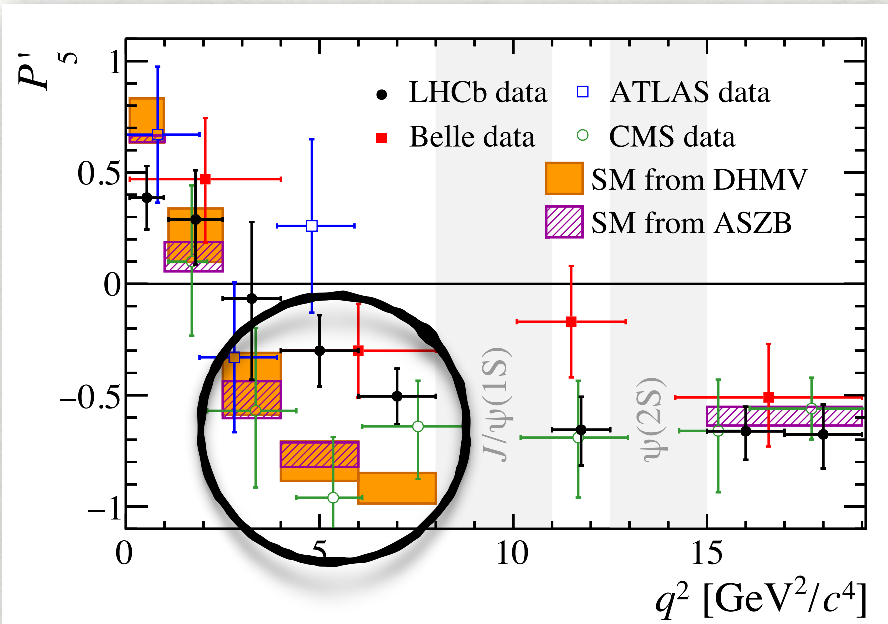
Long distance effects in inclusive rare B decays & phenomenology of $\bar{B} \to X_d \ell^+ \ell^-$

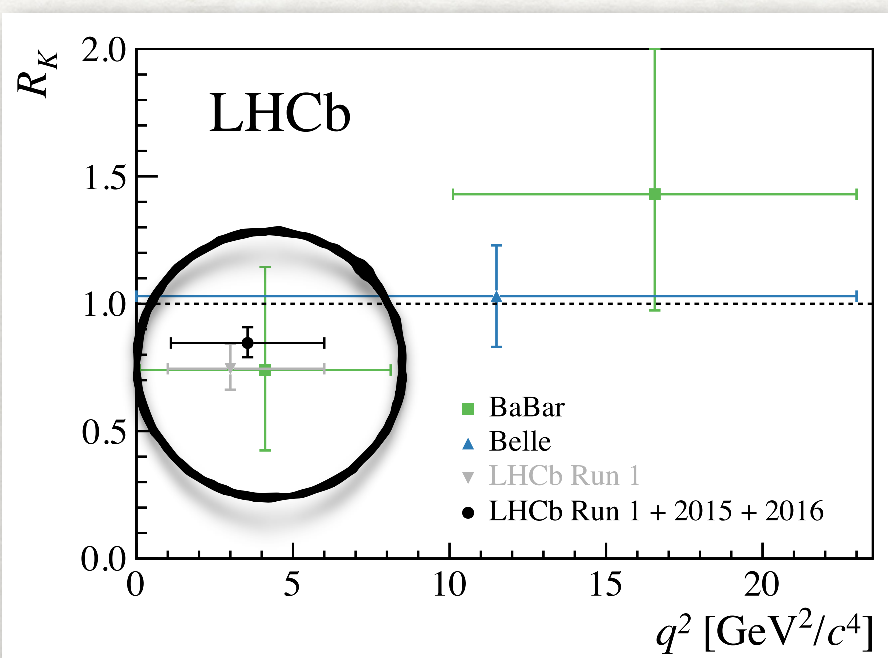
Jack Jenkins

$b \to s\ell^+\ell^-$ anomalies

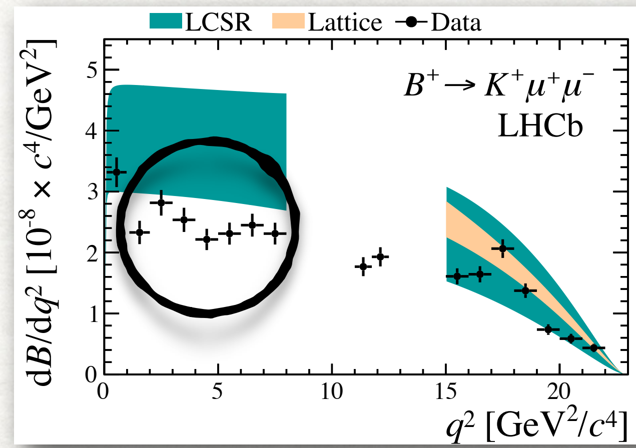
Angular Observables

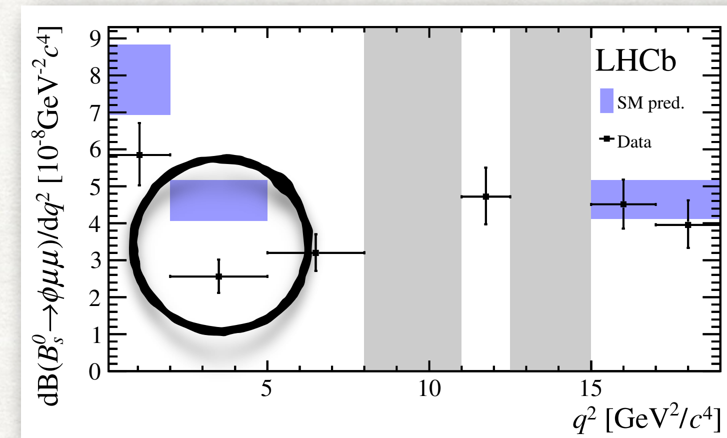


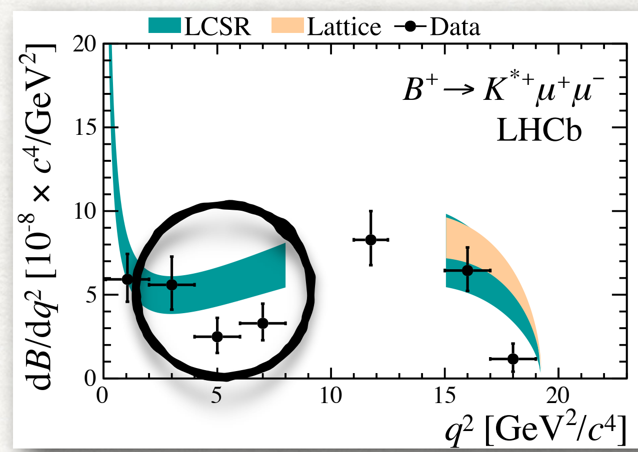
LFV Ratios

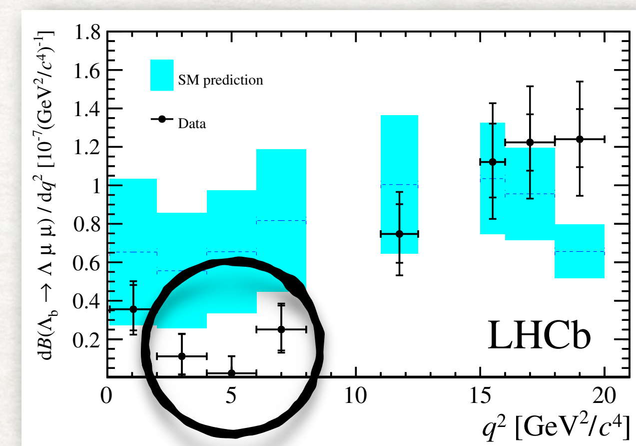


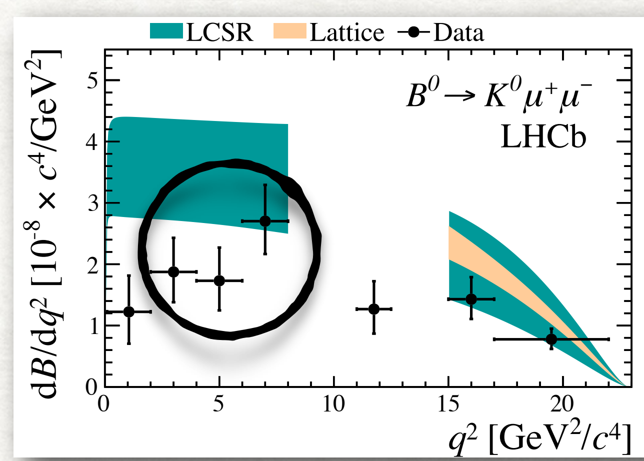
Branching Ratios

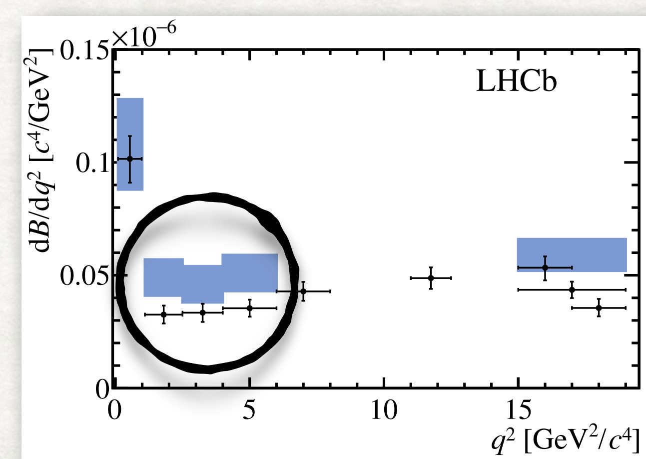












$b \to s\ell^+\ell^-$ anomalies

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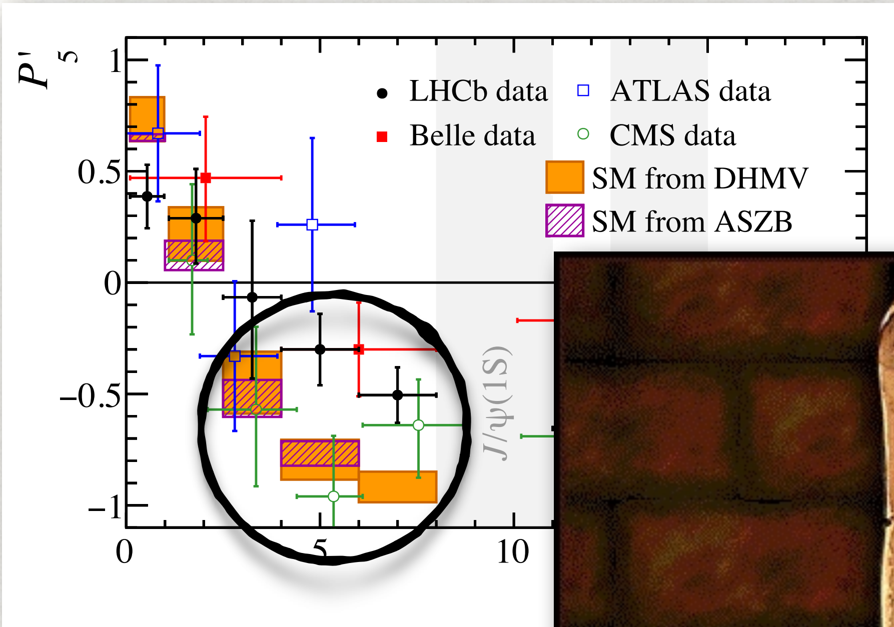
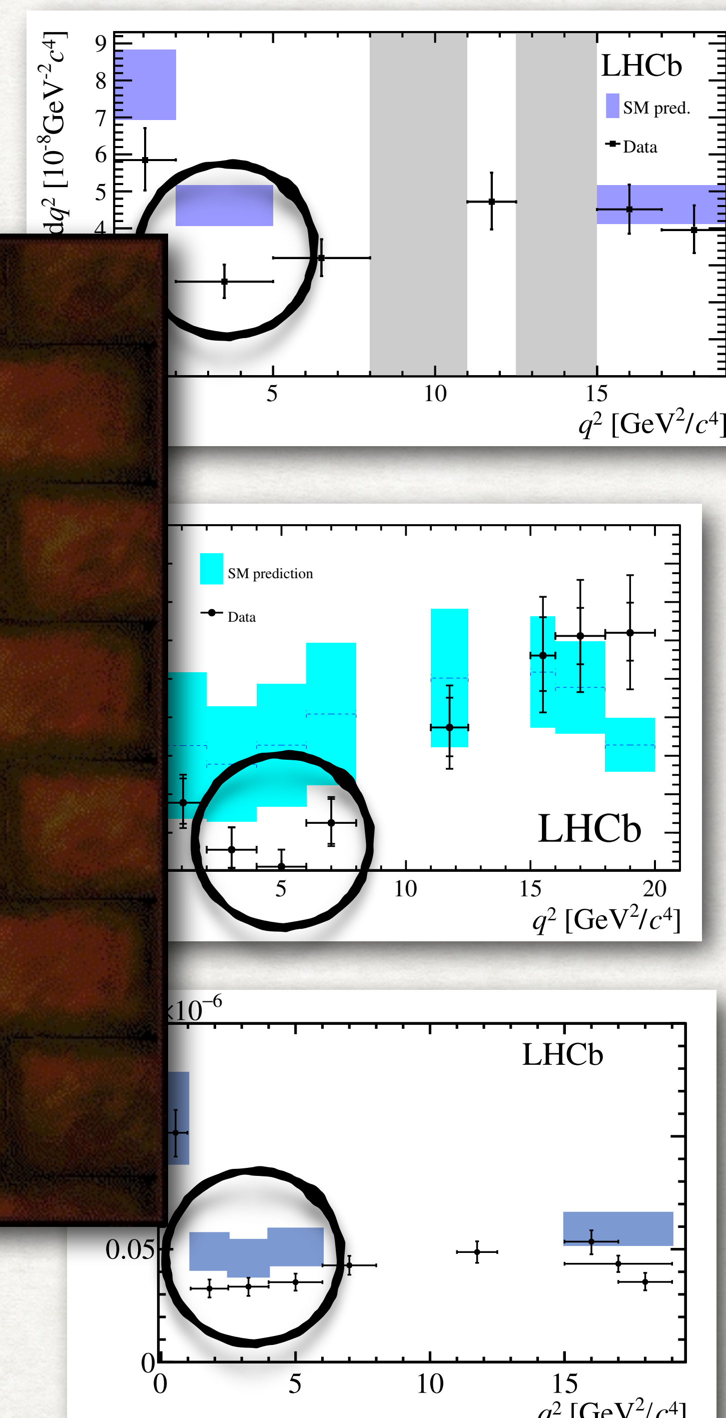
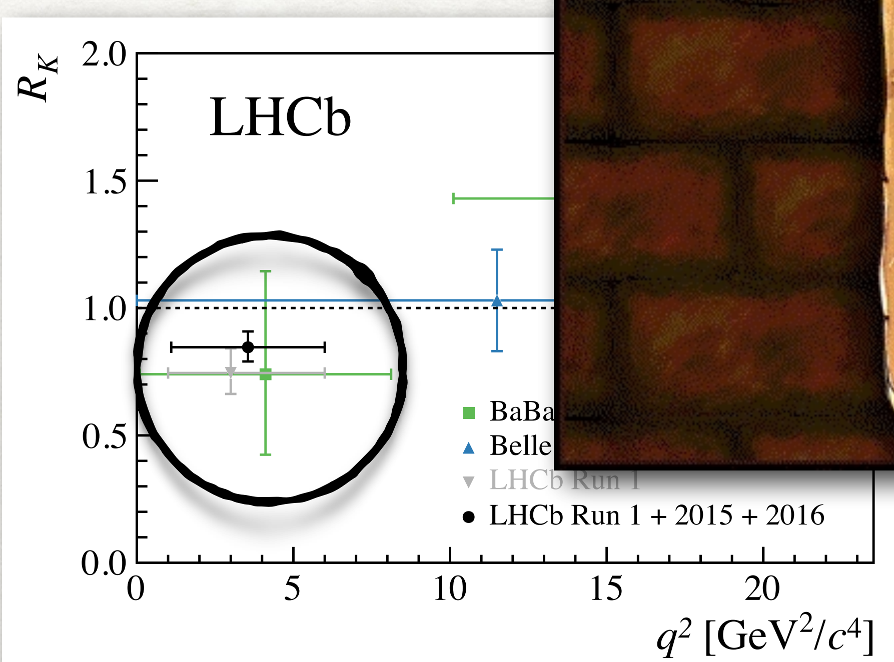


Branching Ratios



LFV Ratio

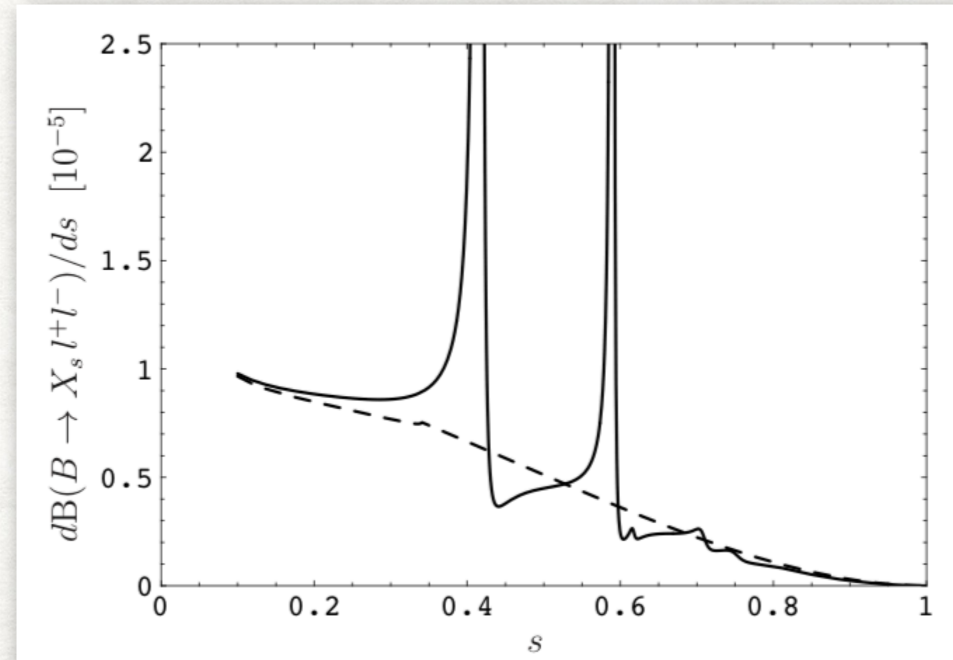


The big picture

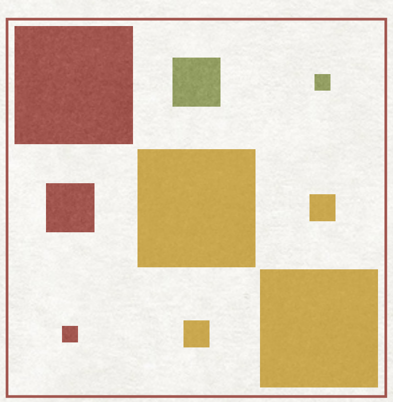
- The SM is renormalizable and perturbatively unitary up to super high scales, so it isn't clear where BSM physics will be found directly.

- BSM physics may leave a fingerprint over the landscape of flavor physics observables.

- Inclusive and exclusive B-decays are treated quite differently (both experiment and theory), and are complementary



$b \to s\ell^+\ell^-$

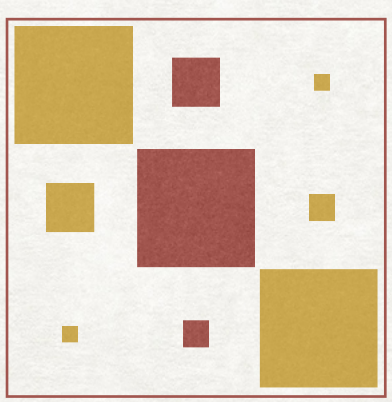


$|V_{ub}V_{us}| \sim \lambda^4$

$|V_{cb}V_{cs}| \sim \lambda^2$

$|V_{tb}V_{ts}| \sim \lambda^2$

$b \to d\ell^+\ell^-$



$|V_{ub}V_{ud}| \sim \lambda^3$

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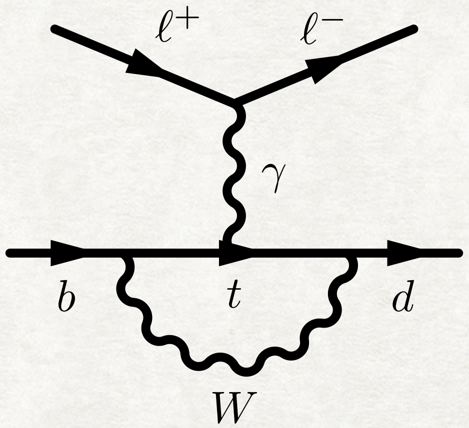

Outline

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- Inclusive FCNC overview
  - Electroweak Wilson coefficients & the OPE
  - Perturbative corrections to the leading power
  - Local & nonlocal power corrections

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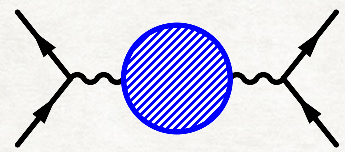
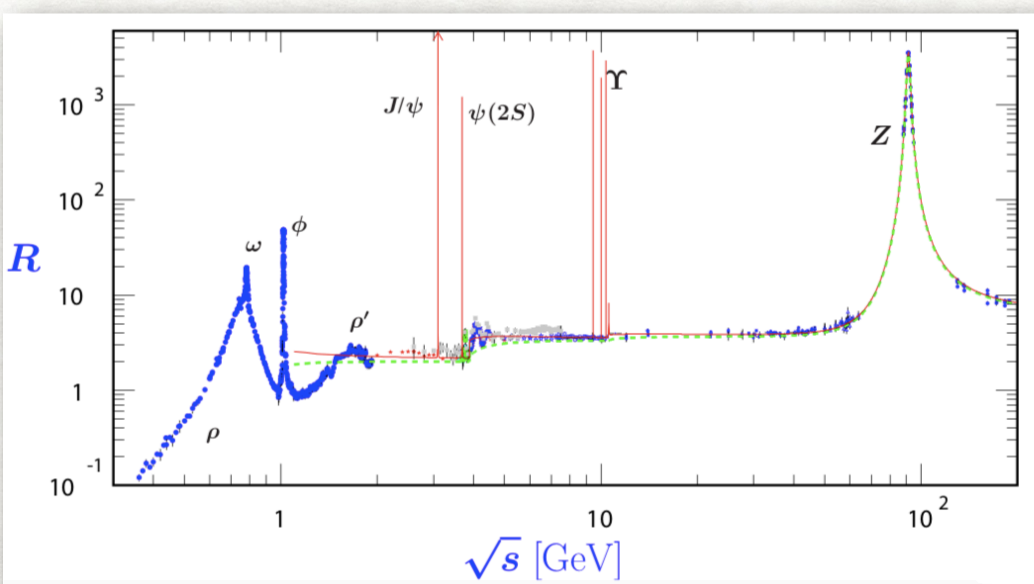
- Predictions for $\bar{B} \to X_d\ell^+\ell^-$ rates and optimized observables

OPE and quark-hadron duality

Unitarity

Quark-hadron duality

$$\sigma(e^+e^- \to \text{hadrons}) \sim \text{Im}\Big[$$

$$\Big] \longrightarrow \int d^4x\; e^{iqx}\big\langle T\{J^{\mu}_{em}(0)J^{\nu\dagger}_{em}(x)\}\big\rangle g_{\mu\nu}$$



OPE and quark-hadron duality

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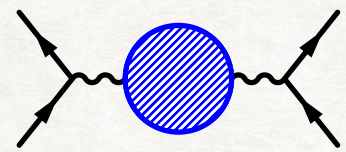
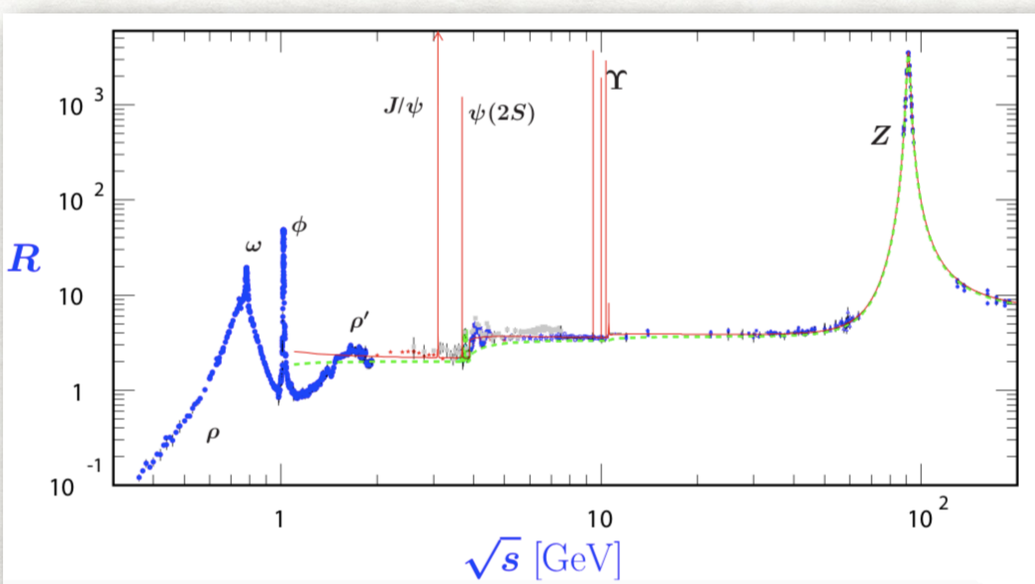
$\sigma(e^+e^- \to \text{hadrons}) \sim \text{Im}\big[$  $\big]$

Quark-hadron duality

$$\int d^4x \; e^{iqx}\big\langle T\{J^\mu_{em}(0)J^{\nu\dagger}_{em}(x)\}\big\rangle g_{\mu\nu}$$

Analyticity & OPE matching with Euclidean momenta

$$C_0(q^2)\big\langle 1\big\rangle + \sum_q C_q(q^2)\big\langle \bar{q}q\big\rangle + C_G(q^2)\big\langle Tr[G_{\mu\nu}G^{\mu\nu}]\big\rangle$$



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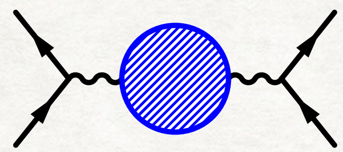
$\sigma(e^+e^- \to \text{hadrons}) \sim \text{Im}\Big[$  $\Big]$

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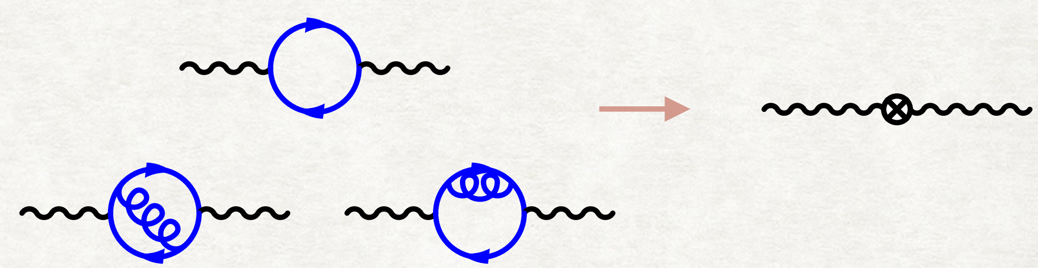
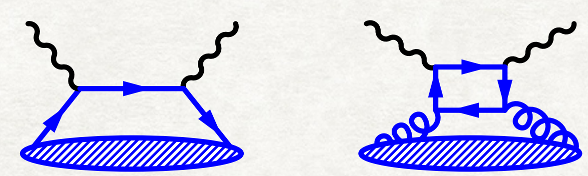
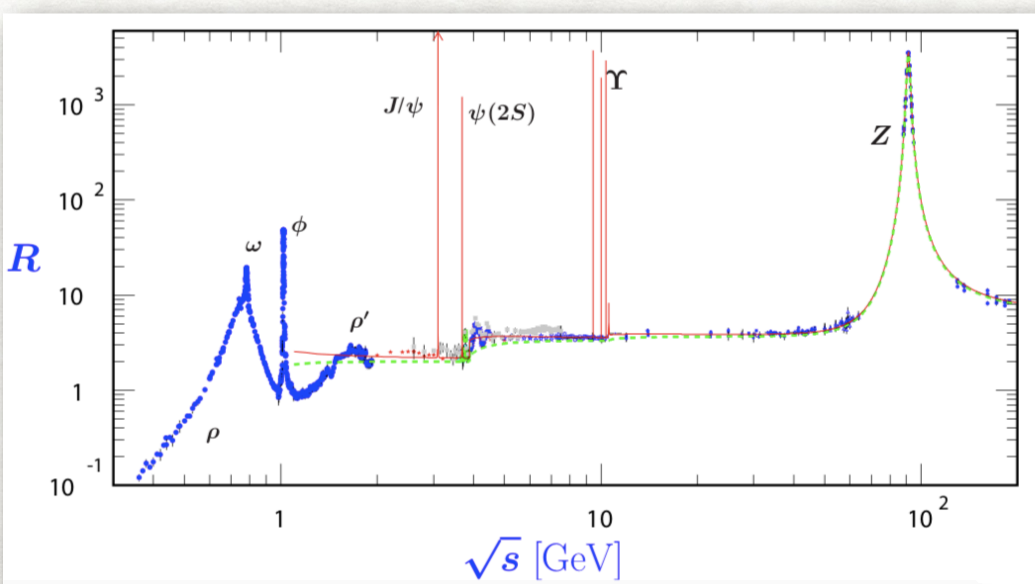




Power corrections



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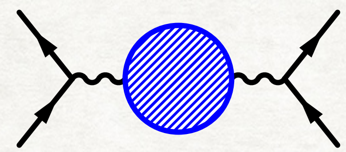
$\sigma(e^+e^- \to \text{hadrons}) \sim \text{Im}\Big[$  $\Big]$

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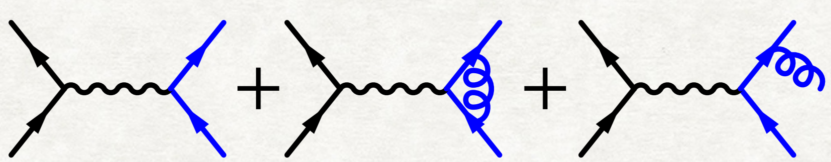
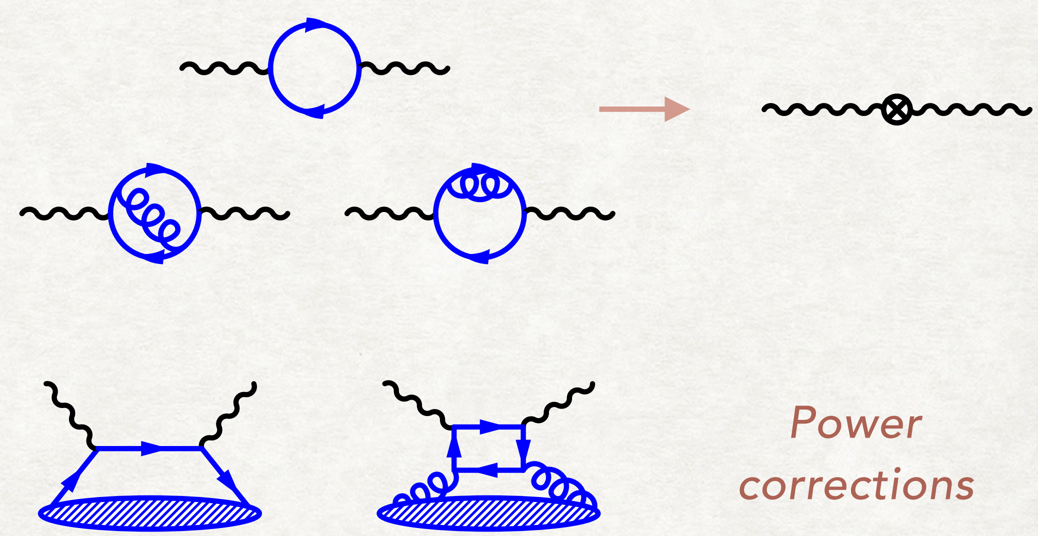
Leading power description

$\sigma \sim \int d\Pi \Big|$  $\Big|^2$



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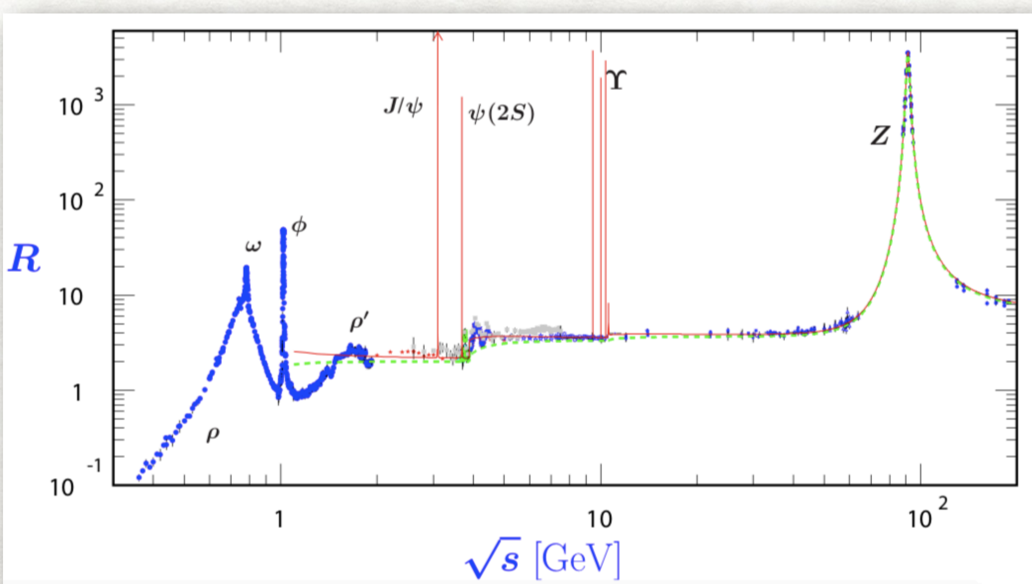
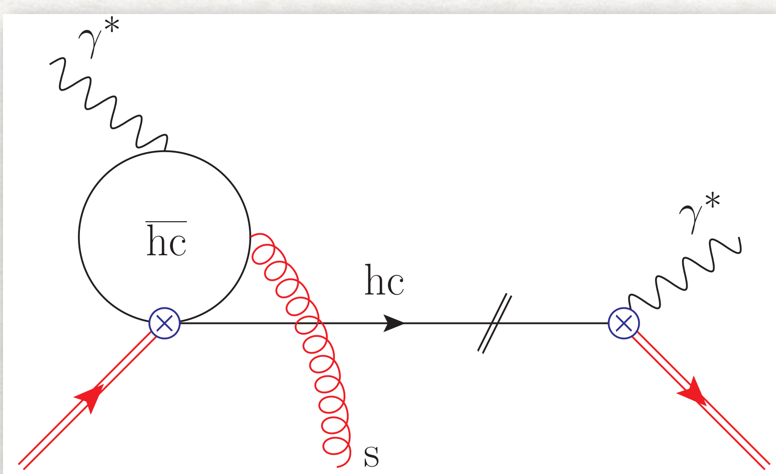


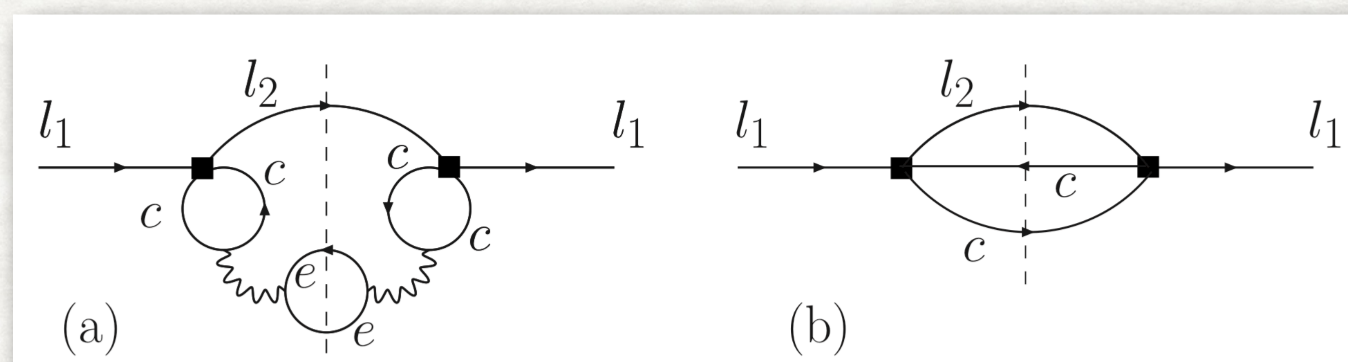
Jack Jenkins

OPE and quark-hadron duality

- Heavy quark inclusive decays:
  - CC: $\bar{B} \to X_c \ell^- \bar{\nu}$, $\bar{B} \to X_u \ell^- \bar{\nu}$, $D \to X_s \ell^+ \nu$
  - FCNC: $\bar{B} \to X_{s(d)} \gamma$, $\bar{B} \to X_{s(d)} \ell^+ \ell^-$, $\bar{B} \to X_{s(d)} \nu \bar{\nu}$
- The OPE here is different (and problematic) for a number of reasons:
  - Hadron in the initial state → power corrections can be nonlocal
  - The matching itself is nonperturbative → input from hadronic amplitudes are needed
  - Kinematic cuts required by experiment → OPE isn't directly applicable



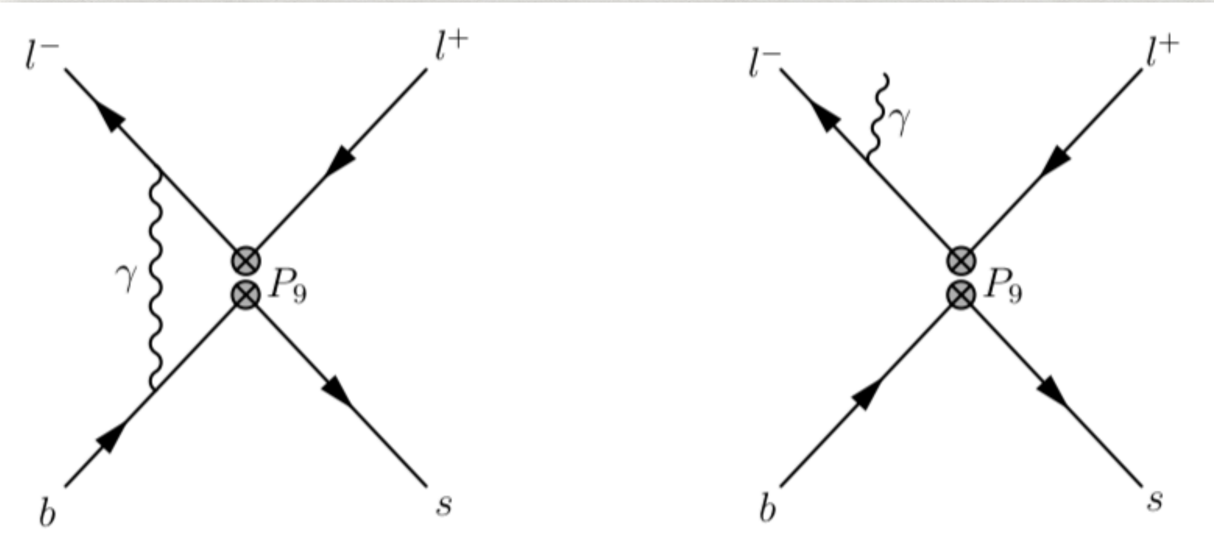
Hurth, Benzke, Fickinger, Turczyk
arXiv: 1711.01162



Beneke, Buchalla, Neubert, Sachrajda
arXiv: 0902.4446

QED corrections

Radiative Corrections



- $\alpha \ln(M_W/m_b)$ logs appear in the Wilson coefficients and are not resummed in the RGEs. The effective expansion is in α_s and $\kappa = \alpha/\alpha_s$.

  Bobeth, Gambino, Gorbahn, Haisch
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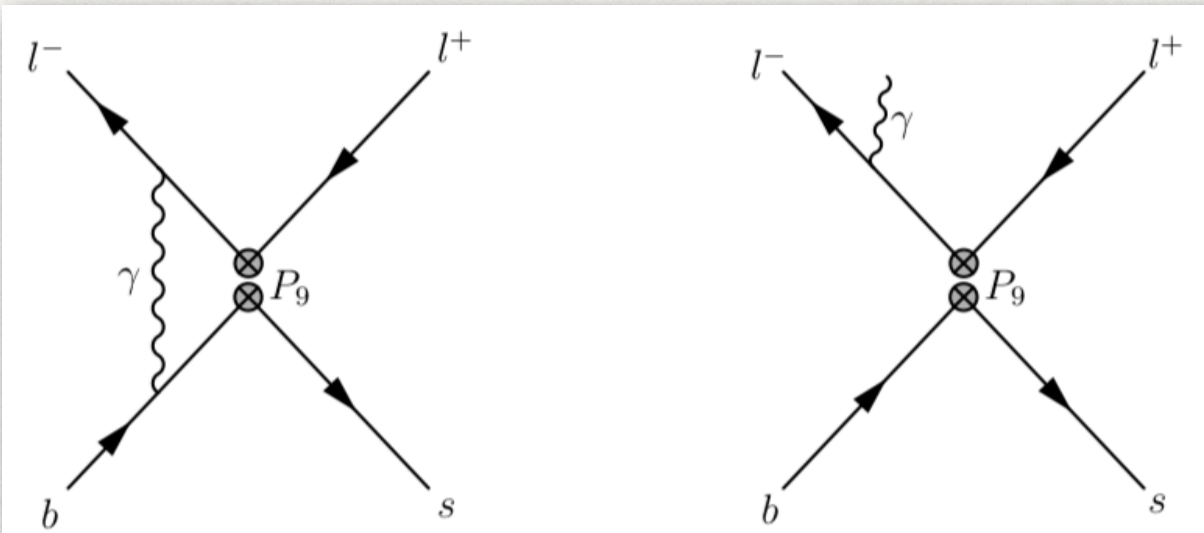

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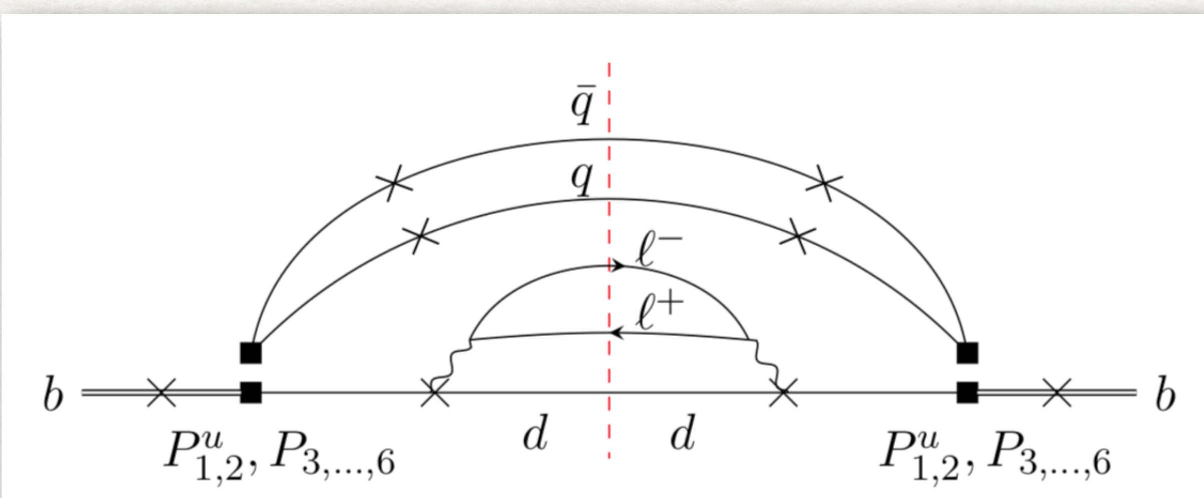
Five-body contributions



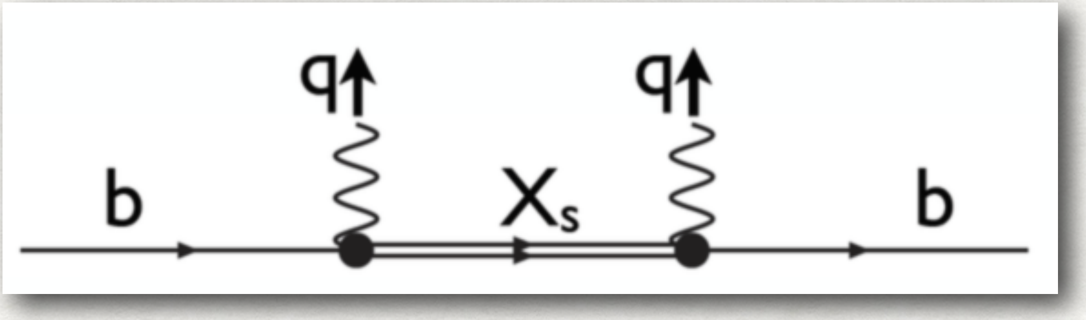
- Suppressed at high q^2 by the phase space
- $P_{1,2} - P_{1,2}$ interference dominates
- More important for $b \to d$ than $b \to s$ (CKM)

Huber, Qin, Vos
arXiv: 1806.11521

Breakdown of the heavy mass expansion

- At large q^2 the power corrections are large due to the kinematics of small invariant hadronic mass

$$p_X^2 = (p_b - q)^2 = m_b^2 + q^2 - 2m_b q_0$$
$$< m_b^2 + q^2 - 2m_b\sqrt{q^2} = (m_b - \sqrt{q^2})^2$$



$$\langle B|\bar{b}b|B\rangle = 1$$
$$\langle B|\bar{b}D^2b|B\rangle = \lambda_1 \longleftarrow \boxed{\text{Fermi Motion from } \bar{B} \to X_c \ell \nu}$$
$$\langle B|\bar{b}\sigma^{\mu\nu}G_{\mu\nu}b|B\rangle = \lambda_2 \longleftarrow \boxed{B - B^* \text{ mass difference}}$$

Matrix elements of $d > 5$ operators:
Gambino, Healey, Turczyk

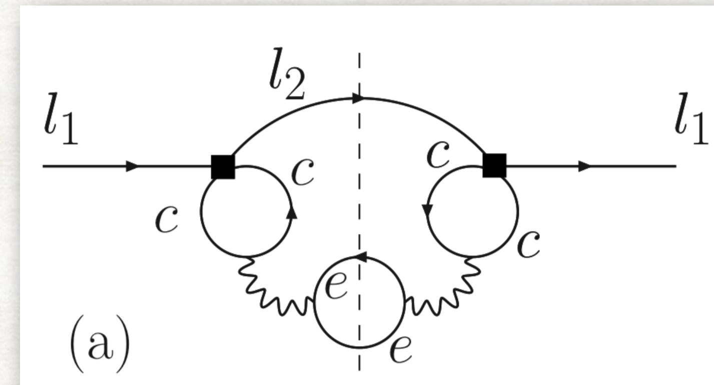
- Normalizing to inclusive $\bar{B} \to X_u \ell \nu$ integrated over the same q^2 region dramatically reduces uncertainty from the power corrections (~40% to ~10%)

- At low q^2 the local power corrections are well-known but there are nonlocal power corrections in SCET (more from TH's talk)

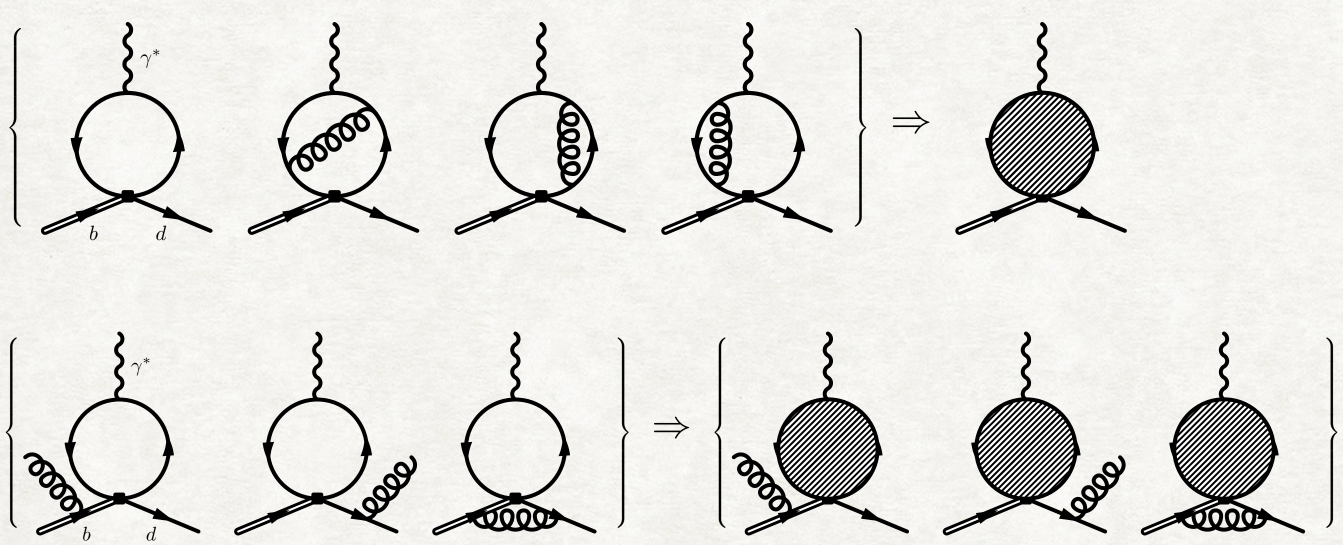
Krüger-Sehgal mechanism

- We would like to expand the terms $\left\langle P_1^c(0), P_9(x) \right\rangle$, $\left\langle P_2^u(0), P_1^u(x) \right\rangle$ etc. as was done for all the others.

- The intermediate state can consist of a $q\bar{q}$ resonance, and matching to pNRQCD in the charm sector is nonperturbative. The OPE isn't dead, but t<u>he Wilson coefficients of the OPE are nonperturbative (!)</u>



- Use data on $e^+e^- \to X_{c\bar{c}}$ and a dispersion relation to calculate these Wilson coefficients $b \to d(c\bar{c} \to \ell^+\ell^-)$



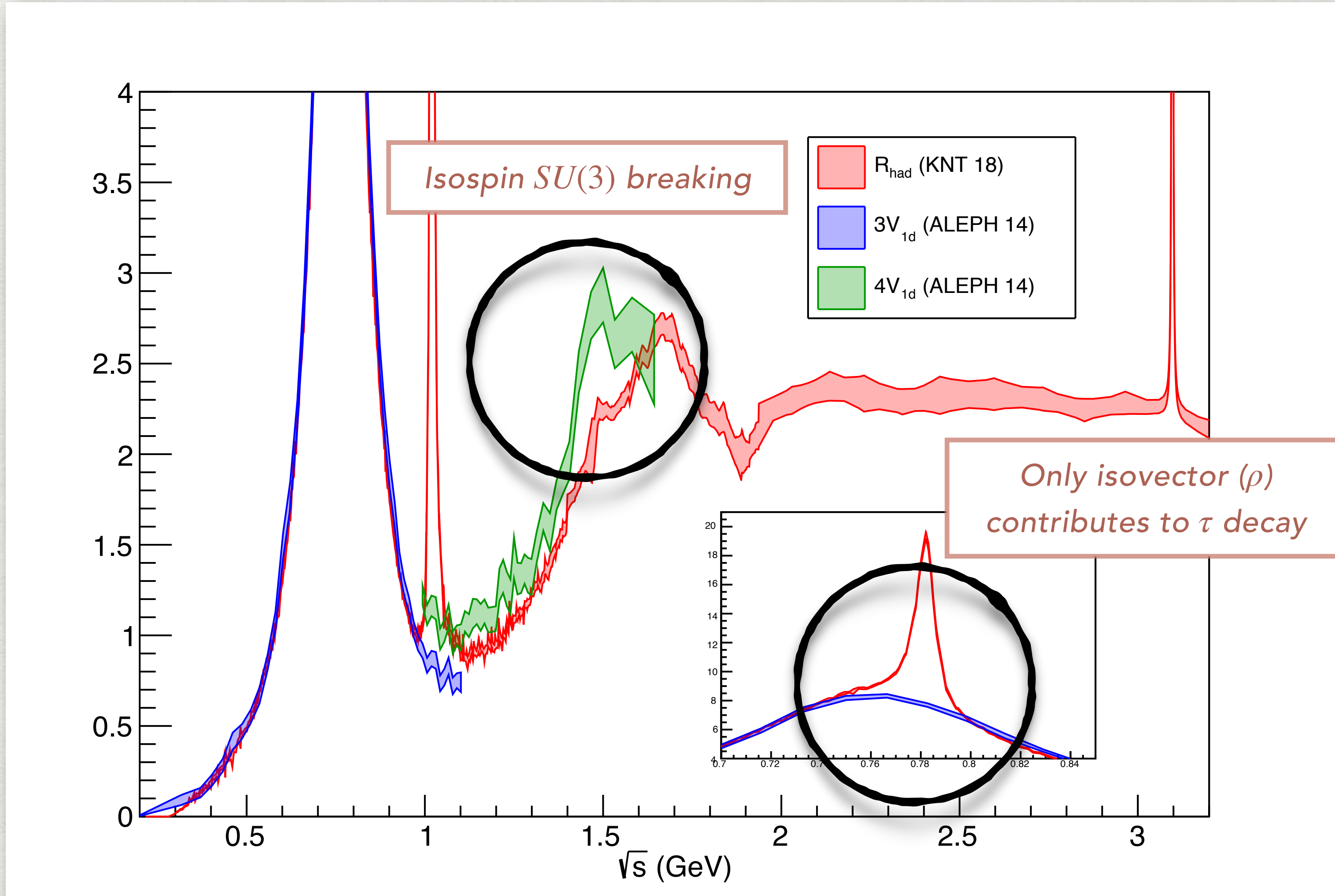
$$\text{Im}[h_c] = \frac{\pi}{3} R_{had}$$

$$\text{Re}[h_q(s)] = \text{Re}[h_q(s_0)] + \frac{s - s_0}{\pi} \int_0^\infty \frac{\text{Im}[h_q(t)]}{(t-s)(t-s_0)}$$

Perturbative for $s_0 \sim -\mu_b^2$

Krüger, Sehgal
arXiv: 9603237

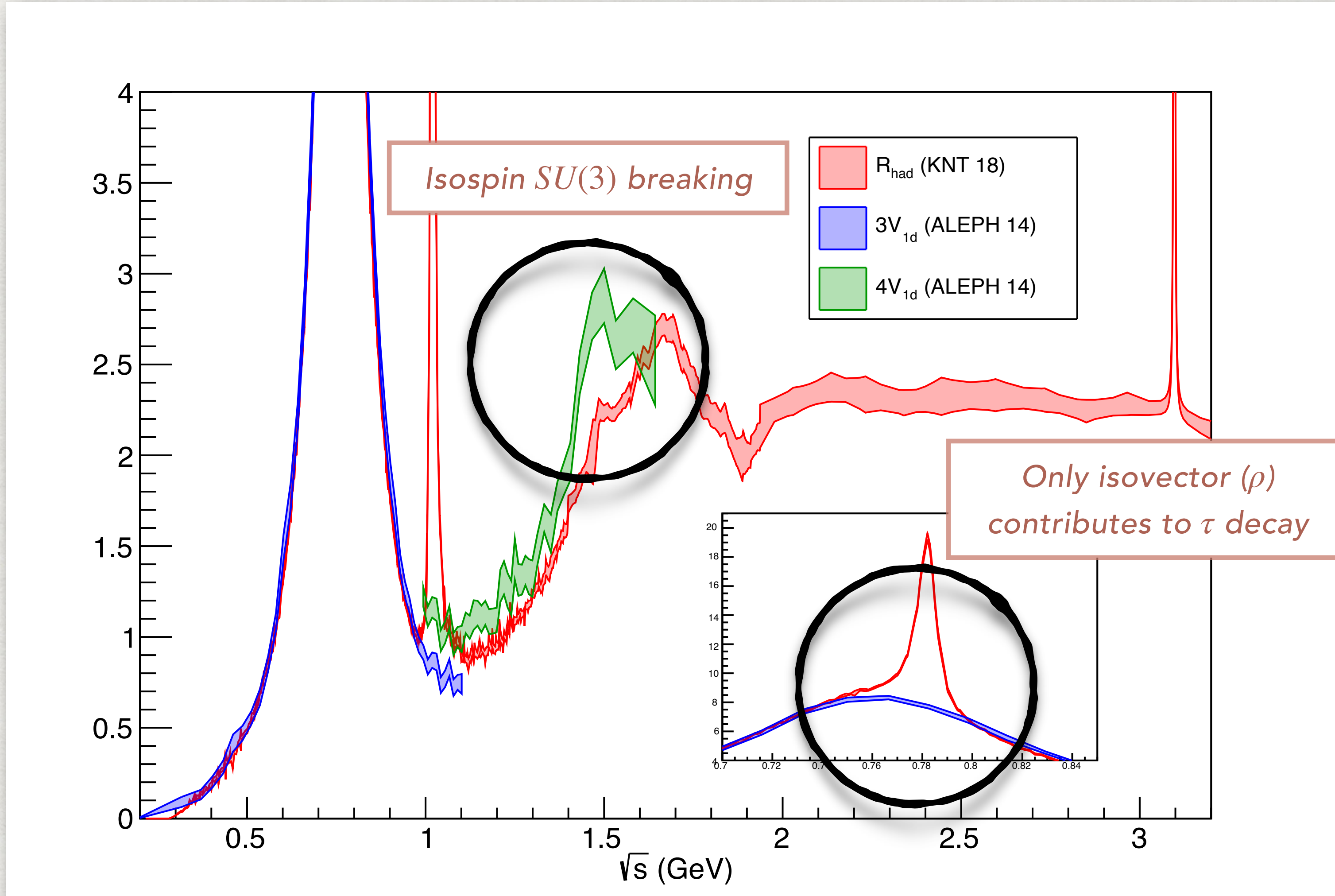
Krüger-Sehgal mechanism



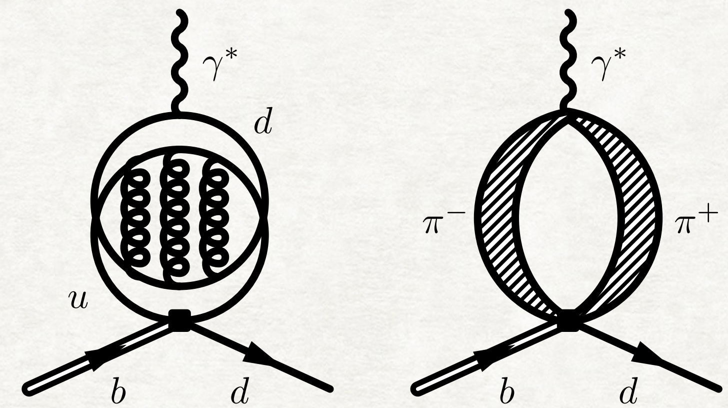
$$\frac{d\mathscr{B}(\tau \to V_{1d}\nu)}{ds} = \frac{6|V_{ud}|^2}{m_\tau^2}\frac{\mathscr{B}(\tau \to e\nu\bar{\nu})}{\mathscr{B}(\tau \to V_{1d}\nu_\tau)}\left(1 - \frac{s}{m_\tau}\right)^2\left(1 + \frac{2s}{m_\tau^2}\right)V_{1d}(s)$$

$$V_{1d} = 2\pi\ \mathrm{Im}[\Pi_{\bar{u}d}] \qquad (q^\mu q^\nu - q^2 g^{\mu\nu})\Pi_{\bar{u}d}(q^2) = i\int d^4x\ e^{iqx}\left\langle T\{J^\mu_{\bar{u}d}(0)J^{\nu\dagger}_{\bar{u}d}(x)\}\right\rangle$$

Krüger-Sehgal mechanism



- The data from $e^+e^- \to$ hadrons in the light quark resonance region corresponds to more than one loop function, not just the up-quark



$$\rho : \omega : \phi = \begin{cases} 9:1:2 & e^+e^- \to \text{hadrons} \\ 3:1:0 & \bar{B} \to X_d \ell^+\ell^- \\ 1:0:0 & \tau \to X\nu \end{cases}$$

$$\frac{d\mathscr{B}(\tau \to V_{1d}\nu)}{ds} = \frac{6|V_{ud}|^2}{m_\tau^2} \frac{\mathscr{B}(\tau \to e\nu\bar{\nu})}{\mathscr{B}(\tau \to V_{1d}\nu_\tau)} \left(1 - \frac{s}{m_\tau}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right) V_{1d}(s)$$

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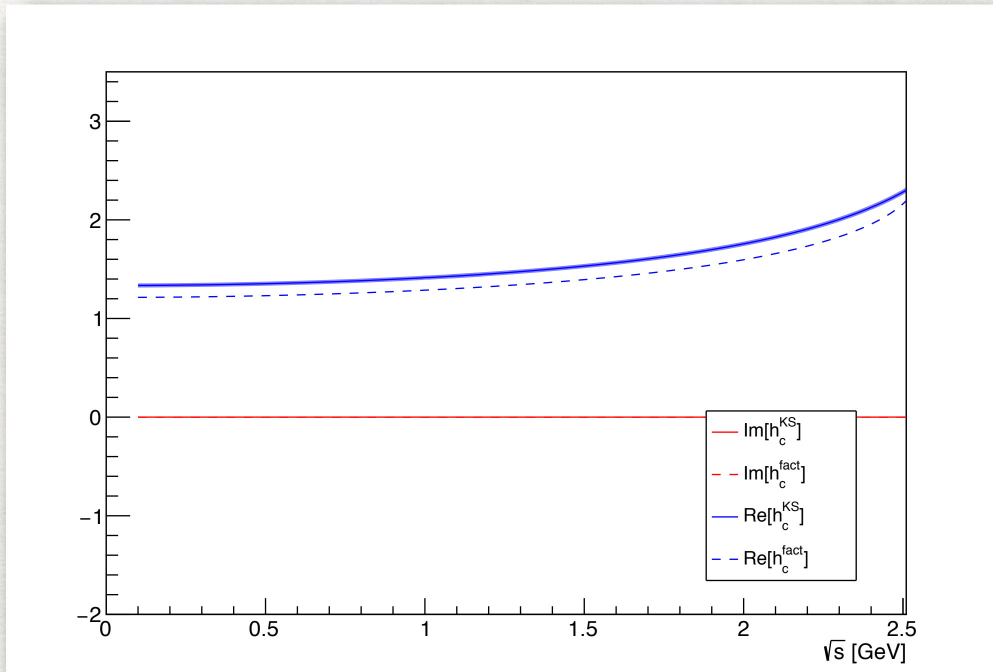
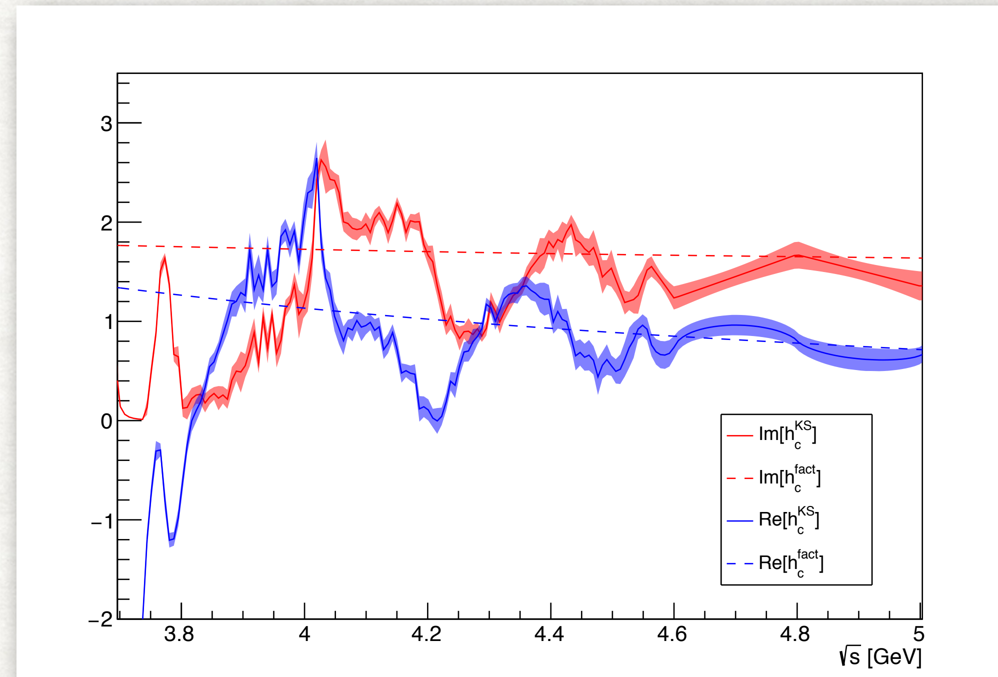
Krüger-Sehgal mechanism

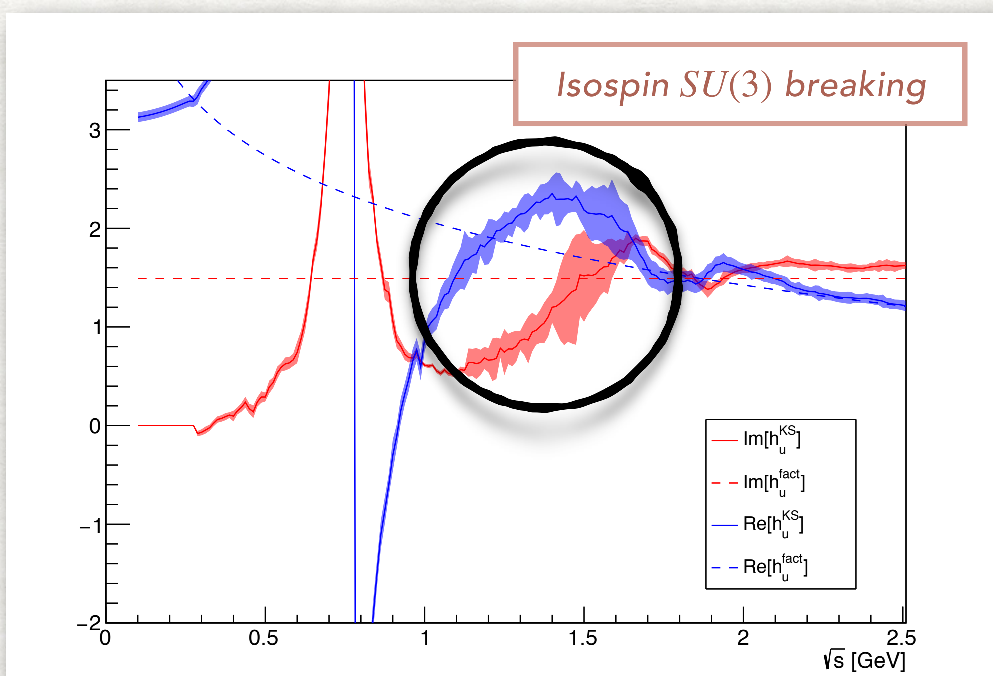
Low q^2 | High q^2

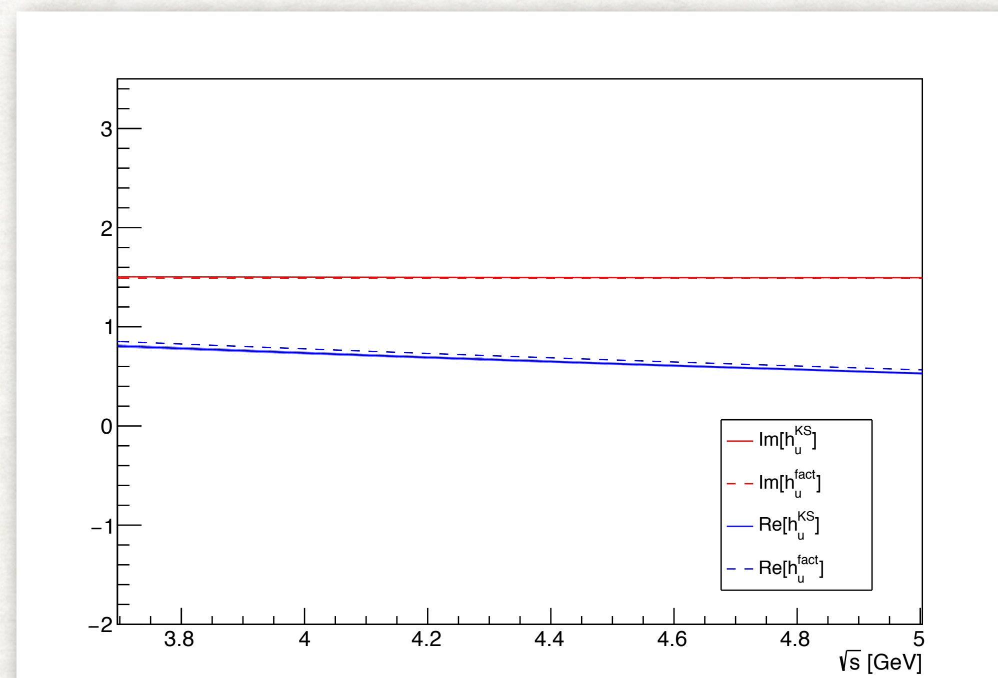
Charm





Up



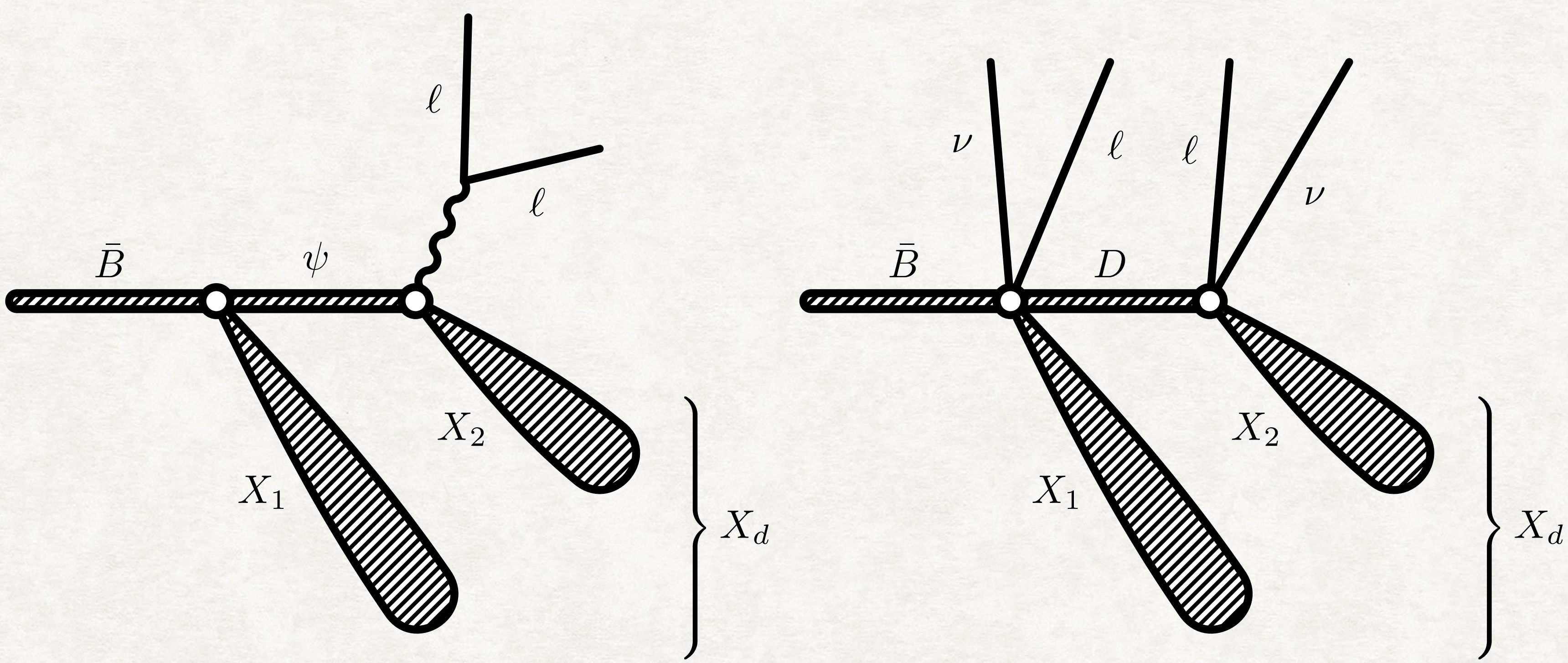


Charmonium cascade contributions

- $\psi \to \ell^+\ell^-$ separates the low-q^2 and high-q^2 regions

- $\psi \to X\ell^+\ell^-$ contributes in the low-q^2 region. Also $(\eta_c, \chi_{cJ}, h_c, \ldots) \to X\ell^+\ell^-$ contributes either directly or through charmonium feed-down, but rates are smaller

- In the narrow width approximation these effects do not interfere with short distance amplitudes sensitive to NP. They can be accounted for using data on $\bar{B} \to X_{s(d)}\psi$ and $\psi \to X\ell^+\ell^-$.



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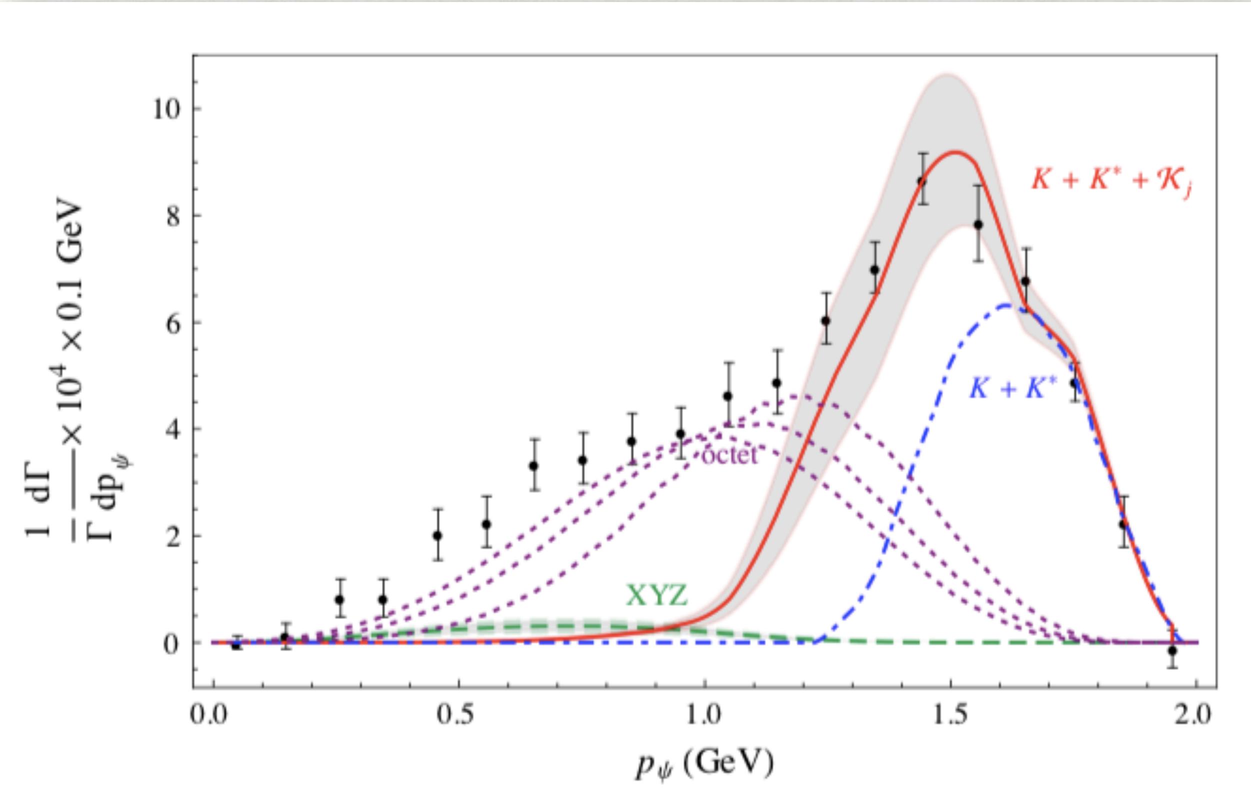
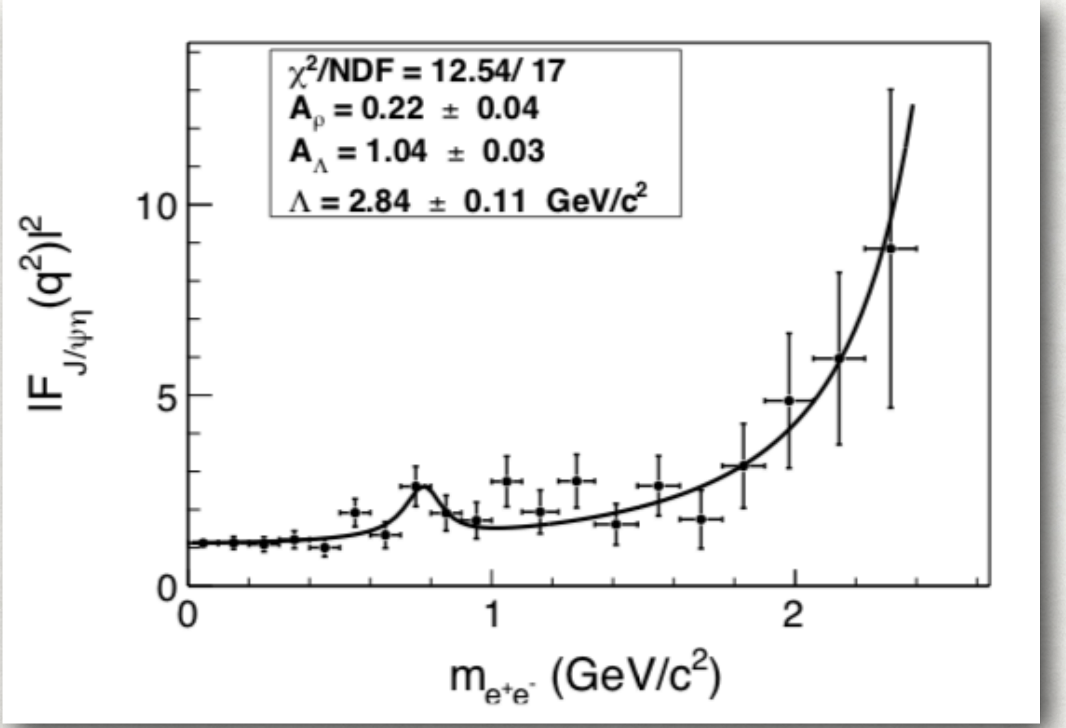
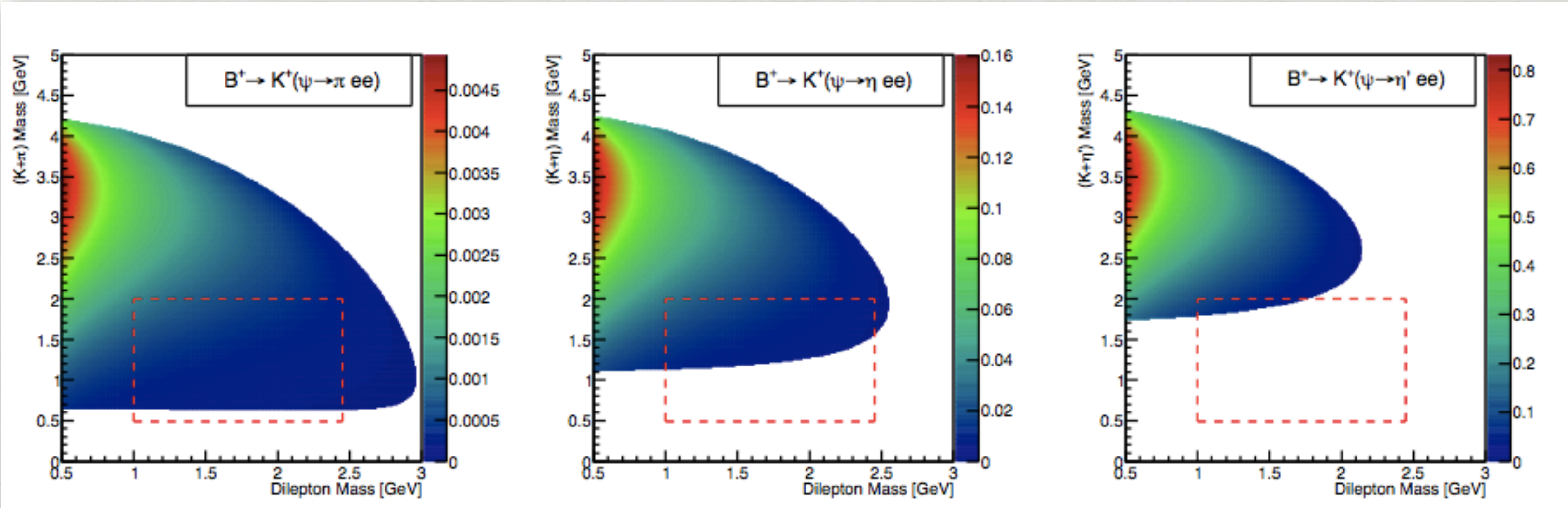




- Small $p_\psi \rightarrow$ large hadronic mass, induced by color octet current
- Charmonium is polarized longitudinally

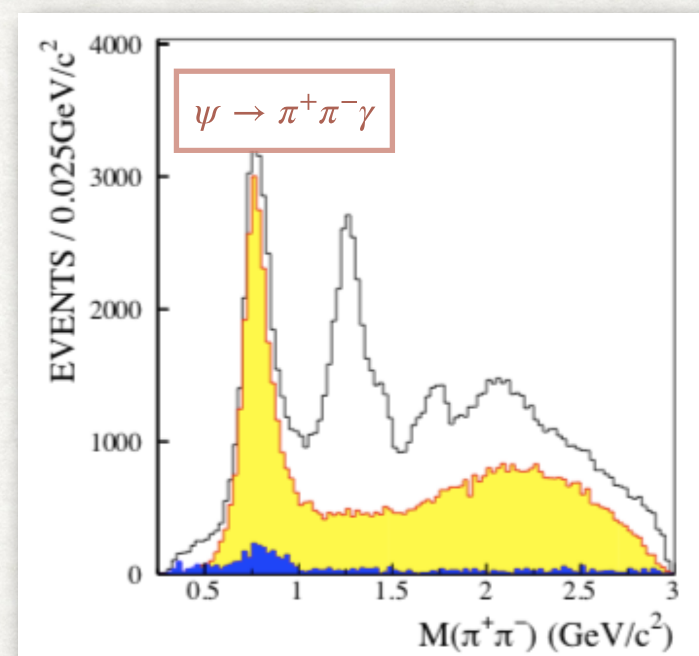
$$\frac{d\Gamma(\psi \to \eta \ell^+ \ell^-)}{dq^2 \Gamma(\psi \to \eta\gamma)} = \frac{\alpha}{3\pi q^2} \left[\left(1 + \frac{q^2}{m_\psi^2 - m_\eta^2} \right)^2 - \frac{4 m_\psi^2 q^2}{(m_\psi^2 - m_\eta^2)^2} \right]^{3/2} |F(q^2)|^2$$

Charmonium cascade contributions



- The cascade contributions form a background for low q^2 but are reduced well below the % level by the M_X cut

- Selected study of missing exclusive modes from $\psi \to X\gamma$ supports this conclusion



Results for $\bar{B} \to X_d \ell^+ \ell^-$

Huber et. al.
arXiv: 1908.07507

Branching ratio, low q^2

$$\mathscr{B}[1,6]_{ee} = (7.81 \pm 0.37_{scale} \pm 0.08_{m_t} \pm 0.17_{C,m_c} \pm 0.08_{m_b} \pm 0.04_{\alpha_s} \pm 0.15_{CKM} \pm 0.12_{BR_{sl}} \pm 0.05_{\lambda_2} \pm 0.39_{res}) \times 10^{-8}$$
$$= (7.81 \pm 0.61) \times 10^{-8}$$
$$\mathscr{B}[1,6]_{\mu\mu} = 7.59 \pm 0.35_{scale} \pm 0.08_{m_t} \pm 0.17_{C,m_c} \pm 0.09_{m_b} \pm 0.04_{\alpha_s} \pm 0.14_{CKM} \pm 0.11_{BR_{sl}} \pm 0.05_{\lambda_2} \pm 0.38_{res}) \times 10^{-8}$$
$$= (7.59 \pm 0.59) \times 10^{-8}$$

Branching ratio and Zoltan ratio, high q^2

$$\mathscr{B}[>14.4]_{ee} = (0.86 \pm 0.12_{scale} \pm 0.01_{m_t} \pm 0.01_{C,m_c} \pm 0.08_{m_b} \pm 0.02_{CKM} \pm 0.02_{BR_{sl}} \pm 0.06_{\lambda_2} \pm 0.25_{\rho_1} \pm 0.25_{f_{u,d}}) \times 10^{-8}$$
$$= (0.86 \pm 0.39) \times 10^{-8}$$
$$\mathscr{R}[>14.4]_{ee} = (0.93 \pm 0.02_{scale} \pm 0.01_{m_t} \pm 0.01_{C,m_c} \pm 0.002_{m_b} \pm 0.01_{\alpha_s} \pm 0.05_{CKM} \pm 0.004_{\lambda_2} \pm 0.06_{\rho_1} \pm 0.05_{f_{u,d}}) \times 10^{-4}$$
$$= (0.93 \pm 0.09) \times 10^{-4}$$

Zero-crossing of the forward-backward asymmetry

$$(q_0^2)_{ee} = (3.28 \pm 0.11_{scale} + 0.001_{m_t} \pm 0.02_{C,m_c} \pm 0.05_{m_b} \pm 0.03_{\alpha_s} \pm 0.004_{CKM} \pm 0.001_{\lambda_2} \pm 0.06_{res})\ \text{GeV}^2$$
$$= (3.28 \pm 0.14)\ \text{GeV}^2$$

Thoughts and conclusions

- Tree-level reactions within the SM ($e^+e^- \to X$, $\tau \to X\nu$, $\bar{B} \to X\ell^-\bar{\nu}$) are interesting ingredients for FCNC processes sensitive to BSM physics

- Interplay between different FCNC processes: normalization, correlations of power corrections, optimized observables …

- At the squared amplitude level, backgrounds from decays of charmonium produced on-shell are removed by the q^2 cut and the cut $M_X <$ 2 GeV

- Color-singlet resonances are captured by the KS approach, while color-octet effects are handled in a model-independent and conservative way, all at the amplitude level.

- There is a hierarchy of scales associated to charmonium intermediate state $E < p \sim \Lambda < m_b$. New EFT approach?

- Naive rescaling of $\bar{B} \to X_s\ell^+\ell^-$ from Belle I to Belle II suggests that $\bar{B} \to X_d\ell^+\ell^-$ might be measured at Belle II with comparable accuracy as $\bar{B} \to X_s\ell^+\ell^-$ at Belle I