## Form factors and charm-loop effect with light-cone

 sum rules
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Rare semileptonic B Decays:
Theory and Experiment
Lyon, 4-Sep-2019

## What's new?

- revisit soft gluon contributions to the "charm-loop" effect in rare $B \rightarrow K^{(*)} l l$ decays
- compute $B_{s} \rightarrow K^{(*)}, D_{s}^{(*)}, \phi$ form factors (FFs)
- method of $B$-meson Light-Cone Sum Rules including NNL twist corrections



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- compute $B_{s} \rightarrow K^{(*)}, D_{s}^{(*)}, \phi$ form factors (FFs)
- method of $B$-meson Light-Cone Sum Rules including NNL twist corrections
- preliminary results show a smaller soft gluon contributions to the charm-loop comparing with the previous calculation
- agreement $B_{s} \rightarrow K^{(*)}, D_{S}, \phi$ FFs with the literature $B_{s} \rightarrow D_{s}^{*}$ FFs computed for the first time [Bharrucha/Straub/Zwicky '15]



## Introduction

## Anomalies in $b \rightarrow$ sll

FCNC are loop, GIM and CKM suppressed in the SM
sensitive to new physics contributions.
tension between experiment and SM in several observables:

- lepton flavour universality ratios $R_{K^{(*)}}^{\mu e}$
- angular observables $B \rightarrow K^{*} \mu \mu\left(P_{5}^{\prime}, \ldots\right)$


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tension between experiment and SM in several observables:

- lepton flavour universality ratios $R_{K^{(*)}}^{\mu e}$
- angular observables $B \rightarrow K^{*} \mu \mu\left(P_{5}^{\prime}, \ldots\right)$
improve theory predictions and reduce uncertainties
problem: non-perturbative QCD effects (soft gluons, mesons...)
observables depends on
- local matrix elements (FFs)

$$
\left\langle K^{(*)}(k)\right| \bar{s} \Gamma^{\mu} b(0)|B(q+k)\rangle
$$

- nonlocal matrix elements
(soft gluon contributions to the charm-loop)

$$
\left\langle K^{(*)}(k)\right| \tilde{O}_{\mu}(0, x)|B(q+k)\rangle
$$

## $b \rightarrow$ sll effective Hamiltonian

transitions described by the effective Hamiltonian

$$
H\left(b \rightarrow s l^{+} l^{-}\right)=-\frac{4 G_{F}}{\sqrt{2}} V_{t b} V_{t s}^{*} \sum_{i=1}^{10} C_{i}(\mu) O_{i}(\mu) \quad \mu=m_{b}
$$

main contribution to $B \rightarrow K^{(*)} l l$ in the SM given by local operators $O_{7}, O_{9}, O_{10}$


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main contribution to $B \rightarrow K^{(*)} l l$ in the SM given by local operators $O_{7}, O_{9}, O_{10}$
$O_{7}=\frac{e}{16 \pi^{2}} m_{b}\left(\bar{s}_{L} \sigma^{\mu \nu} b_{R}\right) F_{\mu \nu}$
$O_{9}=\frac{e}{16 \pi^{2}}\left(\bar{s}_{L} \gamma^{\mu} b_{L}\right) \sum_{l}\left(\bar{l} \gamma_{\mu} l\right)$
$O_{10}=\frac{e}{16 \pi^{2}}\left(\bar{s}_{L} \gamma^{\mu} b_{L}\right) \sum_{l}\left(\bar{l} \gamma_{\mu} \gamma_{5} l\right)$
hadronic matrix elements computed in a previous work ( $B \rightarrow K^{(*)} \mathrm{FFs}$ ) and in this one ( $B_{s} \rightarrow \phi \mathrm{FFs}$ )
[NG/Kokulu/van Dyk '18]


## Charm-loop in $B \rightarrow K^{(*)} l l$

additional non-local contributions come from $O_{1}$ and $O_{2}$ combined with the e.m. current (charm-loop contribution)

$$
O_{1}=\left(\bar{s}_{L} \gamma^{\mu} c_{L}\right)\left(\bar{c}_{L} \gamma_{\mu} b_{L}\right) \quad O_{2}=\left(\bar{s}_{L}^{j} \gamma^{\mu} c_{L}^{i}\right)\left(\bar{c}_{L}^{i} \gamma_{\mu} b_{L}^{j}\right)
$$



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## Light-cone sum rules in a nutshell

# Methods to compute hadronic matrix elements 

[^0]
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QCD perturbation theory breaks down at low energies non-perturbative techniques are needed to compute hadronic matrix elements

## Light-cone sum rules (LCSRs)

quark-hadron duality approximation
universal $B$-meson matrix elements
applicable for both local and nonlocal matrix elements (at low $q^{2}$ )

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## Lattice QCD

numerical evaluation of correlators in a finite and discrete space-time
local matrix elements (usually at high $q^{2}$ )
nonlocal matrix elements still work in progress

## Light-cone Sum Rules in a nutshell 1

LCSRs are used to determine hadronic matrix elements from a correlation function $\Pi(k, q)$

$$
\Pi^{\mu v}(k, q)=i \int \mathrm{~d}^{4} x e^{i k x}\langle 0| T\left\{J_{\text {int }}^{v}(x), \mathcal{O}^{\mu}(0, \ldots)\right\}|B(q+k)\rangle
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consider e.g. $B \rightarrow K^{(*)}$ transitions
$J_{i n t}^{v}=\bar{d} \gamma^{v}\left(\gamma_{5}\right) s$
for the form factors $\mathcal{O}^{\mu}(0, \ldots)=\bar{s} \Gamma^{\mu} b(0)$
for the charm-loop $\mathcal{O}^{\mu}(0, y)=\bar{s} \Gamma b(0) I^{\mu}(y)$
we want to compute the matrix element $\left\langle\boldsymbol{K}^{(*)}(\boldsymbol{k})\right| \boldsymbol{\mathcal { O }}^{\mu}(\mathbf{0}, \ldots)|\boldsymbol{B}(\boldsymbol{q}+\boldsymbol{k})\rangle$


$$
\Pi(k, q)=i \int \mathrm{~d}^{4} x e^{i k x}\langle 0| T\left\{J_{i n t}(y), \tilde{O}_{\mu}(0, x)\right\}|B(q+k)\rangle
$$

1
Hadronic representation for positive $k^{2}$


OPE for large negative $k^{2}$ and small $q^{2}$

## Light-cone Sum Rules in a nutshell 2

two ways to compute the correlator

$$
\Pi(k, q)=i \int \mathrm{~d}^{4} x e^{i k x}\langle 0| T\left\{J_{i n t}(y), \tilde{O}_{\mu}(0, x)\right\}|B(q+k)\rangle
$$

1
Hadronic representation for positive $k^{2}$


2
OPE for large negative $k^{2}$ and small $q^{2}$
the sum rule is obtained matching the result the two different calculations of $\Pi(k, q)$ and using semi-global quark-hadron duality

## Hadronic calculation

for positive $k^{2}$
$\Pi^{\mu \nu}(k, q)=i \int \mathrm{~d}^{4} x e^{i k x}\langle 0| T\left\{J_{\text {int }}^{\nu}(x), \mathcal{O}^{\mu}(0, \ldots)\right\}|B(q+k)\rangle$
insert a complete set
of hadronic states


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insert a complete set
of hadronic states
$\frac{\overbrace{\langle 0| J_{i n t}^{v}\left|K^{(*)}\right\rangle}^{\propto f_{K^{(*)}}}\left\langle K^{(*)}(k)\right| \mathcal{O}^{\mu}(0, \ldots)|B(q+k)\rangle}{k^{2}-m_{K^{(*)}}}+$ continuum


## OPE calculation

for large negative $k^{2}$ and small $q^{2}$, the $B$-meson treated in HQET

$$
\Pi^{\mu v}(k, q)=i \int \mathrm{~d}^{4} x e^{i k x}\langle 0| T\left\{J_{i n t}^{v}(x), \mathcal{O}^{\mu}(0, \ldots)\right\}|B(q+k)\rangle
$$

light-cone expansion $\boldsymbol{x}^{\mathbf{2}} \simeq \mathbf{0}$, factorize hard and soft contributions

$$
\begin{aligned}
\Pi\left(k^{2}, q^{2}\right)=\int_{0}^{\infty} \mathrm{d} s \sum_{n} & \frac{1}{\left(s-k^{2}\right)^{n}}\left(I_{n}\left(s, q^{2}\right)\langle 0| \bar{d}(x) h_{v}(0)|B(v)\rangle\right. \\
& \left.+I_{n}^{\alpha \beta}\left(s, q^{2}\right)\langle 0| \bar{d}(x) G_{\alpha \beta} h_{v}(0)|B(v)\rangle+\ldots\right)
\end{aligned}
$$



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\end{aligned}
$$

- compute $I_{n}$ from a perturbative hard scattering kernel
- $B$-to-vacuum non-local matrix element is a necessary non-perturbative input



## Light-cone distribution amplitudes

express $B$-to-vacuum matrix elements in terms of $B$-meson light-cone distribution amplitudes (LCDAs)
2-particle contribution

$$
\langle 0| \bar{d}(x) h_{v}(0)|B(v)\rangle=-\frac{i f_{B} m_{B}}{4} \operatorname{Tr} \int_{0}^{\infty} d \omega e^{i \omega v \cdot x}\left(\boldsymbol{\phi}_{+}(\omega)+x^{2} \boldsymbol{g}_{+}(\omega)\right)-\frac{x_{\alpha} \gamma^{\alpha}}{2 v \cdot x}\left(\left[\boldsymbol{\phi}_{+}-\boldsymbol{\phi}_{-}\right](\omega)+x^{2}\left[\boldsymbol{g}_{+}-\boldsymbol{g}_{-}\right](\omega)\right)
$$

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$$

3-particle contribution

$$
\begin{aligned}
& \langle 0| \bar{d}(x) G_{\alpha \beta}(u y) h_{v}(0)|B(v)\rangle \\
& =\frac{f_{B} m_{B}}{4} \operatorname{Tr}\left\{\gamma _ { 5 } P _ { + } \left[\left(v_{\alpha} \gamma_{\beta}-v_{\beta} \gamma_{\alpha}\right)\left(\boldsymbol{\Psi}_{A}-\boldsymbol{\Psi}_{\boldsymbol{V}}\right)-i \sigma_{\alpha \beta} \boldsymbol{\Psi}_{\boldsymbol{V}}-\left(y_{\alpha} v_{\beta}-y_{\beta} v_{\alpha}\right) \frac{\boldsymbol{X}_{\boldsymbol{A}}}{v \cdot y}+\left(y_{\alpha} \gamma_{\beta}-y_{\beta} \gamma_{\alpha}\right) \frac{\boldsymbol{W}+\boldsymbol{Y}_{\boldsymbol{A}}}{v \cdot y}\right.\right. \\
& \left.\left.-i \epsilon_{\alpha \beta \sigma \rho} y^{\sigma} v^{\rho} \gamma_{5} \frac{\widetilde{\boldsymbol{X}}_{A}}{v \cdot y}+i \epsilon_{\alpha \beta \sigma \rho} y^{\sigma} \gamma^{\rho} \gamma_{5} \frac{\widetilde{\boldsymbol{Y}}_{A}}{v \cdot y}-\left(y_{\alpha} v_{\beta}-y_{\beta} v_{\alpha}\right) y_{\sigma} \gamma^{\sigma} \frac{\boldsymbol{W}}{(v \cdot y)^{2}}+\left(y_{\alpha} \gamma_{\beta}-y_{\beta} \gamma_{\alpha}\right) y_{\sigma} \gamma^{\sigma} \frac{\boldsymbol{Z}}{(v \cdot y)^{2}}\right]\right\}(\boldsymbol{x}, \boldsymbol{u} \boldsymbol{y})
\end{aligned}
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\end{aligned}
$$

new models and higher twist LCDAs triggered our revisiting of the sum rules
[Braun/Ji/Manashov '17]
organize LCDAs in a twist expansion (twist = dimension - spin)
higher twists are power of $\Lambda_{\mathrm{had}} / m_{B}$ suppressed

$$
\begin{aligned}
& \Pi\left(k^{2}, q^{2}\right)=\int_{0}^{\infty} \mathrm{d} s \sum_{n} \frac{1}{\left(s-k^{2}\right)^{n}}\left(I_{n}\left(s, q^{2}\right)\langle 0| \bar{d}(x) h_{v}(0)|B(v)\rangle\right. \\
&\left.+I_{n}^{\alpha \beta}\left(s, q^{2}\right)\langle 0| \bar{d}(x) G_{\alpha \beta} h_{v}(0)|B(v)\rangle+\ldots\right)
\end{aligned} \quad \begin{aligned}
& \quad \text { insert the LCDAs in the OPE }
\end{aligned} \quad \begin{array}{r}
\Pi\left(k^{2}, q^{2}\right)=f_{B} \int_{0}^{\infty} \mathrm{d} s \sum_{n, t \leq 4} \frac{I_{n, t}\left(s, q^{2}\right)}{\left(s-k^{2}\right)^{n}} \Psi_{t}(x, u y)
\end{array}
$$

$$
\begin{aligned}
& \Pi\left(k^{2}, q^{2}\right)=\int_{0}^{\infty} \mathrm{d} s \sum_{n} \frac{1}{\left(s-k^{2}\right)^{n}}\left(I_{n}\left(s, q^{2}\right)\langle 0| \bar{d}(x) h_{v}(0)|B(v)\rangle\right. \\
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& \text { integrate by parts } \\
& \text { to obtain a dispersive integral }
\end{aligned} \quad \begin{aligned}
& \Pi\left(k^{2}, q^{2}\right)=f_{B} \int_{0}^{\infty} \mathrm{d} s \sum_{t \leq 4} \frac{I_{t}\left(s, q^{2}\right)}{s-k^{2}} \Psi_{t}(x, u y)
\end{aligned}
$$

## The Sum Rule

matching of the hadronic representation onto the OPE result

$$
\frac{\overbrace{\langle 0| J_{\text {int }}^{v}\left|K^{(*)}\right\rangle}^{\propto f_{K^{(*)}}}\left\langle K^{(*)}(k)\right| \mathcal{O}^{\mu}(0, \ldots)|B(q+k)\rangle}{k^{2}-m_{K^{(*)}}}+\text { continuum }
$$

$$
f_{B} \int_{0}^{\infty} \mathrm{d} s \sum_{t \leq 4} \frac{I_{t}\left(s, q^{2}\right)}{s-k^{2}} \Psi_{t}(x, u y)
$$

use semi-global quark-hadron duality $s_{0}=$ effective threshold

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matching of the hadronic representation onto the OPE result


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\frac{\left\langle K^{(*)}(k)\right| \mathcal{O}^{\mu}(0, \ldots)|B(q+k)\rangle}{k^{2}-m_{K^{(*)}}}=\frac{f_{B}}{f_{K^{(*)}}} \int_{0}^{s_{0}} \mathrm{~d} s \sum_{t \leq 4} \frac{I_{t}\left(s, q^{2}\right)}{s-k^{2}} \Psi_{t}(x, u y)
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$$

apply Borel transformation
Sum Rule

$$
\left\langle K^{(*)}(k)\right| \mathcal{O}^{\mu}(0, \ldots)|B(q+k)\rangle=\frac{f_{B}}{f_{K^{(*)}}} \int_{0}^{s_{0}} \mathrm{~d} s e^{\frac{m_{K^{(*)}}-s}{M^{2}}} \sum_{t \leq 4} I_{t}\left(s, q^{2}\right) \Psi_{t}(x, u y)
$$

$$
\begin{aligned}
& B_{s} \rightarrow K^{(*)}, D_{s}^{(*)}, \phi \\
& \text { form factors }
\end{aligned}
$$

## Definition of the form factors

FFs parametrize exclusive local hadronic matrix elements

$$
\begin{aligned}
& \langle P(k)| \bar{q}_{1} \gamma_{\mu} b|B(q+k)\rangle=2 k_{\nu} f_{+}\left(q^{2}\right)+q_{\mu}\left(f_{+}\left(q^{2}\right)+f_{-}\left(q^{2}\right)\right) \\
& \langle P(k)| \bar{q}_{1} \sigma_{\mu \nu} q^{v} b|B(q+k)\rangle=\frac{i f_{T}\left(q^{2}\right)}{m_{B}+m_{P}}\left(q^{2}(2 k+q)_{\mu}-\left(m_{B}^{2}-m_{P}^{2}\right) q_{\mu}\right)
\end{aligned}
$$

FFs are functions of $q^{2}\left(q^{2}\right.$ is the dilepton mass squared)
3 independent $B$ to pseudoscalar $(P)$ FFs
7 independent $B$ to vector ( $V$ ) FFs

We consider here the final transitions $B_{S} \rightarrow K, D_{s}$ and $B_{S} \rightarrow K^{*}, D_{S}^{*}, \phi$


## Form factors results

same analytical results as NG/Kokulu/van Dyk '18
results for all the $B \rightarrow P, V$ FFs at the $q^{2}$ points

- $q^{2}=\{-15,-10,-5,0\} \mathrm{GeV}^{2}$ for $B_{s} \rightarrow D_{s}^{(*)}$
- $q^{2}=\{-15,-10,-5,0,+5\} \mathrm{GeV}^{2}$ for $B_{s} \rightarrow K^{(*)}, \phi$


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Results already available in the literature:

- $B_{s} \rightarrow K^{*}, \phi$ FFs computed also with light-meson LCSRs
- $B_{s} \rightarrow K^{(*)}, D_{s}, \phi$ FFs lattice QCD results available
- $B_{s} \rightarrow D_{s}^{*}$ only the FF $A_{1}$ at $q_{\text {max }}^{2}$ was known the other 6 FFs are computed for the first time
[Bharrucha/Straub/Zwicky '15]
[Horgan et al. '15]
[McLean et al. '19]
[RBC/UKQCD '15]
[HPQCD '17]


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| $B_{s} \rightarrow \phi l l$ FFs | BSZ2015 | this work |
| :---: | :---: | :---: |
| $A_{0}\left(q^{2}=0\right)$ | $0.389 \pm 0.045$ | $0.38 \pm 0.08$ |
| $A_{1}\left(q^{2}=0\right)$ | $0.296 \pm 0.027$ | $0.31 \pm 0.08$ |
| $A_{12}\left(q^{2}=0\right)$ | $0.246 \pm 0.029$ | $0.24 \pm 0.05$ |
| $V\left(q^{2}=0\right)$ | $0.387 \pm 0.033$ | $0.40 \pm 0.11$ |

[Bharrucha/Straub/Zwicky '15]
[Horgan et al. '15]
[McLean et al. '19]
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[HPQCD '17]

## Extrapolation to large $q^{2} B \rightarrow \phi$ FFs



fit our results to the BSZ2015 parametrization to extrapolate the FFs values in the whole spectrum fits done using both only LCSRs and LCSRs + lattice QCD

Charm-loop effect

## soft-gluon contributions to the charm-loop 16/22

$$
\begin{aligned}
& \text { expand correlator near the light-cone }\left(\boldsymbol{x}^{2} \simeq \mathbf{0}\right) \\
& \int \mathrm{d}^{4} y e^{i k y}\left\langle K^{(*)}(k)\right| T\left\{O_{1,2}(0), \bar{c} \gamma_{\mu} c(x)\right\}|B(q+k)\rangle
\end{aligned}
$$

expansion yields new, non-local matrix elements, to be computed


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$$

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$$
C_{1,2}\left\langle K^{(*)}(k)\right| \bar{s} \Gamma^{\mu} b(0)|B(q+k)\rangle g\left(q^{2}\right)
$$


$C_{1}\left\langle K^{(*)}(k)\right| \tilde{o}_{\mu}(0, x)|B(q+k)\rangle I(q)$

## Charm-loop sum rule

compute the soft-gluon contributions to the charm-loop using LCSRs (for $q^{2} \ll \mathbf{4 m}_{\boldsymbol{c}}^{\mathbf{2}}$ )

$$
\Pi(k, q)=i \int \mathrm{~d}^{4} y e^{i k y}\langle 0| T\left\{J_{i n t}(y), \tilde{o}_{\mu}(0, x)\right\}|B(v)\rangle
$$

where

$$
\begin{aligned}
& J_{\text {int }}=\bar{d} \gamma_{\nu}\left(\gamma_{5}\right) s \\
& \tilde{o}_{\mu}(0, x)=I_{\mu \rho \alpha \beta}(\tilde{q}) \bar{s}_{L} \gamma_{\rho} b_{L}(0) G(u x)
\end{aligned}
$$

obtain the sum rule (only the 3-particle LCDAs contribute)

$$
\left\langle K^{(*)}(k)\right| \tilde{O}_{\mu}(0, x)|B(q+k)\rangle=\frac{f_{B}}{f_{K^{(*)}}} \int_{0}^{s_{0}} \mathrm{~d} s e^{\frac{m_{K^{(*)}}-s}{M^{2}}} \sum_{t=3,4} I_{t}\left(s, q^{2}\right) \Psi_{t}(y, u x)
$$

## Charm-loop sum rule

compute the soft-gluon contributions to the charm-loop using LCSRs (for $q^{2} \ll \mathbf{4 m}_{\boldsymbol{c}}^{\mathbf{2}}$ )

$$
\Pi(k, q)=i \int \mathrm{~d}^{4} y e^{i k y}\langle 0| T\left\{J_{i n t}(y), \tilde{o}_{\mu}(0, x)\right\}|B(v)\rangle
$$

where

$$
\begin{aligned}
& J_{\text {int }}=\bar{d} \gamma_{\nu}\left(\gamma_{5}\right) s \\
& \tilde{O}_{\mu}(0, x)=I_{\mu \rho \alpha \beta}(\tilde{q}) \bar{s}_{L} \gamma_{\rho} b_{L}(0) G(u x)
\end{aligned}
$$

obtain the sum rule (only the 3-particle LCDAs contribute)

$$
\left\langle K^{(*)}(k)\right| \tilde{O}_{\mu}(0, x)|B(q+k)\rangle=\frac{f_{B}}{f_{K^{(*)}}} \int_{0}^{s_{0}} \mathrm{~d} s e^{\frac{m_{K^{(*)}}-s}{M^{2}}} \sum_{t=3,4} I_{t}\left(s, q^{2}\right) \Psi_{t}(y, u x)
$$

method already applied in Khodjamirian/Mannel/Pivovarov/Wang 2010 (KMWP2010)

## 3-particle distribution amplitudes

traditional set 3-particle $B$-meson light-cone distribution amplitudes (B-LCDA) [Kawamura et al. '01]

$$
\begin{aligned}
& \langle 0| \bar{d}(y) G_{\alpha \beta}(u x) h_{v}(0)|B(v)\rangle \\
& =\frac{f_{B} m_{B}}{4} \operatorname{Tr}\left\{\gamma_{5} P_{+}\left[\left(v_{\alpha} \gamma_{\beta}-v_{\beta} \gamma_{\alpha}\right)\left(\boldsymbol{\Psi}_{\boldsymbol{A}}-\boldsymbol{\Psi}_{\boldsymbol{V}}\right)-i \sigma_{\alpha \beta} \boldsymbol{\Psi}_{\boldsymbol{V}}-\left(y_{\alpha} v_{\beta}-y_{\beta} v_{\alpha}\right) \frac{\boldsymbol{X}_{\boldsymbol{A}}}{v \cdot y}+\left(y_{\alpha} \gamma_{\beta}-y_{\beta} \gamma_{\alpha}\right) \frac{\boldsymbol{Y}_{\boldsymbol{A}}}{v \cdot y}\right]\right\}(\boldsymbol{y}, \boldsymbol{u} \boldsymbol{x})
\end{aligned}
$$

used in the previous calculation Khodjamirian/Mannel/Pivovarov/Wang 2010 (KMWP2010)

## 3-particle distribution amplitudes

basis with all independent Lorentz structures, 8 independent $B$-LCDAs $[B r a u n / J i / M a n a s h o v ~ ' 17] ~$

$$
\begin{aligned}
& \langle 0| \bar{d}(y) G_{\alpha \beta}(u x) h_{v}(0)|B(v)\rangle \\
& =\frac{f_{B} m_{B}}{4} \operatorname{Tr}\left\{\gamma _ { 5 } P _ { + } \left[\left(v_{\alpha} \gamma_{\beta}-v_{\beta} \gamma_{\alpha}\right)\left(\boldsymbol{\Psi}_{\boldsymbol{A}}-\boldsymbol{\Psi}_{\boldsymbol{V}}\right)-i \sigma_{\alpha \beta} \boldsymbol{\Psi}_{\boldsymbol{V}}-\left(y_{\alpha} v_{\beta}-y_{\beta} v_{\alpha}\right) \frac{\boldsymbol{X}_{\boldsymbol{A}}}{v \cdot y}+\left(y_{\alpha} \gamma_{\beta}-y_{\beta} \gamma_{\alpha}\right) \frac{\boldsymbol{W}+\boldsymbol{Y}_{\boldsymbol{A}}}{v \cdot y}\right.\right. \\
& \left.\left.-i \epsilon_{\alpha \beta \sigma \rho} y^{\sigma} v^{\rho} \gamma_{5} \frac{\widetilde{\boldsymbol{X}}_{\boldsymbol{A}}}{v \cdot y}+i \epsilon_{\alpha \beta \sigma \rho} y^{\sigma} \gamma^{\rho} \gamma_{5} \frac{\widetilde{\boldsymbol{Y}}_{\boldsymbol{A}}}{v \cdot y}-\left(y_{\alpha} v_{\beta}-y_{\beta} v_{\alpha}\right) y_{\sigma} \gamma^{\sigma} \frac{\boldsymbol{W}}{(v \cdot y)^{2}}+\left(y_{\alpha} \gamma_{\beta}-y_{\beta} \gamma_{\alpha}\right) y_{\sigma} \gamma^{\sigma} \frac{\boldsymbol{Z}}{(v \cdot y)^{2}}\right]\right\}(\boldsymbol{y}, \boldsymbol{u} \boldsymbol{x})
\end{aligned}
$$

the LCDAs $\Psi_{A}, \Psi_{V}, X_{A}, Y_{A}, \ldots$ have no definite twist (twist = dimension - spin) higher twists are suppressed by power of $\Lambda_{\text {had }} / m_{B}$
express the LCDAs in the traditional form in LCDAs with definite twist
calculation up to twist 4 , twist 5 or higher give corrections of the order $1 / m_{b}^{2}$

## Preliminary results and comparison

| $\boldsymbol{\Delta C 9}\left(\boldsymbol{q}^{\mathbf{2}}\right)$ |  | KMPW2010 | GvDV2019 |
| :---: | :---: | :---: | :---: |
| factorizable contr. |  | 0.27 | 0.27 |
| $B \rightarrow K l l$ | $\tilde{\mathcal{A}}\left(q^{2}=1\right)$ | $-0.09_{-0.07}^{+0.06}$ | $\left(1.9_{-0.6}^{+0.6}\right) \cdot 10^{-4}$ |
| $B \rightarrow K^{*} l l$ | $\tilde{\mathcal{V}}_{1}\left(q^{2}=1\right)$ | $0.6_{-0.5}^{+0.7}$ | $\left(1.2_{-0.4}^{+0.4}\right) \cdot 10^{-3}$ |
|  | $\tilde{\mathcal{V}}_{2}\left(q^{2}=1\right)$ | $0.6_{-0.5}^{+0.7}$ | $\left(2.1_{-0.7}^{+0.7}\right) \cdot 10^{-3}$ |
|  | $\tilde{\mathcal{V}}_{3}\left(q^{2}=1\right)$ | $1.0_{-0.8}^{+1.6}$ | $\left(3.0_{-1.0}^{+1.0}\right) \cdot 10^{-3}$ |
| $B_{s} \rightarrow \phi l l$ | $\ldots$ | - | $? ? ?$ |

results represented as a $q^{2}$ dependent correction to $C 9$
[ $q^{2}$ is the dilepton mass square] we fully reproduce the results given in KMWP2010

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results represented as a $q^{2}$ dependent correction to $C 9$
[ $q^{2}$ is the dilepton mass square] we fully reproduce the results given in KMWP2010
matrix elements parametrized analogously to the form factors:

$$
\begin{aligned}
& \langle K(k)| \tilde{O}_{\mu}(0, x)|B(q+k)\rangle=\left((k \cdot q) q_{\mu}-q^{2} k_{\nu}\right) \tilde{\mathcal{A}}\left(q^{2}\right)+\ldots \\
& \left\langle K^{*}(k, \eta)\right| \tilde{O}_{\mu}(0, x)|B(q+k)\rangle= \\
& \epsilon_{\mu \alpha \beta \gamma} \eta^{* \alpha} q^{\beta} k^{\gamma} \widetilde{\mathcal{V}}_{1}\left(q^{2}\right)+i\left(\left(m_{B}^{2}-m_{K^{*}}^{2}\right) \eta_{\mu}^{*}-\left(\eta^{*} \cdot k\right)(2 k+q)_{\mu}\right) \tilde{\mathcal{V}}_{2}\left(q^{2}\right) \\
& \\
& \quad+i\left(\eta^{*} \cdot q\right)\left(q_{\mu}-\frac{q^{2}}{m_{B}^{2}-m_{K^{*}}^{2}}(2 k+q)_{\mu}\right) \tilde{\mathcal{V}}_{3}\left(q^{2}\right)+\ldots
\end{aligned}
$$

## Why such different results?

## KMPW10:

$\lambda_{H}^{2}=\lambda_{E}^{2}=0.31 \pm 0.15 \mathrm{GeV}^{2}$
$\Rightarrow$ twist 3 does not contribute
different inputs: $B$-LCDAs models depend on $\lambda_{H}^{2}, \lambda_{E}^{2}$

$$
\begin{aligned}
\text { we use } \lambda_{E}^{2} & =0.03 \pm 0.02 \mathrm{GeV}^{2} \\
\lambda_{H}^{2} & =0.06 \pm 0.03 \mathrm{GeV}^{2}
\end{aligned}
$$

$\Rightarrow \sim 10$ times smaller [Nishikawa/Tanaka '14]

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& \text { [Nishikawa/Tanaka '14] }
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3-particle $B$-LCDAs heavy quark (twist) expansion
$\rightarrow$ expansion was not known
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we use Braun/Ji/Manashov '17
KMPW10: 4 Lorentz structures
Lorentz structures considered in $\langle 0| \bar{d}(y) G_{\alpha \beta}(u x) h_{v}(0)|B(v)\rangle \rightarrow \quad$ all 8 independent Lorentz structures $\Rightarrow$ partial cancelation (new structures come with an opposite sign)

## Summary

$B_{s} \rightarrow\{P, V\}$ form factors (FFs)

- first calculation of $B_{s} \rightarrow K^{(*)}, \phi$ FFs with $B$ meson LCSRs
- first calculation of $B_{s} \rightarrow D_{s}$ FFs with LCSRs
- first calculation of $B_{s} \rightarrow D_{s}^{*}$ FFs (except for $A_{1}$ at $q_{\text {max }}^{2}$ )


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soft gluon contributions to the charm-loop
- update the previous calculation for $B \rightarrow K^{(*)} l l$ decays and give results for $B_{s} \rightarrow \phi l l$ for the first time
- numerical $\sim 100$ smaller comparing with KMPW2010 (inputs, Lorentz structures, twist expansion, new $B$-LCDAs models)


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## EOS

soft gluon contributions to the charm-loop

- update the previous calculation for $B \rightarrow K^{(*)} l l$ decays and give results for $B_{s} \rightarrow \phi l l$ for the first time
- numerical $\sim 100$ smaller comparing with KMPW2010 (inputs, Lorentz structures, twist expansion, new $B$-LCDAs models)
all results will be accessible in the open source software EOS (https://github.com/eos/eos)

Thank you!

## 3-particle LCDAs twist basis

models given for LCDAs up to twist 4 , twist 5 or higher give corrections of the order $1 / m_{b}^{2}$

$$
\begin{aligned}
& \Psi_{A}=\frac{1}{2}\left(\Phi_{3}+\Phi_{4}\right) \\
& \Psi_{V}=\frac{1}{2}\left(-\Phi_{3}+\Phi_{4}\right) \\
& X_{A}=\frac{1}{2}\left(-\Phi_{3}-\Phi_{4}+2 \Psi_{4}\right) \\
& Y_{A}=\frac{1}{2}\left(-\Phi_{3}-\Phi_{4}+\Psi_{4}-\Psi_{5}\right)
\end{aligned}
$$

$$
\begin{aligned}
& \tilde{X}_{A}=\frac{1}{2}\left(-\Phi_{3}+\Phi_{4}-2 \widetilde{\Psi}_{4}\right) \\
& \tilde{Y}_{A}=\frac{1}{2}\left(-\Phi_{3}+\Phi_{4}-\widetilde{\Psi}_{4}+\widetilde{\Psi}_{5}\right) \\
& W=\frac{1}{2}\left(\Phi_{4}-\Psi_{4}-\widetilde{\Psi}_{4}+\Psi_{5}+\widetilde{\Psi}_{5}+\widetilde{\Phi}_{5}\right) \\
& Z=\frac{1}{4}\left(-\Phi_{3}+\Phi_{4}-2 \widetilde{\Psi}_{4}+2 \widetilde{\Psi}_{5}+\widetilde{\Phi}_{5}+\Phi_{6}\right)
\end{aligned}
$$

use to compute the sum rule

- all 8 independent Lorentz structures (four of them considered for the first time)
- results using LCDAs up to twist 4
- new models for the LCDAs


# 3-particle LCDAs models and $\lambda_{H, E}^{2}$ 

## KMPW2010

$\Psi_{A}\left(\omega_{1}, \omega_{2}\right)=\Psi_{V}\left(\omega_{1}, \omega_{2}\right)=\frac{\lambda_{E}^{2}}{6 \lambda_{B}^{4}} \omega_{2}^{2} e^{-\frac{\omega_{1}+\omega_{2}}{\lambda_{B}}}$
$X_{A}=\frac{\lambda_{E}^{2}}{6 \lambda_{B}^{4}} \omega_{2}\left(2 \omega_{1}-\omega_{2}\right) e^{-\frac{\omega_{1}+\omega_{2}}{\lambda_{B}}}$
$Y_{A}=-\frac{\lambda_{E}^{2}}{24 \lambda_{B}^{4}} \omega_{2}\left(7 \lambda_{B}-13 \omega_{1}+3 \omega_{2}\right) e^{-\frac{\omega_{1}+\omega_{2}}{\lambda_{B}}}$

$$
\begin{aligned}
& \Phi_{3}\left(\omega_{1}, \omega_{2}\right)=\frac{\lambda_{E}^{2}-\lambda_{H}^{2}}{6 \lambda_{B}^{4}} \omega_{1} \omega_{2}^{2} e^{-\frac{\omega_{1}+\omega_{2}}{\lambda_{B}}} \\
& \Phi_{4}\left(\omega_{1}, \omega_{2}\right)=\frac{\lambda_{E}^{2}+\lambda_{H}^{2}}{6 \lambda_{B}^{4}} \omega_{2}^{2} e^{-\frac{\omega_{1}+\omega_{2}}{\lambda_{B}}} \\
& \Psi_{4}\left(\omega_{1}, \omega_{2}\right)=\frac{\lambda_{E}^{2}}{3 \lambda_{B}^{4}} \omega_{1} \omega_{2} e^{-\frac{\omega_{1}+\omega_{2}}{\lambda_{B}}} \\
& \widetilde{\Psi}_{4}\left(\omega_{1}, \omega_{2}\right)=\frac{\lambda_{H}^{2}}{3 \lambda_{B}^{4}} \omega_{1} \omega_{2} e^{-\frac{\omega_{1}+\omega_{2}}{\lambda_{B}}}
\end{aligned}
$$

$\lambda_{H, E}^{2}$ definition
$\langle 0| \bar{d}(0) G_{\alpha \beta}(0) h_{v}(0)|B(v)\rangle=-\frac{i}{6} f_{B} \lambda_{H}^{2} \operatorname{Tr}\left[\gamma_{5} \Gamma P_{+} \sigma_{\alpha \beta}\right]-\frac{i}{6} f_{B}\left(\lambda_{H}^{2}-\lambda_{E}^{2}\right) \operatorname{Tr}\left[\gamma_{5} \Gamma P_{+}\left(v_{\alpha} \gamma_{\beta}-v_{\beta} \gamma_{\alpha}\right)\right]$

## Threshold $s_{0}$ determination

$$
\begin{equation*}
f_{K^{(*)}} e^{-\frac{m_{K^{(*)}}}{M^{2}}}\left\langle K^{(*)}(k)\right| \tilde{O}_{\mu}(0, x)|B(q+k)\rangle=f_{B} \int_{0}^{s_{0}} \mathrm{~d} s e^{-\frac{s}{M^{2}}} \sum_{t=3,4} I_{t}\left(s, q^{2}\right) \Psi_{t}(y, u x) \tag{1}
\end{equation*}
$$

$$
\begin{gathered}
\quad \begin{array}{l}
\text { derive with respect to } 1 / M^{2} \\
\text { and divide by (1) }
\end{array} \\
m_{K^{(*)}}=\frac{\int_{0}^{s_{0}} \mathrm{~d} s s e^{-\frac{s}{M^{2}}} \sum_{t=3,4} I_{t}\left(s, q^{2}\right) \Psi_{t}(y, u x)}{\int_{0}^{s_{0}} \mathrm{~d} s e^{-\frac{s}{M^{2}}} \sum_{t=3,4} I_{t}\left(s, q^{2}\right) \Psi_{t}(y, u x)}
\end{gathered}
$$

daughter sum rule to extract $\boldsymbol{s}_{\mathbf{0}}$

## Alignment of the gluon with the $K^{(*)}$ meson

We are interested in the dominant effect of the nonvanishing gluon momenta generated by the exponent in (3.9). Decomposing the covariant derivative in the light-cone vectors

$$
\begin{equation*}
\mathcal{D}=\left(n_{+} \mathcal{D}\right) \frac{n_{-}}{2}+\left(n_{-} \mathcal{D}\right) \frac{n_{+}}{2}+\mathcal{D}_{\perp} \tag{3.10}
\end{equation*}
$$

we retain only the $n_{-}$component, which corresponds to the gluons emitted antiparallel to $q$, that is, in the same direction as the $s$-quark in the $B$-meson rest frame. We then have

$$
\begin{aligned}
& G^{\alpha \beta}(u x) \simeq \exp \left[-i u\left(n_{-} x\right) \frac{\left(i n_{+} \mathcal{D}\right)}{2}\right] G^{\alpha \beta} \\
&=\int d \omega \exp \left[-i u\left(n_{-} x\right) \omega\right] \delta\left[\omega-\frac{\left(i n_{+} \mathcal{D}\right)}{2}\right] G^{\alpha \beta} \\
& {[\ldots] }
\end{aligned}
$$

is represented in a compact unintegrated form, and we use the notation $\tilde{q}=q-u \omega n_{-}$, so that $\tilde{q}^{2} \simeq q^{2}-2 u \omega m_{b}$. Here we take into account that $\omega \ll m_{b}$, after the hadronic matrix element is taken. Note that the neglected components of $\mathcal{D}$ in (3.19) produce small, $O\left(\omega / m_{b}\right)$ corrections to $\tilde{q}^{2}$, hence our approximation is well justified.

$$
\begin{aligned}
& \langle 0| \bar{d}(y) G_{\alpha \beta}(u x) n^{\beta} h_{v}(0)|B(v)\rangle \\
& =\frac{f_{B} m_{B}}{4} \operatorname{Tr}\left\{\gamma_{5} P_{+}\left[\left(v_{\alpha} \gamma_{\beta}-v_{\beta} \gamma_{\alpha}\right)\left(\boldsymbol{\Psi}_{A}-\boldsymbol{\Psi}_{\boldsymbol{V}}\right)-i \sigma_{\alpha \beta} \boldsymbol{\Psi}_{\boldsymbol{V}}-\left(y_{\alpha} v_{\beta}-y_{\beta} v_{\alpha}\right) \frac{\boldsymbol{X}_{\boldsymbol{A}}}{v \cdot y}+\left(y_{\alpha} \gamma_{\beta}-y_{\beta} \gamma_{\alpha}\right) \frac{\boldsymbol{Y}_{\boldsymbol{A}}}{v \cdot y}\right] n^{\beta}\right\}(\boldsymbol{y}, \boldsymbol{u x})
\end{aligned}
$$


[Kawamura et al. '01]

$$
\begin{aligned}
& \langle 0| \bar{d}(y) G_{\alpha \beta}(u x) h_{v}(0)|B(v)\rangle \\
& =\frac{f_{B} m_{B}}{4} \operatorname{Tr}\left\{v_{5} P_{+}\left[\left(v_{\alpha} \gamma_{\beta}-v_{\beta} \gamma_{\alpha}\right)\left(\boldsymbol{\Psi}_{\boldsymbol{A}}-\boldsymbol{\Psi}_{\boldsymbol{V}}\right)-i \sigma_{\alpha \beta} \boldsymbol{\Psi}_{\boldsymbol{V}}-\left(y_{\alpha} v_{\beta}-y_{\beta} v_{\alpha}\right) \frac{\boldsymbol{X}_{\boldsymbol{A}}}{v \cdot y}+\left(y_{\alpha} \gamma_{\beta}-y_{\beta} \gamma_{\alpha}\right) \frac{\boldsymbol{Y}_{\boldsymbol{A}}}{v \cdot y}\right]\right\}(\boldsymbol{y}, \boldsymbol{u} \boldsymbol{x})
\end{aligned}
$$

## $\lambda_{B_{S}}$ extimation

in the exponential model

$$
\lambda_{B}=\frac{2}{3} \bar{\Lambda}
$$

where

$$
\bar{\Lambda}_{q}=m_{B_{q}}-m_{q}+O\left(\frac{1}{m_{q}}\right)
$$

then

$$
\begin{gathered}
\frac{\lambda_{B}}{\lambda_{B_{s}}}=\frac{\bar{\Lambda}}{\bar{\Lambda}_{s}} \\
\Downarrow \\
\lambda_{B_{s}}=\lambda_{B} \frac{\bar{\Lambda}_{s}}{\bar{\Lambda}}
\end{gathered}
$$


[^0]:    QCD perturbation theory breaks down at low energies
    non-perturbative techniques are needed
    to compute hadronic matrix elements

