

Analogues acoustiques de l'horizon d'un trou noir

une mise en évidence du rayonnement de Hawking

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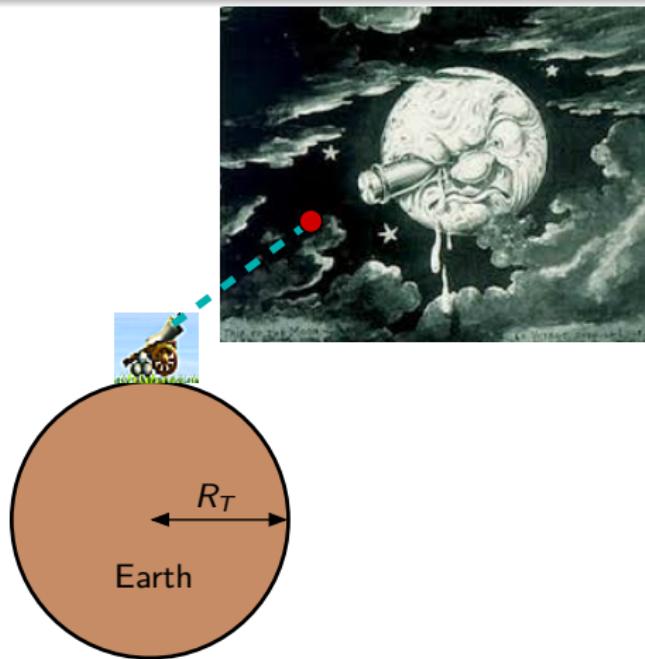
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“Chez les déconstructionnistes, on fait de la théorie à peu près comme on fait de la poésie ou de la musique.”

18th century : naive Black Hole



$$v_{\text{escape}} = \sqrt{2gR_T} = 11.2 \text{ km/s}$$

where $g = GM_T R_T^{-2}$

1783: John Michell

1796: Pierre-Simon Laplace

$$v_{\text{escape}} \propto (M_T/R_T)^{1/2} \propto \rho^{1/2} R_T$$

Un astre lumineux, de la même densité que la Terre, et dont le diamètre serait 250 fois plus grand que le Soleil, ne permettrait, en vertu de son attraction, à aucun de ses rayons de parvenir jusqu'à nous. Il est dès lors possible que les plus grands corps lumineux de l'univers puissent, par cette cause, être invisibles

$$250 \times 107 \times 11.2 = 299600 \text{ km/s}$$

1915 - Einstein's General Relativity

1915 - Schwarzschild solution of Einstein equations. Metric singular at $r = 0$ and at $r = R_s = 2GM/c^2$ ($R_s(\text{earth}) = R_T \times (v_{\text{escape}}/c)^2 = 9 \text{ mm}$)

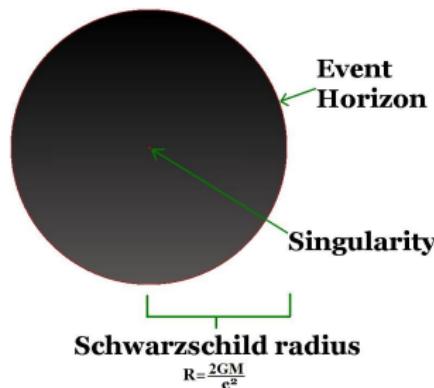
1921: Painlevé then Gullstrand find a solution which is non-singular at R_s

1924: Eddington shows that the singularity at R_s was a coordinate artifact.

1933: Lemaître understands that the singularity at R_s was non physical.

1958 - Finkelstein theorises that R_s is a causality barrier: an event horizon

1964 - Penrose and Hawking prove the existence of a true singularity at $r = 0$



1915 - Einstein's General Relativity

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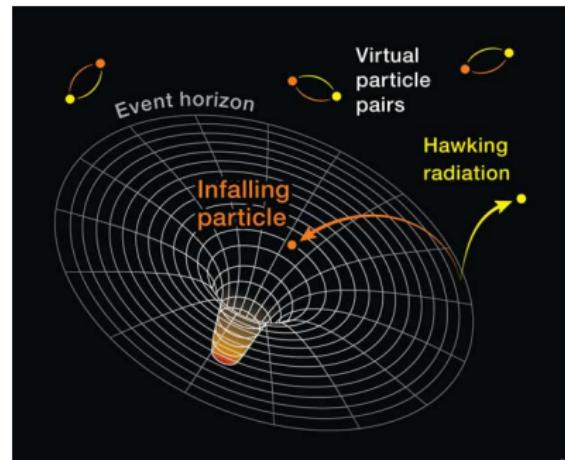
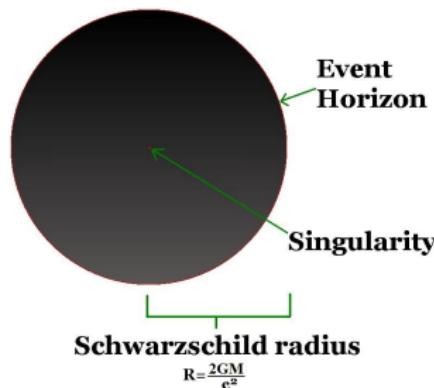
1931 - Chandrasekar : heavy white dwarfs are unstable

1939 - Oppenheimer and Snider : gravitational collapse

Oppenheimer and Volkoff : heavy neutron star → black hole

1958 - Finkelstein theorises that R_s is a causality barrier: an event horizon

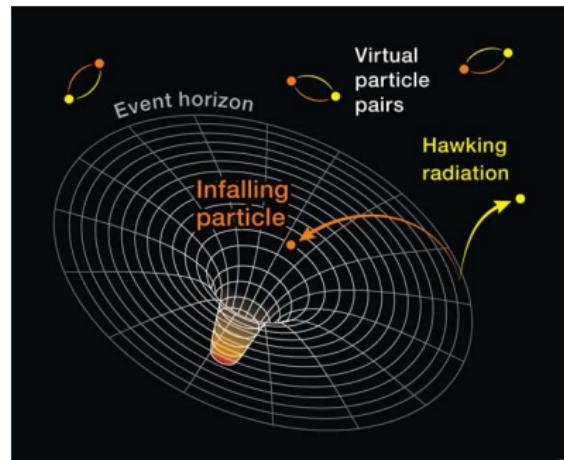
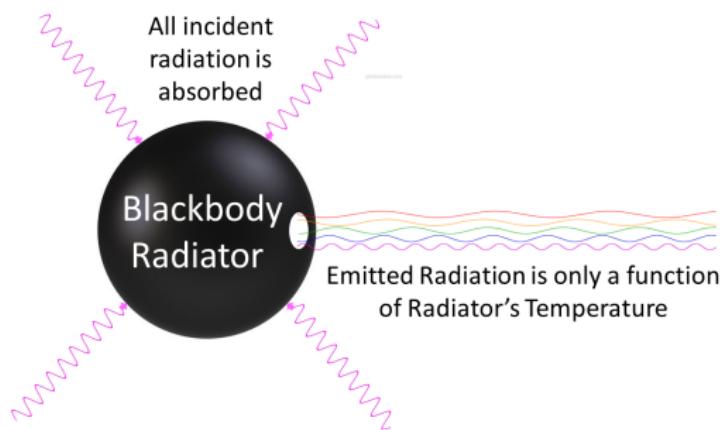
1974 - Hawking radiation: quantum fluctuations near R_s .



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1915 - Schwarzschild solution of Einstein equations. Metric singular at $r = 0$ and at $r = R_s = 2GM/c^2$ ($R_s(\text{earth}) = R_T \times (v_{\text{escape}}/c)^2 = 9 \text{ mm}$)

1974 - Hawking radiation

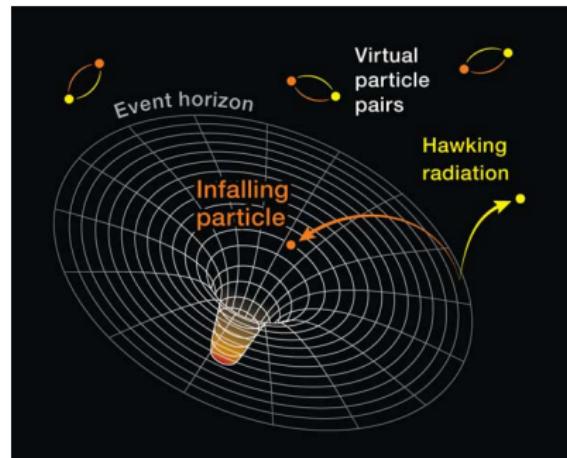


1915 - Einstein's General Relativity

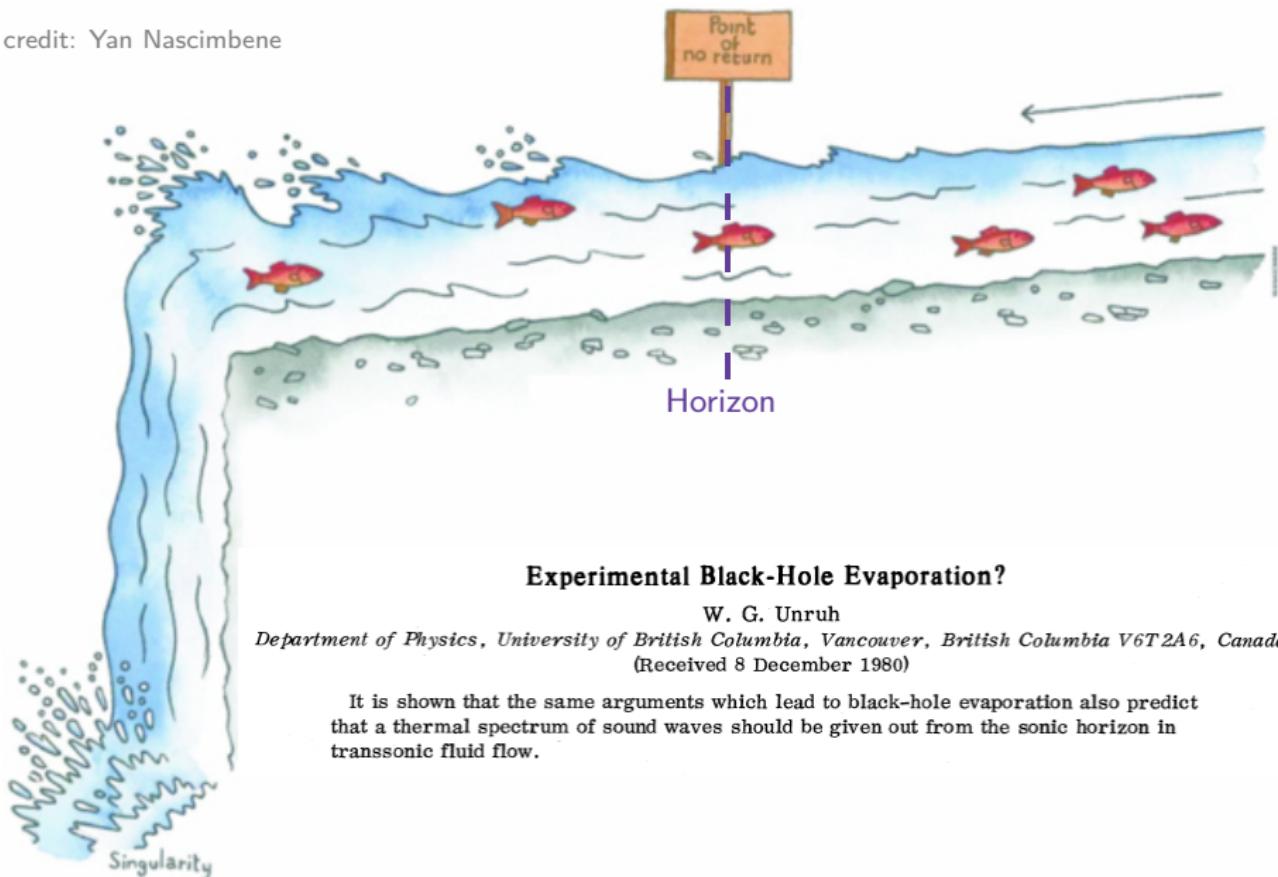
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1974 - Hawking radiation

$$T_H = 10^{-7} M_\odot / M \text{ (K)}$$



credit: Yan Nascimbene



Experimental Black-Hole Evaporation?

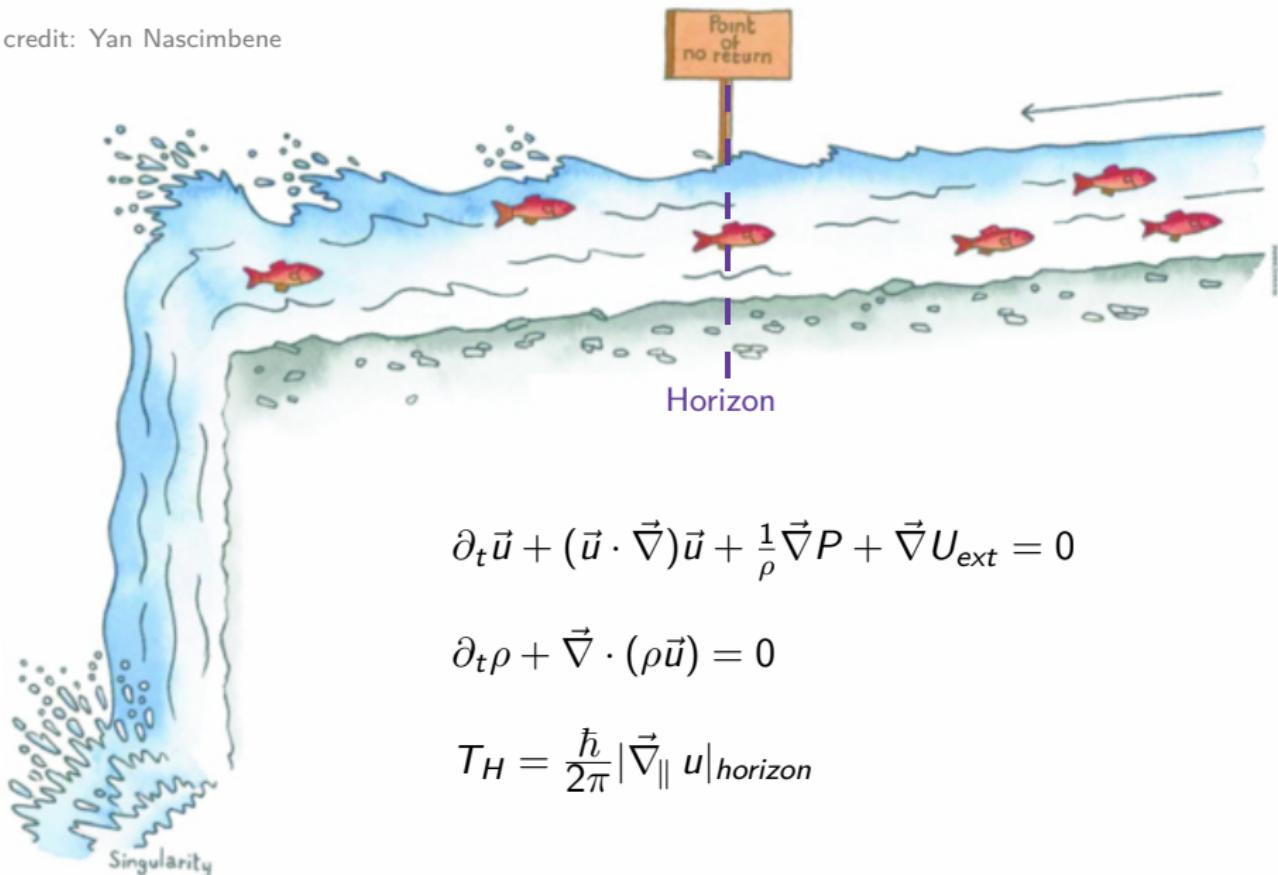
W. G. Unruh

Department of Physics, University of British Columbia, Vancouver, British Columbia V6T 2A6, Canada

(Received 8 December 1980)

It is shown that the same arguments which lead to black-hole evaporation also predict that a thermal spectrum of sound waves should be given out from the sonic horizon in transsonic fluid flow.

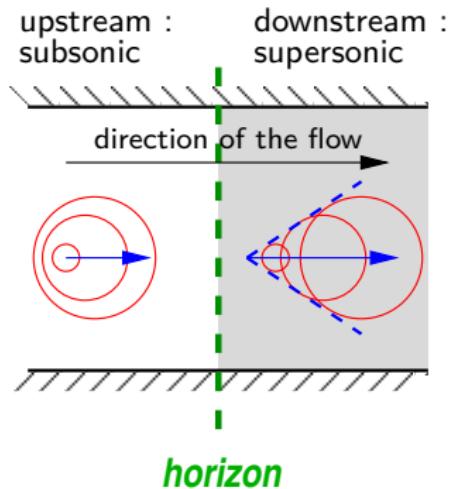
credit: Yan Nascimbene

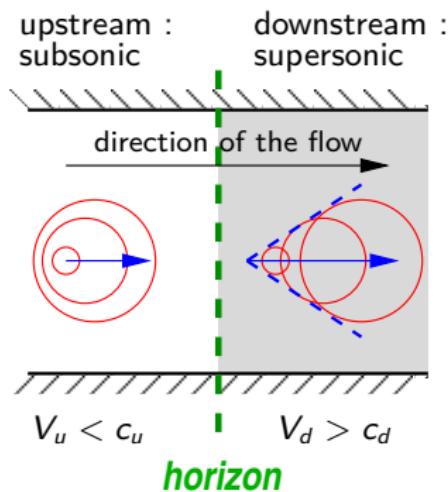


$$\partial_t \vec{u} + (\vec{u} \cdot \vec{\nabla}) \vec{u} + \frac{1}{\rho} \vec{\nabla} P + \vec{\nabla} U_{ext} = 0$$

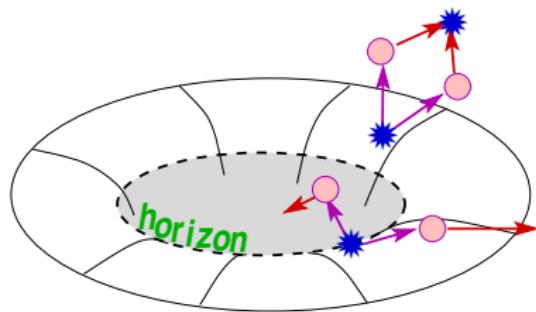
$$\partial_t \rho + \vec{\nabla} \cdot (\rho \vec{u}) = 0$$

$$T_H = \frac{\hbar}{2\pi} |\vec{\nabla}_{||} u|_{horizon}$$

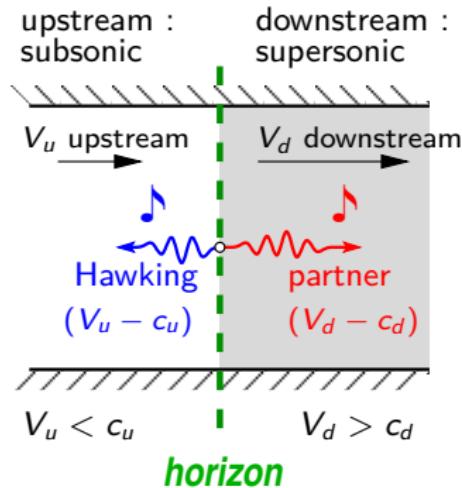
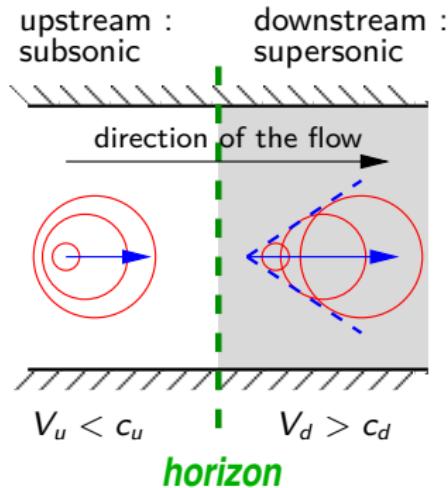


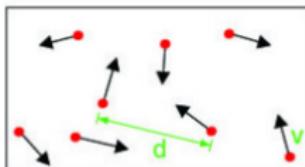


gravitational black hole



Hawking radiation 74'



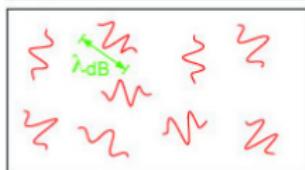


High Temperature T:

thermal velocity v

density d^{-3}

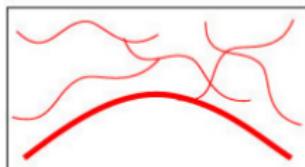
"Billiard balls"



Low Temperature T:

De Broglie wavelength
 $\lambda_{dB} = h/mv \propto T^{-1/2}$

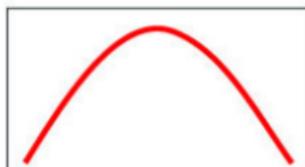
"Wave packets"



$T = T_{crit}$:
Bose-Einstein Condensation

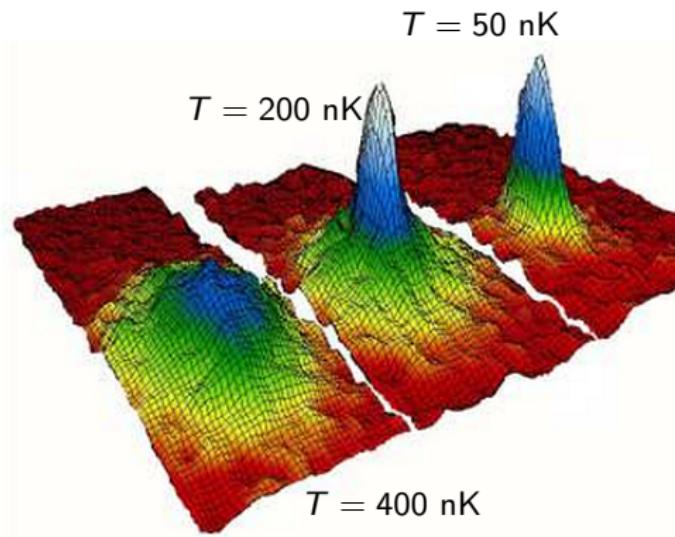
$\lambda_{dB} \approx d$

"Matter wave overlap"

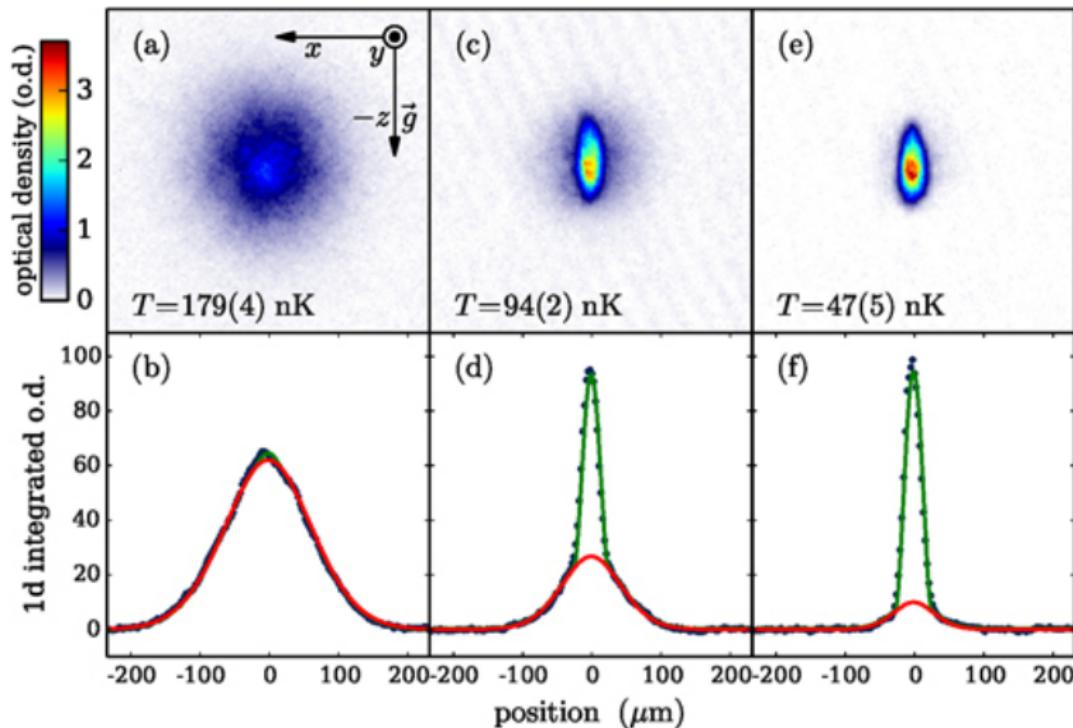


$T=0$:
Pure Bose condensate

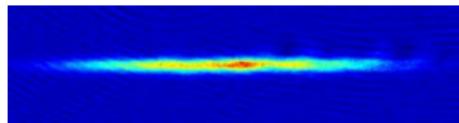
"Giant matter wave"



Cornell & Wiemann (JILA), Ketterle (MIT) : Nobel prize 2001

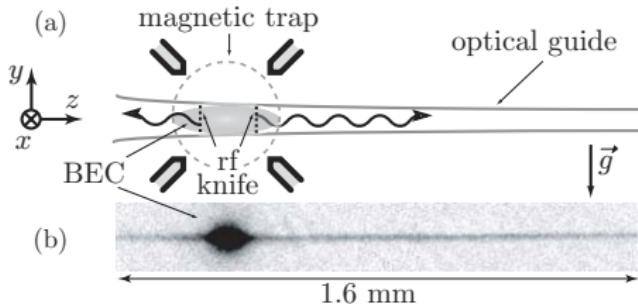


Y. Tang *et al.*, N. J. Phys. (2015) ^{162}Dy , $N \simeq 10^5$ atoms



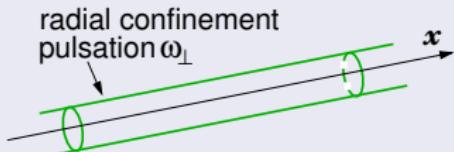
low T , quantum, low c , 1D

quasi-1D condensate
longitudinal size $\sim 10^2 \mu\text{m}$
transverse size $\sim 1 \mu\text{m}$



Guerin et al., Phys. Rev. Lett. (2006)

tight harmonic radial confinement :



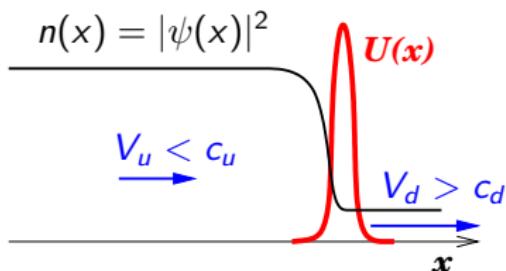
$$V_{\perp}(\vec{r}_{\perp}) = \frac{1}{2} m \omega_{\perp}^2 r_{\perp}^2 .$$

→ 1D model : $\Psi(x, t)$

How to form a sonic horizon ?

stationnary Gross-Pitaevskii Eq.

$$-\frac{1}{2}\psi_{xx} + (U(x) + g|\psi|^2)\psi = \mu\psi$$

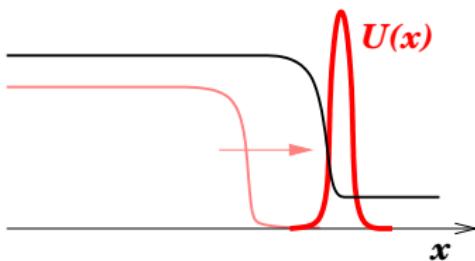


P. Leboeuf and N. Pavloff, Phys. Rev. A (2001)

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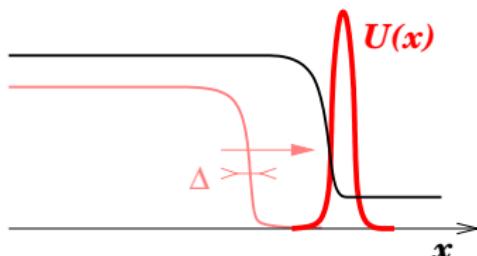


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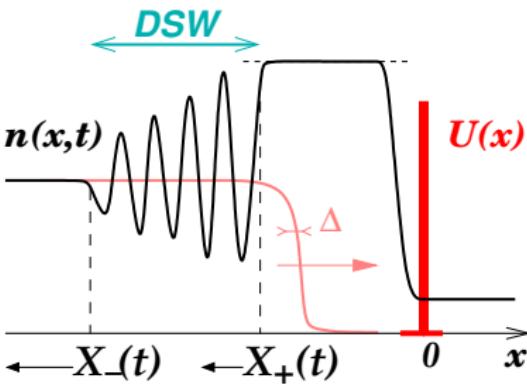
$$-\frac{1}{2}\psi_{xx} + (U(x) + g|\psi|^2)\psi = \mu\psi$$



P. Leboeuf and N. Pavloff, Phys. Rev. A (2001)

time-dependent Gross-Pitaevskii Eq.

$$-\frac{1}{2}\psi_{xx} + (U(x) + g|\psi|^2)\psi = i\psi_t$$

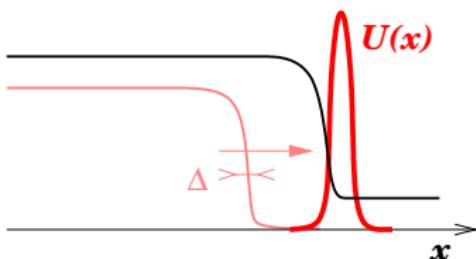


A. Kamchatnov & N. Pavloff, Phys. Rev. A (2012)

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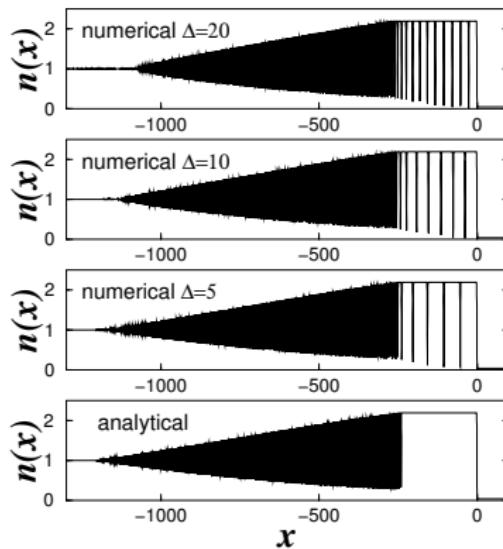
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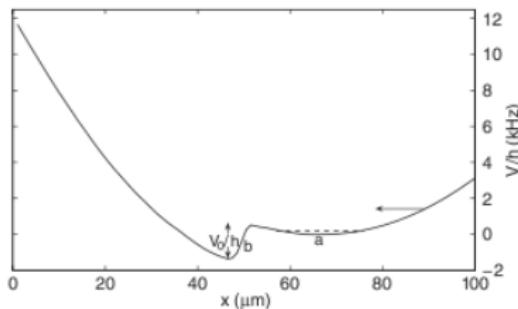
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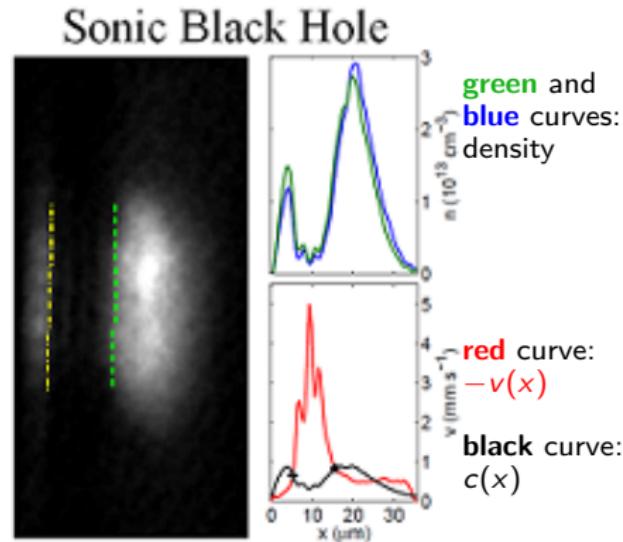


The arrow indicates the direction of the harmonic potential relative to the stationary step like potential ($v \sim 0.3 \text{ mm/s}$).

left plot:

$$v(x) = -\frac{1}{n} \int^x n_t dx' ,$$

$$c(x) = \sqrt{g n(x)} .$$

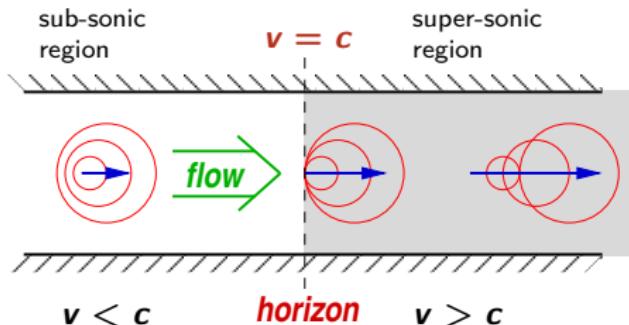


velocity

green dashed line: black hole horizon

yellow dash-dot: white hole horizon

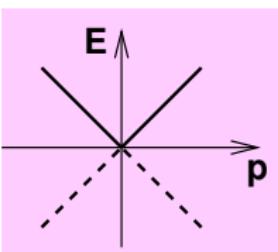
Sonic black holes : “dumb holes”



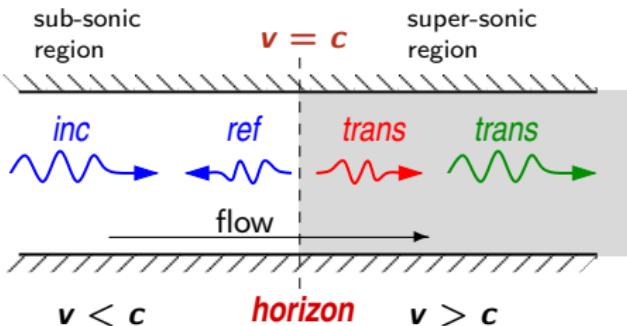
**sound waves
in the comoving frame:**

$$E(p) = c |p|$$

p : momentum in the
comoving frame



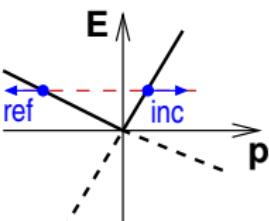
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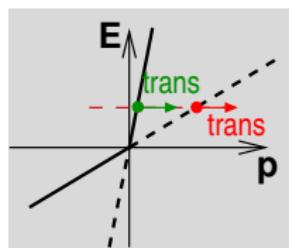
sound waves
in the lab frame:

$$E(p) = c|p| + v p$$

Doppler

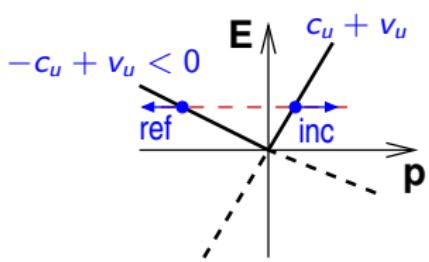


subsonic region

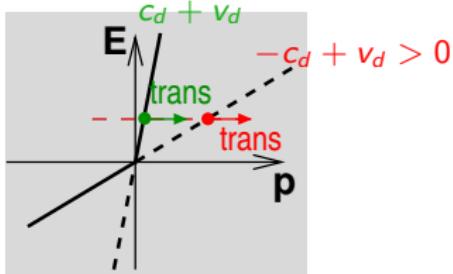


supersonic region

Sonic black holes : “dumb holes”



subsonic region

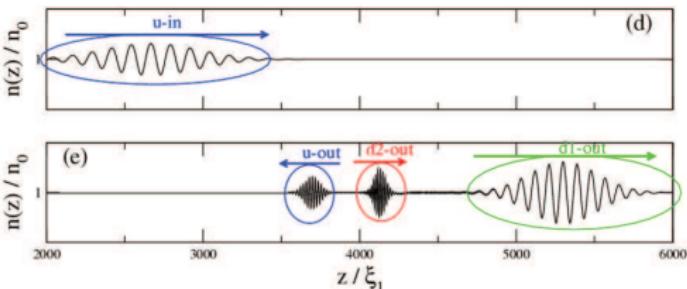


supersonic region

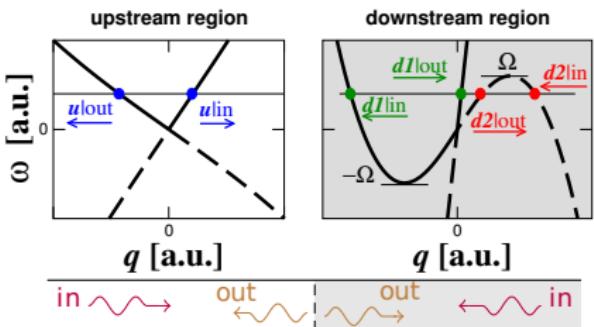
**sound waves
in the lab frame:**

$$E(p) = c|p| + v_p$$

Doppler



Recati, Pavloff, Carusotto, PRA (2009)



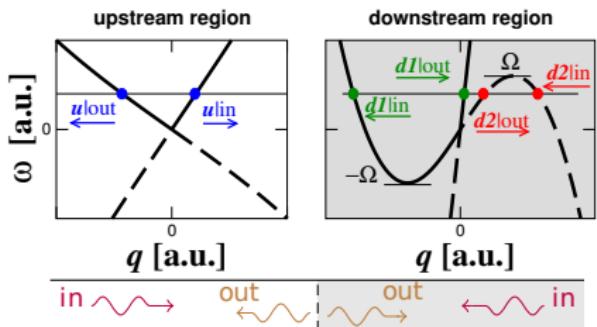
Hawking temperature

$$T_H \simeq 10 \text{ nK} \ll T_{exp} !$$

New theoretical and experimental interest:

study of density correlation on each side of the horizon

$$g^{(2)}(x, x') = \langle n(x)n(x') \rangle - \langle n(x') \rangle \langle n(x) \rangle$$



Hawking temperature

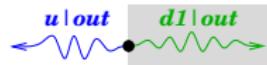
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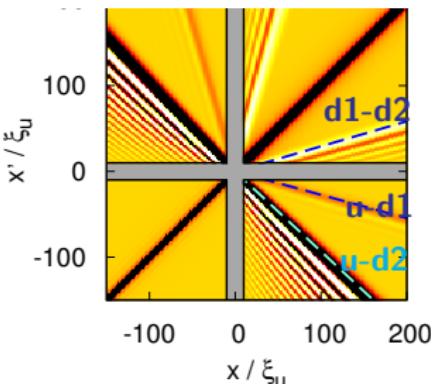
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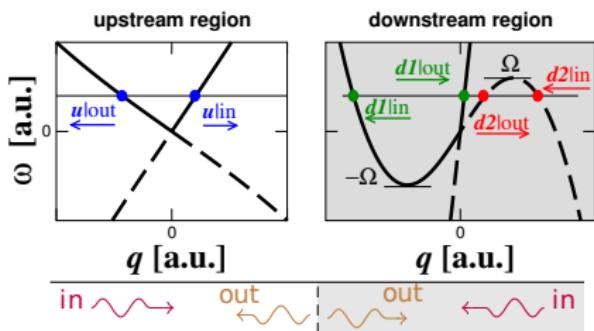
★ example of induced correlation:



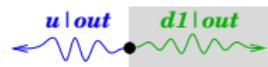
$$\begin{aligned} x &= (v_d + c_d)t && \text{correlates with} \\ x' &= (v_u - c_u)t \end{aligned}$$

★ affects the density correlation pattern





★ example of induced correlation:

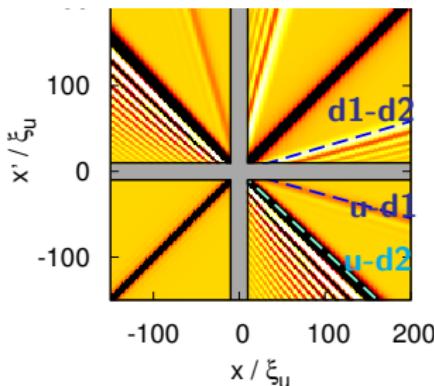
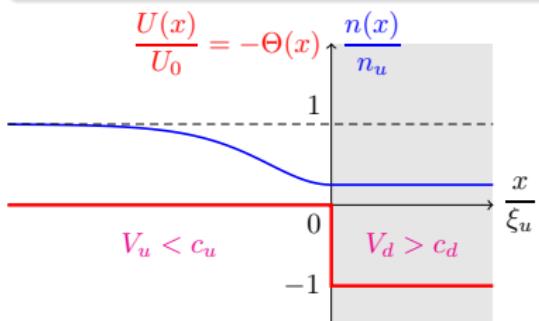


$$x = (v_d + c_d)t \quad \text{correlates with}$$

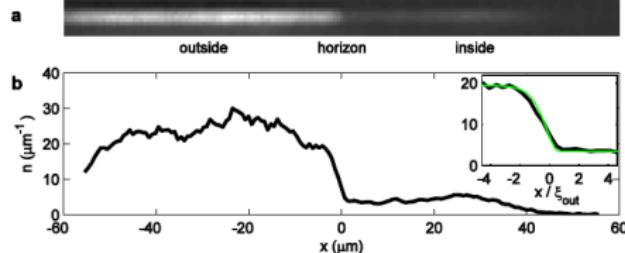
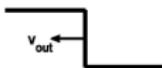
$$x' = (v_u - c_u)t$$

★ affects the density correlation pattern

theoretical “waterfall” configuration :



$$\frac{V_d}{V_u} = \frac{n_u}{n_d} = \left(\frac{c_u}{V_u} \right)^2 = \frac{V_d}{c_d} = \left(\frac{c_u}{c_d} \right)^2$$

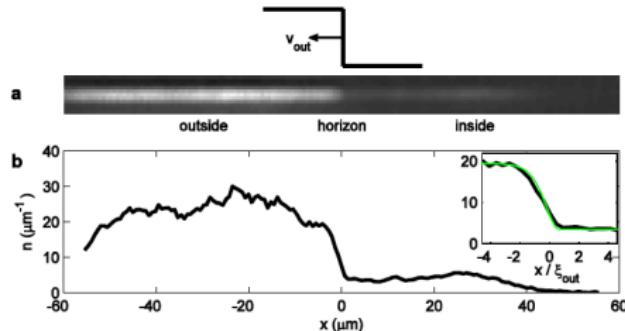


density profile near the horizon \simeq
waterfall $n_u/n_d = 5.55 \ 3.25$

$c_u/c_d = 2.4 \ 1.80$

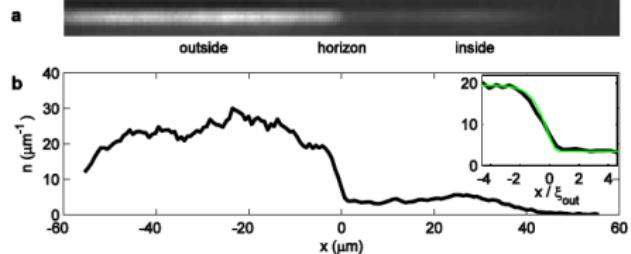
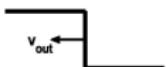
$V_u/c_u = 0.375 \ 0.555 \ V_d/c_d = 3.25 \ 3.25$

$$T_H = 1.0 \text{ nK} \quad \left| \begin{array}{l} T_H/(gn_u) = 0.36 ? \\ T_H/(gn_u)|_{theo} \leq 0.25 \end{array} \right.$$



density profile near the horizon \simeq
 waterfall $n_u/n_d = 5.55$ 5.55
 $c_u/c_d = 2.4$ 2.36
 $V_u/c_u = 0.375$ 0.425 $V_d/c_d = 3.25$ 5.55

$$T_H = 1.0 \text{ nK} \quad \left| \begin{array}{l} T_H/(gn_u) = 0.36 ? \\ T_H/(gn_u)|_{theo} \leq 0.25 \end{array} \right.$$



density profile near the horizon \simeq
waterfall $n_u/n_d = 5.55$ 5.55

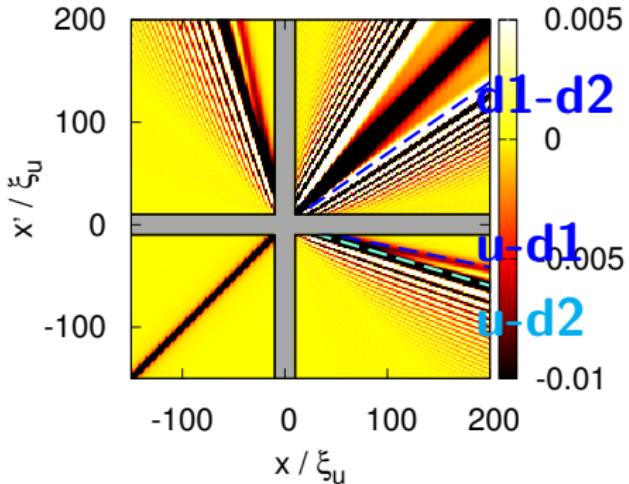
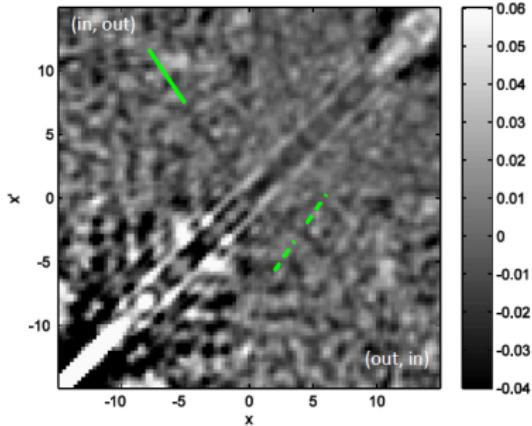
$$c_u/c_d = 2.4 \text{ } 2.36$$

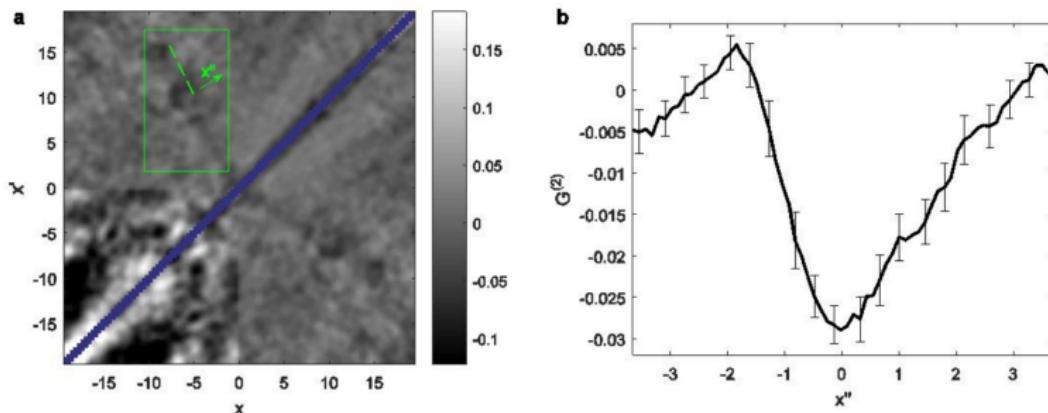
$$V_u/c_u = 0.375 \text{ } 0.425 \quad V_d/c_d = 3.25 \text{ } 5.55$$

$$T_H = 1.0 \text{ nK}$$

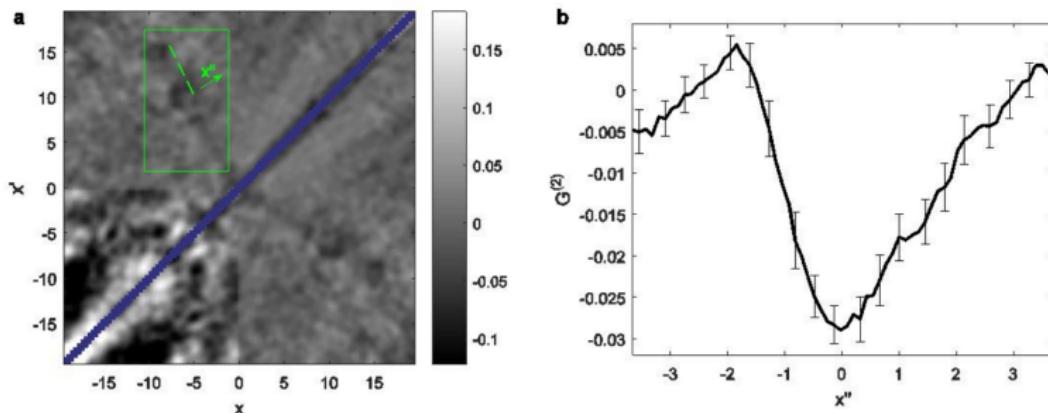
$$\left| \frac{T_H}{(gn_u)} \right| = 0.36 ?$$

$$\left| \frac{T_H}{(gn_u)} \right|_{theo} \leq 0.25$$

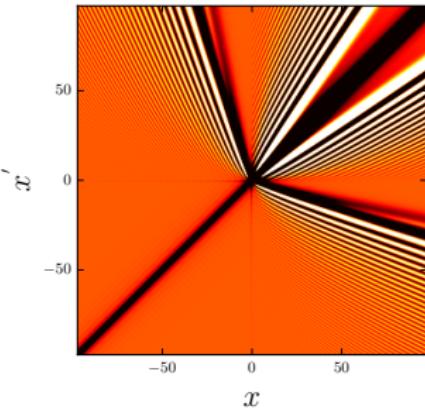


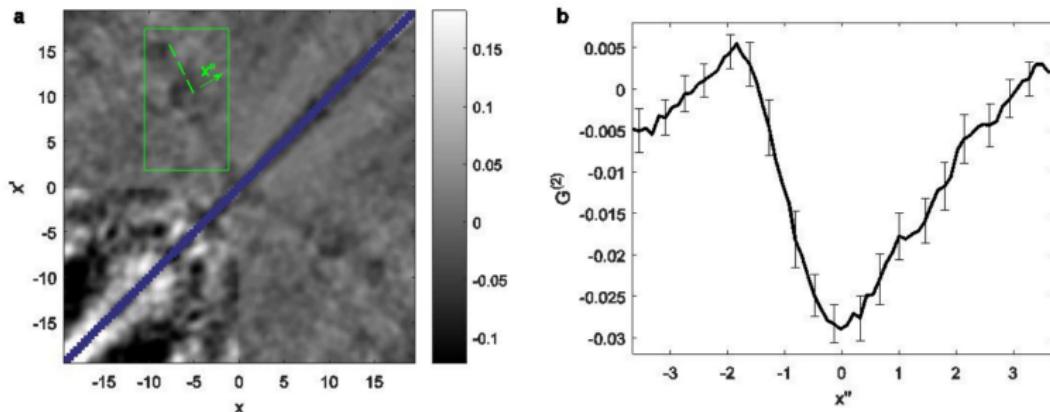


$$T_H|_{exp} = 0.35 \text{ nK} \quad T_H/(gn_u)|_{exp} = 0.12 \quad T_H/(gn_u)|_{theo} = 0.13 \quad (\text{fit: } \frac{V_d}{c_d} = 2.9)$$

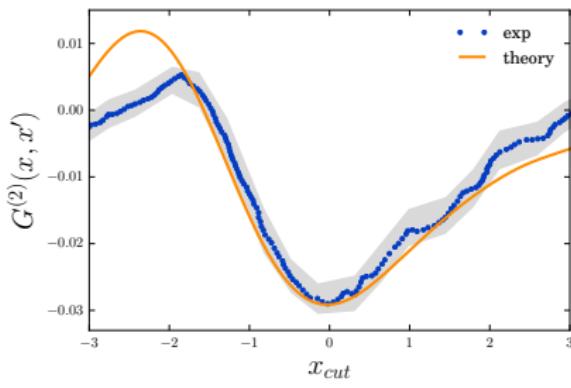
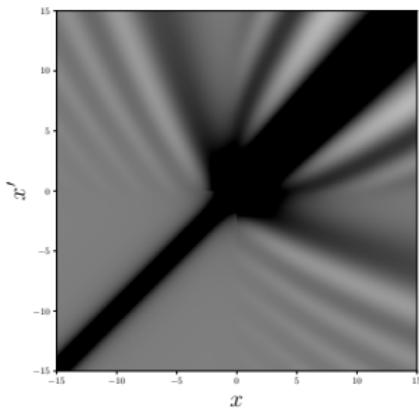


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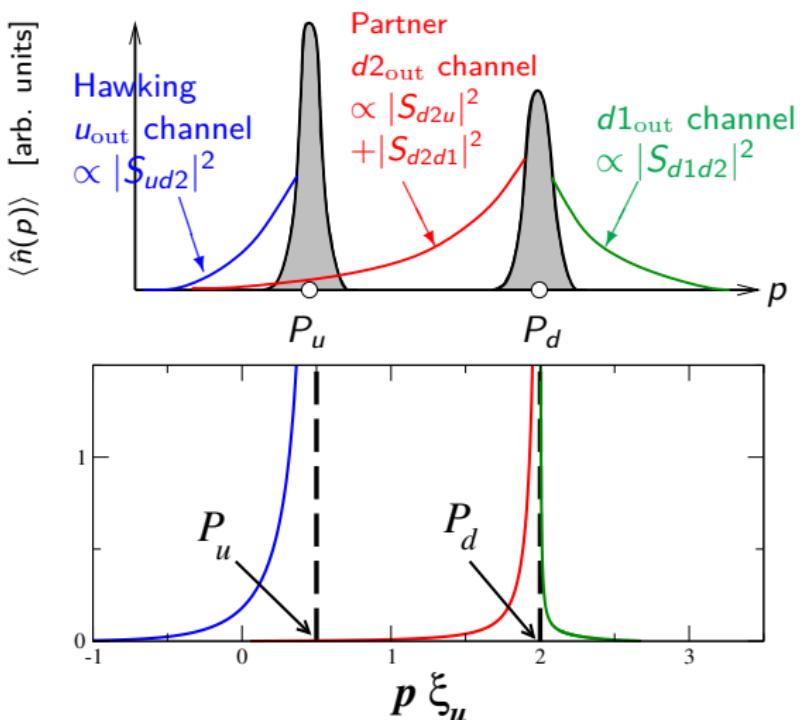
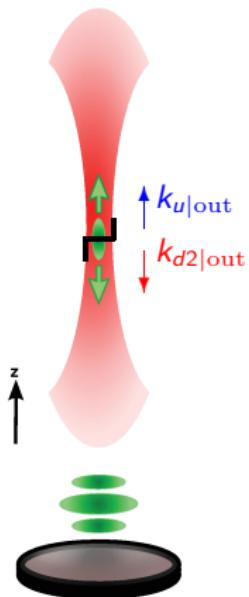
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One body momentum distribution in the presence of a horizon

$T = 0$, adiabatic opening of the trap

Boiron et al. PRL (2015)



Two body momentum distribution in the presence of a horizon

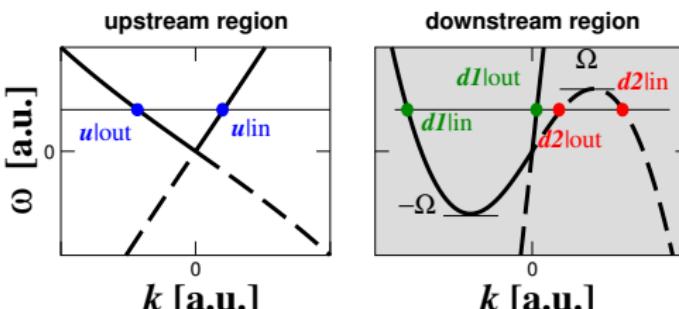
p, q : absolute momenta in units of ξ_u^{-1}

$T = 0$ adiabatic opening

Boiron et al. PRL (2015)

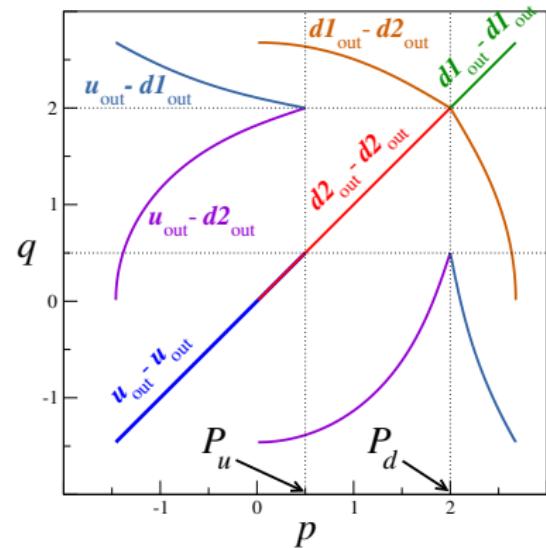
right plot: $g_2(p, q) \rightarrow$

$$\text{where } g_2(p, q) = \frac{\langle : \hat{n}(p) \hat{n}(q) : \rangle}{\langle \hat{n}(p) \rangle \langle \hat{n}(q) \rangle}$$



k : momentum relative to the condensate

$$p = k + P_{(u/d)} \text{ where } P_{(u/d)} = m V_{(u/d)}$$



without horizon: $g_2 \equiv 1$

Two body momentum distribution in the presence of a horizon

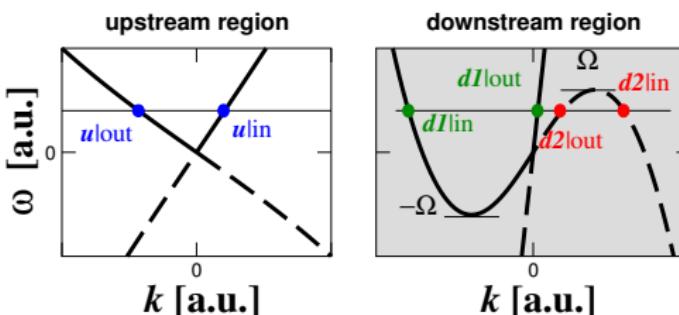
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Boiron et al. PRL (2015)

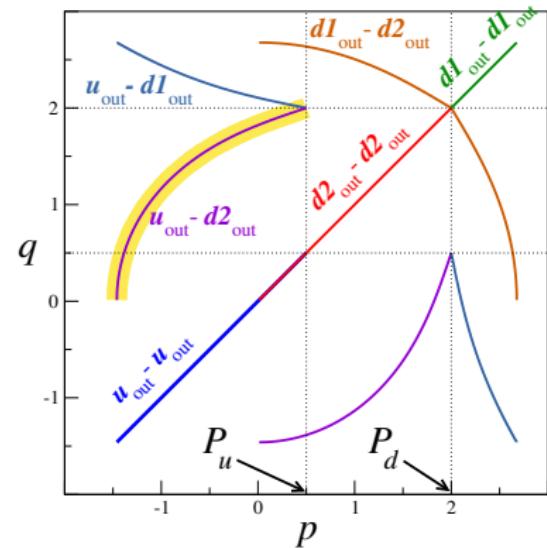
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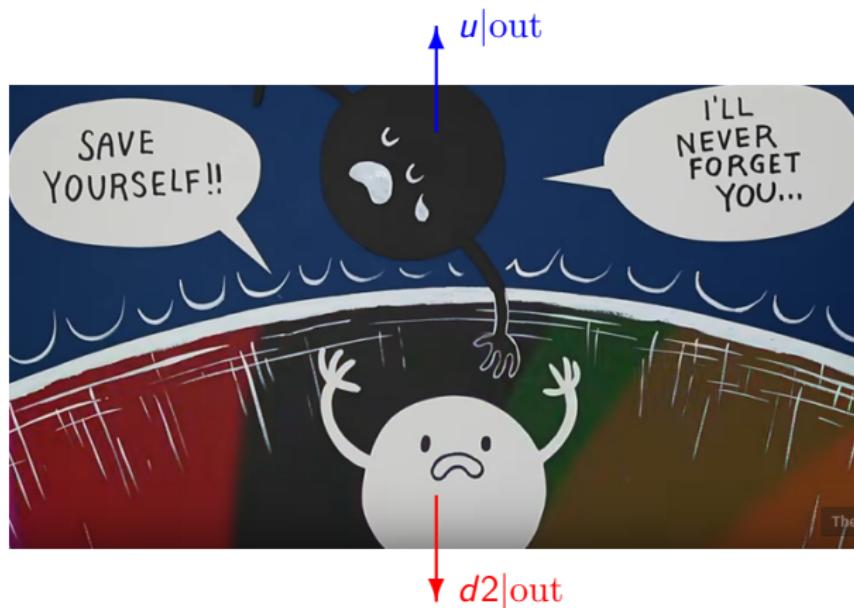
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Violation of Cauchy-Schwarz inequality ($T \neq 0$)

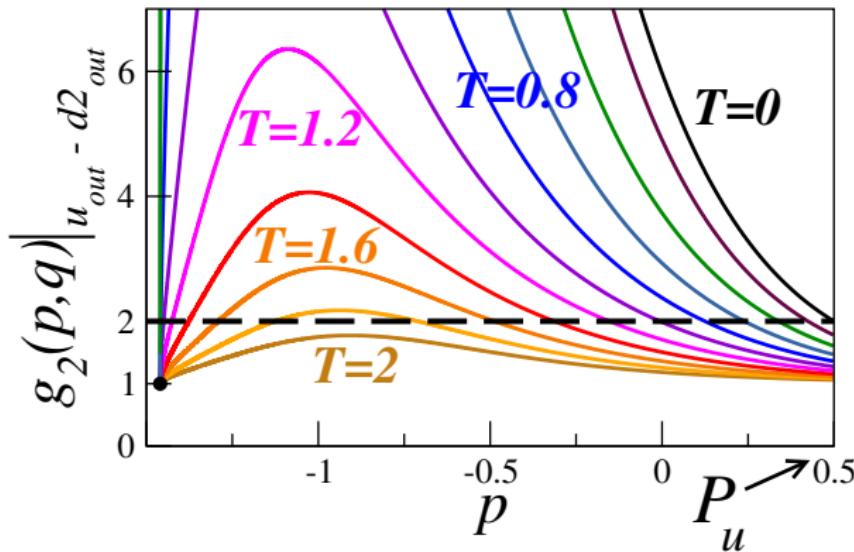
$$\text{C.-S. violation : } g_2(p, q) \Big|_{u_{\text{out}} - d_{2\text{out}}} > \sqrt{g_2(p, p) \Big|_{u_{\text{out}}} \times g_2(q, q) \Big|_{d_{2\text{out}}}} \equiv 2$$



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Boiron *et al.* PRL (2015)



T in units of μ

$$T_H = 0.13$$

$$V_u/c_u = 0.5$$

$$V_d/c_d = 4$$

$$V_d/V_u = 4$$

$$n_u/n_d = 4$$

Gravity wave analogues

Schützhold and Unruh, Phys. Rev. D (2002)

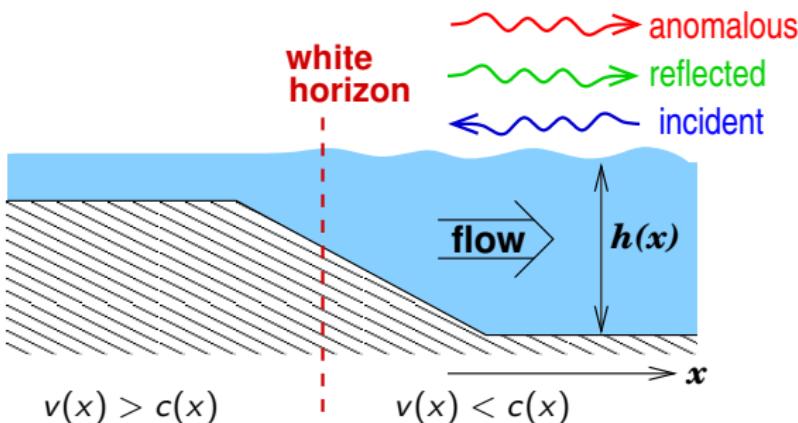
Weinfurtner et al., Phys. Rev. Lett. (2011)

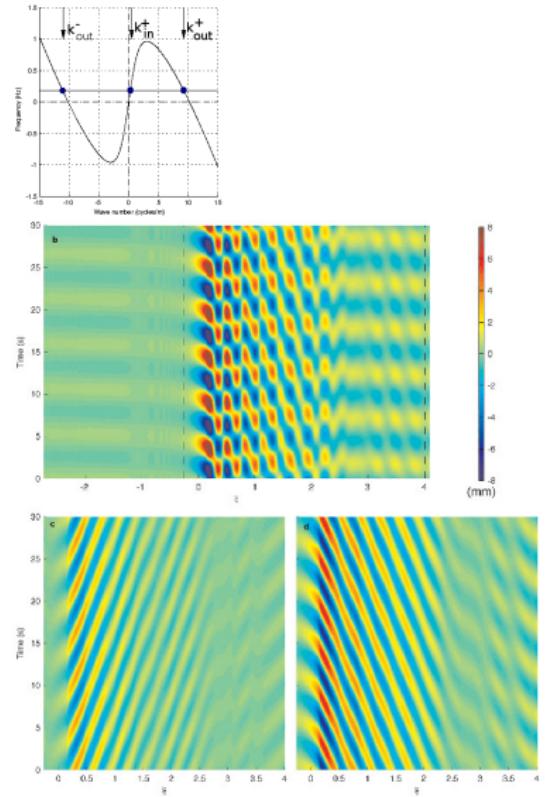
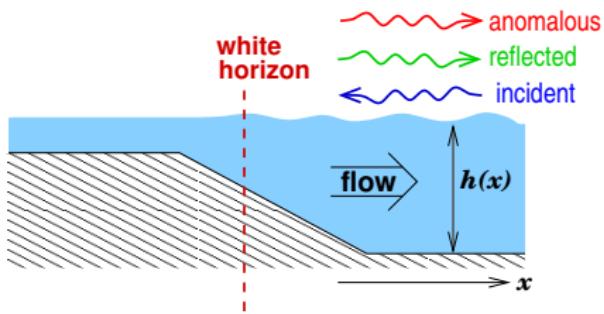
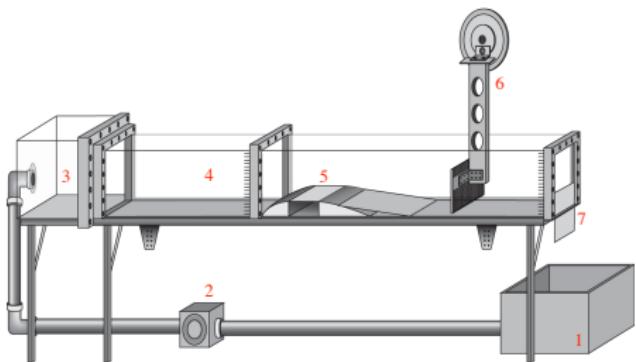
Rousseaux et al., New Journal of Physics (2008)

Euvé et al., Phys. Rev. D (2015), Phys. Rev. Lett. (2016)

in a basin of depth h , the dispersion relation of gravity waves is
 $(\omega - V k)^2 = g k \tanh(k h)$, corresponding to $c = \sqrt{g h}$ when $\lambda \gg h$

Experimental test of mode conversion :



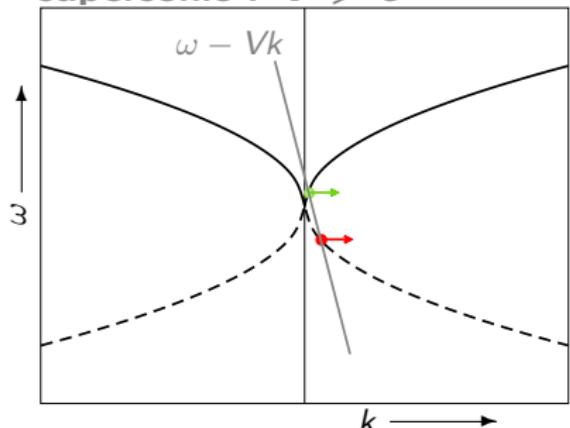


$$\omega - V k = \pm \sqrt{g k \tanh(hk)}$$

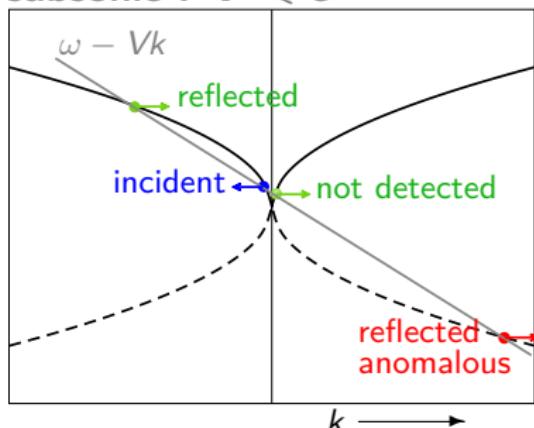


Poitiers experiment

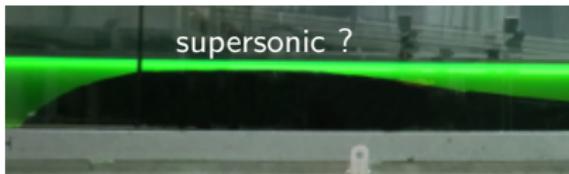
supersonic : $V > c$



subsonic : $V < c$

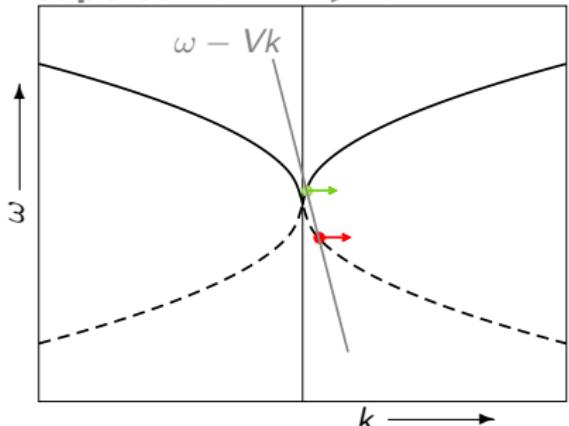


$$\omega - V k = \pm \sqrt{gk \tanh(hk)}$$

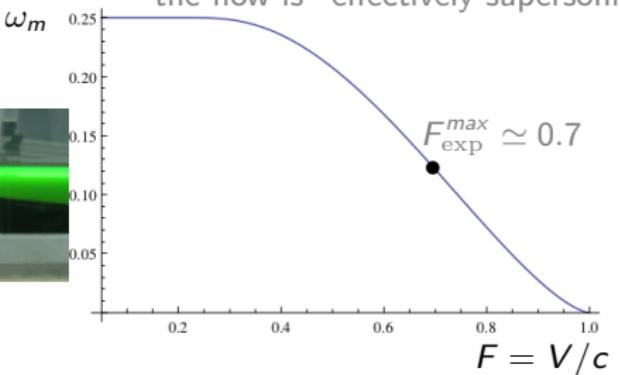


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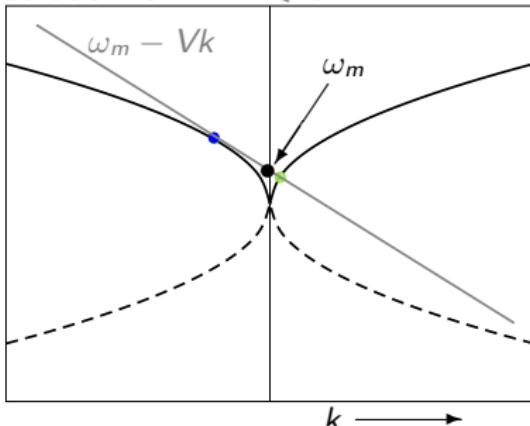
supersonic : $V > c$



for $\omega > \omega_m$: wave blocking
the flow is “effectively supersonic”



subsonic : $V < c$



Conclusion

BECs offer interesting prospects to observe analogous Hawking radiation

general perspective : quantum effects with nonlinear matter waves

Recent experiments (Steinhauer's group) provide clear direct evidences

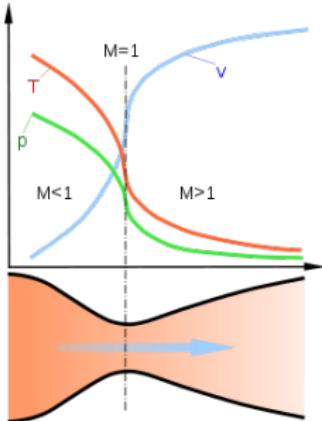
➡ of the occurrence of a sonic horizon.

➡ of the associated acoustic Hawking radiation.

One- and two-body **momentum distributions** accessible by present day experimental techniques should make it possible to flag

👉 the quantum nature of the Hawking process.

😊 The signature of the quantum behavior persists even at temperatures larger than the chemical potential.



Nozzle of a V2 rocket

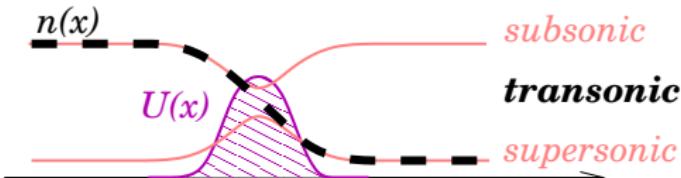
$$F = \dot{m} (v_{\text{out}} - v_{\text{in}})$$

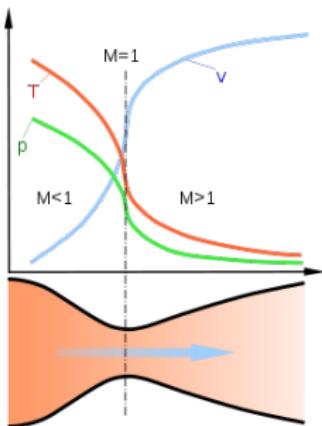
For a **thick** barrier $U(x)$ of width $\gg \xi \sim (gn)^{-1/2}$:

$$\begin{cases} -\frac{(n^{1/2})_{xx}}{2n^{1/2}} + \frac{1}{2}v^2(x) + g n(x) + U(x) = C^{st}, \\ n(x)v(x) = C^{st}. \end{cases}$$

$$\sim \frac{1}{n} \frac{dn}{dx} \left[v^2 - c^2 \right] = \frac{dU}{dx} \quad \text{where } c^2(x) = g n(x)$$

$$v(x) \leqslant c(x) \leftrightarrow \text{sign}\left(\frac{dn}{dx}\right) = \mp \text{sign}\left(\frac{dU}{dx}\right)$$





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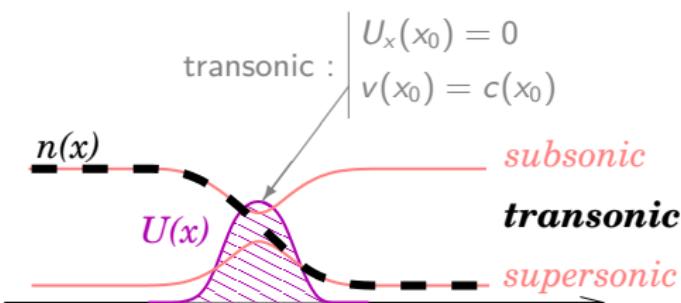
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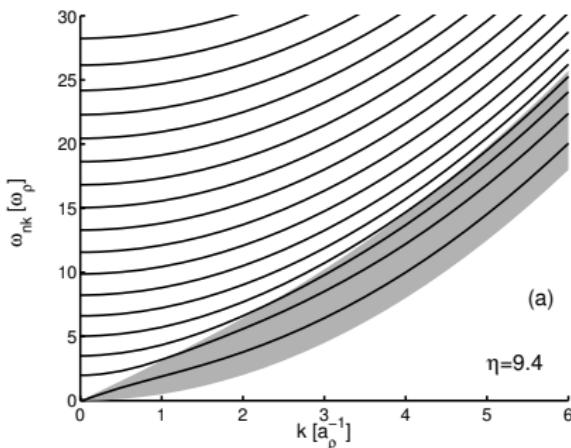


Nozzle of a V2 rocket

$$F = \dot{m}(v_{\text{out}} - v_{\text{in}})$$



when $\hbar\omega_{\perp} \leq \mu$:



Zaremba, PRA (1998)

Stringari, PRA (1998)

Fedichev & Shlyapnikov, PRA (2001)

Tozzo & Dalfonso, PRA (2002)

modified dispersion relation :

$$\omega_0^2(q) = c_{1d}^2 q^2 \left(1 - \frac{1}{48} (qR_{\perp})^2 + \dots \right)$$

new channels :

$$\omega_{n \geq 1}^2(q) = 2n(n+1)\omega_{\perp}^2 + \frac{1}{4}(qR_{\perp}\omega_{\perp})^2 + \dots$$

these new channels will be populated
at $T = 0$

mass term \neq Klein-Gordon

→ new “in” modes