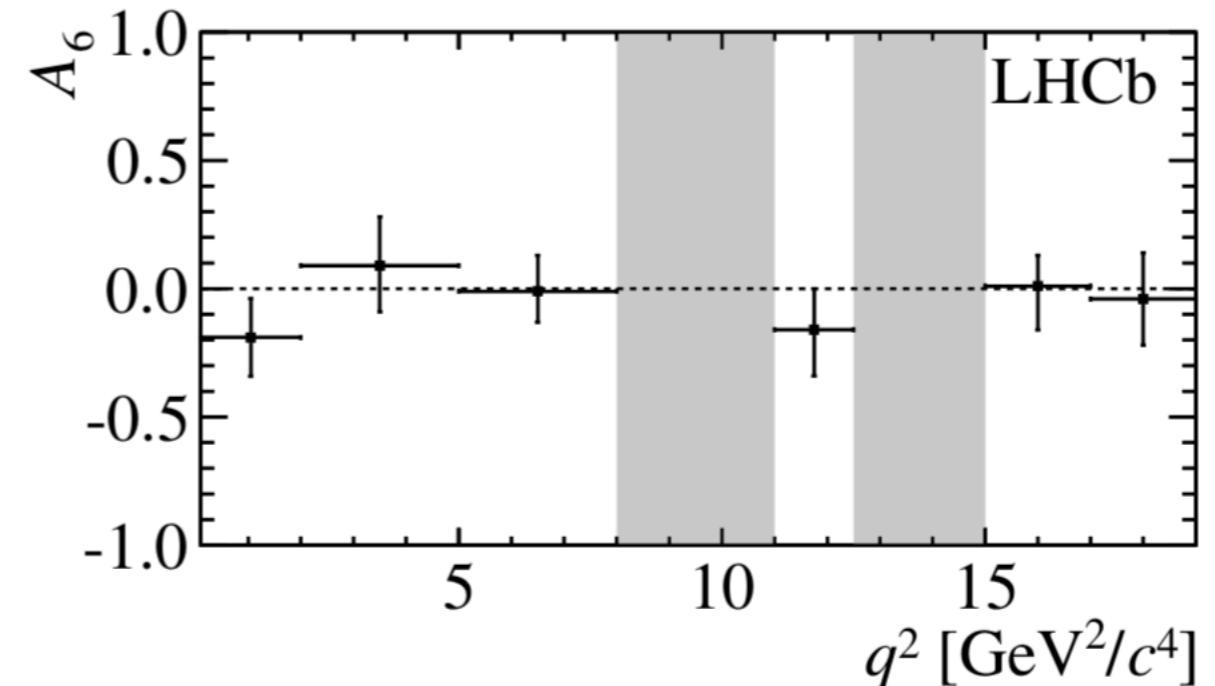
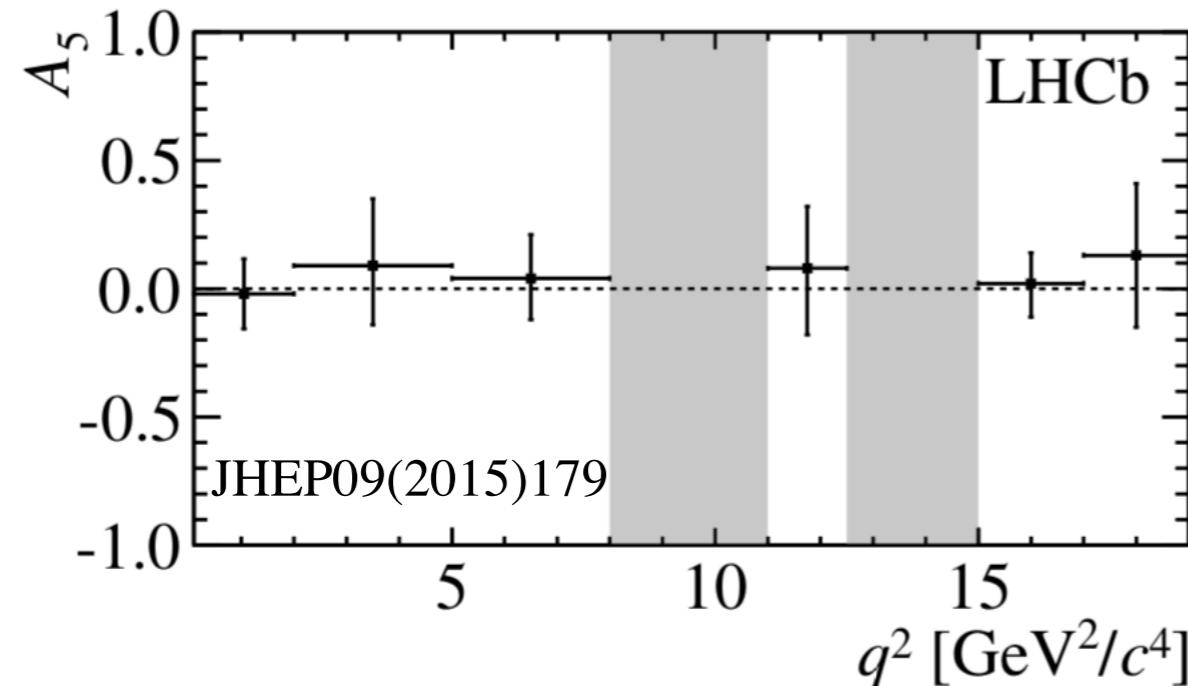
$$\begin{aligned} \frac{1}{d\Gamma/dq^2} \frac{d^3\Gamma}{dcos\theta_l \, dcos\theta_K \, d\Phi} = & \frac{9}{32\pi} \left[ \frac{3}{4}(1 - F_L) \sin^2 \theta_K + F_L \cos^2 \theta_K \right. \\ & + \frac{1}{4}(1 - F_L) \sin^2 \theta_K \cos 2\theta_l - F_L \cos^2 \theta_K \cos 2\theta_l \\ & + S_3 \sin^2 \theta_K \sin^2 \theta_l \cos 2\Phi + S_4 \sin 2\theta_K \sin 2\theta_l \cos \Phi \\ & + A_5 \sin 2\theta_K \sin \theta_l \cos \Phi + A_6 \sin^2 \theta_K \cos \theta_l \\ & + S_7 \sin 2\theta_K \sin \theta_l \sin \Phi + A_8 \sin 2\theta_K \sin 2\theta_l \sin \Phi \\ & \left. + A_9 \sin^2 \theta_K \sin^2 \theta_l \sin 2\Phi \right]. \end{aligned} \quad (1.1)$$

# Example: $B_s^0 \rightarrow \phi(\rightarrow KK)\mu\mu$

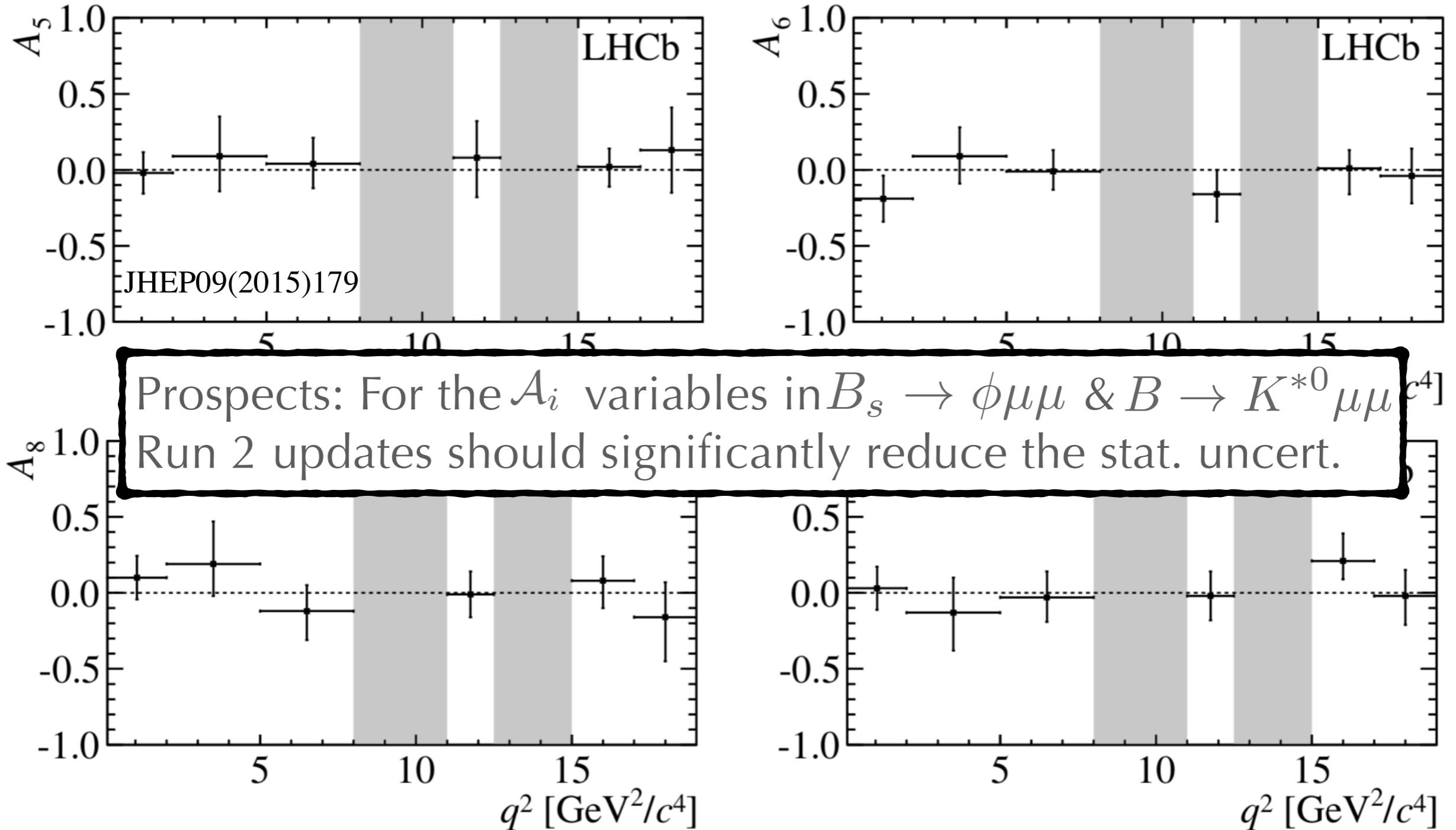


# Example: $B_s^0 \rightarrow \phi(\rightarrow KK)\mu\mu$

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# Example: $B_s^0 \rightarrow \phi(\rightarrow KK)\mu\mu$



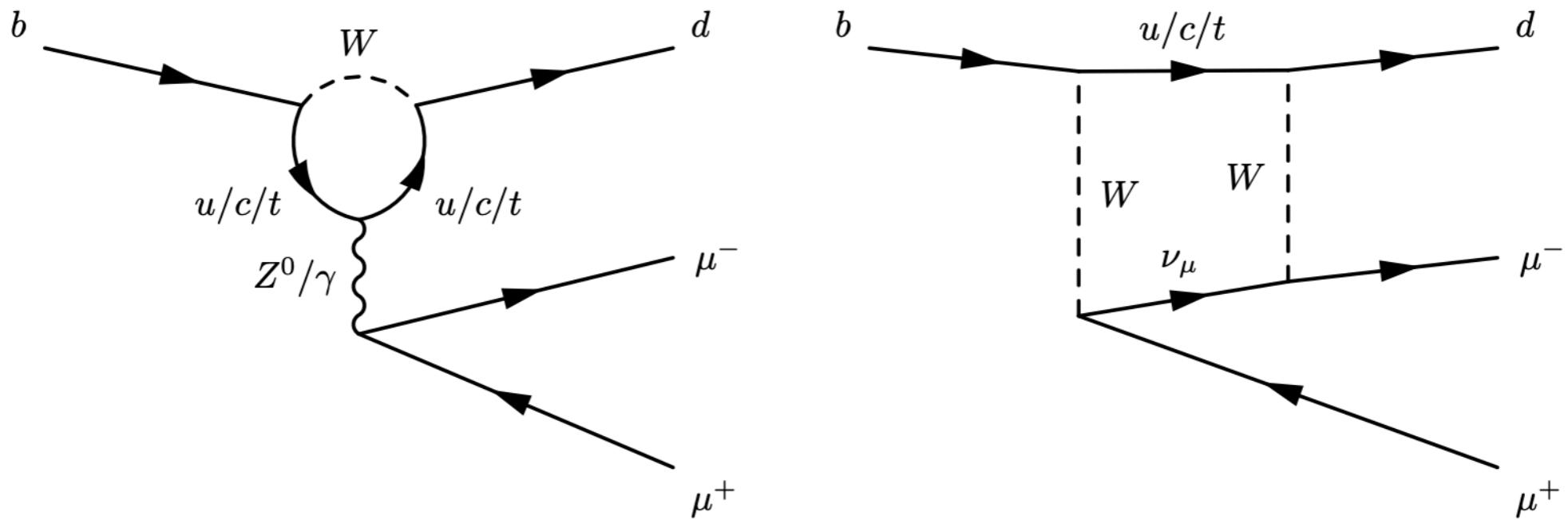
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## CP asymmetries in rates

$$A_{CP} = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}$$

$$\mathcal{A}_{CP}(B^+ \rightarrow \pi^+ \mu^+ \mu^-)$$

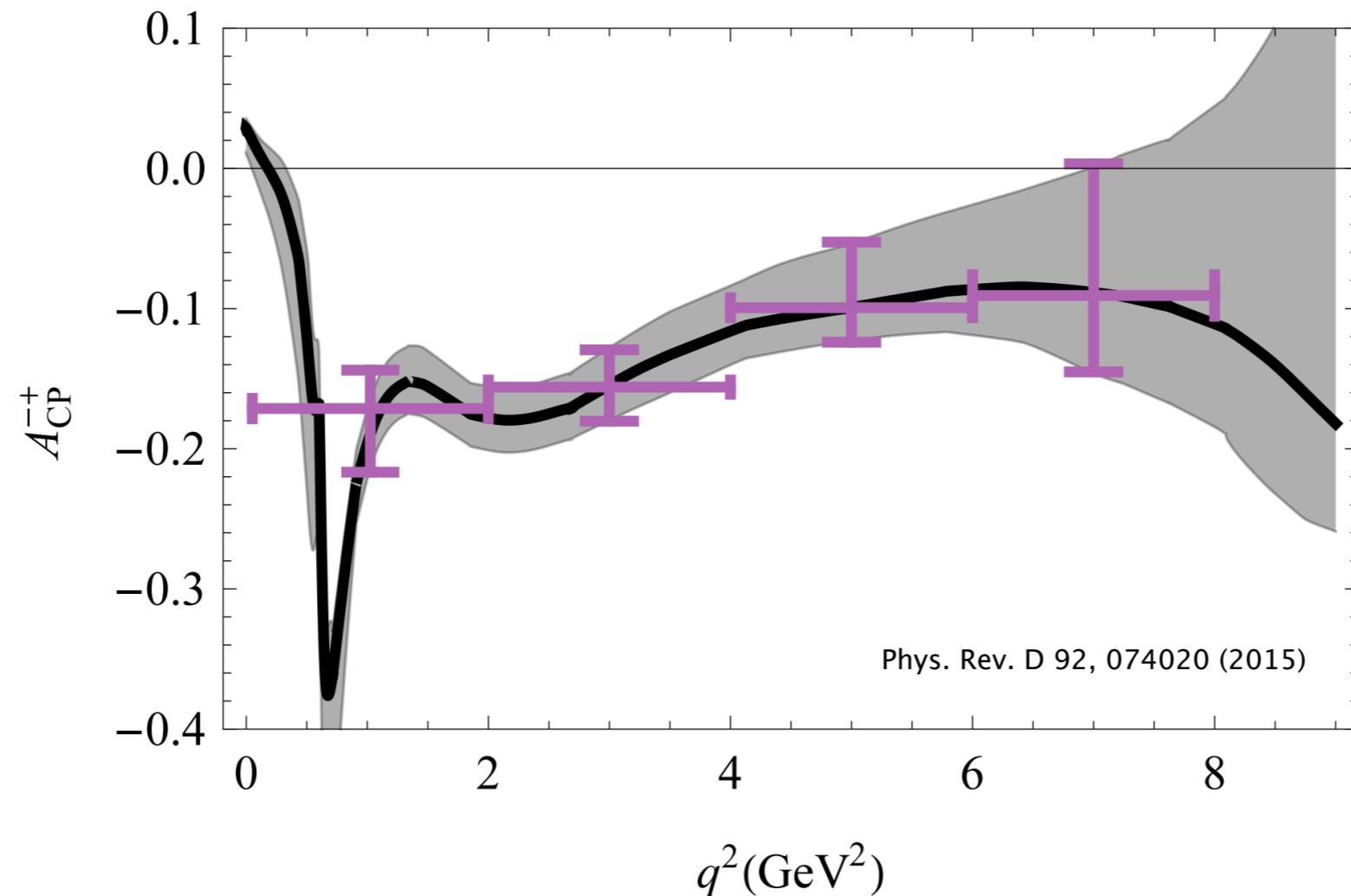
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$b \rightarrow dll$  suppressed wrt  $b \rightarrow sll$  by  $\frac{V_{td}}{V_{ts}}$

$\mathcal{A}_{CP}(B^+ \rightarrow \pi^+ \mu^+ \mu^-)$  expected to be non-vanishing

$$\mathcal{A}_{CP}(B^+ \rightarrow \pi^+ \mu^+ \mu^-)$$



Bin [ $\text{GeV}^2$ ]	[0.05, 2.0]	[2.0, 4.0]	[4.0, 6.0]	[6.0, 8.0]	[1.0, 6.0]
$\mathcal{A}_{CP}^{(-)}$	$-0.171^{+0.027}_{-0.045}$	$-0.156^{+0.027}_{-0.024}$	$-0.099^{+0.047}_{-0.025}$	$-0.091^{+0.093}_{-0.053}$	$-0.143^{+0.035}_{-0.029}$

$$\mathcal{A}_{CP}(B^+ \rightarrow \pi^+ \mu^+ \mu^-)$$

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$$\mathcal{A}_{CP} \equiv \frac{\Gamma(B^- \rightarrow \pi^- \mu^+ \mu^-) - \Gamma(B^+ \rightarrow \pi^+ \mu^+ \mu^-)}{\Gamma(B^- \rightarrow \pi^- \mu^+ \mu^-) + \Gamma(B^+ \rightarrow \pi^+ \mu^+ \mu^-)}$$

$$\mathcal{A}_{RAW} \equiv \frac{\mathcal{N}(B^- \rightarrow \pi^- \mu^+ \mu^-) - \mathcal{N}(B^+ \rightarrow \pi^+ \mu^+ \mu^-)}{\mathcal{N}(B^- \rightarrow \pi^- \mu^+ \mu^-) + \mathcal{N}(B^+ \rightarrow \pi^+ \mu^+ \mu^-)}$$

Raw asymmetries, take from fits

$$\mathcal{A}_{CP}(B^\pm \rightarrow \pi^\pm \mu^+ \mu^-) = \mathcal{A}_{RAW} - \boxed{\mathcal{A}_P} - \boxed{\mathcal{A}_{DET}},$$

Production asymmetry  
 $(-0.6 \pm 0.6)\%$

Phys. Rev. Lett. 114, 041601

Detector asymmetries

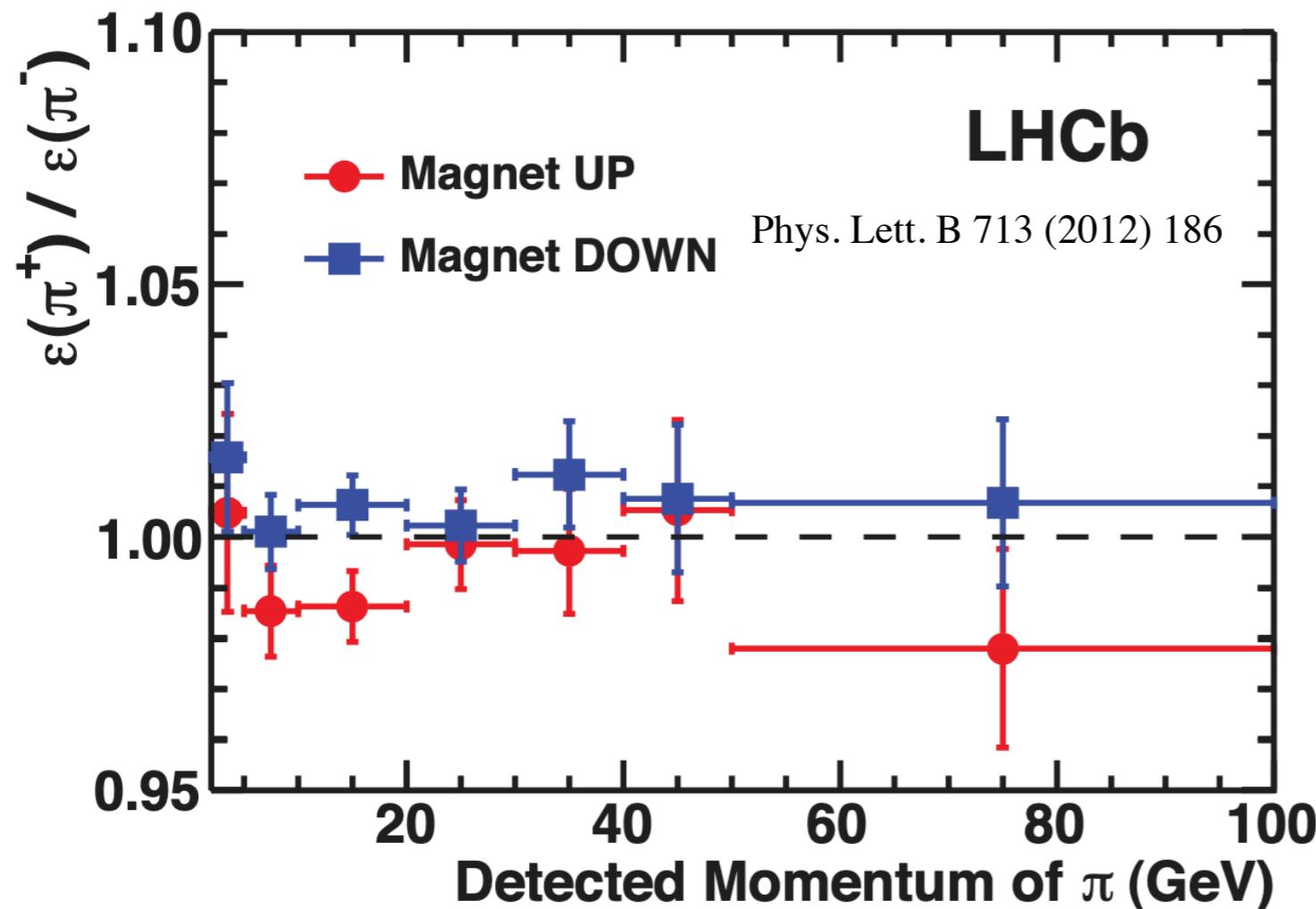
$$\mathcal{A}_{CP}(B^+ \rightarrow \pi^+ \mu^+ \mu^-)$$

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Detector asymmetries: Phys. Lett. B 713 (2012) 186

$$D^{*+} \rightarrow \pi_s^+ D^0 \quad D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$$

Compare fully reconstructed  $D^0$  to partially reconstructed (pion missing)

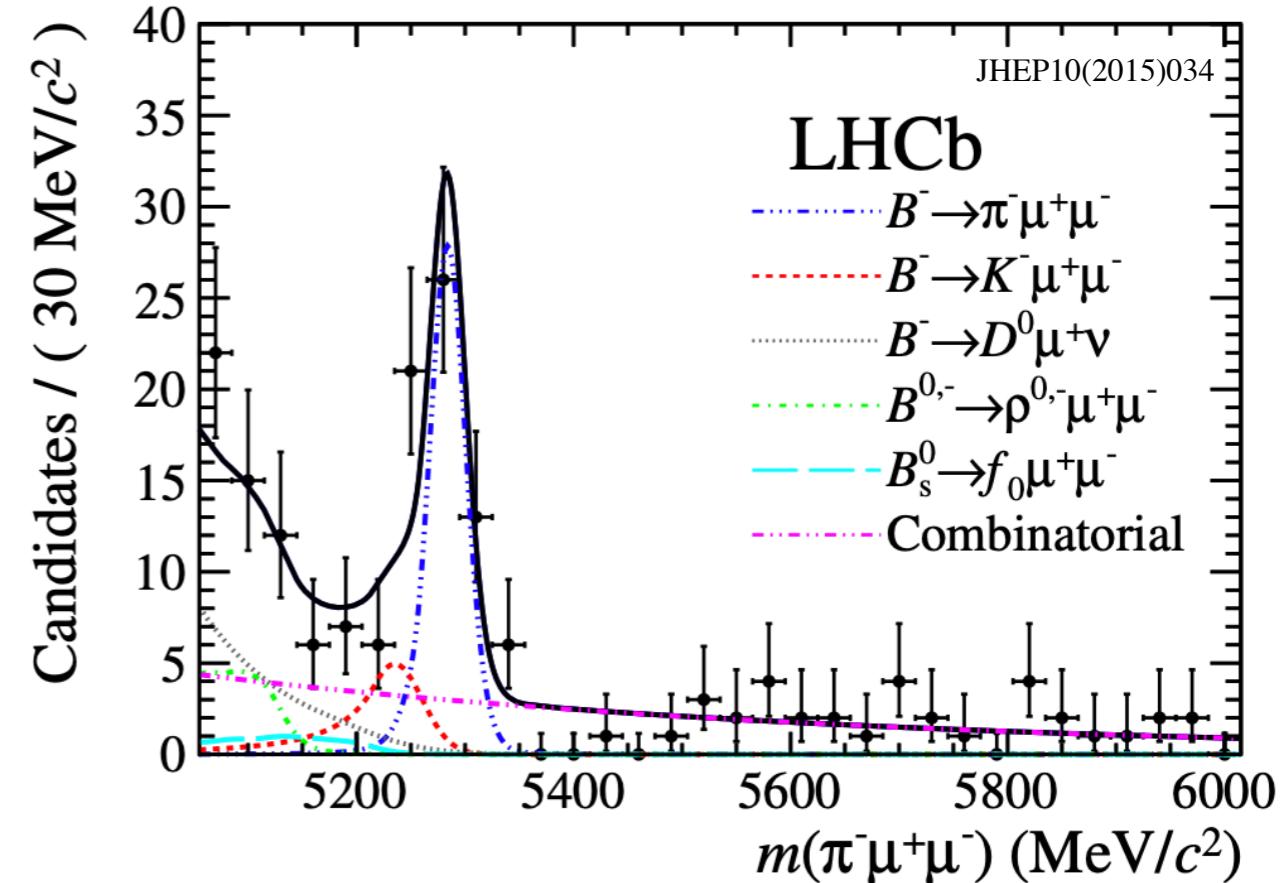
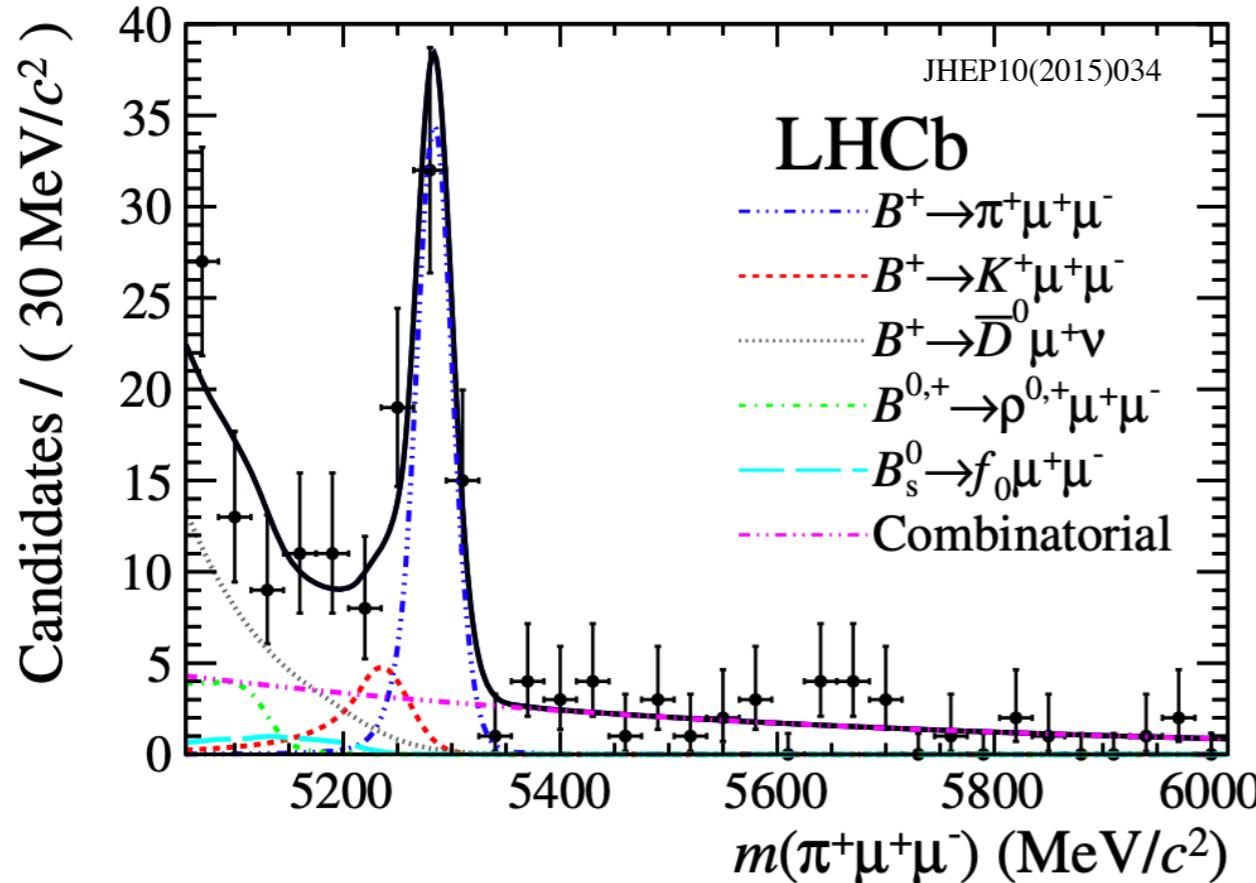


Binned in  $p$ ,  $p_t$  and azimuthal production angle

$$\epsilon_{\pi^+}/\epsilon_{\pi^-} = 0.9914 \pm 0.0040$$

$$\epsilon_{\pi^+}/\epsilon_{\pi^-} = 1.0045 \pm 0.0034$$

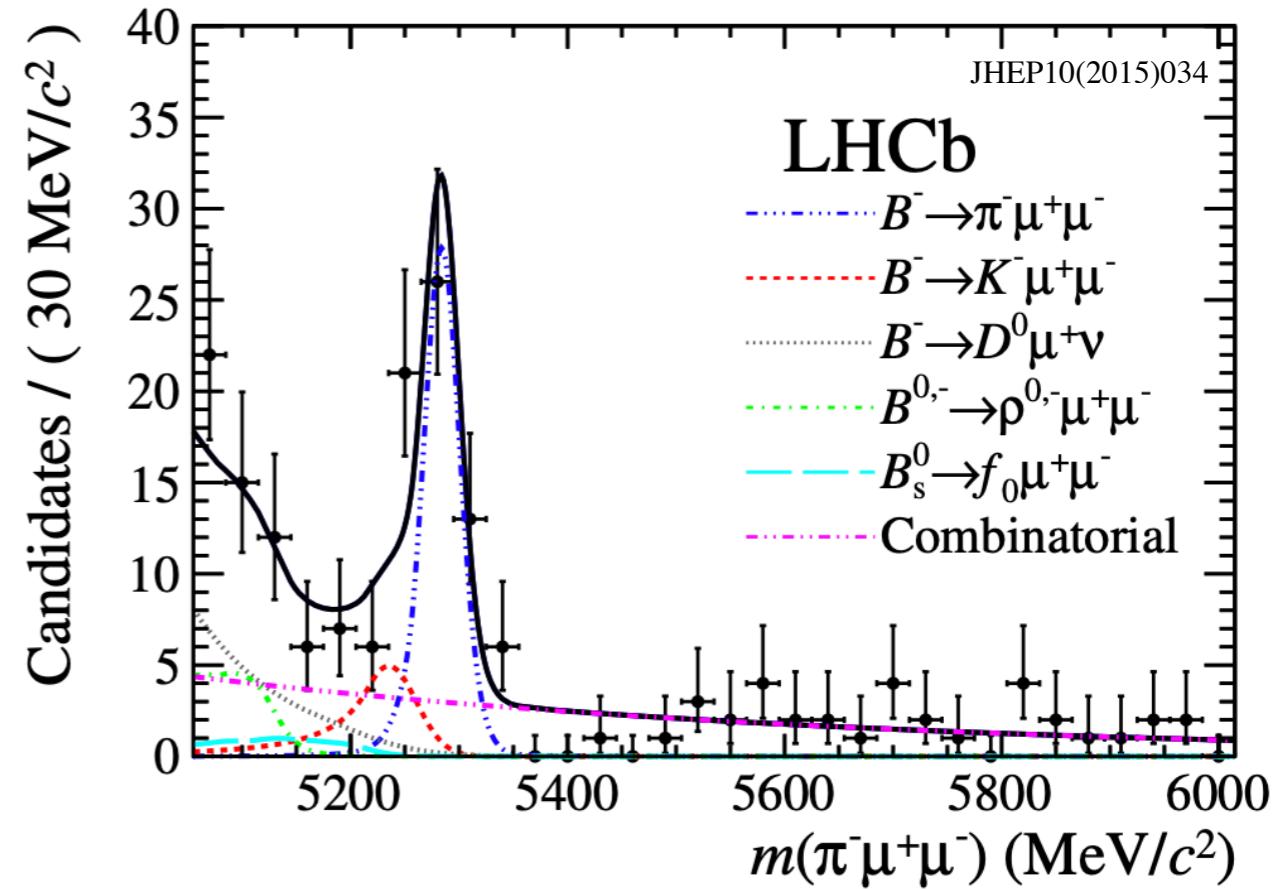
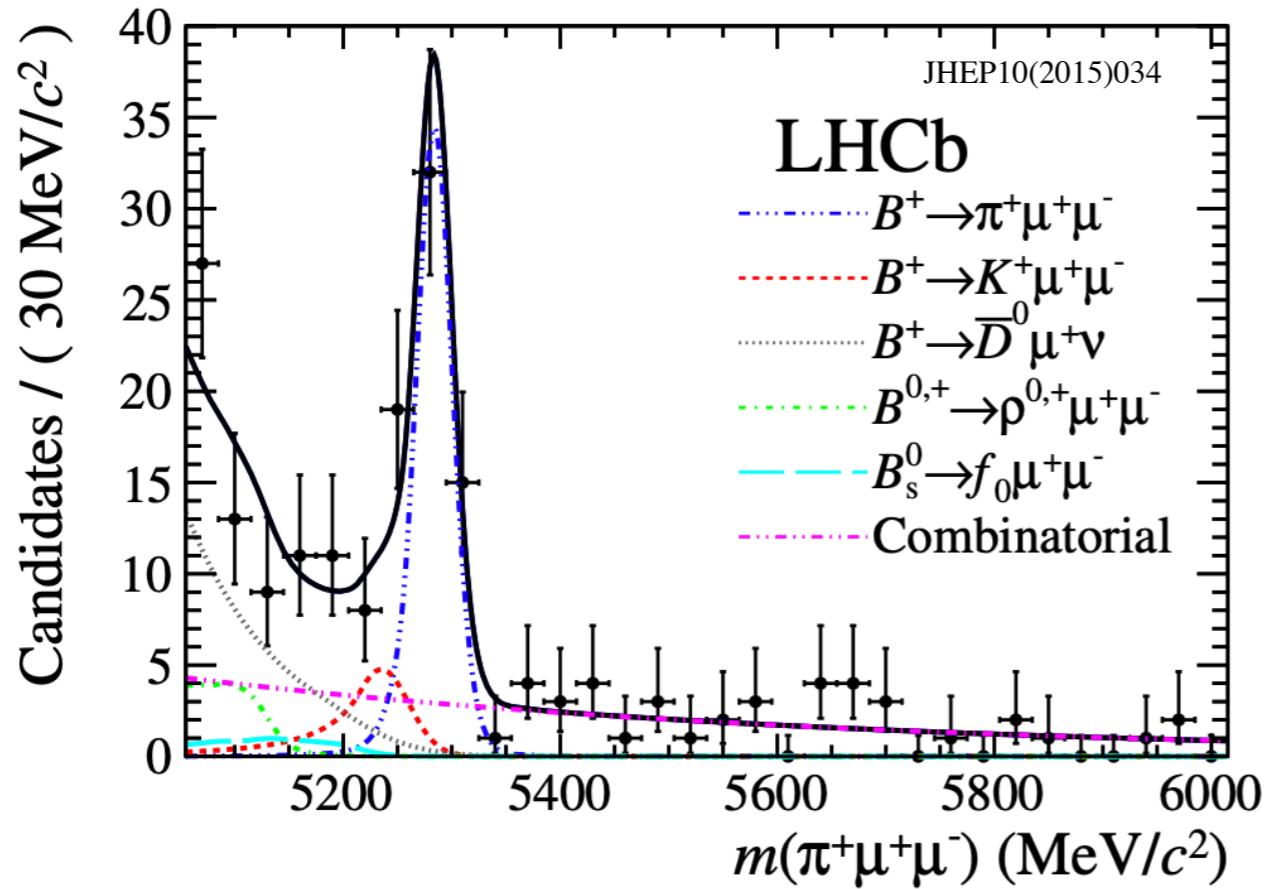
$$\mathcal{A}_{CP}(B^+ \rightarrow \pi^+ \mu^+ \mu^-)$$



$\mathcal{N}(B^\pm \rightarrow \pi^\pm \mu^+ \mu^-)$	$\mathcal{N}(B^+ \rightarrow \pi^+ \mu^+ \mu^-)$	$\mathcal{N}(B^- \rightarrow \pi^- \mu^+ \mu^-)$
$92.7 \pm 11.5$	$51.7 \pm 8.3$	$41.1 \pm 7.9$

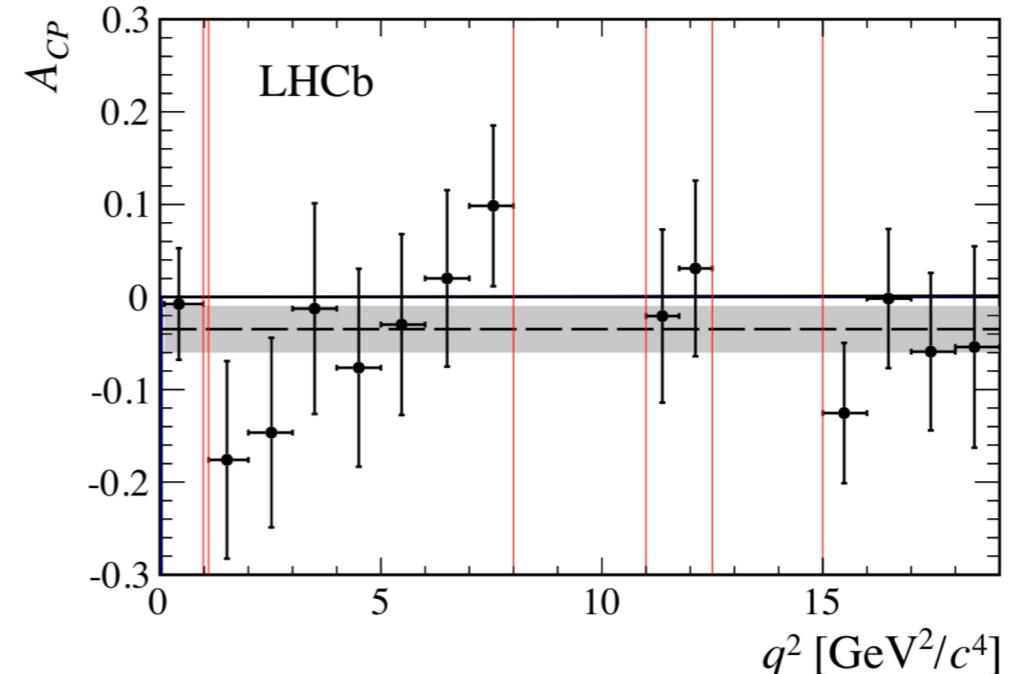
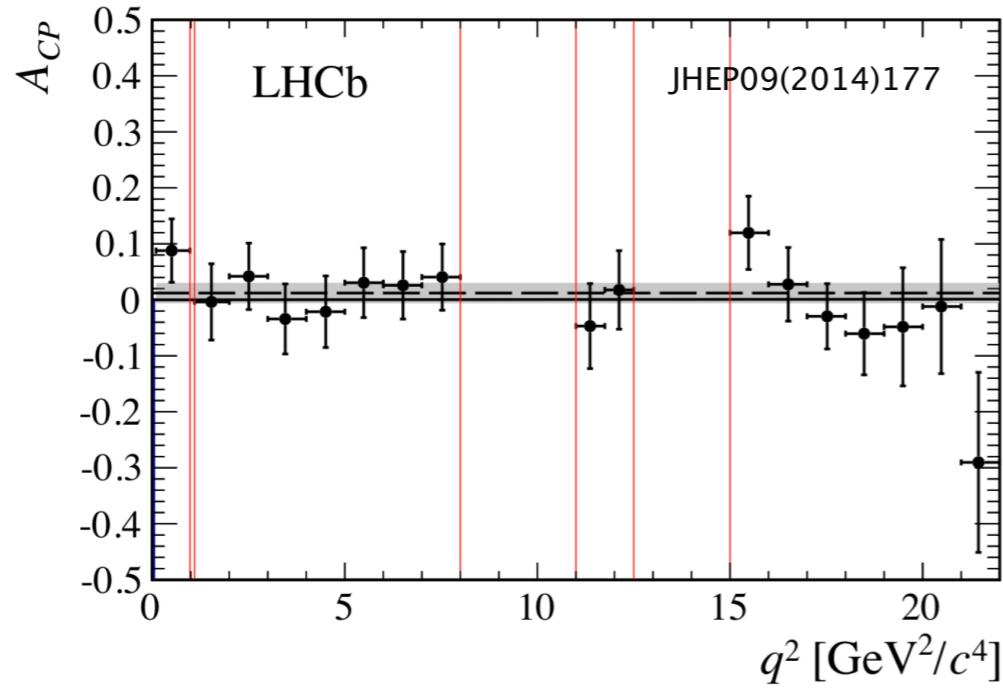
Simultaneous fit between  $m(\pi^+ \mu^+ \mu^-)$  and  $m(K^+ \mu^+ \mu^-)$  constrain cross-feed

$$\mathcal{A}_{CP}(B^+ \rightarrow \pi^+ \mu^+ \mu^-)$$



$$\mathcal{A}_{CP}(B^\pm \rightarrow \pi^\pm \mu^+ \mu^-) = -0.11 \pm 0.12 \text{ (stat)} \pm 0.01 \text{ (syst)}$$

# $\mathcal{A}_{CP}$ in $B \rightarrow K^{(*)} ll$ decays

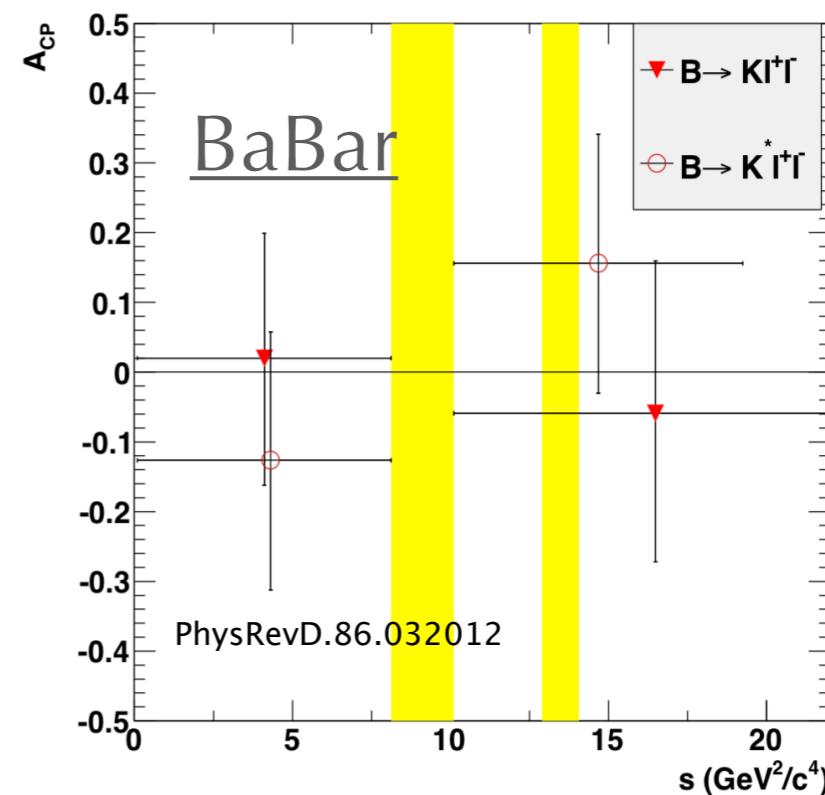


LHCb

$$\begin{aligned} \mathcal{A}_{CP}(B^0 \rightarrow K^{*0} \mu^+ \mu^-) &= -0.035 \pm 0.024 \pm 0.003, \\ \mathcal{A}_{CP}(B^+ \rightarrow K^+ \mu^+ \mu^-) &= 0.012 \pm 0.017 \pm 0.001, \end{aligned}$$

Belle [Phys.Rev.Lett.103:171801,2009]

$$\begin{aligned} A_{CP}(K^* \ell^+ \ell^-) &= -0.10 \pm 0.10 \pm 0.01, \\ A_{CP}(K^+ \ell^+ \ell^-) &= 0.04 \pm 0.10 \pm 0.02. \end{aligned}$$

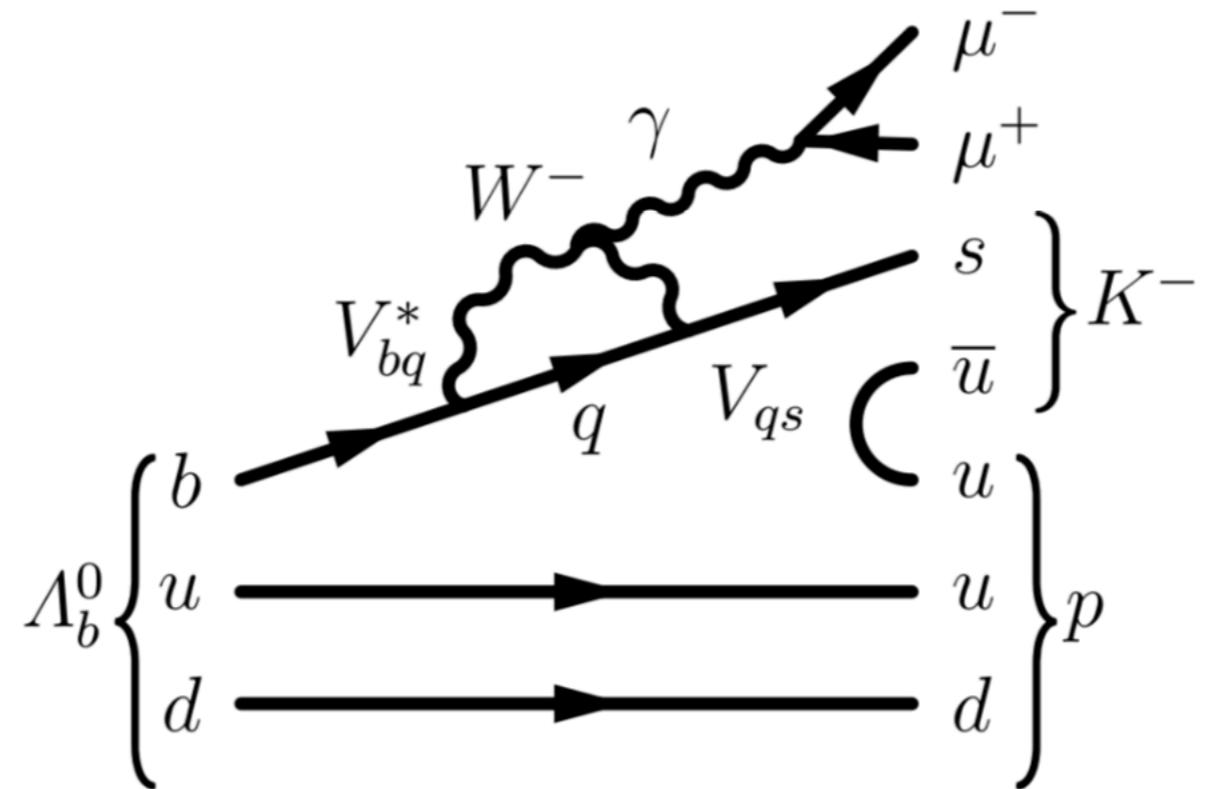


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CP asymmetries in baryons using  
rates and triple products

$$\Lambda_b^0 \rightarrow p K \mu\mu$$

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- $b \rightarrow sll$  transition in baryon sector
- CP asymmetries looked at in rates and triple products

# $\Lambda_b^0 \rightarrow p K \mu \mu$

Expected to be small in the SM [e.g. 10.1093/ptep/ptv017]

$$\mathcal{A}_{CP} = \mathcal{A}_{RAW} - \boxed{\mathcal{A}_P} - \boxed{\mathcal{A}_{DET}},$$

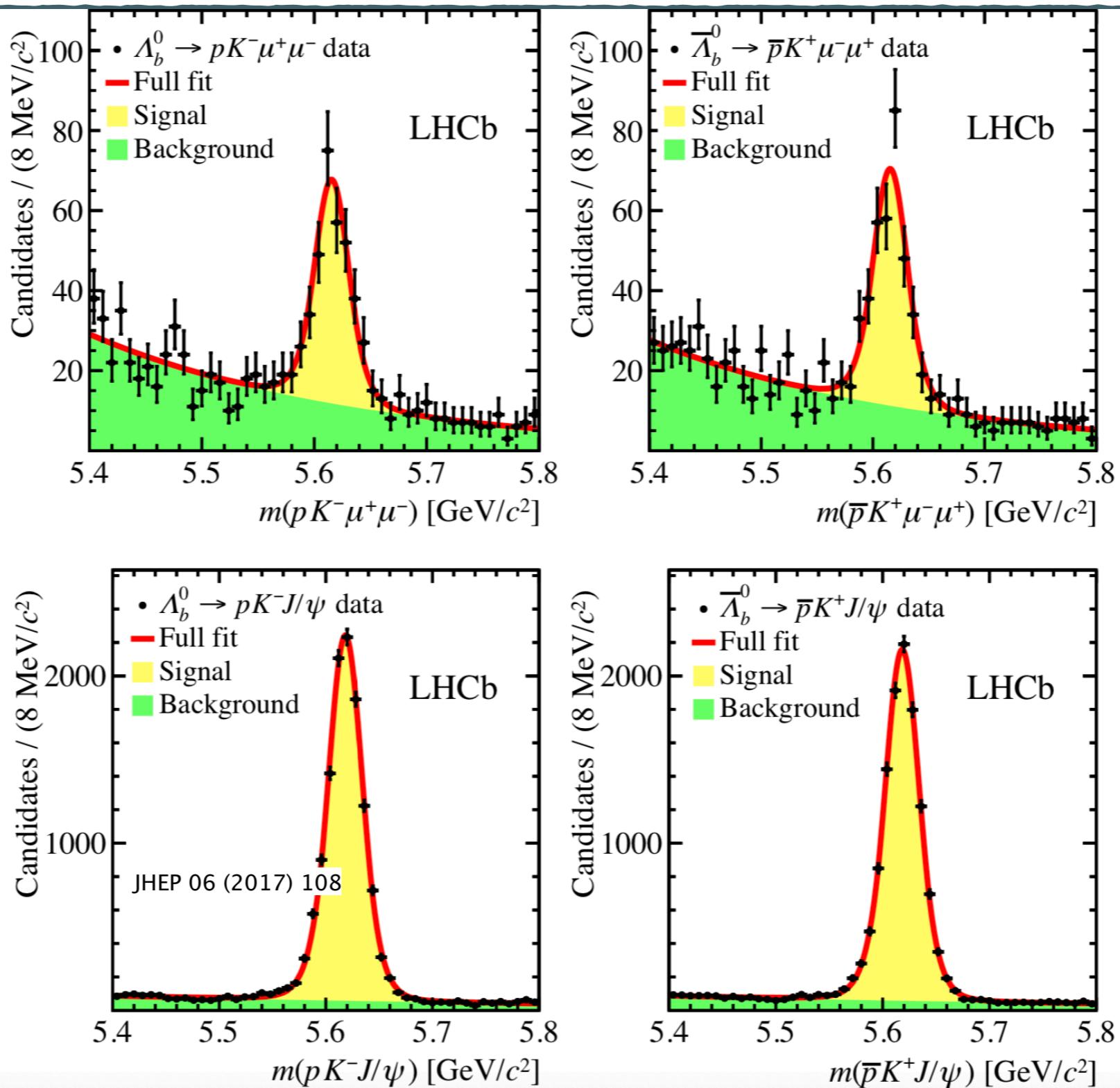
- Detector asymmetries for final state well-measured
- Production asymmetries for  $\Lambda_b$  measured at LHCb for first time in 2017, after this result
- Solution: measure  $\Delta\mathcal{A}_{CP}$  between modes with same mother
- Measure relative to mode:  $\Lambda_b \rightarrow p K J/\psi$  Assume CP-conserving

$$\mathcal{A}_{raw} \approx \mathcal{A}_{CP}(\Lambda_b^0 \rightarrow p K^- \mu^+ \mu^-) + \boxed{\mathcal{A}_{prod}(\Lambda_b^0)} - \boxed{\mathcal{A}_{reco}(K^+) + \mathcal{A}_{reco}(p)}$$

$$\begin{aligned}\Delta\mathcal{A}_{CP} &\equiv \mathcal{A}_{CP}(\Lambda_b^0 \rightarrow p K^- \mu^+ \mu^-) - \mathcal{A}_{CP}(\Lambda_b^0 \rightarrow p K^- J/\psi) \\ &\approx \mathcal{A}_{raw}(\Lambda_b^0 \rightarrow p K^- \mu^+ \mu^-) - \mathcal{A}_{raw}(\Lambda_b^0 \rightarrow p K^- J/\psi).\end{aligned}$$

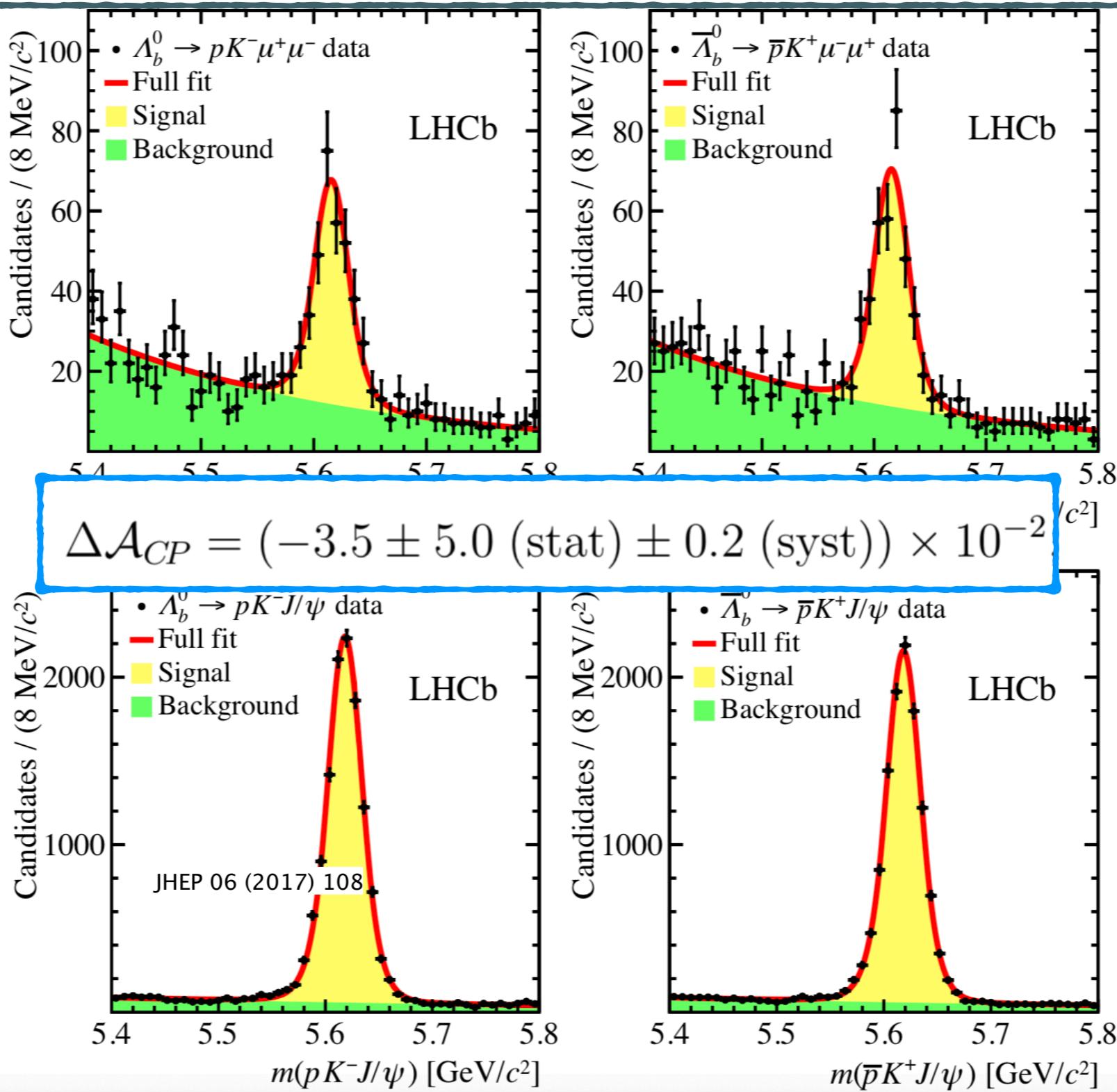
# $\Lambda_b^0 \rightarrow pK\mu\mu$

First  
observation,  
600 events in  
total



# $\Lambda_b^0 \rightarrow pK\mu\mu$

First  
observation,  
600 events in  
total



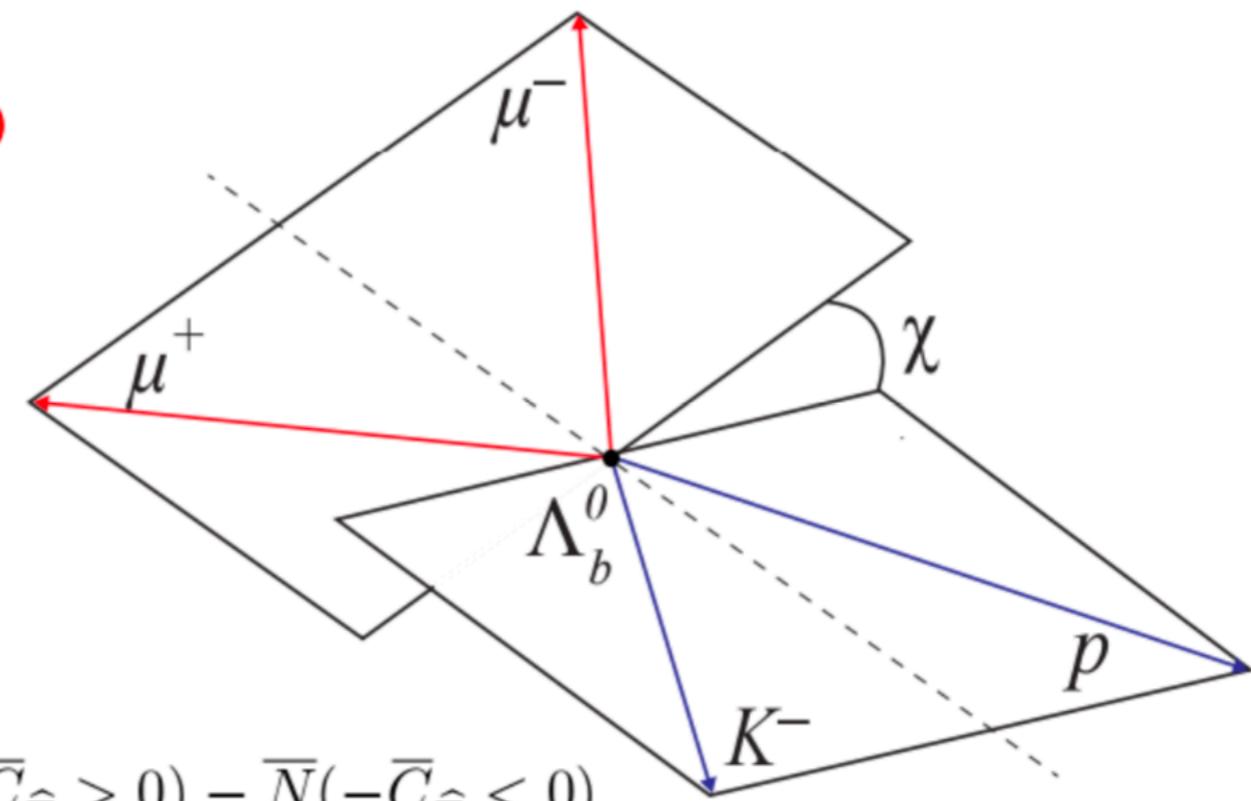
# $\Lambda_b^0 \rightarrow p K \mu\mu$

- Can also look at triple products (TP):

Triple product

proportional to

$$C_{\hat{T}} \equiv \vec{p}_{\mu^+} \cdot (\vec{p}_p \times \vec{p}_{K^-}),$$
$$\bar{C}_{\hat{T}} \equiv \vec{p}_{\mu^-} \cdot (\vec{p}_{\bar{p}} \times \vec{p}_{K^+}),$$



$$A_{\hat{T}} \equiv \frac{N(C_{\hat{T}} > 0) - N(C_{\hat{T}} < 0)}{N(C_{\hat{T}} > 0) + N(C_{\hat{T}} < 0)}, \quad \bar{A}_{\hat{T}} \equiv \frac{\bar{N}(-\bar{C}_{\hat{T}} > 0) - \bar{N}(-\bar{C}_{\hat{T}} < 0)}{\bar{N}(-\bar{C}_{\hat{T}} > 0) + \bar{N}(-\bar{C}_{\hat{T}} < 0)},$$

$$a_{CP}^{\hat{T}\text{-odd}} \equiv \frac{1}{2} (A_{\hat{T}} - \bar{A}_{\hat{T}}),$$

$$a_P^{\hat{T}\text{-odd}} \equiv \frac{1}{2} (A_{\hat{T}} + \bar{A}_{\hat{T}}),$$

# $\Lambda_b^0 \rightarrow p K \mu\mu$

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- Can also look at triple products (TP):
  - Triple products are CP-odd variables, which, because of how they are constructed, are not dependent on production effects
  - Non-zero triple product value  $\rightarrow$  CP-violation
- Also complement  $\mathcal{A}_{CP}$  search for CP violation as:

$\mathcal{A}_{CP}$	Triple product
$\frac{d\Gamma}{d\Phi} \Big _{CP-\text{odd}}^{\hat{T}-\text{even}} \propto a_1^e a_2^e \sin(\delta_1^e - \delta_2^e) \sin(\phi_1^e - \phi_2^e),$ Enhanced with large strong phase differences	$\frac{d\Gamma}{d\Phi} \Big _{CP-\text{odd}}^{\hat{T}-\text{odd}} \propto a_1^e a_1^o \cos(\delta_1^e - \delta_1^o) \sin(\phi_1^e - \phi_1^o),$ Enhanced with small strong phase differences

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# $\Lambda_b^0 \rightarrow p K \mu\mu$

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- Can also look at triple products (TP):
  - Triple products are CP-odd variables, which, because of how they are constructed, are not dependent on production effects
  - Non-zero triple product value  $\rightarrow$  CP-violation
- Also complement  $\mathcal{A}_{CP}$  search for CP violation as:

$$\frac{d\Gamma}{d\Phi} \Big|_{CP-\text{odd}}^{\hat{T}-\text{even}} \propto a_1^e a_2^e \sin(\delta_1^e - \delta_2^e) \sin$$

$\mathcal{A}_{CP}$

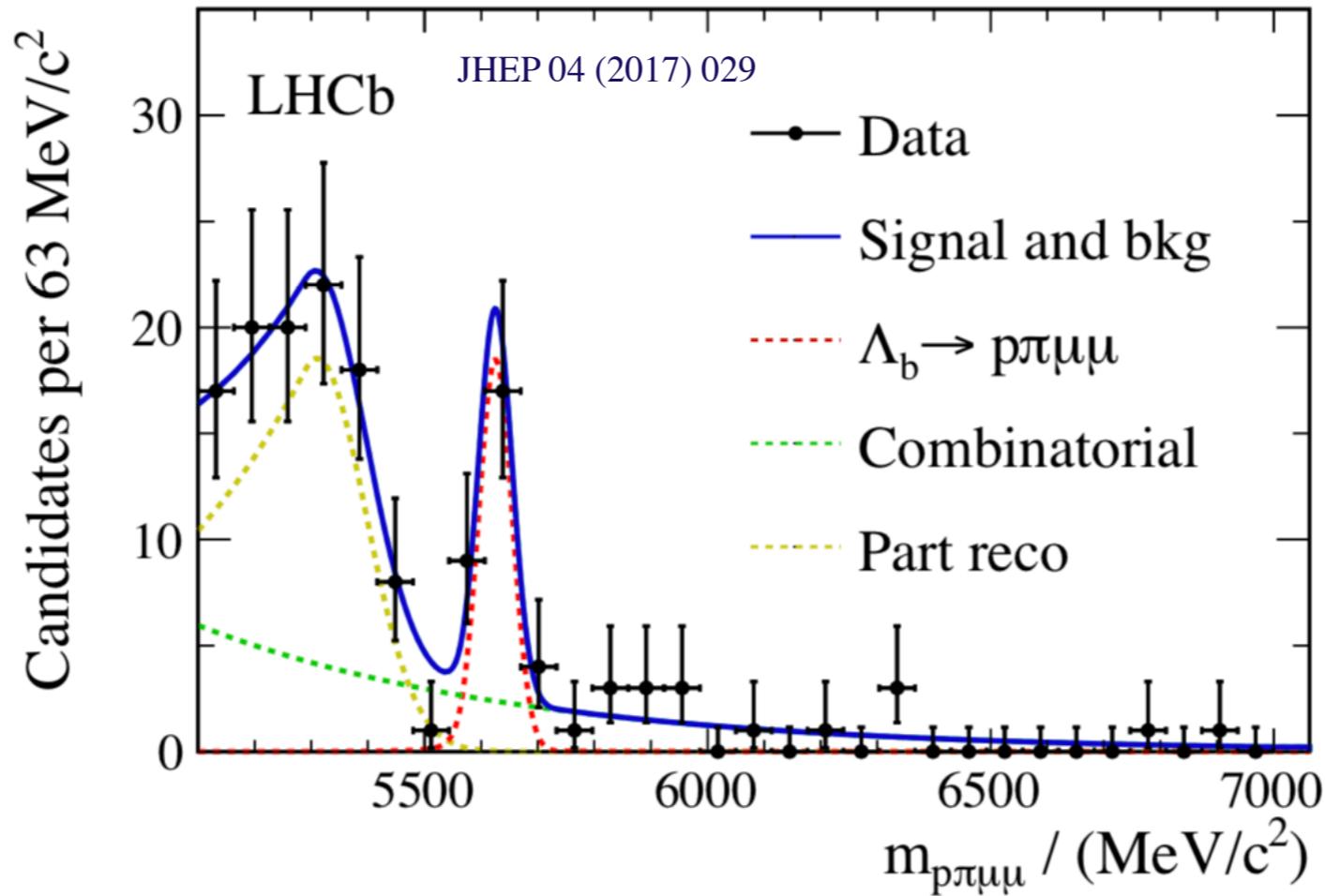
Triple product

$a_{CP}^{\hat{T}\text{-odd}} = (-1.2 \pm 5.0 \text{ (stat)} \pm 0.7 \text{ (syst)}) \times 10^{-2},$  $a_P^{\hat{T}\text{-odd}} = (-4.8 \pm 5.0 \text{ (stat)} \pm 0.7 \text{ (syst)}) \times 10^{-2}.$

Enhanced with large strong phase effect

# $\Lambda_b \rightarrow p\pi\mu\mu$ ?

- Equivalent to  $B^+ \rightarrow \pi^+\mu\mu$  in the meson sector
- Statistics should be high enough in Run 2 to do  $\mathcal{A}_{CP}$



# Summary

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- CP asymmetries offer good tests of the SM
- Different CP-violating observables are complementary with respect to both experimental effects and in relation to strong phase differences
- Angular CP-violating observables are effective at constraining Wilson Coefficient phases

Thank you for your attention!

# Expressions for $A_i$

JHEP 0807:106,2008

$$A_{\text{CP}} = \mathcal{A} \frac{8\hat{m}_b}{3\hat{s}} \text{Re} \left\{ \frac{\xi_{\parallel}^2}{\xi_{\perp}^2} \frac{M_B^2}{M_{K^*}^2} \frac{(1-\hat{s})^2}{8} \left[ \hat{m}_b \frac{|\mathcal{T}_{\parallel}^-|^2}{\xi_{\parallel}^2} - \frac{\mathcal{T}_{\parallel}^-}{\xi_{\parallel}} (C_9 - C'_9)^* \right] + \frac{\hat{m}_b}{\hat{s}} \frac{|\mathcal{T}_{\perp}^+|^2 + |\mathcal{T}_{\perp}^-|^2}{\xi_{\perp}^2} \right. \\ \left. + \frac{\mathcal{T}_{\perp}^+ - \mathcal{T}_{\perp}^-}{\xi_{\perp}} C_9^* + \frac{\mathcal{T}_{\perp}^+ + \mathcal{T}_{\perp}^-}{\xi_{\perp}} C'^*_9 - (\delta_W \rightarrow -\delta_W) \right\} + \mathcal{O}(m_l^2/q^2), \quad (\text{D.1})$$

$$A_3 = \mathcal{A} \frac{2\hat{m}_b \beta_l}{\hat{s}} \text{Re} \left\{ \frac{\hat{m}_b}{\hat{s}} \frac{|\mathcal{T}_{\perp}^+|^2 - |\mathcal{T}_{\perp}^-|^2}{\xi_{\perp}^2} + \frac{\mathcal{T}_{\perp}^+ - \mathcal{T}_{\perp}^-}{\xi_{\perp}} C_9^* + \frac{\mathcal{T}_{\perp}^+ + \mathcal{T}_{\perp}^-}{\xi_{\perp}} C'^*_9 - (\delta_W \rightarrow -\delta_W) \right\}, \quad (\text{D.2})$$

$$A_4^D = -\mathcal{A}^D \frac{\hat{m}_b \beta_l}{2\hat{s}} \text{Re} \left\{ \left( \frac{\mathcal{T}_{\perp}^-}{\xi_{\perp}} - \hat{s} \frac{\mathcal{T}_{\parallel}^-}{\xi_{\parallel}} \right) (C_9 - C'_9)^* - 2\hat{m}_b \frac{\mathcal{T}_{\perp}^- (\mathcal{T}_{\parallel}^-)^*}{\xi_{\perp} \xi_{\parallel}} - (\delta_W \rightarrow -\delta_W) \right\}, \quad (\text{D.3})$$

$$A_5^D = -\mathcal{A}^D \frac{\hat{m}_b}{\hat{s}} \text{Re} \left\{ \left( \frac{\mathcal{T}_{\perp}^-}{\xi_{\perp}} - \hat{s} \frac{\mathcal{T}_{\parallel}^-}{\xi_{\parallel}} \right) C_{10} - \left( \frac{\mathcal{T}_{\perp}^-}{\xi_{\perp}} + \hat{s} \frac{\mathcal{T}_{\parallel}^-}{\xi_{\parallel}} \right) C'^*_{10} - (\delta_W \rightarrow -\delta_W) \right\}, \quad (\text{D.4})$$

$$A_6 = \mathcal{A} \frac{4\hat{m}_b}{\hat{s}} \text{Re} \left\{ \frac{\mathcal{T}_{\perp}^+ + \mathcal{T}_{\perp}^-}{\xi_{\perp}} C_{10}^* - \frac{\mathcal{T}_{\perp}^+ - \mathcal{T}_{\perp}^-}{\xi_{\perp}} C'^*_{10} - (\delta_W \rightarrow -\delta_W) \right\}, \quad (\text{D.5})$$

# Expressions for Ai

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$$A_7^D = \mathcal{A}^D \frac{\hat{m}_b}{\hat{s}} \text{Im} \left\{ (C_{10} - C'_{10}) \left( \frac{\mathcal{T}_{\perp}^-}{\xi_{\perp}} + \hat{s} \frac{\mathcal{T}_{\parallel}^-}{\xi_{\parallel}} \right)^* - (\delta_W \rightarrow -\delta_W) \right\}, \quad (\text{D.6})$$

$$\begin{aligned} A_8^D = \mathcal{A}^D \frac{\beta_l}{2} \text{Im} & \left\{ \frac{2\hat{m}_b^2}{\hat{s}} \frac{\mathcal{T}_{\perp}^+ (\mathcal{T}_{\parallel}^-)^*}{\xi_{\perp} \xi_{\parallel}} - \frac{\hat{m}_b}{\hat{s}} \left[ \left( \frac{\mathcal{T}_{\perp}^+}{\xi_{\perp}} + \hat{s} \frac{\mathcal{T}_{\parallel}^-}{\xi_{\parallel}} \right) C_9^* - \left( \frac{\mathcal{T}_{\perp}^+}{\xi_{\perp}} - \hat{s} \frac{\mathcal{T}_{\parallel}^-}{\xi_{\parallel}} \right) C_9'^* \right] \right. \\ & \left. + C_9 C_9'^* + C_{10} C_{10}'^* - (\delta_W \rightarrow -\delta_W) \right\}, \end{aligned} \quad (\text{D.7})$$

$$\begin{aligned} A_9 = -\mathcal{A} 2\beta_l \text{Im} & \left\{ \frac{2\hat{m}_b^2}{\hat{s}^2} \frac{\mathcal{T}_{\perp}^+ (\mathcal{T}_{\perp}^-)^*}{\xi_{\perp}^2} + \frac{\hat{m}_b}{\hat{s}} \left[ \frac{\mathcal{T}_{\perp}^+ - \mathcal{T}_{\perp}^-}{\xi_{\perp}} C_9^* - \frac{\mathcal{T}_{\perp}^+ + \mathcal{T}_{\perp}^-}{\xi_{\perp}} C_9'^* \right] \right. \\ & \left. - C_9 C_9'^* - C_{10} C_{10}'^* - (\delta_W \rightarrow -\delta_W) \right\}, \end{aligned} \quad (\text{D.8})$$