Resolved contributions to the inclusive decay $\bar{B} \to X_{s,d}\ell^+\ell^-$

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b>sll 2019: 7th workshop on rare semileptonic B decays

Inclusive modes $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$

How to compute the hadronic matrix elements $\mathcal{O}_i(\mu = m_b)$?

Heavy mass expansion for inclusive modes:

$$\Gamma(\bar{B} \to X_s\gamma) \xrightarrow{m_b\to\infty} \Gamma(b \to X_s^{parton}\gamma)\,, \quad \Delta^{nonpert.} \sim \Lambda^2_{QCD}/m_b^2$$

No linear term Λ_{QCD}/m_b (perturbative contributions dominant)

An old story:

– If one goes beyond the leading operator ($\mathcal{O}_7$, $\mathcal{O}_9$):
  breakdown of local expansion

A new dedicated analysis:

naive estimate of non-local matrix elements leads to 5% uncertainty.

Benzke,Lee,Neubert,Paz,arXiv:1003.5012

Analysis in $B \to X_s\ell\ell$ in this talk; Benzke,Hurth,Turczyk, arXiv:1705.10366

Cuts in the dilepton and hadronic mass spectra

- On-shell-$c\bar{c}$-resonances $\Rightarrow$ cuts in dlepton mass spectrum necessary :
  $1\text{GeV}^2 < q^2 < 6\text{GeV}^2$ and $14.4\text{GeV}^2 < q^2$ $\Rightarrow$ perturbative contributions dominant

- Hadronic invariant-mass cut is imposed in order to eliminate the background like $b \to c\,(\to s e^+ \nu) e^- \bar{\nu} = b \to s e^+ e^- +$ missing energy
  - $*$ Babar,Belle: $m_X < 1.8$ or $2.0 GeV$
  - $*$ high-q^2 region not affected by this cut
  - $*$ kinematics: X_s is jetlike and $m_X^2 \le m_b \Lambda_{QCD}$ $\Rightarrow$ shape function region
  - $*$ SCET analysis: universality of jet and shape functions found:
    the 10-30% reduction of the dilepton mass spectrum can be accurately computed using the $\bar{B} \to X_s \gamma$ shape function
    5% additional uncertainty for $2.0 GeV$ cut due to subleading shape functions

Lee,Stewart hep-ph/0511334
Lee,Ligeti,Stewart,Tackmann hep-ph/0512191
Lee,Tackmann arXiv:0812.0001 (effect of subleading shape functions)
Bell,Beneke,Huber,Li arXiv:1007.3758 (NNLO matching QCD $\to$ SCET)

Nonlocal subleading contributions

Benzke, Hurth, Turczyk JHEP10 (2017) 031 and work in progress

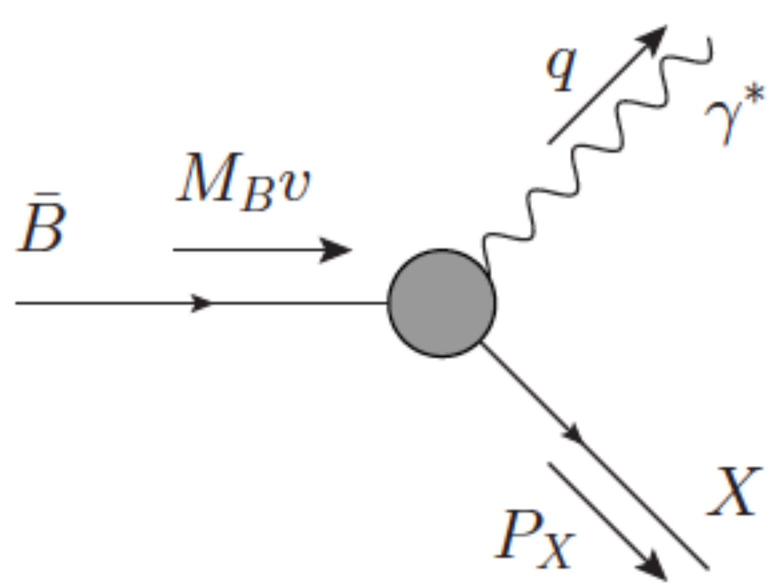
Subleading power factorization in $B \to X_s \ell^+ \ell^-$

Hadronic cut

Additional cut in X_s necessary to reduce background
affects only low-q^2 region.

Hadronic invariant $m_X^2 < 1.8(2.0) GeV^2$

Multiscale problem $\to$ SCET



$$M_B^2 \sim m_b^2 \gg m_X^2 \sim \Lambda_{\rm QCD} m_b \gg \Lambda_{\rm QCD}^2$$

Scaling $\lambda = \Lambda_{\rm QCD}/m_b$

Kinematics

B meson rest frame

$$q = p_B - p_X \qquad 2\, m_B\, E_X = m_B^2 + M_X^2 - q^2$$

X_s system is jet-like with $E_X \sim m_B$ and $m_X^2 \ll E_X^2$

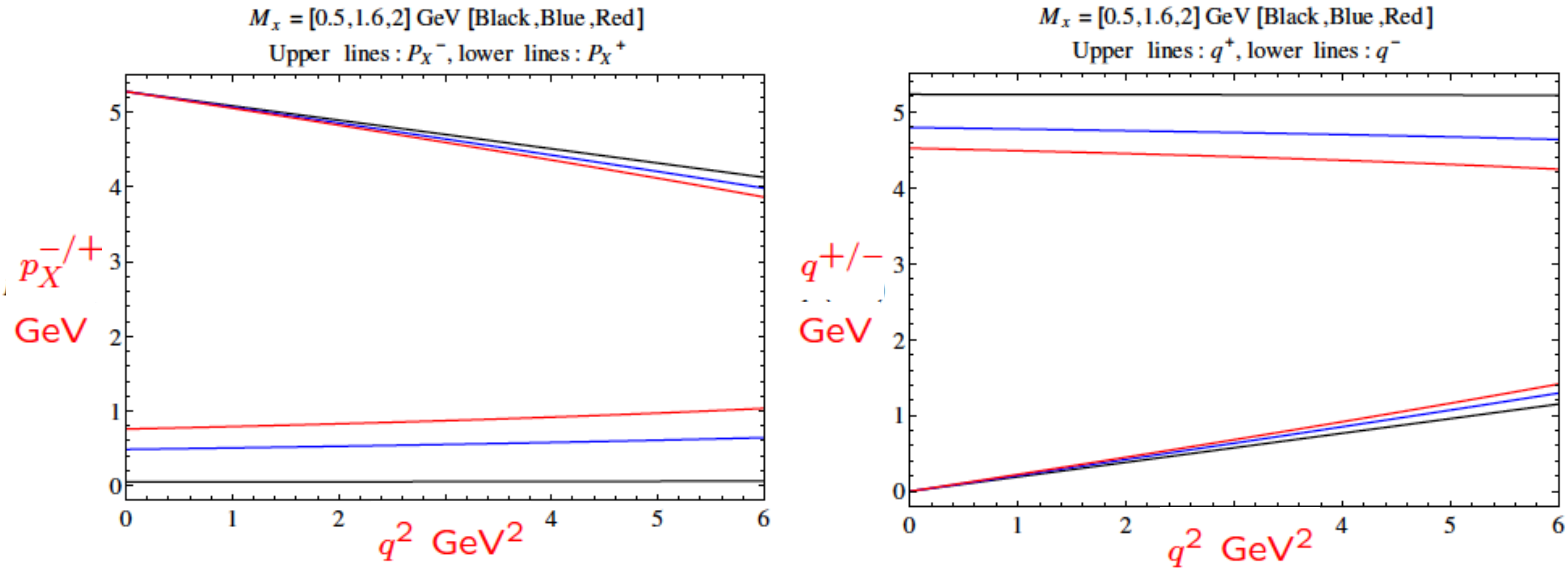
two light-cone components $\qquad p_X^- p_X^+ = m_X^2$

$$\bar{n} p_X = p_X^- = E_X + |\vec{p}_X| \sim \mathcal{O}(m_B)$$
$$n p_X = p_X^+ = E_X - |\vec{p}_X| \sim \mathcal{O}(\Lambda_{\mathrm{QCD}})$$

$$q^+ = nq = m_B - p_X^+ \qquad q^- = \bar{n}q = m_B - p_X^-$$

$\bar{n} = (+1, 0, 0, -1); n = (+1, 0, 0, +1)$

Scaling $\lambda = \Lambda_{\rm QCD}/m_b$ $\quad m_X^2 \sim \lambda \Rightarrow m_b - n \cdot q \sim \lambda$



For $q^2 < 6GeV^2$ the scaling of np_X and $\bar{n}p_X$ implies $\bar{n}q$ is of order λ, means q anti-hard-collinear (just kinematics).

Stewart and Lee assume $\bar{n}q$ to be order 1, means q is hard.

This problematic assumption implies a different matching of SCET/QCD.

Matching

QCD *and* QED fields are included in SCET analysis !

$$\mathcal{O}_{7\gamma} = -\frac{e}{8\pi^2} m_b \,\bar{s}\sigma_{\mu\nu}(1+\gamma_5)F^{\mu\nu}b \quad \to \quad \mathcal{O}^{(0)}_{7\gamma A} = \bar{\xi}_{\rm hc}\,\frac{\bar{\not n}}{2}\,[in\cdot\partial \mathcal{A}^{\rm em}_{\perp}]\,(1+\gamma_5)h$$

$\mathcal{O}_7$ scales as $\lambda^{\frac{5}{2}}$ $\qquad -\frac{em_b}{4\pi^2}\, e^{-im_b\, v\cdot x}$

$$\mathcal{O}_9 = \frac{\alpha}{2\pi}(\bar{s}b)_{V-A}(\bar{l}l)_V \quad \to \quad \mathcal{O}^{(1)}_9 = \frac{\alpha}{2\pi}(\bar{\xi}^s_{\rm hc}[1+\gamma^5]h)(\bar{\xi}^\ell_{\rm hc}\frac{\not n}{2}\xi^\ell_{\rm hc})$$

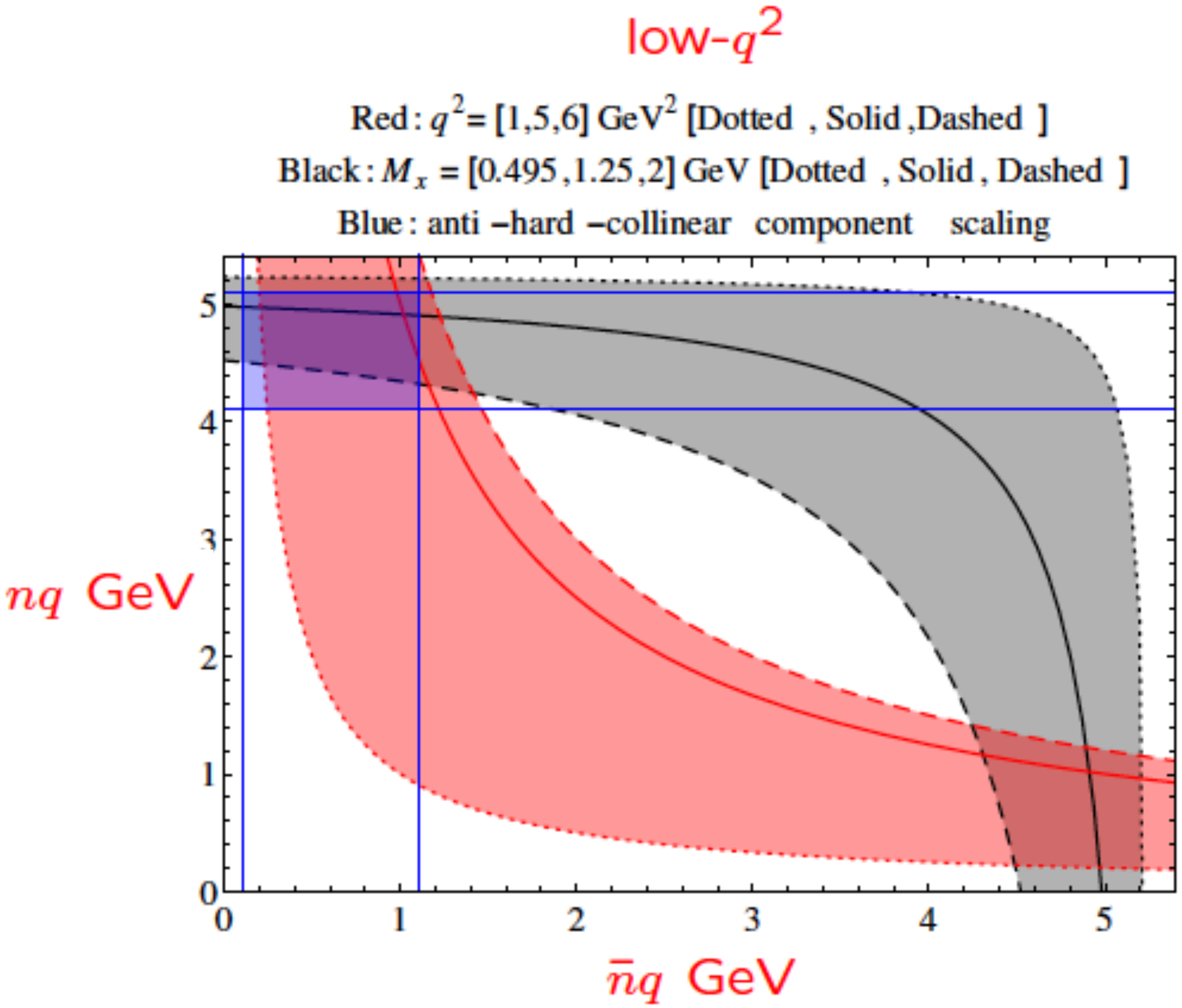
$$\mathcal{O}_{10} = \frac{\alpha}{2\pi}(\bar{s}b)_{V-A}(\bar{l}l)_A \quad \to \quad \mathcal{O}^{(1)}_{10} = \frac{\alpha}{2\pi}(\bar{\xi}^s_{\rm hc}[1+\gamma^5]h)(\bar{\xi}^\ell_{\rm hc}\frac{\not n}{2}\gamma^5\xi^\ell_{\rm hc})$$

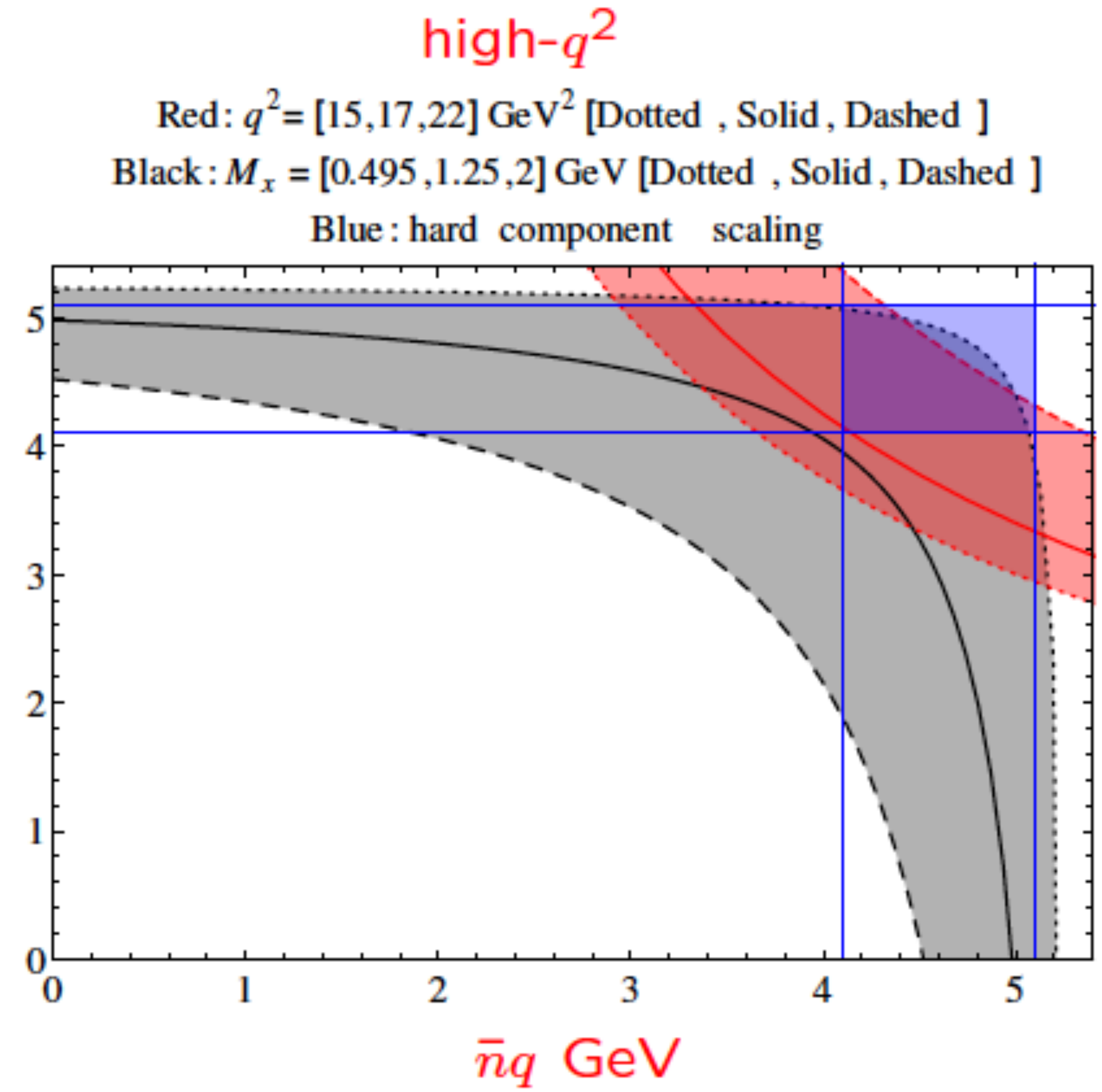
$$\lambda^{\frac{1}{2}+\frac{3}{2}+2\frac{1}{2}} = \lambda^3$$

But in high-q^2 region lepton fields and photon fields are hard and add no suppression

$h, q \sim \lambda^{3/2} \qquad \xi_{\rm hc} \sim \sqrt{\lambda} \qquad \mathcal{A}^{\rm em}_{\perp} \sim \sqrt{\lambda}$

Alllowed regions



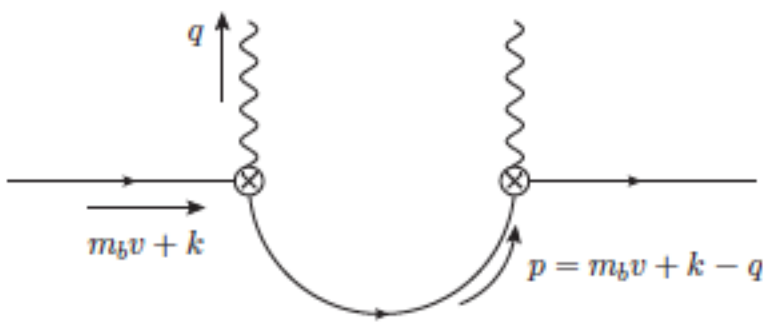


Scaling

$$\lambda = \Lambda_{\rm QCD}/m_b \qquad m_X^2 \sim \lambda \Rightarrow m_b - n\cdot q \sim \lambda$$

Shapefunction region

Local OPE breaks down for $m_X^2 \sim \lambda$:



$$\frac{1}{(m_b v+k-q)^2} = \frac{1}{m_b - n\cdot q}\left(1 - \frac{n\cdot k}{m_b - n\cdot q} + \ldots\right)\frac{1}{m_b - \bar{n}\cdot q}$$

Resummation of leading contributions into a shape function.

(scaling of $\bar{n}q$ does not matter here; zero in case of $B \to X_s\gamma$)

Factorization theorem $\quad d\Gamma \sim H \cdot J \otimes S$

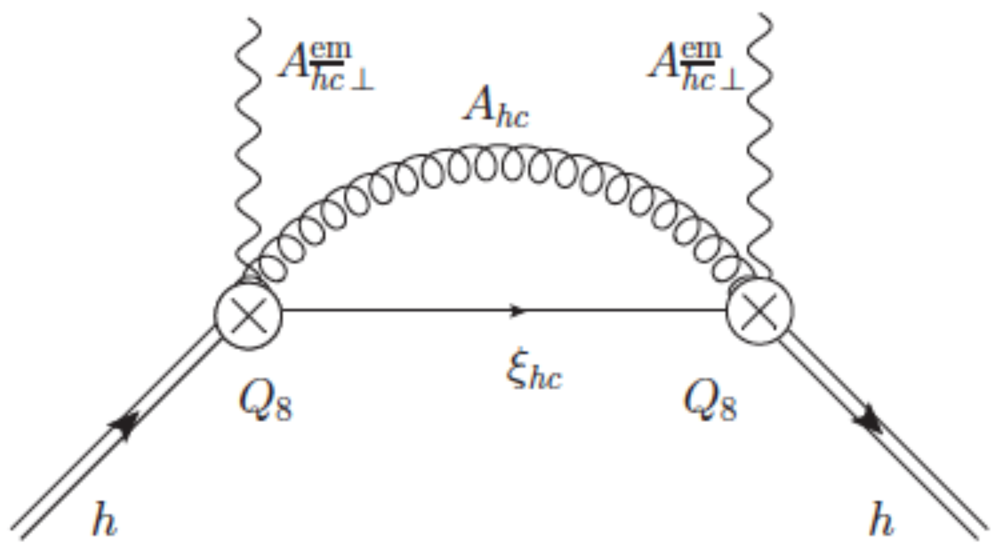
The hard function H and the jet function J are perturbative quantities.

The shape function S is a non-perturbative non-local HQET matrix element.

(universality of the shape function, uncertainties due to subleading shape functions)

Calculation at subleading power

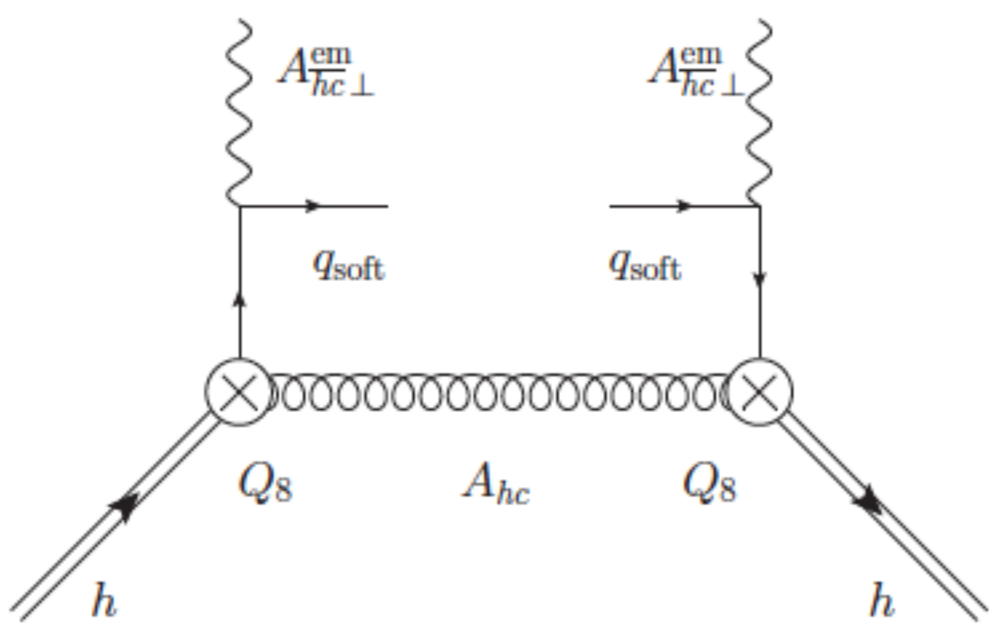
Example of **direct** photon contribution which factorizes $d\Gamma \sim H \cdot j \otimes S$



$\rightarrow \frac{\alpha_s}{m_b}$ in low m_X^2 region

Example of **resolved** photon contribution (double-resolved) which factorizes

$d\Gamma \sim H \cdot J \otimes s \otimes \bar{J} \otimes \bar{J}$



$\rightarrow \frac{\Lambda}{m_b}$

In the resolved contributions the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex.

Resolved contributions

- $1/m_b$ $\mathcal{O}_{7\gamma} - \mathcal{O}_{8g}$, $\mathcal{O}_{8g} - \mathcal{O}_{8g}$, and $\mathcal{O}_1^c - \mathcal{O}_{7\gamma}$, but also $\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$

  $\lambda^{12/2}$ (note additional suppression due to subleading insertion !)

- $1/m_b^2$ for example $\mathcal{O}_1^c - \mathcal{O}_9$, $\mathcal{O}_1^c - \mathcal{O}_{10}$

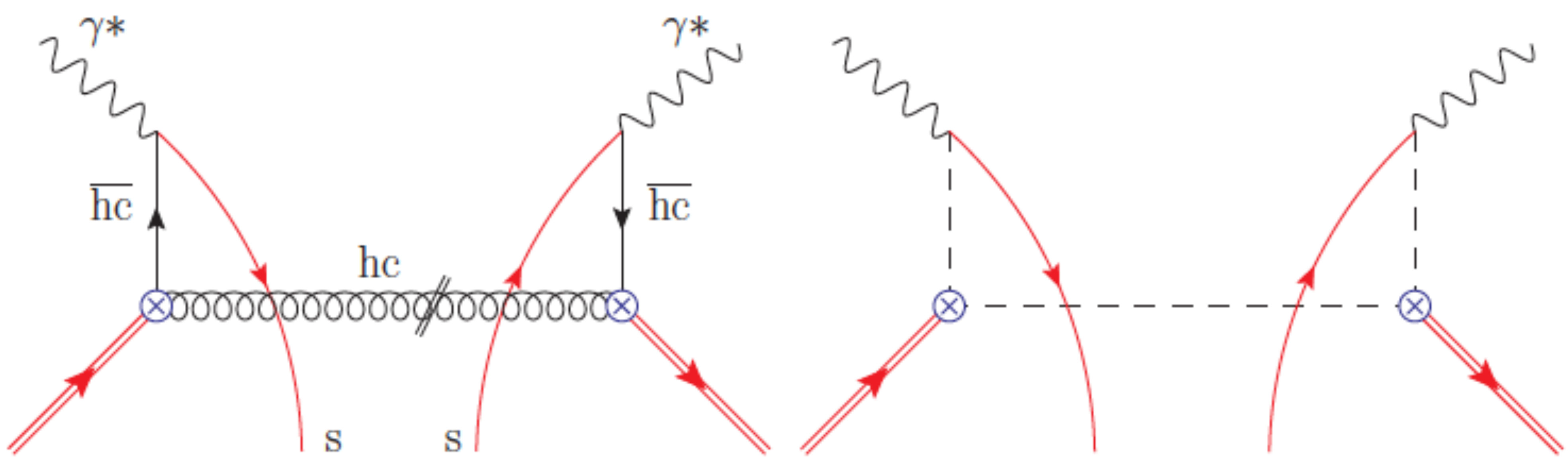
  $\lambda^{14/2}$

  consequence of the fact that the virtual photon is hard-collinear

  these $1/m_b^2$ terms might be numerically relevant due $|C_{9/10}| \sim 13|C_{7\gamma}|$

- Contributions to order $\lambda^{13/2}$ vanish (no transversal components) !

Interference of Q_8 and Q_8

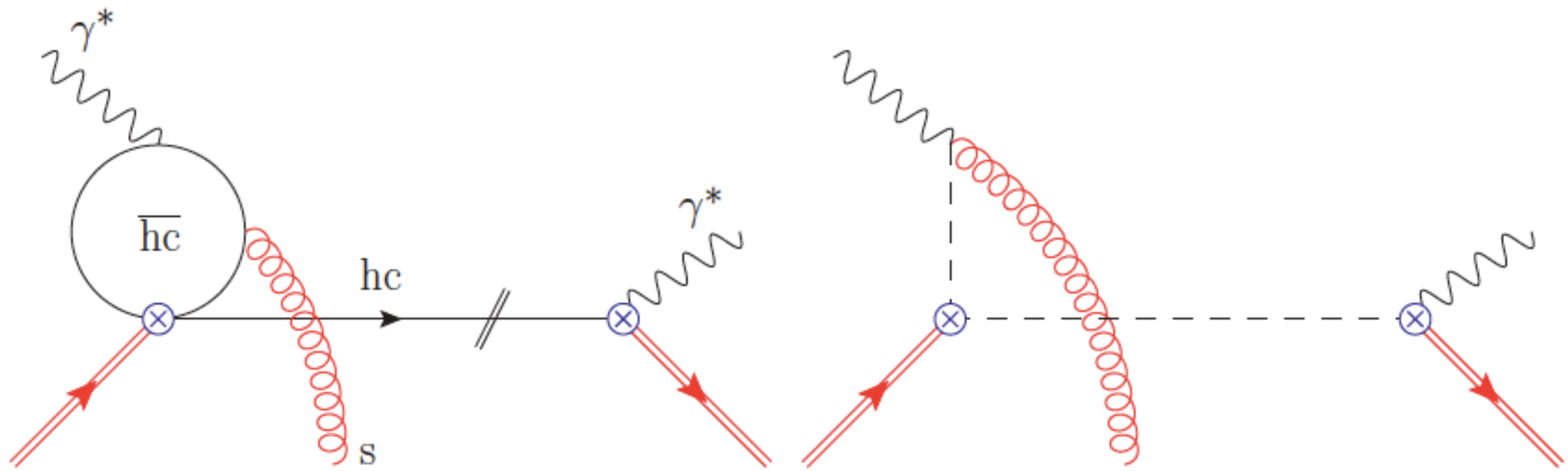


$$\frac{d\Gamma^{\rm res}}{dn\cdot q\, d\bar{n}\cdot q} \sim \frac{e_s^2\alpha_s}{m_b}\int d\omega\, \delta(\omega+p_+)\int \frac{d\omega_1}{\omega_1+\bar{n}\cdot q+i\varepsilon}\int \frac{d\omega_2}{\omega_2+\bar{n}\cdot q-i\varepsilon} g_{88}(\omega,\omega_1,\omega_2)$$

$$g_{88}(\omega,\omega_1,\omega_2) = \frac{1}{M_B}\langle \bar{B}|\bar{h}(\mathbf{tn})\ldots s(\mathbf{tn}+\mathbf{u\bar{n}})\bar{s}(\mathbf{r\bar{n}})\ldots h(\mathbf{0})|\bar{B}\rangle_{\rm F.T.}$$

- Convolution of jet function and shape function
- Scaling: $\lambda^{5/2}\,\lambda^{5/2}\,\lambda^{1/2}\,\lambda^{1/2}$
- No resolved contribution if the photon is assumed to be hard !

Interference of Q_1 and Q_7



$$\frac{d\Gamma^{\rm res}}{dn\cdot q\, d\bar n\cdot q} \sim \frac{1}{m_b}\int d\omega\, \delta(\omega+p_+)\int \frac{d\omega_1}{\omega_1+i\varepsilon}$$

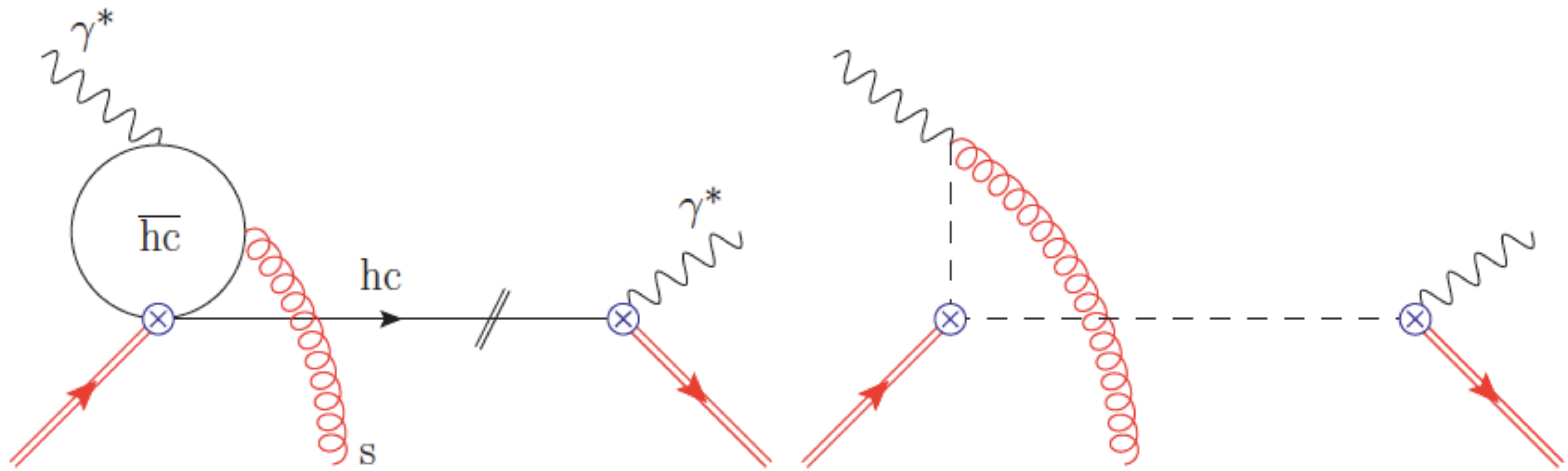
$$\frac{1}{\omega_1}\Bigg[\bar n\cdot q\left(F\left(\frac{m_c^2}{n\cdot q\bar n\cdot q}\right)-1\right)-(\bar n\cdot q+\omega_1)\left(F\left(\frac{m_c^2}{n\cdot q(\bar n\cdot q+\omega_1)}\right)-1\right)$$

$$+\bar n\cdot q\left(G\left(\frac{m_c^2}{n\cdot q\bar n\cdot q}\right)-G\left(\frac{m_c^2}{n\cdot q(\bar n\cdot q+\omega_1)}\right)\right)\Bigg] g_{17}(\omega,\omega_1)$$

$$g_{17}(\omega,\omega_1)=\int\frac{dr}{2\pi}e^{-i\omega_1 r}\int\frac{dt}{2\pi}e^{-i\omega t}\frac{1}{M_B}\langle\bar B|\bar h(\mathbf{tn})\ldots G_s^{\alpha\beta}(\mathbf{r\bar n})\ldots h(0)|\bar B\rangle$$

- Scaling: $\lambda^{6/2}\,\lambda^{5/2}\,\lambda^{1/2}$
- Connection to Voloshin term (see below)

Interference of Q_1 and Q_7



$$\frac{d\Gamma^{\mathrm{res}}}{dn\cdot q\, d\bar{n}\cdot q} \sim \frac{1}{m_b}\int d\omega\, \delta(\omega + p_+)\int \frac{d\omega_1}{\omega_1 + i\varepsilon}$$

$$\frac{1}{\omega_1}\left[\bar{n}\cdot q\left(F\left(\frac{m_c^2}{n\cdot q\bar{n}\cdot q}\right)-1\right) - (\bar{n}\cdot q+\omega_1)\left(F\left(\frac{m_c^2}{n\cdot q(\bar{n}\cdot q+\omega_1)}\right)-1\right)\right.$$

$$\left.+\bar{n}\cdot q\left(G\left(\frac{m_c^2}{n\cdot q\bar{n}\cdot q}\right) - G\left(\frac{m_c^2}{n\cdot q(\bar{n}\cdot q+\omega_1)}\right)\right)\right] g_{17}(\omega,\omega_1)$$

$$g_{17}(\omega,\omega_1) = \int \frac{dr}{2\pi} e^{-i\omega_1 r}\int \frac{dt}{2\pi} e^{-i\omega t}\frac{1}{M_B}\langle \bar{B}|\bar{h}(\mathbf{tn})\ldots G_s^{\alpha\beta}(\mathbf{r\bar{n}})\ldots h(0)|\bar{B}\rangle$$

- Shape function is nonlocal in both light cone directions
- It survives $M_X \to 1$ limit (irreducible uncertainty)
- Due to support properties of the soft functions resolved contributions are almost cut-independent (besides $8-8$).

Angular observables

$$\frac{d^2\Gamma}{dq^2 dz} = \frac{3}{8}\Big[(1+z^2)H_T(q^2) + 2(1-z^2)H_L(q^2) + 2z\,H_A(q^2)\Big]$$

$$\frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2) \qquad \frac{dA_{\rm FB}}{dq^2} = 3/4\,H_A(q^2)$$

$$d\Gamma(\bar{B}\to X_s\ell^+\ell^-) \equiv d\Lambda_{\alpha\beta}\,W^{\alpha\beta}(v,q)\,,$$

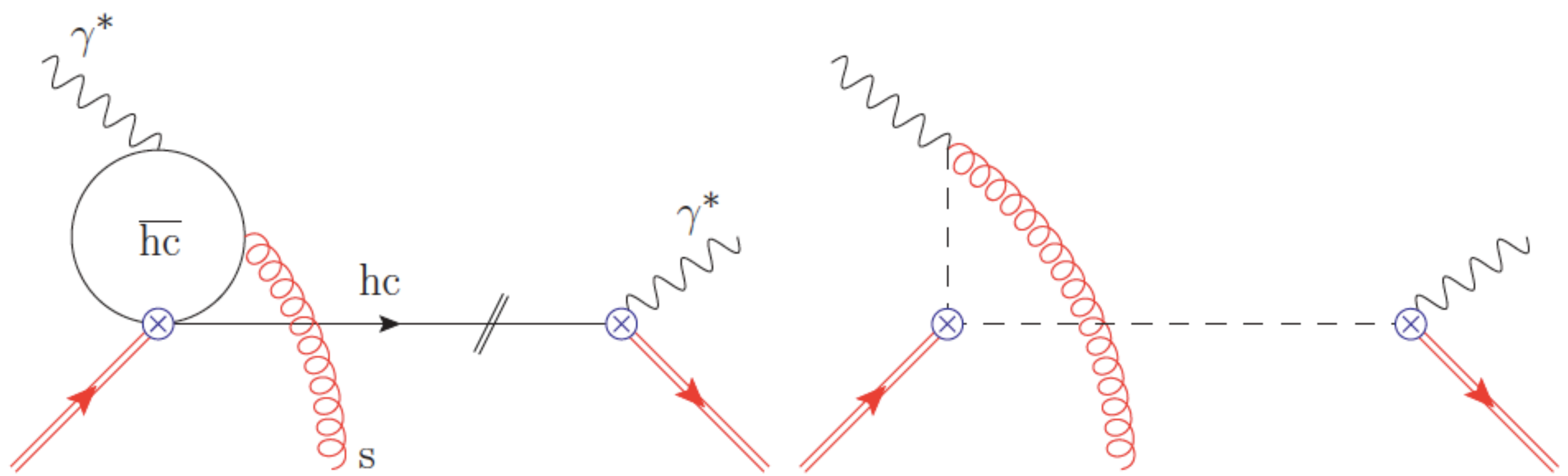
$$d\Lambda_{\alpha\beta;1/m_b} = dn\cdot q\,d\bar{n}\cdot q\,dz\frac{\alpha}{128\pi^3}(1+z^2)\,\frac{n\cdot q}{\bar{n}\cdot q}\,g_{\perp,\alpha\beta}\,.$$

At $O(1/m_b)$ nonlocal powercorrections only to $H_T(q^2)$.

Numerical evaluation

- Subleading shape functions of resolved contributions similar to $b \to s\gamma$
- Use explicit defintion to determine properties:
  * PT invariance: soft functions are real
  * Moments of g_{17} related to HQET parameters
  * Vacuum insertion approximation relates g_{78} to the B meson LCDA
- Perform convolution integrals with model functions

Numerical evaluation $\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$



$$d\Gamma_{17} = \frac{1}{m_b}\,\mathrm{Re}\left[\hat{\Gamma}_{17}\frac{-(\lambda_t^q)^*\lambda_c^q}{|\lambda_t^q|^2}\right]\frac{\alpha}{24\pi^3}\,dn\cdot q d\bar{n}\cdot q\,\frac{(n\cdot q)^3}{\bar{n}\cdot q}$$

$$\times\mathrm{Re}\int d\omega\;\delta(\omega+m_b-n\cdot q)\int d\omega_1\frac{1}{\omega_1+i\epsilon}$$

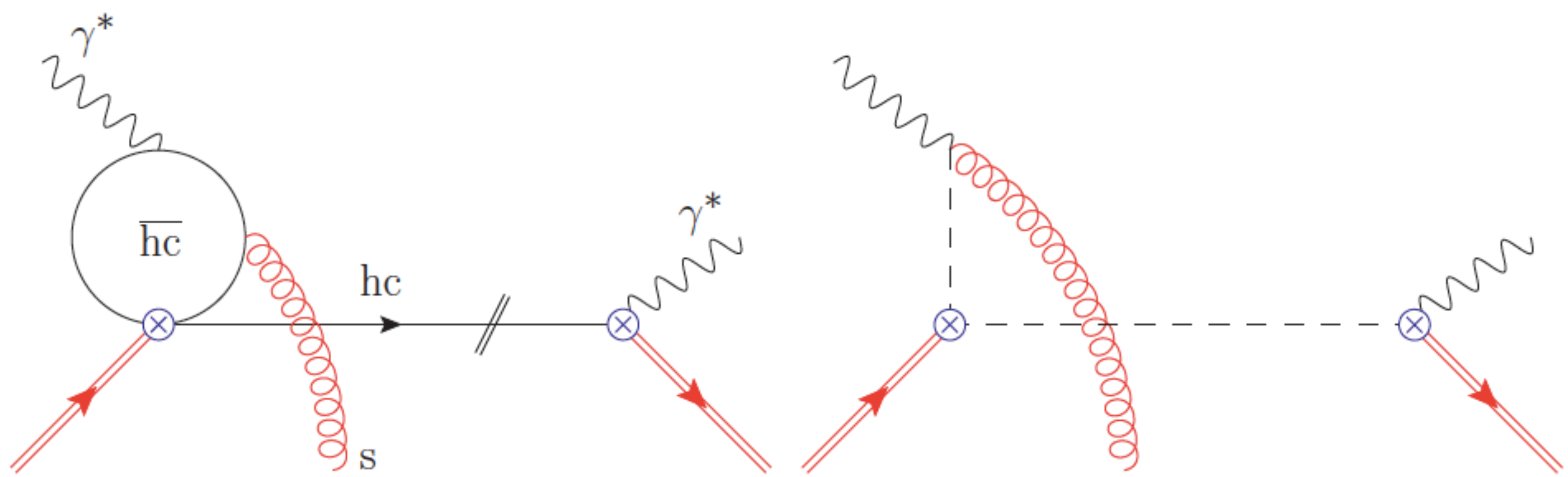
$$\times\frac{1}{\omega_1}\left[(\overline{n}\cdot q+\omega_1)\left(1-F\left(\frac{m_c^2}{n\cdot q(\overline{n}\cdot q+\omega_1)}\right)\right)-\overline{n}\cdot q\left(1-F\left(\frac{m_c^2}{n\cdot q\overline{n}\cdot q}\right)\right)\right.$$

$$\left.-\overline{n}\cdot q\left(G\left(\frac{m_c^2}{n\cdot q(\overline{n}\cdot q+\omega_1)}\right)-G\left(\frac{m_c^2}{n\cdot q\overline{n}\cdot q}\right)\right)\right]\,g_{17}(\omega,\omega_1,\mu)\,,$$

$$g_{17}(\omega,\omega_1,\mu)=\int\frac{dr}{2\pi}\,e^{-i\omega_1 r}\int\frac{dt}{2\pi}\,e^{-i\omega t}$$

$$\times\frac{\langle\bar{B}|\left(\bar{h}S_n\right)(tn)\,\bar{n}\!\!\!/(1+\gamma_5)\left(S_n^\dagger S_{\bar{n}}\right)(0)\,i\gamma_\alpha^\perp\bar{n}_\beta\left(S_{\bar{n}}^\dagger\,gG_s^{\alpha\beta}S_{\bar{n}}\right)(r\bar{n})\left(S_{\bar{n}}^\dagger h\right)(0)|\bar{B}\rangle}{2M_B}$$

Numerical evaluation $\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$



$$d\Gamma_{17} = \frac{1}{m_b}\,\mathrm{Re}\left[\hat{\Gamma}_{17}\frac{-(\lambda_t^q)^*\lambda_c^q}{|\lambda_t^q|^2}\right]\frac{\alpha}{24\pi^3}\,dn\cdot q\,d\bar{n}\cdot q\,\frac{(n\cdot q)^3}{\bar{n}\cdot q}$$
$$\times\mathrm{Re}\int d\omega\;\delta(\omega+m_b-n\cdot q)\int d\omega_1\frac{1}{\omega_1+i\epsilon}$$
$$\times\frac{1}{\omega_1}\left[(\overline{n}\cdot q+\omega_1)\left(1-F\left(\frac{m_c^2}{n\cdot q(\overline{n}\cdot q+\omega_1)}\right)\right)-\overline{n}\cdot q\left(1-F\left(\frac{m_c^2}{n\cdot q\overline{n}\cdot q}\right)\right)\right.$$
$$\left.-\overline{n}\cdot q\left(G\left(\frac{m_c^2}{n\cdot q(\overline{n}\cdot q+\omega_1)}\right)-G\left(\frac{m_c^2}{n\cdot q\overline{n}\cdot q}\right)\right)\right]g_{17}(\omega,\omega_1,\mu)\,,$$

$\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$

- **Limit** $m_c \to m_u = 0$

$$\times\frac{1}{\omega_1}\,[\,\omega_1\,]\;g_{17}(\omega,\omega_1,\mu)$$

$$g_{17}(\omega,\omega_1,\mu) = \int\frac{dr}{2\pi}\,e^{-i\omega_1 r}\int\frac{dt}{2\pi}\,e^{-i\omega t}$$
$$\times\frac{\langle\bar{B}|\left(\bar{h}S_n\right)(tn)\,\bar{\not{n}}(1+\gamma_5)\left(S_n^\dagger S_{\bar{n}}\right)(0)\,i\gamma_\alpha^\perp\bar{n}_\beta\left(S_{\bar{n}}^\dagger\,gG_s^{\alpha\beta}S_{\bar{n}}\right)(r\bar{n})\left(S_{\bar{n}}^\dagger h\right)(0)|\bar{B}\rangle}{2M_B}$$

Numerical evaluation $\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$

- **Trace formalism of HQET:** $\int_{-\infty}^{\bar\Lambda} d\omega\, g_{17}(\omega,\omega_1,\mu) = \int_{-\infty}^{\bar\Lambda} d\omega\, (g_{17}(\omega,-\omega_1,\mu))^*$

$$
\begin{aligned}
d\Gamma_{17} =& \frac{1}{m_b}\,\mathrm{Re}\left[\hat\Gamma_{17}\frac{-(\lambda_t^q)^*\lambda_c^q}{|\lambda_t^q|^2}\right]\frac{\alpha}{24\pi^3}\, dn\cdot q d\bar n\cdot q\,\frac{(n\cdot q)^3}{\bar n\cdot q}\\
&\times \mathrm{Re}\int d\omega\ \delta(\omega+m_b-n\cdot q)\int d\omega_1 \frac{1}{\omega_1+i\epsilon}\\
\times\frac{1}{\omega_1}&\left[(\overline{n}\cdot q+\omega_1)\left(1-F\left(\frac{m_c^2}{n\cdot q(\overline{n}\cdot q+\omega_1)}\right)\right)-\overline{n}\cdot q\left(1-F\left(\frac{m_c^2}{n\cdot q\overline{n}\cdot q}\right)\right)\right.\\
&\left.-\overline{n}\cdot q\left(G\left(\frac{m_c^2}{n\cdot q(\overline{n}\cdot q+\omega_1)}\right)-G\left(\frac{m_c^2}{n\cdot q\overline{n}\cdot q}\right)\right)\right] g_{17}(\omega,\omega_1,\mu)\,,
\end{aligned}
$$

$\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$

- **Limit** $m_c \to m_u = 0$

$$\times\frac{1}{\omega_1}\,[\ \omega_1\ ]\ g_{17}(\omega,\omega_1,\mu)$$

Numerical evaluation $\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$

- **Trace formalism of HQET:** $\int_{-\infty}^{\bar{\Lambda}} d\omega\, g_{17}(\omega, \omega_1, \mu) = \int_{-\infty}^{\bar{\Lambda}} d\omega\, (g_{17}(\omega, -\omega_1, \mu))^*$

- **PT invariance:** g_{17} is real

$$d\Gamma_{17} = \frac{1}{m_b}\,\mathrm{Re}\left[\hat{\Gamma}_{17}\frac{-(\lambda_t^q)^*\lambda_c^q}{|\lambda_t^q|^2}\right]\frac{\alpha}{24\pi^3}\, dn\cdot q d\bar{n}\cdot q\, \frac{(n\cdot q)^3}{\bar{n}\cdot q}$$
$$\times \mathrm{Re}\int d\omega\; \delta(\omega + m_b - n\cdot q)\int d\omega_1 \frac{1}{\omega_1 + i\epsilon}$$
$$\times\frac{1}{\omega_1}\left[(\overline{n}\cdot q + \omega_1)\left(1 - F\left(\frac{m_c^2}{n\cdot q(\overline{n}\cdot q+\omega_1)}\right)\right) - \overline{n}\cdot q\left(1 - F\left(\frac{m_c^2}{n\cdot q\overline{n}\cdot q}\right)\right)\right.$$
$$\left. -\overline{n}\cdot q\left(G\left(\frac{m_c^2}{n\cdot q(\overline{n}\cdot q+\omega_1)}\right) - G\left(\frac{m_c^2}{n\cdot q\overline{n}\cdot q}\right)\right)\right] g_{17}(\omega, \omega_1, \mu)\,,$$

$\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$

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$$\times\frac{1}{\omega_1}\,[\;\omega_1\;]\; g_{17}(\omega, \omega_1, \mu)$$

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$$\times\frac{1}{\omega_1}\left[(\overline{n}\cdot q+\omega_1)\left(1-F\left(\frac{m_c^2}{n\cdot q(\overline{n}\cdot q+\omega_1)}\right)\right)-\overline{n}\cdot q\left(1-F\left(\frac{m_c^2}{n\cdot q\overline{n}\cdot q}\right)\right)\right.$$
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$\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$

- **Limit** $m_c\to m_u=0$

$$\times\frac{1}{\omega_1}\,[\ \omega_1\ ]\ g_{17}(\omega,\omega_1,\mu)$$

- **Integration of** ω_1**:**

Interference term $\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$ vanishes within the integrated rate

Numerical evaluation $\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$

- **Trace formalism of HQET:** $\int_{-\infty}^{\bar\Lambda} d\omega\, g_{17}(\omega,\omega_1,\mu) = \int_{-\infty}^{\bar\Lambda} d\omega\, (g_{17}(\omega,-\omega_1,\mu))^*$

- **PT invariance:** g_{17} is real

$\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$

$$d\Gamma_{17} = \frac{1}{m_b}\,\mathrm{Re}\left[\hat\Gamma_{17}\frac{-(\lambda_t^q)^*\lambda_c^q}{|\lambda_t^q|^2}\right]\frac{\alpha}{24\pi^3}\,dn\cdot q\, d\bar n\cdot q\,\frac{(n\cdot q)^3}{\bar n\cdot q}$$
$$\times \mathrm{Re}\int d\omega\;\delta(\omega+m_b-n\cdot q)\int d\omega_1\frac{1}{\omega_1+i\epsilon}$$
$$\times\frac{1}{\omega_1}\left[(\overline{n}\cdot q+\omega_1)\left(1-F\left(\frac{m_c^2}{n\cdot q(\overline{n}\cdot q+\omega_1)}\right)\right)-\overline{n}\cdot q\left(1-F\left(\frac{m_c^2}{n\cdot q\overline{n}\cdot q}\right)\right)\right.$$
$$\left.-\overline{n}\cdot q\left(G\left(\frac{m_c^2}{n\cdot q(\overline{n}\cdot q+\omega_1)}\right)-G\left(\frac{m_c^2}{n\cdot q\overline{n}\cdot q}\right)\right)\right] g_{17}(\omega,\omega_1,\mu)\,,$$

- **Limit** $m_c \to m_u = 0$

$$\times\frac{1}{\omega_1}\,[\;\omega_1\;]\;g_{17}(\omega,\omega_1,\mu)$$

- **Integration of** ω_1:

Interference term $\mathcal{O}_1^u - \mathcal{O}_{7\gamma}$ vanishes within the integrated rate

Crucial result for all CP averaged inclusive $b\to d\ell^+\ell^-$ quantities

(previously no estimate for this up-quark loop of order Λ/m_b was available)

Numerical evaluation $\mathcal{O}_1^c - \mathcal{O}_{7\gamma}$

$$\mathcal{F}_{17}^q = \frac{1}{m_b}\frac{C_1(\mu)C_{7\gamma}(\mu)}{C_{\rm OPE}} e_c\,\mathrm{Re}\left[\frac{-(\lambda_t^q)^*\lambda_c^q}{|\lambda_t^q|^2}\right]\int_{-\infty}^{+\infty} d\omega_1\, J_{17}(q^2_{\rm min}, q^2_{\rm max}, \omega_1)\, h_{17}(\omega_1,\mu)$$

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$$h(\omega_1,\mu) := \int_{-\infty}^{\bar{\Lambda}} d\omega\, g_{17}(\omega,\omega_1,\mu)$$

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$$J_{17}(q^2_{\mathrm{min}}, q^2_{\mathrm{max}}, \omega_1) = \mathrm{Re}\frac{1}{\omega_1 + i\epsilon}\int_{\frac{q^2_{\mathrm{min}}}{M_B}}^{\frac{q^2_{\mathrm{max}}}{M_B}} \frac{d\bar{n}\cdot q}{\bar{n}\cdot q}\,\frac{1}{\omega_1}$$

$$h(\omega_1,\mu) := \int_{-\infty}^{\bar{\Lambda}} d\omega\, g_{17}(\omega,\omega_1,\mu)$$

$$\left[(\bar{n}\cdot q + \omega_1)\left(1 - F\left(\frac{m_c^2}{m_b(\bar{n}\cdot q+\omega_1)}\right)\right) - \bar{n}\cdot q\left(1 - F\left(\frac{m_c^2}{m_b\bar{n}\cdot q}\right)\right)\right.$$
$$\left. -\bar{n}\cdot q\left(G\left(\frac{m_c^2}{m_b(\bar{n}\cdot q+\omega_1)}\right) - G\left(\frac{m_c^2}{m_b\bar{n}\cdot q}\right)\right)\right].$$

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- One derives normalization of soft function: $\int_{-\infty}^{\infty} d\omega_1 h_{17}(\omega_1,\mu) = 2\,\lambda_2$
- h_{17} should not have any significant structure (maxima or zeros) outside the hadronic range
- Values of h_{17} should be within the hadronic range

Numerical evaluation $\quad \mathcal{O}_1^c - \mathcal{O}_{7\gamma}$

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Further constraints from higher moments of soft function?
Paz et al. arXiv:1908.02812 $\quad \int d\omega\omega \int d\omega_1 g_{17}(\omega,\omega_1) = -\rho_2$

- First trial for a model function for h_{17}, a Gaussian, fullfills all needed properties.

$$h_{17}(\omega_1) = \frac{2\lambda_2}{\sqrt{2\pi}\sigma} e^{-\frac{\omega_1^2}{2\sigma^2}}$$

$\sigma = 0.5\,\mathrm{GeV}$ as typical hadronic scale: $\mathcal{F}^s_{17\mathrm{exp}} \approx +1.6\,\%$

$\sigma = 0.1\,\mathrm{GeV}$: $\mathcal{F}^s_{17\mathrm{exp}} \approx +1.9\,\%$

However, convolution leads only to positive percentages !

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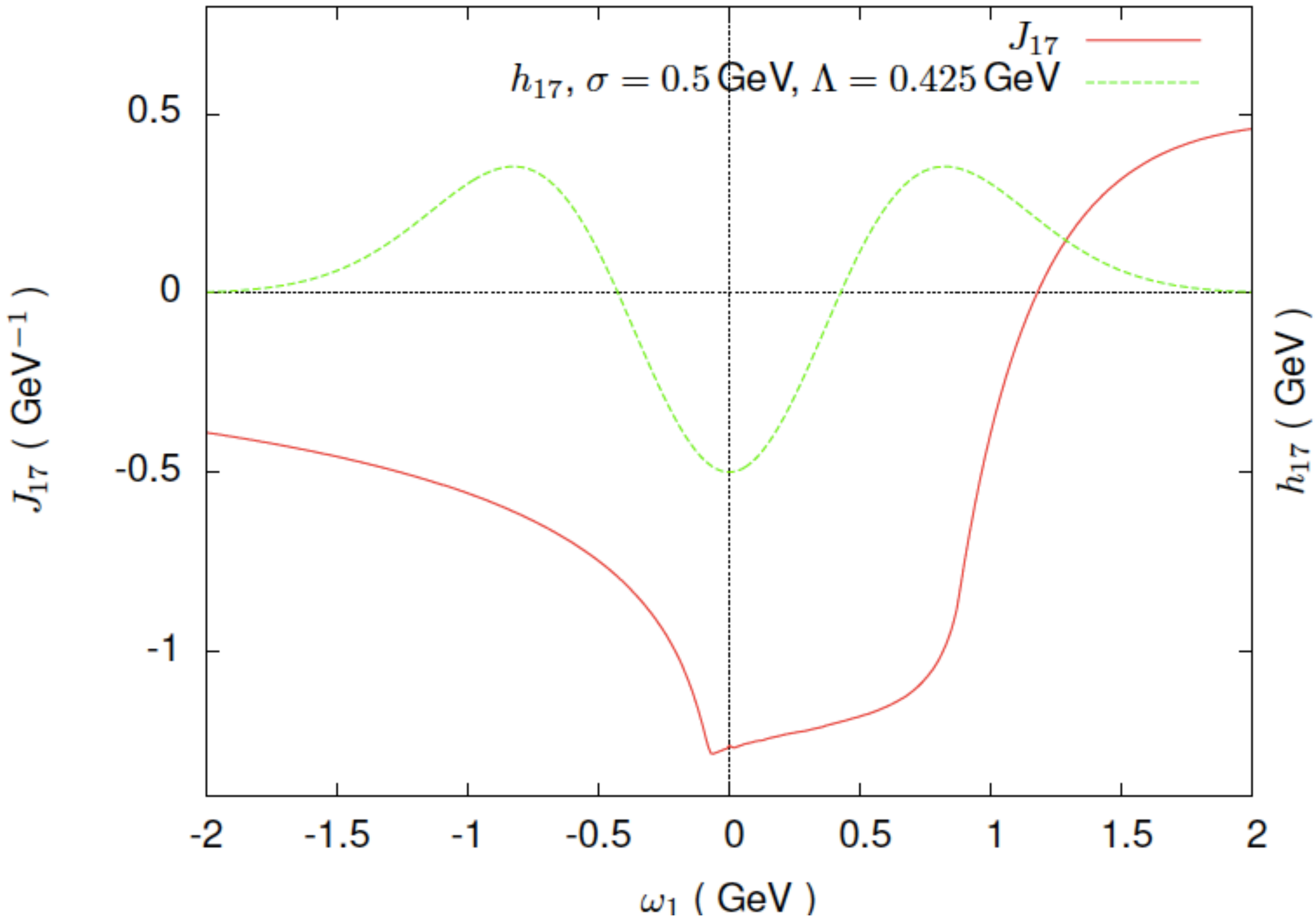
- More conservative estimate with $$h_{17}(\omega_1) = \frac{2\lambda_2}{\sqrt{2\pi}\sigma} \frac{\omega_1^2 - \Lambda^2}{\sigma^2 - \Lambda^2} e^{-\frac{\omega_1^2}{2\sigma^2}}$$

With Λ and σ of order Λ_{QCD} all general properties of h_{17} are fulfilled.

$\sigma = 0.5\,\mathrm{GeV};\ \Lambda = 0.425\,\mathrm{GeV}$: $\mathcal{F}^s_{17} = -0.5\%$

$\Lambda = 0.575\,\mathrm{GeV}$: $\mathcal{F}^s_{17} = +3.4\%$

$$\mathcal{F}^s_{17} \in [-0.5, +3.4]\,\%, \qquad \mathcal{F}^d_{17} \in [-0.6, +4.1]\,\%$$



$$\mathcal{F}_{17}^{s} \in [-0.5, +3.4]\,\%, \qquad \mathcal{F}_{17}^{d} \in [-0.6, +4.1]\,\%$$

- Relation to the Voloshin term: Voloshin 1997, Buchalla,Isidori,Rey 1997

We can rederive the leading Voloshin term under the following assumptions:

One starts with a narror enough Gaussian as shape function, so that one can expand the jet function around $\omega_1 = 0$ assuming $\Lambda_{\mathrm{QCD}} m_b/m_c^2$ to be small $((m_b\omega_1)/m_c^2$ corresponds to $t = k \cdot q/m_c^2$ in Buchalla et al.):

$$[\ldots] = \omega_1^2 \bar{n}\cdot q \left[\frac{1}{2\bar{n}\cdot q^2} - \frac{2m_c^2}{\bar{n}\cdot q^2} \frac{1}{4m_c^2 - m_b \bar{n}\cdot q} \sqrt{\frac{4m_c^2 - m_b\bar{n}\cdot q}{m_b \bar{n}\cdot q}} \arctan \frac{1}{\sqrt{\frac{4m_c^2 - m_b \bar{n}\cdot q}{m_b\bar{n}\cdot q}}} \right]$$

$$= -\frac{m_b \omega_1^2}{12 m_c^2} F_{\mathrm{V}}(r)\,, \quad r = q^2/(4m_c^2)$$

However:

Voloshin term significantly underestimates the possible charm contributions.

Final result to $O(1/m_b)$

Our final estimates of the resolved contributions to the leading order:

(normalized to OPE result)

$$\mathcal{F}_{17}^{s} \in [-0.5, +3.4]\,\%, \;\; \mathcal{F}_{17}^{d} \in [-0.6, +4.1]\,\%,$$

$$\mathcal{F}_{78}^{d,s} \in [-0.2, -0.1]\,\%, \;\; \mathcal{F}_{88}^{d,s} \in [0, 0.5]\,\%$$

$$\mathcal{F}_{1/m_b}^{d} \in [-0.8, +4.5]\,, \;\; \mathcal{F}_{1/m_b}^{s} \in [-0.7, +3.8]$$

Numerical relevant contributions to $O(1/m_b^2)$

$$\mathcal{F}_{19}: \quad O(1/m_b^2) \text{ but } |C_{9/10}| \sim 13|C_{7\gamma}|$$

- Interference of Q_1 and Q_9: Subleading power correction to BR

Indications that additional suppression in all terms are within the jet function!
$\rightarrow$ Q_1 and Q_7 and Q_1 and Q_9 terms could have the same shape function

- Interference of Q_1 and Q_{10}: First contribution to A_{FB}

$\mathcal{F}_{1/m_b^2}$

Power corrections in the inclusive mode

- For q anti-hard-collinear we have a new type of subleading power corrections.

- In the resolved contributions the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex.

- They constitute an irreducible uncertainty because they survive the $M_X \to 1$ limit.

- If q was hard then these resolved contributions would not exist

Nonlocal power corrections of $O(1/m_b^2)$ numerically relevant

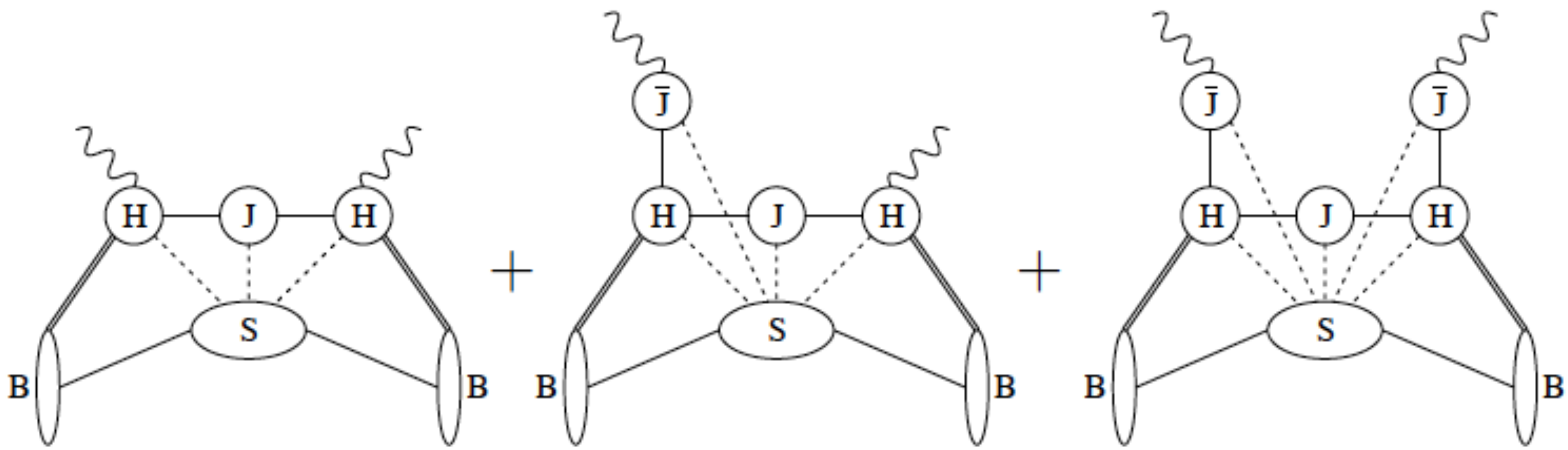
M_X cut effects in the low-q^2 region with q^2 anti-hard-collinear

Extra

Factorization formula

In the $m_X^2 \sim \lambda$ and $q^2 \sim \lambda$ region we have the following factorization formula

$$
\begin{aligned}
d\Gamma \quad \sim \quad & H \cdot J \otimes S + \frac{1}{m_b} \sum H \cdot j_i \otimes S + \frac{1}{m_b} \sum H \cdot J \otimes s_i \\
& + \frac{1}{m_b} \sum H \cdot J \otimes s_i \otimes \bar{J} + \frac{1}{m_b} \sum H \cdot J \otimes s_i \otimes \bar{J} \otimes \bar{J} + \mathcal{O}\left(\frac{1}{m_b^2}\right)
\end{aligned}
$$



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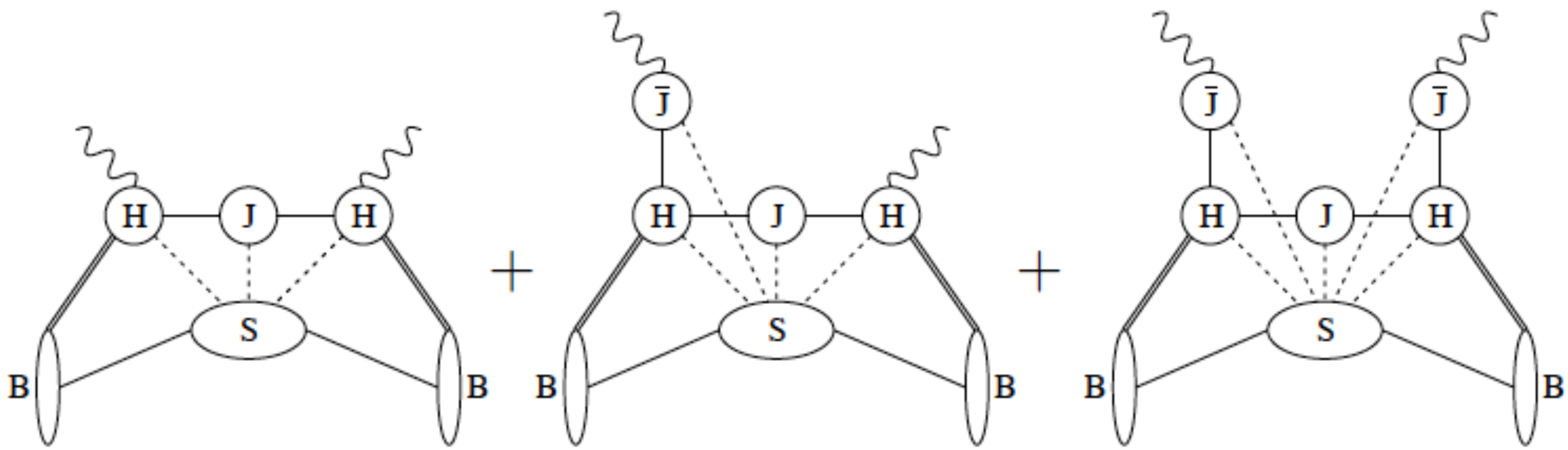


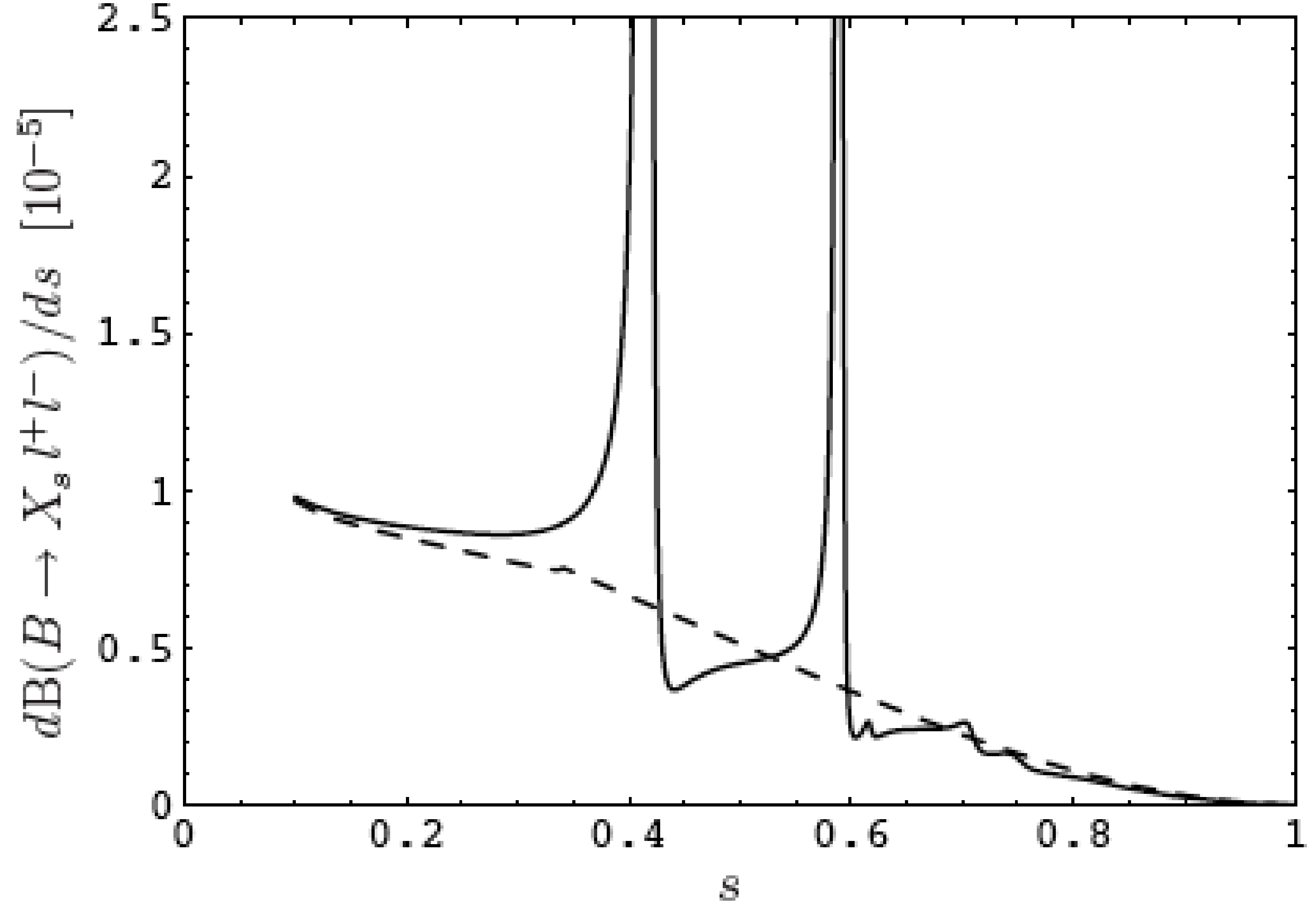
Subtlety in the Q_8 and Q_8 contribution: convolution integral is UV divergent

- This subtlety implies that there is no complete proof of the factorization formula.
- Nevertheless one shows that scale dependence of direct and resolved contribution cancel.
- No direct analogy to the problem of IR divergent convolution integrals in power-suppressed contributions to exclusive B decays.

Quark-hadron duality violated in $\bar{B} \to X_s \ell^+ \ell^-$?

BBNS, arXiv:0902.4446

Within integrated branching ratio the resonances J/ψ and ψ' exceed the perturbative contributions by two orders of magnitude.



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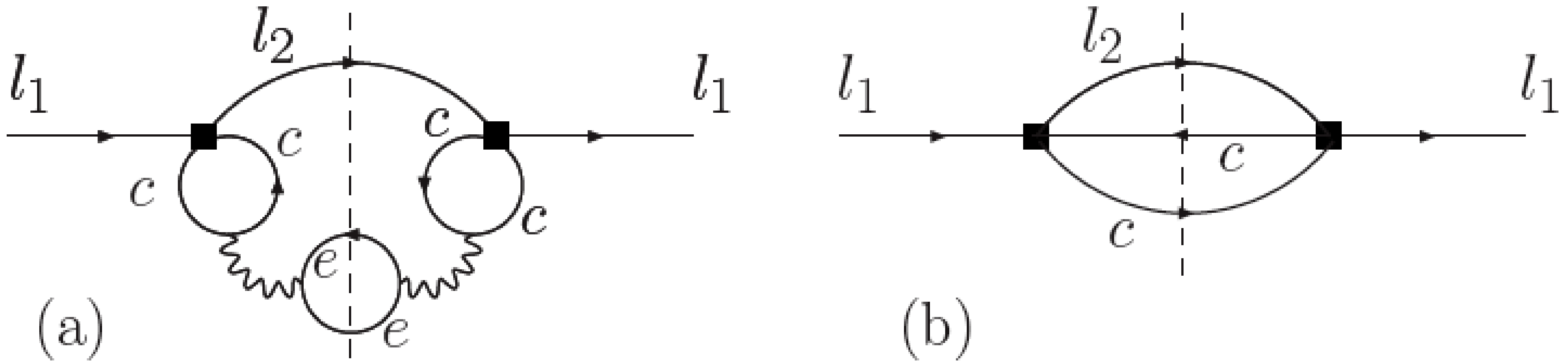


The rate $l_1 \to l_2 e^+ e^-$ (a) is connected to the integral over $|\Pi(q^2)|^2$ for which global duality is NOT expected to hold.

In contrast the inclusive hadronic rate $l_1 \to l_2 X$ (b) corresponds to the imaginary part of the correlator $\Pi(q^2)$.

Soft-collinear effective theory (SCET) Bauer et al. 2001

Effective field theory for external states with a large energy ($P_X^0 \sim m_b$) but small invariant mass ($P_X^2 \sim m_b \Lambda_{\mathrm{QCD}}$) $\rightarrow$ jets

Concepts

Light cone vectors n, $\bar{n}$

hard-collinear momentum $\bar{n} \cdot p_{hc} \sim m_b$, $n \cdot p_{hc} \sim \Lambda_{QCD}$, $p^2 \sim m_b \Lambda_{\mathrm{QCD}}$

scale parameter $\lambda \sim \Lambda_{\mathrm{QCD}}/m_b \rightarrow p \sim (n \cdot p, \bar{n} \cdot p, p_\perp) \sim (\lambda, 1, \lambda^{1/2})$

The SCET Lagrangian is generated by integrating out the hard modes, as well as fluctuations around the light cone. The theory still contains hard-collinear and soft fields.

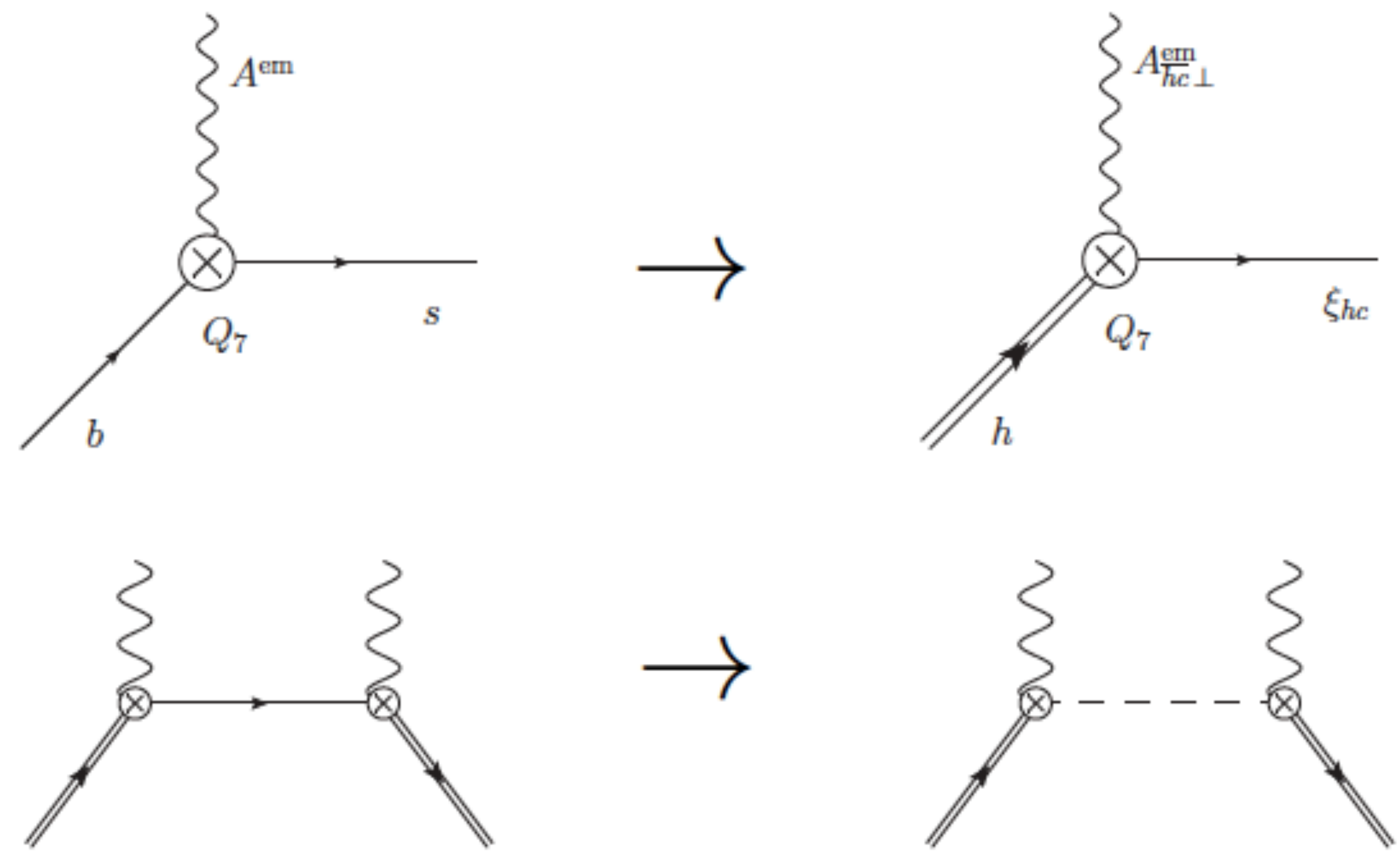
Each field scales with a certain power of λ

$\rightarrow$ **systematic expansion in λ**

Matching QCD → SCET → HQET

(QCD) → (SCET)

(SCET) → (HQET)



Factorization in SCET

Two matching steps: QCD $\to$ SCET $\to$ HQET
$\to$ Factorization at leading power: $d\Gamma^{\mathrm{LO}} \sim H\, J \otimes S$

Korchemsky, Sterman 1994, Bauer et al 2001

The hard function H and the jet function J parameterize physics at the scales m_b and $\sqrt{m_b \Lambda_{\mathrm{QCD}}}$, respectively and are perturbatively calculable

The shape function S is the fourier transform of a **non-local** HQET matrix element
$S(\omega) = \int \frac{dt}{2\pi}\, e^{-i\omega t}\, \langle \bar{B}(v)|\bar{h}(\mathbf{tn}) \ldots h(\mathbf{0})|\bar{B}(v)\rangle$
The leading shape funktion can be determined from the photon spectrum in $\to \bar{B} \to X_s \gamma$