III] Beyond the Standard Model

1) Problems of the SM

a. Data that the SM can't describe

*** Neutrino masses :** known experimentally that neutrino are massive fermions (ν-oscillations)

- remember Dirac mass term : $m\,\overline{e_L}\,e_R$
  $m\,\overline{\nu_L}\,\boxed{\nu_R}$
  ↳ does not exist ⇒ no Dirac mass possible

- Rq : - for fermions charged under gauge grp → Dirac mass
  - ____ uncharged ________ → Majorana mass : $m\,\nu_R\,\nu_R$
  ↳ $\psi^{\overset{\uparrow}{(c)}} = \psi$ (charge conjugat°)
  ↳ we don't know if it exists in Nature ...

× Dark matter :

- Astrophysical observations : - 5% of the total mass energy budget of the universe is matter we know
  - 27% is dark (no interact° with EM)
  ↳ observed by gravitational effects

⇒ DM is presumably composed of a new class of particle

famous candidate : WIMP (Weakly Interacting Massive Particle)

× The missing anti-matter :

- The observed abundance of matter over anti-matter is not explained in the SM.

b. Unexplained features that lead us to believe that there must be physics BSM :

× Cosmological constant vs vacuum :

- Vacuum in QFT : in SM, remember $V(\Phi_{min}) = -\frac{\lambda}{4}v^4$
  ↳ experiment : $v \sim 246$ GeV

  in natural unit $1\,GeV^3 \sim 1{,}25 \times 10^{47}\,m^{-3}$ $([E]=[L])$

  $$|V(\Phi_{min})| \simeq \frac{V}{vol} \simeq 10^{54}\,\frac{GeV}{m^3}$$

- should correspond to the vacuum energy density ≝ dark energy density = energy which forces the expansion ~~of the~~ rate to accelerate

  $\Lambda_{cosmo} \sim 70\%\ \rho_c$
  ↳ critical density $\simeq 5\,\frac{GeV}{m^3}$

⟹ disagreement of 54 orders of magnitude ...

× The hierarchy problem :

- one expects $\underbrace{\text{QM + relativity}}_{SM}$ to collapse du to quantum effects of gravity at the Planck scale $M_p \sim 10^{19}$ GeV

- Why $M_p \gg M_W$

- because $G_F \propto \frac{1}{M_W^2} \gg G_N \propto \frac{1}{M_p^2} \longrightarrow$ Why $G_N \ll G_F$?

- because $V_{Coulomb} \propto \frac{e^2}{r}$ and $V_N \propto G_N \frac{m^2}{r} \overset{\text{in atoms}}{\Longrightarrow} \vec{F}_N \ll \vec{F}_{Coulomb}$

but why ? ...

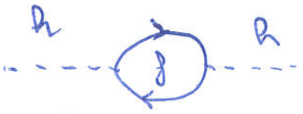
$\rightsquigarrow$ translates to why $M_{Higgs} \ll M_p$?

$\frac{M_h^2}{2} h^2$

× in SM, $M_h = \sqrt{2}\,\lambda v = \sqrt{2\mu^2}$ is an input !

× However, if SM is an EFT of a more complete theory with a Λ cutoff :

Radiative correct° :



$(\Lambda \sim m_t)$

or

$\Delta M_h^2 \propto \Lambda^2 \longrightarrow M_h \sim M_p$?

(if SM valid up to M_p)

× in BSM $M_h \ll M_p$ is a big issue : → hiérarchie pb

→ naturalness issue

↳ fine tunning, because we need a large cancelat° to accomodate $M_h = 125$ GeV with a large Λ

2) Supersymmetry

1. Extension of the Poincaré symmetry

Symmetries are crucial : each continuous symmetry ⟷ conserved quantity (cf. Noether th.)
and Nature follows many of them !

enlarge
- Quantum physics : Translations + Rot° in 3D spatial space
- QFT : Poincaré group (Lorentz tf $[M^{\rho\sigma}]$ & translat° $[P^\sigma]$) defined by : $[P^\rho, P^\sigma] = 0$, $[M^{\cdot\cdot}, M^{\cdot\cdot}] \neq 0$, $[P^{\cdot}, M^{\cdot\cdot}] \neq 0$

More symmetries $\boxed{\text{but}}$ initial space is bigger (+ time !)   (new time is everywhere)

* Natural question : could we enlarge this new symmetry group ?

- Yes : what we did with gauge theory ex : SU(N) (T^a)
  → but trivial extension since all generators commute with the Poincaré algebra

$$[T^a, T^b] \neq 0$$

extended group = Poincaré ⊗ Gauge group $\begin{cases} [T^a, M^{\cdot\cdot}] = 0 \\ [T^{\cdot\cdot}, P^{\cdot\cdot}] = 0 \end{cases}$

↳ very useful to describe particle interactions

- OK, but is it possible in a "non trivial" way ?   or $\left[T^a, \begin{matrix} M^{\cdot\cdot} \\ P^{\cdot\cdot} \end{matrix}\right] \neq 0$?

No-go th. of Coleman-Mandula : No

But in QFT, we have assumed that those $T^a, M^{\cdot\cdot}, P^{\cdot\cdot}$ have to be bosonic op. ie don't change the spin of the state they act on ...

→ instead, let's imagine operators Q, which change the spin of states by $\frac{1}{2}$

↳ introduce supersymmetry tf :

$$Q|\text{boson}\rangle = |\text{fermion}\rangle , \quad Q|\text{fermion}\rangle = |\text{boson}\rangle$$

$$\left.\begin{array}{l} \{Q,\bar{Q}\} \neq 0 \ (?) \\ \rightarrow [M^{\cdots}, Q] \neq 0 \\ [Q, P] = 0 \\ \vdots \end{array}\right\}$$

→ SUSY nice candidate to replace SM, only way to increase the Poincaré symmetry respected by Nature

(1st motivat° historically)

2. Minimal Supersymmetric SM (MSSM)

× keep SM gauge group : $SU(3)_c \times SU(2)_L \times U(1)_Y$

× 1 Q op. (could have more N susy)

SM particles $\xrightarrow{Q}$ SUSY particles

+ spin 0 (Higgs) → spin 1/2 Higgsinos

+ spin 1/2 (lepton quark) → spin 0 (sleptons squarks)

+ spin 1 (gauge boson) → spin 1/2 (Wino Bino gluino)

" $N^{MSSM}_{particle} = 2 \times N_p^{SM}$ "

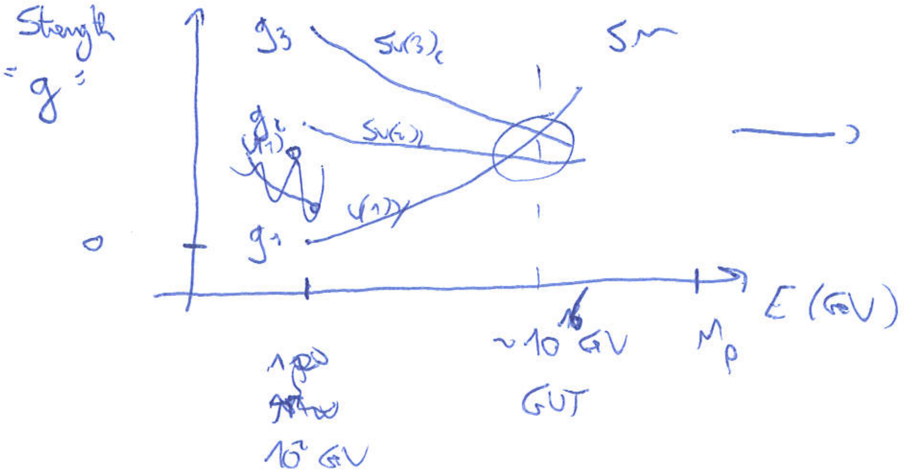
→ Rich phenomenology

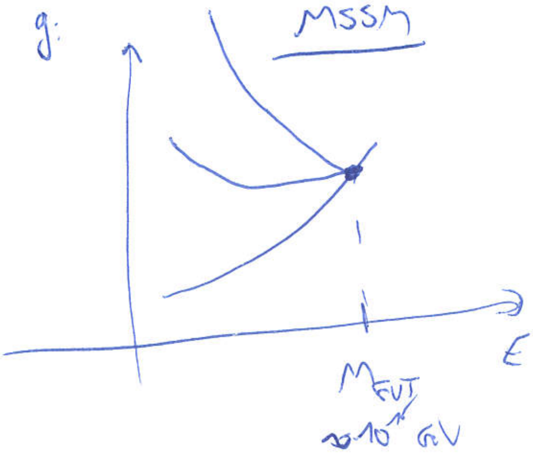
× Benefits of MSSM (SUSY) :

- Protection of the Higgs mass i.e solve the hierarchy problem

$\Delta M_H \propto \Lambda^2$ $\quad -\Lambda^2 \quad = 0$ ⇒ <u>M_H is protected</u>

- Unification of gauge couplings





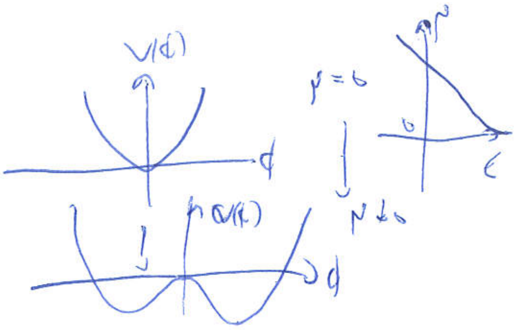
- Explanation of the EW symmetry breaking

$V^{SM}_{(\Phi)} = -\mu^2|\phi|^2 + \lambda|\phi|^4$: <u>ad-hoc</u>

× SUSY : × start from unbroken $V = \lambda|\phi|^4$

× radiative correct° generate "μ" (which evolves with E-scale)

⇒ <u>SUSY provides the mechanism of radiative EW symmetry Breaking</u>



- DM candidate :

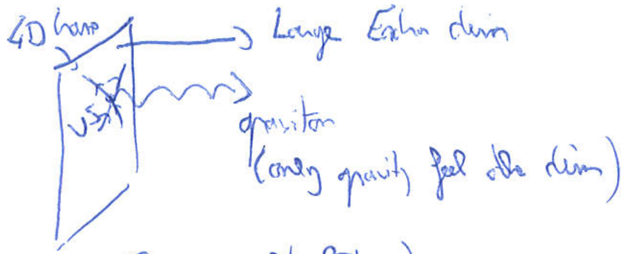
Higgsino + inos ⇝ neutralinos. The lightest one is a good DM candidate (WIMP)

2) Extra dimensions

* g^{al} idea: expansion of the Poincaré symmetry: ~~by adding generators to the one of translations~~ and additional generators associated with translation invariance in each extra spatial dim.

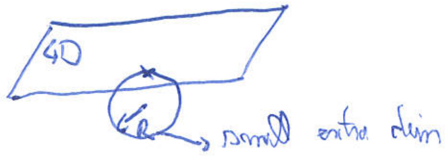
2 cases

• we are stuck on a brane



(Brane World Picture)

• The extra dim are curled up on themselves



(Kaluza Klein Picture)

1. Large Extra dimensions

× 3D spatial : $F = G_N \dfrac{m_1 m_2}{r^2}$

3D + d spatial $F = G_{grav} \dfrac{m_1 m_2}{r^{2+d}}$

$$\Rightarrow \boxed{G_N = \frac{G_{grav}}{r^d}} \rightarrow M_{Pl}^2 = M_F^{D-2} V_{D-4} = M_F^{D-2} r^{D-4}$$

Hierarchy Pb $\overset{\text{why}}{G_N \ll G_F}$? → illusion, $G_{grav} \sim G_F$

BUT : r^d is "big"

so at the end $\dfrac{G_{grav}}{r^d} \ll G_F$ ...

$D = N+1$

$4 = 3+1$

• $M_{Pl}^2 = M_F^{D-2} r^{D-4}$

if $M_F = 1\,\text{TeV}$, 5D : $r \sim 10^8$ km → ruled out by exp. (gravity)

6D : $r \sim 0,1$ mm → OK (consistent with measurement of $\vec{F}$ on small scale distance)

rephrase

• SM : → scale distance $\sim 10^{-16}$ cm

rephrase the hierarchy pb

why the extra dim are much larger than $\underbrace{10^{-16}\,\text{cm}}_{5n}$?

$\boxed{\text{(large extra dimension)}}$

2. Warped or Randall-Sundrum space-times (small extra dim)
(curved)

× if extra dim are warped / curved gravitons would behave differently than gauge bosons

→ explain their ≠ coupling to matter.

× metric is exponentially warps along the extra dim y:

Minkowski 4D: $$ds^2 = \underbrace{\eta_{\mu\nu}}\, dx^\mu dx^\nu \longrightarrow \underbrace{e^{-|ky|}\eta_{\mu\nu}}\, \overset{\uparrow}{\underset{3+1}{dx^\mu dx^\nu}} + \underset{+1}{dy^2} = 4+1 = 5D$$

~~metric for y=0 ⟶ for y=πr ημν~~

metric for $y=0$ $\qquad \propto e^{-k\pi r}\eta_{\mu\nu}$

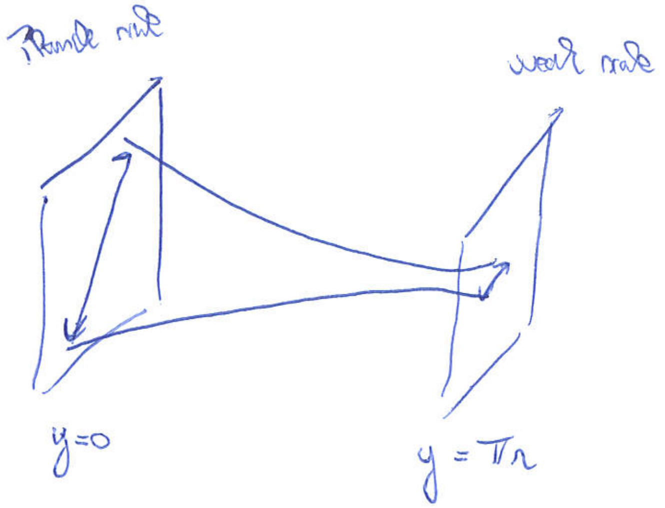
metric for $y=\pi r$

Planck scale ~~m~~ gets warped down to the weak scale:

$$\Lambda(\sim \text{TeV}) \simeq M_{Pl}\, \underbrace{e^{-k\pi r}}$$

$$r \sim \frac{10}{k} \longrightarrow \sim M_p \sim \frac{10}{M_{Pl}}$$

↳ small extra dimension



3. Compactification and a scalar field in 5D

- Take compactified 5th dim
- massless 5D scalar field, ψ, action $S_{5D} = \int d^5x \, \partial^M \psi \partial_M \psi$ $\quad M = 0, \dots, 4$
- $x^4 = y$, defines a circle of radius r, $y \equiv y + 2\pi r$, periodicity → discrete Fourier expansion

$$\psi(x^\mu, y) = \sum_{n=-\infty}^{\infty} \psi_n(x^\mu)\, e^{\frac{i n y}{r}}$$

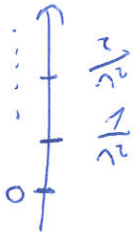
∞ set of 4D scalar fields

E.O.M : $\partial^M \partial_M \psi = 0$

$$\boxed{\partial_\mu \partial^\mu \psi_n(x^\mu) - \frac{n^2}{r^2} \psi_n(x^\mu) = 0}$$

- ∞ set of Klein-Gordon eq. for massive 4D fields.
- each Fourier mode ψ_n is a 4D particle with mass : $m_n^2 = \frac{n^2}{r^2}$

Kaluza Klein Tower



→ zero mode $(n=0)$ is massless

: rich phenomenology!

Rq: "extension" for vector field: $S_{5D} = \int d^5x \frac{1}{g_{5D}^2} F_{MN} F^{MN}$

→ we end up with a 4D theory of a massless gauge particle $F^{\mu\nu}_{(0)}$, a massless scalar, ϕ_0, and an infinite towers of massive vector and scalar fields.

3) Grand Unified Theory

× Internal symmetries in which the symmetries of the SM are themselves the result of the breaking of a yet larger symmetry group:

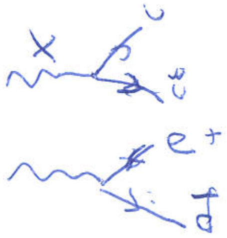
$$G_{GUT} \to \underset{SU(3)_c \times SU(2)_L \times U(1)_Y}{G_{SM}} \longrightarrow SU(3)_c \times U(1)_Y$$

$$\text{or} \begin{pmatrix} SO(10) \\ SU(5) \\ \vdots \end{pmatrix}$$

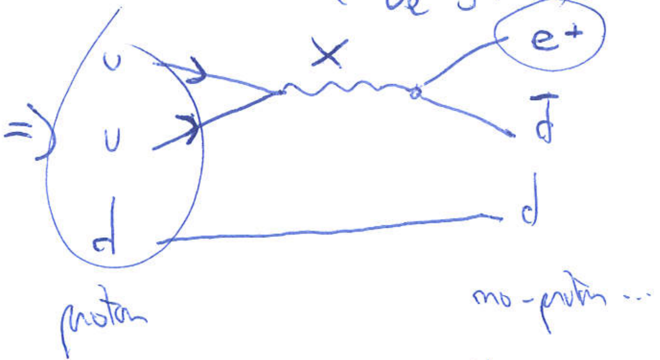
× Proton decay

• for $SU(2)_L$: we found multiplets $\begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix}$ meaning

• for SU(5) for ex, we form multiplets $\begin{pmatrix} \left.\begin{matrix} d^c_{red} \\ d^c_{blue} \\ d^c_{green} \end{matrix}\right\} SU(3) \\ \left.\begin{matrix} e \\ \nu_e \end{matrix}\right\} SU(2) \end{pmatrix}$ meaning



$\Rightarrow$



× experimental bound $\underset{\text{half-life}}{\tau(\text{proton})} > 1{,}4 \times 10^{34}$ years

$$\tau \nearrow \Rightarrow M_{GUT} \; (M_X^{min}) \nearrow$$

4) Effective Field Theory

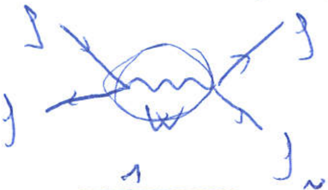
× go BSM → seems like an ∞ possibilities opens up (write down any $\mathcal{L}$ we like)

up (UV)
|
bottom (SM)

× Let's start by SM (bottom → up)

$\mathcal{L}_{EFT} = \mathcal{L}_{SM} +$ general operators
(could be produce by many BSM models)

× Remember Fermi Theory:



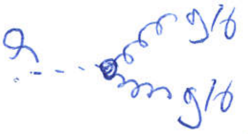
$$\frac{1}{p^2 - M^2 + iM\Gamma} \underset{\substack{p^2 \ll M^2 \\ \& \ \Gamma \ll M}}{\simeq} -\frac{1}{M^2}$$

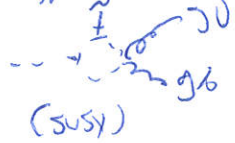
$$\mathcal{L} \supset g\,\bar{e}\,\boxed{\gamma_\mu}^{e} W_\mu \nu_e + g\,\bar{\nu}_\mu\,\boxed{\gamma}^{e} W e_\mu$$

$$\downarrow$$

$$\mathcal{L} \supset (\bar{e}\,\Box^{e}\,\nu_e)(\bar{\nu}_\mu\,\Box_{e}\,\mu) \longleftarrow \text{4f. op.}$$

× We have a new particle above TeV, Higgs boson → build EFT with H

$\mathcal{L}_{EFT} \supset$


→ $\mathcal{L}_{UV} \supset$


(SUSY)
e.g.

$$O_{GG} = g_s^2 |H|^2 G^a_{\mu\nu} G^{a\,\mu\nu}$$

$$O_{BB} = g'^2 |H|^2 B_{\mu\nu} B^{\mu\nu}$$

$$\mathcal{L}_{EFT} = \mathcal{L}_{SM} + \sum_i c_i \frac{O_i}{\Lambda^2}$$

Eg:
$$\frac{1}{p^2 - m^2} = \frac{-1}{m^2}\left(\frac{1}{1 - \frac{p^2}{m^2}}\right) \simeq \frac{-1}{m^2}\left[\overset{\text{dim } 4}{1} + \underset{\text{dim } 6}{\left(\frac{p}{m}\right)^2} + \overset{\text{dim } 8}{\left(\frac{p}{m}\right)^4} + \dots\right]$$